# Fluid Mechanics

**Prof. T.I. Eldho**
**Department of Civil Engineering**
**Indian Institute of Technology, Bombay**

## Lecture – 7
## Kinematics of Fluid Flow

Welcome back to the video course on fluid mechanics. In the last lecture we were discussing about the kinematics of fluid flow. So in the kinematics of fluid flow as we discussed, we are studying fluid mechanics without much consideration to the forces up on which the fluid flow is governed. So, without much concern to the forces, we are trying to derive various equations and various theories as far as the fluid flow is concerned.

We have already seen the various velocity field then the acceleration field and also we have seen how the flow can be expressed in one dimension, two dimension and three dimension flow that can be steady state or unsteady state; all these aspects we have seen in the previous lecture, also we have seen the fluid flow can be described in terms of either lagrangian description or eulerian description.

In the lagrangian description, we are just tracking certain fluid particles and seeing with respect to time what happens to those fluid particles. But in eulerian approach what we are doing is we are taking a particular point or particular section of the fluid flow and then with respect to space and time we are just taking what is happening to the fluid particles passing through the particular point or section.

Generally, in fluid flow problems, we are generally using eulerian approach. Since it is much easier and we are much interested what happens to the fluid flow at the particular section or particular point. So, we have also seen that as far as the fluid flow is concerned, we can describe with respect to the acceleration is concerned we can have a local acceleration and also we have conductive acceleration. So total acceleration with respect to the local acceleration, conductive acceleration and with respect to this we have also discussed about some numerical examples.

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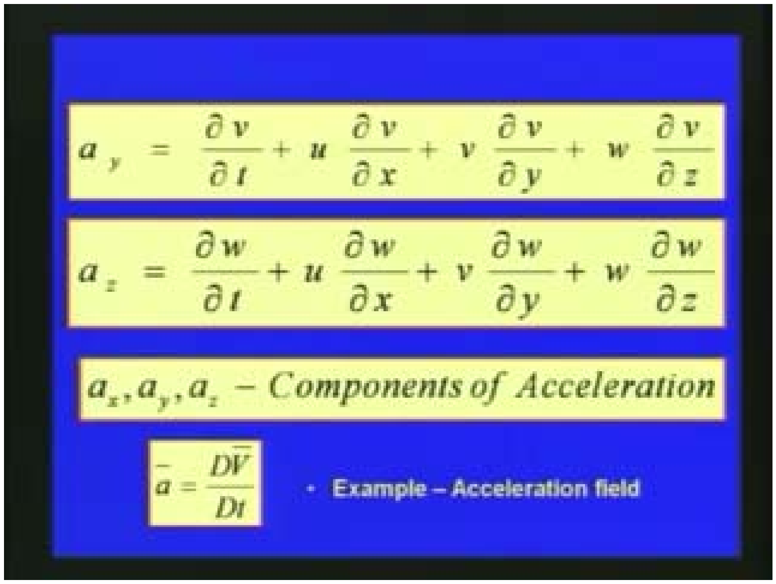


And also we have seen when we discuss the fluid flow parameters we will be describing the time rate of change for given particle with respect to Local and with respect to conductive cases as shown in this slide and this is called as the material derivative.

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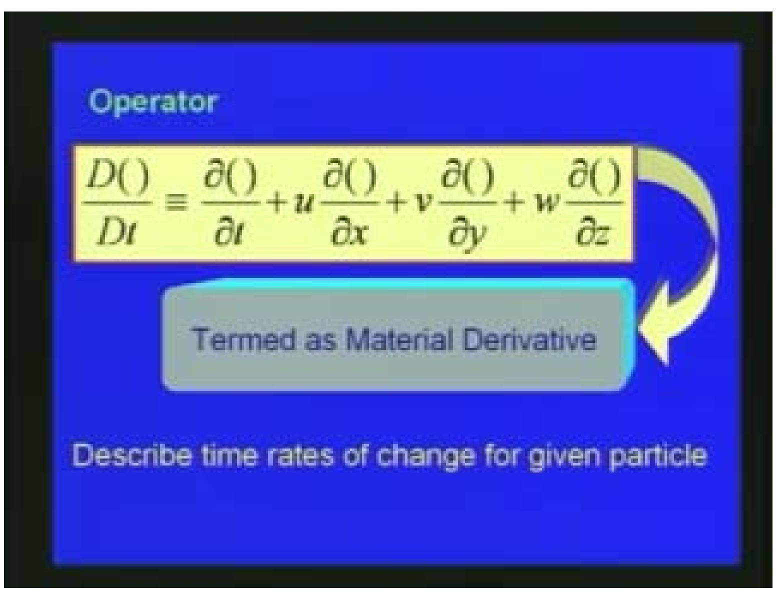

These things we have discussed in the last lecture, now we have seen that in the fluid flow can be described in three dimensions with respect to x, y and z coordinates and with respect to time, but many times it will be very convenient if we can describe fluid flow with respect to the stream lines, since stream line as we have seen earlier, stream line is a continuous line drawn tangential to the velocity vector at every point that is known as stream line.

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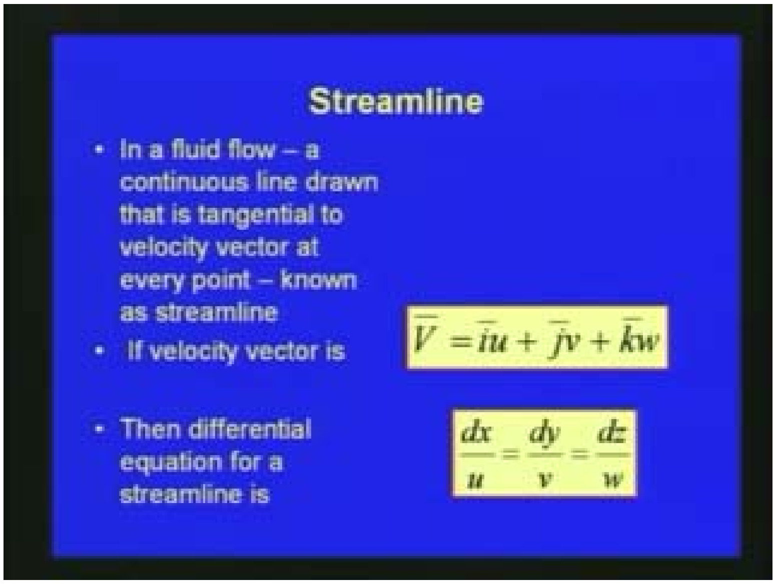


Since the flow is with respect to the direction we are describing the stream lines, it is always advantages if we can describe the fluid flow with respect to the stream line coordinate system. If the velocity vector is defined as v is equal to the unit vector I into u plus unit vector j into v plus unit vector k into w in three dimensions, then the differential equation for stream line we can defined as dx by u is equal to dy by v is equal to dz by w. So this is as far as the three dimensional flow in x y z directions the velocity component u v w are concerned.

The differential equation for stream line is generally described as dx by u is equal to dy by v is equal to dz by w, so the stream line is the continuous line drawn that is tangential to the velocity vector at every point. So this will be very usefully if we can say most of the fluid flow if you can describe in terms of the with respect to stream line coordinates it will be very advantages in many of the fluid flow problem.

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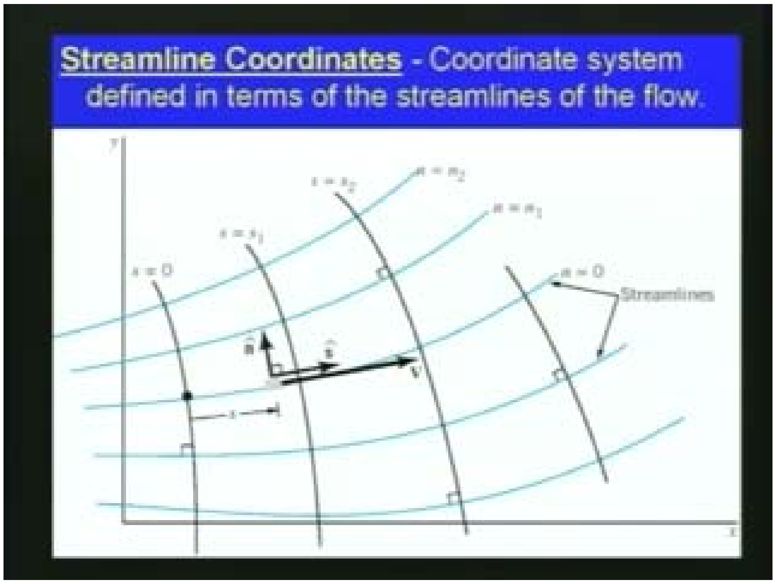


Here in this slide the stream line coordinates with respect to stream line coordinates the flow is described so stream line is a coordinate system defined in terms of the stream lines of the flow, here in this slide you can see this stream lines are drawn with x and y axis in two dimensions. Here, the stream lines shows with respect to flow stream lines are shown here with respect to n is equal to $n_1$ $n_2$ like that, so its normal direction with respect to that is shown as $s_1$ $s_2$ like that so you can see just like a flow net where the stream lines and then its normal lines which is the potential line generally.

