Soil Mechanics
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Lecture – 31
Stress Distribution in soils
Lecture No.4

Students, so we meet again. Once again we have a lecture on stress distribution in soils. This is the fourth in the series; we have covered three lectures already. And apart from seeing the importance of stress distribution in soil mechanics, we have also got started with the computation of stress distribution using the method of theory of elasticity. It is worth recapitulating that theory of elasticity is based on certain very basic assumptions such as the medium is homogenous, isotropic, elastic and a continuum. We also assume or we ensure rather that any typical adjacent elements in the medium, they undergo deformation in a compatible way. So taking together the equilibrium equations, the relationships between strains and displacements, the stress strain elastic relationships and the compatibility conditions, we found that in all there are 15 equations and 15 unknowns and they can be solved to get what is known as a Laplace's equation.

This equation can be solved with the help of so called Airy's stress approach. The Airy's stress function is a function, which is a function of position. That means it is function of the co ordinates x y z of the medium, which means that it gives the distribution of the stresses, the overall pattern of the distribution of the stresses in the medium, both with respect to x y the horizontal direction and also with respect to the z direction. This stress function is a function when suitably differentiated will give you the stress components sigma x, sigma y and sigma z at any point, at which we desire to get the stresses. That is precisely what Boussinesq's had done. He evolved a stress function, the credit goes to him for evolving that stress function phi which is applicable to a three dimensional problem of stress distribution in a medium subjected to a concentrated load on the surface.

Let us take a quick look at few slides to recapitulate all these, what ever we had discussed in the last few classes. So in the last lecture we continued with the theory of elasticity, saw how theory of elasticity has been used by Boussinesq's and how Boussinesq's has evolved his own approach based on theory of elasticity. And we also saw how to use Boussinesq's approach to determine stress distribution due to one, a point load and two, a line load. Why did we choose a point load and a line load? A point load is a simplest form of loading, one can imagine a concentrated load at a point in a medium. See for example if this is the medium, at any point there can be a load. This is known as a point load or a concentrated load. We have a method to determine stress at any point P or say A due to this concentrated load. In other words if we have a number of concentrated loads and if we assume super position to be valid, then we can find out at this point A the stress due to each one of these concentrated loads which means in effect a line load.

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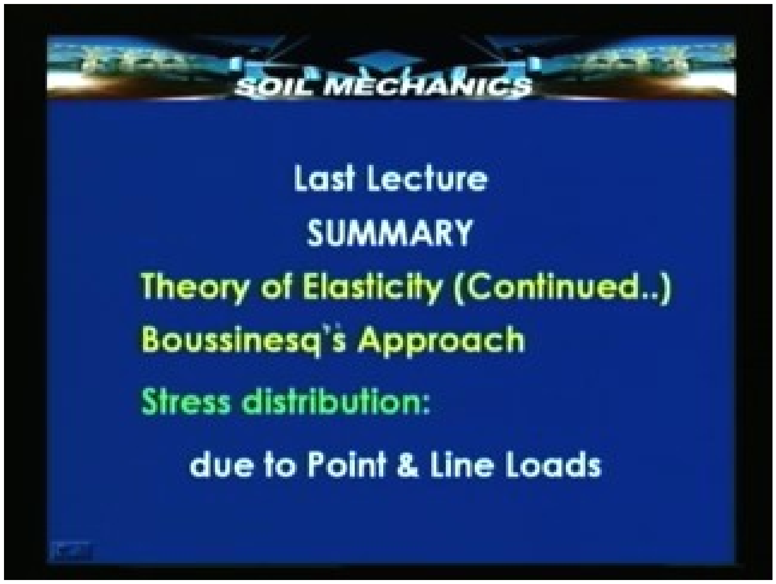

After all a line load is nothing but a load consisting a number of concentrated loads along a single line. So if we get the solution for one concentrated load it means that we can get the solutions for every concentrated load acting at every point along a line. And all these when summed up or in other words when it is integrated over a finite length, we get the stress at the point A. These two are fundamental and very basic with the help of these concentrated and line load and the method for stress distribution determination due to these two loads we will be able to solve any problem.

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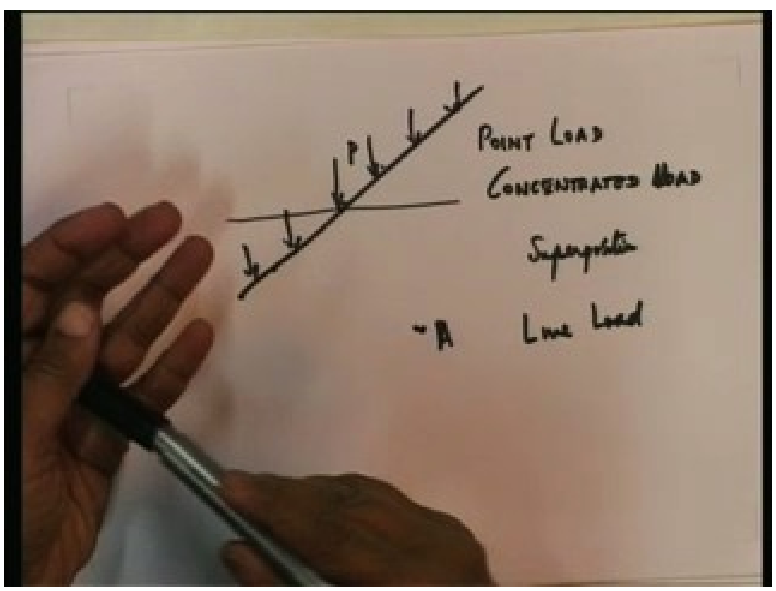

Suppose we have rectangular area which has got a uniformly distributed load, knowing the distribution due to the concentrated load or a line load we can extrapolate it to a number of lines and cover the entire area. So the stress distribution due to a uniformly distributed load over a large area can be calculated. If the area is circular all that will happen is the pattern of summing up the effect that is integrating will change but the basic principle, concept the approach will still remain the same. So once we know the stress distribution procedure for concentrated load and hence the line load, we can apply it to any area. And there are of course methods available to also tackle arbitrarily shaped areas.

We shall be now proceeding further to see once again very quickly the equations and the method of computation of stresses under point loads and line loads. In this lecture we shall quickly take a look at the calculation of stresses due to a concentrated and line load as I said just now and then continue with application of those to a strip load, two a rectangular area and three a circular area.

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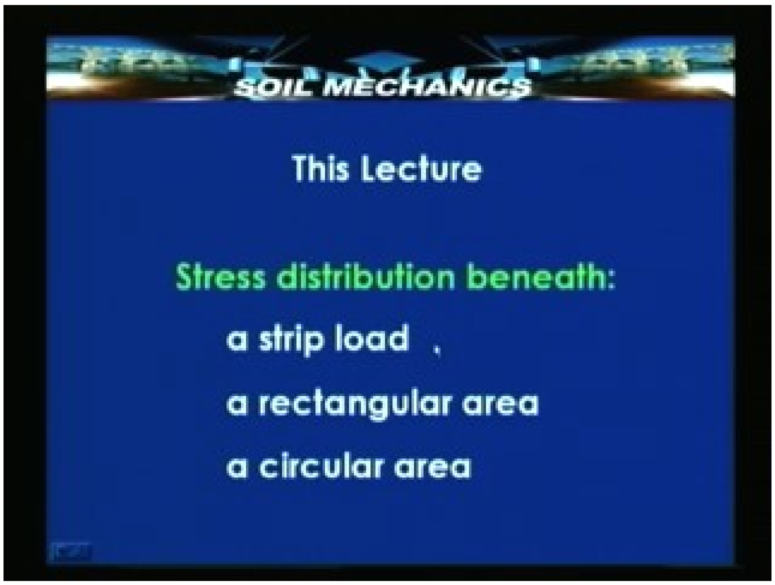

These as you can see or visualize represent three very typical cases in practice. Imagine as I have said a few times in the earlier lectures as well, a long compound wall. This has got a foundation and under this foundation there is going to be stresses induced due to the compound wall and this can be very well be taken as a small strip. Because it has got a very large length compared to the small dimensions in the cross sectional direction and this can very well be considered as a strip. So this is nothing but what is known as a strip load. On the other hand if we have a regular foundation under a column and if we consider the load being distributed under this foundation, then the area of this foundation the base area of the foundation acts as a rectangular area over which there is uniformly distributed load where B is the width of this foundation and L is the length of the foundation, it could be square.

