Soil Mechanics

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Lecture – 24

Flow of water through soils-V

Welcome to lecture five of flow of water through soils. In the previous lecture we have learned in detail about factors affecting permeability of soils. In this lecture we will be looking into flow of water through stratified soil deposits and we also looked into the theoretical aspects in deriving or in computing seepage problems particularly by using laplace equation of continuity. Let us look at the review. In the previous classes we have started fluid flow through soils and then we have also discussed methods for determining the coefficient of permeability of soils. Then we said that the two methods one is constant head permeameter test other one is falling head permeameter test which are there in the laboratory. And we discussed the methods, merits and demerits of each methods and their suitability to a particular type of soils.

In this lecture as informed earlier we will be looking of the permeability of stratified soils and equation of continuity. So if you recall the list of the factors affecting permeability were listed around 7 factors. We said that shape and size of soil particles are an influencing factor and void ratio and k increases with increase in void ratio, also the degree of saturation we said that k increases with increase in the degree of saturation.

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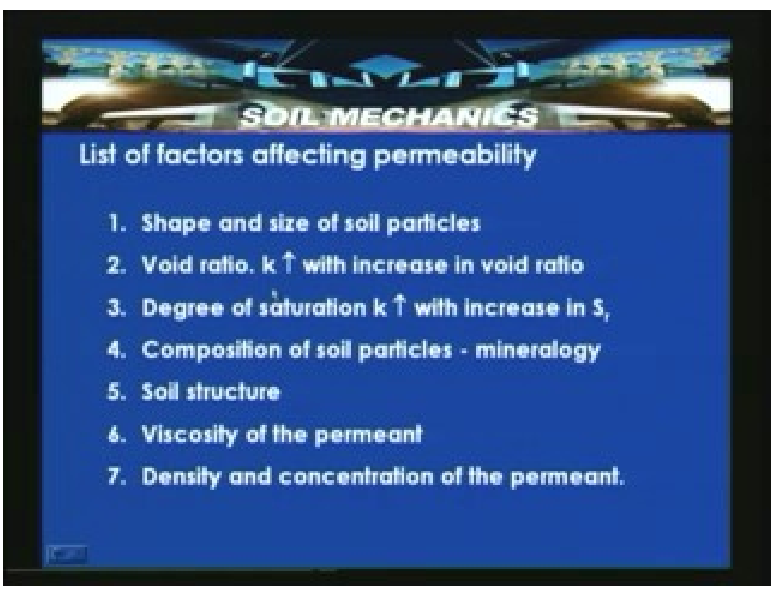

In the composition of soil particles we said that the mineralogy also place a great role in influencing the coefficient of permeability for fine grain soils. Soil structure that is whether it has flocculent structure or dispersed structure, the effect of soil structure also

we have discussed. Then viscosity of the permeant, density and concentration of the permeant are the other factors affecting the coefficient of permeability.
Based on this permeability value which is computed, the soil can be classified. This classification of soils according to their coefficient of permeability can be carried out as follows.

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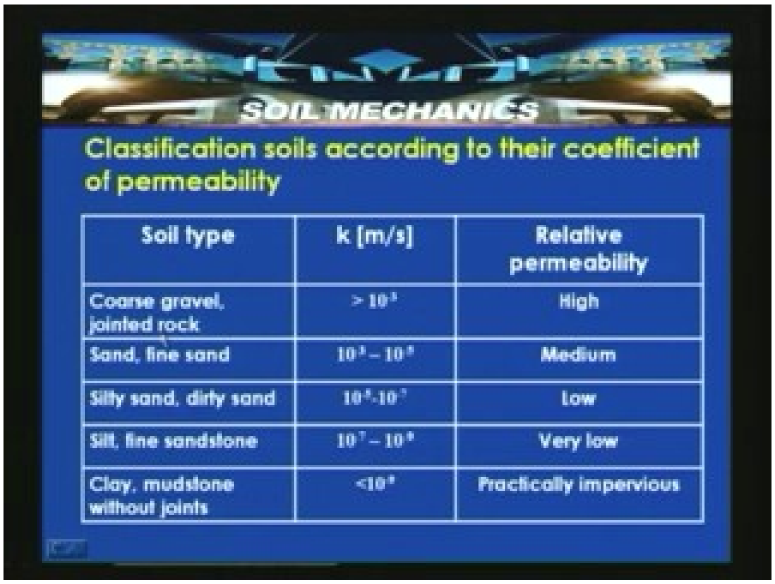

In this table if you look the first column tells about soil type and second column gives coefficient of permeability in meter per second and third column gives relative permeability. So if the soil is of coarse gravel or jointed rock then it can have permeability greater than 10 to the power of minus 3 meter per second. And it is said that it is having high permeability. Sand which is fine sand can have permeability of the order of 10 to the power of minus 3 to 10 to the power of minus 5 meter per second. It is said that it is relatively of medium permeability. Silty sand with dirty sand with mixed matter and with organic constituents we can say that the permeability is of the order of 10 to the power of minus 5 to 10 to the power of minus 7, we can say it is of low permeability. Silt and fine sandstone can have permeability in the range of 10 to the power of minus 7 to 10 to the power of minus 10 meter per second, we can say that very low permeability.

When it comes to some clay and mudstone without joints can also exhibit permeability less than 10 to the power of minus 9 meter per second also. Then in that case by all practical means we will say that the soil is practically impervious. So particularly this classification is important in deciding say for construction of some earthen barriers or water barriers or compacted clay barriers for arresting the flow of contaminants from one zone to other zone. This can be taken up by using this important property of clay.

So let us look how to arrive at the effective coefficient of permeability of stratified soils. So as we said in general natural soil deposits are highly stratified. In the beginning of this soil mechanics lectures we have seen that the soil deposits, during the process can have layers of soils. An example can be like warlord clay it can have silt and clay deposits.

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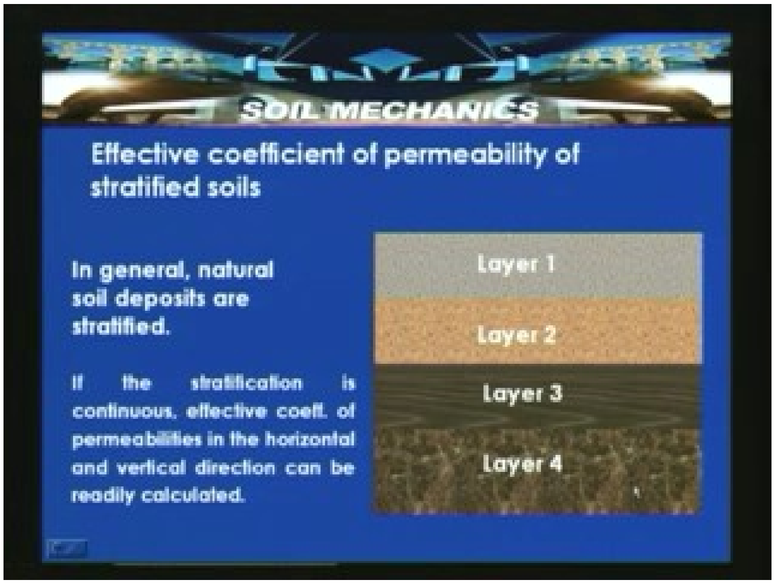

So in general, natural soil deposits are all stratified. So in this particular figure there are some 4 layers of soils have been shown. Here a particular soil in this layer 1 is said to be a sand of certain type and layer 2 is sand of certain type and layer 3 is clay and layer 4 is also another clay of another type. So if this stratification is there and if this stratification is continuous then our primary goal is how to arrive at the coefficient of permeability of the soils if they are stratified. So if the stratification is continuous and effective coefficient of the permeabilities both in horizontal direction and vertical direction can be computed again by using the fundamental principles of flow of water through soils.

So let us now consider first of all to find out the equivalent coefficient of permeability if the flow is occurring in the horizontal direction. That is parallel to the direction of stratification. So flow in the horizontal direction that is parallel to layers. Let us assume that we have got a permeameter in which n number of layers which are having coefficient of permeability k_1, k_2, k_3, k_4 to k_n are placed horizontally at thicknesses H_1, H_2 to H_n. If the length of the flow which is allowed in the permeameter is say L, so k_1, k_2, k_3, k_4 to k_n are the coefficient of permeabilities of stratified layers of thickness H_1, H_2, H_3, H_4 and H_n. And here if you assume that the flow is taking place from this point to this point through a head loss H_L over a length L (Refer Slide Time: 7:11).

So in this case though we have got n layers that flow entering the soil layer is q which is divided into q_1, q_2, q_3, q_4 to q_n. The q_1 is the rate of flow passing through the first layer and q_2 is the rate of flow occurring through the second layer and q_n is the rate of flow occurring to the nth layer. So this is the q which is entering the soil mass that is stratified soil mass and this is the q which is coming out of the soil mass. So q is equal to $q_1+q_2+q_3+\ldots+q_n$. So condition 1 here is that $q= q_1+q_2+q_3+\ldots+q_n$ and head loss is uniformly constant. That means all layers will have the same head loss because horizontally though the layers are stratified the same head which is deriving the flow from left side to the right hand side.

