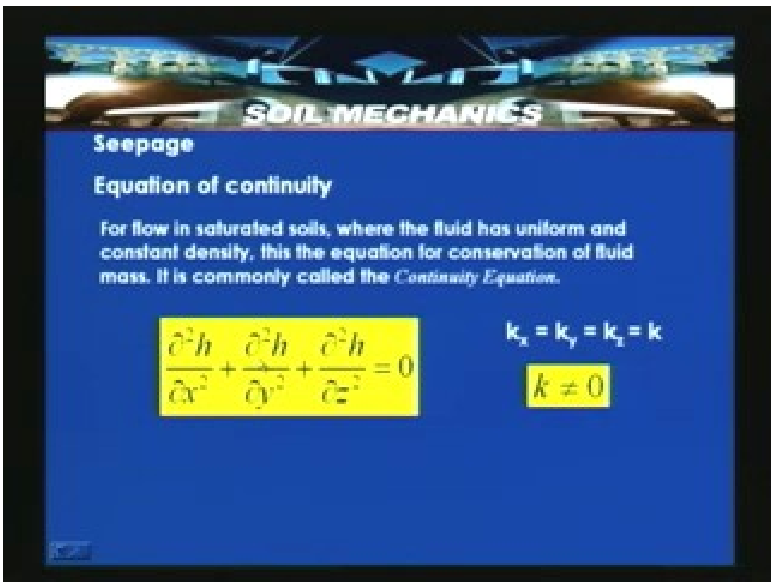
Soil Mechanics
Prof. B.V.S. Viswanathan
Department of Civil Engineering
Indian Institute of Technology, Bombay
Lecture – 25
Flow of water through soils-VI

Welcome to lecture number six of flow of water through soils. So in the previous lecture we have solved Laplace equation which is theory of continuity for computing the seepage. In this lecture we will be looking into the flow net methods. In the previous lecture we have introduced methods for solving Laplace equation of continuity. So in this lecture we will be discussing one of the methods that is flow net methods and flow net construction and some example problem.

(Refer Slide Time: 02:03)

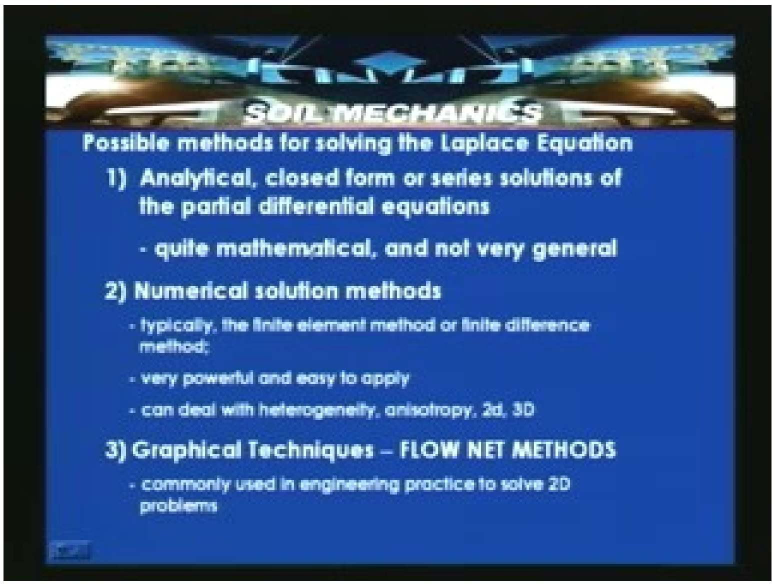


So in the equation of continuity when we define in the previous lecture, for flow in saturated soils where the fluid has uniform and constant density, this the equation of conservation of fluid mass. It is commonly called as continuity equation and this is for soils which are having identical permeability in all directions that is k_x, k_y, k_z= k. So with this dell square phi is equal to zero in terms of three dimensional flow considerations, we can write like dow square h by dow x square + dow square h by dow y square + dow square h by dow z square is equal to zero.

This also can be indicated as dell square is equal to zero as a Laplace equation of continuity and we mentioned about methods for solving this type of Laplace equation. And then we identify that the popular methods for solving the Laplace equation of continuity is the flow net method which is based on the graphical method of construction. So in this lecture we will be looking into the flow net method details and then its construction details and then some example problems for computing seepage.

So the possible methods for solving the Laplace equation are given here in this slide. We are reviewing whatever we have discussed in the previous lecture. So analytical, closed form or series of solution of the partial differential equations. So quite mathematical and not very general, that is what we have discussed. And another method which is possible for solving the Laplace equation of continuity is a numerical solution method. Typically the finite element method or finite difference method are used, very powerful and easy to apply, can deal with heterogeneity, anisotropy conditions and 3 D conditions. Then the third method which we are going to discuss today is a graphical technique that is flow net methods commonly used in engineering practice to solve two dimensional problems.

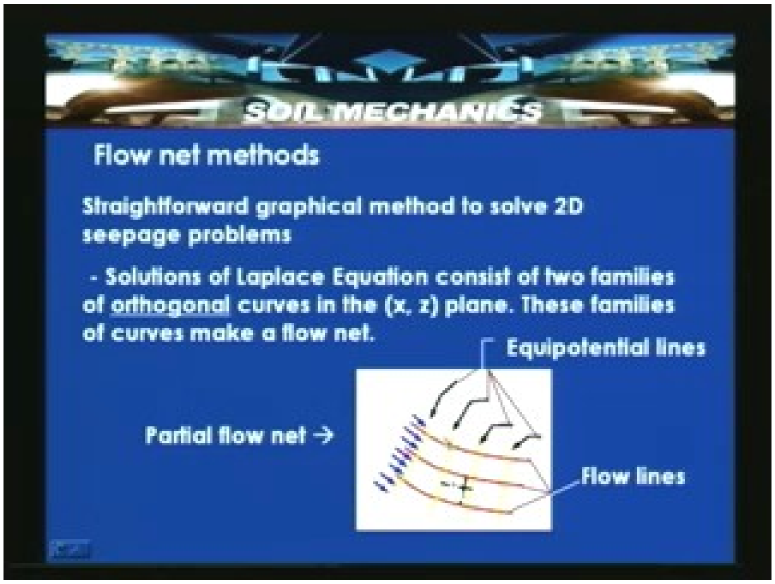
(Refer Slide Time: 02:58)



So while introducing this, we also discussed many flow types like one dimensional flow type, two dimensional flow and three dimensional flow considerations. So most of our problems for computing seepages will be adopting the two dimensional flow considerations. Basically the flow net method is based on the two dimensional flow considerations. We assume that the structure is continuing per meter length, perpendicular to the plane of the figure under consideration. So here we have discussed that the flow net is nothing but a family of two lines. One is called flow line another one is called equi-potential line. And we discussed, shown and noted that a partial flow net has got different equi-potential lines which are shown in yellow color and this red one which is the flow taking place to this, so they are called flow lines. The space between any two flow lines is called flow channel.

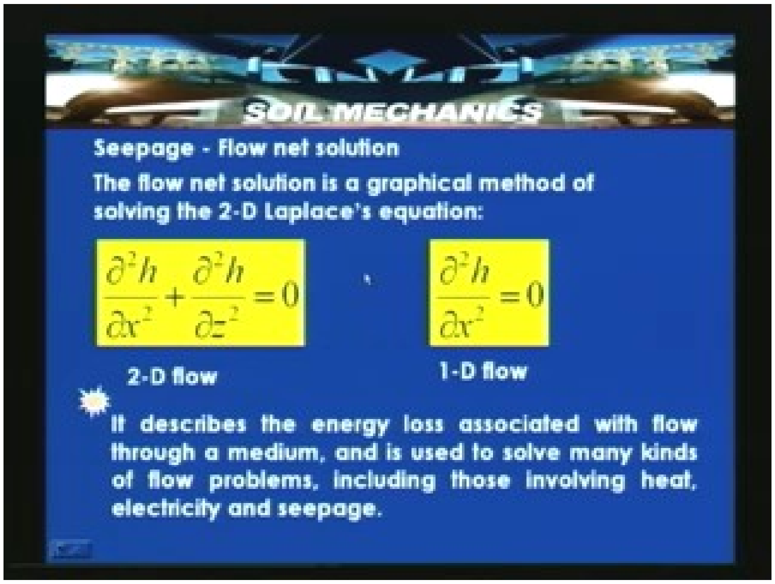
And between any two equi-potential lines a head loss will occur. The potential between here and here, the flow is occurring in the direction then there will be a loss of head. As what we mentioned earlier that flow net method is a straight forward graphical method to solve two dimensional seepage problems. So solutions of Laplace equation consist of two families of orthogonal curves in the x z plane. So x z plane in the sense x is the horizontal distance and z depth, so what we are considering is the two dimensional problems under consideration.

