Fluid Mechanics

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Lecture – 8

Kinematics of Fluid Flow

Welcome back to the video course on fluid mechanics. In the fluid kinematics which we were discussed in the last few lectures, the last lecture we discussed about the Reynolds transport theorem, then the conservation of mass, the continuity equation which we have derived based up on the integral formed and also seen that we can use either integral form of differential form. Today we will discuss about the linear motion deformation then rotational flow and then we will go to the potential flows and further we will see the applications of the potential flow.

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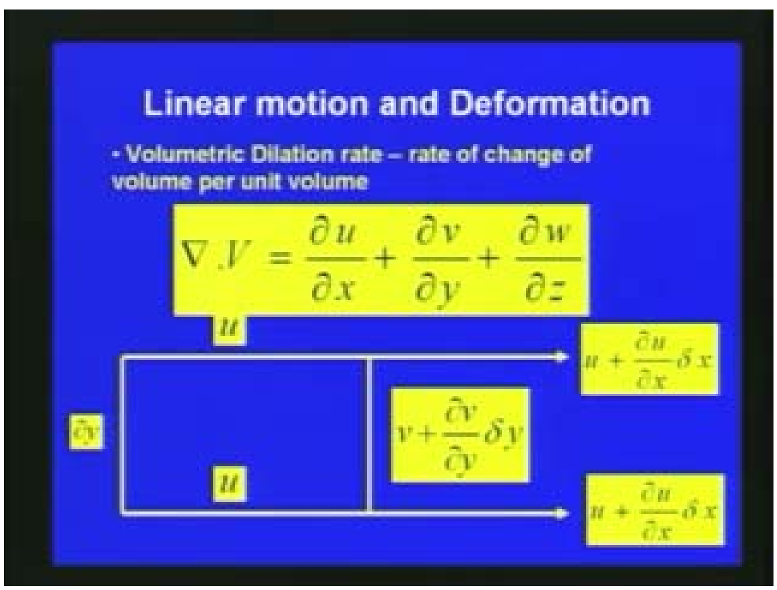

We can see here, linear motion deformation, so volumetric dilation rate as we discussed in the last lecture, it is the rate of change of volume per unit volume, say del v that means the velocity vector. We can express this as del u by del x plus del v by del y plus del w by del z, where u v and w are the velocity component in xyz direction.

As we can see in this figure, if it is a 2D problem, linear motion with respect to linear motion deformation you can see that Vu is the velocity here at location at delta x, a way the velocity will be u plus del u by del x plus delta x and v is the velocity direction here then the other say at a distance delta y the v velocity will be v plus delta v by delta y. Like this we can see the linear motion with respect to fluid movement and deformation.

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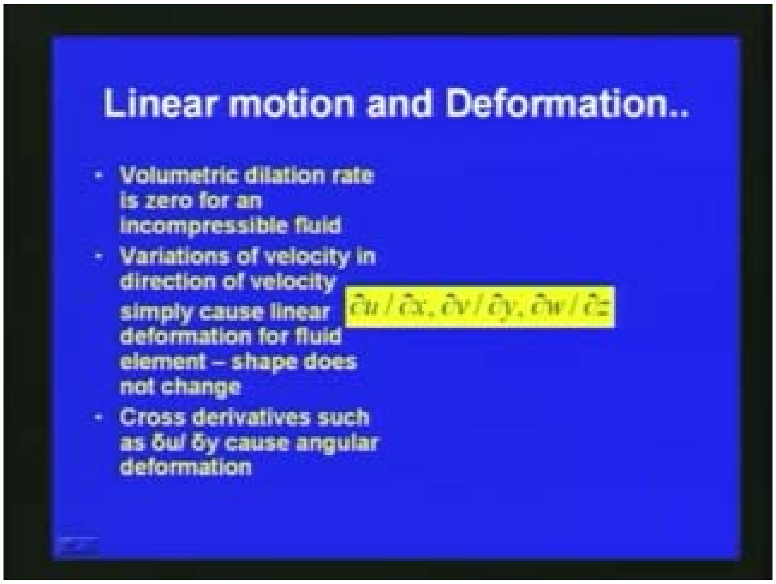

Here we can see that with respect to the deformation the volumetric dilation rate is 0 for an incompressible fluid so that we can see that the variation of velocity of the direction of the velocity simply cause linear deformation or fluid element shape does not change, that means the volumetric dilation rate is 0, since the fluid is incompressible and then variation is already in the direction of the flow that means shape does not change so that we can cross derivative such as del u by del y only cause angular deformation. But if you take the direction of flow like del u by del x, that means the velocity u is in the direction of x and del u by del x is the deformation in the x direction and if you take the velocity component y direction then if you take del v by del y or the velocity component z direction del w by del z.

So, all this in the direction of the velocities or in the direction of the corresponding Cartesian coordinate so that we can say that only linear motion takes place there is no angular deformation, so with respect to this, here we can say this del u by del x plus del v by del y plus del w by del z is the total volumetric dilation. Now, this we will further elaborate in the case of angular motion and deformation.

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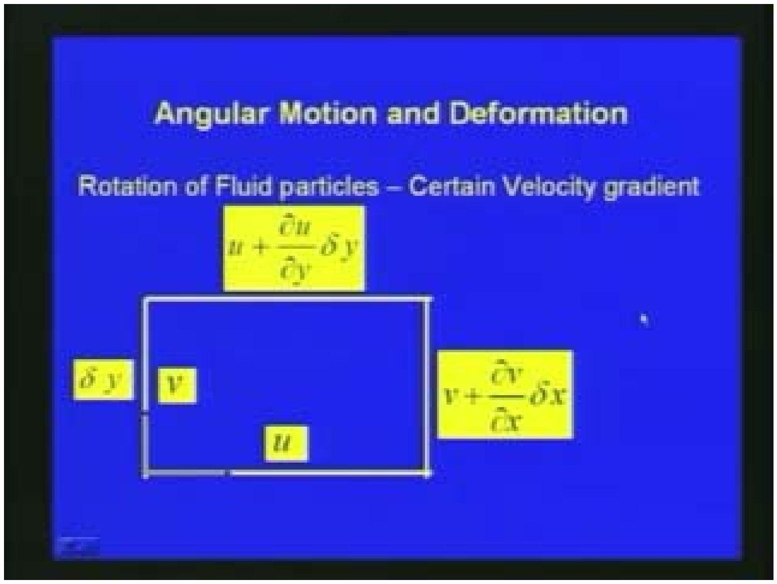

If the velocity is the dilation or in the change in velocity changes in the direction of that coordinate system then it is only linear motion, but if it is in the other direction say for example, del u by del x if we are considering del u by del y, it will be an angular motion or it will make an angular motion or deformation. So rotation of fluid particles certain velocity gradient like u plus del u by del y or delta y, here we can see that the change is in the other coordinate systems. So, u is in this direction of x that means x is in this direction and velocities are in that direction and v is in the y direction so but if the deformation is with respect to y.

The deformation change in u is with respect to y means del u by del y and say for two dimensional problem if the change in velocity for v in the direction of x or del v by del x then we can see that there will be rotation of fluid particle and then certain velocity produced which may cause angular motion and the deformation. This aspect is very important when we see whether there is any rotation or there is no rotation that means rotational flow or irrotational flow. So whether there is any angular deformation takes place that is very important to see based up on that we can say that the flow is rotational or the flow is irrotational.

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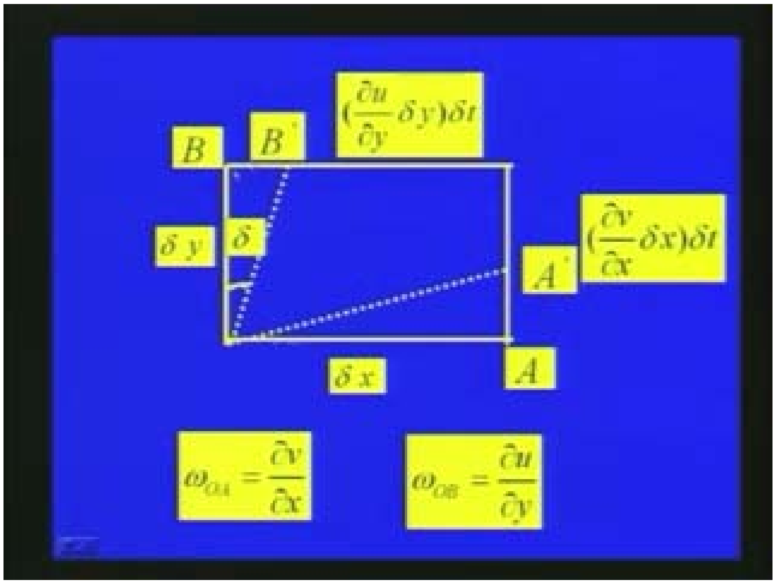

For example, if you take this fluid element, say which is in 2D we are considering, so delta x is the length of the fluid element delta y is the width of the fluid element which we are considering. Initially the shape is say just like in a rectangular shape shown here but due to deformation that means there is change of velocity in y direction for u that means del u by del y and velocity v in y direction changes with respect to x are del v by del x.

Then you can see that the there is angular deformation takes place at the rate of delta here in this figure and then the angular deformation is in this direction so A is stricter to A dash and B is V to B dash and then with respect to this we can say that there is say rotation and the rotation rate is return say omega$_{OA}$ that means the rotation of this fluid element with respect to this OA is the omega$_{OA}$ is del v by del x and the rotation of the fluid element with respect to OB if this is origin O.

So, omega$_{OB}$ is the rotation is del u by del y. If it is del u by del x or del v by del y, then it is only linear motion and there is no deformation place in the other direction as spread here. But if the deformation is with respect to other axis that means u is changing with respect to y and v is changing with respect to x for a two dimensional problem as explained here, so omega$_{OA}$ is del u by del x and omega$_{OB}$ is del v by del y.

