Soil Mechanics
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Lecture – 26
Flow of water through soils-VII

Welcome to lecture seven of flow of water through soils. In the previous lecture we have introduced the flow net concepts and then we constructed methodology for predicting the rate of seepage through isotropic materials. In this lecture let us look into how we can solve some problems by using the concepts which you have discussed in the previous lecture. Then we extend our knowledge for calculating the rate of seepage through anisotropic material. Being anisotropic in the sense from the permeability point of view having different permeabilities in horizontal and vertical direction. Thereafter we will look in to how we can calculate the rate of seepage through an earthen dam. So once again let us look into this particular concept of the flow net method of construction.

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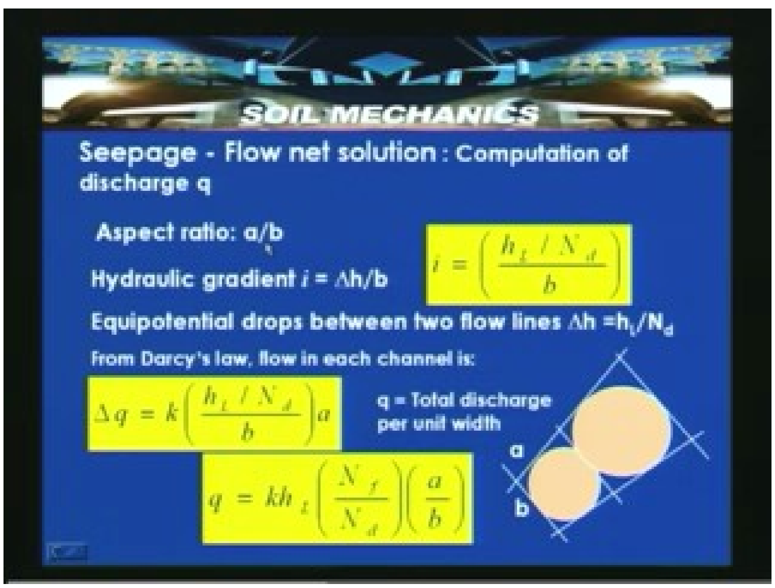

Let us look into Seepage- Flow net solution. The computation of the rate of seepage we have discussed that it is formed by the two families of curves; one is formed by equi-potential lines and flow lines. The space between any two flow lines we call it as flow channel and the head difference between any two equi-potential lines we call it as potential drop. So based on this the equi-potential drops between two flow lines is delta h is equal to h_L by N_d , N_d is the number of the potential drops. So from the Darcy's law if we again apply then we said that q is equal to k into h_L, h_L is the total head loss which is occurring N_f by N_d that is number of flow channels to number of this potential drops in to a by b which is nothing but a aspect ratio. $q = kh_L(N_f/N_d)\ (a/b)$.

So we said that while constructing the curvilinear squares this aspect ratio is to be maintained as one. So let us look into a problem, Flow net around horizontal drain. Let us consider in this particular slide, what you are seeing is a drain which is found at 3 meter from the top surface which is placed in sand having coefficient of permeability 5 into10 to the power of minus 7 meter per second, isotropic in nature. And the center of this drain is 3 meters from the top surface, the water table is found to be at the top surface itself.

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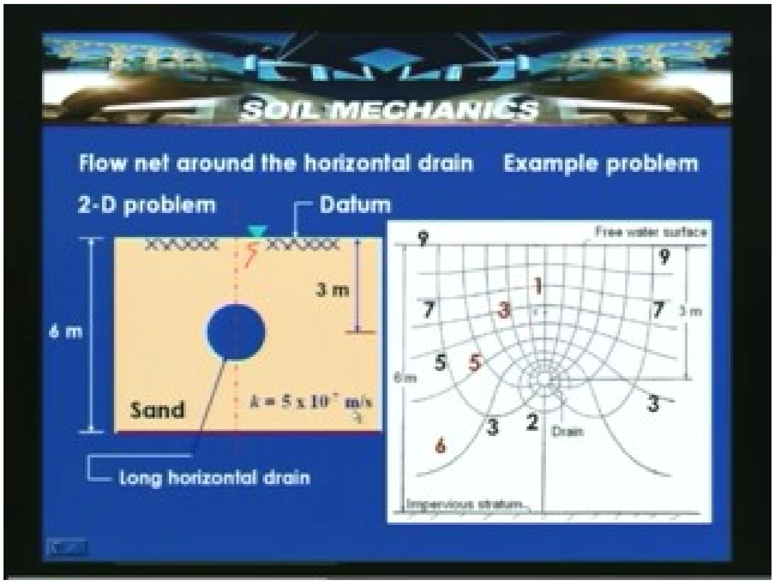

The free water surface is found to match with the top surface of the soil. So the depth of the soil which is under consideration is around 6 meters. So this is the long horizontal drive. So here what we are considering in this problem is a long horizontal drain. So this particular aspect also can be studied as a two dimensional problem. So in this case if this is the x direction, this is the z direction then this again turns out to be a two dimensional problem. Now consider this top surface of the soil or top surface of the water as datum. By using this flow net concept if you look into it we can draw flow nets. Keeping in view of the symmetry, the center line, we can see the stream of equi-potential lines and flow lines. So the flow takes place like this in this direction because the head loss occurs in this direction.

So the flow occurs in this direction and then the drain tries to divert the water from this. So we are interested in finding out the rate of seepage and then this particular pore water pressure at this particular location x which is shown in this slide and also the velocity of the flow. So this 3 meter is the center and this is the flow net and this is the symmetry. So you have got a two symmetrical mirror image type flow nets which are shown here. So this is the first equi-potential line, second, third, four, five, six, seven, eight, nine. So by the time it comes to 9, the head which is there for the flow to takes place is around 3 meters. So the flow is actually commencing from here, so the flow is in this direction. So as for our conventional numbering we numbered like 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Similarly this side (Refer Slide Time: 04:30-05:40).

So this is the first flow channel, second flow channel, third, four, five and six. So these are the number of the flow channels which are determining this particular flow. Flow net construction for example a long horizontal drain and that can be considered as a two dimensional problem. So let us look into the solution. Flow net around the horizontal drain is the example under consideration. Find the discharge through the drain in meter cube per day per meter length of the drain. Suppose if the drain is of 100 meters then we can calculate by using this concept. So considering the datum at the free water surface, so this makes total head at top of the ground is 3 meters. With that the total head at the top of the ground is around 3 meters. So H_L that is head loss from the surface to the drain is around 3 meters.

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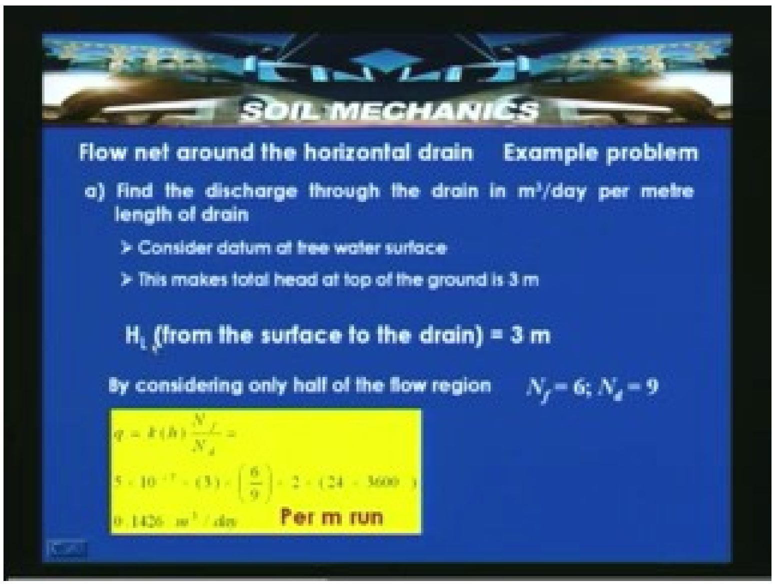

So by considering only one half of the flow region. So keeping in view of the symmetry, consider only one half of the flow region. If you count then number of flow channels are 6 and number of potential drops are 9. Being isotropic in nature we can use q is equal to k (h_L) into N_f by N_d, being curvilinear squares the aspect ratio a by b can be assumed as one. So by substituting these values where k is the coefficient of permeability and h is the head loss and number of flow channels, number of potential drops and multiplied by two because we have the two flow regions determining this longitudinal drain.