(Refer Slide Time: 04:52)



So these families of curves make a flow net that is what we have seen. Let us look into this seepage and flow net solution. The flow net solution is a graphical method for solving the two dimensional Laplace equation. So in the two dimensional case this Laplace equation of continuity reduces to dow square h by dow x square plus dow square h by dow z square is equal to zero. Because we are considering x z plane. So the equation of continuity is dow square h by dow x square + dow square h by dow z square is equal to zero. Incase of one dimensional flow like dow square h by dow x square is equal to zero. So if the flow is taking place only along the x direction then it is called dow square h by dow x square is equal to zero.

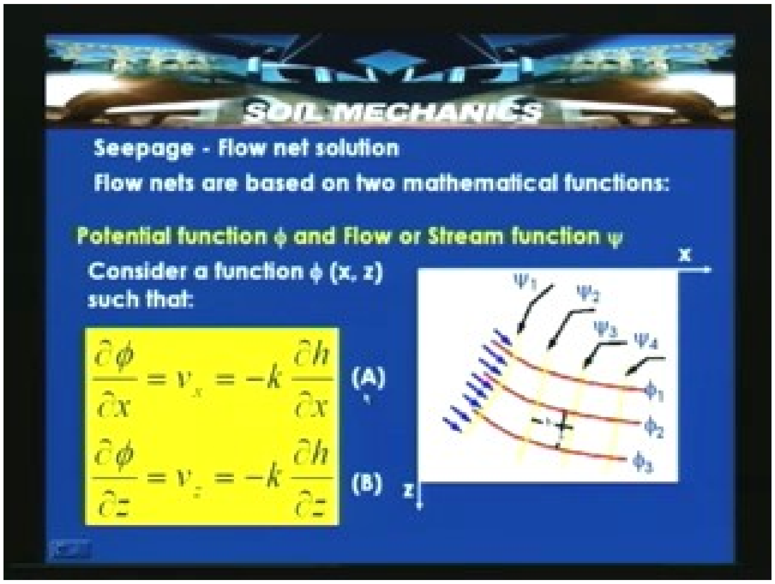
(Refer Slide Time: 05:36)



So it describes the energy loss associated with the flow through a medium and is used to solve many kinds of problems. So this particular concept is used in solving heat transfer, electricity transfer and seepage. That is from electrical transfer from one potential to another potential, again the similar concept and similar logic is used to explain this particular theory. So this flow net solution basically describes the energy loss associated with the flow through a medium and is used to solve many kinds of flow problems basically for solving flow problems including those involving the heat, electricity and seepage.

So when you come to this particular flow net, we said that the two families of curves. So one is flow lines and another one is equi-potential lines. We will come to this details of definitions later, but consider one of this equi-potential line is represented by potential functions say phi. Similarly this flow lines are represented by a stream function or flow function psi. So here flow nets are based on the two mathematical functions potential function phi and flow or stream function psi.

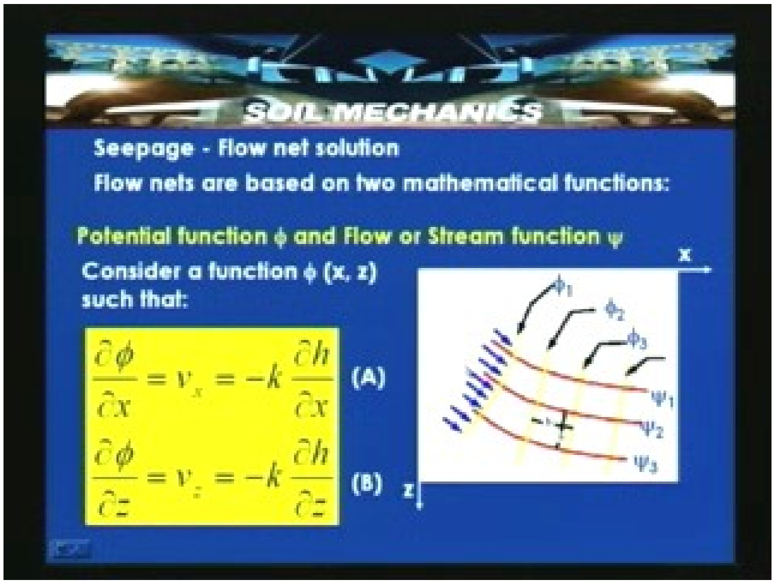
(Refer Slide Time: 07:58)



So consider a function phi (x, z) such that dow phi by dow x = v_x that is the velocity in the x direction is equal to minus k dow h by dow x. This is negative because the head loss is occurring in the direction of the flow. Similarly dow phi by dow z= v_z is equal to minus k dow h by dow z. So consider a function phi (x, y) such that dow phi by dow x is equal to v_x and dow phi by dow z is equal to v_z. What we consider is that these families of curves are represented by function called psi 1, psi 2, psi 3, psi 4 or phi 1, phi 2, phi 3. So let us consider the flow nets are based on two mathematical functions, one is potential function phi and flow or stream function psi.

So consider a function phi (x, z) such that dow phi by dow x is equal to v_x is equal to minus k dow h by dow x and dow phi by dow z is equal to v_z is equal to minus k dow h by dow z. So this is negative because the head loss is occurring in the direction of flow.

(Refer Slide Time: 09:11)

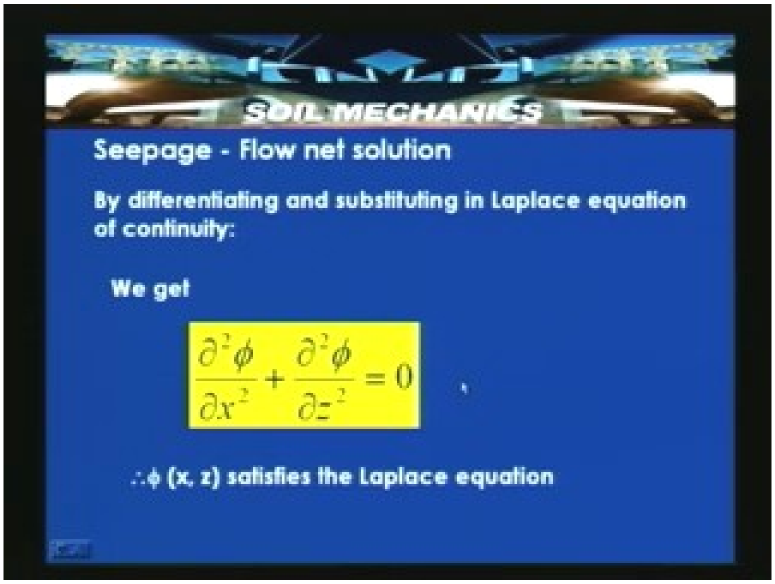


So in the slide above a part of the partial flow net is shown. The red lines indicates stream lines that is flow lines and represented by a function called psi 1, psi 2, psi 3 and this equi-potential lines which are represented by a function called phi 1, phi 2, phi 3. So the flow is occurring in this direction, the space between any two stream lines or flow line is called flow channel. So here the distance between two flow lines is A and length of the flow line is B. Ratio of A to B is defined as an aspect ratio. So here if you say phi 1, phi 2, phi 3, suppose if the flow is occurring is here the head loss is taking place. So the head loss which is occurring between any two adjacent potential lines is also called potential drop.

In a flow net, number of potential drops will be there from starting point to the ending point where the flow is ending. So in order to see whether this potential function and stream function satisfy the Laplace equation of continuity or not. And also in order to prove that how we can say that they are orthogonal. So that also we can try to derive based on the considerations. So for that consider a function phi (x, z) such that dow phi by dow x is equal to v_x is equal to minus k dow h by dow x. Let us indicate this particular equation as A and dow phi by dow z is equal to v_z is equal to minus k dow h by dow z, let us indicate this as B.

