Fluid Mechanics

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Lecture – 42

Pipe Flow Systems

Welcome back to the video course on fluid mechanics. In the last lecture, we were discussing about the pipe network systems and unsteady flow through pipes or closed conduits. So we have seen that as far as unsteady flow for closed conduits are concerned, most of the time we have to deal with the water hammer or hydraulic transients when we deal with the water or if it is oil then oil hammer or it stream it is stream hammer - all this we have discussed in the last lecture.

So today, we will discuss with respect to hydraulic transience or water hammer; we will discuss the basic principles; we will discuss the governing equations and some of the method of solutions as far as this unsteady pipe flow through pipes or closed conduits are concerned. Also we will discuss say to reduce this water hammerer effects you may give as surge tank especially in the case of hydro power projects when we connect turbines through a [penstock] and then the water first comes through the tunnel and then passes through a [penstock] and then especially in the case of turbines, when the turbine is switched off or turbine is not working, that means, sudden shut down [type pace], that there is a very large water hammer or very high intensity water hammer may produce, so in that we produce surge tank. So we may discuss today also the surge tank and its related theories.

So, we will now comeback to the unsteady flow in closed conduits or pipelines.

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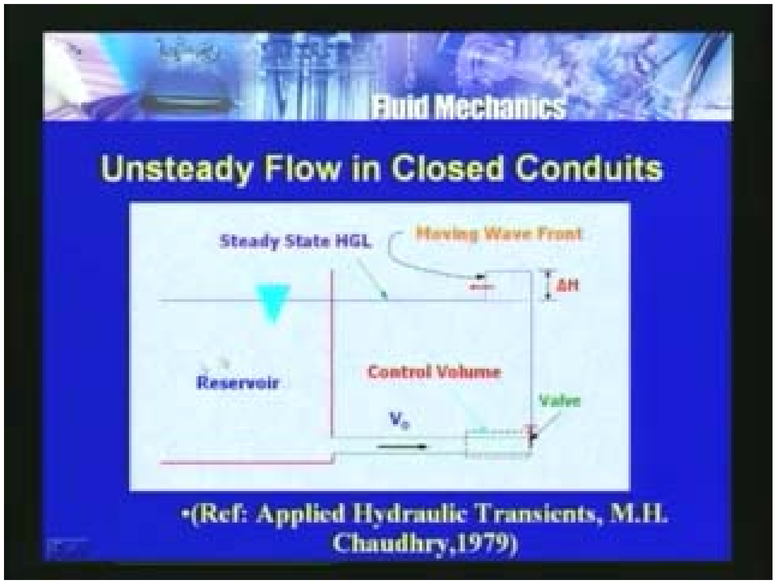

If we consider simple system like this here, there is a reservoir and then the reservoir pipe is connected or a conduit is connected like this and then to develop the theories with respect to such a system - a simplified system - say we assume here there is a valve and then due to sudden showdown of the valve, what happens with respect to the water hammer or hydraulic transients.

So you can see that where ever there is sudden closure of this valve, there is say moving wave front will be produced and then it will be moving to the reservoir; then again, it may reflect back to the closed valve back and then it will again return back to the reservoir; so like that oscillatory motions with respect the water hammer will take place.

So, with respect to this, we will discuss now some of the theories and some of the governing equations with respect to this unsteady flow and hydraulic transience initially with respect to a single pipe system connected to reservoir and then with respect to the sudden closure of the valve what happens?

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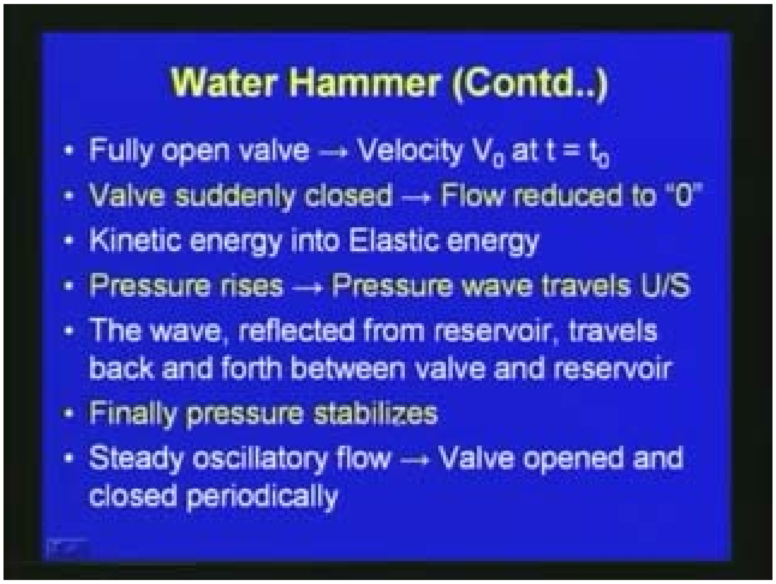

Here we assume that for the fully open valve the velocity V_0 is velocity V_0 at time t is equal to t_0 and then valve is suddenly closed. So flow is reduced to 0; so here v is equal to V_0 at t is equal to t_0 time and with sudden closure, the flow is reduced to 0. Then we can see that with respect to the movement of fluid or with respect to the flow, there is kinetic energy; so due to the sudden closure, this kinetic energy is actually transformed into elastic energy. Then what happens? The pressure rises and then pressure wave travels up stream. So here due to sudden closure, the pressure rises and then a pressure will be traveling to the up stream side; then the wave again reflected back, reflected from the reservoir travels back and forth between valve and reservoir.

So here again it goes back and then it returns back; like that it will happen for some time until the intensity of the pressure reduces or the hydraulic transience water hammer effects reduces, this wave travels will take place and finally the pressure stabilizes.

So here, we can see that there is steady oscillatory flow once the valve is opened and closed periodically. So we can observe, say, first the valve is opened and then closed. So with respect to this valve opening and closing, we can see that periodically a steady oscillatory flow takes place.

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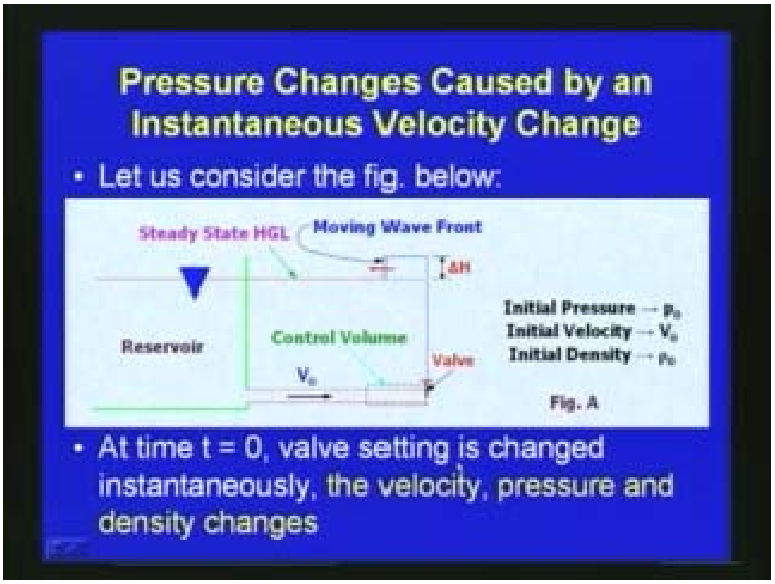

Now, we want to find out some relationship to connect with respect to the water hammer the velocity of the wave front and then how this can be connected with respect to the pressure which is building up, with respect to the sudden valve closure - so that is what we want to find out.

So let as consider the typical system which we have seen with respect to the figure here. We want to get the pressure changes caused by an instantaneous velocity change. So, we consider the same system. Here there is reservoir and then a pipe is connected and there is a valve which we suddenly close. So here, the initial pressure in the pipe let it be p_0 before the valve closure; initial velocity be V_0 before valve closure; initial density let it be rho_0 - the density of fluid.

Now time - t - is equal to 0 value setting is changed instantaneously and then we can see that due to sudden closure, the velocity, pressure and density changes, as we discussed in the previous slide.

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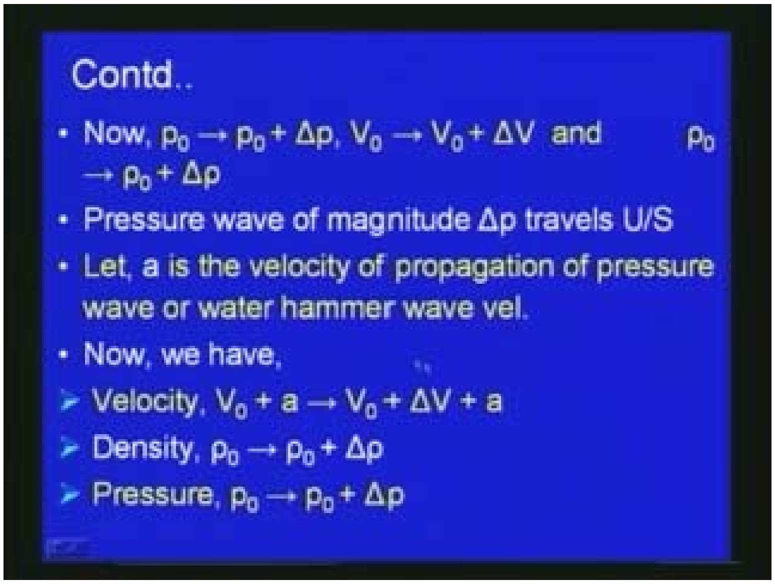

Now due to the sudden changes, let as assume that with respect to valve closure, so let the pressure change from V_0 - which is the initial pressure - to p_0 plus delta p. Then similarly, velocity changes from V_0 to V_0 plus delta v; then density changes rho_0 to rho_0 plus delta rho; like in this slide and the pressure wave of magnitude, say, delta p travels upstream, as we have seen. Now we can see that this wave which moving to upstream and then coming back like that, so it has the high velocity. So let ‘a’ be the velocity of propagation of pressure wave or water hammer wave. This pressure wave it is called water hammer wave.