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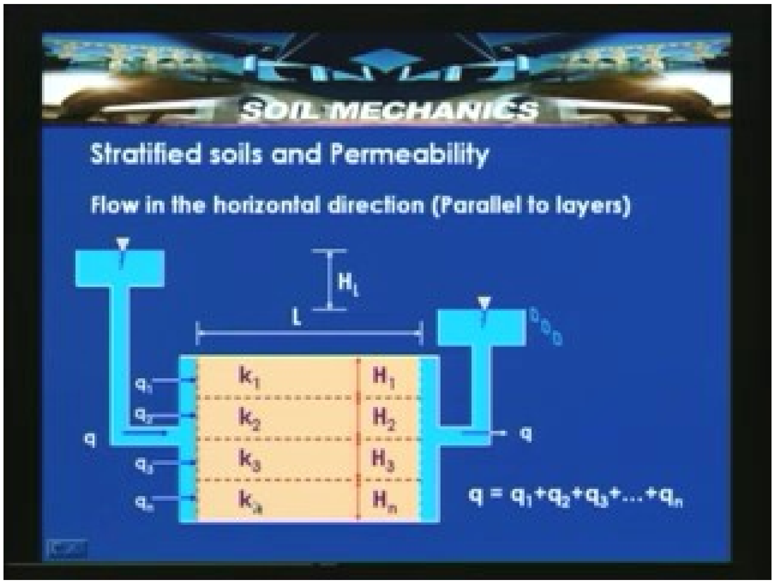

Let us look how this flow in the horizontal direction and parallel to layers can be obtained. So for horizontal flow, the head drop H_L over the same flow path length L will be the same for each layer. So if the head loss is same then we can say that i_1 that is the hydraulic gradient which occur in the first layer and hydraulic gradient which occur in the second layer and hydraulic gradient which occur in the last layer that is the nth layer can be same. So a condition is that $i_1 = i_2 = i_3 = i_n$. So the flow rate through a layered block of soil of breadth B is therefore: So assume that this permeameter has got a breadth of B perpendicular to the plane of the figure. So in this case by again using the Darcy's law we can write the flow rate k_h which is on the left hand side that will show the coefficient of permeability of all the layer in horizontal direction. That is the equivalent coefficient of permeability in a horizontal direction k_h and i is the hydraulic gradient which is nothing but in this case H_L by L and B is the width perpendicular to the plane of this figure which is shown in this slide and H is the total thickness of this layers.

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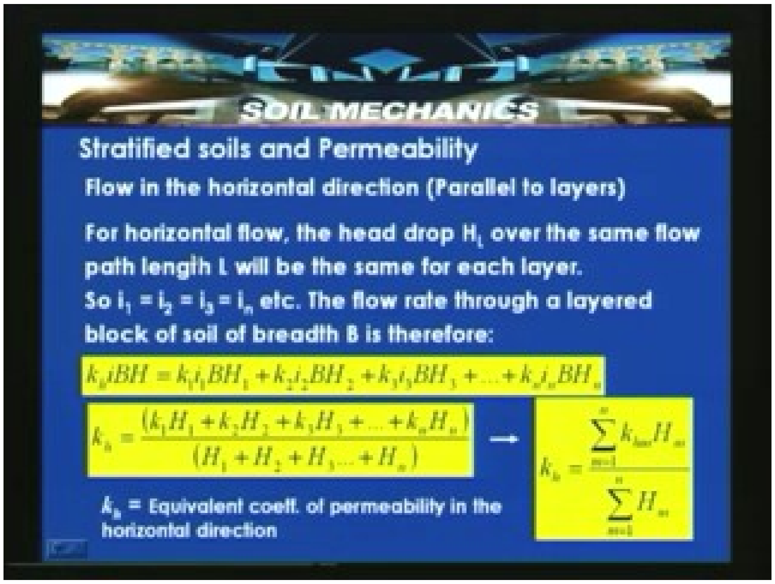

That means suppose if you have got 4 stratified layers then we will have $H = H_1 + H_2 + H_3 + H_4$ or incase of n number of layers the stratified thickness will be that is the total thickness of all the layers $H = H_1 + H_2 + H_3 + H_4$ to H_n. So similarly writing for the layer1, layer 2, layer 3 to the layer n we can write $k_1 i_1 B H_1 + k_2 i_2 B H_2 + k_3 i_3 B H_3$ to $k_n i_n B H_n$. So k suffix h that is the equivalent coefficient permeability in the horizontal direction is equal to $k_1 H_1 + k_2 H_2 + k_3 H_3 + \ldots + k_n H_n$ divided by (H_1 to H_n). So the summation of the thickness of the m number of layers and multiplication with k the respective coefficient of permeability of that particular layer with the thickness of the layer is going to give the coefficient of permeability.

That is equivalent coefficient of permeability of all the layers under consideration in the horizontal direction that is in the direction of the flow. So here when we have got number of layers, when the flow is occurring under a confined medium that is it should be confined in both top and bottom and then in that case the flow will occur with a head loss at a starting point say left point 1 to point on the right hand side to 2. So flow will occur from 1 to 2 with some certain head H_L. So when the flow conditions which are occurring parallel to the layers then we said that one condition $q = q_1 + q_2 + q_3 = q_n$.

Then another one we said that $i_1 = i_2 = i_3 = i_n$. So with that we tried to use the coefficient of permeability and hydraulic gradient and superficial velocity relationship v= ki. And then which is also extended to write a flow rate = kiA where A is the area over which the flow is occurring. For the thickness of H_1 having a width B perpendicular to its plane, then H_1 B is the area over which the flow is occurring. So we have written k_1 i H_1B. So that is what we are seeing here. So by simplifying will be able to get this. So this is expression we have derived for equivalent coefficient of permeability in the horizontal direction that is in the direction of the flow.

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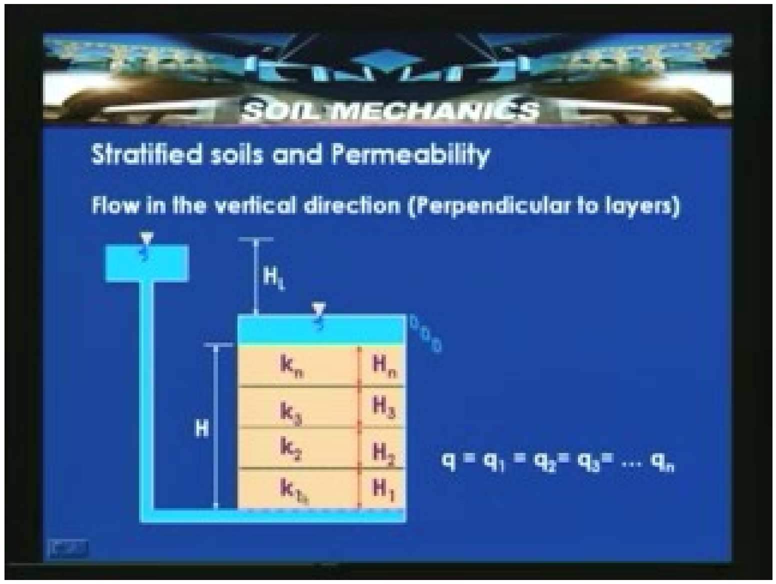

Similarly let us assume that we have got similar stratification k_1, k_2, k_3, k_4 to k_n but the flow is occurring in the vertical direction. So in this case it can be said that because of some artesian pressure at the bottom the flow is occurring in the vertical direction in the upward flow direction. So the head loss which is deriving the flow is H_L that is H_L is the head drop. So flow in the vertical direction perpendicular to layers that means the flow is occurring in this direction. So k_1, k_2, k_3 to k_n are the number of layers which are placed in this permeameter and H is the thickness of all the layers under consideration here $H = H_1 + H_2 + H_3 + \ldots + H_n$.

The entire flow which is taking place instead of getting divided into q_1, q_2, q_3, q_4 and in previous case we expressed as a summation. Here it will be $q = q_1 = q_2 = q_3 = q_n$. So $q = q_1 = q_2 = q_3 = q_n$ turns out to be one condition because the same water is entering and then erupting out once the head is elapsed. So when this particular condition arises for a vertical flow, the flow rate q through area A. So the flow is occurring in the vertical direction say upward direction, so the area over which the flow is occurring say A. And each layer being the same so the head drop occurs over a series.