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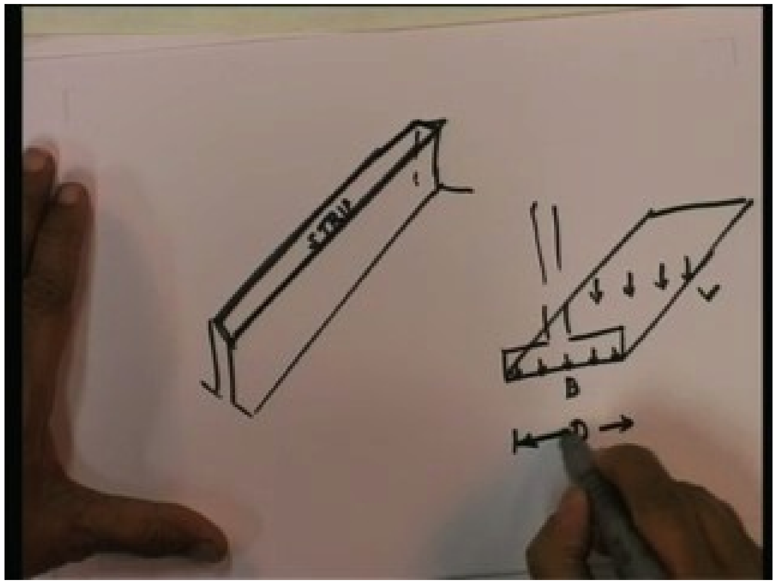

If it is circular of diameter say D then again this principle can be applied and we can study the stress distribution below a circular load as well and that is what we are going to see today. We shall be taking a strip load extending it to a rectangular area and then we shall also see how to compute stresses below a circular area. A word about super position which I have mentioned briefly. If we have two loads the effect of one load and the effect of the other load can be determined separately and they can be added together to get the effect of the total load and this is known as the principle of the super position.

Let me just briefly explain why this is important and how this can be justified. Let us take the stress strain relationship for the soil below the foundation. Let us say it is linear and in fact if you look deep into the method that we have used, you will find that we have assumed the elastic properties E and new to be constants which means that this modulus E is a constant or it implies that the stress strain relationship is linear, sigma E relationship is linear. If this is linear then this area of elasticity that we have used is called generally as linear elasticity theory. When we employ this theory of linear elasticity, only then the principle of super position is valid.

Imagine we are applying a load and a corresponding stress say which we call as P_1. This stress P_1 is going to produce a strain that is epsilon one. Suppose I apply an additional stress of delta P that is going to produce an additional strain say delta epsilon. This means that the total stress applied is P_1 + delta P, say P_2 and the total strain is epsilon one plus delta epsilon say epsilon two. Now imagine that we are applying straight away the load P_2 then this load P_2 according to the linear theory of elasticity is going to have a strain epsilon two which is going to be exactly the same as this epsilon two. This means that whether we apply P_1 and then an incremental stress delta P or whether we apply P_2, the final strain remains the same.

This means that the principle of super position is valid rather in case we want to determine the stress due to a load P_2 which consists of two components P_1 and delta P. It is possible to calculate the effect of P_1 separately, effect of delta P separately and add and get exactly the same effect that P_2 would have created if it had been applied directly. This is the principle of super position and very soon we shall see an example where this will be useful to us.

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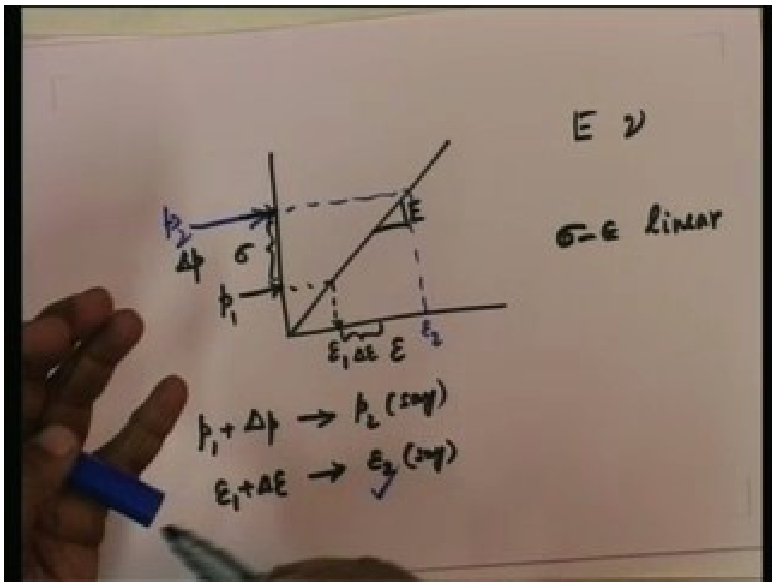

Let us proceed further. Let me recapitulate, emphasize that we are dealing with an x y z co ordinate system as shown here. This is z, this is x y and there is an arbitrary point P and an element surrounding that in the medium, the centre of this parallelepiped or the element has got co ordinates x y z.

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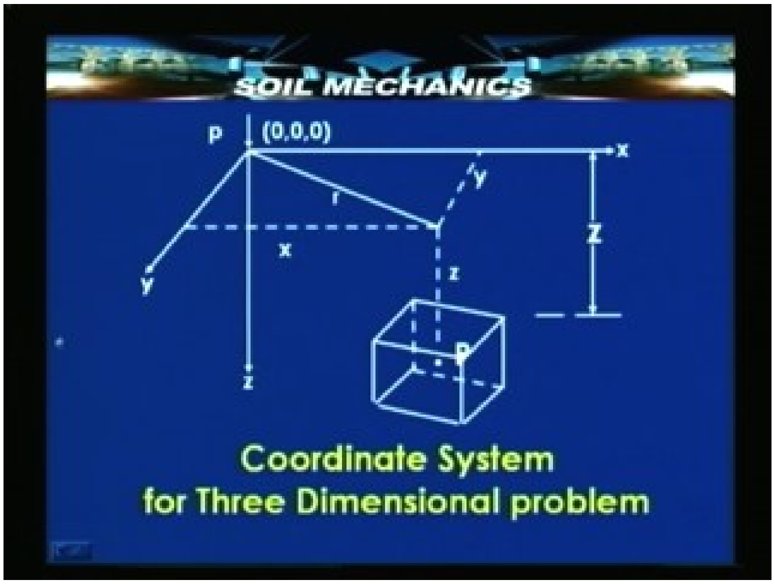

We have already seen that due to a concentrated load P, the stress sigma z at any point inside the medium is going to be given by this expression. And in this expression if we take out P by z square, we can write the stress sigma z in terms of a factor called the influence factor which is incidentally equal to this. Now these three expressions are valid for a concentrated load and therefore for stress at any point due to a concentrated load P the influence factor is this.

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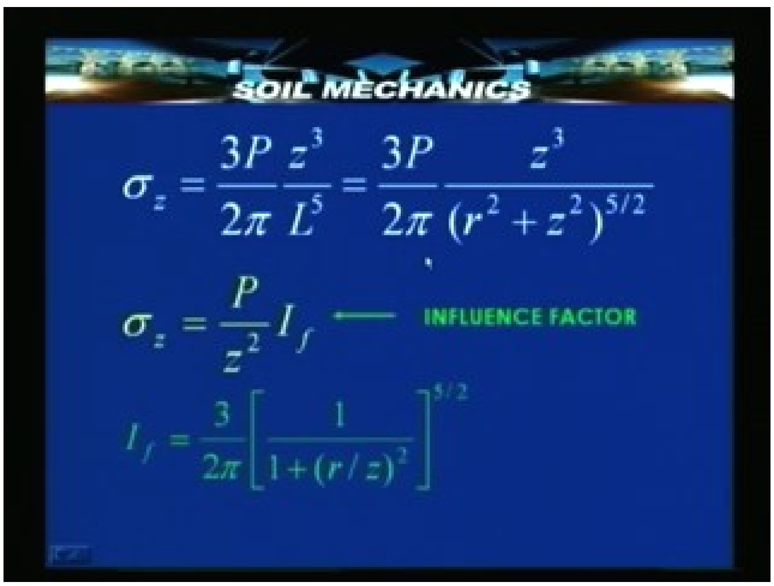

These are the influence factors derived by substituting different values of r upon z in the expression for sigma z shown in the previous slide. So a series of influence factors can be obtained for a range of r by z varying from zero to 2.9 and therefore if you know in any given problem where the point at which you want the stress is lying. That means if you know the coordinates r and z, you can calculate I_f, I_f multiplied by P by z square will give you sigma z which we had seen in the earlier lecture and let us take a quick look at the example which we saw last time.