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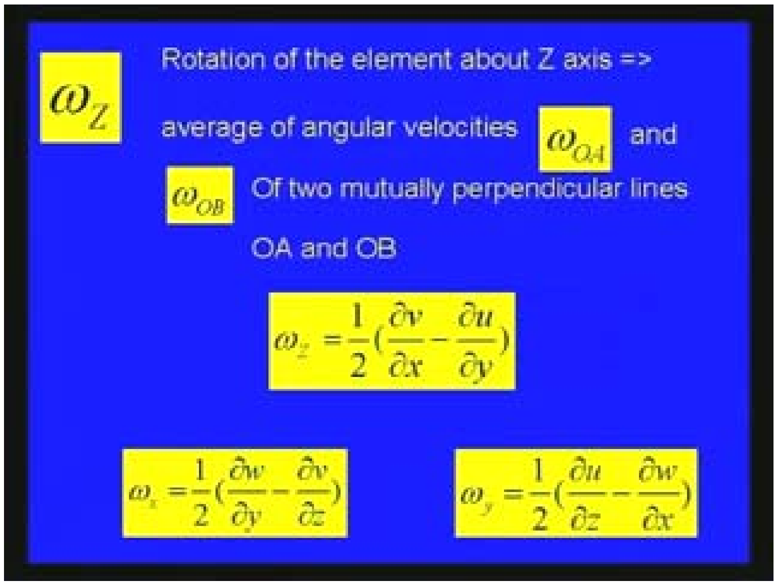

This rotation of the element say about the z axis, if we consider z axis also then the average of the angular velocity so omega$_{OA}$ and omega$_{OB}$ of two mutually perpendicular lines OA and OB we can write omega$_z$ is equal to half del v by del x minus del u by del y with respect to the angular deformation if this is xy and this is z direction, so we can see which we consider this is OA and this is OB and then deformation takes place and then omega$_{OA}$, we have already seen here omega$_{OA}$ is equal to del v by del x and omega$_{OB}$ is equal to del u by del y.

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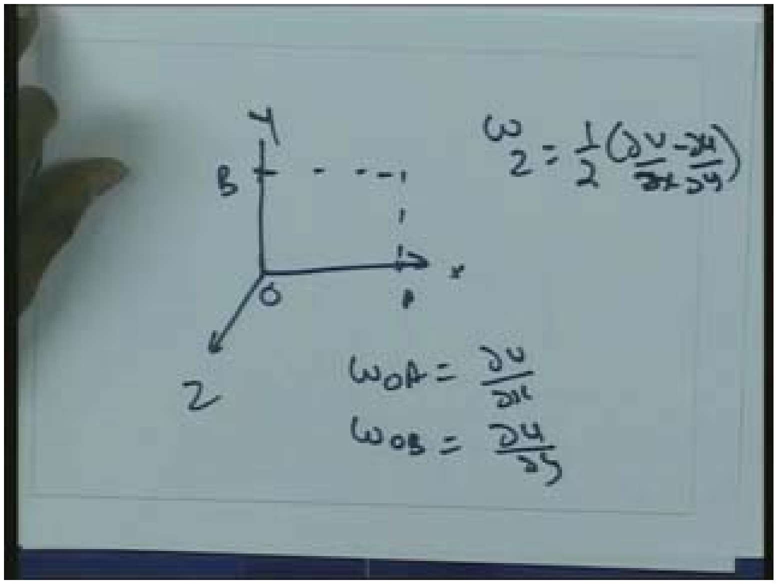

Here, we can see that omega$_z$ is the rotation of the element about z axis and then we can write this omega$_z$ which is the rotation about the perpendicular axis with respect to x and y is obtained as half del v by del x minus del u by del y, so omega$_z$ is equal to half of del v by del x minus del u by del y.

Like this we have the rotation that we defined in the direction of z that means with respect to OA and OB in this figure we have found the rotation omega$_{OA}$ and omega$_{OB}$. Similarly, like this we can derive the rotation of the element in the x axis that means omega$_x$ can be defined as half del w minus w by del y minus del v by del z and then similarly the rotation of the element about y axis will be half del u by del z minus del w by del x.

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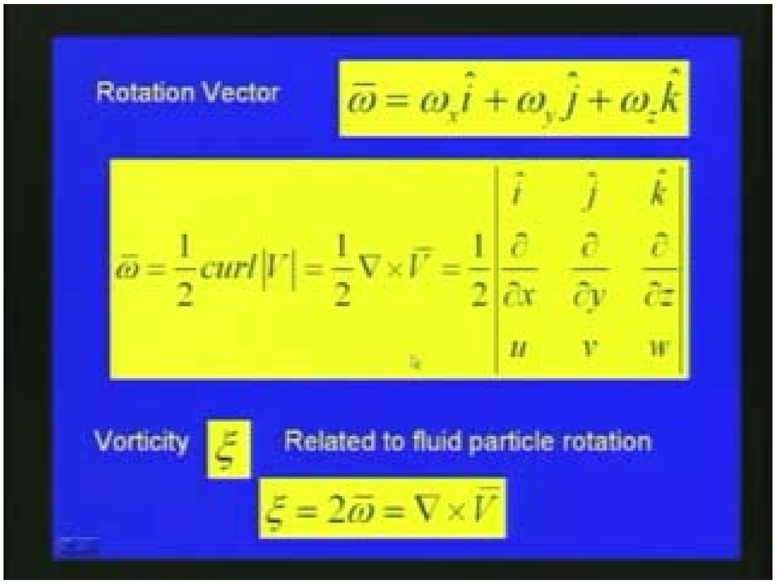

Now, the rotation of the element, finally the rotation vector for rotational flow can be represent as omega is equal to as shown in this slide we can write omega is equal to omega$_x$i with respect to ijk triode, that means with respect to the xyz direction if you define unit vector i j and k then the rotation vector can be written as omega bar is equal to omega$_x$i plus omega$_y$j plus omega$_z$k where ijk are the unit vectors in xyz direction and omega$_x$ omega$_y$ and omega$_z$ are the rotation components in xyz direction as we find earlier.

The rotation vector finally we can write as half curve of the velocity vector written like this so that is equal to half del cross V bar, that is equal to half of say with respect to ijk del by del x del by del y del by del z of uvw so it can be written in this vectorial rotation like this and then the rotation vector can be written as a curl of the velocity vector like this. Now with respect to this rotational vector we can define a term called the vorticity and this vorticity is related to the fluid particle rotation.

So the vorticity can be defined as in the case of a rotational type of fluid, the vorticity we can define as say as psi vorticity psi is equal to two times say the omega vector so this is equal to del cross v bar, so this is called as the vorticity of the fluid moment so for rotational flow we can define the vortices as two times the rotational vector which is here or that is equal to del cross the velocity vector. This is defined as vorticity. So rotational flow determination of the rotational vector vorticity is very important as we had defined in the previous slide, so first we are starting with respect to a fluid element and then we are giving some rotation and say for example, for the velocity change in the x direction u with respect to del u by del y and v is with respect to del v by del x.

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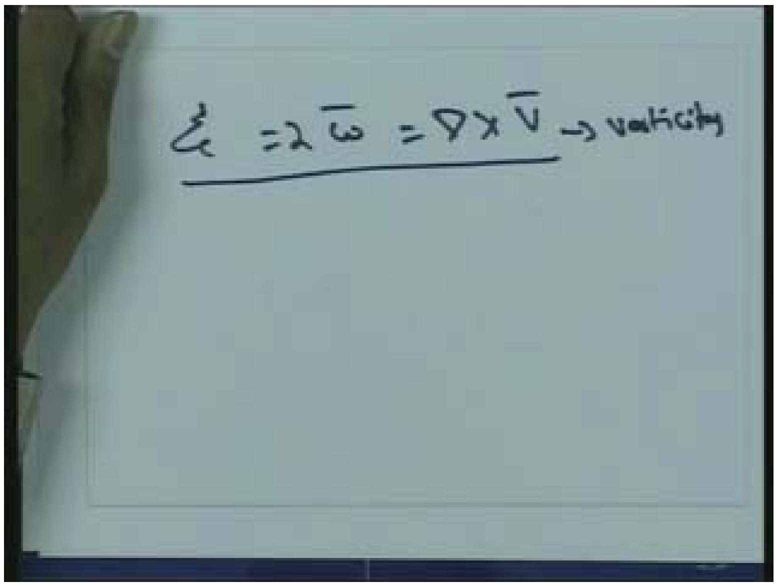

Like that we are taking and then finally we are defining the rotational vector with respect to z axis as half of del u by del x minus del u by del y and the rotational vector in the direction of x axis half of del w by del y minus del v by del z and finally the rotation in the y direction is half of del u by del z minus del w by del z del w minus del x and then finally the rotational vector is defined and finally we are defined the vorticity with respect to this rotational vector as two times of omega bar or del cross v.

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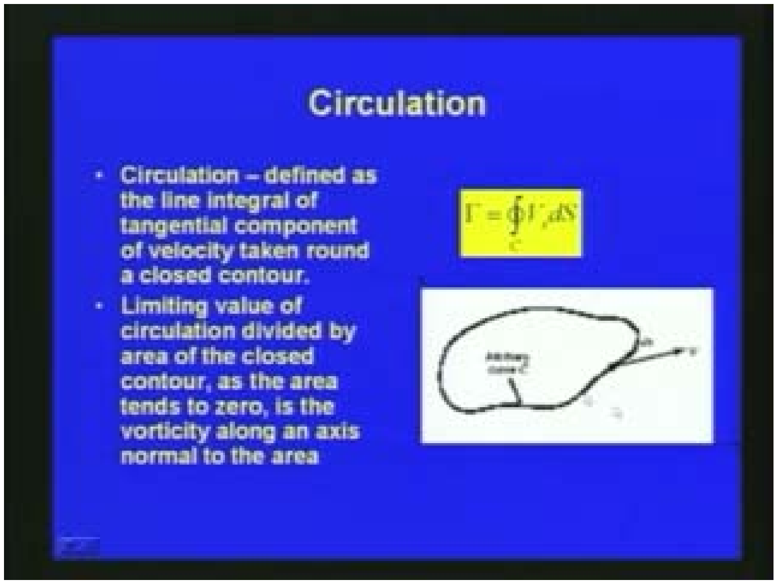

Now, with respect this say wherever rotational flow is concerned with respect to the rotation and vorticity we can also define another term called Circulation. So, circulation is defined as the line integral of the tangential component of velocity taken round a closed contour. If you consider a closed contour, the term circulation is defined as circulation is equal to capital gamma is equal to the integral $V_S dS$.