Instead of describing the fluid flow with respect to this xy coordinate system, here we can describe the fluid flow with respect to this s and n, so the stream line and then its normal into direction of the fluid flow, so we have much advantages as shown in here, so the flow is described with respect to the stream line coordinates.

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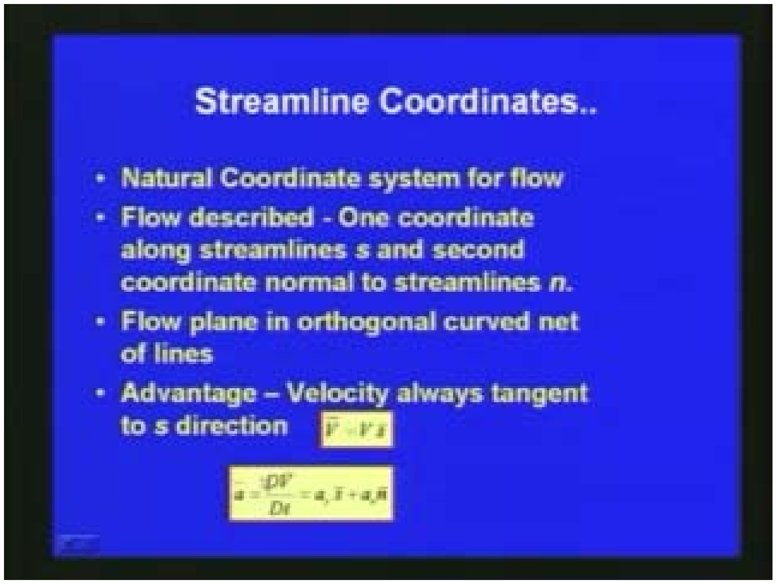


This has got the advantages as for as the fluid flow is concerned, the stream lines give the natural coordinate system. That is one of the important advantages of the stream line coordinate system since in natural coordinate system itself is flow, so the direction itself taken as the coordinate system so we can describe many of the fluid flow properties with respect to this coordinate system and flow is described as one coordinate along the stream line s and then second coordinate normal to the stream lines n.

As described in normal lines and then stream lines s, here in the previous figure, so s indicates the stream line and then this normal line are also drawn. So with respect to this the flow is described with respect to along the stream line s and second coordinated normal to the streamline n. So flow plane is orthogonal curved net of lines, instead of the Cartesian coordinates system which we generally described here, the flow plane is orthogonal in orthogonal curved net of lines and the advantages that velocity is always tangent to the s direction or the stream line directions.

So that we can describe the velocity vector as v bar is equal to v into s bar which is the stream line coordinate system. So many times we will be describing the flow system with respect to this stream line coordinates since it has got advantages compare to the other coordinate system which we generally use in mechanics.

The acceleration is described as DV bar by Dt. So that is here as into x bar plus a n into n barso where as and an are the acceleration components in the direction of stream line and its normal direction so the acceleration is described as x bar plus an n bar, so this stream line coordinate system is most of the time used since it has got it soon, advantage in fluid flow description. Now, before going to the some of the fundamental theories and principles with respect to the fluid kinematics, we will discuss the control volume and system representations. This we have seen earlier, but since we are going to derive these equations, before that we will further discuss, what the control volume is and what is the system representation? So, a system is a correction of matter of fixed identity.

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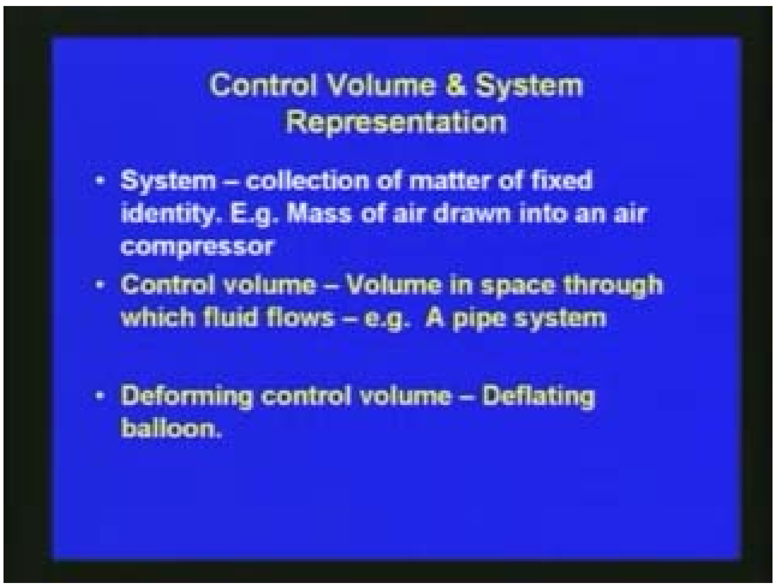


If we consider, for example, here in this board what is inside whether it is fluid either it is water or gas what ever inside in this ball, this is a fixed it has its own fixed identity and it is a collection of matter inside fixed boundaries so for example as the mass of the air drawn into an air compressor or the air inside this board. So all this system approach is very much used in fluid mechanics, in most of the time we will be interested to see what happens to the fluid inside a system and then a control volume as we have discussed earlier control volume is approach is used in fluid mechanics. So control volume is the volume in space through which the fluid flows, for example, if we consider a small pipe like this here, when the fluid is for from one direction to another.

So the control volume if we consider between one section to another section so in between that space through the fluid flow it is the control volume here considering as for as the derivation of various fluid flow theories are concerned and also some times we can have deforming of control volume. Here, you can see this is just like a balloon so it can deform so that one time there is a very less fluid or for gas is inside and then if you blow it up then you can see that it is full. So this is deform so this is called a just like in a balloon it is called a deforming control volume so it either a deflating balloon or just like a deflating plastic material as shown in here so this shows the deforming control volume. So many times instead of a fixed control volume we can also use deforming control volume as far as the fluid flow theories of development of the equations depending up on the problem. Now before going to the derivation of the continuity equation based up on the consideration of mass and other equations we will discuss one theorem called Reynolds transport theorem.

This Reynolds transport theorem is one of the fundamental principles used in a fluid mechanics based up on which many of the fundamental theories or fundamental principles are derived. According to the Reynolds transport theorem, according to this laws governing fluid motion using both system concepts and control volume concepts so we have seen in the case of system concept and control volume concept, so most of the time this Reynolds transport theorem is the law which governs the fluid motion either in system concept or control volume concept.

To do this we need an analytical tool to shift from one representation to the other, that means when we are describing the fluid flow from a system approach to the control volume approach or the from the control volume to the system so we need a analytical tool which can easily used so that we can shift from one representation to the other. So, this Reynolds transport theorem provides this tool. Reynolds transport theorem is generally used, as we have seen most of the time we will be using either a control volume approach or a system approach so when we want to shift from one approach to one representation to another representation we can use this Reynolds transport theorem which is described here.

According to the Reynolds transport theorem it gives the relationship between the time rate of change of an extensive property for a system and for a control volume. We can describe a fluid flow with respect a system or a control volume. For example, take any properties like velocity pressure or any of the other properties of fluid flow then Reynolds transport theorem use a relationship between the time rate of as returning this slide this gives the time rate of change of an extensive property for a system and for the control volume. If there is a system inside on which we are dealing, then we are considering control volume between that how with respect to time the property is changing the Reynolds transport theorem describes. Here, we can see a control volume.