Let the velocity of the water hammer wave be ‘a’. Now, what can we do? Here, we can see that this a transient system. If we super impose this water hammer velocity ‘a’ with respect to the existing earlier velocity V_0, we can see that we can transform the transient system into steady state system by superposing this water hammer velocity. So that we can write the velocity V_0 is represented here as V_0 plus a and then V_0 plus delta v plus a, that is after the valve closure which changes. So density is say rho_0 to rho_0 delta rho and pressure is p_0 to p_0 plus delta p.

So what we do here it is a transient phenomenon that means the wave goes and come back; so it is transient phenomenon so that we transform into a steady state phenomenon by adding this water hammer velocity with respect to the velocity as in this slide.

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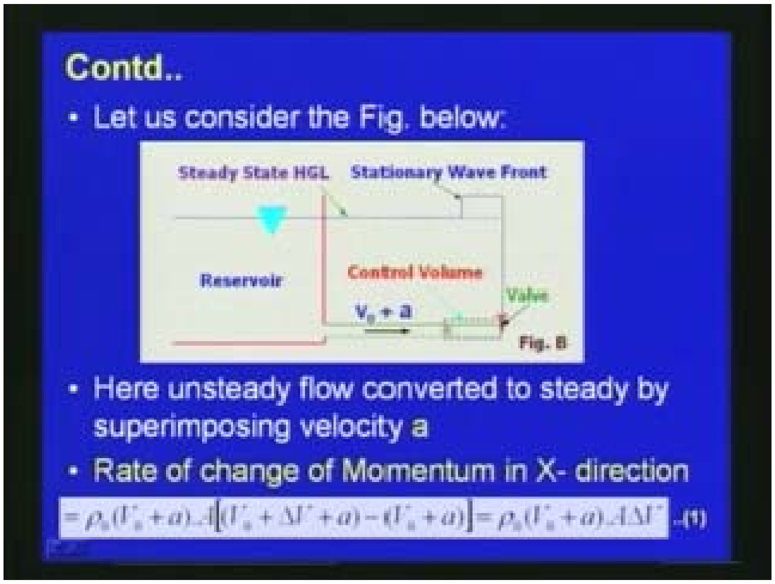

Now here say if we consider the same systems as we discussed, so here now the velocity here is V_0 plus a and then here at the velocity changes. Here, the unsteady flow, as I mentioned is converted to steady state by superimposing the velocity ‘a’. So here, that is what we are done. Now we can find out what is the rate of change of momentum. Here we can see that with respect to the change in velocity there will be rate a change of momentum.

The rate of change momentum in x direction we can write as rho_0 into V_0 plus a into a into the velocity change. So the velocity changes V_0 plus delta v plus a minus V_0 plus a; so this is in bracket; so that is equal to rate if change of momentum with respect to change in velocity will be rho_0 into V_0 plus a into a into delta v. So, this equation number 1 gives the rate of changes momentum in x direction, which is the direction we consider.

Here actually we consider the flow as one-dimension transient, So that is what we now transformed into a steady state by super posing this water hammer velocity ‘a’. So the rate of change of momentum in x direction is obtained as rho_0 into V_0 plus a into a into delta v, as in equation number 1.

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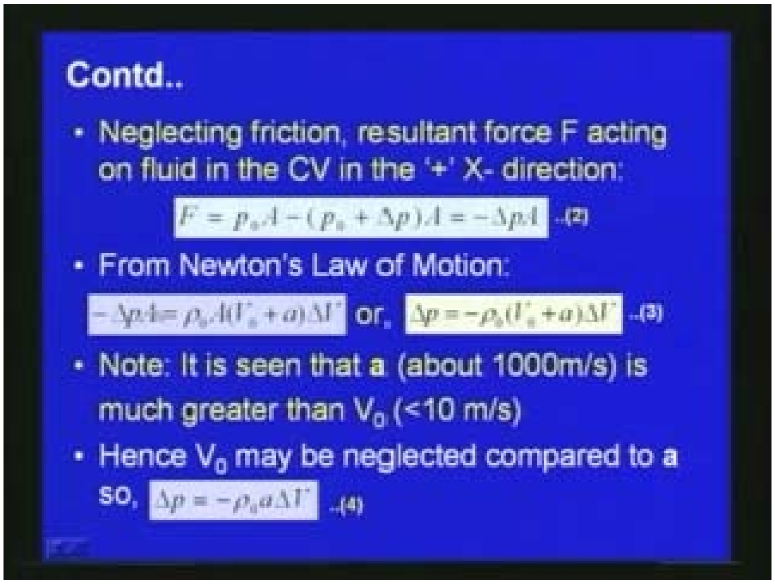

Now, if we neglect the friction resultant force acting in fluid in the control volume, in the x direction, so here if we consider control volume, then the resultant force F acting on fluid, in the control volume, in the positive x direction, will be F is equal to p_0 into a minus p_0 plus delta p into a. So we consider the control volume between two sections and then if the one section it is p_0 a and the other section, that means, with respect to the valve closure for the change in velocity we get V_0 plus delta. So the change in resultant force will be p_0 minus a minus p_0 plus delta p into a, that is equal to minus delta p into a as in equation number 2.

Now we can use Newton's law of motion. The resultant force is equal to rate of change of momentum as we done here in this equation number 1. So we can equate equation number 1 and 2, so that from Newton's law motion we get minus delta p into a is equal to rho_0 into a into V_0 plus a into delta V. So net resultant force is equal to rate of change of momentum or we will get this delta p - change in pressure - as delta p is equal to minus rho_0 into V_0 plus a into delta V as in equation number 3. So we get the pressure change delta p by the equation number 3.

So here we can see that through the experiment, through various analysis, we can show that is water hammer velocity is much higher compared to the normal velocity of fluid through the pipe over the closed conduit.

This ‘a’ the value of ‘a’ is the order of about 1000 meter per second, but generally the value of V_0, which is the velocity of the fluid moving through the pipe, it may be to the range of 10 meter per second. We can see that here with respect to this delta p is equal to minus rho_0 into V_0 plus a into delta v, if we consider this V_0 is much smaller compared to V, the water hammer velocity it is about 1000 meter per second, so we can neglect this V_0 here, so that we can write delta p is equal to minus rho_0 into a into delta v; so equation number 4. We neglect this V_0 since ‘a’ is much higher than compared to V_0. We get delta p is equal to minus rho_0 into a into delta v as in equation number 4.

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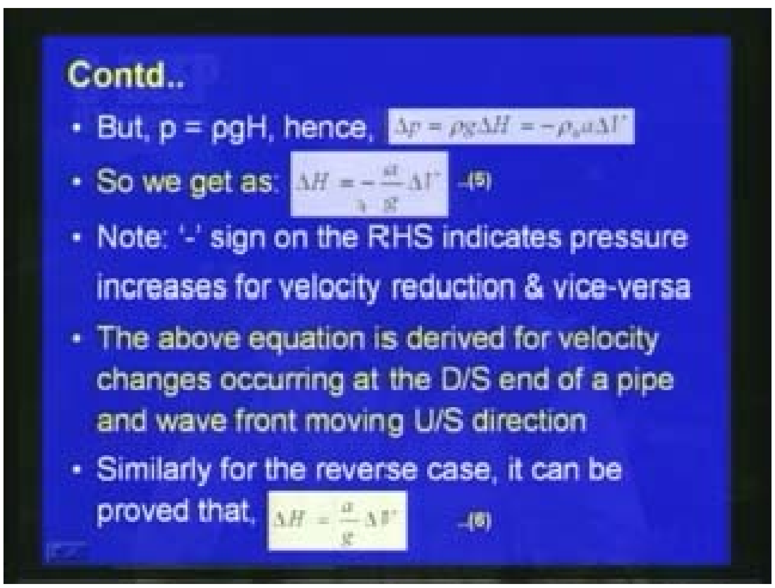

So now we want to find out what will be the change in the pressure head, so that we can represent this p as rho into g into H. Delta p can be written as rho g into delta H, which is equal to as we as seen, that is equal to minus rho_0 into a into delta V. Finally we get this delta H is equal to this. Here we can see that if we consider the change in density as negligible, we can cancel this rho_0 and rho, so that we get delta H is equal to minus a into delta V by g, as in equation number 5.

If we consider water hammer or with respect to water movement, then we can see that this change in density much less. So, we can cancel rho with respect rho_0. So we get delta H is equal to minus a by g into delta V as in equation number 5. Here please note that this negative sign on the right hand side indicates pressure increases for velocity reduction and vice-versa.

Here the pressure increases, when the velocity is reducing and then vice versa. This equation is derived for velocity changes occurring at the down stream end of the pipe and wave front moving up stream directions.

In a similar way, if the cases is reverse, that means, here the cases if the velocity change occurring at the up stream and then the wave front moving down stream, then same expression, sign changes; we can write delta H is equal to a by g into delta V as in equation number 6.

Here in this equation number 5 or 6 we get a relationship between the change in velocity and the water hammer velocity and then with respect the pressure change or the head change delta H. So that is the significance of this relationship, since we get now a relationship between delta V, delta H and the water hammer velocity 'a'. Here the g is the acceleration due to gravity as in equation number 6.