So, with respect to this figure it is defined here, we are considering say a closed contour and there circulation is defined as the line integral of the tangential component of the velocity taken. The limiting value of circulation divided by area of the closed contour, as the area tends to 0, is the vorticity along an axis normal to the area. So with respect to circulation also we can define the vorticity, that means, it is the limiting value of circulation divided by the area of the closed contour as the area tends to 0 is the vorticity along an axis normal to the area.

As for as rotational flow is concerned, the rotational say vectors $omega_x$ $omega_y$ $omega_z$ and then the vorticity and then circulation is very important term with respect to this. generally we will be describing the rotational fluid motion.

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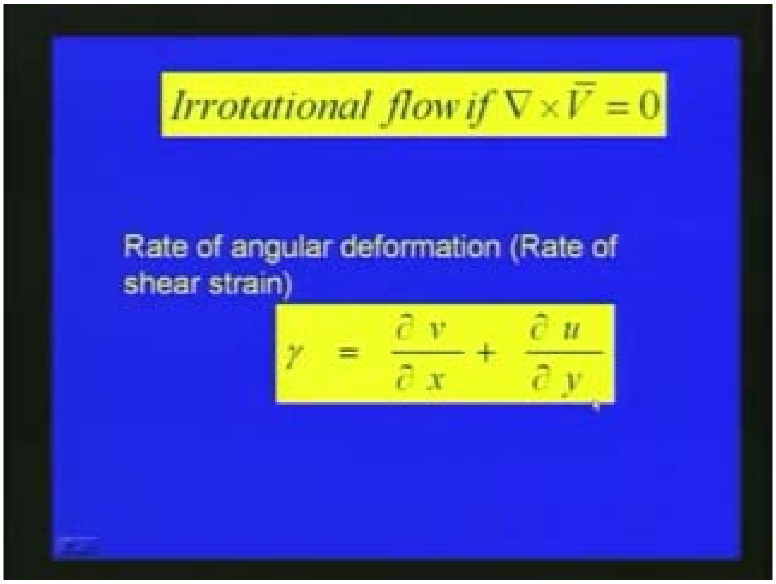

Now, after this rotational flow, we discuss about the irrotational flow. Irrotational flow that means here as we have seen in the previous slide, here in the case of irrotational flow there is no scope for rotation, that means fluid is only in the direction of the changes which is only in the direction of the velocity changes takes place in the direction of x for velocity component u and say in the direction of y for velocity component v and in the direction of z for velocity component w.

So, irrotational flow where we can define this del cross v bar is equal to 0, that means the rate of angular deformation or rate of shear strain say here this gamma is del v by del x plus del u by del y so there is no rate of angular deformation rate of shear strain is to be neglected, this tend to 0. So, the irrotational flow is defined where cross v is equal to v bar is equal to 0.

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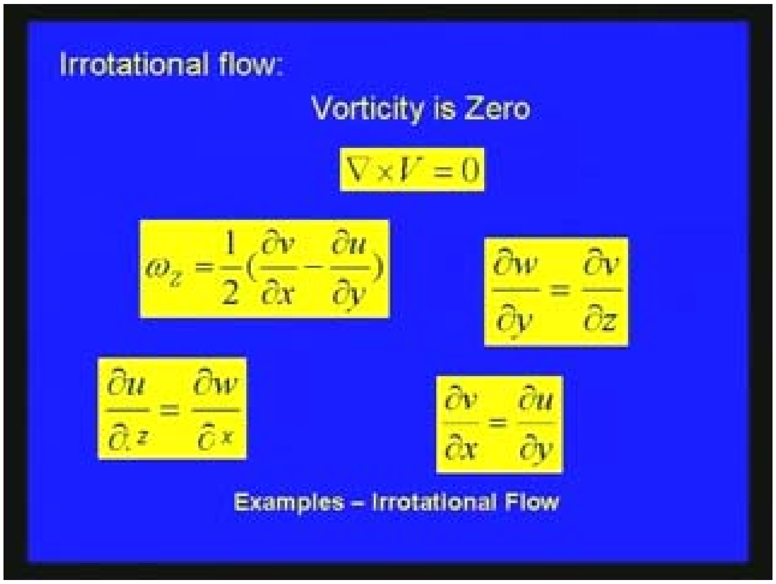

So, for the irrotational flow we can see that we do not have to consider the rotational components like omega$_x$ omega$_y$ omega$_z$ as we have seen earlier, so for irrotational flow the vorticity is 0 or del cross v is equal to 0 that means if we consider the rotational component z direction so here, all the rotational components are 0. So omega$_z$ is equal to 0, that means say half del v by del x minus del u by del y is equal to 0 or we can write say del v by del x is equal to del u by del y.

Similarly, we can write with respect to omega$_x$ omega$_y$ and omega$_z$, that means for irrotational flow we can write for irrotational flow omega$_x$ is equal to 0 omega$_y$ is equal to 0 and omega$_z$ is equal to 0, this omega$_z$ is equal to 0 this gives say as we have defined omega$_z$ is half del v by del x minus del u by del y that gives say this is equal to 0 that means we can write del v by del x is equal to del u by del y.

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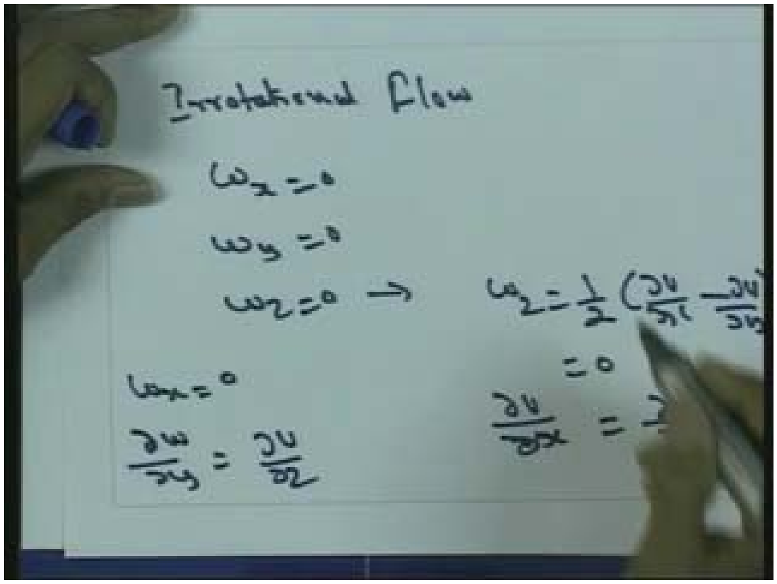

The other components like say as we have defined earlier, if you can take the component omega$_{x}$ is equal to 0 that will give del w by del y is equal to del v by del z and similarly if we consider the component in the direction on omega$_{y}$ is equal to 0 del u by del z is equal to del w by del x, so here for irrigational flow all the rotational components are 0s.

So that we can write del w by del y is equal to del v by del z and then del v by del x is equal to del u by del y and then del u by del z is equal to del w by del x. Like this we can define all the terms say when omega$_{x}$ is equal to 0 del w by del y is equal to del v by del z when omega$_{y}$ is equal to 0 say del u by del z is equal to del w by del x and when this omega$_{z}$ is equal to 0 then del v by del x is equal to del u by del y, that means all the vorticity components are 0.

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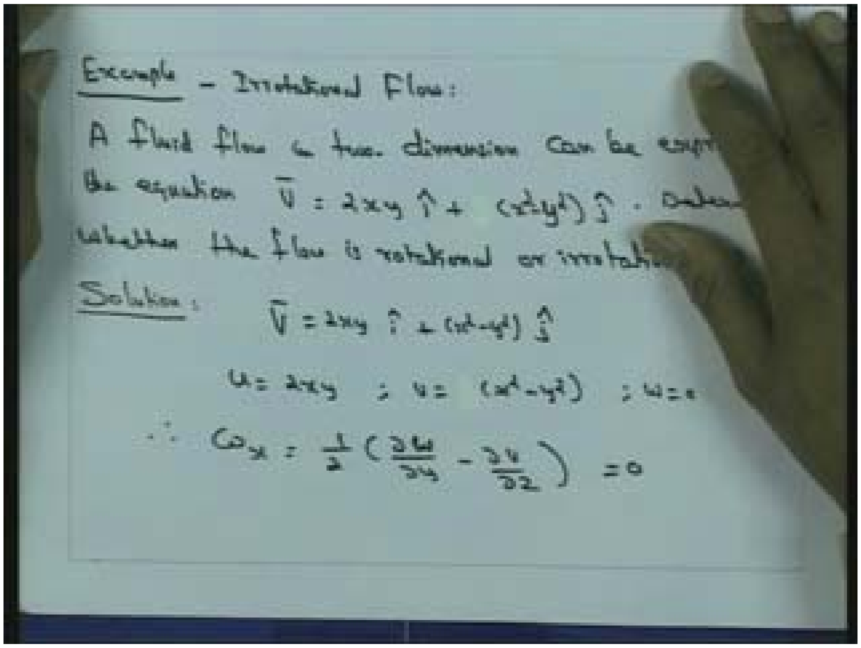

Now, with respect to this we will just see one example with respect to this irrotational fluid flow, we will discuss a small example here, so here we want to see there is a two dimensional flow for here the example we are going to discuss is the irrotational flow example. For irrotational flow, the fluid flow in two dimension can be expressed by the equation v bar is equal to 2 xy I plus x square minus y square j so this 2D flow v bar is 2 xy i hat plus x square minus y square j hat, so we are determine whether the flow is rotational or irrotational.

Since the velocity vector is now defined as 2 xy i hat plus x square minus y square into j hat, this is the velocity in two dimension, so u can be in the velocity in x direction can be written as say u is equal to 2 xy and then v is equal to x square minus y square, here we are considering the two dimensional flow, so w is equal to 0, u is equal to 2 xy v is equal to x square minus y square and w is equal to 0. Now, we will take each component of what we have already defined rotational components omega$_x$ omega$_y$ and omega$_z$ as per our definition on omega$_x$ is equal to half del w by del y minus del v by del z is equal to 0.

Since the flow we want to show whether the flow is rotational or irrotational to show that if it is totally irrotational, then all the components so omega$_x$ omega$_y$ and omega$_z$ should be 0. For this particular problem is concerned, here you can see that w is already 0 and then we don’t consider z direction since z component is not there.