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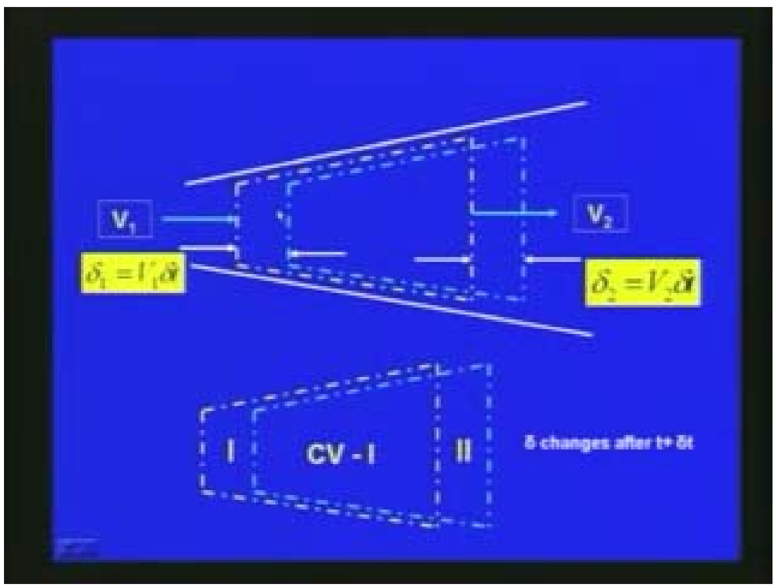


On this section, the velocities $V_1$ and at the second section here velocities $V_2$ and here $delta_1$ is $V_1$ into delta t, that means the fluid movement with respect to time $delta_1$ is $V_1$ into delta t and here $delta_2$ is equal to $V_2$ into delta t, then with respect to the changes taking place here is the control volume of the fluid, initially it is like this and after delta t so the control volume is shifting like this in this slide, delta changes after t plus delta t, so delta is a control volume which we are dealing.

The control volume is changing after delta t time, so after t plus delta t this is the position. So, this is the initial position number 1 and this is after sometime delta t the position is number 2. Finally we can see that transfers taking place between the control volumes minus 1, this is the change taking place with respect to this.

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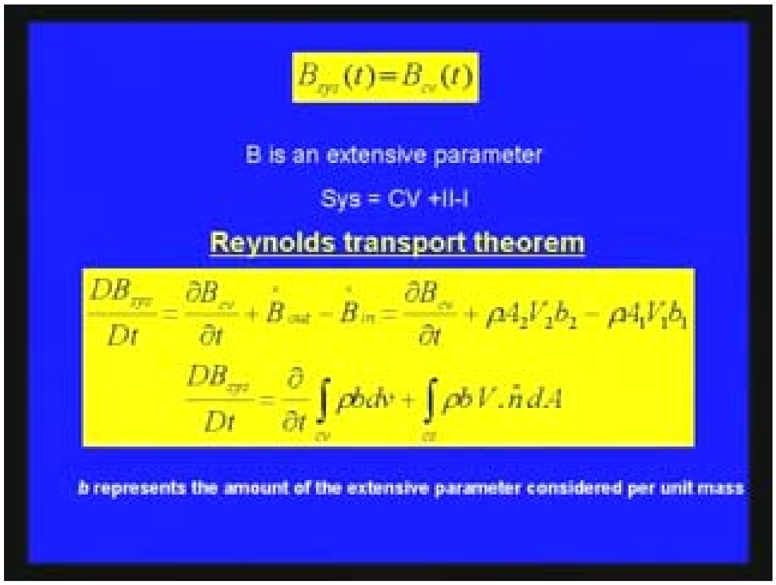


According to the Reynolds transport theorem, if we consider the system property B is the extensive parameter which we are dealing. Here, B system t is equal to B control volume t, so in the previous slide if B is the extensive property we are dealing with so B system at time t is equal to B control volume at time t. so the system is equal to control volume plus 2 minus 1, so here in the slide the system is control volume plus 2 minus 1 gives the system as far as between this, as mentioned here, the system is control volume plus 2 which is described in this figure and then minus 1 as shown in that figure, so B is the extensive parameter.

According to the Reynolds transport theorem we can describe with respect to the control volume and system like this, so DB system by Dt the total derivative of the extensive property DB divided by Dt or DB by Dt is equal to a partial derivative of the extensive property B with respect to control volume B, del $B_{cv}$ by del t plus the property what is the something adding to the system from the buoying to the out of the system B out minus B in that is equal to del B by del t plus rho $A_2$ $V_2$ $b_2$ minus rho $A_1$ $V_1$ $b_1$.

So, we have seen here $V_1$ is the velocity at this section and $V_2$ is the velocity at this section and so the control volume is considered here. Now, for the extensive parameter which we are described here according to the Reynolds transport theorem we can write $DB_{sys}$ divided by Dt with respect to system is given as with respect to the control volume del $B_{cv}$ by del t plus changes taking place with respect to the inflow and out flow so B out minus B in so that is with respect to time what happens whether how much is coming to the system and how much is going out of the system, so that is the Reynolds transport theorem.

Finally DB system by Dt can be represent as del $B_{cv}$ by del t plus roh $A_2$ $v_2$ $b_2$ minus rho $A_1$ $V_1$ $b_1$, where rho is the density of fluid which we are considering here and $A_2$ is the cross sectional area at section 2 and $A_1$ is the cross sectional area section 1 and $V_2$ is the velocity at section 2, $V_1$ is the velocity at section 1.

Finally, we can write this in an integral form this Reynolds transport theorem can be written like this DB system DB by Dt is equal to del by del t of we can integrate with in the control volume del by delt of integral roh bdv plus on the surface what happens in the control surface integral rho b v n hat which is the union vector dA, that means finally what happens to the system is given such a way that here you can see that with respect to system if you are considering just like a pipe like this so what happens to the extensive property which we are dealing is inside the control volume what happens and then on the surface whether with respect to surface something is going inside something is going outside from the system through the surfaces so what happens to that surface so that gives Reynolds transport theorem. The total changes with respect to time for the extensive property which we are considering here that means DB by Dt so B is the extensive property.

So the total change with respect to time what happens to the extensive property is expressed as we are considering within the control volume what happens with respect to time as far as the fluid inside is concerned that is given by del by del t of integral with respect to control volume rho DB and then on the surface what happens that means what is something is come adding to the system or something is going out of the system, so that gives the integral over the control surface rho b V n had d, so this is the same as the above expression so Reynolds transport theorem like as described in this slide so it gives what happens with respect to system and then the control volume inside the system. This way we can easily represent same you can easily transform or we can change one system that means the approach is based up on the system then we can get equations or we can transform that into a control volume based up on this Reynolds transport theorem the parameters are given with respect to the control volume we can also transform to the system.

Finally the Reynolds transport theorem what happens with respect to total system is for an extensive property what happens with respect to time is described in terms of the control volume, inside the control volume and then with respect to surface what is happening what is something is coming through or going out of the surface, so that gives the Reynolds transport theorem. This theorem is very useful in derivation of most of the fluid flow theories and fundamental principles which we will be discussing later so finally as we can seen in this slide.

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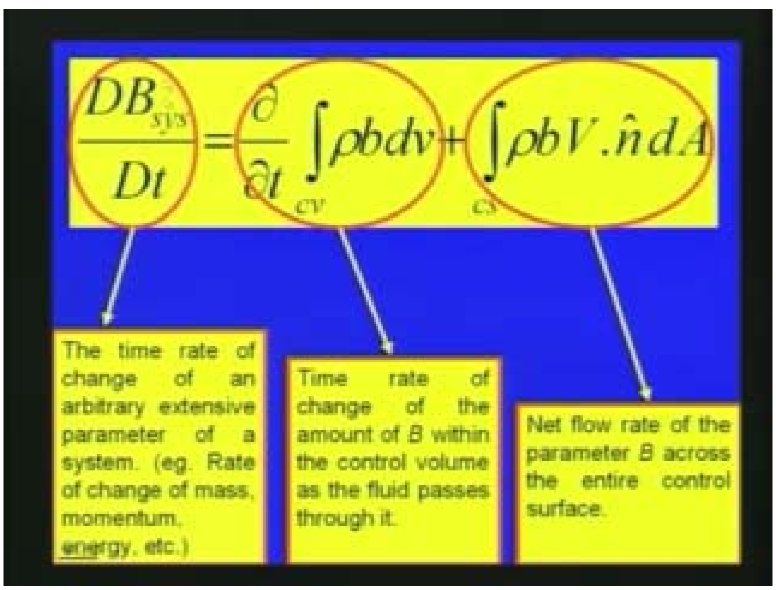


So, DB system that is gives the time rate of change of an arbitrary extensive parameter of a system. For example, rate of change of mass momentum energy extra so this is the DB by Dt and that is equal to say in del by del t of integral rho bdv so that is equal to time rate of change of the amount of B within the control volume as the fluid passes through it plus the net flow rate of the parameter B across the entire control surface. This slide gives the Reynolds transport theorem.