So multiplied by two which gives us 0.1426 meter cube per day, per meter run of the pipe. This is the rate of seepage or rate of discharge which can occur in per meter run of the pipe. So by using this concept of flow net construction simple problems like the drain problem which we consider and we treated as a two dimensional problem and it can be solved. Flow net around horizontal drain that is the same problem which is continued. Find the pore water pressure at x, 1.5 meter into the soil, directly above the drain. That means if you go into the previous slide, this is x which is 1.5 meter below the ground surface and directly above the center of the drain. So there we are required to determine the pore water pressure.

So if you look into this potential drop between two equi-potential lines is around 3/9. The N is the number of total potential drops and head loss is around 3, so potential drop between any two equi-potential lines is around 3/9. The total head at the location x which is 1.5 meter below the ground surface and above the center of the drain, that can be given by total head at x is equal to 5.6/9 ×3. The total head at x which is available is around 1.867. That means it started with three, by the time it came here that much head is already dissipated in the form of a flow which is taking place through the soil.

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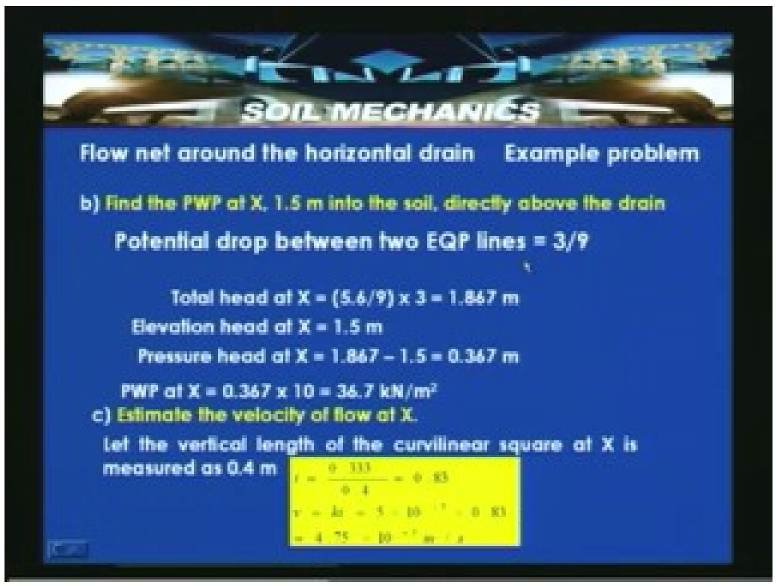

So elevation head at x is 1.5 meter because the datum is at the ground surface. Elevation head is 1.5 meter. By using total head is equal to pressure head plus elevation head, from there we get 0.367 meter as the pressure head at x. So pore water pressure at x is equal to 0.367×10, it is around 36.7 kilo Newton per meter square, by treating this unit weight of water as 10 kilo Newton per meter cube. So that is how we can determine the pore water pressure at different locations. If you know where exactly the pore water pressure is required and from this construction we can estimate what is the head at particular point. From there by looking into the elevation head we can calculate the pore water pressure at a particular location. So this is an example that demonstrates and by using the same concept we can determine the pore water pressure at different location.

Let us estimate the velocity of the flow at x. We are also asked to determine the velocity of flow at the location x. So let the vertical length of the curvilinear square at x is around 0.4 meters. Let us assume this particular flow net is drawn to a scale, we can easily get the vertical length of the curvilinear scale at location x, if that happens to be 0.4 meters. Then now the hydraulic gradient is nothing but 0.33 that is because head drop between any two equi-potential lines is around 3/9 that is 1/3 around 0.33, that divided by the length of the flow that is around 0.4 meters. So with that the hydraulic gradient is obtained around 0.83. By using the Darcy's law v= ki where k is knows to us and i is known to us. From that we can get the velocity of the flow at x which is given by a 4.75 ×

10 to the power of minus 7 meter per second. So in this problem we try to determine the seepage or a discharge through phi per meter length and then by treating that long horizontal drain as a two dimensional problem. We constructed a flow net and it has got two flow regions. So by using the concept of the method of flow net construction, the flow nets are constructed by approximating as curvilinear squares. And then we determined the number of flow channels and number of potential drops, then we tried to obtain the seepage and from there we tried to calculate the pore water pressure at a particular location which has been asked in this particular problem as x. Then we tried to calculate the velocity at a particular location x.

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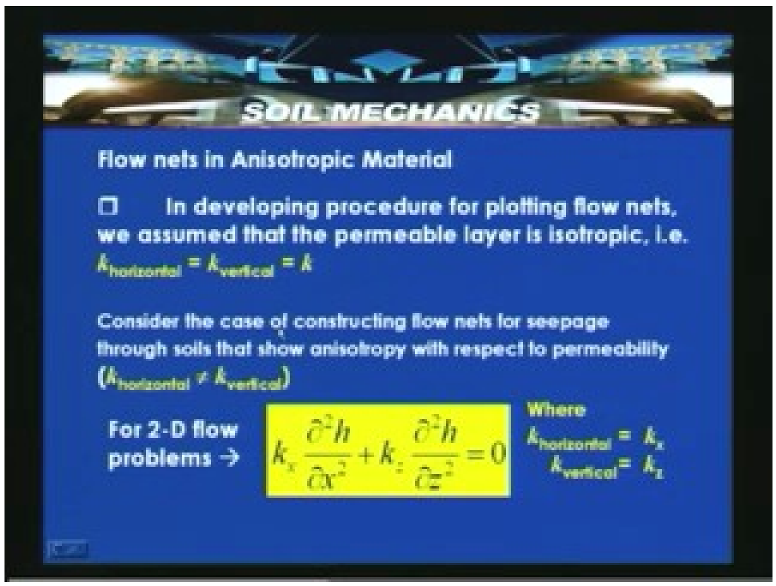

So having discussed the flows in isotropic materials, let us try to look into the flow in anisotropic material. That is the flow nets in anisotropic material, how to deal about this. So in developing the procedure for plotting flow nets, we assumed that permeable layer is isotropic. That means k horizontal is equal to k vertical, being a two dimensional k horizontal is equal to k vertical, we assumed which is equal to k. So that is isotropic as far as permeability is concerned. Consider the case of constructing flow nets for seepage through soils that show anisotropic with respect to permeability. Anisotropic with respect to permeability is concerned means the k horizontal is not equal to k permeability. So in that case what we should do and how to construct the flow nets? So for two dimensional problems say k horizontal if it is equal to k_x, k vertical if it is equal to k_z then we can write from laplacian equation as k_x into dow square h by dow x square plus k_z into dow square h by dow z square is equal to zero. So that is for two dimensional problems.

Incase of isotropic problems what we said is that k_x is equal to k_z is equal to k and not being equal to zero, we said that dow square h by dow x square plus dow square h by dow z square is equal to zero is the Laplace equation for a two dimensional problem. But incase of anisotropicity as far as the permeability is concerned, k_x into dow square x by dow x square plus k_z into dow square h by dow z square is equal to zero is a laplacian equation. Now the same equation by rearranging the terms we can write in the form of equation A which is shown here as k_x by k_z into dow square h by dow x square plus dow square h by dow z square is equal to zero. A closer analysis of the above equation indicates that a simple laplacian equation may be endangered if an adequate change of variable is used. So let us try to write the same equation by substituting x is equal to that is capital X is equal to small c times x where c is a constant. Now by writing this dow square h by dow x square plus dow square h by dow z square, with that by substituting for x is equal to cx we can write the same equation as dow square h by dow cx whole square plus dow square h by dow z square which can be written as one by c square into dow square h by dow x square plus dow square h by dow z square is equal to zero. So let us call this as an equation of the form which is indicated by B.