So, the flow is in two dimension so that we can see that omega_x is equal to half omega del w by del y minus del v by del z so w is already 0 so this component will be 0 and del v by del z since there is no variation with respect to z so this will be also 0 so that we can show that omega_x is equal to 0. Similarly, we will now do it for omega_y.

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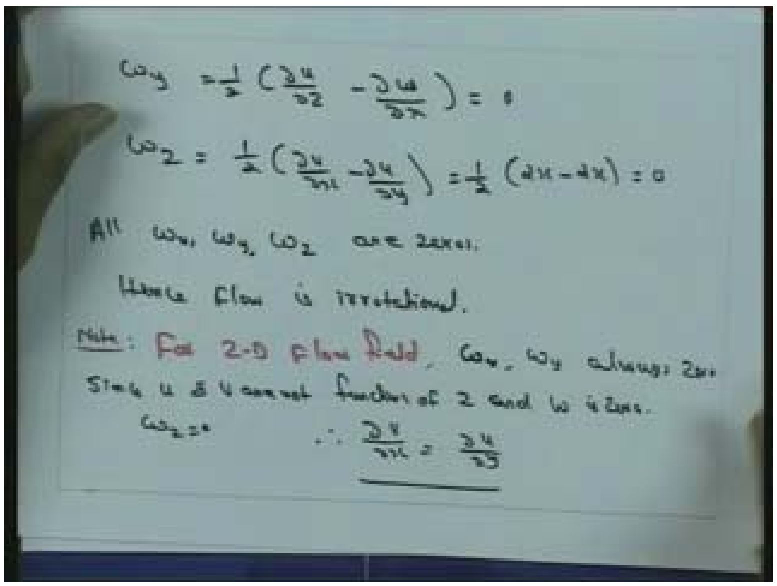

So, omega_y as per our definition is omega_y is equal to half del u by del z minus del w by del x, here again del u by del z, there is no change of velocity in the direction of z, this also tends to 0 and w is already 0. Finally del omega_y is equal to 0 and then the third rotational component omega_z is defined as half of del v by del x minus del u by del y, so that we can write this is equal to, so here the v is as far as our definition is concerned u bar is equal to v bar is equal to 2 xy i plus x square minus y square j.

So u is defined as 2 xy and v is defined as x square minus y square so that del v by del x if you differentiate v with respect to x, del v by del x we will get as say when we differentiate this will be 2 x and the del u by del y will be again 2 x, so half of del u by del x minus del u by del y it will be ha lf of 2 x minus 2 x so that also is equal to 0. So that finally we can see that all the rotational component omega$_x$ omega$_y$ and omega$_z$ are 0 and here the flow is irrotational.

Here, we have approved for the given velocity for two dimensional case say we have shown that omega$_x$ is equal to 0 rotational component omega$_x$ is equal to 0 omega$_y$ is equal to 0 and omega$_z$ is equal to 0 and hence we can conclude that the flow is irrotational flow and then say for two dimensional flow field you can see that omega$_x$ and omega$_y$ always 0 since say as for the definition say for two dimensional flow w is the velocity component direction w is equal to 0 and then other say the velocity variation of v as per our definition here velocity variation v with respect will also 0.

For two dimensional flow omega$_x$ and for omega$_x$ is equal to 0 so similarly with respect to omega$_y$ direction omega$_y$ del w since w is 0 del w by del x already 0 and since there is no variation of u with respect to z so del u by del z is also 0 so for two dimensional flow we can say that omega$_x$ or omega$_y$ are always 0, since u and v are not functions of z and w is 0. Finally for irrotational flow, for two dimension flow we have only to check generally omega$_z$ is whether 0 or not. If it is irrotational flow, then we can say that del v by del x is equal to del u by del y, this is the observation for two dimensional irrotational flow. If it is flow is rotational, there will be the values for omega$_x$ omega$_y$ and omega$_z$ so like this we can differentiate whether the given flow is rotational or irrotational depending up on the problem.

This is about the rotational flow and irrotational flow, so most of our say when we discuss about the fluid kinematics it is important that we should determine whether the flow is rotational or irrotational since have to changes as per the principle so the theories which will be using well if it is rotational flow then it will be different and if it is irrotational flow it will be different and for rotational flow as we have seen we have to determine the rotational flow component omega$_x$ omega$_y$ and omega$_z$ and also we have to determine the vorticity and circulation. But as far as irrotational flow is concerned we are dealing with only in the variation with respect to xyz direction with respect to del cross v is equal to 0 and then we can define the various terms as far as irrigational flow is concerned.

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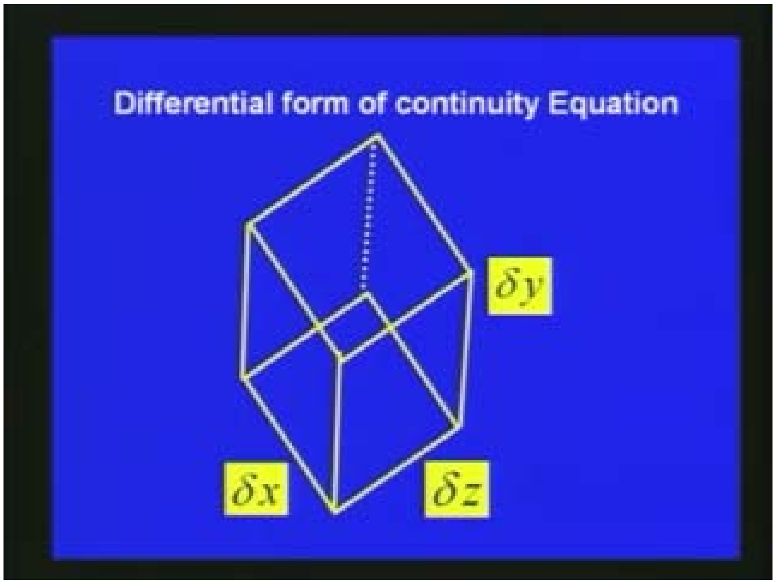

So before going with respect to the rotational flow further will be discussing about the potential flow later but before going to the potential flow here we will discuss the differential form of the continuity equation. We have already seen earlier that when we derive the general equation, fundamentally given equation, we can either use the differential approach or integral approach depending upon problem concerned.

For the conservation of mass or the continuity equation, in the last lecture we have already discussed about the integral approach how to derive the continuity equation and then we have also investigated some problem of the integral approach of the continuity equation.

Now, we will discuss about differential form of the continuity equation. To derive the differential form of the continuity equation, let us consider a fluid element just like in the slide so here xyz direction the velocities are u p and w and so let us consider the fluid element of delta x where delta y by delta z.

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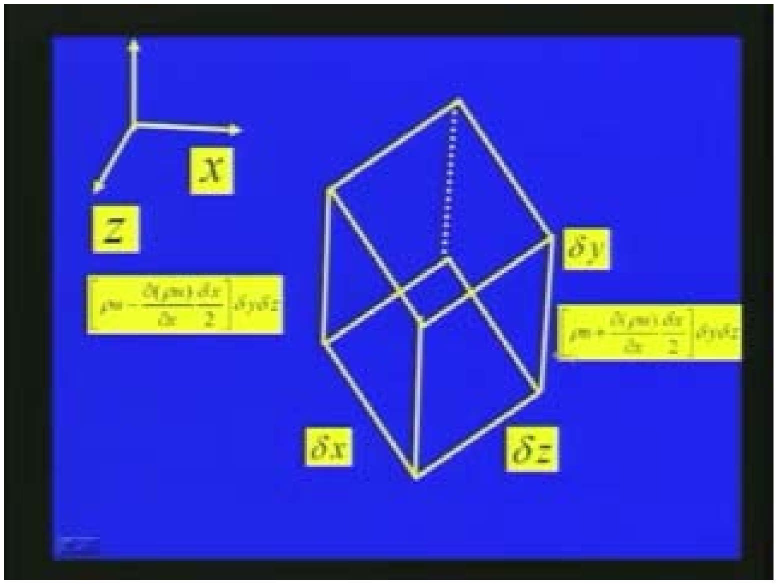

So with respect to xyz direction, let the velocity be u p and w, with respect to this say the cubic element which we are considering on this face of the flow with respect to the fluid flow which we have concerned. On the left hand side, the velocity let it be rho u minus del rho u by del x into delta x by 2 into delta y into delta z and then other side it will be rho u plus delta rho u by delta x into delta x by 2 into delta y into delta z.

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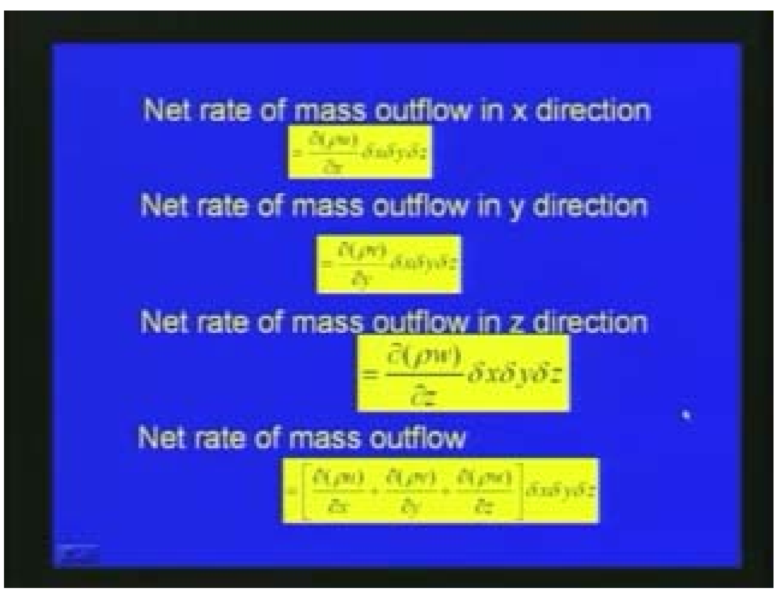

Now, we are considering the x direction, so similarly for y direction z direction also we can write and then finally we can find the net rate of mass outflow in xyz direction. If we consider the net rate of mass out flow in x direction we can write del rho u by del x into delta x into delta y into delta z as for as the fluid element which we considered here. So the rate of change of flow what is the possible outflow with respect to this element the control volume which we are considering, we can write net rate of flow will be is equal to del rho u by del x into delta x into delta y into delta z and then net rate of mass outflow in y direction can be written as del rho v by del y into delta x into delta y into delta z and similarly in z direction you can write with respect to the velocity component w del rho w by del z into delta x into delta y into delta z.