The time rate of change of an arbitrary extensive parameter of a system is equal to time rate of change of the amount of B within the control volume as the fluid passes through it plus the net flow rate of the parameter B across the entire control surface so this theorem is very useful many of the fundamental development of the fluid flow theories and principles, so with respect to the Reynolds transport theorem the integral counter part of material derivatives that we have seen.

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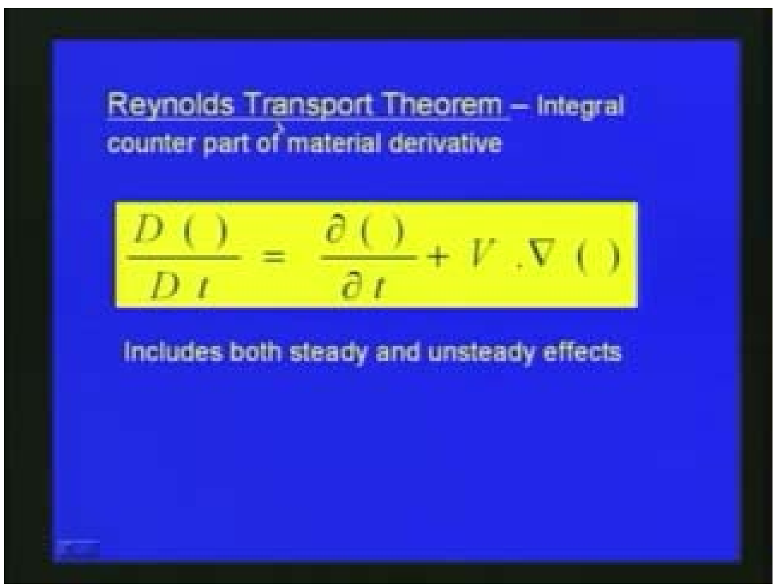


Reynolds transport theorem with respect to the counter part of the material derivative, we can write as D of Dt of the particular parameter which we are dealing equal to del by del t of the parameter plus V dot product del of the parameter, so this include both the steady and unsteady effects we have seen, so this gives the Reynolds transport theorem with respect to the material.

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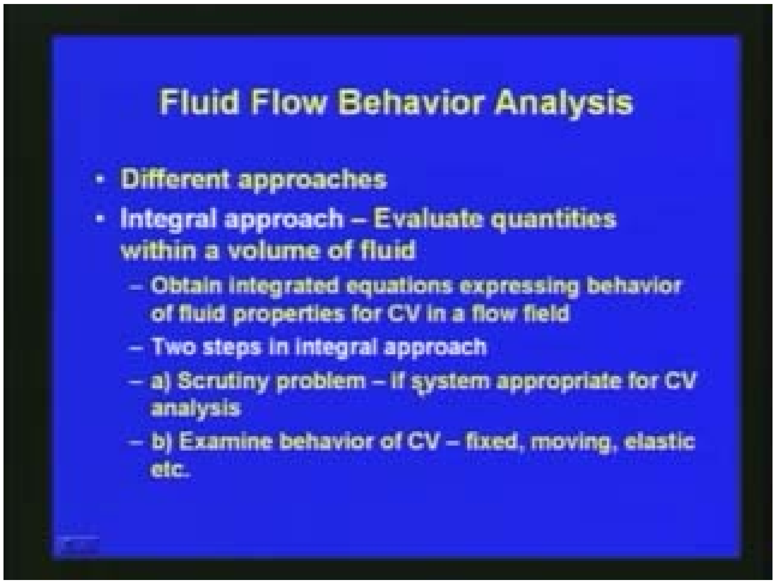


With respect to the material derivative, you can see that here steady and unsteady conditions are described. Reynolds transport theorem will be using many times as far as the fundamental theories development and demonstration is concerned. Before going to the continuity equation based up on the consideration of mass, we will initially discuss different approaches fluid flow behavior, so how we can analyze fluid flow behavior with respect to different approaches.

In fluid flow analysis, generally we can have two types of approaches: first is called integral approach and second one is called differential approach. Here, the fluid flow behavior, we are analyzing with respect to either a system or with respect to a control volume so with in this we can analyze either we can go for a integral approach or we can go for a differential approach, so what is an integral approach so in an integral approach what we are doing is we are trying to evaluate the quantities within a volume of fluid? For example, if you consider the pipe flow here between this section 1 and 2 what happens so what we will be discussing? We are trying to evaluate or we are trying to identify within the volume of fluid.

So through this we get integrated equations expressing behavior of fluid flow for control volume in a flow field so that is what we are going in the integral approach. So in the integral approach there are basically two steps first one is we have to check the problem or we have to verify whether the system is appropriate for a control volume analysis, so depending up on the problem sometimes we can go for a control volume analysis sometimes it may not be possible.

We have to see the problem in detail whether we can go for a control volume analysis or we have to go for other types of analysis so we have to scrutiny the problem whether the system as appropriate for the control volume analysis and second step in the integral approach is we have to examine the behavior of the control volume. So we have to see whether the control volume is just like a fixed control volume like shown in this ball inside it is a fixed control volume or we have to see just like a in an aeroplane with respect to when we doing the analysis the aeroplane also moving, so whether we are trying to analyze a moving control volume or we have to see whether just like we have seen deforming or an elastic control volume. So these aspects also we have to analyze or we have to evaluate before we choose either an integral approach or whether we have going for a differential approach

The two essential steps before going for integral approach is we are to scrutiny the problem we have to verify the problem whether the system is appropriate for control volume analysis and secondly we have to see the behavior of the control volume whether it is fixed one or whether it is moving one or whether it is an elastic type of control volume. So first we have to do these two steps of analysis and then only we will be going for the integral approach or whether we will be going for other types of approach.

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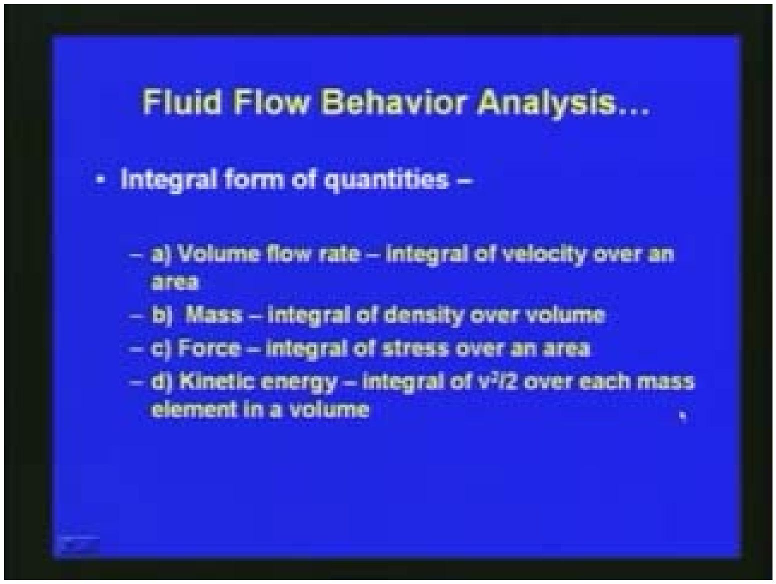


So integral form of quantities we can see that most of the quantities we can described in terms of integral form just like for example the volume flow rate it is the integral of velocity over an areas so if we consider the volume of the discharge passing through integral of velocity over an area that is the discharge for volume of flow rate and second for example the mass if we consider the mass it is the integral of density over volume.

So the mass, when we are dealing with the consideration of mass we can go for integral form of approach and then if you consider the force, so it is the integral of stress over an area and kinetic energy it is the integral of v square, that means the velocity square by 2 over each mass element in a volume.

Like this we can analyze the problem and see whether it is this kind of quantities whether it is a discharge of volume flow rate or it is mass or force or kinetic energy what we are going to deal and then we can choose the integral approach or differential approach the other kinds of approach which will be discussing later so this integral approach is very useful in many problems where this kinds of say we can use the appropriate depending up on the problem.