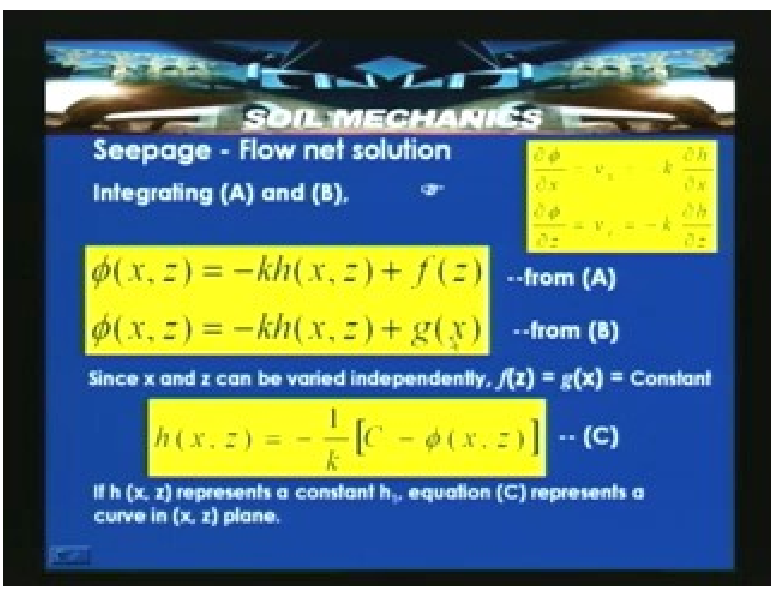
By differentiating equations A and B and substituting in Laplace equation of continuity. For two dimensional flow we have defined that Laplace equations of continuity is dow square h by dow x square plus dow square h by dow z square is equal to zero. Substituting equations A and B or after differentiating if you get dow square phi by phi x square plus dow square phi by dow z square is equal to zero. So this indicates that the potential function whatever we have considered phi (x, z) satisfies the Laplace equation of continuity.

(Refer Slide Time: 11:18)



Similarly when you integrate say A and B, previously we have introduced the A and B. So that A and B equations are represented here again, dow phi by dow x is equal to v_x is equal to minus k dow h by dow x, dow phi by dow z is equal to v_z is equal to minus k dow h by dow z. If you integrate from A we will get phi (x, z) is equal to minus kh (x, z) plus function of z and phi (x, z) is equal to minus kh (x, z) plus g(x) that is g (x) is a function of x. So by integrating this A and B, this is A and this is B we get this particular equations. Since x and z can be varied independently, function of z is equal to g(x) is equal to constant.

(Refer Slide Time: 11:57)



So now by simplifying we can write the equation C which is h (x, z) is equal to minus 1 by k into [C - phi of function of (x, z)], that is phi (x, z). So if h (x, z) represents a constant h_1, then the equation C represents a curve in (x, z) plane. So what we have deduced is that the equation of this potential line, if h(x, z) represents a constant h_1, so for that particular point this equations C represent a curve in a (x, z) plane. So what we did is that we considered dow phi by dow x is equal to v_x is equal to minus k dow h by dow x and dow phi by dow z is equal to v_z is equal to minus k dow h by dow z.

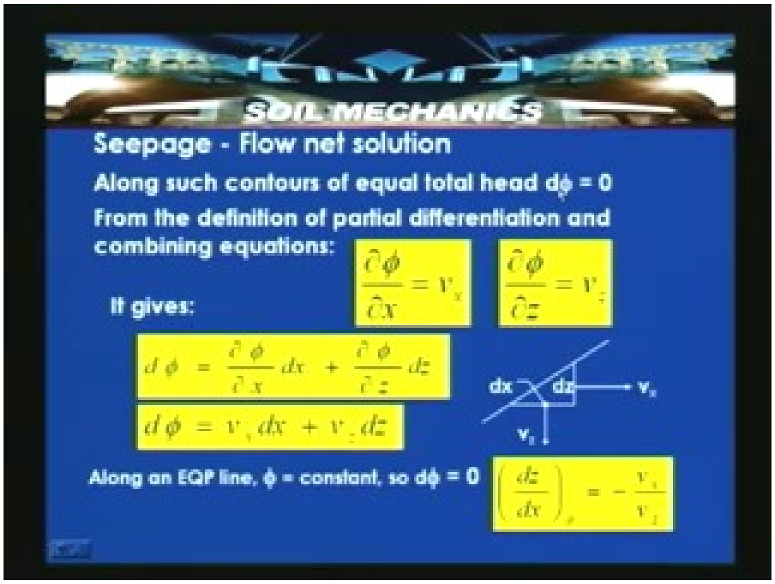
So first we tried to see whether this potential function fulfils the Laplace equation of continuity or not. Then what we did is that after fulfilling we tried to obtain whether this potential function line represents the equation that is the line or not. So for that we integrated A and B equations and then because x and z can be varied independently, we have written that function of z is equal to g(x) is equal to constant. So with that we have written h (x, z) is equal to minus 1 by k C minus phi (x, z). So here if the (x, z) represents a constant h_1 then we said that equation C represents a curve in (x, z) plane.

So equi-potential line is a curve in the cross section such that phi is constant along the curve. That is equi-potential lines is a curve drawn in the cross section of a particular structure under consideration where the flow is taking place in a two dimensional plane. So equi-potential line is a curve in the cross section such that phi is constant along that curve. That means the potential will be constant. Suppose if it has got a head h_1, the h_1 will be a constant along that particular potential line. So total head is constant along that equi-potential line. That is what it says that total head is constant along that equi-potential line. So thus the family of curves is similar to the contour lines on a topographical map.

(Refer Slide Time: 14:26)

So if you compare these potential lines, these families of curves of potential lines are analogous to contour lines in a topographical map. But only thing is that except that it is drawn in a vertical section and the lines represent equal heads (not equal elevations). Incase of contour lines it represents equal elevations and it is drawn in a plan, but in this case of potential lines it is drawn in a cross section and it represents equal heads.

(Refer Slide Time: 15:27)

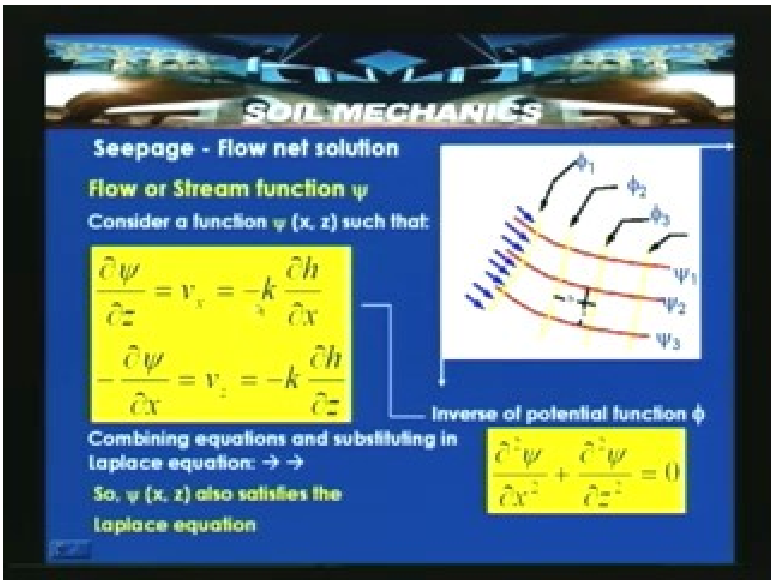


So we will try to determine the slope of the potential lines. Whenever we have the contours of equal total head, we can say that d phi is equal to zero. That is the change in potential head along particular potential lines is zero that is d phi is equal to zero. So from the definition of partial differentiation and combining equations dow phi by dow x is equal to v_x and dow phi by dow z is equal to v_z, it gives d phi is equal to dow phi by dow x dx + dow phi by dow z dz. Now substituting for dow phi by dow x =v_x and dow phi by dow z= v_z, we can write d phi is equal to v_x dx + v_z dz. Along the equi-potential line d phi is equal to zero, so equating d phi is equal to zero here, we can write dz by dx of potential line is equal to minus v_x by v_z that is minus v_x by v_z . So v_x is the velocity in the x direction where dz is the depth and perpendicular to its plane the length is one meter.