Finally with respect to the xyz direction we can add this all net rate of mass of flow we can write as del rho u by del x plus del rho v by del y plus del rho w by del z multiplied by delta x into delta y into delta z. so with respect to this, now for the system concerned or for the control volume concerned, now the mass is concerned so that we can say that now use in the equation d the total derivative of the mass of the system DM of the system by DT is equal to should be equal to 0 since with respect to the conservation of mass then the rate of change of total rate of change should be DM by Dt is equal to 0.

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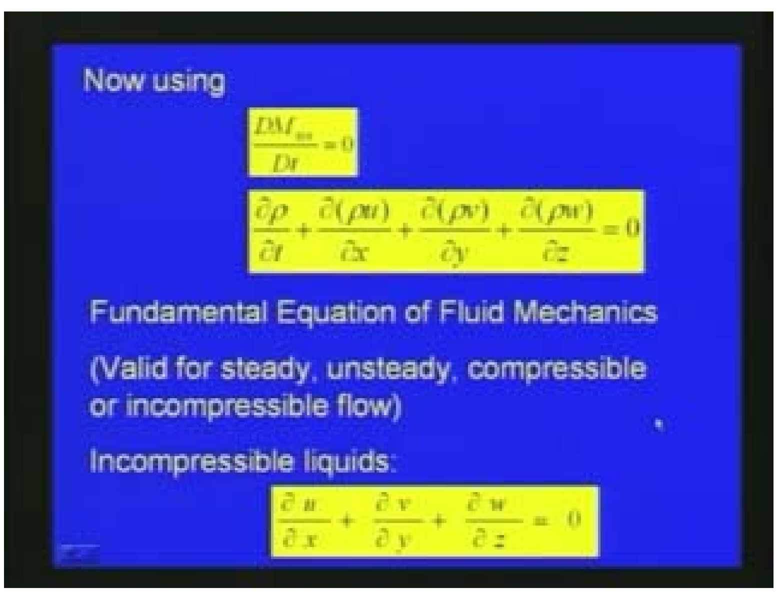

Now, if you write with respect to the earlier formations so this variations we have to consider with respect to time that is del rho by del t and then with respect to the xyz changes which we have discussed in the previous slide so finally with respect to DM sys by Dt is equal to 0 can be written as del rho by del t plus del rho u by del x plus del rho v by del y plus del rho w by del z is equal to 0, so this is called the continuity equation in the differential form so this is one of the fundamental equation of the fluid mechanics this is varied for steady, unsteady compressible or incompressible flow. This is the continuity equation using the differential form of the formulation derived based up on the conservation of mass principle. If the flow is the fluid is incompressible then we can see that there is no change in with respect to rho the density.

So that density this rho can be taken no need to consider so that there is no change with respect to time of with respect to space. For incompressible fluid, incompressible liquid we can write del u by del x plus del v by del y plus del w by del z is equal to 0, so this is the differential form of the continuity equation or the differential form of the conservation of mass as far as incompressible fluid is concerned so del u by del x plus del v by del y plus del w by del z is equal to 0 at any point of the fluid mass, fluid which we are considering for the particular domain which we are dealing with.

Earlier we have seen the continuity equation based up on the conservation of mass for with respect to integral approach therefore we have shown that A_1 V_1 is equal to A_2 V_2 where A_1 and if we consider a system like this say a fluid flow between section 1, 1 and 2, 2 with respect to the integral approach, where A_1 is the cross sectional area section 1, 1 and A_2 cross section 2, 2 and V_1 is the velocity of flow at section 1, 1 v_2 is the velocity of flow at section 2, 2 then with respect to the continuity with respect to the conservation of mass based up on the integral approach we have seen that A_1 V_1 is equal to A_2 V_2 and then here as we derived so with respect to differential approach we can say if it is 2D flow then we can write del u by del x lus del v by del y is equal to 0.

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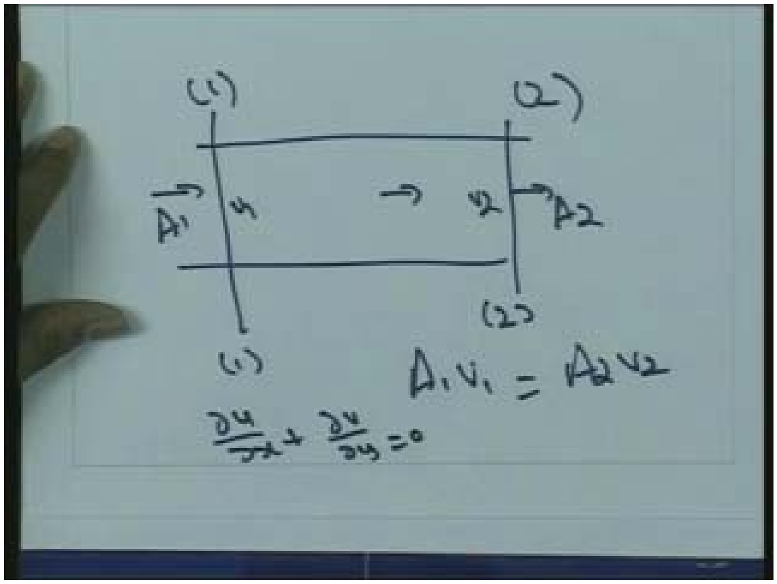

If it is 3D flow, as we have already derived it is del u by del x plus del v by del y plus del w by del z is equal to 0 so this is the continuity equation in the differential form. The same continuity equation if you consider say sometimes we have to deal with the polar coordinate system or cylindrical coordinate system in terms of the radial direction R and theta and z, in that case the continuity equation can be derived as shown in this slide.

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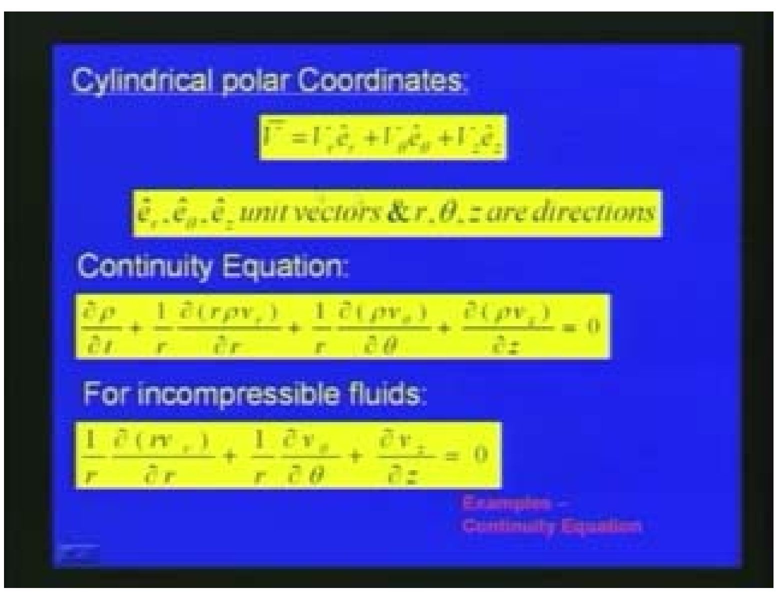

The velocity component can be written as say V bar is equal to V_r unit vector e r plus V theta unit vector e theta plus V_z unit vector e z. Here this e_r e theta e_z are unit vector with respect to r $theta_z$ and r $theta_z$ are directions. Theta is the angle and r is the radial direction z is the vertical direction, so corresponding to this as we have derived earlier in the Cartesian coordinate system we have derived the continuity equation in the cylindrical polar coordinate system as say in the final equation is del rho by del t plus 1 by r del r rho v r by del r plus 1 by r del rho v theta by del theta plus del rho v z by del z is equal to 0.

As we have seen earlier, for the incompressible fluid there is no change with respect to density rho, we can write this equation as 1 by r del r v_r by del r plus 1 by r del v theta by del theta plus del v z by del z is equal to 0 so when we are solving some particular problems where the cylindrical coordinate system should be used say for example called cylinder or in a pipe flow it is to be considering cylindrical or polar coordinate system then we can use this kind of continuity equation which is derived here for general equation and for the incompressible fluid.

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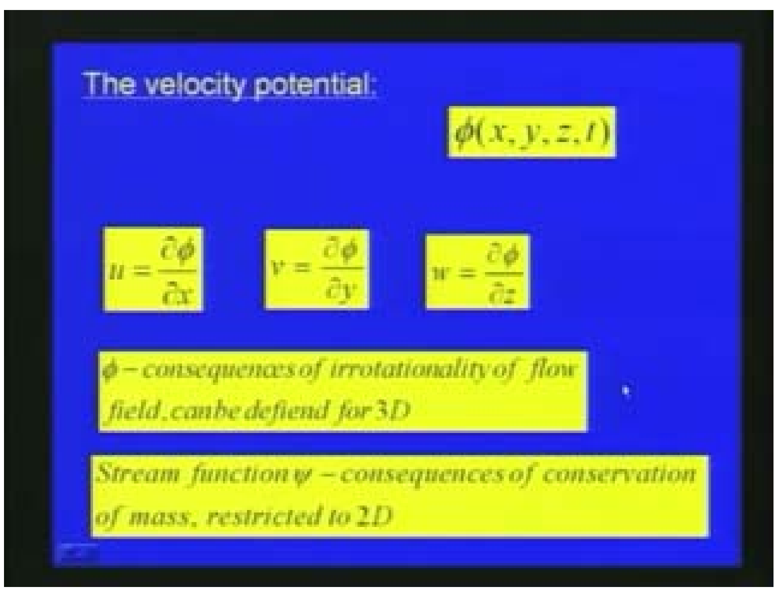

Now, with respect to this continuity equation we will just discuss one problem, so the problem here is say an example for the continuity equation in the differential form.