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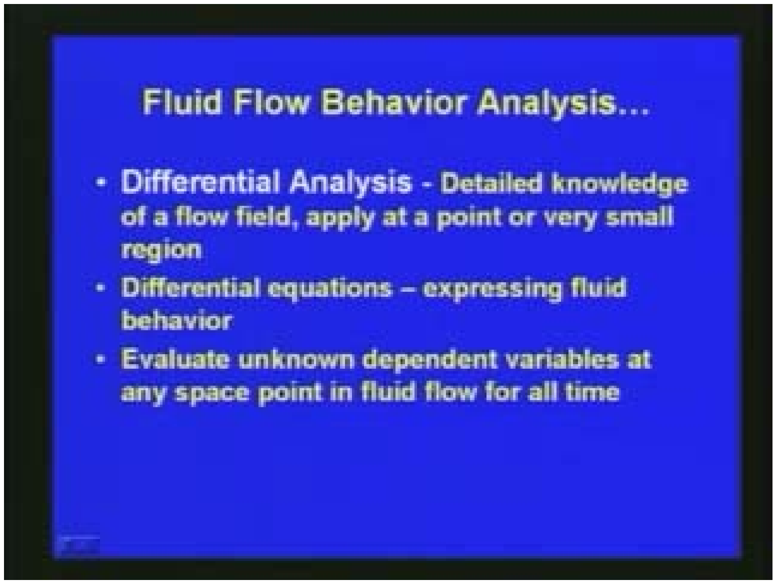


Now, the second approach commonly used in fluid flow behavior analysis is called differential analysis. In the differential analysis, what we are trying to do is we are trying to get detailed knowledge of a flow field apply at a point or very small region so we have already seen that most of the time we will be going for an eulerian approach so we will be discussing what happens in the fluid flow what happens at a particular point or particular section what happens.

So, that is our major concern this differential analysis the second kind of analysis very important. So what we are doing here is as far as the flow field concerned which we will be choosing at particular point or a very small regions trying to see the variations of the differential of various fluid flow properties at that particular point. In this differential analysis we will be deriving differential equation which expresses the behavior or fluid flow behavior.

Differential equations are derived with respect to theories in differential analysis and then we are trying to evaluate the unknown dependent variable at any space point in fluid flow for all the times, that is what we are doing the differential analysis so we are trying to evaluate the unkown depending parameter dependent variable with respect to space for all the time.

These two approaches the integral approach or differential approach, for both of the approaches are very commonly used in fluid flow problem so fluid mechanics problem we will be discussing both the integral approach and the differential approach for our problems for the development of fundamental theories in fluid flow and fluid mechanics which will be discussing later. Initially, we will be discussing the integral approach for the consideration of mass or the derivation of the continuity equation.

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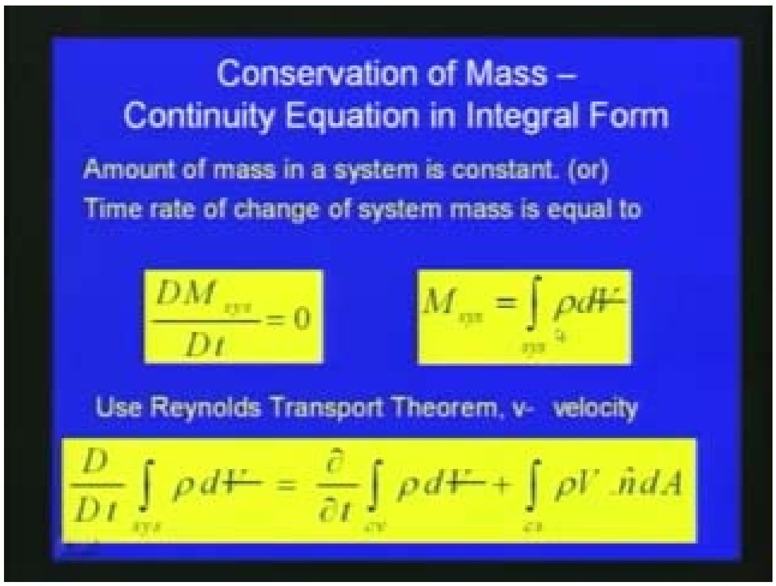


Now, we discuss the conservation of mass, the continuity equation with respect to the integral form of approach. We have seen that just like in fluid mechanics also the fundamental theories are based up on the consideration of mass, consideration of momentum and consideration of energy, so most of the basic principles of basic theories are developed based up on conservation of mass momentum and consideration of energy.

First, we will be discussing here is conservation of mass and the continuity equation is derived based up on the conservation of mass. First we will see the continuity equation with respect to the integral approach which we have seen earlier so as I mentioned earlier here say the mass has the integral of density over volume. so we can see that here we can easily use when we scrutinize the problem we can see that we can easily use the integral approach so but any way as for as conservation of mass is or the continuity equation is concerned we will be discussing also in terms of the differential approach.

First we will see the integral approach. Amount of mass in system is according to the conservation of mass, amount of mass in a system is constant or the time rate of change of mass is equal to 0 that is the basic principles of the conservation of mass. Conservation of mass is the amount of mass in a system is constant or the time rate of change of system mass is equal to 0 or we can describe with respect to the total derivative DM with in the system by Dt is equal to 0, where t is the time M is the mass D indicates the total derivative.

As far as the system is concerned, if the fluid inside this ball or inside in a container here we can see the fluid inside this container so with respect to the conservation of mass we are saying that the amount of mass inside the system inside this the gas inside this ball or inside this container it is constant, so that is the basic principle, so that we can say that if there is any change of gas inside this ball or with respect to this container here we can say the time rate of change of mass is equal to 0, so that is the conservation of mass or the continuity equations says so that we can write this mass of the system is concerned with respect to whether the air inside or water inside in a container.

The mass can be defined with in the system we can defied as we can integrate with respect to the density and then the volume so the mass of the system can be written as integral of rho dv so now have seen the Reynolds transport theorem earlier So within the system what happens we have to describing here if we consider the pipe flow which we have seen.

Within the system we will be describing in terms of the control volume what happens and then with respect to the control surface or the boundaries say whether something is entering to this system or something is dealing from the system so that gives the Reynolds transport theorem so here as far as conservation of mass is concerned say if we apply the Reynolds transport theorem we can write D by Dt the total derivative of the system with respect to the mass rho dV is equal to del t with in the control volume which we are considering here of rho dV plus with respect to the control surface or the series that means exceed time inlet of the system plus the control integral of the control surface of rho with V n hat dA.

Here, this V indicates the velocity and this v indicates the volume. Now, here to derive the basic equation of the continuity based up on the conservation of mass we are using the Reynolds transport theorem, so D by Dt of integrals with respect to system rho dV can be written as del by del t of the control volume rho dv plus with respect to control surface integral of rho V n hat dA, so this gives the Reynolds transport theorem as far as the mass which we are considering here.

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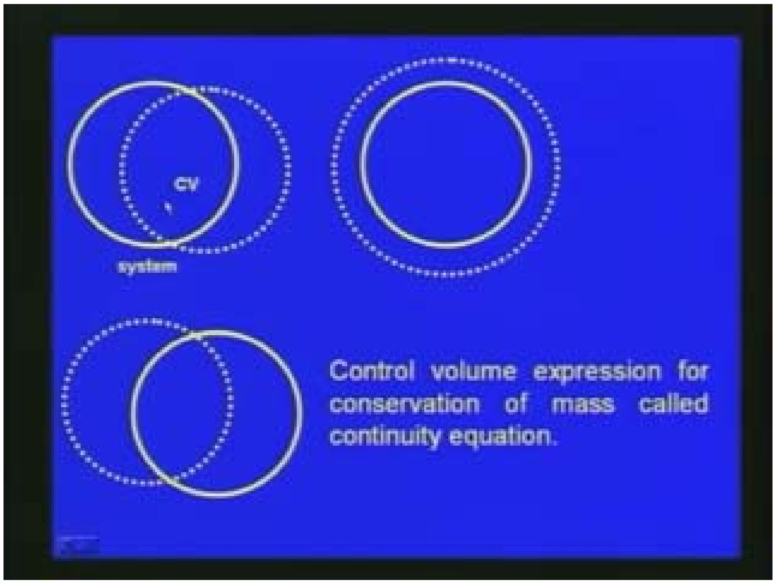


Now, as we can see in this slide the control volume it can here, this is the system which we are considering and the control volume is inside what is there and then what changes taking place with respect to whether with in the control surface that is what we are describing so control volume expression for conservation of mass called the continuity equation.