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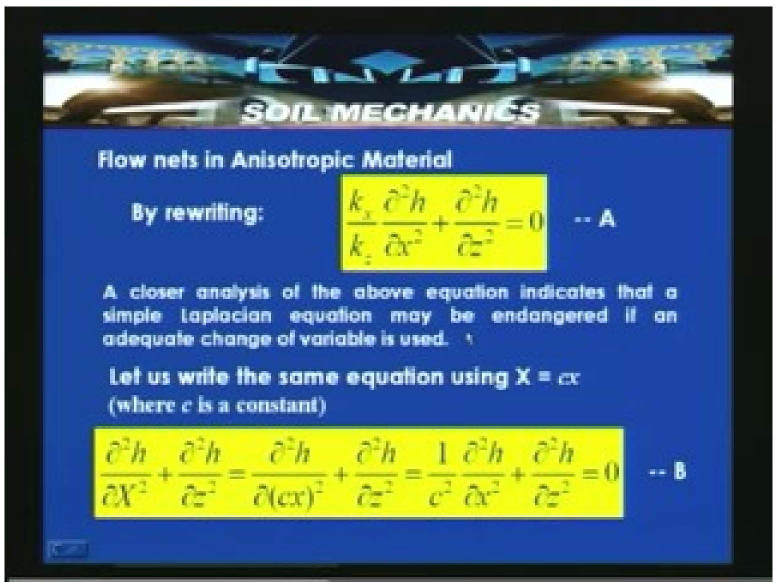

So comparing the equations A and B, the constant c can be identified as c is equal to root over k_z by k_x. By comparing the equation A and B from the previous slide we can obtain the constant c which is identified by comparing the equations A and B as c is equal to root over k_z by k_x. So equation B is of the same form as dow square h by dow x square plus dow square h by dow z square is equal to zero. Which governs the flow in isotropic soils and should represent two sets of the orthogonal curves in capital X small z plane (Xz plane). So the z is again same but x is different because we are trying to convert anisotropicity into some transformed section.

The x is equal to c times x, if you substitute for c that is root over k_z by k_x, we can write capital X is equal to root over k_z by k_x into small x. Let us consider the flow nets in anisotropic material. So if you take the flow through the natural anisotropic section and this will look like, which is L and then same depth D assuming that the flow is taking place in the vertical direction that is along the z- axis. The k_z is the permeability along the z axis and the flow is occurring over L into one meter that is per meter length of a plane which is considered.

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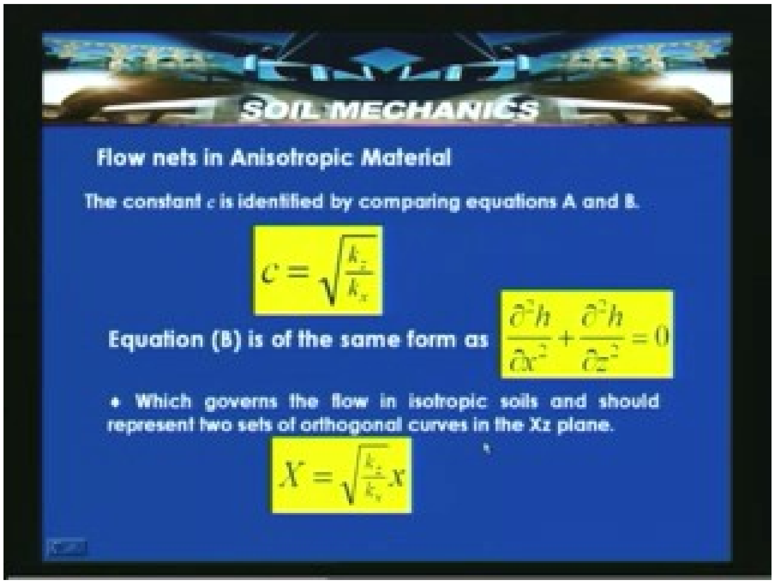

So in this case a flow is occurring over an area of L into one. So assumption that anisotropy merely causes a linear distortion of the flow net and the flow through natural anisotropic section which is shown here, D is the depth and L is the length and k_x is the flow which is along the x direction, k_z is in the vertical direction that is z direction and q is the amount of flow which is occurring through this. Now if you look into the flow through the transformed section because we have to convert that into a permeability which is isotropic in nature. So for that let us assume that k_t is the transformed permeability which is both in horizontal direction as well as in the vertical direction. So if that is the case then the flow through the transformed section which is again converted into isotropicity, that anisotropic is converted into isotropicity by considering a transformed permeability.

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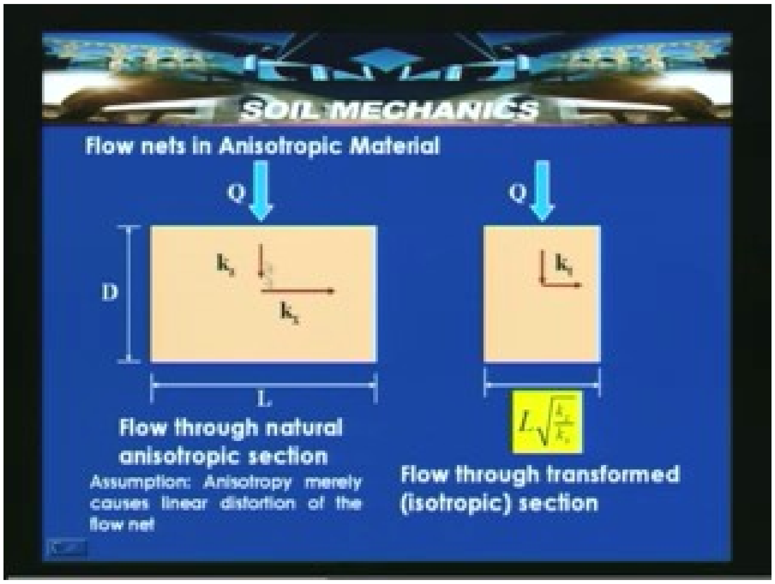

So this length is taken as L into root over k_z by k_x. Even with this transformation or whatever we are doing we should see that the laplacian equation is satisfied. So here the length is L into root over k_z by k_x and which is being small because we have already discussed that k_x is always more than k_z. That is our assumption that the horizontal permeability is more than vertical permeability.

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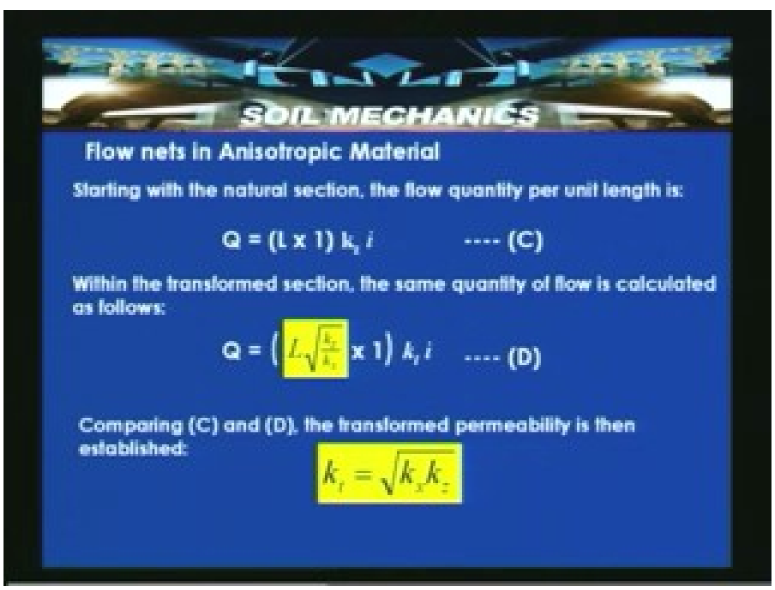

Now starting with the natural section that is on the left hand side in the previous slide. The flow quantity per unit length can be written as Q= (L×1) that is area over which the flow is occurring into k_z that is the permeability and i is the hydraulic gradient with which the flow is occurring. That is let us write that equation as c. Within the transformed section, the same quantity or the flow is calculated as follows. But that L which is the transformed length which is nothing but L into root over k_z by k_x into one, that is the area through which the flow is occurring with a permeability k_t but with same hydraulic gradient i. Now comparing C and D, with the transformed section the same quantity of the flow is calculated and if that is indicated as C, and comparing C and D the transformed permeability can be obtained as k_t = root over k_x k_z.

So the transformed permeability for example if the anisotropicity is there and if k_x is not equal to k_z or k horizontal is not equal to k vertical, the transformed permeability can be obtained by considering a square root of k_x and k_z or k horizontal into k vertical. The same expression could have been found using the flow in horizontal direction. So in the previous slide if you have observed, you would have considered the flow in the vertical direction. The same thing would have been considered if you consider the flow in the horizontal direction. But note, then the hydraulic gradient changes from i = H/L in the natural section to i= H/L into root over k_z by k_x in the transformed section. So steps involved in the construction of the flow net, incase of a soil which is having anisotropicity in nature as far as the permeability concerned is to plot the section of the hydraulic structure, adopt a vertical scale.