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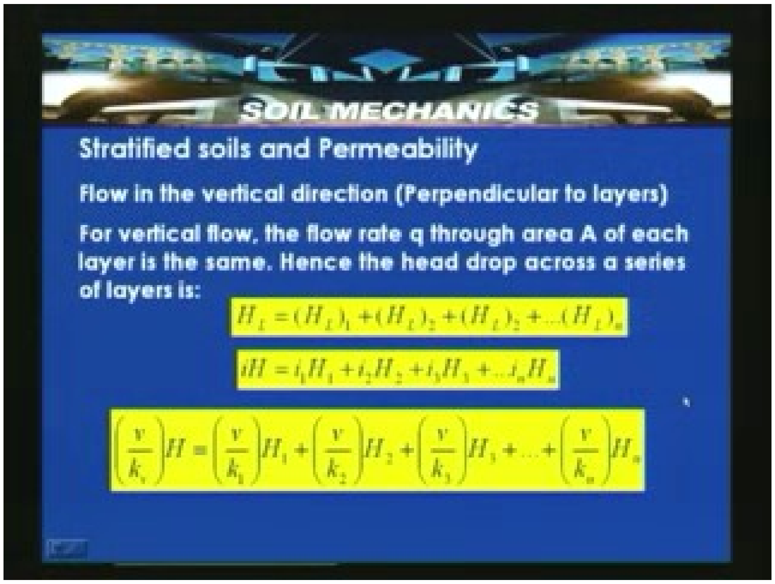

So this head drop across a series of layers can be said as H_L is equal to a head loss in one first layer, head loss in second layer, head loss in third layer. Total head loss is equivalent to total head loss occurred in all the layers. So it need not be the same for other layers depending upon the type of soil, then different head losses can occur. Suppose if a sand layer is there in the stratified then the flow is occurring vertical direction, then you can have a certain type of head loss. If a clay layer is there then it can have a certain type of head loss. But condition here is that for vertical flow the flow rate q through an area A of each layer is same. That is what the first condition we have said. The second condition is hence the head drop across a series of layers, because that is happening. With that the series can be written as $H_L= (H_L)_1+ (H_L)_2+ (H_L)_3+…+ (H_L)_n$.

So that can be simplified by using H_L which can be written as i in terms of H. The H is the total thickness of the layer and i_1 is the head loss occurred in the first layer, then H_1 is the thickness of that particular layer then H head loss occurred in the first layer can be written as i_1 times H_1 where i_1 is the hydraulic gradient in first layer and H_1 is the thickness of the first layer under consideration. Similarly we can write $iH = i_1 H_1 + i_2 H_2$ so on i_3H_3 to $i_n H_n$. So by using Darcy's law v= ki, that can be expressed as i= v by k. Then by writing now v by k_v H where k_v is the equivalent coefficient of permeability in the vertical direction. So this represents equivalent coefficient of permeability for all the layers under consideration. So v by k_v where v is the superficial velocity. Being the flow is same, it will have the same velocity $v = v_1 = v_3 =v_n$. So (v by k_v) H = (v by k_1) H_1+ (v by k_2) H_2+ (v by k_3) H_3+…+ (v by k_n) H_n.

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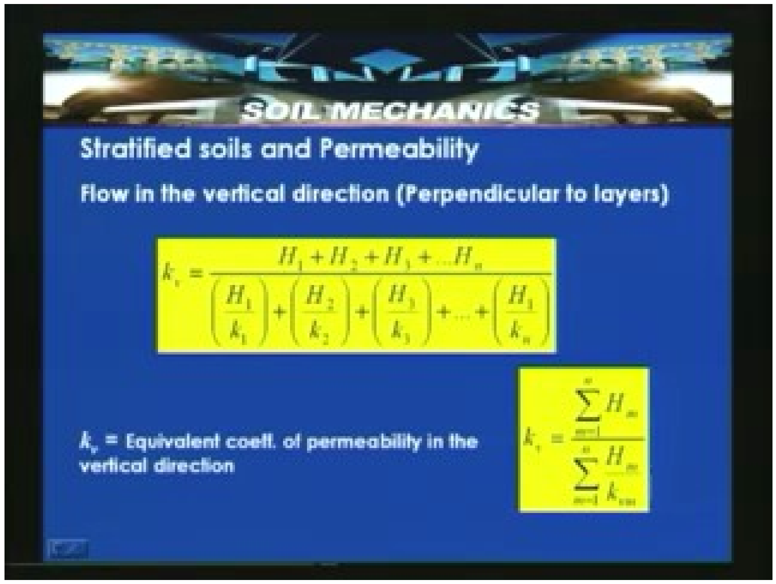

So by simplifying this we will get $k_v = H_1 + H_2 + ... + H_n$ divided by (H_1 by k_1) + (H_2 by k_2) +…+ (H_n by k_n). So here what we get is the summation, where equivalent coefficient of permeability in vertical direction k_v is equal to summation of all the thickness of layers where the vertical flow is occurring to the summation from m= 1 to n, H_m by k_{vm}. So k_{vm} is the vertical permeability of that particular layer, for example if m= 1 then it will happen to the first layer. If m happens to be the nth layer, then it happens to be for the nth layer.

So we have tried to determine the equivalent coefficient of permeability for stratified layers and we tried to use again the fundamentals whatever we have discussed in deliberating this discussion. Main points about stratified soils which are looked for consideration are, if you summarize in general for stratified soils k_h is not equivalent to k_v. That is equivalent coefficient of permeability in horizontal direction is not equivalent to coefficient of permeability in vertical direction or horizontal permeability is not equivalent to vertical permeability.

As we all know in the case of vertical permeability there is say geostatic stress or because of its own self weight soil tend to have increase in vertical stress with depth. So with that the number of the voids which are available for the flow to takes place will keep on decreasing. In this case if you look into the horizontal direction the scenario is different. Because the horizontal stresses that is depending upon the stress history of a soil, we can have different conditions, particularly for certain type of soils where this coefficient of say at pressure rest is say less than one.

In that case the sigma H that is horizontal geostatic stress is less than vertical stress. Because of that reason generally what will happen is that k_h is said to be more than k_v. That is coefficient of permeability in horizontal direction is more than coefficient of vertical direction. So if this particular condition like k_h is not equivalent to k_v is said, so such type of soil deposits are said to be anisotropic soils. Particularly from the flow point of view if the permeability is not identical in both the direction under consideration then we can say that it is anisotropic condition.

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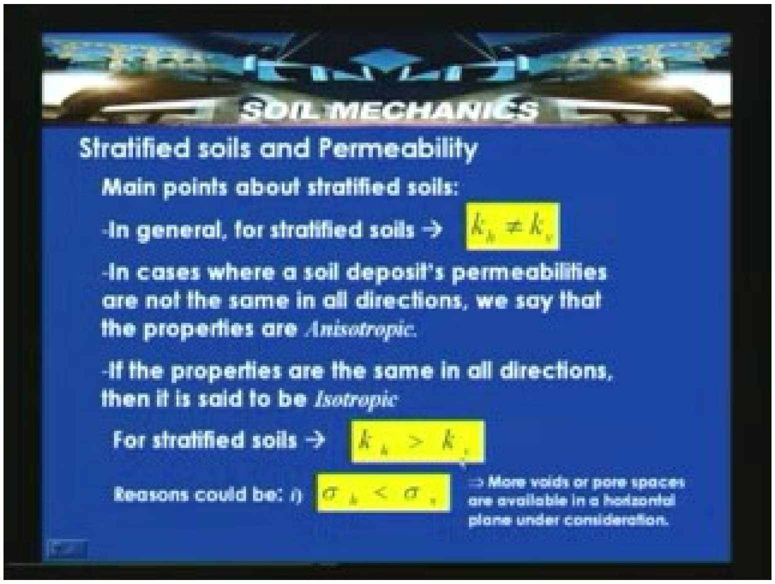

For example if you are considering a flow in 3 directions say permeability in x direction, y direction and z direction. If the permeability is not same in all the direction say kx is not equivalent to ky is not equivalent kz. Then we will say that it is an anisotropic condition. So here incase where soil permeabilities are not the same in all directions we say that the properties are anisotropic. If the properties are the same in all direction then it is said isotropic from permeability point of view.

As I said because of the variation in geostatic stresses a sigma H is less than sigma v, we can say that generally it is observed that k_h is greater than k_v. The reason it can be attributed to, one is that sigma H is less than sigma v, the other one in a more detailed way we can say that the more voids or pore spaces are available in a horizontal plane under consideration for the flow to takes place.

Because of this reasons we can explain that k_h is reasons for higher permeability in the horizontal direction than vertical direction. So we have discussed about the stratified soils and then how the equivalent permeability can be computed and we also said generally for stratified soils k_h is not equivalent to k_v. But it is observed that k_h is greater than k_v. The reasons we have explained through the geostatic stress principles.

So let us now try to look into an example where the flow is occurring in the horizontal direction. So in this slide a steady flow parallel to soil layers example was considered. There are 3 types of soil layers which are placed in between 2 impermeable layers, one is one impermeable layer at the top one impermeable layer at the bottom. So if this conditions are there the flow occurs parallel to this stratification.