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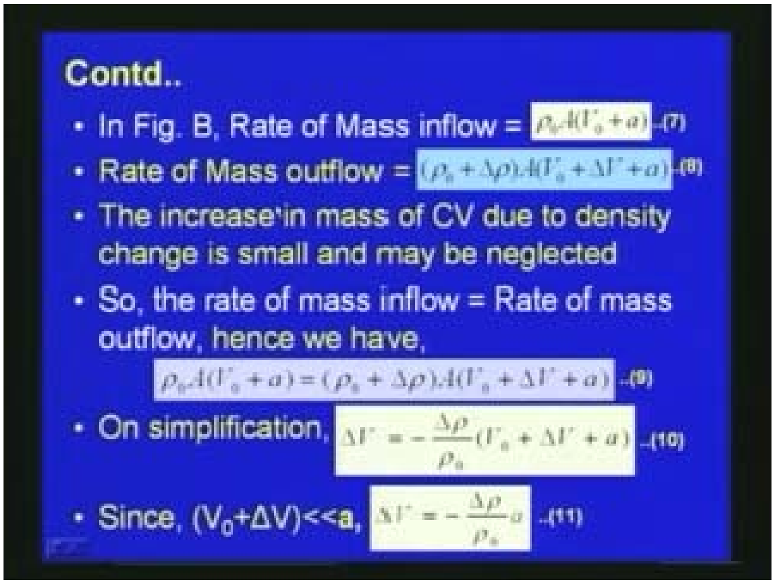

Now, with respect to this figure here (Refer Slide Time: 15:19 min), this figure B; so here if we consider this figure B, we can see that the [rate] of mass inflow; we can see here rate of mass inflow to the system, the control volume which we consider rho_0 into a into V_0 plus a; since we consider now with respect to steady state systems as in equation number 7.

Mass inflow is rho_0 into a into V_0 plus a and rate of mass out flow from the control volume is say with respect to change in density and velocity. It is rho_0 plus delta rho into a into V_0 plus delta V plus a as in equation number 8.

Now the increase in mass control volume due to density change is small and may be neglected, so that the rate of mass inflow is equal to rate of mass outflow. With the small control volume which we consider between section one and two in the pipe are the close conduit. We can see that the rate of increase or decrease the mass will be much less. We can neglect the rate of mass inflow to rate of mass outflow, so that we can write rho_0 into a into V_0 plus a is equal to rho_0 plus delta rho into a into V_0 plus delta V plus a as in equation number 9.

Here we can simplify this equation to get delta V as delta V is equal to minus delta rho by rho_0 into V_0 plus delta V plus a as in equation number 10.

So here again we can see that even this V_0 plus delta V will be much smaller compared to water hammer velocity a, so that here in this relationship we can write delta V is equal to minus delta rho by rho_0 into a as in equation number 11.

So this reason is that water hammer velocity it is range of about 1000 meter per second. So even the change in velocity of the fluid will be, which is flowing through the pipeline, will be much less, it will range of 10 meter per second; so that we can approximate delta V is equal to minus delta rho by rho_0 into a as in equation number 11.

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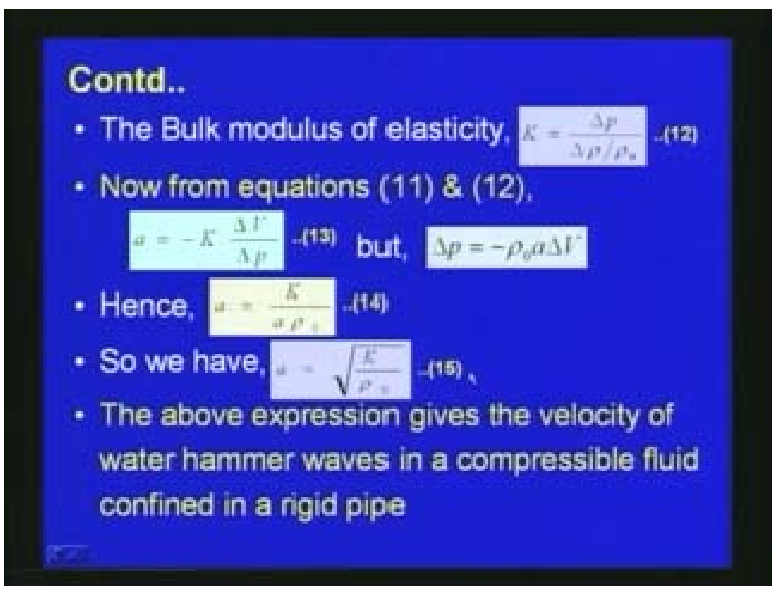

Now let us introduce this with respect to the fluid movement, let us introduce this Bulk modulus of elasticity of the system. If you write the Bulk modulus of elasticity K is equal to minus delta p by delta rho by rho_0 as in equation number 12.

We introduced k is equal to delta p by delta rho by rho_0. Now with respect to equation number 11 and 12, we can write the water hammer velocity a is equal to minus K into delta V by delta p as in equation number 13.

Here we can see that now this relationship for the water hammer velocity with respect to the Bulk modulus of elasticity K and then change in velocity delta V and change in pressure delta p. So a is equal to minus k into delta V by delta p as in equation number 13. We have already seen here earlier this delta p is equal to minus rho_0 a into delta V.

If you substitute back here (Refer Slide Time: 18:43 min) we get a is equal to K by a rho_0 or we get the water hammer velocity a is equal to square root of K by rho_0; rh as in equation number 15.

So water hammer velocity a is equal to square root of K by rho_0 where K is the Bulk modulus of elasticity of the fluid considered and rho_0 is the density of the fluid. So we finally what to we get here the water hammer velocity a is we can see that it is a parameter which depends on the Bulk modulus of elasticity and the fluid density as in equation number 15.

So a is equal to square root of K by $rho_{0.}$ The above expression gives us the velocity of water hammer waves in compressible fluid confirmed in a rigid pipe. So if the pipe is rigid, we can use this relationship, but if the pipe is not rigid, then if it is elastic pipe then we have to change this equation. So far the derivations we have considered the pipe to be rigid. So a is equal to square root of K by rho_0 as in equation number 15.

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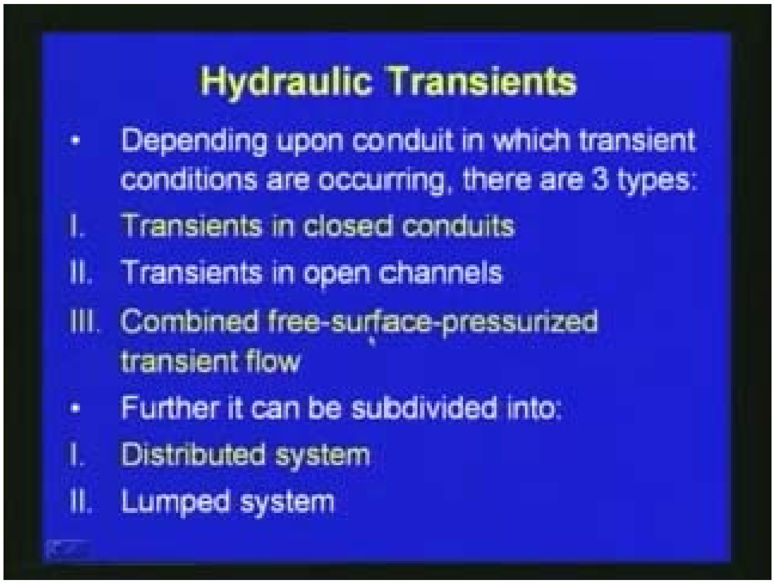

Now say we have seen with respect to a sudden valve closure in the case of a pipe or closed conduit connected to reservoir, so a water hammer or hydraulic transient is generated. So, with respect to the hydraulic transient the velocity water hammer we have found relationship, we have found the relationship with respect to delta H delta, the water hammer velocity and change in velocity. Also, we have found the relationship for the water hammer velocity as square root of K by rho_0 as in equation number 15.

Now we can see that as I mentioned in the case of a sudden valve closure, then wave is generated, water hammer, the velocity is happening and the wave generated and it is going back and again it is coming back; so there will be transitory moment with respect to the valve closure and reservoir and then periodically a wave action takes place. We have seen what is the velocity of the water hammer and we have derived relationship. So with respect to this, now we want to analyze … we want to be derive the governing equations with respect to this hydraulic transients or the water hammer. Now hydraulic transients we have seen depending upon the conduit in which transient conditions are occurring we can have three type systems.

Hydraulic transients in closed conduits which we will be discussing here, like in the pipes and also the case of open channels also we can have transients, so that is in transient so in open channels and then we can have combined free-surface-pressurized transient flow - that is the third kind of system.

We can classify the hydraulic transient with respect to what kind of systems we dealing, we can have hydraulic transient closed systems, we can have hydraulic transient in open systems or we can have at work transients in combined free-surface-pressurized transient flow system.

Here in this lecture we are only discussing the transients in closed conduits and then also with respect to how whether the fluid masses, whether it is distributed as in the case of a pipe flow or whether it is lumped as in the case surged tank. We can also classify the hydraulic transients into distributed system and the lumped system. These are the some classifications with respect to the hydraulic transient.