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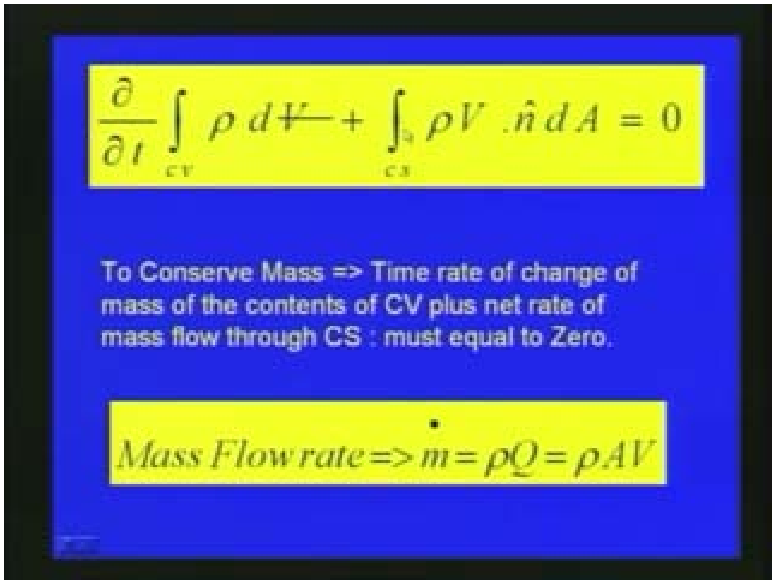


Now from the Reynolds transport theorem and since according to the conservation of mass we can write DM by Dt is equal to 0 so now if you use the Reynolds transport theorem here we can write del by del t of integral of control volume rho dV plus integral over the control surface rho V n hat dA is equal to 0. So, that means, to conserve the mass the time rate of change of mass of the contents of the control volume plus net rate of mass flow through the control surface must be equal to 0 so that gives the Reynolds transport theorem with respect to the conservation of mass. Time rate of change of the mass of the contents of control volume plus net rate of mass flow through the control surface must be equal to 0.

From this we can write the mass flow rate is equal to rho into Q so that we can express as if V is the velocity and A is the area cross section so this s equal to rho into A into V, where rho is the density of the fluid, so the integral expression from the integral approach we have seen the continuity equation can be described by the Reynolds transport theorem as given this equation del by del t of integral of control volume of rho dV plus integral over the control surface rho Vn dA. So this gives the integral form of the conservation of mass for the continuity equation and finally it is steady state condition you can say that mass change can be written as rho into AV where V is the velocity here.

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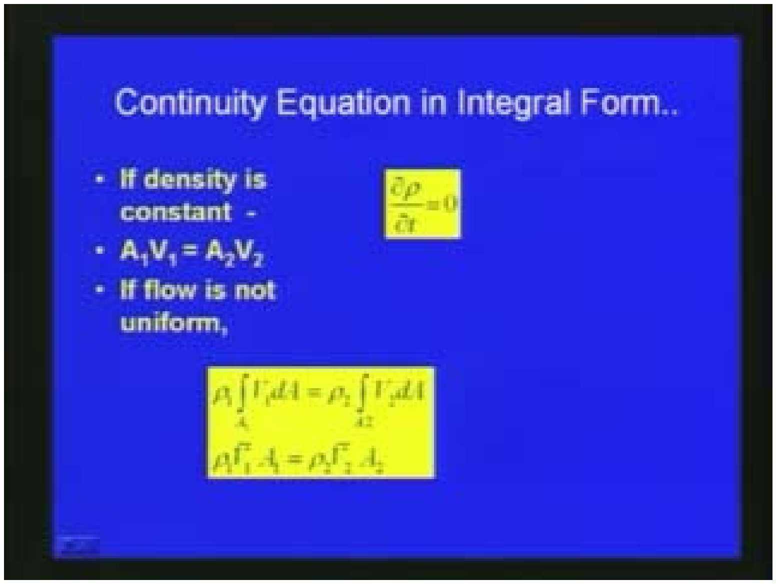


Finally, the continuity equation integral form if the system is at steady state or if there is no change with respect to the density that means there is del rho by del t that means no change of density with respect to time this can be written as 0.

In most of the fluid flow which will be discussing will not be any change as far as rho density concerned, so we generally dealing with incompressible fluid flow so that del rho by del t is equal to 0 so finally if the density is constant we can write $A_1$ $V_1$ is equal to $A_2$ $V_2$ for the system which we are considered and the system if it is non uniform or it is not uniform then we can write $rho_1$ integral over area $A_1$ $V_1$ d A is equal to $rho_2$ integral $A_2$ $V_2$ d A or the non uniform flow we can write $rho_1$ $V_1$ bar $A_1$ is equal to $rho_2$ $V_2$ bar $A_2$ so this gives the integral form of the continuity equation.

As we have seen here we can the general expression if you deal with compressible flow also and in the case of incompressible flow you can write the equation since here del rho by del t is equal to 0 that we can write the integral form of the continuity equation as rho $V_1$ bar $A_1$ is equal to $rho_2$ $V_2$ bar $A_2$ so here what it says is with respect to this is the system is just like in a pipe system which we are dealing so the fluid entering with respect to system that should be equal to the fluid leaving from the system so that we have to use this equation here.

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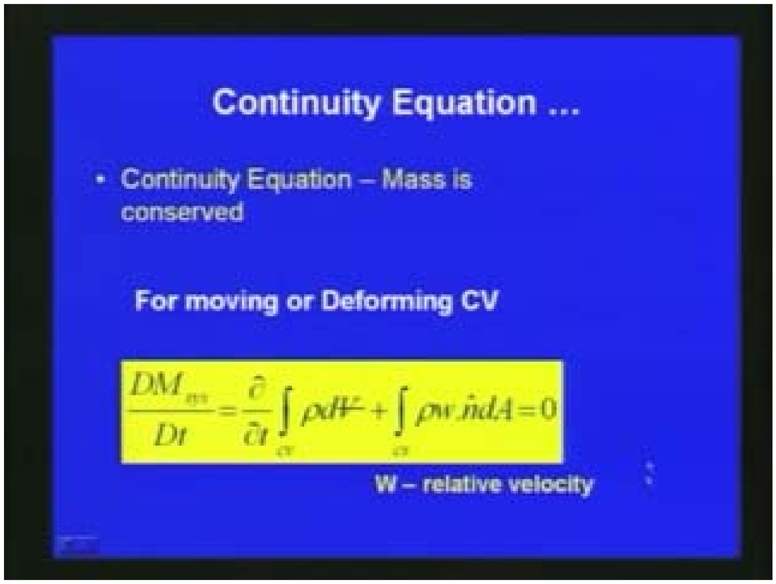


The continuity equation as we have seen here the mass is concerned and we have also seen that there can be or deforming a control volume so in that case the with respect to the continuity equation of the conservation of the mass we can write DM system Dt is equal to del by del t of control volume rho dv plus all the control surface rho w instead of V here we are using rho w n hat dA is equal to 0.

For the case of the moving or deforming the control volume this w is the relative velocity so the system itself in the control volume is moving or deforming with respect to that we have to consider changes taking place with respect to the relative velocity. So, for moving or deforming control volume we can write the continuity equation as DM by Dt is equal to del by delt of integral of rho dV plus all the integral over the control surface rho w, where w is the relative velocity n hat dA where n hat dv is the unit vector so this gives the continuity equation in the integral form.

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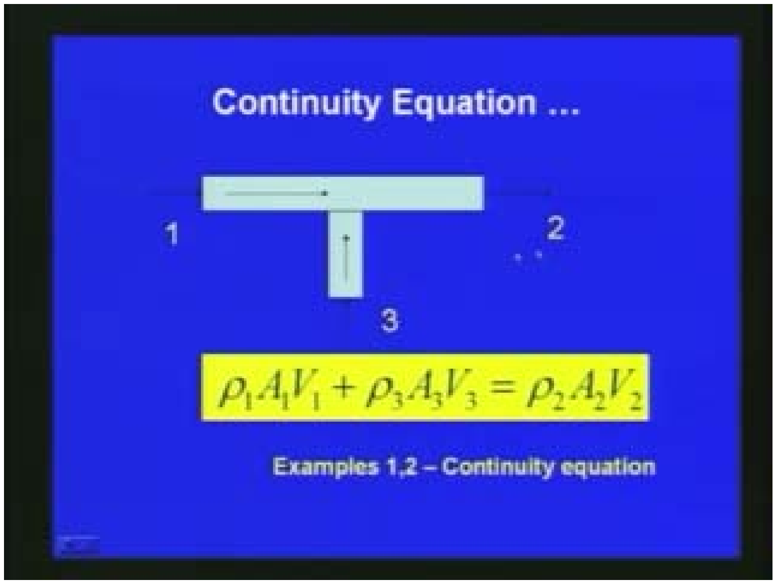


For example, if you consider a pipe with branch like in this slide so here we can express the continuity equation like here. At the section 1 if $V_1$ is the velocity and area of cross section is $A_1$ and the density here is $rho_1$ and at section here at three if the velocity is $V_3$ and area of cross section is $A_3$ and density is $rho_3$ and then here this location the density is $rho_2$ and $A_2$ is the area cross section $V_2$ is the velocity.