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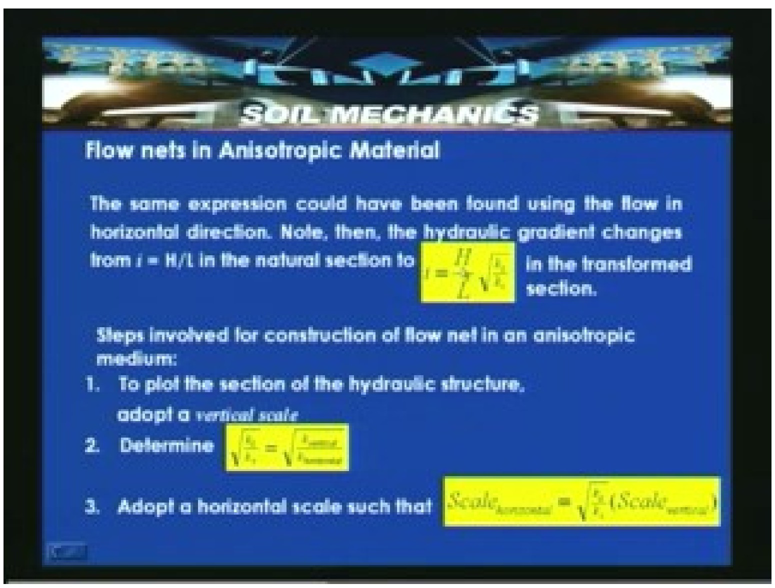

So first any hydraulic structure under consideration is there, so plot the structure with vertical scale and determine ratio of root over k_z by k_x that is nothing but root over k vertical by k horizontal. Once the ratio is obtained now adopt a horizontal scale such that horizontal scale is root over k_z by k_x times the vertical scale. That is the horizontal scale is root over k_z by k_x times the vertical scale. The steps involved for the construction of the flow net in an anisotropic medium are as follows: To plot the sections of the hydraulic structure adopt a vertical scale. Determine root over k_z by k_x is equal to root over k vertical by k horizontal and adopt a horizontal scale such that scale horizontal is equal to root over k_z by k_x into scale vertical. The next step, with the scales adopted in step 1 and step 3, plot the cross section of the structure.

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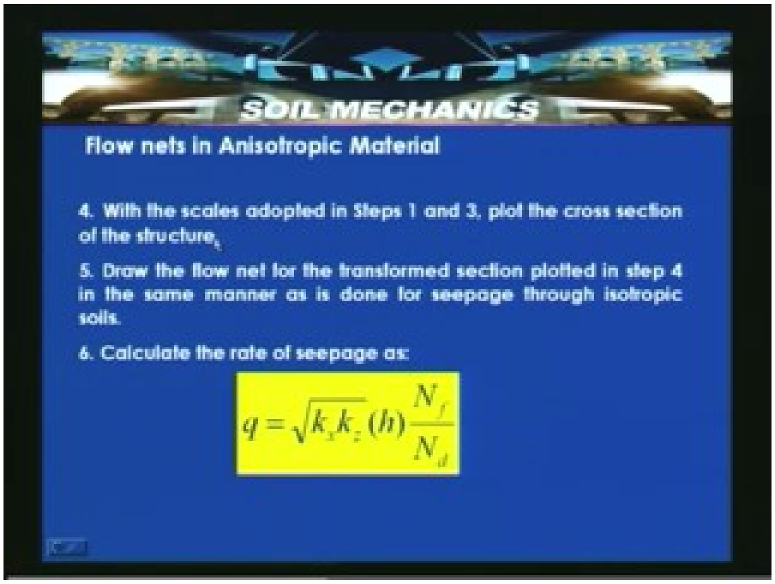

Now with the scales adopted in step one that is vertical scale and it is same. In the previous step 3 we said that scale horizontal is equal to root over k_z by k_x into scale vertical. So draw the flow net for the transformed section plotted in step 4, in the same manner as is done for the seepage through isotropic soils. So now we converted this anisotropic medium into a transformed section representing the isotropicity and for that we accounted the variation in horizontal direction, vertical direction by considering transformed permeability k_t is equal to root over k_x and k_z. So calculate the rate of seepage as q is equal to root over k_x k_z because now the transformed permeabilities under consideration which is nothing but the square root of k_x and k_z and h is the head loss into N_f by N_d. So by using this we can determine the rate of seepage.

Having looked into the theoretical aspect let us look into an example problem for the flow nets in anisotropic material. So consider a concrete dam which is like this and a cut-off sheet pile wall is shown here. Basically these types of cut-off sheet pile wall are provided to control seepage pressures. As indicated here in this slide, to control seepage pressure this cut-off sheet pile wall are driven. Sometimes you have got this cut off sheet pile walls even in the upstream side as well as on the downstream side. In this case only it has got an upstream side and sometimes you practice even to provide cut-off sheet pile walls on the downstream side also. So consider a concrete dam on a layer of a homogeneous anisotropic clay having k_x =16×10 to the power of minus 8 meter per second and k_z = 1×10 to the power of 8 meter per second.

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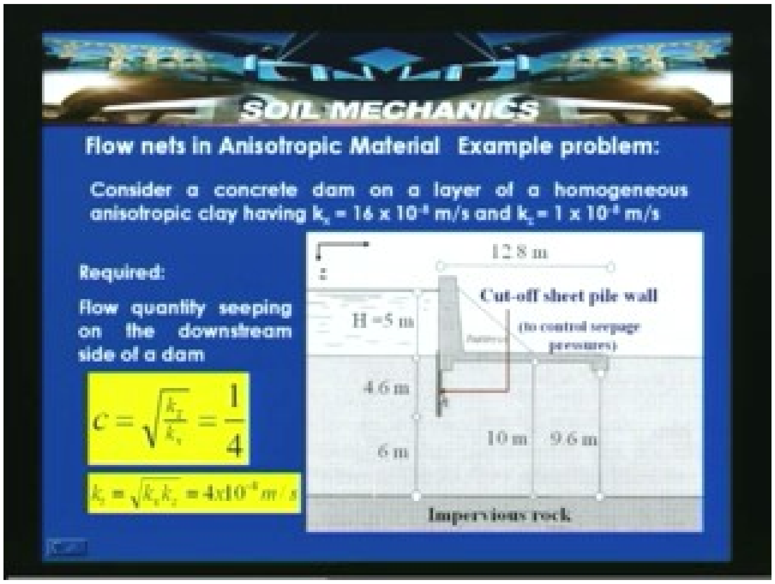

So x is the horizontal direction and z is the vertical direction. The length from here to here is 12.8 meters and 5 meters is the head which is available on the upstream side and the depth of the cut of sheet pile wall is 4.6 meters and the tip is 6 meters above this impervious rock under consideration (Refer Slide Time: 23:36). So from the flow net what we require is the flow quantity seeping on the downstream side of a dam that is seeping through the soil.

We have got permeabilities, k_x value which is around 16×10 to the power of 8 meter per second and k_z is 1×10 to the power of 8 meter per second. So determine c, that is nothing but root over k_z by k_x which is nothing but 1/4. Now the transformed permeability is nothing but root over k_x and k_z that is 4×10 to the power of minus 8 meter per second. Now as per the procedure if you look into it, if you draw with an anisotropic medium you will get the flow net like this and the curvilinear rectangles of a sides ratio root over k_x by k_z. So what we get here is not squares or orthogonal. The reasons we are going to discuss but the curvilinear rectangle of a sides ratio is shown here.

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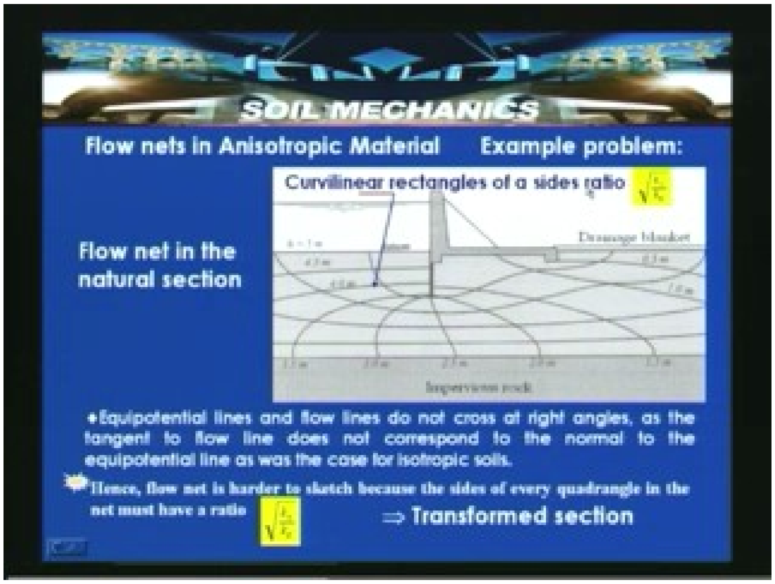

And what you seeing here is again flow nets but in a natural section with anisotropic. The effect of anisotropicity reflected like in this form now. This equi-potential lines and flow lines do not cross at right angles, as the tangent to flow line does not correspond to the normal to the equipotential line as was the case for isotropic soils. Hence the flow net is basically harder to sketch because the sides of every quadrangle in the net must have a ratio of a around root over k_x by k_z. Hence it is better to resolve to the transformed section. How to construct a transformed section? First construct a hydraulic structure by adopting a vertical scale.