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r/z	I_f	r/z	I_f	r/z	I_f
0.00	0.478	1.00	0.0844	2.00	0.0085
0.10	0.466	1.10	0.0658	2.10	0.0070
0.20	0.433	1.20	0.0513	2.20	0.0058
0.30	0.385	1.30	0.0402	2.30	0.0048
0.40	0.329	1.40	0.0317	2.40	0.0040
0.50	0.273	1.50	0.0251	2.50	0.0034
0.60	0.221	1.60	0.0200	2.60	0.0029
0.70	0.176	1.70	0.0160	2.70	0.0024
0.80	0.139	1.80	0.0129	2.80	0.0021
0.90	0.108	1.90	0.0105	2.90	0.0018

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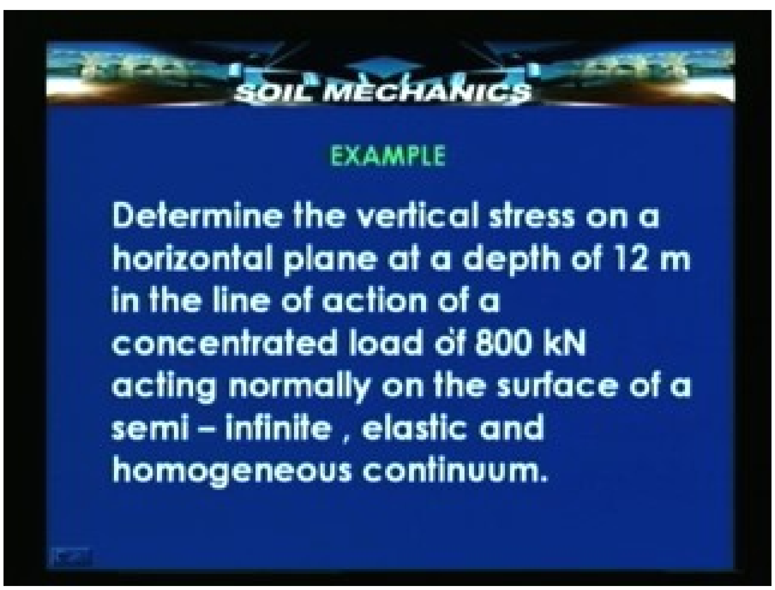

Determine the vertical stress on a horizontal plane at a depth of 12 meters due to a concentrated load 800 kilo Newtons in an elastic medium. So the solution would be find out r by z, it so happens that if r/z is zero I_f is 0.47745. This is happening because we want the stress on the surface where z is equal to zero, then the influence coefficient is this. Therefore the stress sigma z is this influence coefficient into P divided by z square and that is 2.653 kilo Newtons per meter square.

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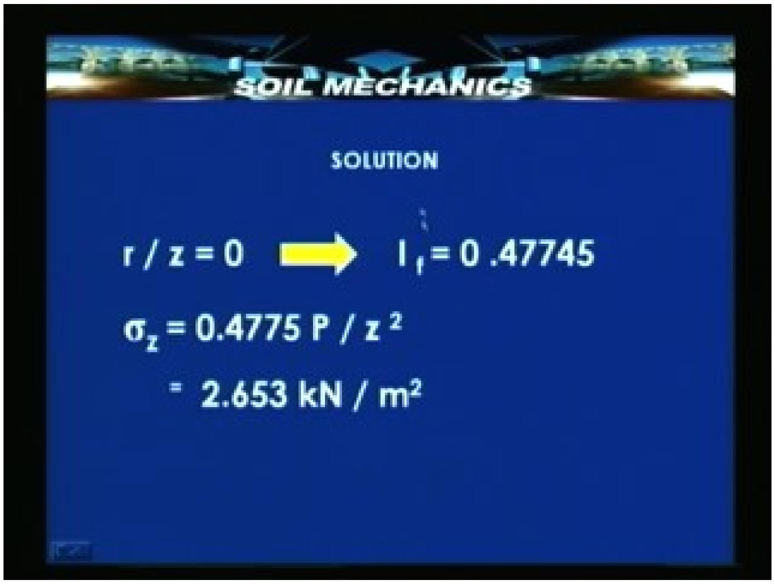

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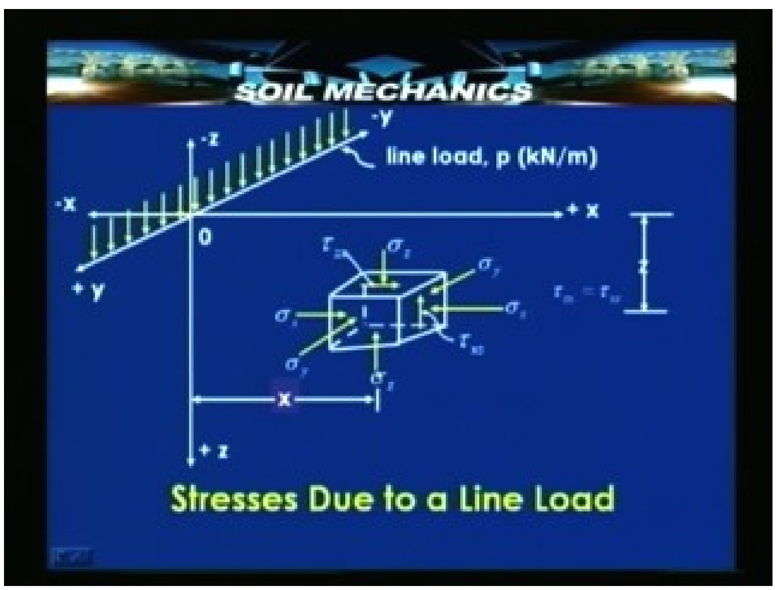

If we have a line load, the effect of concentrated load can be summed up over this entire length and the effect of the line load at the point in the parallelepiped can be computed. The expression for sigma z will then be this and if we want to determine the stress due to a line load of magnitude small p equal to 400 kilo Newton per meter where small p is the intensity of loading along the line. Then for an x of 5 meters and z of 5 meters, we can compute the sigma z value as 12.73 kilo Newtons per meter square.

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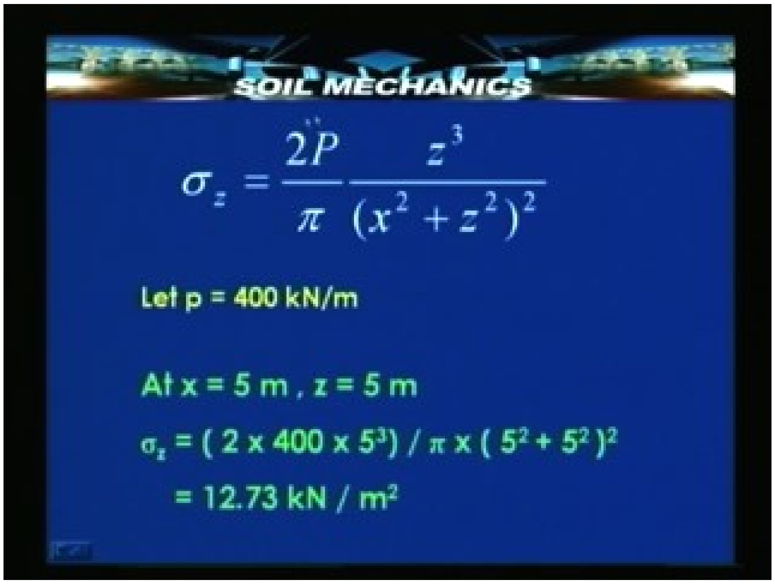

Now comes the importance of the use of sigma z computation by the principle of super position. Take for example two concentrated loads P_1 and P_2 and any point A in the medium P_1 is 400 kilo Newton per meter. This is a line load actually 400 kilo Newton per meter in a direction like this and P_2 1000 kilo Newton per meter again a line load along this.