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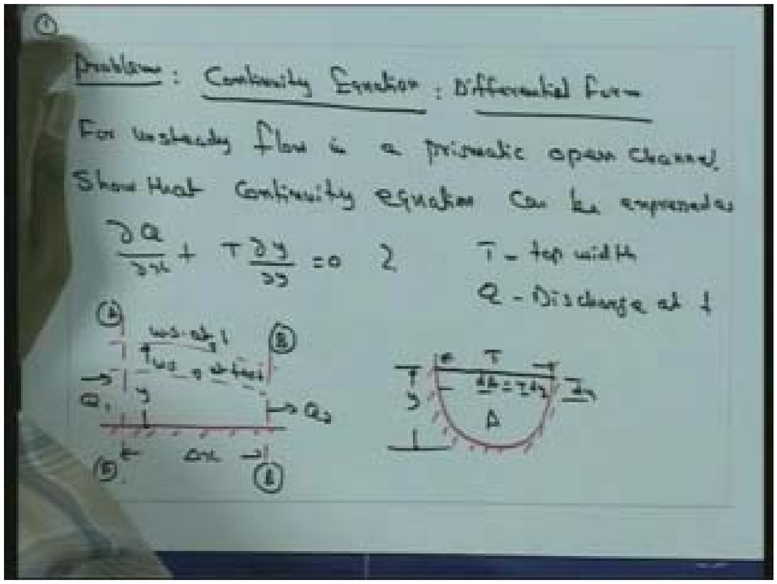

For a fluid flow in open channel we want to derive for unsteady flow in a prismatic open channel. We want to show that the continuity equation can be expressed as del q by del x plus t into del t into say want to prove that del y by del t, so this is del y by del t del q by del x plus t into del y by del t is equal to 0, where t is the top width of the channel which is considered, Q is the discharge of the time t and x and y are the directions as shown in this figure.

We want to derive the unsteady flow equation or we want to shown that if Q is the discharge flow through a channel or we want to show that del Q by del x plus t into del y by del t is equal to 0 with respect to the continuity equation. Let us consider the open channel flow which we is in a prismatic channel, so the flow depth is y and area of cross section is A and top width is t as shown in this figure and then let us consider a small strip like v I is equal to t into dy of depth y and then let us consider the two section of the open channel at AA and at BB.

Here, the discharge entering at section AA is Q_1 and passing through at section BB is Q_2 and at time t is the depth of flow is y and then it is changing at delta t say change of flow and now we are considering the section between AA and BB at distance delta x as shown in this figure.

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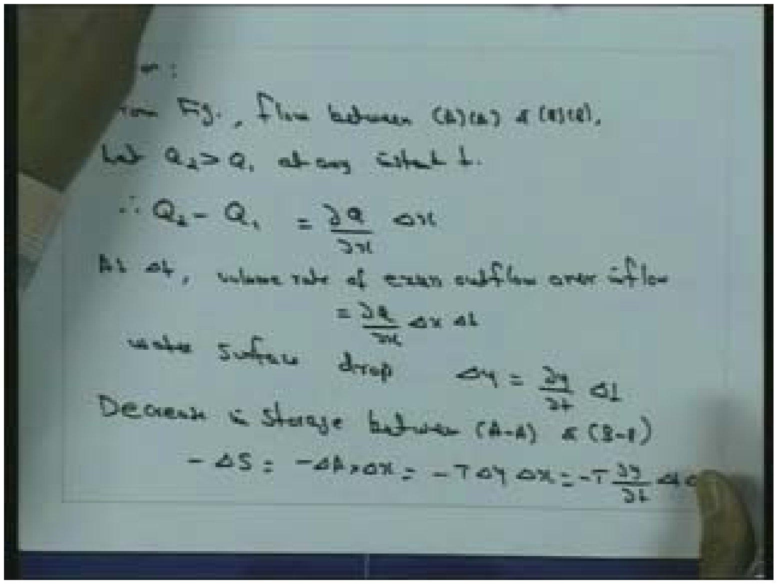

From the figure which we have seen here, from this figure we can write the flow between section AA and BB, let us assume this Q_2 is greater than Q_1 at any instant of time t. so Q_2 minus Q_1 we can write as the change on discharge like between section A and section BB so Q_2 minus Q_1 is equal to del Q by del x into delta x we are considering. Now a time step delta t, at delta t volume rate of excess outflow over inflow can be represent as del Q by del x into delta x into delta t, so delta t is the time difference delta x is the distance between section AA and BB.

The volume rate of excess outflow over inflow is del Q by del x into delta x into delta t and then the water surface drop say as shown in this figure water surface drop delta y can be written as delta y is equal to del y by del t in to delta t and finally decrease in storage between AA and BB can be written as minus delta s is equal to minus delta A into delta x so this is equal to minus t into delta y into delta x is obvious from this figure here, so this is equal to minus t del y by del t into delta t into delta x so the decreasing storage between AA you can write as delta s is equal to minus delta A into delta x so this is equal to minus t into delta y into delta x as obvious this figure here so this is equal to minus t del y by del t into delta t into delta x.

Now by continuity equation which we have derived by working the conservation of mass this decreasing in storage between A should be equal to this change in storage which is described here that is volume rate of excess outflow over inflow so we can equate both this.

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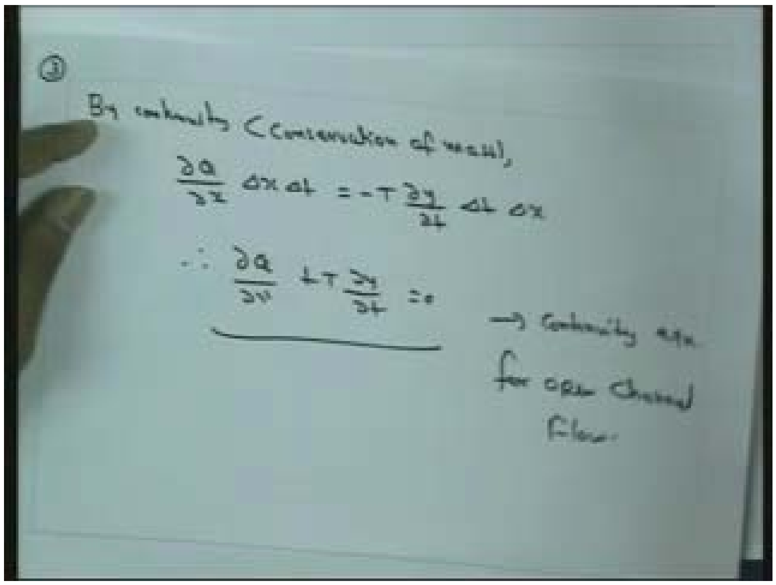

So that we can find write as del Q by del x into delta x delta t is equal to minus t del y by del t into delta t multiplied by delta x, delta x delta t be cancel and finally we can obtained del Q by del x plus t into del y by del t is equal to 0 which is what we have to ask to show. For unsteady flow in prismatic channel here we have shown that del Q by del x that means the discharge change of with respect to x del Q by del x plus t into the top width del y that means the change of depth with respect to time del y by del t that is equal to 0.

So that is continuity equation differenced form for open channel flow so that we have shown that del Q by del x is equal to t into del y by del t is equal to 0 which is the continuity equation for this is the continuity equation for open channel flow. Before proceeding to the potential flow we will solve one more example with respect to the continuity equation the differential form which we have derived here, so here the problem is for a steady state incompressible fluid flow.

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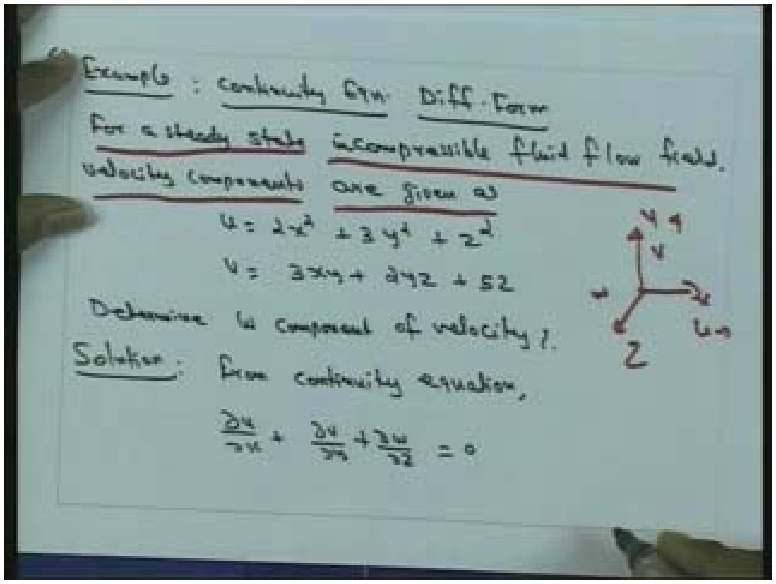

For a steady state incompressible fluid flow velocity components are given as u is equal to 2 x square plus 3 y square plus z square, if x is in the direction y and z is defined here, u is equal to velocity component u is defined as 2 x square plus 3 y square plus z square and velocity component v is defined as 3 xy plus 2 yz plus 5 z, we have to determine the w component of the velocity.

This u is in this direction v is in the y direction and the w is in the z direction, here with respect to u and v components are given with respect to continuity equation we want to determine the velocity component w. From the continuity equation which we have derived we can write del u by del x plus del v by del y plus del w by del z is equal to 0 so now with respect to since u is given in this problem.

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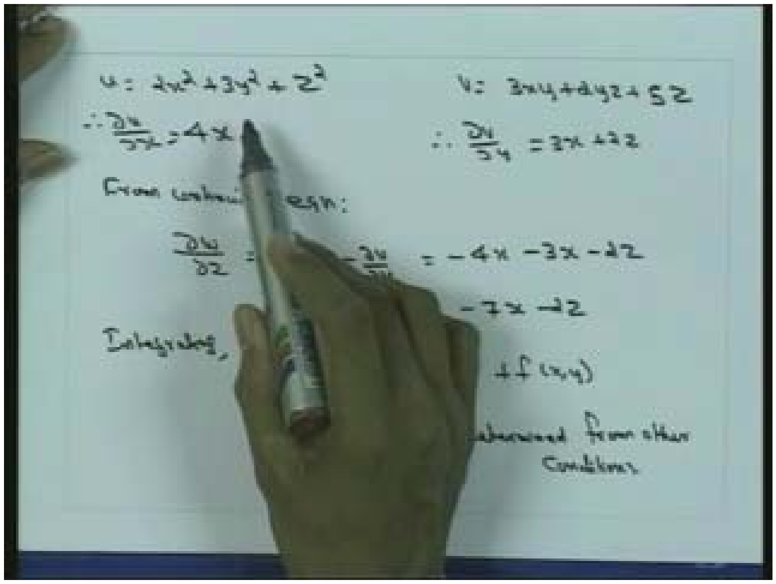

U is given as 2 x square plus 3 y square plus z square so that we can write del u by del x is equal to if you differentiate this function we will get del u by del x is equal to 4 x and it is also given v is equal to 3 xy plus 2 yz plus 5 z so that we can write del v by del y is equal to 3 x plus 2 z. Now from the continuity equation here we have written del u by del x plus del v by del y plus del w by del z is equal to 0.