So the flow area over which that v_x is taking place that is dz into one. Similarly v_z which is erupting out of x that is dx, so perpendicular to that area is say one unit. So dx into one is the area over which the flow is occurring. So v_z and v_x are the velocities in x direction and z direction, so the slope which is given by dz by dx that is the potential as minus v_x by v_z. That is dz and dx are the small lengths of the element under consideration. So what we try to do is that along such contour of equi-potential lines, we have defined equi-potential lines and we said that where the potential is constant in the particular equi-potential line. That is total head is constant so in that case d phi is equal to zero on an equi potential line. So from the definition of partial differentiation and combining equation dow phi by dow x is equal to v_x, dow phi by dow z is equal to v_z we obtain d phi = v_x d x + v_z dz.

So along an equi-potential line phi is equal to constant, so d phi is equal to zero. With that what we did is that we determined a slope of an equi-potential line that is dz by dx is equal to phi that is potential function is equal to minus v_x by v_z. Similarly having seen potential function, let us consider whether this stream function or a flow function psi satisfies Laplace equation or not. So for that consider a function phi (x, z) such that dow psi by dow z = v_x = minus k dow h by dow x, minus dow psi by dow x is equal to v_z is equal to minus k dow h by dow z. Note that this psi is a inverse of potential function.

(Refer Slide Time: 18:33)

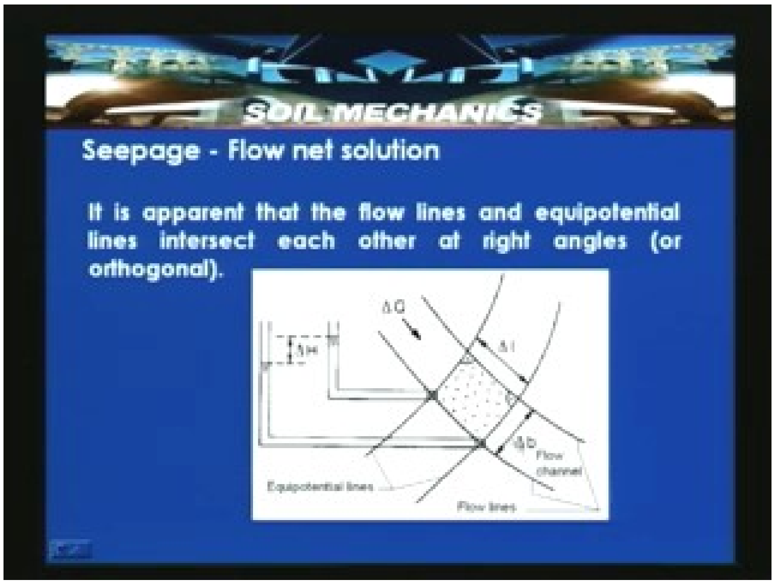


So consider a stream function psi (x, z) such that dow psi by dow z is equal to v_x is equal to minus k dow h by dow x and minus dow psi by dow x is equal to v_z is equal to minus k dow h by dow z. So combining equations and substituting in the Laplace equation again we get dow square psi by dow x square plus dow square psi by dow z square is equal to zero. So with that we can also say the stream function also fulfills the Laplace equation of continuity. So psi (x, z) also satisfies the Laplace equation. That is what we are saying here so that psi (x, z) also satisfies the Laplace equation.

So from the definition of the partial differentiation and combining equations minus dow psi by dow x is equal to v_z and dow psi by dow z is equal to $v_{x,}$ it gives total differential if you determine for a psi (x, z) d psi is equal to dow psi by dow x into dx plus dow psi by dow z into dz. So by substituting for minus dow psi by dow x is equal to v_z, dow psi by dow z is equal to v_x we can write the total differential of psi (x, z) as d psi is equal to minus v_z dx plus v_x dz. For a given flow line if psi is constant then d psi is equal to zero. So equating this equation what we deduced for total differential of the psi (x, z) to zero, we will get d psi is equal to zero is equal to minus v_z dx plus v_x dz. So with that what we get is that the slope of a stream function dz by dx of psi that is the stream function is equal to v_z by v_x. Now we can see that this is the stream line or flow line, so v_x is the velocity in x direction and then v_z is the velocity in the z direction.

So with this, what we can say is that if you note down the dz by dx of phi is what we obtained as minus v_x by v_z and dz by dx of psi we obtained as v_z by v_x. So they appear as orthogonal. So with this deliberation it is apparent that the flow lines and equi-potential lines intersect at angles to each other. So from the deliberation or whatever we have deduced from the theoretical considerations we can say that the flow lines and equi-potential lines intersect each other orthogonally.

(Refer Slide Time: 21:33)



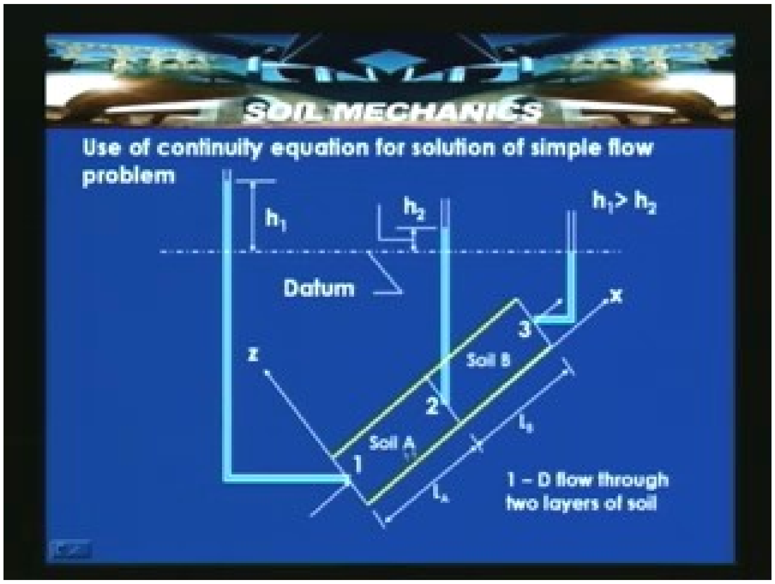
So here in this slide this is a streamline (Refer Slide Time: 21:24) which is shown here and delta B is the distance between two stream lines or flow lines. The distance between any two flow lines be described as flow channel. So this is the flow channel which is indicated, delta Q is the small flow occurring through this flow channel and delta L is the distance that is the length over which the flow is taking place. That is distance between any two equi-potential lines. So if you consider this is an equi-potential line (Refer Slide Time: 22:00) and this is equi-potential line, so the total head along these two points is zero. So if I consider a total head at this point it will be somewhere here (Refer Slide Time: 22:11) with respect to certain datum.

Similarly if I take an equi-potential line that is if I tap and measure the total head we get the same total head. That indicates the head total head is constant along this. But if you come down here because the flow is taking place, so that particular energy is transferred in the form of a frictional drag to soil grains. So when the flow is taking place through soil the head loss has to occur. So here the drop which is taking place for a distance that is delta L flow to take place is around delta h. You can also say that the hydraulic gradient is delta h by delta L, for steady state seepage conditions the i is equal to constant. So here if you see this is the total head for this equi-potential line and this is another equi-potential line having a loss because this experienced a loss where the total head is less than delta h.

That means delta h is the head loss taken place between these two equipotential lines. So if you note down from the theoretical deliberation that the angles where they both meet that is the flow lines and equi-potential lines are orthogonal. So this particular aspect that is orthogonality of flow lines and equipotential lines have to be kept in mind while constructing flow nets. So we will be using that concept and then we will be considering the different types of structure with confined seepage conditions and unconfined seepage conditions. Confined seepage conditions are basically with sheet pile walls or a mesentery structure embedded in permeable soil medium. And incase of unconfined seepage conditions a flow through an earth dam can be considered as an uncombined seepage conditions.