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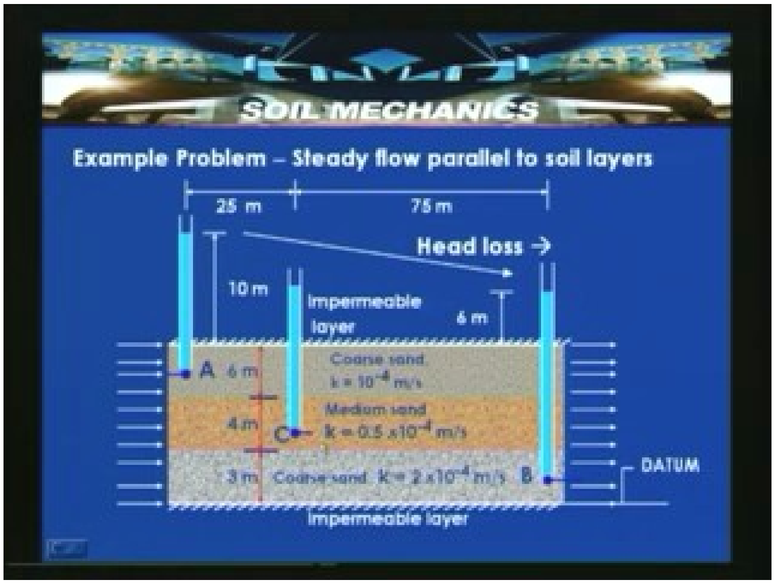

So here there are 3 types of sands, one is core sand which is having the permeability k= 10 to the power of minus 4 meter per second. In the medium sand k = 0.5 into 10 to the power of minus 4 meter per second and a core sand k = 2 into 10 to the power of minus 4 meter per second. So it has got a head of water which is 10 meters above this particular level. And at a distance of 100 meters from here, the head which is available is around 6 meters. That means the head loss is occurring from point A to point B. But if you look into the head which is available at C, so this is the head which is available at this particular point.

So if you look into this particular thickness of the layers, this is the top layer which is having say H_1 = 6 meters, H_2= 4 meters, H_3= 3 meters. And consider this particular bottom surface of the impermeable layer as a datum, the flow which is taking place from point A to point B, so that is with a head of around 4 meters this particular flow is occurring. What we are interested in estimating the equivalent coefficient of permeability in horizontal direction? So the direction is equivalent to the flow of water through layers. So the flow which is occurring here, q entering here and then by confirming to the conservation of the mass, q entering the flow is equal to the q coming out of the soil. So this flow can get divided into q_1, q_2, q_3. So total flow which is occurring $q = q_1 + q_2 + q_3$ is equal to q which is coming out once the flow is taking place. Let us look into the solution how it runs. So the solution for this particular problem having selected datum at bottom impermeable layer. Compute total head at A that is if you look into the previous slide once again which is this 7 meters + 6 that is 13 meters is the total thickness of this layer.

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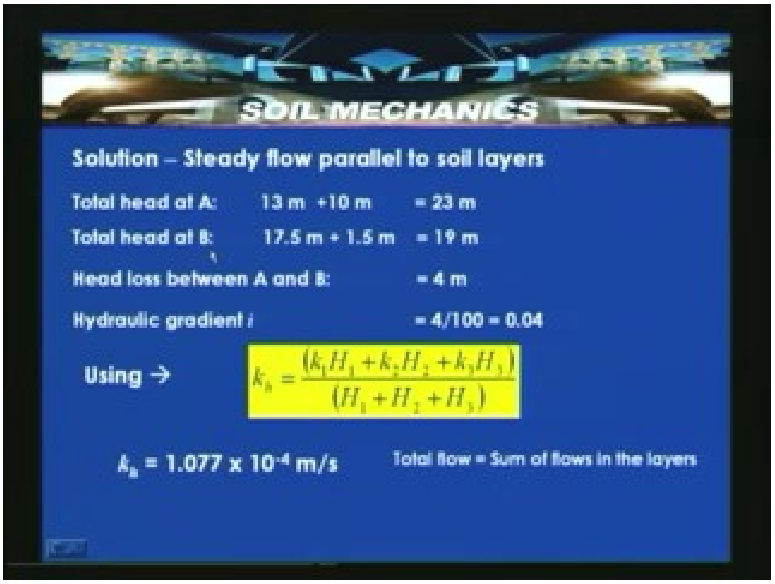

So if you look into this, the total head which is there is elevation head plus pressure head which is around 23 meters. Total head at B which is said here at 1.5 meters from this is the elevation head and then the rest which is available is the pressure head. So we can say the total head available at B is 19 meters. So head loss between A and B is difference between head A and head B that is nothing but 23- 19 that is 4 meters. Hydraulic gradient we can express because the horizontal length over which this flow is occurring is around 100 meters. So hydraulic gradient is now 4 by 100 is 0.04.

So using the expression whatever we had derived that is equivalent coefficient of permeability in horizontal direction is equal to $k_1 H_1 + k_2 H_2 + k_3 H_3$ divided by $H_1 + H_2 + H_3$. Because we are having 3 layers on the consideration so we limited that n= 3. So by simplification total flow is equal to sum of the flows in the layers. So total flow can be obtained by again confirming to Darcy's law q = kiA. So you know the k_1, i and then you know the A_1. So with that you will able to calculate the sum of the flow which is occurring in the 3 layers. Then equivalent coefficient of permeability can be determined as 1.077 into 10 to the power of minus 4 meter per second. So these are the solution has to be worked out for the case of a stratified soil having 3 different types of soils. So what we tried to get is that equivalent coefficient of permeability for all the 3 sand layers given in the problem.

Before introducing the equation of continuity and the theory behind this and then solution which is available for an equation of continuity. Let us try to look into different types of fluid flow which are possible in soils. If you look into it, basically they are divided like one dimensional flow, two dimensional flows and some three dimensions flow and some spherical flows. But most permanent what we discussed till now are all one dimensional flows.

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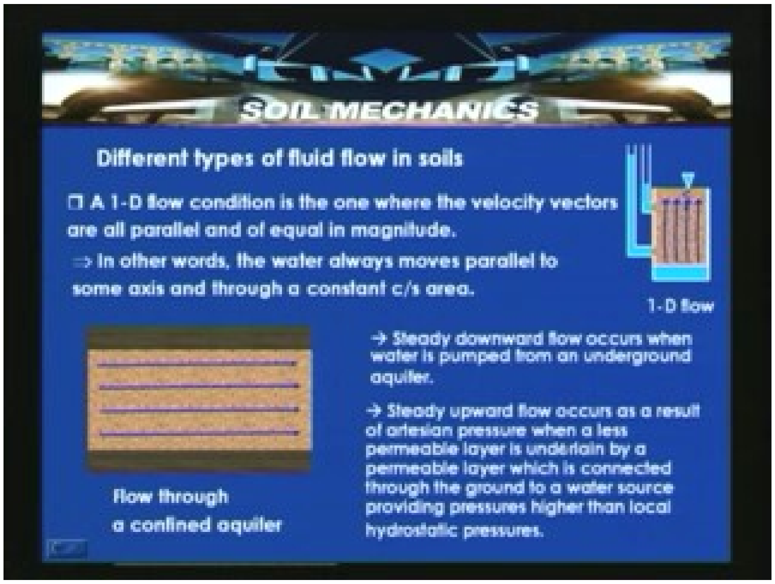

A one dimensional flow condition is the one where the velocity vectors are parallel and equal in magnitude. So in this case the flow is occurring with a head loss. So from 0.1 to 0.2 is the flow direction where the flow is taking place. A one dimensional flow condition is the one where the velocity vectors are all parallel and of equal in magnitude. In the other words the water always moves parallel to some axis and through a constant cross sectional area. So this is said to be a one dimensional flow. Steady downward flow occurs when water is pumped from an underground aquifer. So if the water is being pumped from an underground aquifer then there will be a pressure drop, then the water flow can take place in the downward direction. Steady upward flow can occur if there is say a location of a sand layer and which is receiving a source of water because of the artesian conditions.

If above the sand layer relatively impermeable soil is there, then the water can flow through that particular soil because of the artesian pressure. So steady upward flow occurs as a result of artesian pressure when a less permeable layer is underlying by a permeable layer which is connected through the ground through water source providing the pressure higher than the local hydrostatic pressures. So these pressures which are driving the flow in upward direction, they are receiving these higher pressures because of some artesian conditions, natural conditions which are possible. We have discussed also the different aquifer conditions which are possible.

So steady upward flow occurs as a result of artesian pressure when a less permeable layer is underlain by a permeable layer which is connected through the ground to a water source providing the pressures higher than the hydrostatic pressures. So the flow can be one dimensional. So in this slide basically we have seen a one dimensional flow which can occur due to flow from 0.1 to 0.2 because of the head loss occurring over that particular zone. In other words we can say that the water always moves parallel to the some axis through a constant cross sectional area. So this is an example for one dimensional flow.

Another example is that flow through a confined aquifer. Suppose if I have got a flow which is occurring through a confined aquifer having say constant area of cross section then we can say that again the flow is one dimensional. So here in the above slide in that particular picture what you are seeing is the flow which is occurring through two impermeable layers and the flow is occurring through a sand layer. So this can be said as again a one dimensional flow, not necessarily vertical direction it can be because of any confined aquifer condition which is prevailing and which can also have a one dimensional flow.

Another type of the flow what we said is around two dimensional flow. Basically it can occur like it occurs when all the velocity vectors are confined to a single plane but vary in direction and magnitude within that plane. So here a flow which is adjacent to a sheet pile wall, generally we keep this sheet pile wall to retain water in other end. In this case water flow occurs from higher head to lower head. So higher head if you call it as upstream end and lower head is called as a downstream end, the flow occurs from the upstream end to downstream end.