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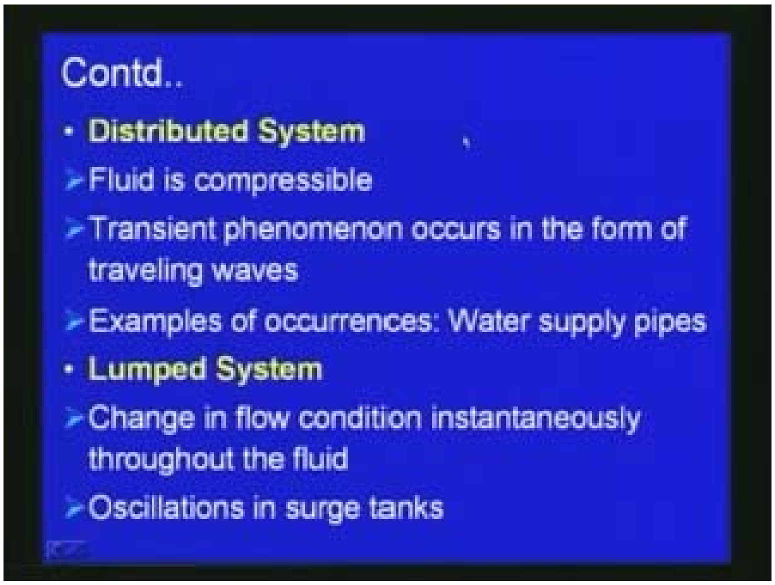

Now here we will discuss the hydraulic transient in closed conduits or pipe systems. Also we will say we can see that the pipe system which we discussed is the distributed system; also we shall briefly discuss the hydro transients with respect to lumped systems as in the case of a surge tank. As far as distributed systems are considered, we derived the basic equations, the fundamental equation by assuming the fluid is compressible and then transient phenomenon occurs in the form of traveling waves.

For example, in water supply pipes when due to sudden closure of a valve we get the hydraulic transient; so that is the case of distributed systems. Then as far as lumped systems are considered, it is change in flow conditions is instantaneously throughout the fluid. So there is a sudden mass of fluid with respect to that change what happens; so that the example is oscillations in surge tanks. Now with respect to these fundamentals which we have discussed so far, only hydraulic transients we will derive the fundamental governing equations with respect the hydraulic transient.

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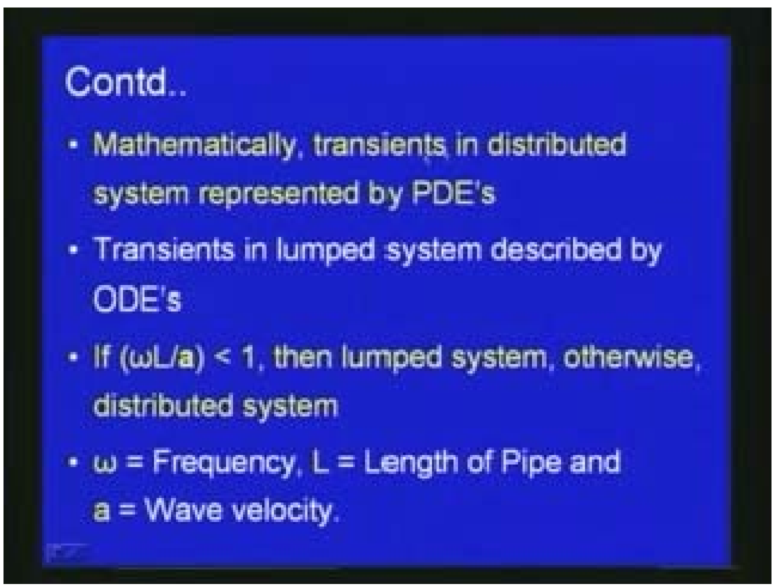

The governing equations are the dynamic equations and the continuity equations. So mathematically the transient and distributed system like in the pipe flow or close conduit represented by partial difference equations and as far as transients in lumped system is generally described by ordinary differential equations.

How to classify whether the system is distributed system or the system is lumped system? There we can use symbol relationship between this omegaL and a, where omega is frequency of the which the wave travels; omega is frequency L is the length of pipe and a is the wave velocity. If you find this relationship omegaL by a if it is less than 1, then the system is lumped system and if it is greater than equal to 1, then we call it say distributed system. So this classification we can have with respect to the frequency of the wave movement and also the water hammer velocity or wave velocity and the length of the pipe.

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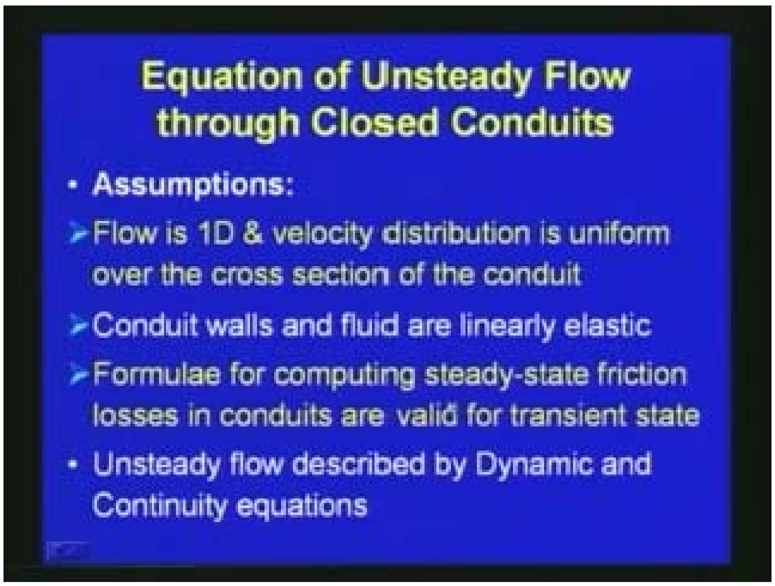

Now we want to derive, we want to see - what are the governing equations with respect to the unsteady flow over the hydraulic transients for closed conduit or unsteady flow in pipes. Here we discuss equation of unsteady flow through closed conduits. As we have seen earlier, here also we use some fundamental assumptions to make the system similar and then to derive the governing equations. Here since we are dealing with the closed conduits or pipes as far as unsteady flow is considered, we assume that flow is one-dimensional and velocity distribution is uniform over the cross section of the conduit.

The flow which we considering here - the pipe flow - we consider the one-dimensional flow and then also the considered that the cross section, the velocity distribution is uniform over cross section of the conduits. The second assumption is that the conduits walls and fluid are linearly elastic - this is second assumption. The third assumption is formula for computing the steady state friction losses in conduits are valid for transient state also.

Based upon these three assumptions here we derive the unsteady flow equations, which we have described by the dynamic and the continuity equations.

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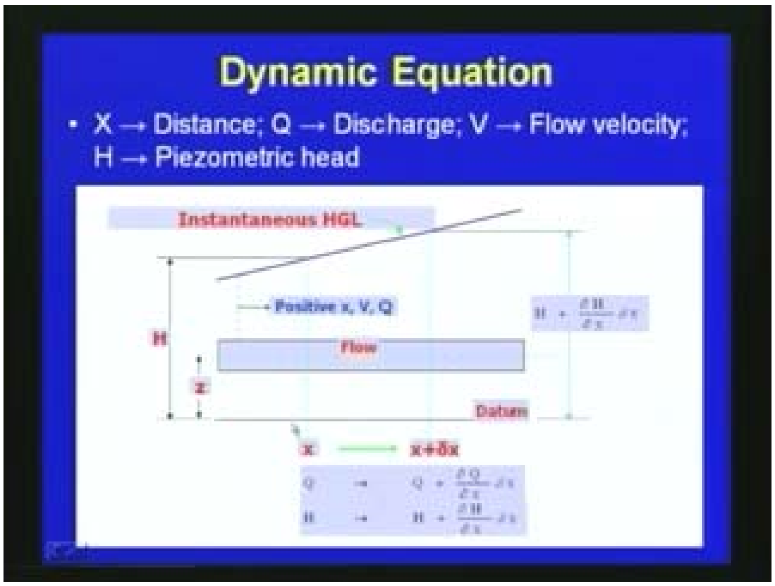

To derive this dynamic equation and continuity equation we consider here say there is the flow through a pipe here over the close conduit and flow is going this direction. Here the radius of the pipe is r and here is datum and here the total head is the piezometric head here H and the here it is plotted the instantaneous hydraulic gradient line is plotted for the system. Here we consider two sections: at section a11 here at distance x from the origin.

So it is the discharge is Q and head is H and then it section two here at distance delta x; so x plus delta x the discharge is changing to Q plus del Q by del x into delta x and H is changed to h plus and del h by del h into delta x. So here the head is H plus del H by del x into delta x. So this is the control volume which we considered here, where H is the distance, Q is discharge, V is the flow velocity and H is the piezometric head. We consider the control volume between section one and two to derive these dynamic equations.

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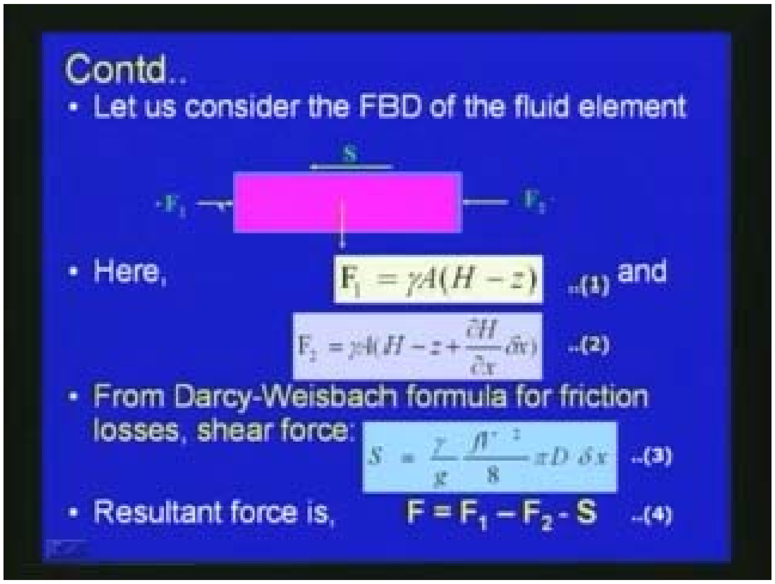

Now if we consider the free board diagram of the fluid diagram element, you can see that here we consider a system between the section one and two. If we consider the fluid element between section one and two, we can see that here the forces acting are the force F_1this direction with respect to the pressure force F_1 and here on this direction here up to F_2 and here there is a shear force S and then weight of the fluid elements.