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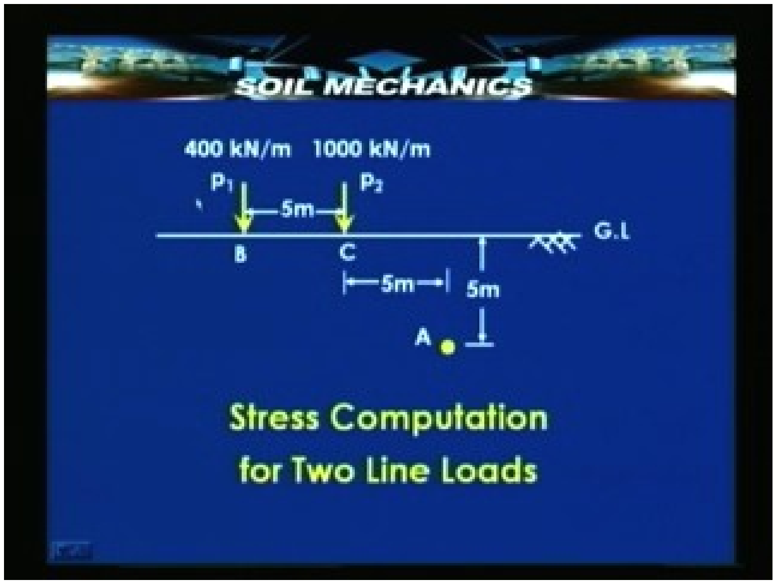

So we will compute this stress due to two line loads. At a point A which is at a distance of 10 meters horizontally from the load P_1 and 5 meters horizontally from the load P_2 and at a depth of 5 meters. Proceeding further we will compute the stress due to each load separately and apply the principle of super position. Suppose we take the first load, the first load is P_1 we know its magnitude, we know the magnitude in the sense the intensity of loading, we know the point A its coordinates and therefore we can use the formula which we saw 2 slides earlier for computing sigma z due to a line load at any depth and the corresponding influence factor. We can find if we apply that principle where the stress due to P_1 in the slide is sigma z due to P_1 at the point A will be this. If I substitute the value of P_1 which is 400 hundred kilo Newton per meter and if I substitute the values of z and x where z is 5 meters and x is 10 meters, I will find that sigma z due to P_1 is 2.037 kilo Newton per meter square.

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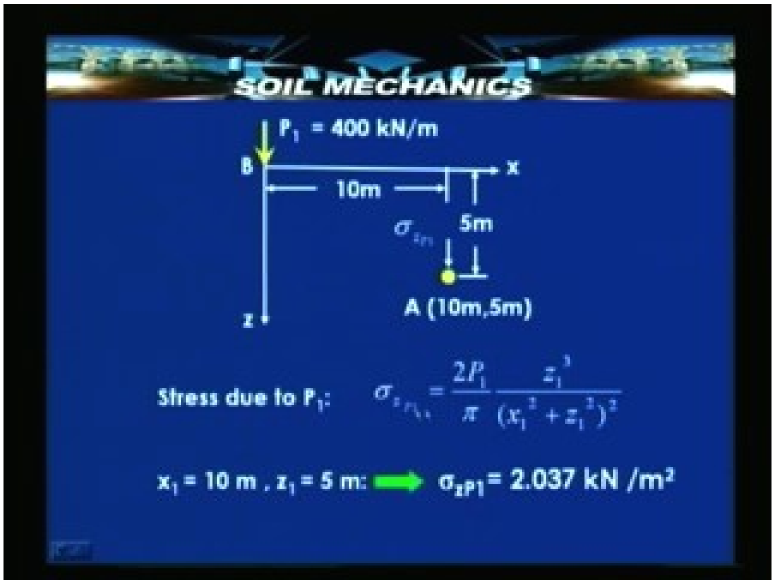

Take for example the second load. If the second load is 1000 kilo Newton per meter and if the point A with respect to this load is at 5 meters horizontal and 5 meters deep, then this stress due to P_2 this sigma z P_2 will given by a similar formula where now we will have the load P_2 and the coordinates x and z equal to 5 and 5 respectively. This when you substitute you get the stress due to second load as 31.835 kilo Newton per meter square. Since principle of super position is justifiable because of the approach being a linear theory of elasticity approach. We can get the total stress as the sum of these two stresses which is equal to the stress sigma z at the point A and that this is 33.872 kilo Newton per meter square.

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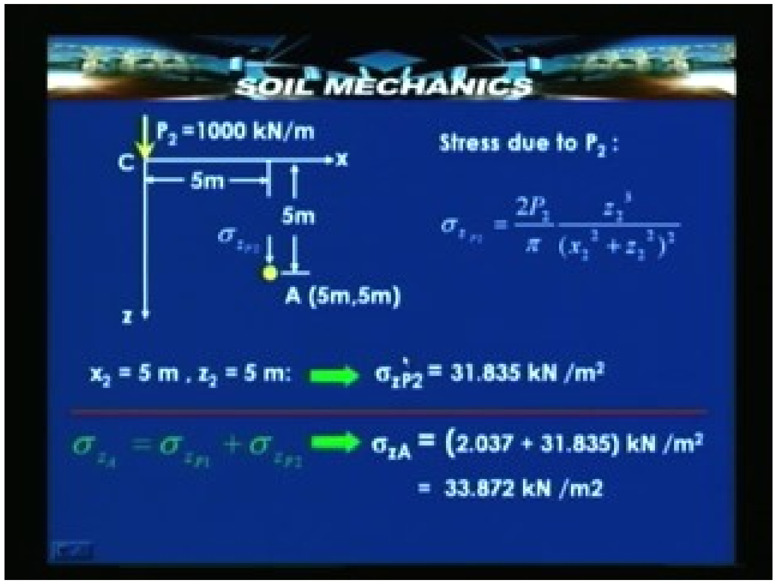

Now we come to subject matter of today's lecture which requires the knowledge of the previous lecture and that is the reason why we went through recapitulating what we have discussed in the last class.

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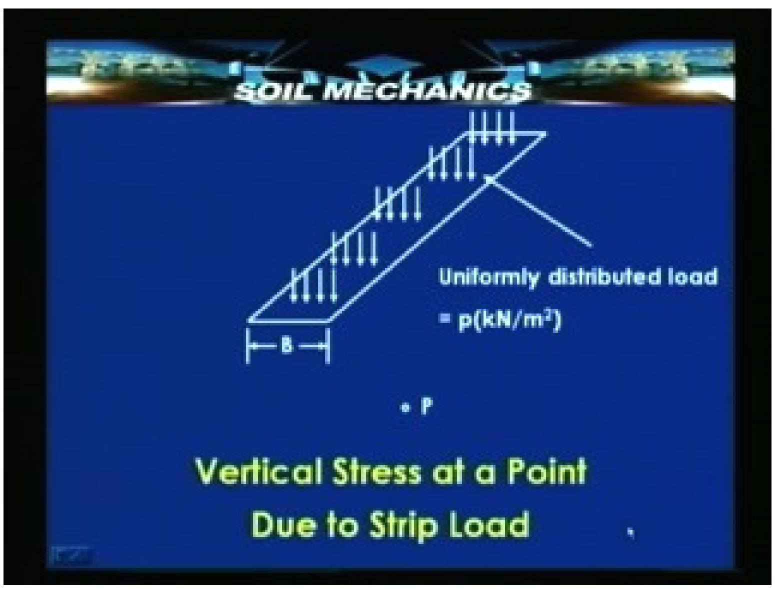

We shall see how to compute the stress below a strip load. Let us take a look at this slide. The strip load here is defined by a width B and a very long length. The length is very large and that is why no specific length is mentioned here. For all practical purposes it can be taken as infinitely long compared to B. And now if you want the stress due to this load at point P then we can proceed using the line load formula.

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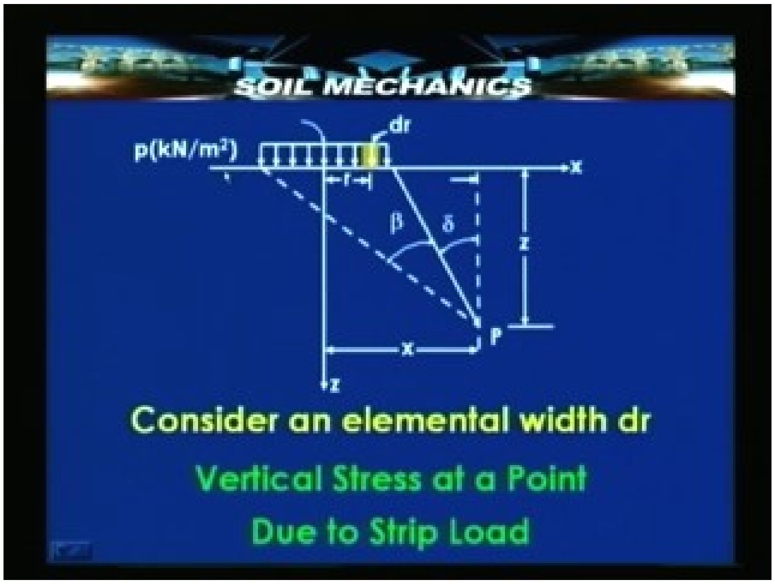

Suppose this is the load, we take a cross section along the load then this is the point P, suppose this is the load distributed over a length B, capital B as we saw in the previous diagram. In order to compute the effect of this stress at point P what we shall do is, take a very small element dr at a distance r, find out its effect and the stress due to it at the point P and then integrate it over the width capital B. So suppose this is the z axis taken at the centre then this r is the distance at which the element dr lies. Due to symmetry we can do the summing of or integration of the stress at P over the length minus B/2 to plus B/2.