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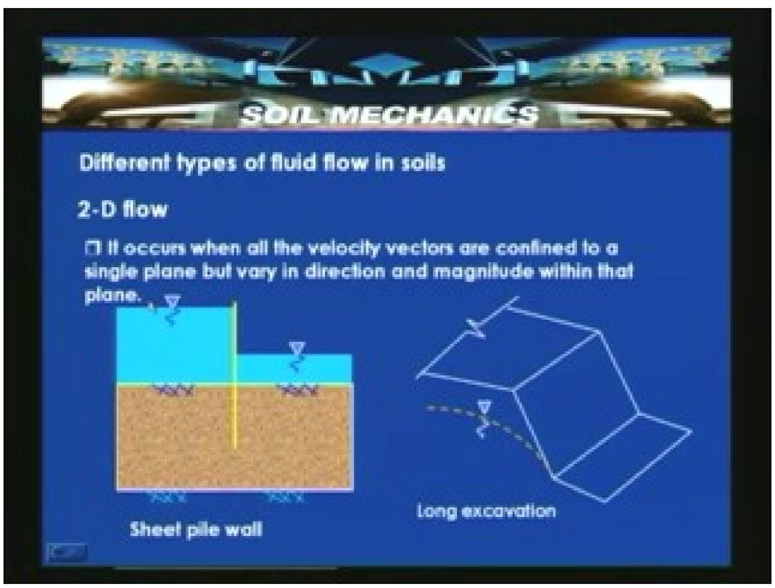

So the yellow color member is designated as sheet pile wall and if it is placed in a saturated sand layer then we can say that this is the head which is driving the flow. So the flow occurs from this point to this point (Refer Slide Time: 30:01). So the pattern of the flow generally changes in two directions that is x direction and z direction. That means if you call this one as x plane and the z plane. Then we can say these types of particular flows are called two dimensional flows where v_z are the flow occurring in the z direction is almost zero. So in that case this particular types of flows are called a two dimensional flows. So it occurs when all the velocity vectors are confined to a single plane but vary in the direction and magnitude within that plane.

You consider an example where the long excavation for a canal excavation is being carried out. So here in this case the flow takes place at a particular cross sectional, if you cut. So here the flow is occurring in this direction. So perpendicular to this plane suppose to this axes is said to be z axis along the length of the canal or length of the excavation. So there the flow which is occurring in that direction is very minimal. So this also can be said as an example for a two dimensional flow conditions. So what we are doing is that we are discussing about the different types of fluid flows in soils and we are also trying to understand with some salient examples.

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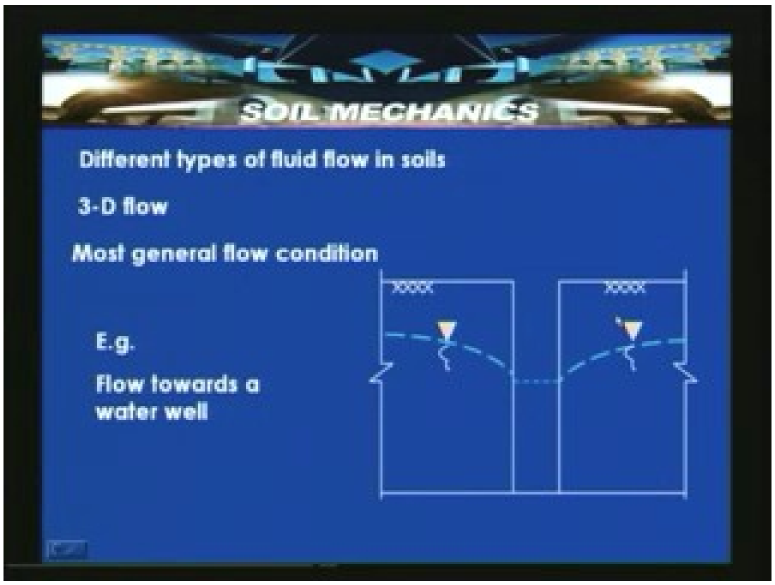

A three dimensional flow is the most general flow condition which can be said as a flow of water towards a water well. Flow of water towards water well can be said as an example. An example for a three dimensional flow where you can see a borehole where the water was driven into the ground. You can say that the flow which is occurring in all directions from spherically and which is coordinating or converging towards the center that is borehole can be set as an example for flow towards a water well which is designated as a three dimensional flow. So as we have seen and then we understood that the multidimensional fluid flow is possible in soils.

So as of now we said that the flow which is occurring in one dimensional is not the case where we can have two dimensional flows or three dimensional flows. But in most of the geostatic structures the two dimensional flows are very common. So in the case of our analysis and then further we are looking into the details of the two dimensional flow conditions for calculating the flow of water through an earth dam or leakages in a reservoir or so. All these things are for a two dimensional flow conditions. As of now we have considered one dimensional flow in soils where all fluid is flowing in the same direction. So in most cases, however fluid in different regions will be flowing in different directions. Then we can say that it is a multi directional flow or multi dimensional flow.

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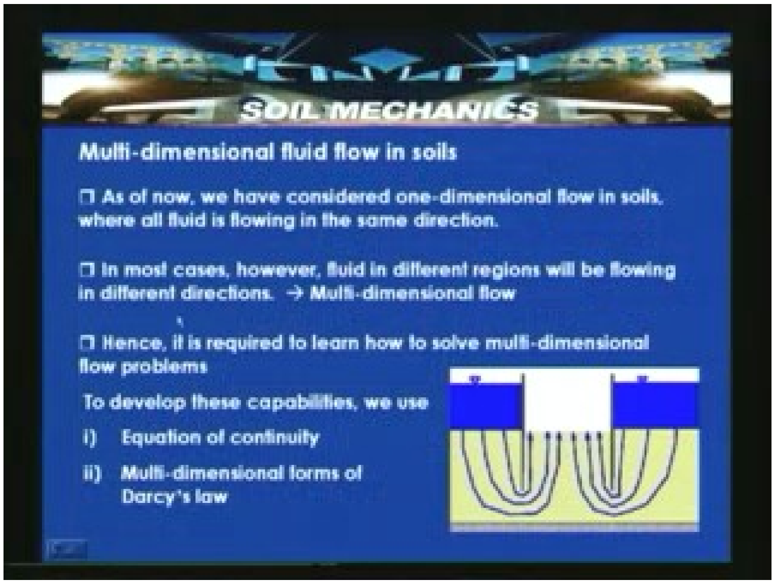

Hence it is required to learn how to solve the multidimensional flows. So we have to develop our knowledge now about how to solve this multidimensional flow problem in soils. So to develop these capabilities we use the equation of continuity, this is based on the conservation of the mass and the multidimensional forms of Darcy's law. So again we will be using the multidimensional forms of Darcy's law and equation of continuity, with the help of these things we will try to arrive at the equation of the continuity and then the solution. And then the application of this can lead to evolve at calculating the seepage problems involved in earth dams or sheet pile walls.

So an example which can elucidate this multidimensional flow is shown here where you have got two sheet pile walls which are placed. It is possible that the flow can occur from this upstream end to this. So this is a cofferdam type where this particular situation has been created to facilitate construction. This is generally done for peer constructions within the riverbed or so. Then in that case here this cofferdams are constructed which are nothing but either spherical or having a particular shape depending upon the requirement. And their main importance is to keep the water away, but water has always got a tendency to flow from higher head to lower head. So in the problem the flow pattern where the flow which is occurring in these directions. So when the flow is taking place in this direction the head loss is also occurring from the starting point to the ending point. Similarly in the symmetric which is shown here.

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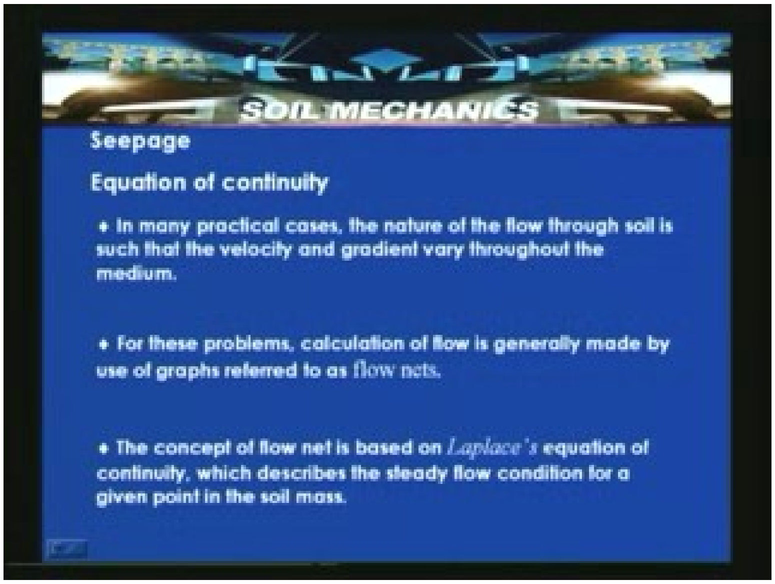

So coming to the seepage and equation of continuity, in many practical cases the nature of the flow through soil is such that velocity and gradient vary throughout the medium. So for these problems calculation of flow is generally made by the use of a graphical procedure called flow net methods. So this flow net method is nothing but the solution of the equation of continuity which we are going to derive very shortly.