Here we assume that pipe is horizontal, so we do not consider the derivation, the weight of the fluid element. With respect to F_1 we can write here for this system F_1 can be written as F_1 is equal to gamma A into H minus z as in equation number 1 and F_2 we can write gamma A into H minus z plus del H by del x into delta x as in equation number 2.

Now as far as shear force is considered we can use the Darcy-Weisbach formula. So from which we have seen earlier we can write the shear forces S is equal to gamma by g into fV square by 8 into pi D into delta x; where D is the diameter of the pipe and then g is the acceleration due to gravity, gamma is the specified H; f is the friction factor and V is average velocity across section as in equation number 3.

So the resultant force for the system we can write F is equal to F_1 minus F_2 minus S in equation number 4.

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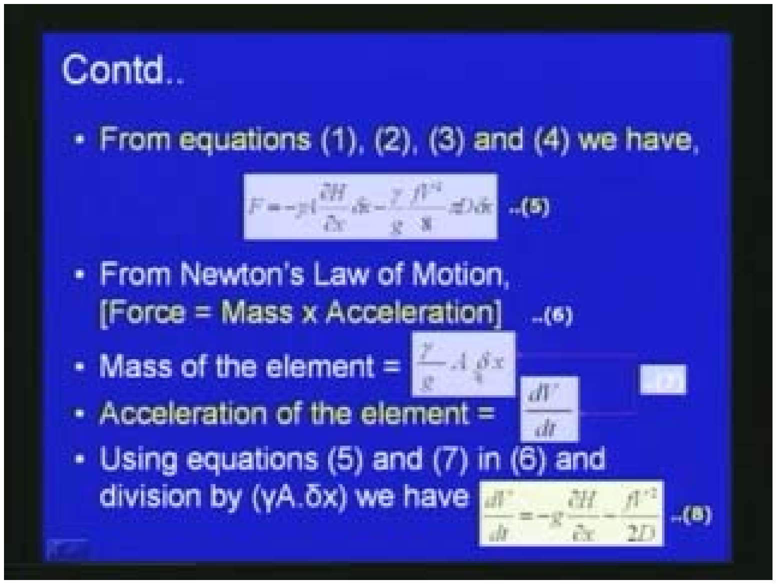

So from equations 1, 2, 3, and 4 we can write F is equal to minus gamma A del h by del x into delta x minus gamma g into fV squared by 8 into pi D into delta x.

For the given system here we considered here this section one and two; so this is F_1, F_2, S; so here F is equal to F_1; F_1 is this direction; this is F_2; so F_1 minus S. We finally we got the resultant force F is equal to minus gamma A del H by del x into delta x minus gamma by g into FV square by 8 into pi D into delta x as given in equation number 5.

Here if we use Newton's law of motion, we can write force is equal to mass into acceleration. So mass of the element with respect to the fluid between section one and two we can write - mass of the element is equal to gamma by g into a into delta x; so we consider between section one and two the distance is delta x, so gamma by g into a into and delta x. Then acceleration is considered we can write acceleration of the element is equal to dv by dt so that this we can write as with respect this, the acceleration is dv by dt. So now if we use equation number 5 and 7, this relationship for acceleration, this is mass and acceleration together. We can write and if you divide by gamma A into delta x we have dv by dt is equal to minus g into del H by del x minus f V squared by 2 d as in equation number 8.

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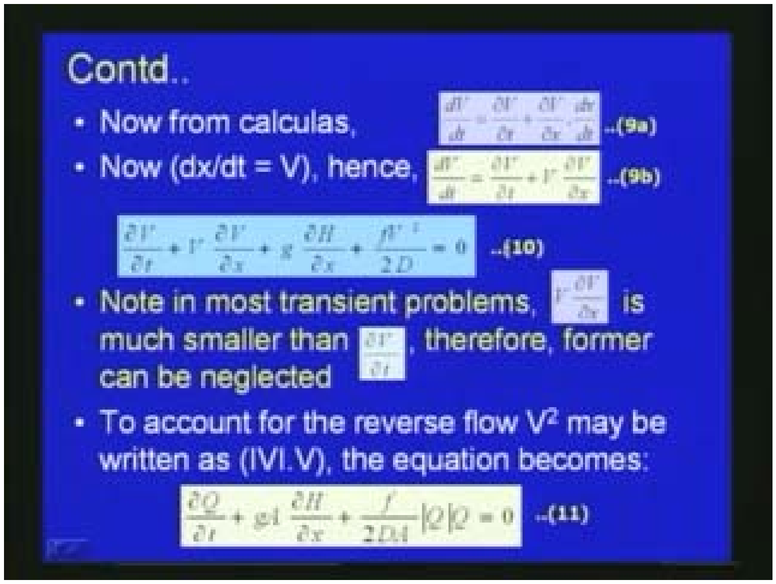

So now from calculus we can write this dv by dt, which is acceleration is equal to del V by del t plus del V by del x into dx by dt, since dx by dt is equal to velocity V, we can write dv by dt is equal to del v by del t plus v into del v by del x as in equation number 9 b. So that finally we can write the dynamic equation as del V by del t plus V into del V by del x plus g into del H by del x plus fV square by 2D is equal to 0, as in equation number 10.

Here we can see that in most of the transient problem this V in into del V by del x is much smaller than the acceleration del V by del t, local acceleration del v by del t; therefore, we can neglect this term if this neglect this term and if we account for the reverse flow V square, we can represent by writing modulus V into V, so that and then transforming Q is equal to a into V. So finally this equation number 10, we can write as del Q by del t plus g into A into del H by del x plus f by 2 DA into Q modulus Q into Q that is equal to 0 as in equation number 11.

This is the dynamic equation with respect to the unsteady flow and transient flow through the closed conduit or the pipe line. Finally, we got the relationship in terms of discharge. Here this V is put in terms of Q by A. Finally, we got the relationship in terms of discharge as in equation number 11.

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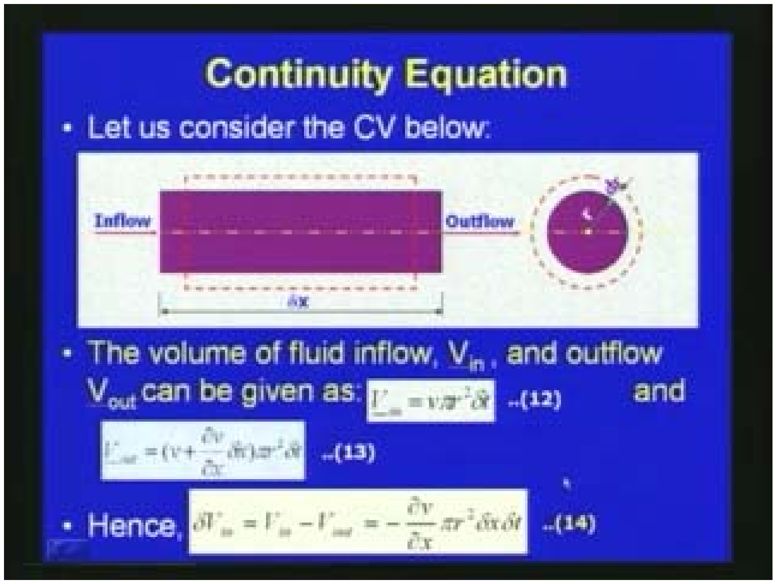

This is the dynamic equation which we have derived based upon the Newton second law. Now the second equation for the unsteady flow through closed conduits or pipe we can obtain from the continuity equation; so to derive the continuity equation let as consider a system like this.

So here we consider a control volume, say, the flow is taking place with respect to pipe which we consider. So this is the control volume; so inflow is here and out flow here; the radius of pipe is r and with respect to change let us assume at section two in this radius is changed to r plus Dr.

Now the volume of fluid inflow at this location V under bar V_{in} and outflow V under bar V_{out} can be given as V under bar is equal to the velocity is small v into pi r square into delta t as in equation number 12 and V bar V_{out} is equal to V plus del V by del x into delta x into pi r square into delta t as in equation number 13.

With respect to inflow and outflow the change in volume is delta V_{in} is equal to delta V_{in} minus V_{out}, that means, the change in volume is equal to minus del V by del x into pi r square into delta x into delta t as in equation number 14.

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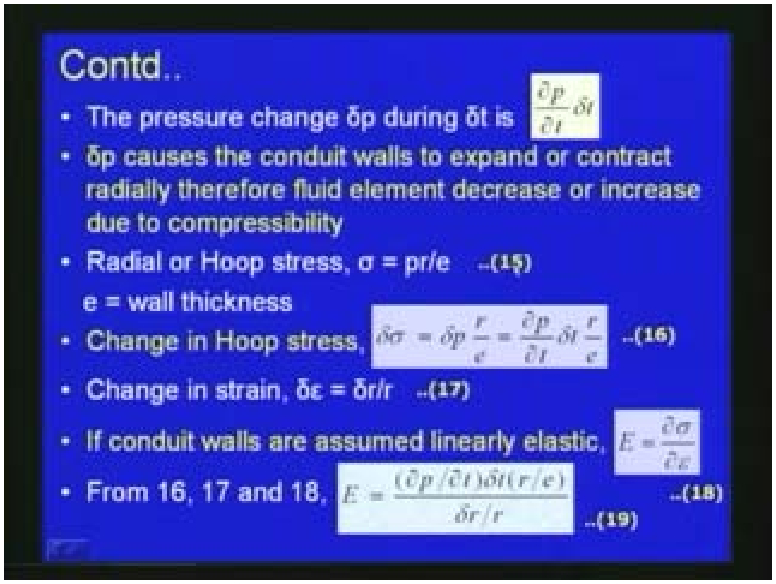

Then the pressure change delta p during delta t the time, change in time we can write as del p by del t into delta t. So the delta p causes the conduit walls to expand or contract radially. Therefore, the fluid element decrease or increase due to the compressibility effect.