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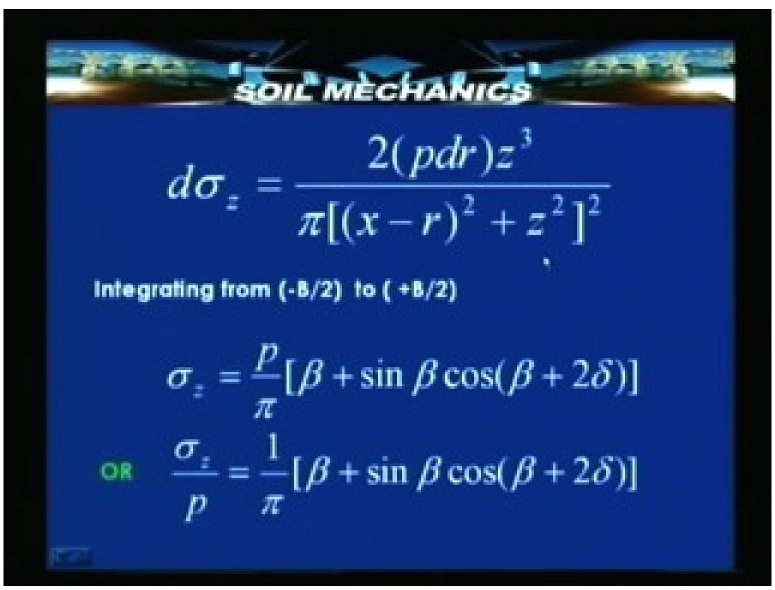

Actually this is amounting to applying the principle of super position repeatedly for every point load that is acting over this length capital B. There lies the importance of the principle of super position and the knowledge of what is the stress due to a point load. If you add all the stresses due to each one of these then you get the vertical stress at the point P due to the entire load. Suppose we derive an expression for the stress due to the small element and we call this stress as d sigma z. Then the stress due to the small element at the point capital B will be given by this. It is possible to derive this based on Boussinesq's theory. Now by integrating this dz from minus B/2 to plus B/2, we will get the stress sigma z at the point p and it will turn out to be sigma z equal to small p by pi into beta plus sine beta cos beta plus two delta where these angles beta and delta are shown in the previous diagram.

Let us take a look at it again. This is the angle beta made by the point B with respect to the edges of the loaded area and delta is the angle made with respect to the vertical by the line joining the nearest end of the loaded area. So knowing beta, delta it will be possible to find out sigma z at P. Suppose we find sigma z at P like this then by dividing this value by the unit load small p per unit length we get what is known as stress concentration. This stress concentration can be represented one upon pi into this (Refer Slide Time: 22:12). This means that for any applied load small p is possible to find out the corresponding stress sigma z using this formula. And for this purpose we can derive a table which gives sigma z by p as a function of x non-dimensionalized and z non-dimensionalized, the stress sigma z is also non-dimensionalized. The distances x and z the coordinates or the lengths x and z also non-dimensionalized. This non dimensionalization is with respect to the width of loading that is capital B.

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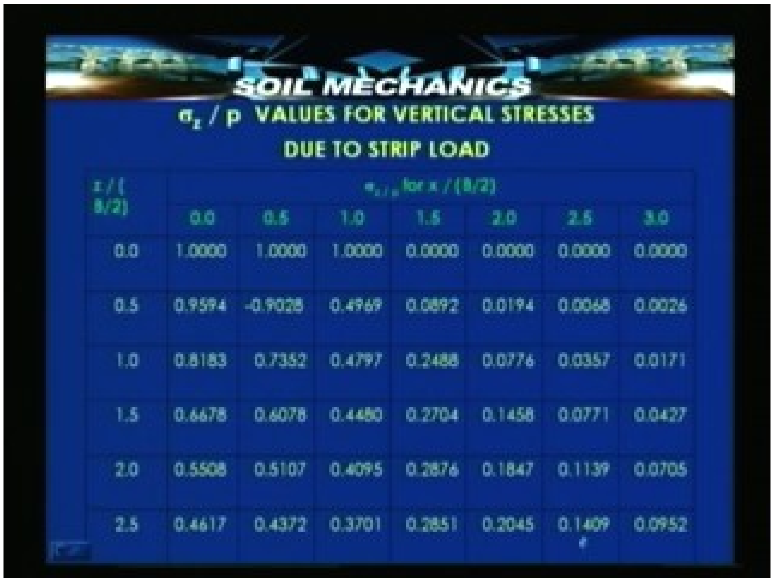

z / (B/2)	σ_z/p for x / (B/2)						
	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0.0	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.5	0.9594	-0.9028	0.4969	0.0892	0.0194	0.0068	0.0026
1.0	0.8183	0.7352	0.4797	0.2488	0.0776	0.0357	0.0171
1.5	0.6678	0.6078	0.4480	0.2704	0.1458	0.0771	0.0427
2.0	0.5508	0.5107	0.4095	0.2876	0.1847	0.1139	0.0705
2.5	0.4617	0.4372	0.3701	0.2851	0.2045	0.1409	0.0952

Now this table gives you the sigma z by p values, you can see that these values range from zero in this range, as we go further away and as we go deeper they gradually decrease. So from this it is possible to calculate the stress at any point laterally as well as with respect to depth due to a strip load. Having understood how the stress is computed due to a strip load, let us take an example. The data that is required for computing stresses due to a strip load or the width of the strip, the load per unit area, the coordinate system the x coordinate and the z coordinate of the point at which we want the stresses. And that is precisely what we have in this example.

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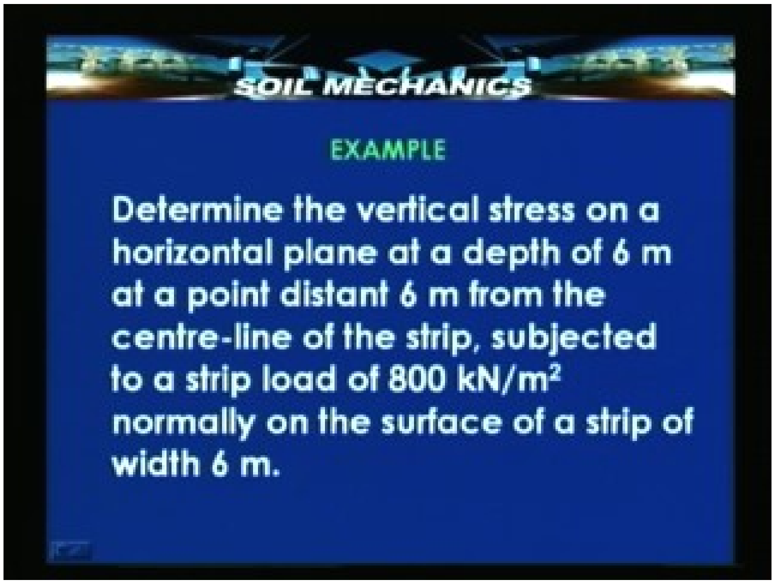

Determine the vertical stress on a horizontal plane at a depth of 6 meters at a point distance 6 meters from the centre line and the strip itself is subjected to 800 kilo Newton per meter square and the width of the strip is 6 meters. Let us take the stress per unit length that is small p, in our case it is 800 kilo Newton per meter square. Next the width of the strip which is loaded is 6 meters which means that B/2, the factor which is half the width which is used for non dimensionalization will be 3. The coordinates of the point at which we want the stresses are x and z.