So the concept of flow net is based on the Laplace equation of continuity, which describes the steady flow condition for a given point in the soil mass. It is valid at a given point and which is occurring in a saturated soil medium. So the concept of flow net is based on Laplace equation of continuity. So the equation of continuity which we are going to use is named as Laplace's equation of continuity and which describes the steady flow condition for a given point in the soil mass. So the Laplace's equation of continuity describes the steady flow condition for a given point in a soil mass.

Let us try to look into this equation of continuity. Let us assume that you have got a dam which is retained by mesentery peer which is constructed with concrete wear or so. And which is having a head of water H_1 on the upstream side, H_2 on the down stream side and it is embedded in a sand soil or a saturated soil. Just beneath the structure, suppose if you select at this particular point A, h is the head which is available there. That means here when it is retaining, the flow is occurring from H_1 to H_2. So here the head available at all these points is around H_1 and H_2. So these types of problems which are possible and which are called two dimensional problems where the flow can occur in this direction.

So the flow can be said that v_x and v_z, suppose if you call this as the x-axis and this is the z-axis then we can say that the v_x and v_z are the x and z-axis over which the flow occurs. And then perpendicular to this axis we call it as a y, then v_y= 0 and this is for a two dimensional case.

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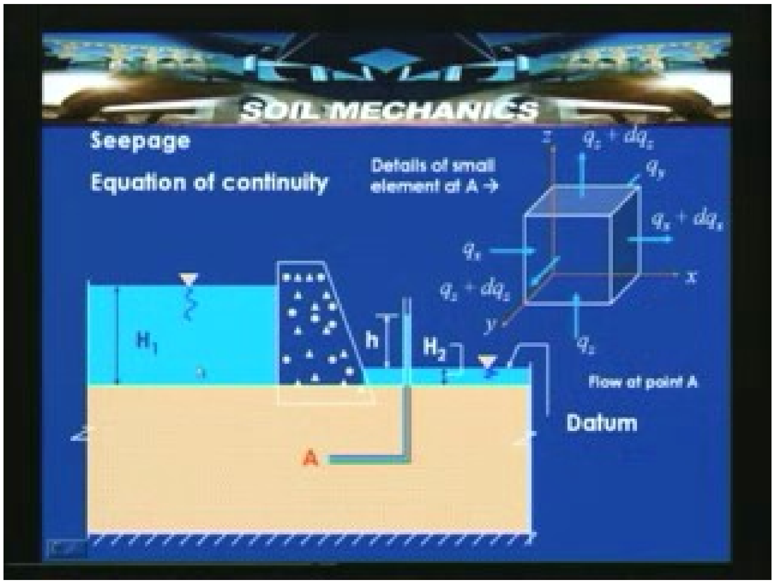

But while deriving the equation of continuity let us assume perfectly anisotropic condition like k_x is not equivalent to k_y is not equivalent to k_z where k_x, k_y, k_z are the coefficient of permeabilities in x direction, y direction and z direction. So let us consider a small element at A which is a cube there and which is considered with the flow occurring here. Let us assume the flow at point A. The flow at point A we can say that q_z is the inflow which is occurring over a plane x y.

So let us assume that this small element which is considered at element A has got dx, dy, dz direction. So volume of the element is dx, dy, dz where dx is in x direction, dy in y duration and dz in z direction. And $q_z + dq_z$ is the flow which is coming out. So the net flow is around dq_z and q_x is the flow which entered and $q_x + dq_x$ is the flow which has come out. Similarly from other phase that is xz plane where q_y and q_z is entering and $q_y + dq_y$ is coming out.

So in this case, in the x direction, y direction and z direction you have seen that the flow entering into the element and the flow coming out the element. Let us try to apply these conditions and again apply the various forms of Darcy's flow and then try to derive. So having seen at the element A, now the flow is entering in x direction, y direction and z direction. So now let us consider how we can compute this flow entering the soil prism which is under consideration, which is having dimension dx, dy and dz. The dx is the dimension in x direction, y direction and z direction. Then by using Darcy's law we can write $q_x = k_x\, i_x$ into A_x where K_x is the coefficient of permeability in x direction, i_x is the hydraulic gradient in the x direction and A_x is the area which is perpendicular to x direction that is nothing but dy dz and h is the hydraulic head at point A under consideration.

So similarly in case of y direction $q_y = k_y\, i_y\, A_y$ and incase of z direction we can write $k_z\, i_z\, A_z$ where k_z into (dow h by dow z) (dxdy) where dow h is the small head drop which is occurring over a length here in this case in the z direction dz. So the dow h by dow z is representing the hydraulic gradient in z direction.

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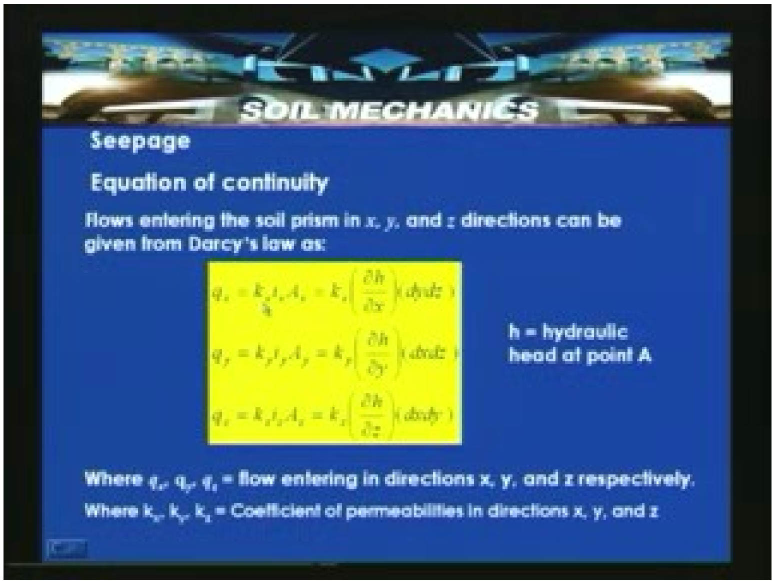

So these are the flows entering the soil prism. Similarly now we can also write the flows leaving the soil prism, particularly soil element under consideration. Where here the q_x, q_y, q_z that is the flows entering in x, y and z direction respectively and k_x, k_y, k_z are the coefficient of permeabilities in x, y and z direction. Our problem is that we are considering in x direction, y direction, z direction. The flow which is occurring in x direction, y direction, z direction is not equal and that means because of the anisotropic conditions we said that we are considering k_x, k_y, k_z.

If $k_x= k_y= k_z$ then for isotropic case we can say that $k_x = k_y = k_z= k$. So the equation of the continuity can be derived based on the flows leaving the soil prism at x, y and z directions can be written like $q_x+ dq_x$ where the dq_x is the small flow change which is occurring when the flow is entering and leaving the soil prism. So that can be written as k_x into $i_x + di_x$ where di_x is the change in the hydraulic gradient. That can be written as dow square h by dow x square over a length which is dx.

So this expression now can be simplified as k_x into (dow h by dow x + dow square h by dow x square into dx) and this is occurring over an area (dy by dz). So now $q_x+ dq_x$, similarly we can write for $q_y+ dq_y$ and $q_z + dq_z$. What we have got is that by using the Darcy's law flows entering the element and flows leaving the element. By using the principle of conservation of the mass let us try to derive the equation of continuity. Here by using the principle of the conservation of fluid mass: For steady flow through an incompressible medium, the assumption is that the medium is incompressible. The flow entering the element is equivalent to flow leaving the element.
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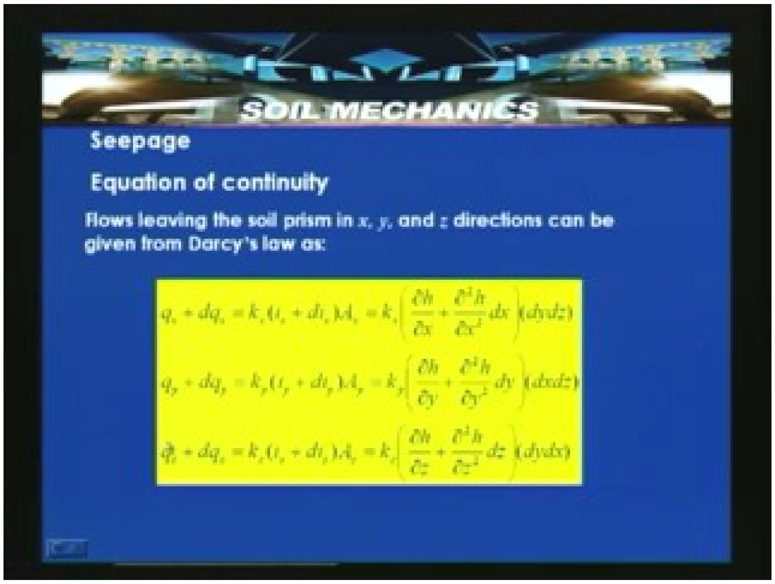

So $q_{in}= q_{out}$, that is the principle we can say in nutshell. As the principle of the conservation of the fluid mass, for steady flow through an incompressible medium the flow entering the element is equal to the flow leaving the element. So from this we can write the equation of continuity by equating the flows entering the element. The sum of the flows entering the element in x, y and z direction can be written as $q_x+ q_y+ q_z$. Similarly the sum of the flows which are leaving the element which is summation of $q_x+ dq_x+ q_y +dq_y+ q_z+ dq_z$.