Finally we can see that the radial or hoop stress for the pipe or the closed conduit which we consider we can write sigma is equal to p into r by e, where p is the pressure and r is the radius of the pipe and e is the valve thickness; so that we can write sigma is equal to pr by e as in equation number 15.

Then the change in hoop stress we can write delta sigma is equal to del p into r by e. So that is equal to del p by del t into delta t into r by e as in equation number 16.

So the change in strain we can write; so strain change will be delta epislon that is equal to delta r by r as in equation number 17 and if the conduit walls are assumed linearly elastic, so that Young modulus we can write as e is equal to del sigma by del epsilon; that means change in stress by change in strain; so e is obtained from this equation number 16, 17 and 18 as e is equal to del p by del t into delta t into r by e divided by del r by r as in equation number 19.

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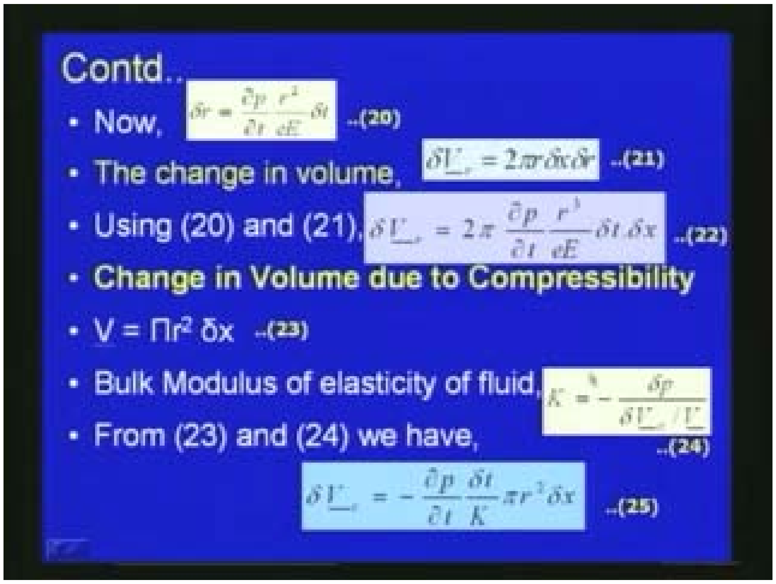

Now we can write delta r is equal to del p by del t into r square by e into capital E which is the Young modulus into delta t as in equation number 20.

So change in volume delta V_r V under bar r is equal to 2 pi r into delta x into delta r as in equation number 21.

Now using 20 that means equation to 20 and 21, we get the delta V is equal to 2 pi into del p by del t into r cube by e into e small e into E into delta t into delta x as in equation number 22.

So change in volume due to compressibility we can write V under bar is equal to pi r square into delta x as in equation number 23.

The Bulk modulus of elasticity of fluid is K is equal to minus del p by the change in rate in change in volume, so that is delta V under bar c divided by delta V bar.

Finally, we can use this equation 23 and 24 we will get this delta V_c bar is is equal to minus del p by del t into delta t by K into pi r square delta x as in equation number 25.

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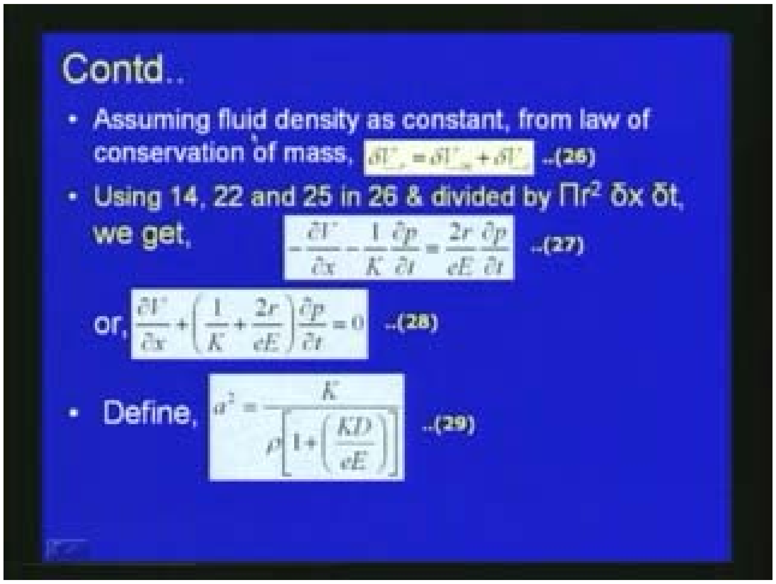

Finally, if you assume that fluid density is constant and now from the law of conservation of mass, the total rate of change in volume V under bar r is equal to del V under bar in plus del V under bar c as in equation number 26.

So if we use equation number 14 which we have derived here; then if we use equation number 22 is this equations for del V under bar r and then and 25 is del V under bar c, in equation number 26 and if we divide by pi r square, the cross section area of pipe into delta x into delta t; delta x is length is we considered; delta t is time step. So we will get minus del V by del x c minus 1 by K del p by del t is equal to 2 r by small e into capital E into del p by del t as in equation number 27. Or finally, we get del V by del x plus 1 by K plus 2 r by e e small e into capital E into del p by del t is equal to 0 as in equation number 28.

So if you defined here this with respect to this square of the water hammer velocity, a square as K by rho into 1 plus KD by e e small e into E, so this is with respect to elastic pipe. Earlier we have seen relationship for the rigid pipe a is equal to square root of K by row; for elastic pipe we have to write an like this equation number 29; we can see this relationship is derived in hydro transience by Hanif Chaudry; so this a square is equal to K by rho into 1 plus KD by e into E as in equation number 29.

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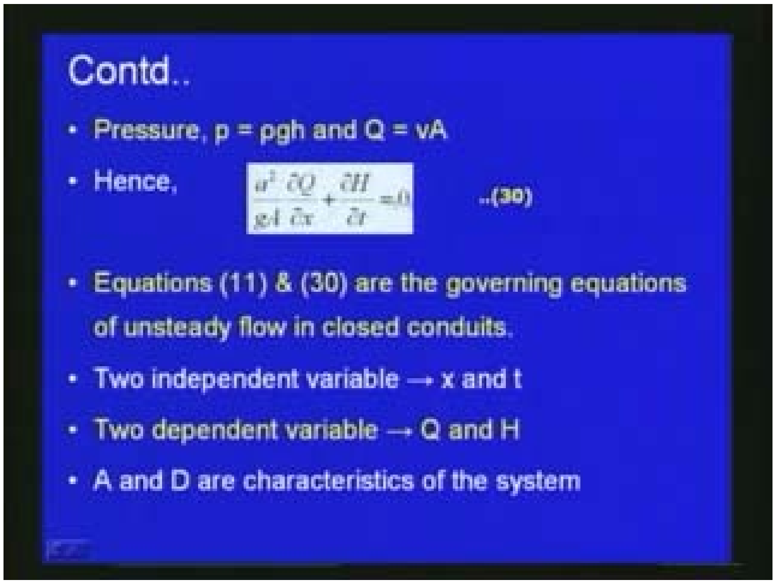

So after putting this and then if I substitute pressure p is equal to rho into g into H and Q is equal to V into a we get A square by g A into del q by del x plus del H by dt is equal to 0. So this gives the continuity equation. So here A is the water hammer velocity, capital A is the area of the cross section of the pipe; Q is the discharge; H is the head and t is the time. This relationship gives the continuity equation. Here equation number 30; the 11 is dynamic equation and equation number 30 is the continuity equation. So this equation number 11 and 30; so equation number 11, which we have derived here, this equation number 11 and then equation number 30. These equations both are the governing equations in unsteady flow in closed conduits. So here we can see that both equations are partially equations and then there are two independent variables here - x and t; special dimension and one-dimensional x in the x direction is x and then time t and we have two dependent variables - the discharge Q and the head H and here the other parameters like a A -the cross section area and D - the diameter, are characteristic of the systems.

So we can solve the unsteady flow in closed conduits of pipelines; we can solve using the dynamic equation and continuity equations, which we have derived now.

So now to solve this equation… we can see that we are analyzing the unsteady or transience in the pipe line system or the close conduit systems, we can say the equation is governing equations, which have this derive are the partial differential equations like the continuity equation here - equation number 30 or the dynamic equation number 11; so these are partially [differential] equations.