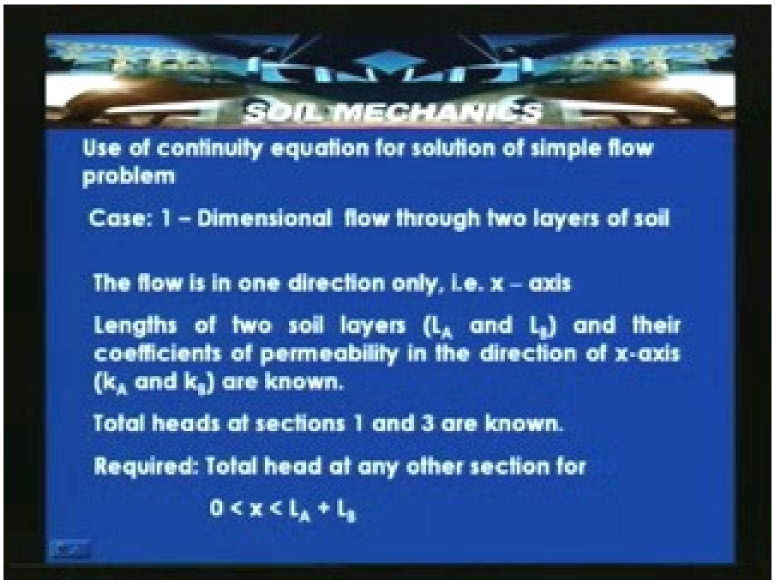
So if you take these two considerations we can estimate the flow which is taking place through an entire medium then we can estimate the total discharge. With this we can also determine another benefits like the safety against some uplifts and safety against some piping and these aspects can be considered into this. So based on this we will be able to design and arrive at the remedial measures. Having derived this potential functions and stream function, let us try to apply this continuity equation for the solution of simple flow problems. Let us assume that we have got a container whose total length is $L = L_A + L_B$ and two soils are filled in this container. Soil A is having a length L_A, so soil A is between 1 to 2 points and from 2 to 3 points soil B is there.

(Refer Slide Time: 25:26)



So it is assumed in a frictionless medium and then the flow is taking place in one direction. That is the x direction is here which is defined and z direction is here. So the flow which is actually taking place or confined between two container walls. The flow is taking place along the x direction, so this is one dimensional consideration. So if you consider at point 1 the total head is h_1, at point 2 the total head is h_2 which is to be determined and here already the head loss has occurred (Refer Slide Time: 25:48). That means the total head loss which is driving the flow from 1, 2, 3 that is the total head which is driving the flow is around h_1. So this one dimensional flow through two layers of soil and after having deduced the expression for two layers of soil we can apply easily for one dimensional flow through a single layer soil. So h_1 is greater than h_2 that is actually required to be noted because naturally when the flow is taking place the head loss will occur, so the h_2 will be less than h_1.

(Refer Slide Time: 26:49)

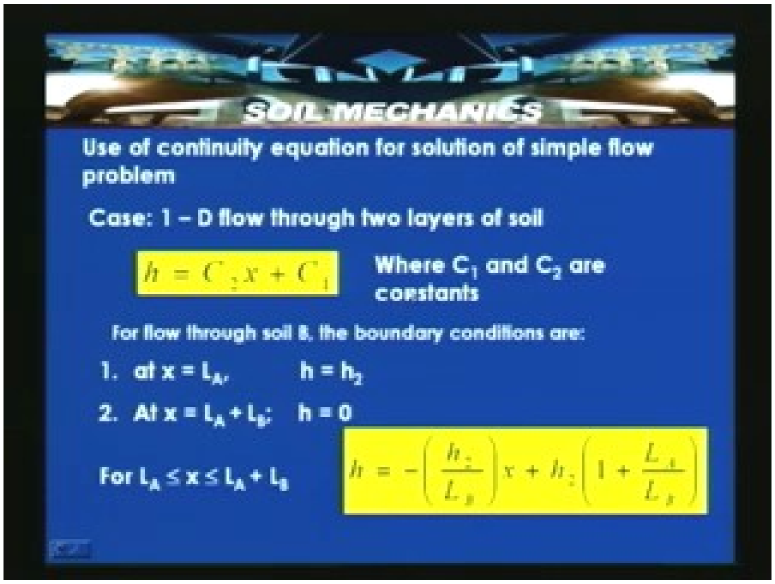


Let us look into the use of continuity equation for solution of simple flow problems. So case 1 is one dimensional flow through two layers of soil. The flow is in one direction only because it is in x-axis that is x-axis. The lengths of the two soil layers L_A, L_B and their coefficient of permeability in the direction of x axis are known. That is lengths are known and Darcy's coefficient of permeabilities K_A and K_B are known to us. And total heads at section 1 and 3 are known, that means we do not know the h_2 in between but we can determine by using this particular deliberation which we are going to discuss. But total head at 1 and total head at point 3 which is shown in the figure in the previous slide can be noted. So total head at section 1 and 3 is known to us. So required now is the total head at any other section for distance between zero and $L_A + L_B$.

Let us consider for one dimensional flow through two layers of soil, the integration of the laplace equation in one dimensional flow consideration that is dow square h by dow x square is equal to zero. Because that is the equation for Laplace equation of continuity for one dimensional flow. So with that we can say that by integration of that we will get $h = C_2 x + C_1$ where C_1 and C_2 are constant, x is the distance along the x axis. So for the flow through soil A the boundary conditions if you wanted to dictate then at $x = 0$, $h = h_1$. That is when $x = 0$ at point 1, $h = h_1$ the total head is h_1 that is the commencement of the flow is taking place at point 1 and at x is equal to L_A, $h = h_2$ where h_2 is unknown. So by solving, now substituting in $h_2 = C_2 x + C_1$ where $x=0$ then we can say that $C_1 = h$. Similarly substituting for $h = h_2$ and $x = L_A$ and solving and simplifying we will get h= - (h_1- h_2 by L_A) into x+ h_1, which is valid for distance between zero and L_A which is valid for zero and

$L_{A=.}$ We did not go to the soil B, we are still in the soil A which is having a length completely L_A.

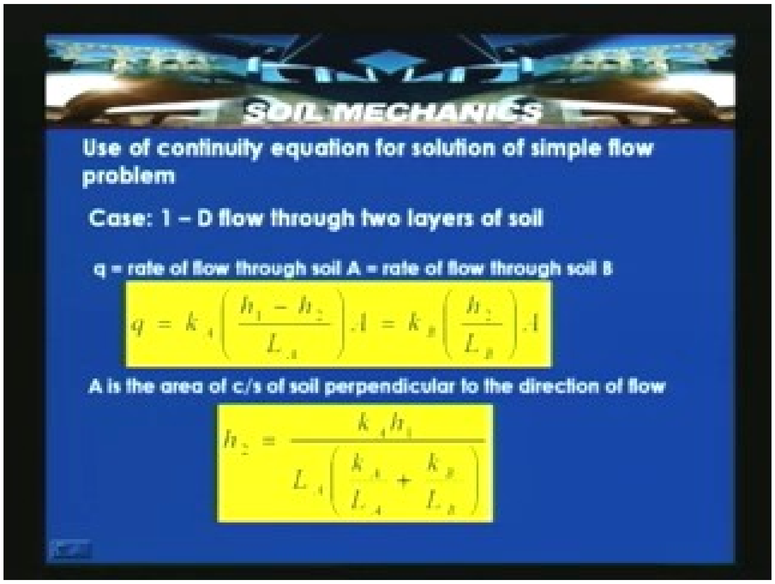
(Refer Slide Time: 29:01)



So with this the head at any point can be determined in terms of say h_1, h_2, L_A, x and h_1. Similarly consider the flow which is taking place from point 2 to 3 that is in the region of the soil B. So in the previous slide we have shown a figure having 2 points with the 2 entry which is again shown here. So this is soil B where the flow is taking place between 2 and 3 that is L_B. Let us now consider the boundary conditions, again the flow is taking place in one dimensional direction. So the dow square h by dow x square is equal to zero and integrating we get $h = C_2 x + C_1$, for flow through soil B the boundary conditions are at $x = L_A$, $h = h_2$. At the end of $x = L_A$, the head is $h = h_2$ and x is equal to $L_A + L_B$ that is from left hand side to right hand side that h = 0. So now substituting this and solving for boundary conditions we will get, for x ranging from L_A to $L_A + L_B$ we get h = - (h_2 by L_B) x + h_2 (1+ L_A by L_B). So this is from the region, flow through soil B, so this is valid from distance L_A to $L_A + L_B$. That is the right hand side of the container end under consideration.