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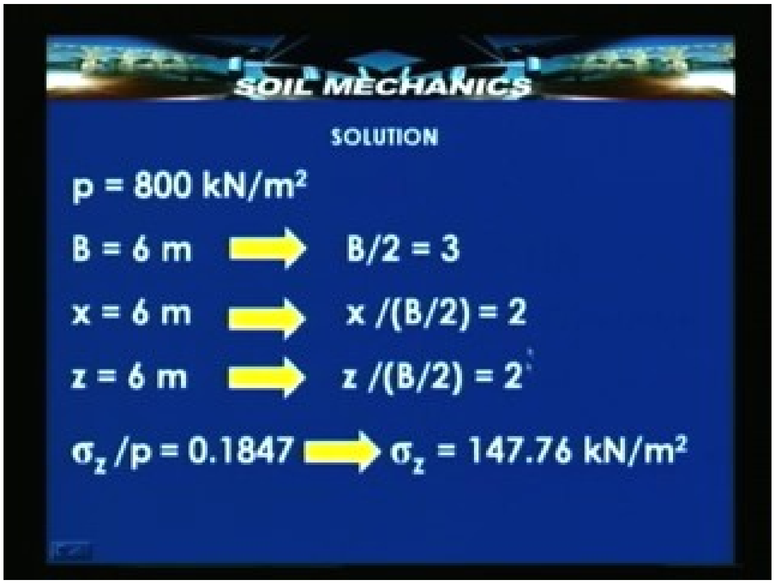

That means we want a stresses on a horizontal plane at a depth of 6 meters at a point which is also at 6 meters from the central line. This can be found out by first working out the non dimensionalized coordinates that is x divided by B/2 and z divided by B/2 which as you will see or also respectively 2 and 2. This means that sigma z by p can be computed from the table which we had in the previous slide. Let us see what value of sigma z by p we get for an x = 2 and z upon B/2 equal to 2. That is the non dimensionalized coordinates are two each. Let us go back to the previous slide (Refer Slide Time 22:24). See here at a value of x upon B/2 equal to 2 and z upon B/2 equal to 2, we find that sigma z upon p is 0.1847 that is the stress concentration.

So going back to our calculation sigma z by p is 0.1847, how ever the load actually applied is 800 kilo Newton per meter square. This means the stress that will arise will be merely this multiplied by the applied stress p that means 0.1847 into 800 which will turn out to be 147.76 kilo Newton per meter square. This is all about strip; once we have the load per unit area it will be possible to calculate stress at any point. With the help of this we can calculate the stress on an entire horizontal plane, we can calculate the stress on a vertical plane, below the centre of the strip, away from the strip and this we will be able to get the stress distribution on a horizontal plane very easily. How the stress varies from the loaded area as we go away or as we go below. In other words with just one expression by applying it repeatedly to different points it is possible to get the entire pattern of distribution of stresses below the foundation or below the loaded area at any depth or any point away from the loaded area including points outside the loaded area.

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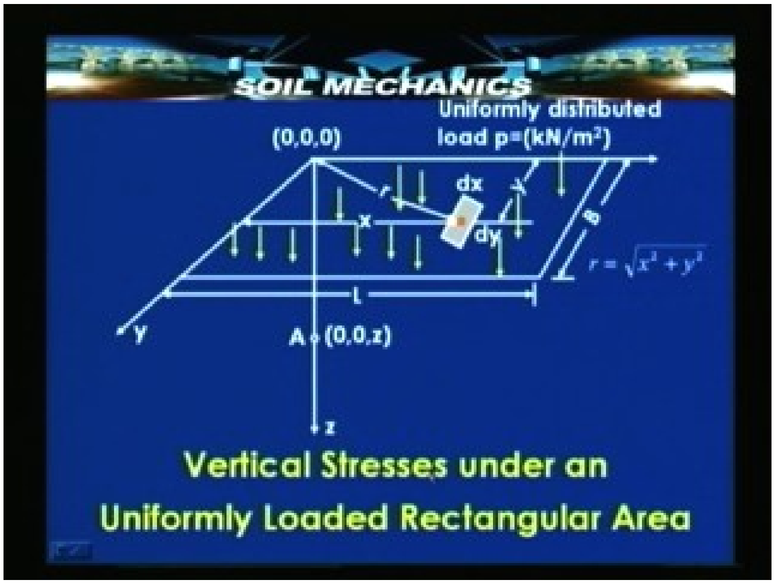

This can be now extended to rectangular areas. Let us see how. This slide shows a typical rectangular area, this is the x coordinate, this is the y coordinate and this is the rectangular loaded area. The dimensions of the loaded area are L and B, L in the direction x and B in the direction y. In order to compute the stress due to this loaded area at any point at depth, we assume uniform distribution of the load above. All these yellow lines indicate a uniformly distributed load over the entire rectangular area.

There are two approaches to this problem, we can divide the rectangular area into a number of strips and apply the concept of loading on a strip and sum them up or alternately which will amount to the same thing. We can once again proceed from fundamentals, take a small area find out the stress due to this loading over the small area at the point A or p where we want the stresses and then integrate it over the length L and the length B. This later one, this approach is what has been preferred and expressions have been derived already. Very well known expressions are already available based on theory of elasticity to compute the stress at any point A due to uniformly distributed load over this entire area. The result will be the same whether we compute the stress due to various strips and add them or we compute due to any one element and sum it up over length and breadth L and B.

And you can imagine that it has to be the same because after all everywhere basically we are applying the principle of super position repeatedly for point loads or concentrated loads acting at every point on the loaded area. So the final result will be given by the same expression and in this case the expression will be due to an elemental area, the stress is this where small p is the applied load per unit area, dx dy are the dimensions of the small elemental loaded area where dx is the dimension in the x direction and dy is the dimension in the y direction, z is the depth at which we desire to have the stress, x and y are the other coordinates. As I said the approach that we shall be preferring is one in which this elemental stress d sigma z at the point p is integrated over the length and the breadth. So this is integrated over B and integrated over L, this expression (Refer Slide Time: 30:16) and finally the resulting expression is expressed in terms of the applied load per unit area p into a factor called the influence factor. This influence factor is represented as I_{mn} where m is nothing but a ratio of the width B divided by a two with respect to the depth z and n is the ratio of the length L and the depth z.

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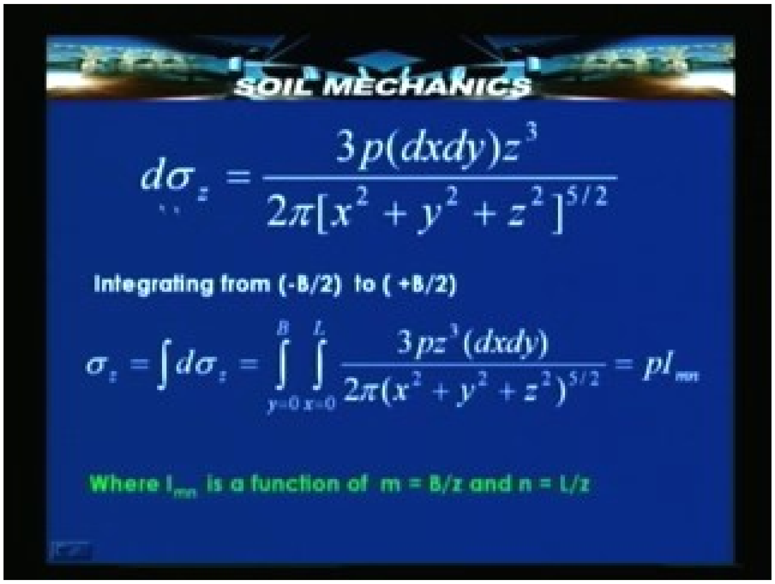

Once again non dimensionalization, this influence factor by definition as we had seen in the earlier cases also is nothing but the stress that is rising due to applied load that is the unit applied load. So if the influence factor is multiplied by the actual applied load we get the stress due to the applied load p and this influence factor is called as I_{mn} because we are now dealing with influence factor over an area or for stresses over a rectangular area. Since a rectangular area has two dimensions which are unequal, the corresponding non dimensionalized dimensions at every depth z will also be two in number m and n. And therefore influence coefficient will depend upon the value of m and the value of n and therefore it is denoted as I_{mn} and will vary as m and n vary.

If we have plot that expression pI_{mn}, what ever expression we have derived pI_{mn}. In this suppose we take p as unit load, we take the influence coefficient I_{mn}, calculate it for several dimensions L and B of typical rectangular areas. Then we get the complete range of influence coefficients and a graph with the help of which rather than computing every time the influence coefficient we can merely measure it of from the graph. See for example in the previous slide once again. This previous slide indicates that this influence coefficient I_{mn} is nothing but integration with small p which is a constant taken outside. That means the influence factor I_{mn} is nothing but the double integral of 3 into z cube dx dy by 2 pi into this (Refer Slide Time: 32:56). This entire integration operation can be considerably simplified, can be used repeatedly if we can develop non dimensional charts in advance. And that is what we are having in the next slide.