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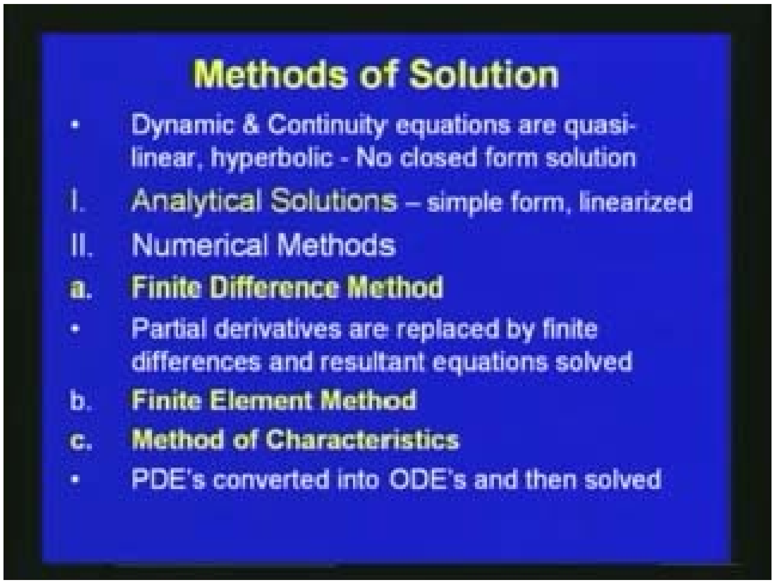

So, to get closed form solution or analytical solution is difficult; so the dynamic and continuity equations are quasilinear, hyperbolic; so no closed form solutions is generally possible, but say if we simplify the system and then if you linearize the equations as we discussed earlier, so the equations which are quasilinear we can linearize and then the boundary conditions are simple, the consider system is simple, then we can derive some simplified form of the analytical solutions which are available in literature.

So as far as the methods or solutions for the unsteady flow in closed conduit of pipes are considered, we can have simplified analytical solutions or so-called closed form solutions for simple system and a linearised system, but generally this may not give accurate results when we deal with practical problems. So analytical solution is generally used for the verification of the numerical models or other kinds of model which we generally develop; otherwise, analytical solutions we cannot use for practical problems.

Then second one is generally used methodology is numerical method. So numerical methods are considered; here we have dynamic and continuity equations. So these equations we can approximate using various using numerical techniques use like a finite difference method, finite element method or finite volume method or boundary element method or method of characteristics like that. You can see that most of the computational fluid dynamic is given to packages, also with all this unsteady flow in closed conduit of pipes, using various techniques is like finite difference method, finite element or finite volume methods….

Some of the commonly used methodology are final difference method, say, like implicitly or explicitly finite difference method. Here as far as finite difference method is consider partial derivatives are replaced by finite differences and then resultant equations are solved. Then finite element method we use either Gallerkin or the method of [weight] received or the other difference forms finite element method we can variation principle like different finite element methods we can use. So actually this is an integer form with respect to… we will approximate the governing equation with respect to say an interpolation function and then we will integrate with respect to this system and then we will find out the waiter received and then we will approximate. So that is basic principles behind finite element method.

In finite difference method we may discritize the dominate to grids with respect to one-dimensional x direction and then time with respect to delta t and delta x. Finite element method is considered, we use one-dimensional elements - it can be linear or quadratic or cubic depending upon the case and then generally, you may use the time is considered, you may use the finite difference scheme for time discritization generally. So that is finite element method.

Another commonly used method is called method of characteristics. Here the partial differential equations are converted into ordinary differential equations and then solve. So by using either the finite difference method or finite element method or say other methods like boundary element method, finite element method or method of characteristic that are number of substitute packages available for the unsteady flow through pipes or closed conduits, which we can directly utilize or based upon the fundamental equations which we discussed - the dynamic equations and the continuity equations - we can develop a computer a code by using one of the methodology and then we can have a computer program to get a solution for the problem which we consider.

So different methods of solutions are available for unsteady flow problem through the closed conduits. So this is as for unsteady flow through pipes or closed conduits by considering the distributed system. Then we have also seen the another system is the lumped system, where the use say as in the case of surge tank instead of the distributed system here, we have lot lumped system, where a large mass is say either transferred to the surge tank or going from the surged tank.

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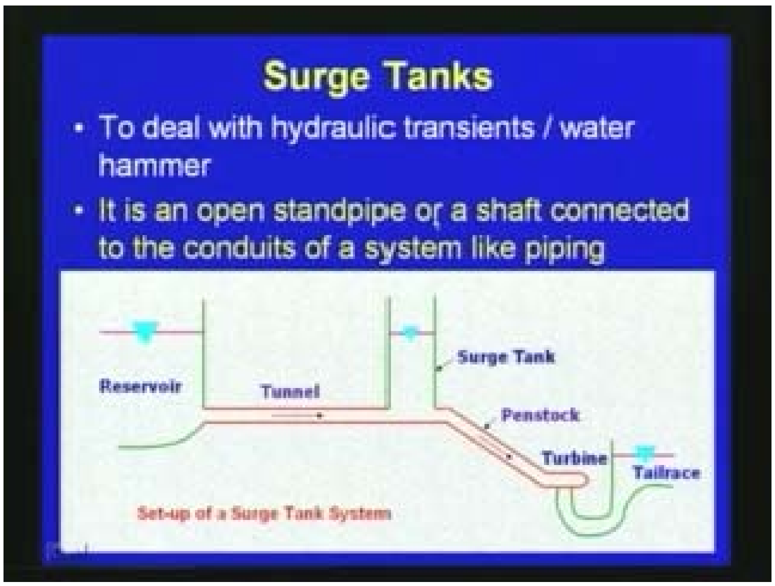

Here we will be briefly discuss the surge tank which we discussed earlier. So surge tanks we used to deal with the hydraulic transients or water hammer. As we discussed say here there is reservoir and then it is tunnel and then there is a penstock and here there is a turbine. After water passes through the turbine, it is goes to the tailrace here. So due to a sudden closure of the turbine, you can see that hydraulic transient or water hammer is generated and then so to avoid of any kind of say fracture in the penstock or the tunnel to take care of the water hammer, we generally the provide a surge tank like this. So this is a symbol kind of a surge tank. Here you can see that the surge tank is an open standpipe or a shaft connected to the conduits of a system like piping. Here typical surge tank is shown.

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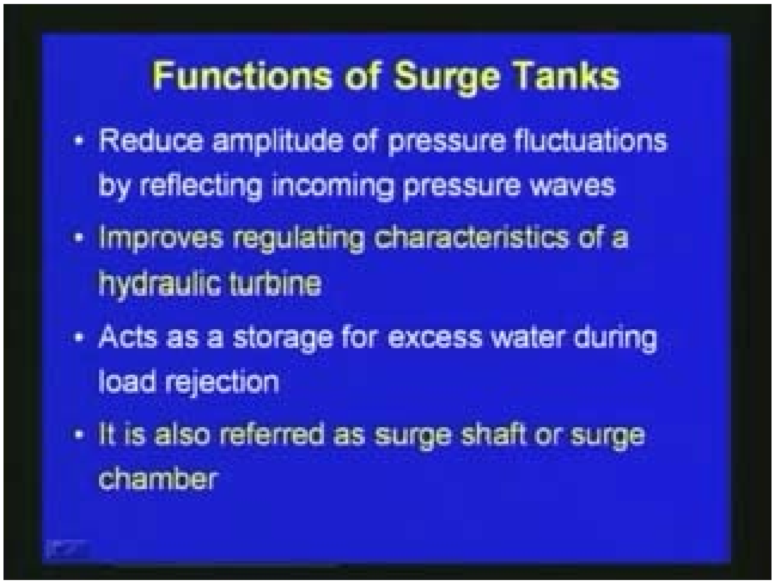

Some of the functions of the surge tanks include: it reduces the amplitude of pressure fluctuations by reflecting the incoming pressure wave; so what ever incoming pressure wave it say reflects the incoming pressure waves, so that the amplitude of pressure fluctuations are reduced; the surge tanks improves the regulating characteristic of a hydraulic turbine, since here we have large stand pipe or so-called surge tanks for shaft here we can see that we can easily control what is with respect to the turbine here by using [oars].

So it improves the regulating characteristic of the hydraulic turbine and also it acts as a storage for excess water during load rejection. It is also refer as surge shaft or some times surge chamber depending upon which way we deal.

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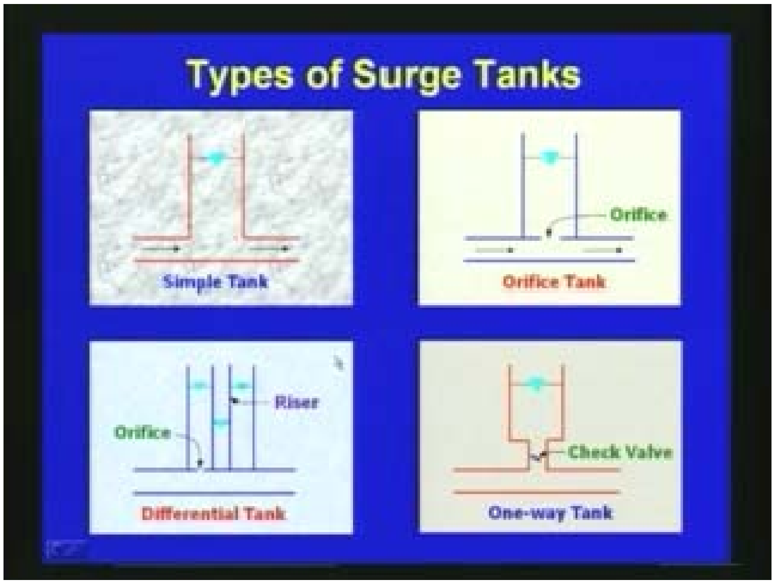