So by adopting that if you draw, the vertical scale is same but if you look into it this is a flow net which is for a transformed section. So here what we did is that the horizontal scale is 12.8 meters which is now reduced to 1 by 4 times. Because if the transformed section scale which we adopted that is scale horizontal is equal to root over k that is 1 by 4 times scale vertical. So if you put into it, what we get is around 3.2 meters in a transformed section. The vertical scale we have not touched up. Based on this if you construct flow lines again by fulfilling the same requirement, this orthogonality and all these things like in isotropic problems. This transformed section is nothing but an isotropic problem. So with this if you compare now 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, so number of potential drops are 10, so N_d =10. Now if you look into this 1, 2, 3, 4, 5. So this is also approximated as a flow channel. So we have got number of flow channels in this particular flow region for this structure in the transformed section. There are around 5 and the total head loss is around 5 meters.

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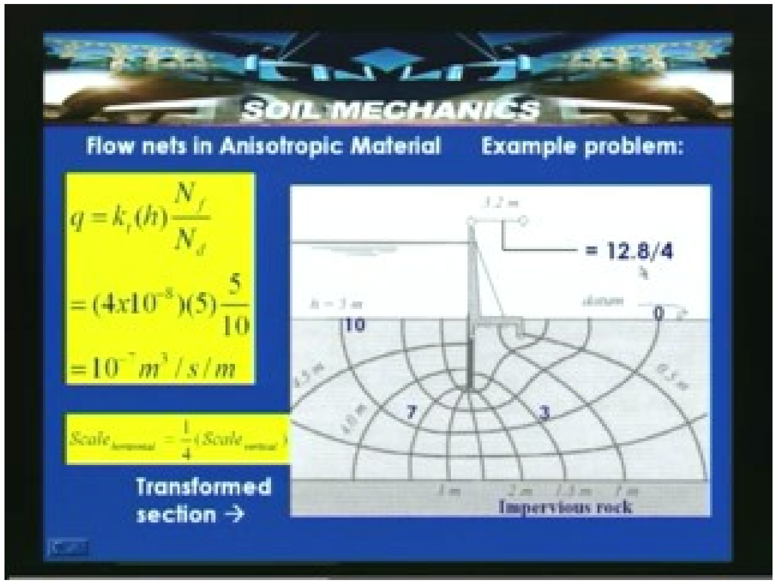

So q = k_t (h) N_f by N_d which is around 10 to the power of minus 7 meter cube per second per meter. So this is how we treat soils with anisotropicity in nature. And particularly in two dimensional problems what we do is that we convert them into a transformed section and draw that same flow nets and then determine the rate of seepage. The procedure is again very similar to the one which we discussed already. Now having discussed for confined problems in the previous slides, how we can determine seepage through dams or earth dams or embankment dams. The flow in the embankment dam or earth dam is an unconfined seepage.

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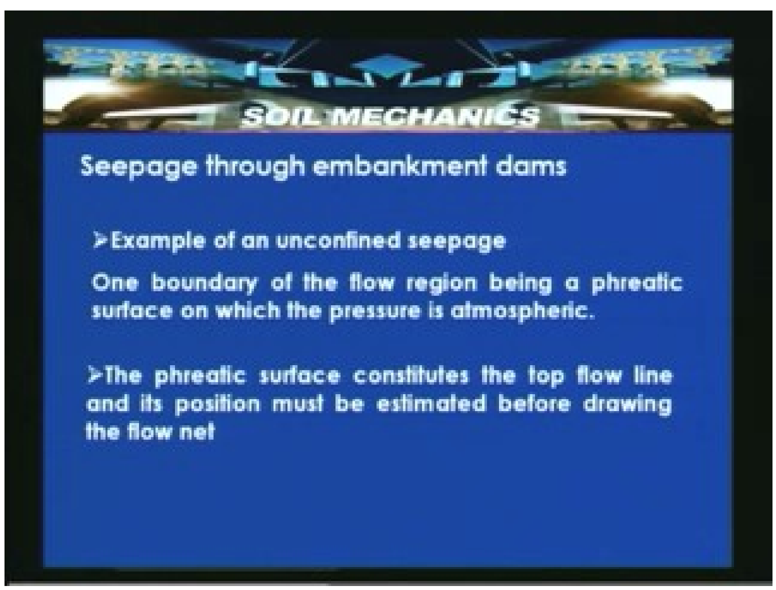

So the example of an unconfined seepage means the seepage through embankment dams is an unconfined seepage. For example if you consider in the previous slide, like flow through a sheet pile wall. That again comes out as confined seepage. In this case what happens is that in the unconfined seepage one boundary of the flow region being a phreatic surface on which the pressure is atmospheric. So the phreatic surface constitutes the top flow line and its position must be estimated before drawing the flow net. So here in this flow through earth dams, arriving at the phreatic line itself involves a tedious procedure. Otherwise the concept of again determining seepage or rate of flow through the dam is very similar. But the tedious procedure which is involved is to arrive at the phreatic surface, to locate the phreatic surface and to draw the phreatic surface and then construct the flow lines and equipotential lines and then again we can use the same concept to determine the rate of seepage through earth dam.

Let us look into the calculation of this seepage through an earth dam resting on impervious base. If this dam which is resting on an impervious base, then several solutions have been proposed for determined the quantity of seepage through a homogeneous earth dam. The popular solutions which were discussed are Dupuit's solution, Schaffernak's solution and Casagrande's solution. In this lecture, in the same order we will try to discuss this Dupuit's solution and Schaffernak's solution and then the graphical method which is involved for the Schaffernak's method and Casagrande's solution which is widely used for locating the phreatic surface.

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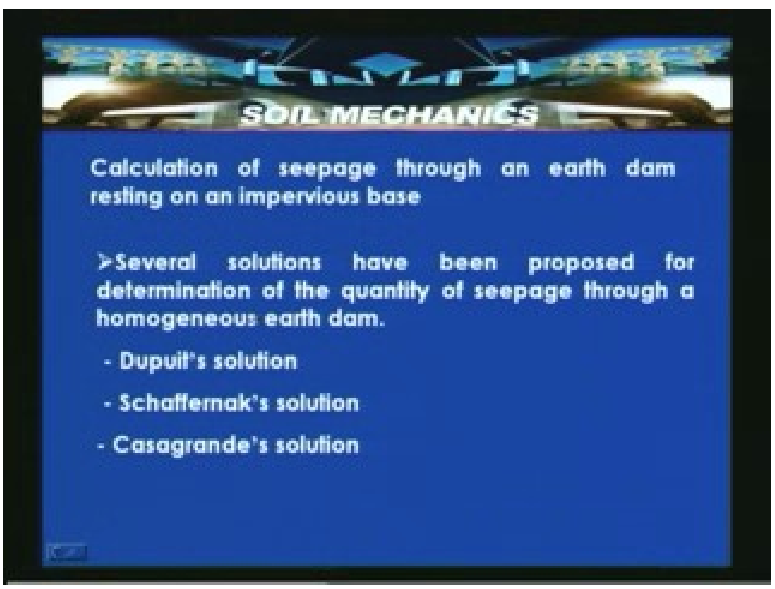

Let us look into the Dupuit's solution for the flow through an earth dam. The Dupuit's solution for the flow through an earth dam can be given like this. The assumption which is involved in this slide that is a cross section of an earthen dam is shown. And this is the crust and this is the base and this is the upstream face and this is the downstream face (Refer Slide Time: 30:55). This is called upstream water with a head H_1 and downstream that is also called tail water level which is called H_2. This is z direction and this is x direction and this resting on an impervious base.