In this slide we are having non dimensionalized charts which will give the influence factors for different values of dimensions of the loaded area and depths. Suppose the longitudinal dimension is expressed as m times the depth where z is the depth and the dimension in the lateral direction is n into z. Then if you look at this graph we have m along the x axis, we have the influence factors along the y axis and all these yellow lines here they are all plotted for different values of n. If I know m and if I know n, I can read of from this graph the corresponding influence factor and the influence factor multiplied by the applied load per unit area small p will give you the stress at this point A.

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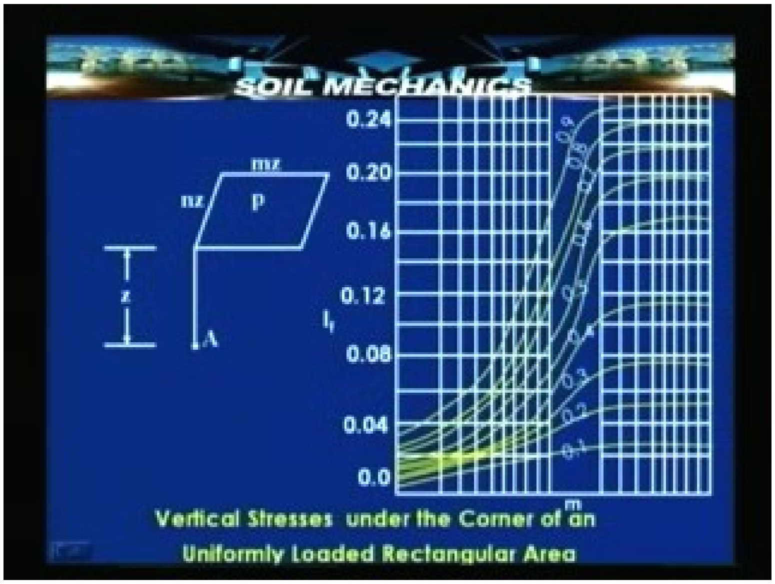

We have several points under this loaded area, all of them at the depth say z. Then for which point shall we compute the influence factors? It is found convenient to develop influence factor charts for a point such as A below the corner of the loaded area. And that is precisely what is used to compute stress below any point below the loaded area. If we know the stress at any point below the corner it is found that it can be used again and again to compute the stress at any other point below the loaded area.

Let us see how. Take this loaded area xy. This is the loaded area xy, this loaded area xy is like this where this length is L this length is B. Then suppose I want the stress at a point below this corner, I can directly use the influence coefficient chart which we saw in the previous slide. Because they have been derived specifically for computing stress below the corner. If I want the stress below let us say any other point such as B it is easy to visualize that the stress at this point B is going to be the stress due to an area like this, due to the remaining area and also due to this and finally this (Refer Slide Time: 36:06). In other words I have divided this into 4 smaller areas 1, 2, 3 and 4 as shown in the slide here.

Once I divided the given area into 4 parts 1, 2, 3, and 4. In this diagram they are shown as equal areas but in general they need not be equalized. Once I divide the area into four parts then suppose this is the point below which I want the stresses say that is at point B. Then B happens to be at the corner of area one, at the corner of area two and also similarly at the corner of area three and area four. It is just that it is at a different corner of each one of these loaded areas. So if I want the stress at this corner of area one, I shall appropriately choose m and n. See for example the previous slide (Refer Slide Time 32:32) once again you find here by definition in this sketch, if the point A is below this corner then this dimension is m, this dimension is n. If I apply that concept to area one here then this dimension is going to be the dimension mz and this direction is going to be the dimension nz. If I extend this to the area two, area two happens to the identical to definition sketch in the slide and that means this dimension is going to be mz and this dimension is going to be nz.

We can extrapolate this, we can apply this same concept or same principle to the other two areas and compute this stress repeatedly due to each one of these rectangular areas at one of their corners. And then we have been using theory of linear elasticity and therefore the principle of super position is going to be valid. All that we need to do now is having calculated the stress at point B due to each one of these loaded areas, we simply sum them up to get the stress at B due to the entire loaded area. It so happens that we have assumed uniform loading over the entire area but suppose the loading also varies, we can apply different loading intensities to each one of these rectangular areas.

But we must remember that within any one rectangle the intensity of loading has to remain same, because the formulae that we are applying are all meant for uniform loading. That means for this area one, the intensity could be different in principle from the intensity in this area $p_{2,}$ this is $p_{1.}$ However it does not affect the method of computation, the method of computation remains same, it is just that we shall be having different intensities to deal with in each one of these loaded areas and the influence factor multiplied by the corresponding intensity of loading will give us the stresses at the point B. So this diagram which we have already seen tells us the principle behind computation of stress at any point beneath the loaded area based on the stress computed at one of the corners of any one of these loaded areas.

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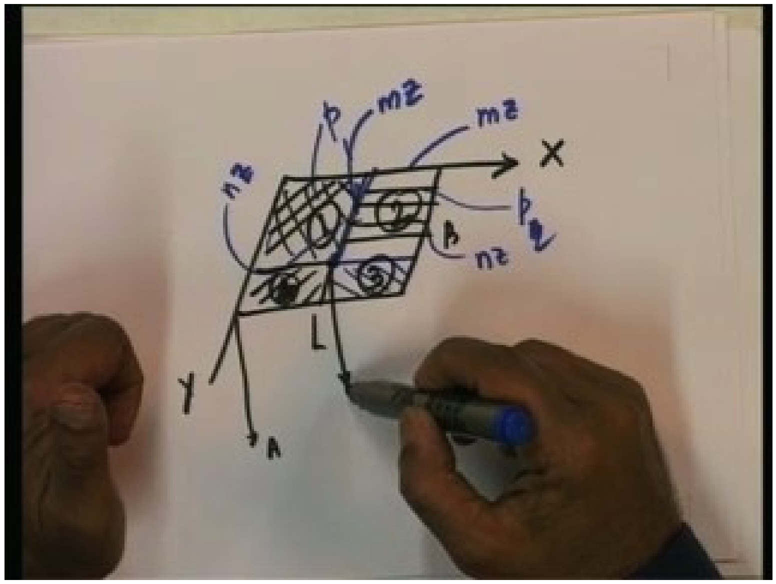

Now comes the extension of the principle to circular areas. It is not uncommon to find circular foundations. Where do we find circular foundations? Where we have pillars which are circular in shape, it is good to have circular foundations. We have bridges, we have even buildings or structures which all have let us say possibility of having a circular shape. A water tank for example could be circular in shape, it will have a few columns and it is possible to use circular foundations for these columns. And these circular foundations will also be amenable to computation of stresses by the same principles that we have been using all these time.

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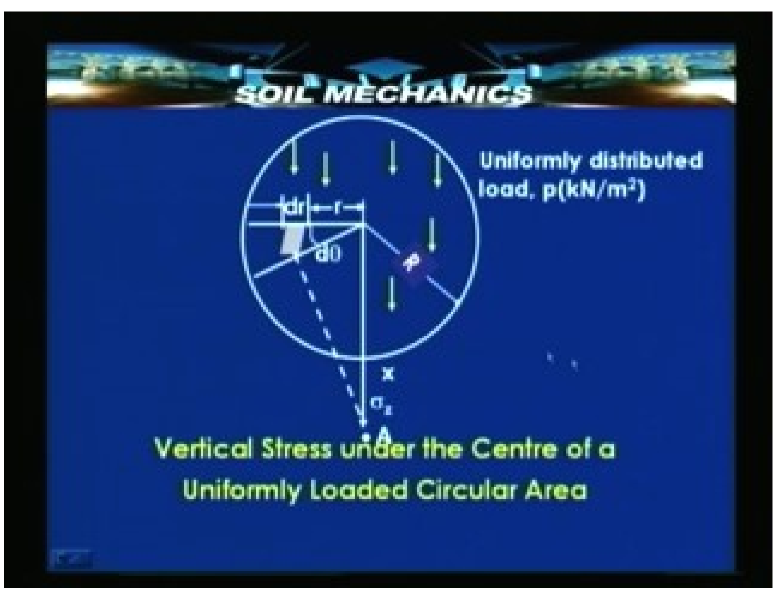

For example what we have been doing all these while was when ever we wanted to compute stresses beneath an area, we took an elemental area. Within that small elemental area the applied load intensity is constant and by using the concept of stress beneath a very small loaded area which is as good as a point load. We now take the expression for that loaded area and integrate it over the dimensions of the loaded area. We can do the same thing with a circular area. Look at this circular area it has got a radius capital R, it has an uniformly distributed load against denoted by small p in kilo Newton per meter square let us say and we want the stress at any point A. This point A in general could be anywhere beneath the loaded area but in this particular diagram I have chosen the point directly beneath the centre. This is the radius capital R, in order to find out the stress due to this loaded area we take a small elemental area here at an angle or rather at a distance small r with a width dr and having a length given by rd theta. This area will impose a stress over the point A which will be given by an expression shown in the next slide. The expression for stress beneath the loaded area, beneath its centre is given in the form of influence factor sigma z by p equal to I_f equal to one upon R by z the whole square plus 1 raised to the power of 3/2.