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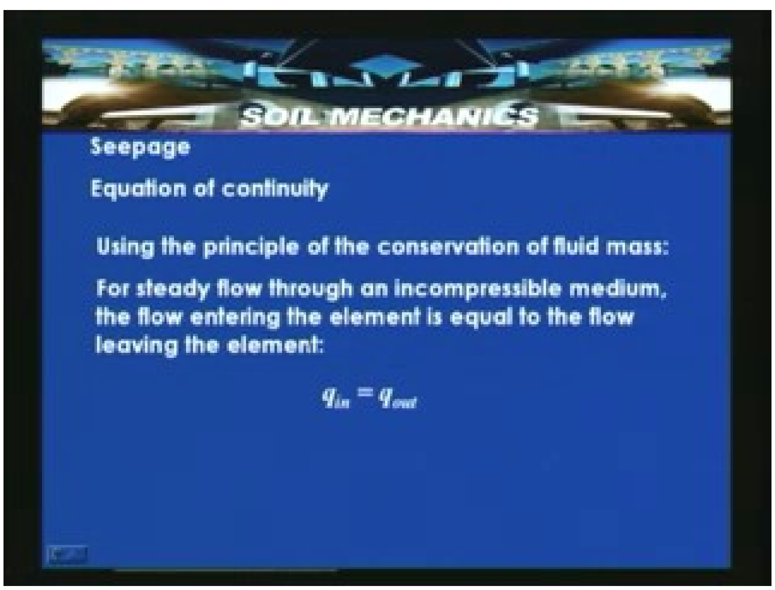

So by substituting from the previous discussion and by simplifying we will get an equation q_x which is $q_x+ q_y+ q_z= q_x+ dq_x+ q_y+ dq_y+ q_z+ dq_z$. The simplification of that yields k_x into dow square h by dow x square + k_y into dow square h by dow y square + k_z

into dow square h by dow z square is equal to zero, where h is the head loss at that particular point under consideration. So the k_x, k_y, k_z are the coefficient of permeabilites in x, y, z direction respectively. And in this case this is the generalized expression, incase of isotropic case where $k_x = k_y = k_z$ then we can say that where k_x is not equal to k_y is not equal to k_z is not equal to k is not equal to zero. Then we can say that dell square 5 is equal to zero. So dell square h is equal to zero is the condition of simplified way of expressing the Laplace equations.

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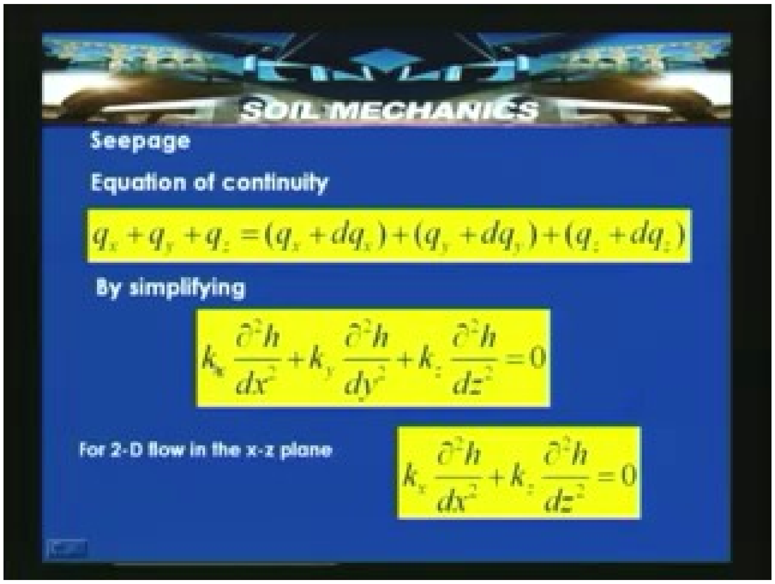

So the 2- D flow condition in x y z plane. So as I said for flow through earth dam which is a two dimensional condition or flow through a sheet pile wall structure is again a two dimensional structure or a long excavation is a two dimensional flow. So for such type of flows, 2 dimensional flow in the x z plane we can write k_x into dow square h by dow x square + k_z into dow square h by dow z square is equal to zero.

So the equation of the continuity can be simplified, if the soil is isotropic with respect to the permeability k_x, k_y, k_z then the continuity equations simplifies to, because $k_x = k_y = k_z =$ k. So in that case as k is not equivalent to zero, coefficient of permeability can not be equivalent to zero in that case we can say that dow square h by dow x square + dow square h by dow y square + dow square h by dow z square is equal to zero which is the simplified laplace's equation of continuity. It is a very important expression which helps in solving the some of the two dimensional problems for flow of water through soils.

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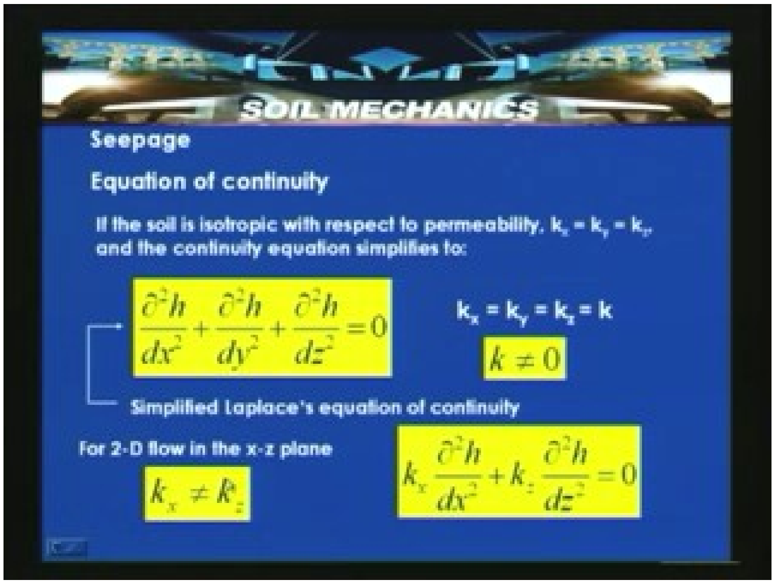

So for two dimensional flow in x z plane, if the k_x is not equivalent to k_z say an anisotropic, as far as two dimensional flow is concerned then we can say that k_x into dow square h by dow x square + k_z into dow square h by dow z square is equal to zero. So here we can say that for a dimensional flow in x z plane the k_x is not equivalent to k_z.

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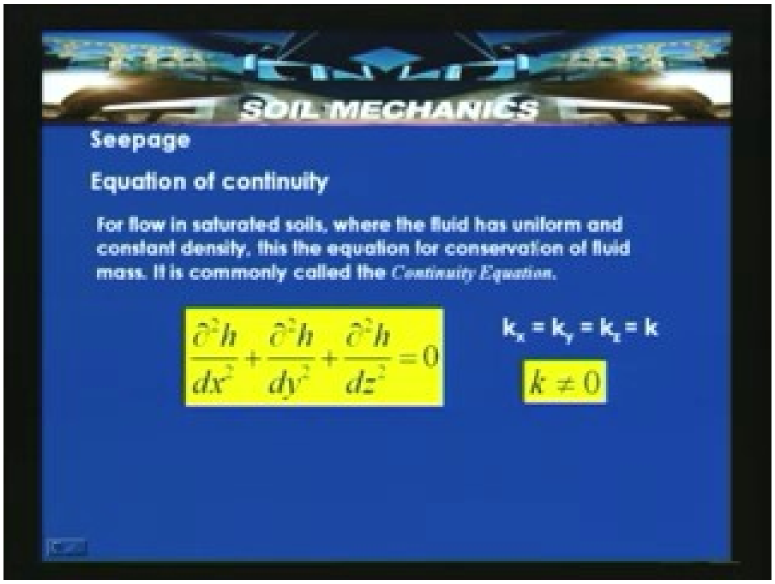

For equation of continuity where we have seen that the flow in saturated soils where the fluid flow has uniform and constant density, the equation for conservation of fluid where it is commonly called continuity equation, where dow square h by dow x square + dow square h by dow y square + dow square h by dow z square is equal to zero. $k_x = k_y = k_z = k$ where k is not equal to zero. So for flow through a constant head permeameter in the laboratory also turns out to be a one dimensional flow example. In such equation, the governing equation can be written as dow square h by dow z square is equal to zero. So in this case if h is the depth which is here, so this particular discussion which we had in the last lecture where z is the direction over which the flow is taking place from top to bottom.