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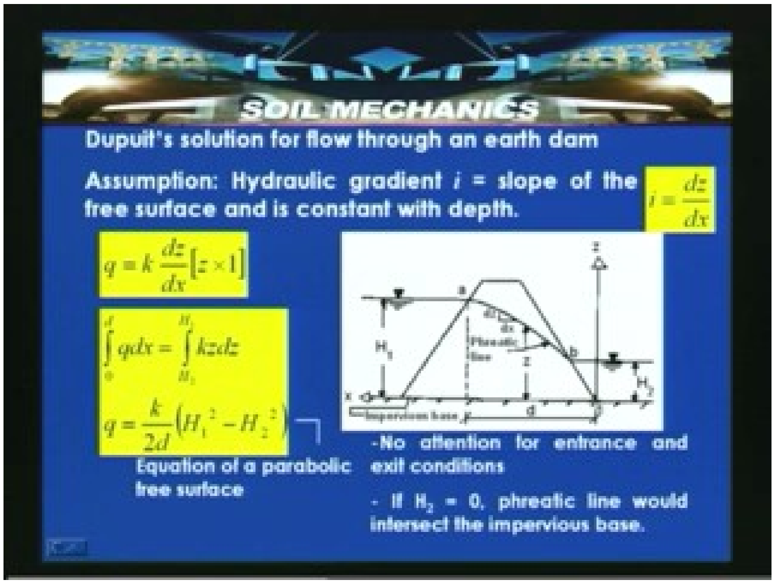

So this is the toe of the dam and b is the location here and the slope which is dz by dx and this is the phreatic surface. So the assumption which is involved in the Dupuit's case is that hydraulic gradient i is equal to slope of the free surface and is constant with the depth. So i = dz by dx is considered by Dupuit. So if it is assumed that flow is taking place again as a two dimensional problem and q = k into dz by dx, where dz by dx is nothing but the slope of that particular phreatic surface. That is dz by dx into [z ×1]. So z is this depth here and one is the per meter length of the dam. So q = k into dz by dx into [z ×1] where k is the coefficient of permeability which is again isotropic. That is what it is assumed because it is constructed with a homogeneous material. So it can be assumed as kz is equal to k. By separating the variables we will get q dx and kz dz. If you integrate between 0 to d that is d is this particular distance from here to here and H_2 to H_1. Suppose if the tailwater level is zero then you will integrate from zero to H_1, in this case there is a H_2, so H_2 to H_1 and kz dz.

So with that after integration and then simplification we get an expression for q that is for calculating the flow through a earth dam by using the Dupuit's solution which is given by q = k by 2d into (H_1 square minus H_2 square) which again represents the equations for a parabolic free surface. So the Dupuit's assumes that this particular phreatic surface is a parabolic free surface. But some limitations in Dupuit's solution are there, that is there is no attention for entrance and exit conditions. So no attention has been paid for entrance and exit conditions because there are certain cases like, if you look this is an equi-potential line where the orthogonality which is required between this flow line that is this stop phreatic surface which is not maintained here. Similarly at the exits conditions also a similar phenomenal reflects and if H_2 is equal to zero the phreatic line would intersect the impervious base. So in the Dupuit's solution if H_2 is equal to zero, the phreatic surface will come and intersect with the impervious base which again happens to be a flow line, so that cannot happen. So these are the some of the limitations with the Dupuit's solution. Let us look into the Schaffernak's solution and how it works out. The Schaffernak's solution for the flow through an earth dam: It is an assumption that phreatic surface will be like line ab and will intersect at a distance l from the impervious base. So in the previous case we have discussed about the limitation, that if H_2 is equal to zero the phreatic surface will intersect the impervious base. So here what the Schaffernak's has considered is that it is intersecting at a distance L from that point O. So here phreatic surface will be like a line ab which is shown here and will intersect at a distance L from the impervious base. So we are interested to determine what is this length L. And this is beta which is that slope and dz by dx and a is this point, so from here to here that is the distance d and H is the upstream head that is H_2 is equal to zero and H_1 is equal to H and now this is converted to H.

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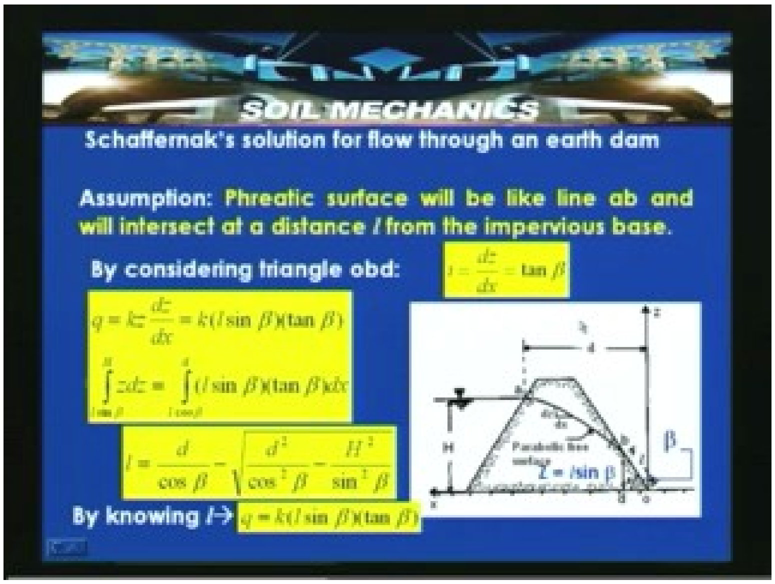

So by considering the triangle OBD, so the z= L sin beta. By considering the triangle obd, Schaffernak's assumes that i= dz by dx = tan beta. So by considering the triangle obd, q = kz into dz by dx, we can write k into z is written as L sin beta that is here into tan beta. The dz by dx the hydraulic gradient is assumed by Schaffernak as tan beta. So by simplifying now, we obtain L so by knowing L we can determine q= k into (l sin beta) into (tan beta). So q = k (l sin beta) (tan beta).

So L is given after simplifying this expression we get L is equal to d by cos beta minus square root of d square by cos square beta minus H square by sin square beta. So this is an expression where d is this distance and H is the head, beta is this slope inclination. So by knowing L we can determine q= k into (l sin beta) (tan beta). This is a theoretical method for determining L, that is the location where this is intersecting. And again he assumes that ab is the phreatic surface and hydraulic gradient is assumed as i=dz by dx =tan beta.

The graphical construction procedure for the Schaffernak's solution is also given for locating the same L that is the distance bc. So in this figure a cross section of an earthen dam is shown and ab is the phreatic surface which is indicated here and bc is the distance above which it is located. That is bc is equal to L which is required to be determined and this is the impermeable base. So the procedure which is involved with that graphical construction procedure for the Schaffernak's solution is extend bc downwards.

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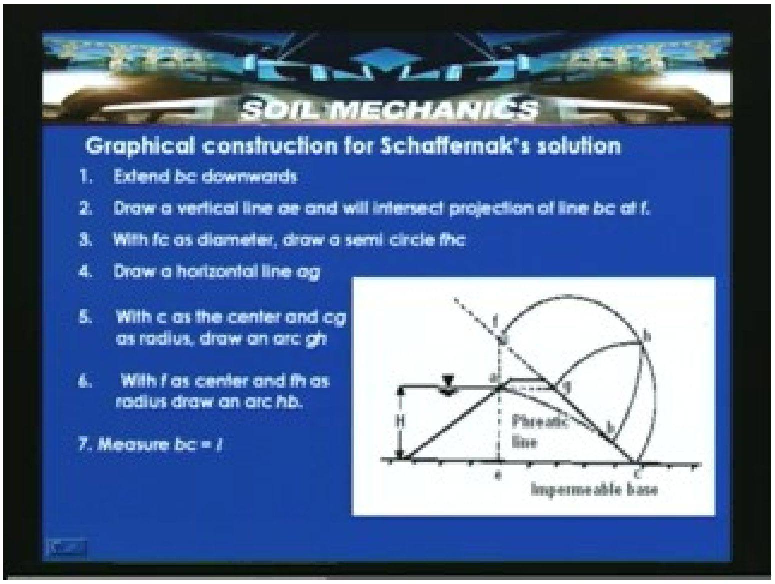

So extend bc downwards and draw a vertical line ae and will intersect the projection of line bc at f. So draw a vertical line ae and extend backwards through this bc, so that it meets at f, with fc as diameter draw a semicircle fhc. Draw a horizontal line ag. With c as center and cg as radius, draw an arc gh. With f as center and fh as radius, draw an arc bh. So with this once you locate that distance bc is nothing but L, then by knowing L we can determine the rate of flow through an earth dam by using Schaffernak's solution.