We can substitute del u by del x and del v by del y so that we can obtain del w by del z, after substitution for del u by del x and del v by del y we will get del w by del z is equal to minus del u by del x minus del v by del y, this is equal to minus 4 x minus 3 x minus 2 z so del w by del z is equal to minus 7 x minus 2 z.

Now we want to determine the velocity component in the z direction w, to get w we can integrate this del w by del z, so on integration get minus 7 xz minus z square plus constant f of xy so this f of xy can be determine from the other condition which we will be given for the problem. So now from this continuity equation, differential from is used to determine the one component the z component of the velocity, but the velocity component the x and y is directions are given.

Like this we can use this continuity equation the differential form for various problems as one of the fundamental equation. Now we have seen the rotational flow irrotational flow and the continuity equation in the differential form. Now we will go to the potential flow, we discuss the mass of the potential flow. Before going to the potential flow let us see what is the velocity potential?

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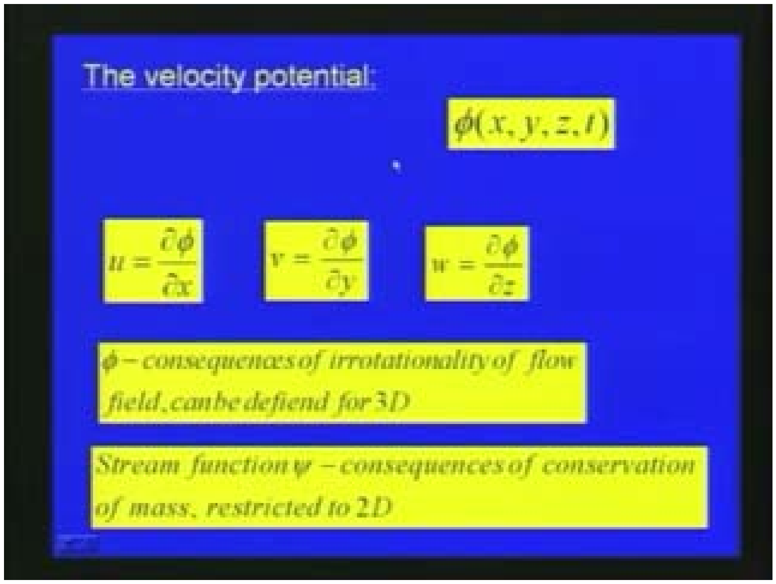

Generally as for as fluid mechanics concerned say if we define terms like velocity potential function etc., these can be used to represent the fluid flow in a terms expressed in fluid kinematics. Here we are going to define the velocity potential and then its various application of velocity potential. Generally it is expressed as phi as a function of xyz and t.

In three dimension xyz and with respect to time so this is the velocity potential is represented as phi and it is defined, so the velocity potential is defined as the velocity component in x direction u is equal to del phi by del x that means the variation of phi the velocity potential with respect to x and velocity component in y direction, if we take the differential of say the velocity potential y direction that velocity component y direction and the velocity component w z direction w is represent as the del phi by del z.

The velocity potential is defined such that the velocity in xyz direction, if we consider the fluid flow and here if xy and z are the Cartesian coordinate system and then the velocity in x direction is u, velocity in y direction is v and velocity in z direction is w. So we are

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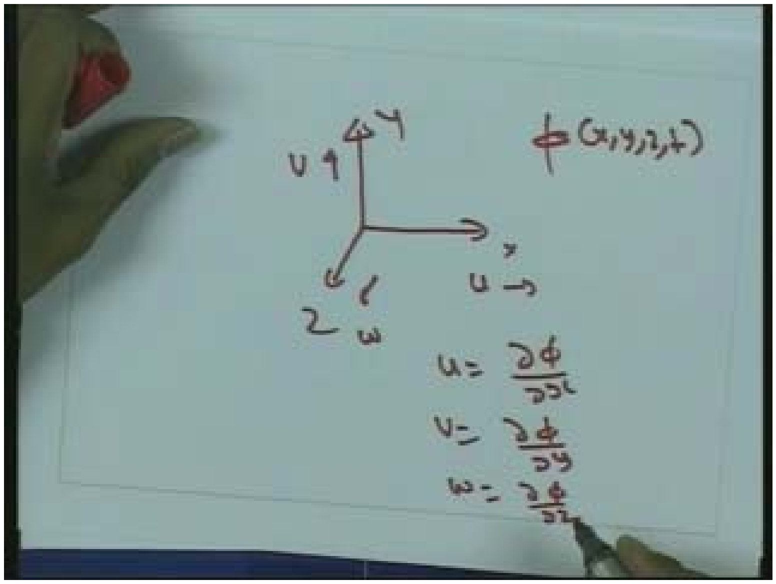

defining term called velocity potential which is varying with respect to xyz and t such that this velocity variation in x direction u we can write as del phi by del x and velocity variation y direction we can write as del phi by del y and velocity variation z direction we can write as del phi by del z. That is the way which we defined this velocity potential the consequence of rotationality or the flow field and it can be defined for 3D flow. We can also define as tem called stream function which is consequence of conservation of mass restricted to 2D, so that will be discussing later so the velocity potential here phi is defined such that velocity component in xyz direction u p and w can be defined as del phi by del x del phi by del y and del phi by del z.

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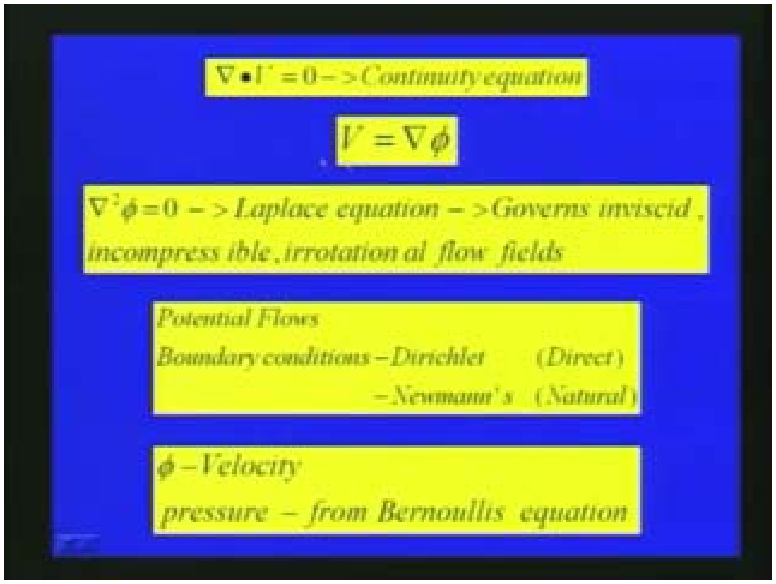

Now if we use the continuity equation the differential form of the continuity equation which we have derived earlier so that for in 3D we have derived that del u by del x plus del v by del y plus del w by del z is equal to 0 for xy and z direction.

So, now if we apply this say we have also defined U is equal to del phi by del x, v is equal to del phi by del y and w is equal to del phi by del z. So we will substitute for uvw in this equation of the continuity equation so if you substitute here u v and w, that we will get as del square phi by del x square plus del square phi by del y square plus del square phi by del z square so that is equal to 0.

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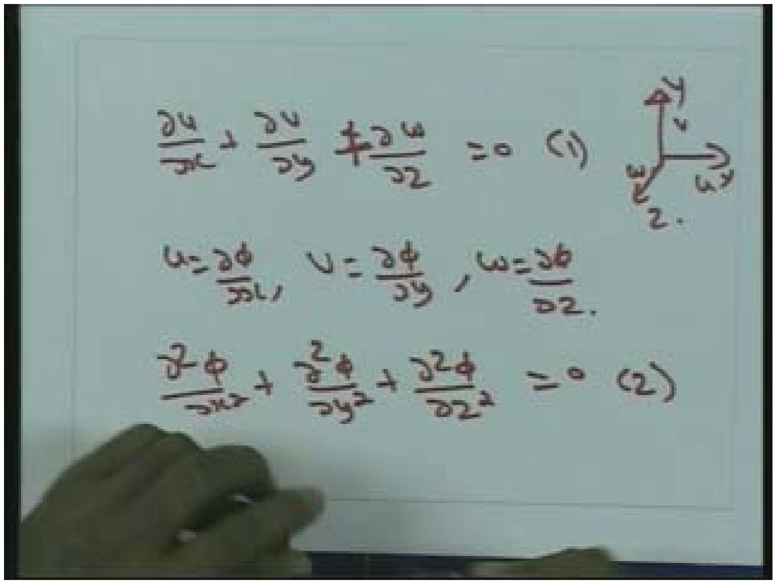

This equation is called the Laplace equation represented as in this slide del square phi is equal to 0, it is Laplace equation and it governs inviscid incompressible irrotational of flow field so the potential this kind of flow is called potential flow and the theories related to this kind of flow is called potential flow theory and the govern equation for this inviscid or nonviscus incompressible fluid flow irrotational inviscid incompressible and irrotational flow field is the Laplace equation defined as l square phi is equal to 0 as derived here.

So, this type of flow is called potential flow and for this kind of flow we can have say if we consider any domain where we are considering the potential flow, the given equation is del square phi is equal to 0 and then say for example, if we consider the flow in a homogeneous isotropic aciform system like this, we can have two types of boundary conditions. Now the given equation is defined and we can have two types of boundary conditions generally defined are: one is the Dirichlet boundary conditions say which is also called Direct boundary conditions. So here we can describe say the potential phi is equal to phi_1 bar and on this direction at this place we can phi is equal to phi_2 bar and then another type of boundary conditions we can also defined del phi by del n is equal to this direction imperil neighbour[49:35] there is no flow, so here also del phi by del n bar is equal to 0.