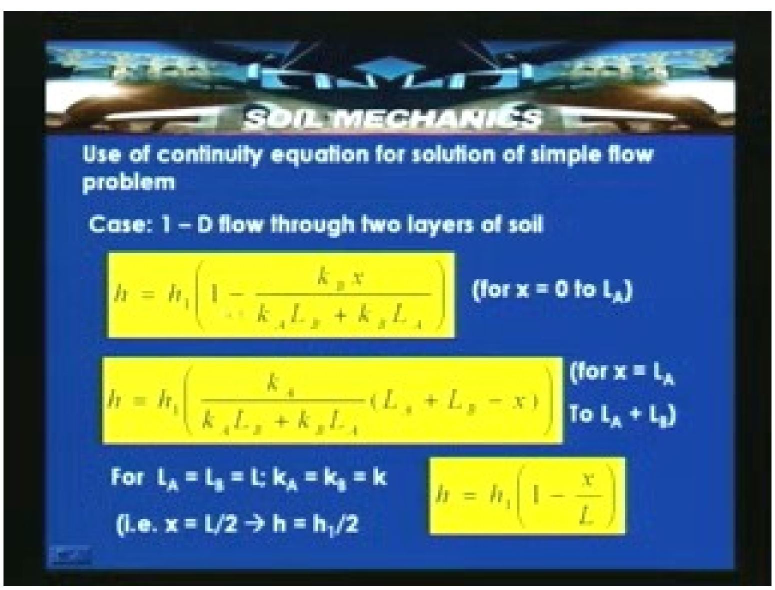
Let us consider q is the rate of the flow of the soil through area of cross section A. A is perpendicular, that is x along the length and perpendicular to the plane of the figure if the unit is one. The area of cross section is indicated as A. The rate of flow through soil B is equal to rate of flow through soil A, that has to happen. So q = k_A (h_1 - h_2 by L_A) A = k_B (h_2 by L_B) A. So because at point 2 the head which is available is h_2, at point 1 head available is h_1. So the total loss which is occurring between point 1 and 2 is h_1- h_2. So the hydraulic gradient in this case is h_1 minus h_2 by L_A and k_A is the coefficient of permeability for soil A in the x direction and A is the area of cross section. Similarly k_B is the coefficient of permeability in the x direction for soil B and h_2 is the head for the loss which is taking place from point 2 to point 3. So h_2 by L_B, L_B is the length of the soil B and A is the area of the cross section.

(Refer Slide Time: 32:00)



So A is the area of the cross section of soil perpendicular to the direction of flow. So by equating these two and solving we get h_2= k_A h_1 divided by L_A (k_A by L_A + k_B by L_B). With this now if you simplify and substitute in the previous expression what we derived we get one dimensional flow through two layers of soils, for x = 0 to L_A we get expression like h= h_1 (1- k_B x by k_A L_B + k_B L_A). Similarly for x =L_A to L_A + L_B that is h is equal to h_1[k_A by k_A L_B + k_B L_A (L_A + L_B –x)]. So if you consider this is an expression for two soil layers that is two soil types A and B having lengths L_A and L_B placed in a container having the cross section area A where the flow is occurring from point 1 to point 3.

(Refer Slide Time: 32:26)

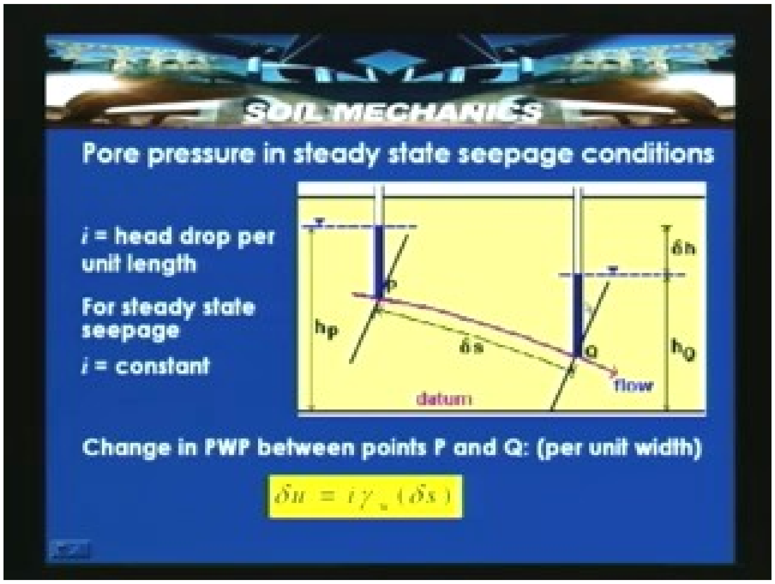


So we tried to determine by using the continuity equation the head at any point. So the same concept can be applied because we have discussed in the beginning like when the flow is taking place over a length L say let us assume that the same container of L_A and L_B the length is say L and if it is having a one type of soil then $L_A = L_B = L$ and $k_A = k_B = k$. At x = L by 2 that is x = 0 that is $h = h_1$. The total head is remaining there, because the no loss has taken place because the flow is about to commence. At x = L once the flow has taken place that is the point where the flow has already taken place that is at the end of the length of the flow that is x= L. Then in that case h= h_1 into 1- 1 that is zero. So the total head available is zero for the flow to take place, that means the flow has already taken place the total head has already been reduced to zero.

For in between points say x= L by 2, we can determine by using this expression h = h_1 into 1- x by L. So this is reduced by substituting in this expressions and then we got this expression h= h_1 into 1- x by L. So if you say that x= L by 2 that is once the 50 % length of the flow, then we can say that the total head already lost is 50 %. That means h_1 is reduced to around h_1 by 2. Similarly if you wanted to say at one quarter distance that the head is again 25 % of the head loss will occur, say three fourth distance of the length L then 75 % of the head loss would have occurred. At the end of the length of flow 100 % head loss would have taken place. So that is what we discussed and then we used the continuity equation to deduce these expressions.

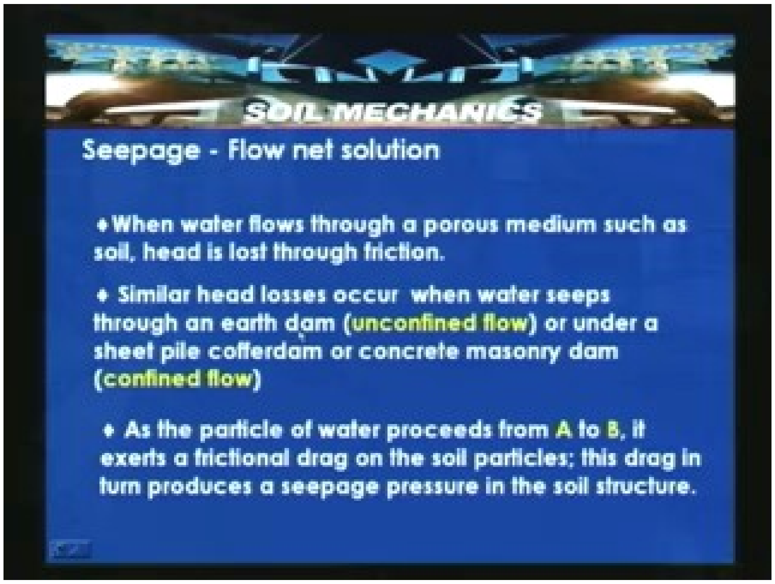
Before explaining the problems relating to flow net construction and then the methodologies for the flow net construction, we should also try to remember the pore pressure variation in a steady state seepage conditions. Incase of a steady state seepage conditions the hydraulic gradient is constant. So consider in a permeable medium where the flow is taking place, p and q is a flow line where the flow is taking place in this direction assume that this is a potential line here and potential lines which is approximately drawn which is orthogonal here and delta s is the length of the flow line over which the flow is taking place. So delta h is the head loss which is taking place or a drop which occurred between potential lines passing through point p and q.

(Refer Slide Time: 35:56)



So that is delta h by delta s which is nothing but a hydraulic gradient. So here we can say that for steady state seepage conditions i is equal to constant. So change in pore water pressure between two points P and Q per unit width, we can write as delta u is equal to i gamma w into delta s into 1. That is per meter length of the cross section say one then we can say that the small change in pore water pressure delta u is equal to i which is hydraulic gradient that is constant, then gamma w is a unit weight of water and then delta s is the distance between these two flow lines. So pore pressure in the steady state flow conditions which is defined here. So when the water flows through a porous medium such as soil, head is lost through friction. So if you recollect again when water flows through a soil medium or a porous medium head is lost through friction.