The third method or third solution which is available for locating the phreatic surface which is also widely used is Casagrande's1937 solution. Casagrande 1937 shown experimentally that the parabola ab actually starts from point a dash which is indicated in this slide here. So previously all other investigators Dupuit's and Schaffernak's were not given enough attention to the entrance and exit conditions. Schaffernak's to some extend considered this exit of the phreatic surface, but as far as Dupuit's is concerned, were the entrance and the exit conditions are ignored. So Casagrande experimentally showed that the parabola ab actually starts from the point a dash. So a dash and a, the distance which is experimentally shown that is around 0.3 times delta, where delta is this particular length. This horizontal distance delta is from this point to this point (Refer Slide Time: 40:32). With this modification value of d, then a dash c is the phreatic surface. So Casagrande's suggested i = dz by ds because dz is that vertical distance and ds is the length along which the flow is taking place. So the Casagrande's considered i= dz by ds which is equal to sin beta instead of i = dz by dx.

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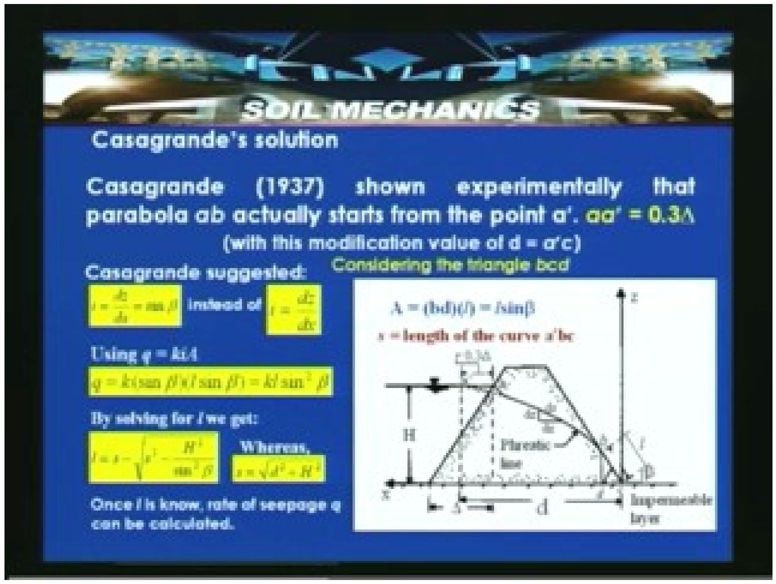

So using again q= kiA, q= k (sin beta) into (l sin beta) where it is kl sin square beta. So A which is area over which the flow is taking place and it is nothing but bd into l where bd is nothing but which is sin beta and which can be written as A= bd into l = l sin beta. So here with this we can write now q= k sin beta that is because of the dz by dx into l sin beta. So q= kl sin square beta, by solving for l if you again rearrange the variables and then integrate, then by solving we get l= square root of s square - H square by tan square beta, where s is the length of the curve a dash bc. That is from here a dash bc is the length of the curve s. This is the length of the curve a dash bc which is represented by small s here.

So with 4 to 5 percent error, we can determine s as root over d square plus H square, where with 4 to 5 percent error we can approximate s is equal to root over d square plus H square. So by knowing s, we can estimate now l. By knowing l we can estimate q is equal to k l sine square beta that is l is known and beta which is the angle shown here which is the slope of inclination beta. So with that once l is known the rate of seepage q can be calculated. In Casagrande's solution the procedure what we discuss is that the plotting the phreatic line for seepage through earth dams.

So for construction of flow net for seepage through earth dams, the phreatic line needs to be established first. So what we discussed in the flow through an earth dam, for the construction of flow nets the most difficult part is the construction of a phreatic line. Though we have discussed about some methods, they suffer from this limitations. But this particular Casagrande's solution where to some extend, both entrance and exit conditions have been treated correctly. So we can arrive that is the phreatic line which is going to occur or representative phreatic line which may occur in the field.

So curve a dash efb dash c is a parabola with its focus at c. So in this particular slide a dam which is shown is having a coefficient of permeability k and again this distance is 0.3 delta and this distance is delta, h is the height and that is the head which is causing the flow and d is the distance from vertical axis to this particular point a dash. So here we assume that the curve a dash efb dash c is a parabola with a focus at c and the phreatic line coincides with the parabola but with minor deviations at the upstream and downstream faces.

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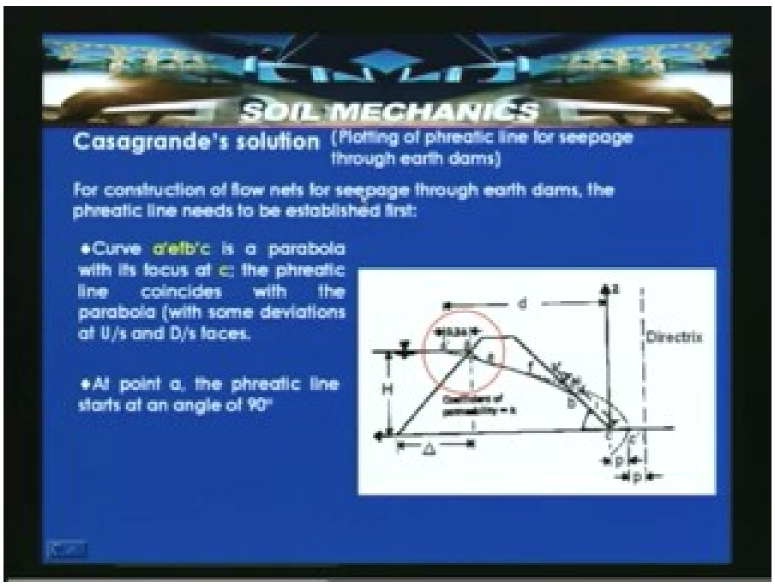

At point a, the phreatic line starts at an angle of 90 degrees because this being the equi-potential line and this being the top most flow line, so this orthogonality has to be maintained. So at point a, the phreatic line starts at an angle of 90 degrees. So here the circled portion indicates that the entrance conditions which is actually has been considered and it is shown here. So while drawing this phreatic line it has to be noted that while drawing from upstream face that this phreatic surface has to be started with 90 degree angle with the upstream face.

So here portion of the phreatic line coincides with the parabola and then this is the c which is the focus and c c dash is the length p and this distance is p and this is the directrix of this (Refer Slide Time:46:18). So b and b dash, this distance is delta L and this is the small l. We will come to this particular point later because some observations have shown that when the beta angle which is greater than 30 degrees then it has got some deviations. So Casagrande's has given some methodology to locate b, b dash and bc incase if beta is more than 30 degrees. So Casagrande's solution if you look into this procedure for constructing parabola a dash efb b dash c. That is a portion of the parabola is shown here, c is the focus and d is this directrix and c c dash is the distance p.

Let c c dash is equal to p, then a is the point on the parabola with coordinates x and z and c with focus coordinates having 0 to (0, 0) and ad is this horizontal distance and ac is this particular distance which is represented as the distance between point c and a, that is focus of this parabola and a is a point on the parabola. Based on the properties of the parabola we can say that ac is equal to ad, but we can write from this figure ac has root over x square plus z square and ad is equal to 2p+ x. Because this being x horizontal distance, we can write ad as 2p+ x. So here by equating ac= ad from the properties of the parabola, we can write root over x square plus z square is equal to 2p+ x. So we can write with ac is equal to ad, we can say the root over x square plus z square is equal to 2p+ x.

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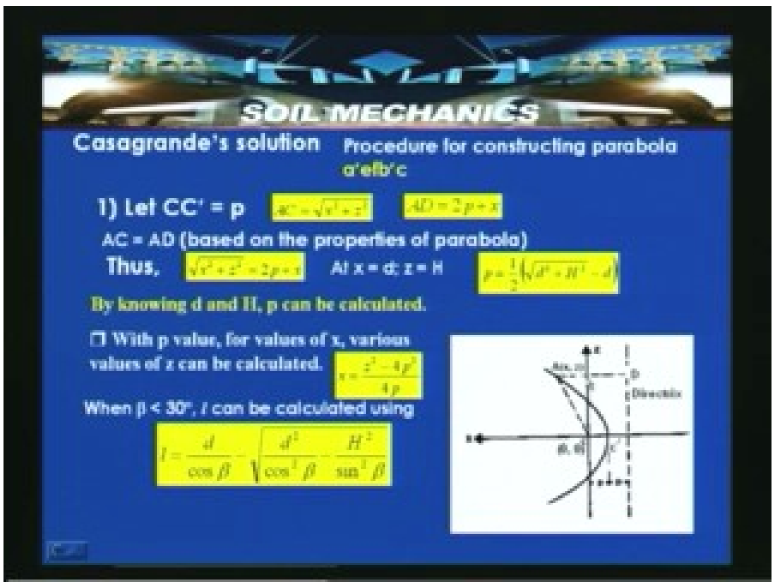

At x = d and z= H, we can write p=1 by 2 root over d square plus H square minus d. That is here in this slide this is for z= H and x =d, now we can write for x= d and z= H, p= 1 by 2 into root over d square plus H square minus d (Refer Slide Time: 48:44). So by knowing d and H, p can be calculated that is the distance from the focus to this c c dash can be estimated. With p value, for values of x, various values of z can be calculated. So again by using this we can obtain x= z square minus 4 p square by 4p. So with p value for values of x, various values of z can be calculated. So when beta less than 30, l can be calculated using l= d by cos beta minus root over d square by cos square beta minus H square by sin square beta. This we have derived earlier, so the same thing can be applied for beta less than 30 degrees.