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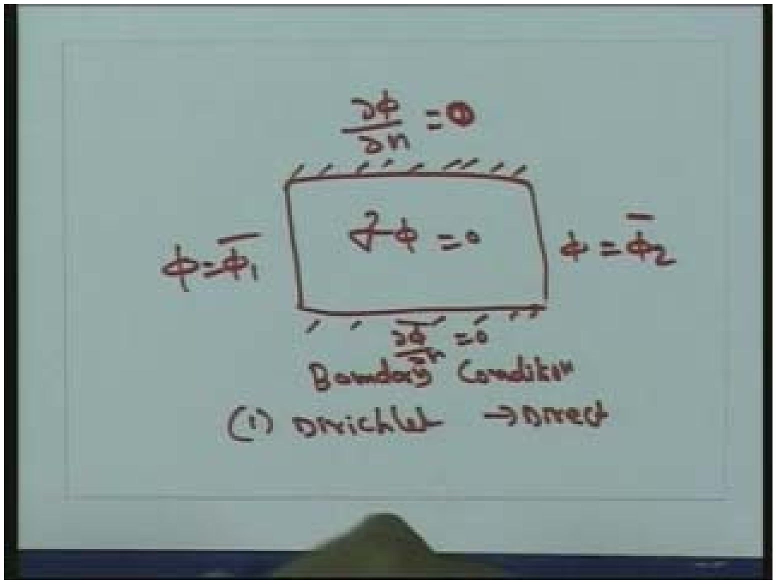

So two types of boundary conditions can be defined this kind of problem, one is the Dirichlet boundary conditions and then Newman boundary conditions and sometimes also mixed form boundary conditions can be used and now this velocities to obtained from the expression from the potential which is defined from the Laplace equation and then the pressure flow can be obtained from the Bernoulli's equation it should be discussing later part so for the potential flow the velocities obtained from the laplace equation and the pressure is obtained from the Bernoulli's equation.

Potential flow as we have defined it is inviscid this potential flow is used for inviscid incompressible and it is irrotational flow and the flow fields are governed by the Laplace equation and called as potential flows and the lines are constant potential is called the equipotential lines and it forms the orthogonal grids with stream lines to form a flow nets which will be discussing later.

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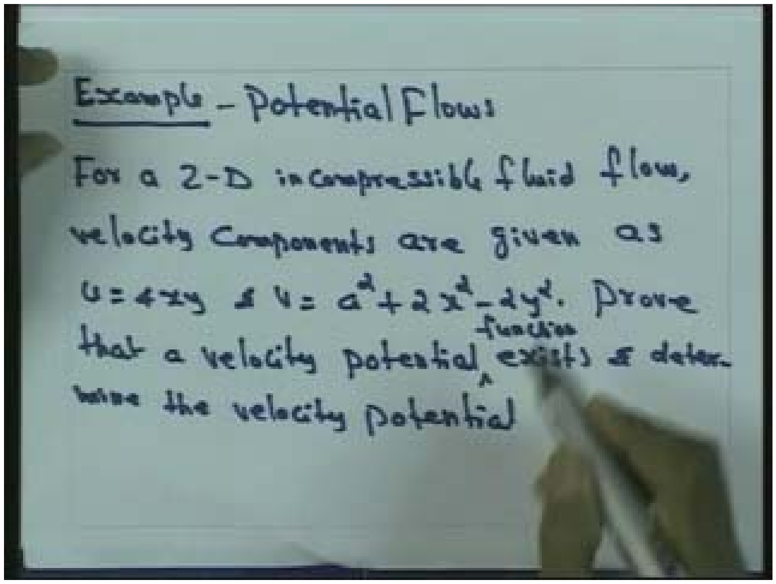

With respect to the potential flow we will just discuss a small example here a potential flow for 2D incompressible fluid flow velocity components are given as u is equal to 4 xy and v is equal to a square plus 2 x square plus 2 y square we have to show that velocity potential function x is and we are determine the velocity potential, so for this problem the potential phi exists for irrotational flow.

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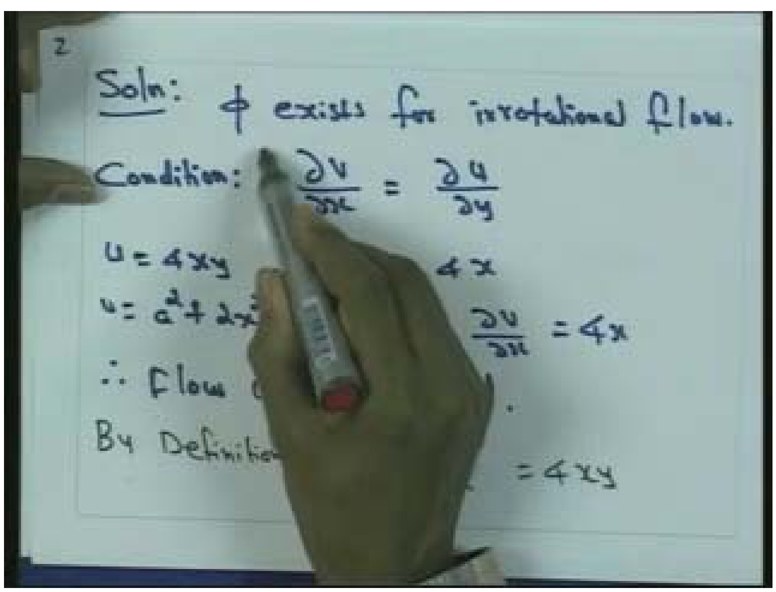

Only the condition is del v by del x is equal to del u by del y, so with respect to this now we can show that u is equal to u is given as 4 xy and if you differentiate with respect to y del u by del y is equal to 4 x with respect to when you differentiate and this function v when we differentiate del v by del x is equal to again 4 x.

So that we can see that del v by del x is equal to del u by del y that flow is irrotational and then as per the problem we want to determine that first we have to show that velocity potential function is exists and then since the flow is irrotational, the velocity potential is exists and then we have to determine the velocity potential. By definition u is defined as del phi by del x and u is here defined as 4 xy and then phi is also equal to 2 x square y plus say phi is d phi by d x.

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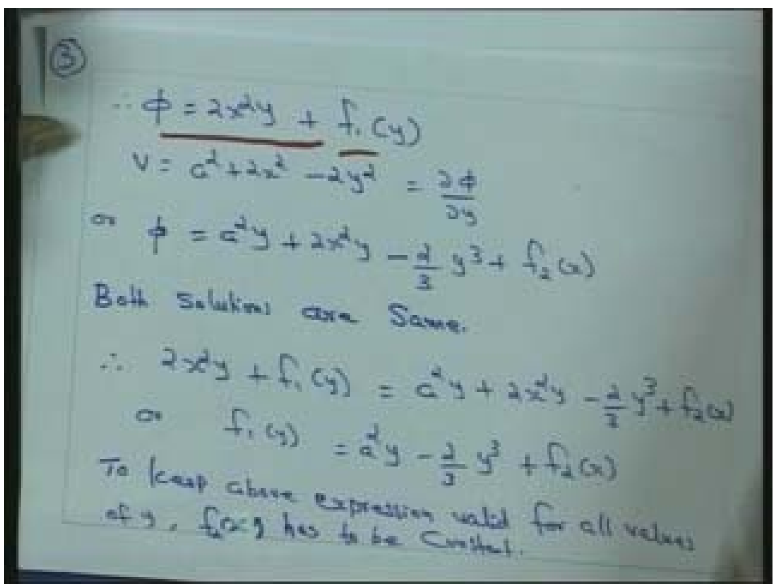

So we integrate this we will get say here one integration will get 2 x square phi plus f_1 y and then V is defined as a square plus 2 x square minus 2 y square and which is equal to del phi by del y or integration of this function phi is equal to a square y plus 2 x square y minus 2 by 3 y cube plus f_2 x and both solutions should be same since phi is obtained with respect to x here phi is obtained as 2 x square y plus f_1 y and with respect to v we got phi is equal to this function in a both solution should be same since phi is same.

So we can write 2 x square y plus f_1 y is equal to a square y plus 2 x square y minus 2 by 3 y cube plus f_2 x or we can write f_1 y equal to you get f_1 y is equal to a square y minus 2 by 3 y cube plus f_2 x, so to keep the above expression valid for all values of y it should be f_2 x has to be constant in this equation. So, finally we can write the velocity potential phi is equal to a square y plus 2 x square y minus 2 by 3 y cube plus constant.

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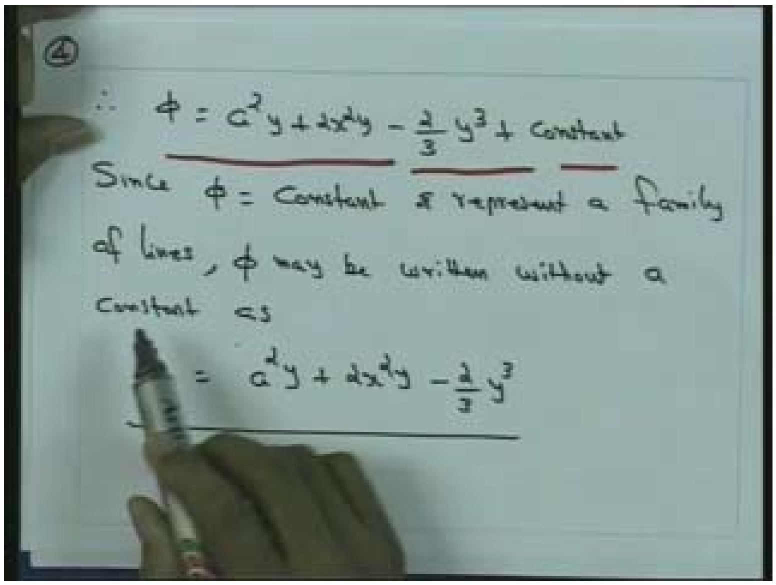

And since phi is the velocity potential constant and represent a family of lines, phi may be written without a constant as finally phi can be written as a square y plus 2 x square y minus 2 by 3 y cube. So like this the definition of potential we can use to determine the velocity or the velocity is given we can determine the potential function and this is valid the potential flow is valid for inviscid incompressible and irrotatinal flow and the field are given by Laplace equation and the flows are called the potential flow. So, further we will be discussing about the potential flow and related theories in the next lecture.