(Refer Slide Time: 37:18)

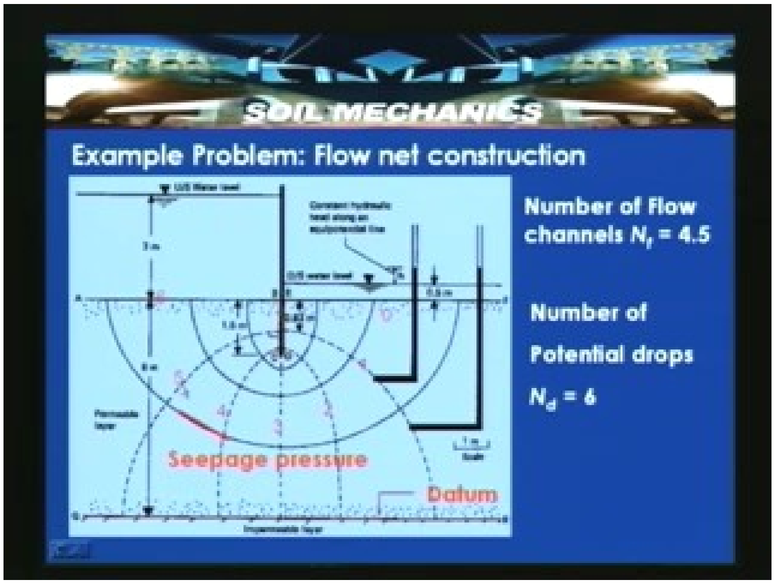


So similar head losses occur when water seeps through an earth dam. That is example for an earth dam we will discuss that in unconfined flow conditions or example for unconfined flow we can say is an earth dam. And under a sheet pile cofferdam or a concrete masonry dam the confined flow conditions can prevail. So as the particle of water proceeds from A to B, so A to B are the two points over which the flow is taking place with some potential difference, then it exerts a frictional drag on the soil particles and this drag in turn produces a seepage pressure in the soil structure. So this seepage force always acts in the direction of a flow that is along the flow lines. So as the particle of water proceeds from A to B it exerts the frictional drag on the soil particles, so this drag in turn produces a seepage pressure in the soil structure.

So let us try to consider and apply this knowledge on flow nets to construct the flow net construction. As indicated a confined flow condition which is a sheet pile wall problem is shown here. So let us assume that we are having a line of sheet pile walls which are driven into a soil to maintain upstream water levels and difference between upstream water level and down stream water level. And if particularly this sheet pile wall which is driven into the ground and which is perpendicular to the plane of the figure, the length continues.

Then the flow takes place from higher head to lower head, so in this case the upstream water level and downstream water level is the one. So that is the difference between upstream water level and downstream water level. The head loss which is the difference between upstream water level and downstream water level is somewhere around 2.5 meters. So if you see the first and foremost thing in constructing the flow nets, one has to draw this cross section to a scale on a graph paper. And another assumption is that whatever we are discussing in this lecture they are isotropic condition, where k_x is equal to k_z is equal to k. That is how we can do this problem incase of anisotropic conditions like k_x is not equivalent to k_z.

(Refer Slide Time: 42:40)



Generally we discussed that k_x is greater than k_z that is the permeability in x direction is more than z direction, the reasons we have discussed horizontal direction and vertical direction. So here in this case k_x is equal to k_z is equal to k. So the k of the medium is say known then by using constructing flow nets how we can estimate the discharge. So for that the methodology has to be build and then we should have a sufficient knowledge about constructing a flow net. So here if you consider A B, A B is represented as an equi-potential line, if certain datum is there then that is the head which is available. Then E F is again potential line and B C D E is the first flow line. So this is the second flow line which is drawn here and third flow line and fourth flow line and the fifth flow line is along the impermeable barrier (Refer Slide Time: 40:40).

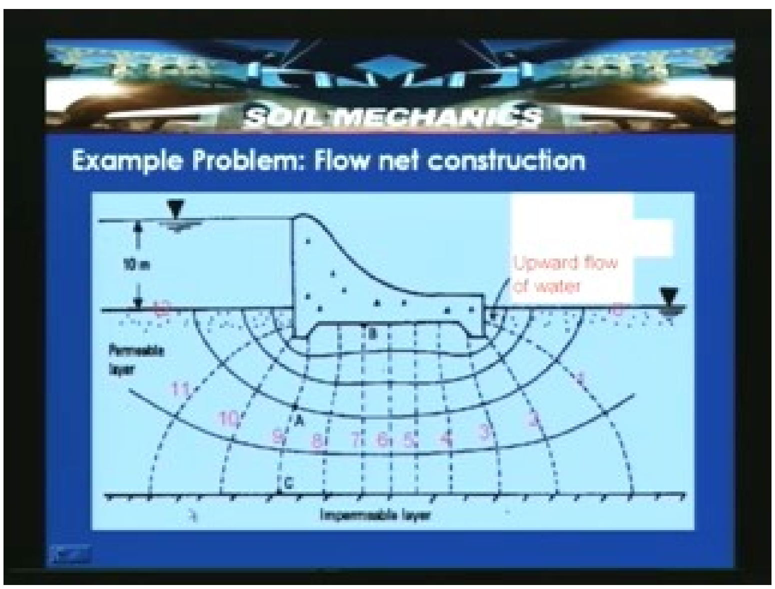
So here the flow nets are constructed approximately by fulfilling that string functions or flow lines and equi-potential lines or orthogonal. So here if you insert a circle you should be able to do it. So it is not necessary that if you are inserting a circle all should be having equal diameters but they should be able to insert approximately equal circles. So that in the curve linear square can be obtained. So these are the potential lines, generally it is numbered like this. So here this is zeroeth potential line, first potential line, second, third, four, five, six. So we will say that, this flow net has got number of potential drops. The total numbers of potential drops are six.

So six potential drops have come that is 1, 2, 3, 4, 5 and 6 so 6 potential drops have come here. So this particular methodology or a notation is very useful. Here the head loss is already taken place. That means there is no head which is available here. The head total head which is available that is 2.5 meter is available for the flow to take place. So at this particular point if I say that 6 by 6 into the total head that is a 2.5 meter head is there. So here 5 by 6 into 2.5 meter is there, that means when the flow is taking place from point 6 to point 5 the drop which has occurred that is nothing but 6 by 6 into 2.5 minus 5 by 6 into 2.5. So that is the drop which is taking place. Similarly between 5 and 4, 4 and 3, 2 and 1, 1 and 0. Finally we will see that the total head is consumed.

This is the first flow channel and being a flow line the water creeps along the boundaries. So you can see that this is a first flow line and then equi-potential line is constructed dropping perpendicular from here seeing that the curve linear squares are achieved (Refer Slide Time: 43:20). Then 1, 2, 3, 4 so some approximation is involved in estimating the number of flow channels. The number of flow channels is approximated for this case as 4.5 where it can lead to some errors. But the number of flow channels can be approximated in this example as 4.5 and number of potential drops as 6. So in the direction of the flow the flow is taking place from A to say B, if it is B here and if it is A here the flow which is actually taking place and seepage pressure is actually acting along the along the flow lines.

So the head loss which is occurring for example suppose if you consider equi-potential line 5, the potential that is total head at any point along this particular line will be same. But the drop between 5 and 4 occurs because as the flow is taking place that is distance between 5 and 4 then it reduces to a drop over a length that is the distance between the flow which is taking place. This is an example problem for the flow net construction for a masonry dam where this is the upstream water level and this is the masonry dam which is resting here, retaining this water and maintaining this water level.

(Refer Slide Time: 45:05)



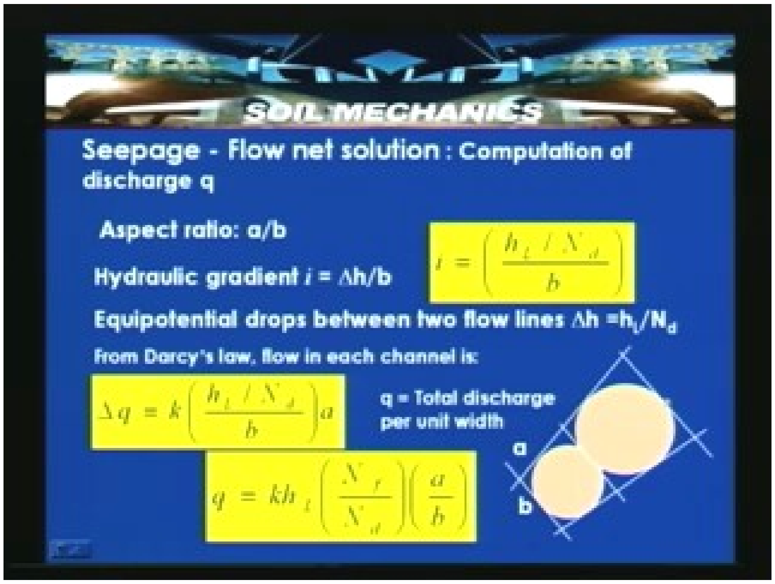
Because of the permeable layer the flow takes place like this. In this plane if this happen to b x-axis and this apposite to be z-axis then this is x-z plane and again the two dimensional flow can take place. So this is an example of confined flow with a masonry dam rushing on a permeable soil medium. So below there is an impermeable soil layer, the flow line can take place along this boundary. So we will be able to get a flow line passing through this particular boundary. So flow channel one, flow channel two, flow channel three, flow channel four and flow channel five, so 1, 2, 3, 4, 5. This being an equi-potential line and this being an equi-potential line and this being a flow line where at this junction here the orthogonality is again maintained (Refer Slide Time: 46:55). By maintaining the orthogonality approximately to 90 degrees, 11 has been constructed 10 has been constructed.