Here, we can see as far as system is concerned there are 2 inflow at 1 and 3 and then there is an outflow at 2 so finally you can write $rho_1$ $A_1$ $V_1$ plus $rho_3$ $A_3$ $V_3$ is equal to $rho_2$ $A_2$ $V_2$ so if the density say we can write this equation as $A_1$ $V_1$ plus $A_3$ $V_3$ is equal to $A_2$ $V_2$ so this way we can write with respect to the integral form of the continuity equation based up on the conservation of mass we can derive the continuity equation for various system. Now before discussing the differential approach for continuity equation we will see the example problems as for as the continuity equation is concerned.

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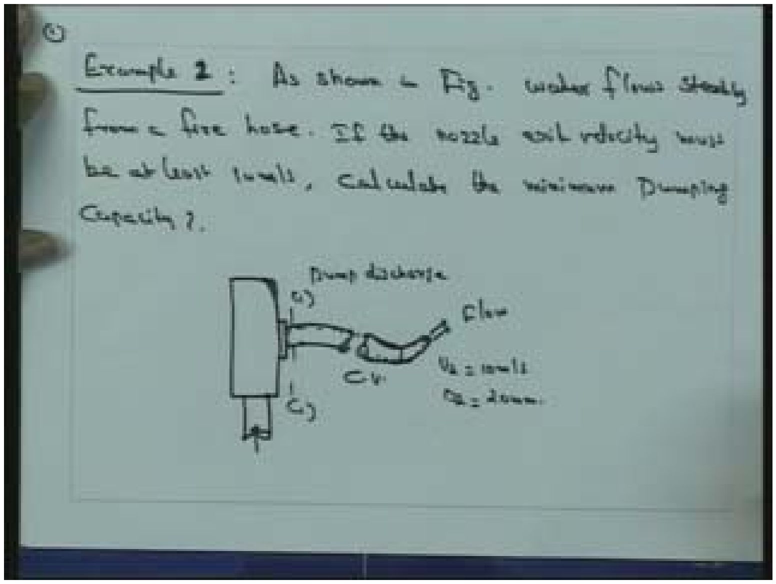


Now, simple example initially which we are discussing here is say example 1 is as shown in figure, water flows steadily from a fire hose. So, there is a pipe through which a fire hose is connected there is a constant supply to the fire hose here and from which we are talking through a pipe system connected with the fire hose we are taking water through this for various purpose so if the nozzle exit velocity must be at least 10 meter per second. Here, the exit velocity is 10 meter per second we want to determine the minimum pumping capacity for which here is the flow, so here this is the direction of the flow we want to determine the minimum pumping capacity for this simple system so here this is the fire hose and this is through which the flow type displace and if we consider the control volume now the control volume here you can see that this is the control volume with respect to the fire hose which we are considering.

So, here the exit velocity is given as 10 meter per second and here the diameter of the pipe the hose is given as 20 millimeter so $V_1$ to find minimum pumping capacity such that the system is supplying a minimum of 10 meter per second with respect to the continuity equation which we have seen earlier.

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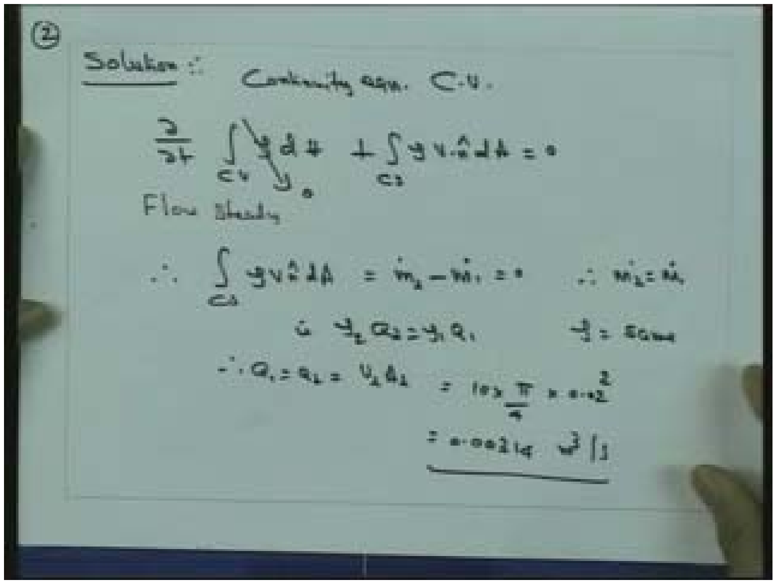


Here you have a control volume and then control system that means the control volume you can see here in this figure so this gives the control volume is here and then with respect to the inlet this is the control surface what is going inside and what is going outside one inlet and exit for the system is concerned, so the continuity equation which we have seen earlier we can write with respect to integral form which we are discussed so as del by del t of integral over the control volume rho dv plus integral over the control surface through V n hat dA is equal to 0 according to the continuity equation, so here the time rate we are not considering.

So this term goes as 0, since a flow is considered as study, so that we can write the final system as integral over the control surface rho Vn dA that means with respect to the system is concerned with respect to what is going inside to this control volume inside the fire hose there should be going out through the exit so what is entering go through the exit.

If you consider the second part here, the continuity equation is integral c over the control surface rho Vn dA that can be written as $m_2$ dash minus $m_1$ dash that means what is entering that is equal to $m_1$. Finally, we can write $rho_2$ $Q_2$ is equal to $rho_1$ $Q_1$ that means through the system what changes that should be through the exit from the system.

So $rho_2$ $Q_2$ is equal to $rho_1$ $Q_1$ so here if you assume that since here we are considering water so rho density so that we can write $Q_1$ is equal to $Q_2$ that is equal to $V_2$ into $A_2$ so here before the fire force which we are consider the radius is the diameter of the hose is given and the velocity is givenso that we can write finally the discharge that means the minimum pumping capacity should be $Q_1$ is equal to $Q_2$ is equal to $V_2$ $A_2$ that is equal to the velocity is 10 meter per second 10 multiplied by the area cross section of the pipe that is phi by 4 d square so d is 0.02 or 20 millimeter.

Finally, this is the a pumping capacity for the fire hose system which we have seen in this particular figure so here Q is equal to 0.00314 meter cube per second so this is the simple problem where we can use this integral form of the continuity equation. We will also discuss another simple example which is related to the conservation of mass based up on the continuity equation.

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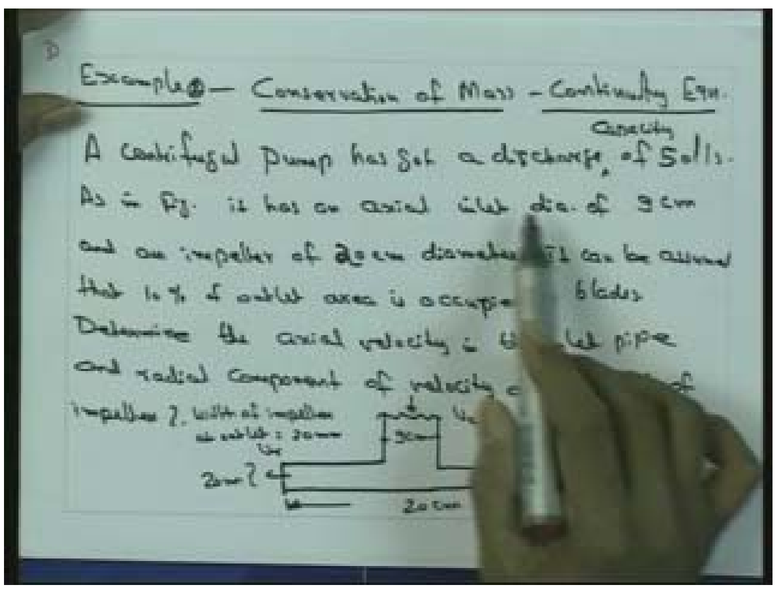


So a problem here is a centrifuged pump has got a discharge capacity of 50 liters per second as shown in figure it has an axial inlet diameter of 9 centimeter and then impeller of 20 centimeter diameter so it can be assume the 10 percent of outlet area is occupied by blades so we have to determine the axial velocity in the inlet pipe and the radio component of velocity like the outer impeller.