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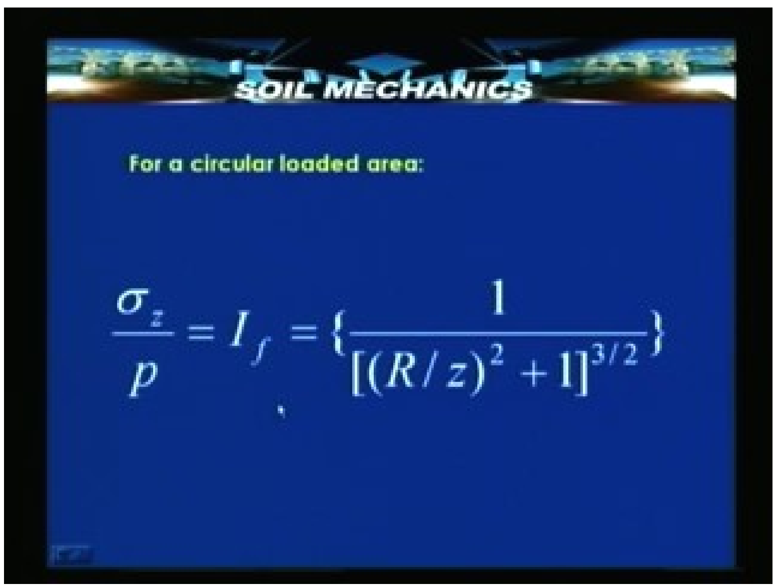

This means that once again we have non dimensionalized the stress, we have non dimensionalized the lengths in this case the radius and therefore we have an expression which is general which can be used for all circular loaded areas and it is possible to evolve. Obviously charts, so that we can avoid repeated computation of the stresses using this formula. If you see in the next slide this is graph which has been precisely derived in the manner which I mentioned a while ago. The z upon R is plotted along the vertical dimension that is vertical axis and the non dimensionalized stress sigma z by p is plotted along the horizontal axis and this is therefore a non dimensional of vertical stress or vertical stress concentration with respect to non dimensional depth z upon R. And this has been computed specifically for a point below the centre of the uniformly loaded circular area. It can be extrapolated also to point away from the centre and even away from the loaded areas in a manner similar to what we had done for rectangular areas.

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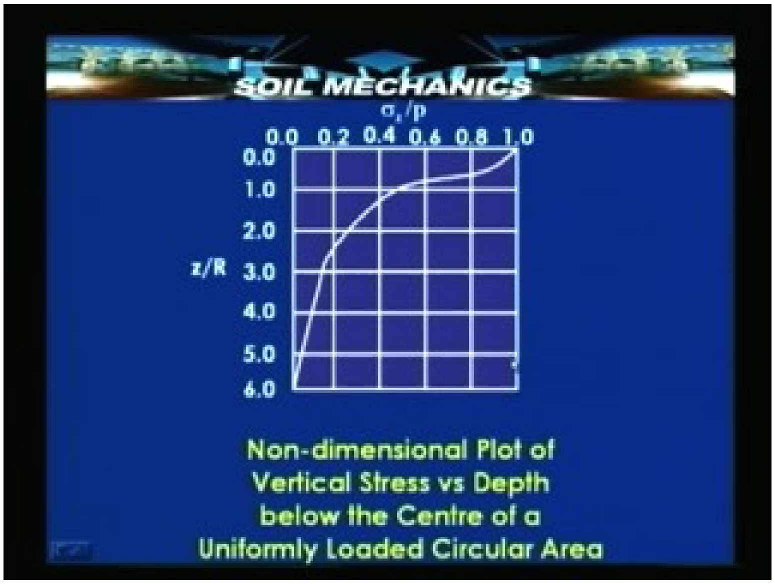

We are armed with if you take stock of what we have done, we are now armed with methods for computing stresses beneath a point load which is extrapolated to a line load which is then extrapolated to a strip load which is then generalized for a rectangular area or for a circular area. This means that almost all regular shapes have been taken care of. Wherever we have regularly shaped foundations it is possible to apply this simple concept of theory of elasticity and repeatedly apply the principle of super position of stresses due to a number of point loads and cover the entire area. And in each case we have simplified the process of repeated computation by taking a small element and integrating over the area. And not only that we have further simplified the process of computing the influence factor by developing non dimensional charts in which influence factor can be merely rid off by knowing non dimensional lengths and widths of the loaded area, non dimensional depth or non dimensional diameter or non dimensional radius in the case of a circular loaded area.

So this in sum and substance is the theory of elasticity approach which is used for computing stresses below loaded areas. We have of course used the assumption that the load intensity is uniform. How ever it is also possible to compute stresses for non uniform loading intensities, all that is required is to divide our given area into areas of uniform loading into a number of areas of different uniform loading intensities, apply the principle that we have seen repeatedly each one of these sub areas, apply the principle of super position and get the total stress. So what we have done is in this lecture, to summarize we have seen this stress distribution beneath a strip load, beneath a rectangular area and beneath a circular area. And as I said we will be in a position to generalize all these and extrapolate it to more generalized instances.

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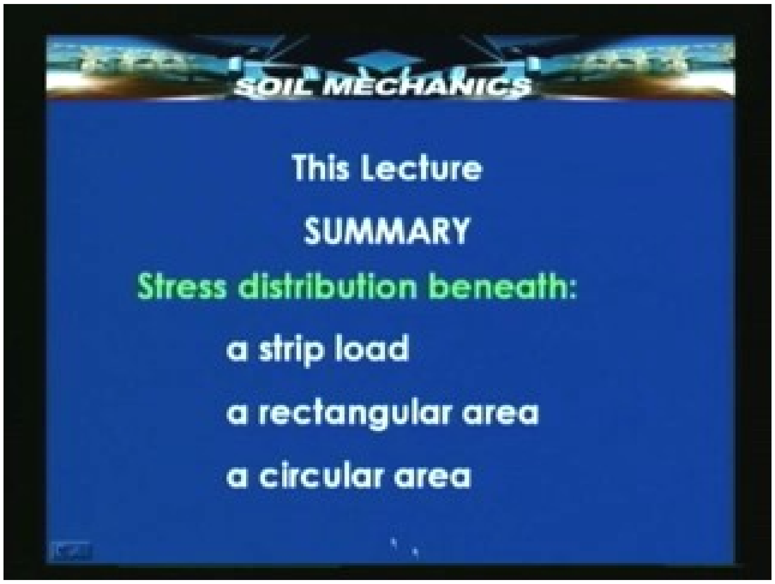

For example in the next lecture we will see how exactly to use this concepts which we have developed, the influence charts which we have seen, in order to compute stresses beneath a rectangular area. We will see a number of examples of different types of loadings or different types of loading intensities of uniform and non uniform, of examples of computing stresses beneath or at points which need not necessarily be located below the corners of the rectangulated area. We will extrapolate all those methods to circular areas and then finally we will see how best we can calculate the stress beneath an area of any arbitrary shape. We will see as I said a number of numerical examples as well in order to reinforce the understanding of the methods that we have used.

So with this I conclude today's lecture. I have necessarily gone into detailed explanation of the procedure, I have gone into detailed explanation of the principle of super position and the concept that has been used in the computation of the stresses beneath each one of the areas. This is basic to computation of the stresses in any generalized situation such as loading below an arbitrarily shaped area. And what is more, in the next class we will not only be seeing a number of numerical examples, we will also be seeing a very generalized chart, a graphical procedure which can be used for any shape circular, rectangular or even arbitrary shaped. Now the understanding that we have developed in the today's lecture is very essential to understand how the chart has been developed. In fact the chart has been developed by a person by name newmark, an engineer by name newmark whose name I have mentioned in the one of the earlier lectures. So with this I will conclude today's lecture.
Thank you.