Similarly here this entire issue that you have to ensure is that the orthogonality between equi-potential lines and flow lines is maintained through out. So this is the direction of the flow in this two dimensional plane which is happening. So we can determine now 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. So total potential drops are 12. That means if you number here so first drop, second drop, third, four, five. So if you want to determine the total head at this particular point then it is nothing but 10 by 12 that is the tenth equi-potential line out of where the total potential drops are 12. So 10 by 12 into the head loss which is occurring from upstream to downstream is 10 meters. So 10 by 12 into 10 meters. Here if you see 12 by 12 into 10 meters that means the total head which is available here for the flow to take place is around 10 meters. By the time when it comes to 12 to 11 meters a head drop which occurs that is nothing but 12 by 12 into 10 – 11 by 12 into 10. So that is the head loss which is occurring over a length.

So when the flow is actually taking place the effective stress increases, when the flow which is taking place at this particular point the effective stress decreases. So that actually creates upward flow of water particularly at the tips of this structures can create piping failure. Then we have got a method for estimating this particular consideration and then stabilizing these particular problems. So in the flow net solution if you look into the computation of the discharge q, we said that aspect ratio which is defined as a and b. So these are the flow lines which are shown and these are the circles which are inscribed in such a way that the curve linear squares are obtained. And these are the flow lines and these are the two potential lines which are orthogonal and hydraulic gradient i is delta h by b. So delta h is the head loss occurring between these two points.

So we can say that b is that length over which the flow is taking place. So i= h_L by N_d. So h_L is the total by N_d that is number of potential drops. So the potential drops between two potential lines can be determined by total head loss divided by the total number of potential drops. N_d is the total number of potential drops, so that we can say the delta h. So we can write hydraulic gradient as i is equal to delta h by B, so being delta h = h_L by N_d we can write i= h_L by N_d by B. Equi-potential drops between two flow lines is delta h is equal to h_L by N_d that is what we discussed. So from the Darcy's law the flow in each channel is delta q is equal to kia. So here we can say k into h_L by N_d by b that is nothing but hydraulic gradient, a is the area of cross section that is the flow which is taking place.

(Refer Slide Time: 48:12)



$$i = \left(\frac{h_L / N_d}{b}\right)$$

$$\Delta q = k\left(\frac{h_L / N_d}{b}\right)a$$

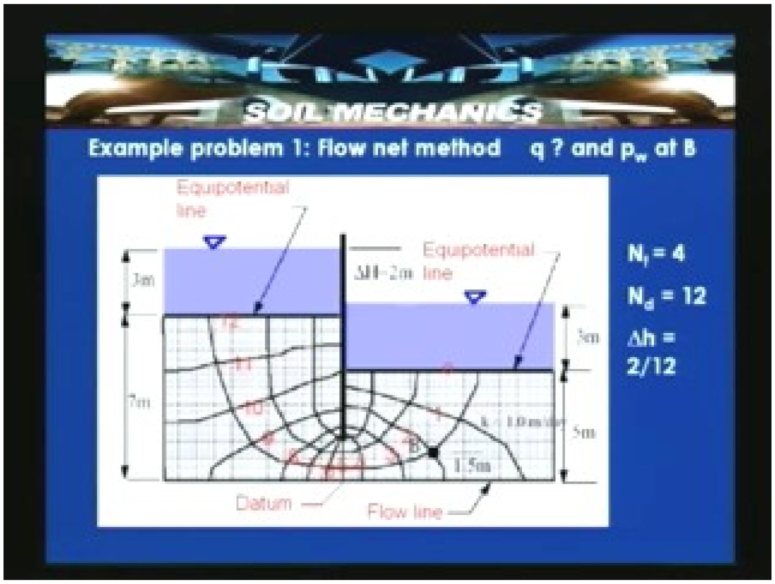
$$q = kh_L\left(\frac{N_f}{N_d}\right)\left(\frac{a}{b}\right)$$

So a into one is the area of cross section. This is the small flow which is taking place through a channel between two flows in a flow channel (Refer Slide Time: 49:35). So now when you have got n number of flow channels that is total number of flow channels if N_f happens to be the total number of flow channels. So what we are trying to do is that while computing the discharge we are trying to determine for the one single channel and then we are trying to see that what will happen for enough channels. That is the total number of channels. So that we can get q = kh_L (N_f by N_d) (a by b). So generally it is seen that this aspect ratio is maintained as 1, a by b is maintained as 1.

So in this case N_f by N_d that is number of flow channels to total number of potential drops. This N_f by N_d ratio is also defined as a shape factors of a flow net and a by b is defined as a aspect ratio of a flow net. Generally it is seen that a by b is equal to one. So in that case q=kh_L into N_f by N_d is the expression for an isotropic soil having identical permeability in x-axis and z-axis. So in that case q = k h_L into N_f by N_d into a by b where a by b is happened to be one then we can write q = kh_L into N_f by N_d. So having discussed for isotropic conditions, let us try to apply this knowledge for constructing a flow net for this type of problem which is shown here.

Let us assume that we are having a confined flow with the help of a sheet pile wall cofferdam and this upstream flow and downstream flow is separated by a head difference that is two meters which is the head loss occurring between upstream side to the downstream side. If the structure is drawn onto a graph paper which is represented and 7 meter is the depth and 3 meter here and three here and it is embedded into the ground. Then now if you draw the flow net, so in constructing flow nets first the approximate flow net has to be constructed then it has to be seen that the flow net will not intersect the impervious boundaries. So that is one thing that you can not draw a flow net which is cutting impervious boundary.

(Refer Slide Time: 51:17)



So here the first flow line is drawn and second flow line, third flow line. So it is seen that the number of flow channels are restricted to 5, 6, 7 something like that, approximately it is a thump rule which is required to be followed. And the potential drops have to be numbered against the flow direction. So if you see here 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. So here if you say that the total head available for the flow is 2 meters. And 0 by 12 into 2 meters that means at this particular point the equi-potential lines with head is zero that is the total head is zero.

We can say that number of flow channels, so 1, 2, 3 and 4 this is approximated as a 4 channels and number of potential drops that is 12. So 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, so total numbers of potential drops are 12. So delta h is equal to that is the drop which is occurring that is h_L twelve that is 2 by 12 meters. So we can even determine by knowing the elevation at this particular point and by using total head is equal to pressure head plus elevation head. So once you draw on a graph paper with this scale then by knowing the elevations we will able to estimate the pore water pressures at any point.

And so with that we will able to estimate the uplift trust exerted on a particular structure and also can be used to calculate the stability against the pipe and other failures. So in this lecture what we try to understand is the application of the Laplace equation in the sense solution of the Laplace equation of continuity that is nothing but the flow net method which is a graphical method. And then we tried to show that the two functions that is potential function and stream function or a flow function. They are orthogonal and also we discussed that they fulfill the Laplace equation of continuity in x z plane.

So based on that we discussed some problems for what is the procedure which are required to be followed for constructing a flow net. So flow net we have described and then we tried to use this concept of flow net for isotropic soils where $k_x = k_z = k$ in two dimensional consideration for determining the discharge which is taking place in a particular structure, basically what we have seen today under confined seepage conditions. So in the next lecture what we will be discussing is that how we can do for anisotropic conditions and with different soils and how this particular concept can be used for calculating stability against uplift and other considerations.