So, this is the problem here. There is a flow takes place to the centrifuged pump in this direction and here the inlet is given as 9 centimeter and here you can see that there will be outlet area blades will be there with respect to the centrifuged pump so this way once it enters here and then it should go through this direction and this direction.

Here, the diameter is given as 20 millimeter and here also 20 millimeter and here this impeller total diameter is given as 20 centimeter, so we want to find the axial velocity entering through the inlet pipe also the radio component of the velocity at the outlet of the impeller, so this is the problem which we are discussing here. Here the discharge for the problem is given as 50 meters per second.

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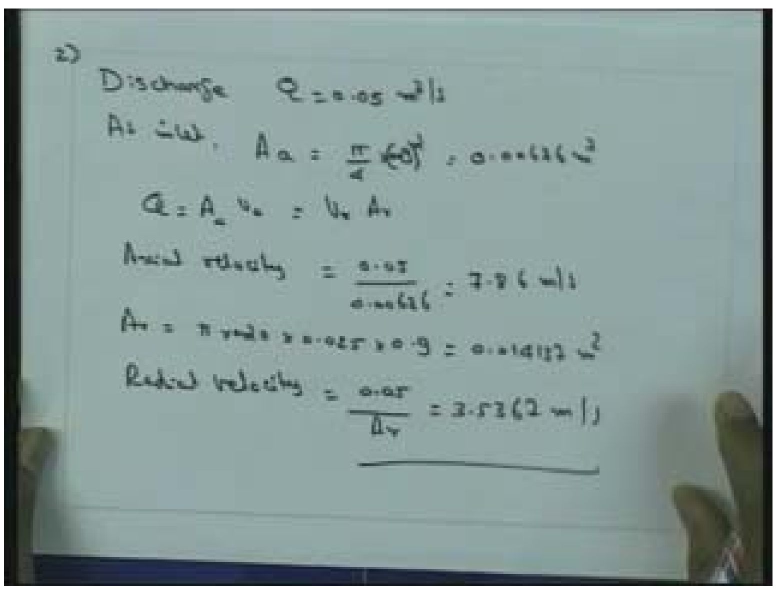


So, the discharge can be written as Q is equal to 0.05 meter cube per second Q is equal to that is discharge is equal to 50 liter per second or it can be written as 0.05 meter cube per second at inlet. So, the inlet is here. At inlet we can write, the discharge is equal to area of cross section multiplied by the axial velocity so that what is entering that should go through impeller.

We can write Q is equal to $A_a$ into $V_a$ is equal to $V_r$ into $A_r$, $A_a$ is the area cross section of the inlet and $V_a$ is the axial velocity and $V_r$ is the radio velocity and $A_r$ is the area which we will be considering, so the axial velocity can be obtained as the discharge Q is known, so that is given as 0.05 meter cube per second. At inlet the area of cross section can be written as phi by 4, the diameter is phi by 4 into 0.09 square, so that gives the inlet area cross section and then axial velocity $V_a$ is equal to Q divided by $A_a$ so this is equal to 0.05 divided by this $A_a$ that is 0.00636 so that gives the axial velocity as 7.86 meter per second. To find the radial velocity we can calculate the effective area with respect to the 10 percent of the outlet area keeper by blades. We can find the effective area by taking the diameter of the impellers is equal to phi into 0.025, since 20 centimeter is given here and then 0.025 is this depth.

So phi into 0.2 into 0.025, so this is actually see 0.025 and this is 25 millimeter so phi into 0.2 into 0.025 into 90% is the effective area since the 10% is occupied by the blades into 0.9 so this gives the $A_r$ so from that we can get the radial velocity is equal to Q by $A_r$ so that is equal to 3.5367 meter per second. Like this we can use this integral approach of the continuity equation of the consideration of mass to show various examples.

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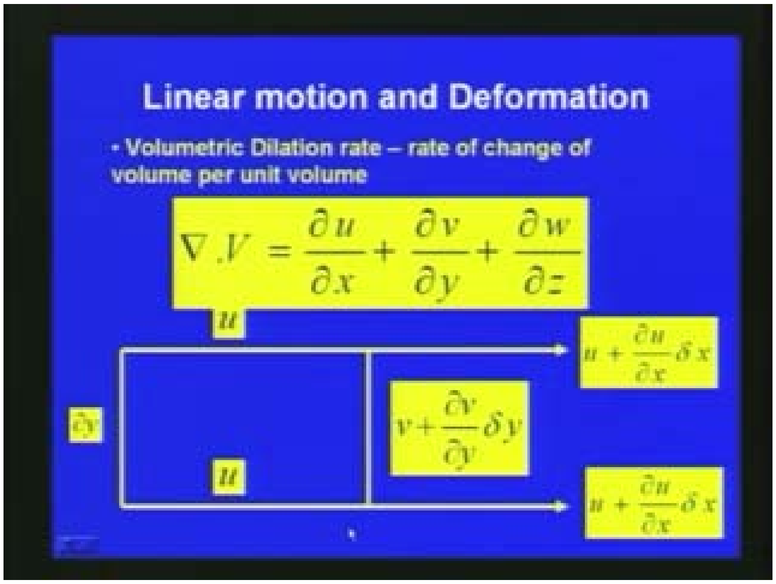


This is the conservation of mass of the continuity equation which is one of the basic equations which is used in all the fluid flow analysis, so this is the integral form. Now we will be discussing about the differential form later. Before going to the differential form of the continuity equation here you can we will see in this slide the linear motion and deformation before going to the differential on the continuity equation will see with respect to the fluid moment how the linear motion and deformation takes place.

Here, we can see that a container here delta y is the size, delta x is the direction and velocity in the x direction u, velocity in y direction is v, so with respect to this say here from one section to another, the velocity change is u plus del u plus del x into delta x and velocity change in the from one section to another, the y direction is v plus del v by del y into delta y. So the volumetric dilation rate means with respect to the volume inside the system dilation, that means the change or the rate of change of volume per unit volume can be expressed by this equation.

Since we are dealing with the velocity in x y and z direction, del v is equal to del u by del x plus del v by del y plus del w by del z, where u v w are the velocity components in xyz direction. So, the total the rate of change of volume per unit volume is expressed as del u by del x plus del v by del y plus del w by del z.

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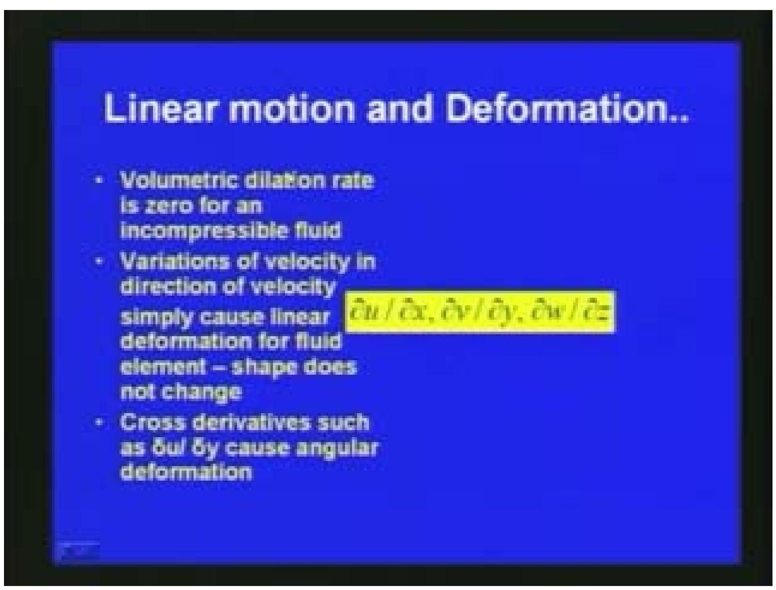


Now, we can see the volumetric dilation rate is 0 for an incompressible fluid, so when we are dealing with incompressible fluid volumetric dilation rate is 0, so we can write the variation of velocity in direction of velocity is del u by del x, del v by del y and del w by del z, since we are dealing the velocity components in xyz direction.

So del u by del x gives the velocity x direction, its radiation, next del v by del y gives the velocity change in the direction of y, del w by del z gives the velocity change in the direction of z for w. So, variation of velocity in direction of velocity simply cause linear deformation for fluid element or shape does not change for the linear variation, so cross derivative such as del u by del y only cause the angular deformations, but otherwise the linear motion is concerned, the variation is the velocity in direction of velocity simply cause the linear deformation. So, shape does not change, now the next lecture we will be discussing the deformation and then we will be discussing the [56:40] and then further we will be going to the differential approach of the continuity equation or the conservation of mass.