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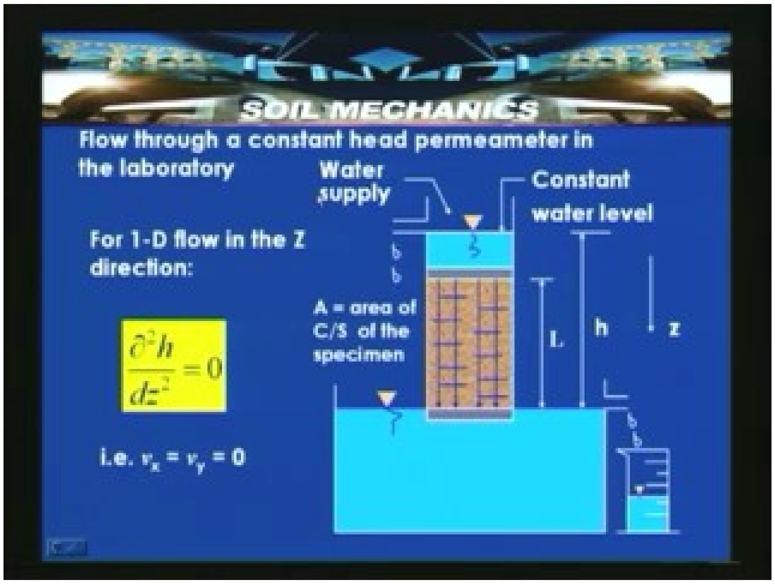

Then for one dimensional flow in the z direction we can say that the governing equation of continuity is dow square h by dow z square is equal to zero. So that means the flow in x direction y direction is zero. So flow is occurring only in the direction that is v_z. So this is an example for a one dimensional flow condition in the laboratory constant head permeameter. The solutions which are available particularly for the laplace's equation basically there are analytical and closed form series solutions of the partial differential equations.

So there are several solutions which are available and this analytical and closed form solutions are quite mathematical and not very general in nature. And numerical solution methods, typically finite element or finite difference methods they do exists and they are very powerful and easy to apply and can deal with heterogeneity, anisotropy and two dimensional and three dimensional flow conditions.

So the solutions for this equation of continuity what we said is that you have got analytical and closed form solutions. Basically quiet mathematical and then not very general. Then numerical solutions are typically based on finite element method or finite difference method, very powerful and easy to apply, can deal with heterogeneity. But the graphical techniques called flow net methods which are widely used for estimating the flow of water through soils incase of a two dimensional structures, commonly used engineering practice to solve two dimensional problems.
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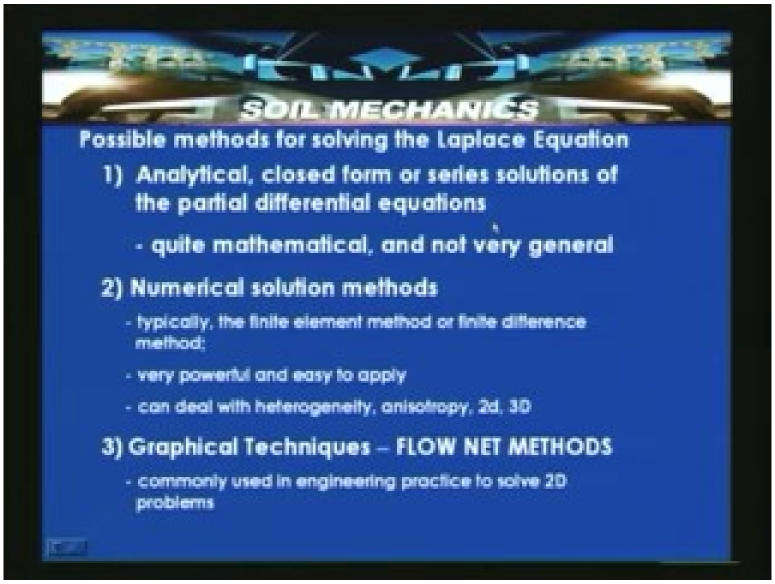

So the flow net methods which are the one which will be applied for solving and computing the flow through a two dimensional plane and so that you will be able to compute the seepage or leakages which are occurring at the times and all these things which are applied through the two dimensional flow conditions. So the solutions if you are listing out means we have got analytical closed forms solutions or a series of solution of the partial differential equations or numerical solution methods or a graphical techniques which are based on the flow net methods which are quite common and commonly used in engineering practice to solve two dimensional flow problems of the geotechnical structures.

So this flow net methods basically they are straight forward and graphical method to solve two dimensional seepage problems. So flow net methods are two dimensional straight forward methods to solve two dimensional flow problems. Solutions of Laplace equation consist of basically, if flow net means it is a two families of orthogonal curves in x and z direction. And these families of curves are called as a flow net. So in this particular slide a partial flow net is shown here.

The red lines which are representing the flow lines, so these are called a flow lines and these are called some equi-potentical lines. So if you see here the aspect ratio is the ratio of the distance between two flow lines that is A and B is the distance between two equi-potential lines. So ratio of A to B is called aspect ratio, generally the flow nets are need to be constructed by maintaining or it should be possible for us to insert a circle which is having certain dimension not necessarily that it should have the identical circular dimensions, but here is the flow which is occurring in this direction. So the space between any two flow lines is called as a flow channel.

In this case of the partial net the two flow channels have been shown. And here equi-potential line in the sense what does it indicate is that in the direction of the flow if you measure a head here, it will be same at each and every point on this line. So the lines which are having equal potential or equal head they are defined as equi-potential lines. So the family of this equi-potential lines and flow lines is said to be a flow net which is used for calculating or computing or solving an application, for solving the flow problems in the two dimensional case. So solutions to Laplace equation consist of the two families of orthogonal curves and they are called equi-potential lines or flow lines which is put together in network called as a flow net.

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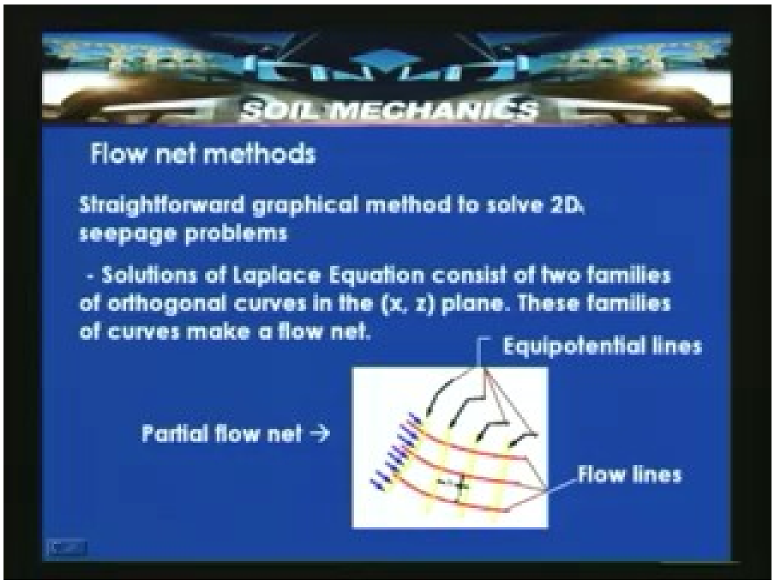

So in this class we tried to understand about the stratified soil deposits and how to determine the equivalent coefficient of permeability in horizontal direction and in the vertical direction. Then we also discussed the reasons why we will have the coefficient of permeability in horizontal direction is more than coefficient permeability in vertical direction. Then we tried to obtain the equivalent coefficient of permeabilites in horizontal direction and vertical direction. Then we discussed the different fluid flows which are possible in soils. We said basically the one dimensional flows, two dimensional flows and then three dimensional flows. Two dimensional flows are quite common basically for the constructions of the gen geotechnical structures where the flow occurring in two planes that is x and z directions.

Then based on the principles of the conservation of mass we tried to drive the equation of continuity and that is dow square h by dow x square + dow square h by dow y square + dow square h by dow z square is equal to zero for x and y z directions. In two dimensional case where with an isotropic conditions that is k_x is equal to k_z as for as permeability is concerned, we said that in that case the laplace equation of continuity can be said as dow square h by dow x square + dow square h by dow z square is equal to zero.
Incase of one dimensional problem like in the case of say constant head permeameter the example where one dimensional flow can occur, where we can say if that is the flow which can occur in the z direction, then we can say that dow square h by dow z square.

So these are the different flows which are considered and based on that we have derived equation of continuity. Then we have discussed the possible solutions which are available for solving the Laplace equation of continuity. And out of these we said that the flow net methods of the graphical methods which are popular for solving the two dimensional flow problems in soils. So in the next lecture we will be discussing more about this particular graphical technique as a solution of the Laplace equation of continuity and we will try to see the principles of this method and try to look into the theoretical aspects as well as applications in the form of some selected problems.