Incase for beta greater than or equal to 30 degrees, Casagrande proposed an approximation which is the distance delta l that is this distance here and l. So from the directrix, this location of this particular thing, so delta l by l+ delta l for different betas which is given. So for 30 degrees delta l by l+ delta l is around 0.36, 60 degrees is around 0.32, 90 degrees is around 0.26 and around 150 degree beta it is around 0.10 and 180 degree beta it is around 0.

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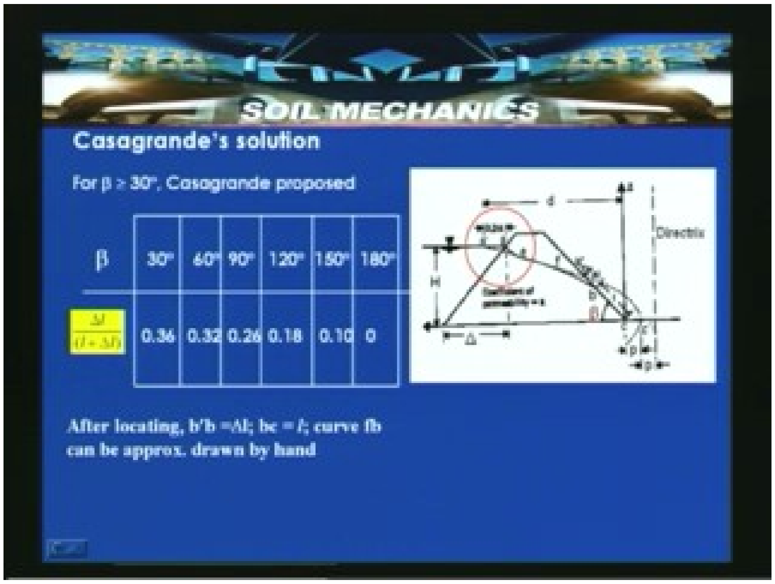

So after locating b dash b=delta l and bc = l, curve f b can be approximated by drawing in hand approximately. So by adopting the procedure that is from this upstream face, setting a distance 0.3 of the delta, that is this particular distance which is 0.3 times the delta and maintaining this orthogonality and drawing a free curve here for joining fb, we can construct this phreatic surface. Once this phreatic surface is obtained, we can use this concept to adopt the concept of constructing flow net for determining rate of seepage through an earth dam. We have come to the stage for the flow net construction of an earth dam. So we said that drawing the phreatic line or location of the phreatic line is very much required. The procedures which are discussed are based on the Dupuit's solution or Schaffernak's solution or by adopting this Casagrande's method we can determine this phreatic line.

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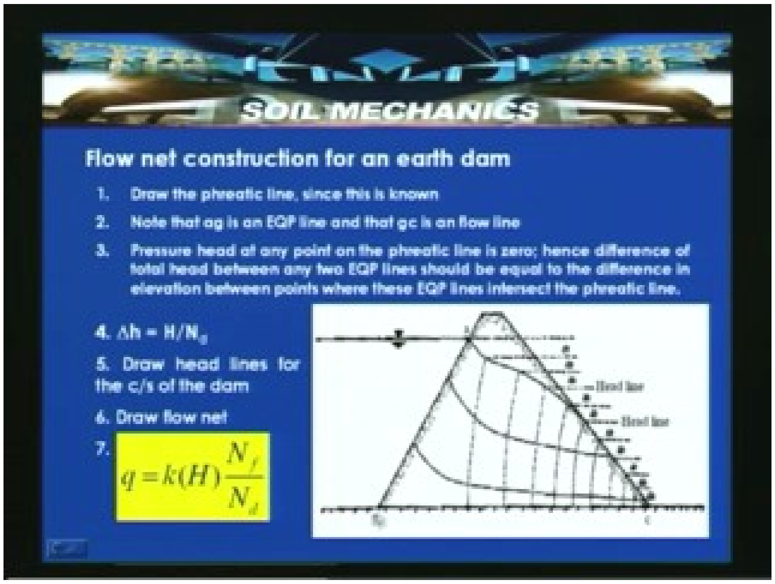

Draw the phreatic line, so this being known to us and note that here in this cross section a dam is shown and ag is equipotential line and gc is a flow line. So this is the bottom most flow line because the flow cannot cross this impervious base, it flows along the impervious base. So with that in mind the gc is flow line and ab is equipotential line, there are some deviations where the orthogonality is not maintained. So draw the phreatic line, since this is known. Note that ag is an equipotential line and gc is flow line. Pressure head at any point on the phreatic surface, so this top most line by the procedure which are discussed going to yield this particular line which is the phreatic surface. So pressure head at any point on the phreatic line is zero because being atmospheric pressure, the pressure head is zero. So pressure head at any point on the phreatic surface is zero.

Hence the difference of the total head between any two equipotential lines should be equal to the difference in elevation between points where these equi-potential lines intersect the phreatic line. So pressure head at any point in the phreatic line is zero, hence the difference of total head between any two equipotential line should be equal to the difference in elevation between points where these equipotential lines intersect the phreatic line. So here for determining the number of potential drops, total head which is H divided by say number of potential drops which you are trying to give, number of potential drops say N_d. So total head is say head loss which is upstream water head in this case is H then H by N_d is going to give the delta H which is the potential drop.

So first procedure is to draw the embankment to the scale and then constructing the phreatic surface and establishing the phreatic surface by adopting the procedure which are discussed in the previous slides, then drawing the headlines. Draw head lines for the cross section of the dam. So simply draw the head line for the cross section of the dam with delta h. It just divide in to the number of equipotential drops and from there draw the flow net again saying that this being a flow line, so maintain this orthogonality and see that these curvilinear squares are maintained.

So in this case now if you look into it the flow is taking place like this. So again the number of the potential drop lines are numbered from this direction to this direction from 0, 1, 2, 3, 4, 5, 6 like this it will come. And when it comes to the flow channels they are 1, 2 may be around 2.5 flow channels. So here by estimating the number of flow channels, number of potential drops and having known the k that is coefficient of permeability of the soil which is in the earth dam, we can estimate the rate of seepage which is actually occurring through this.

So in this lecture we tried to understand how a flow net concept can be applied for solving some typical problems, like we discussed about a long horizontal drain. And we also tried to see how the anisotropicity of soil as far as the permeability is concerned can be considered while drawing flow nets. Then we discussed the methodologies which are involved in arriving at a transformed section and then how this anisotropic can be converted into transformed section. Thereafter we tried to discuss how the rate of flow can be estimated through an earth dam.

So for that we discussed about three methods Dupuit's solution, Schaffernak's solution and Casagrande's solution where main impedance is to locate the phreatic surface and then the procedure is very simple for determining and particularly for the rate of seepage through earthen dam. So here one concept which is required to be kept in mind is that being this pressure along the phreatic line the pressure it is being zero. So whatever this total elevation which is there divided by that number of potential drops directly it is easy to draw equipotential lines in this case.

So the same concept can be used by determining the rate of seepage through this particular medium in this case having a homogeneous soil which is having an isotropic permeability k. In the next class we will try to discuss about how this problems can be prevented by providing some filters and there are some typical failures which are called piping and boiling conditions which are required to be considered for this hydraulic structures and then factors safeties which are involved for this hydraulic structures again and piping failure and boiling failure. Based on this we will try to solve some problems by using this flow net concept for determining the uplift which can be exerted on a particular structure and how this uplift can be relived by using some filters and other possible immediate measures will be discussed in the next lecture.