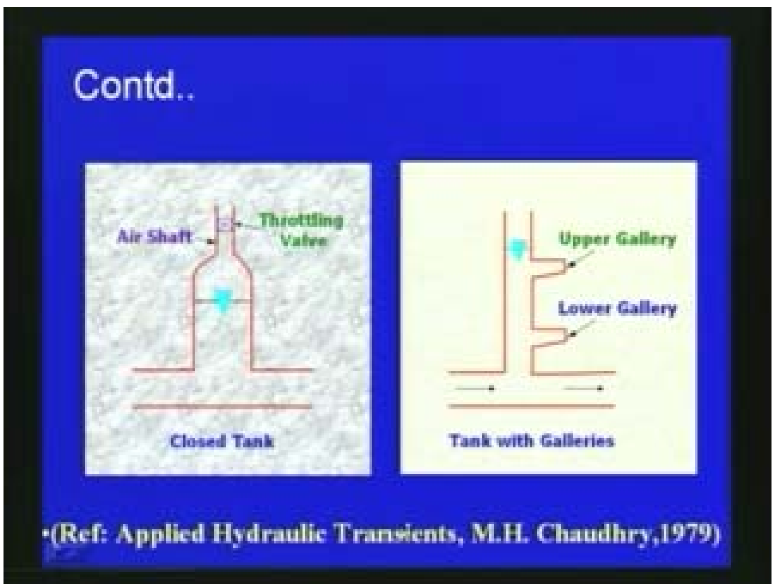

Tere are different types of surge tanks: a simple surge tank is shown here; this is same simple surge tank, connected with the closed conduit or pipe system; this is called an orifice tank, so here there is orifice arrangement here; then there can be differential tank like this, we can see the water is at different level with respect to the system say this is riser; so this is one orifice arrangement and also we can have one-way tank like this with respect to check valve, it is only flow is allowed this direction, so there can be one way tank also.

There can be different kinds of surge tanks, different shpaes of surge tanks; so that it depending upon which type of problem we want to deal with you can use. Also here we can see a closed tank with air shaft totaling valve, so that with respect to we can keep pressure; here it is called closed tank and then we can also have a tank with galleries, here we can see lower gallery, upper gallery like this. All these details we can see in the text book by Hanif Chaudhry - Applied hydraulic transients.

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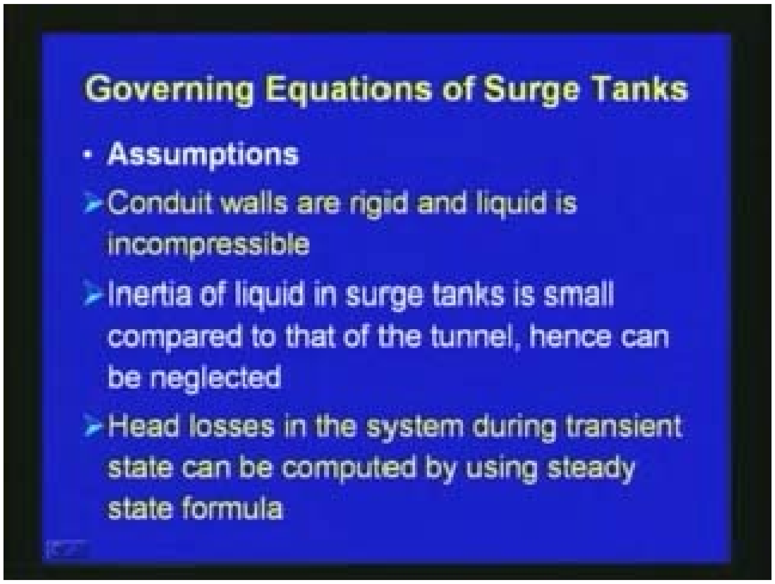

Now, we will just briefly discuss what are the governing equations for such a system or the lumped systems.

Here also we use some of the assumptions like conduit walls are rigid and liquid is incompressible and inertia of liquid in surge tanks is small compared to that of the tunnel; hence we can neglect the inertia of the surge tank and then also the head losses in the system during transient state can be computed by using steady state formula. As we as seen the case of unsteady flow through pipes or closed conduit which we have derived earlier so very similar we use some of the assumptions here.

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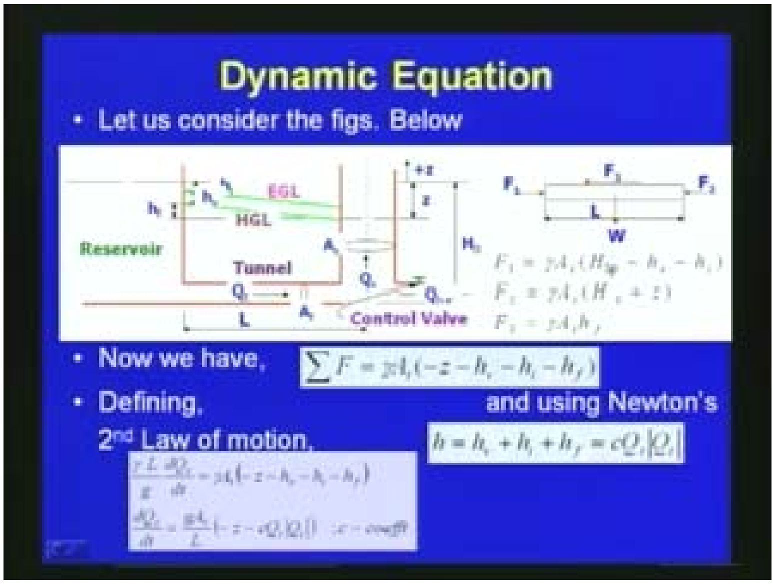

Then we consider a system reservoir and then here we have got pipe line and then there is a surge shaft or a simple surge tanks and here then we have a valve and that goes to turbine. If we consider simple system like this, so here we can see to derive the dynamic equation we consider control volume like here, so that here we consider control volume for the forces acting so the pressure between section one and two - here is the section one here section two - here the force on section one is given as F_1 is equal to gamma A in at; a t is the cross section of the tunnel; so gamma A into S_0 minus h_V minus h_i. This here this S_0 is the initial head and then Hf is the head loss due to friction and h_p is the... if we consider the energy gradient line also, h_p is the energy loss or the velocity head loss and then if with respect to initial loss h_i.

Now F_1 is obtained is gamma At into S_0 minus h_p into h_i and then F_2 this side is considered. We can write with respect this figure At into h_0 plus z; so and then F_3 that means with respect to the friction loss here we can write F_3 is equal to gamma into At into h_f , which is the friction head h_f .

Now the resultant for sigma f is equal to with respect is three equation we can write sigma f is equal to gmma into At into minus z minus h_p minus h_i minus h_f. So here now if you using Newton's second law of motion we can equate this force resultant force to and the mass into acceleration.

Here if we consider this mass as gmma L by g and acceleration we consider as mass gamma L into g by g into a and then we write with respect acceleration dv by dt, so that is represent by d Q_t by dt so that may be we can write comma L by g into d Q_t by dt is the discharge through the tunnel. So that is equal to this gmma At the net resultant force gmma At into minus z minus h_v minus h_i minus h_f , where here h is equal to h_p plus h_i plus h_f. So that we can write this is equal with respect to Darcy-Weisbach equations which we have seen earlier we can write C into Qt into Qt.

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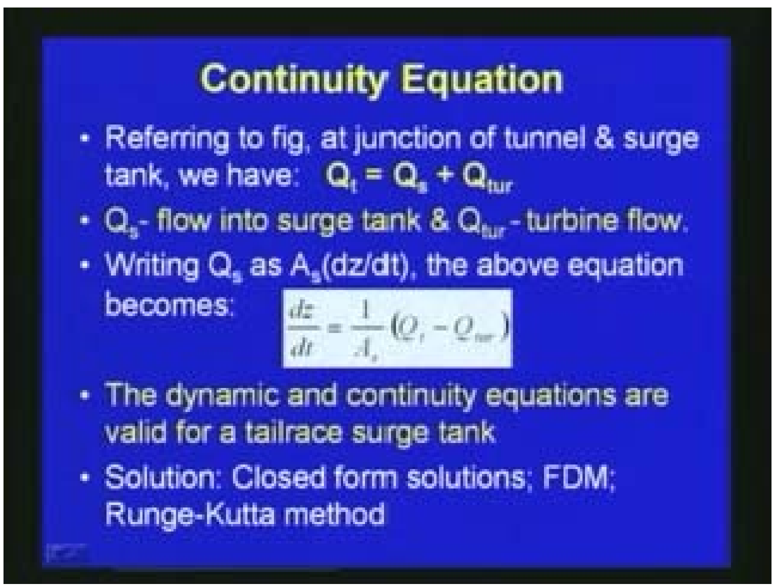

So that here this relationship finally we can write as d Q_t by dt is equal to g into At by L into minus z minus c Q_t into modulus Q_t to take care the direction of flow where c is a coefficients. Here Q_t is the discharge through the tunnel A_t is the process area of the tunnel of L_e is the length we consider from here to here and then c is the coefficient and z is the the head difference here with respect to this is the datum here this z is represented here. Now all the parameters at defined here; so we get this as the relationship for the dynamic equation with respect to the surge tank here.

The then continuity equation we can simple write the total discharge throw the tunnel Q_t is equal to Q what is going to the surge shaft Q_s is plus what is going to turbine $Q_{turbine}$.

So Q is the flow into surge tank; $Q_{turbine}$ is turbine flow; so writing Q_s as A_s into with respect this Q_s what is the surge tank Q_s as A_s into area cross section of the surge tank is A_s so A_s into dz by dt. This equation becomes dz by dt is equal to 1 by A_s into Q_t minus $Q_{turbine}$ the discharge in turbine.

Here Q_t is the tunnel discharge Q_t r by discharge to turbine, z is define here, this head difference and A_s is the area flow section of the surge tank and t is the time. This gives the continuity equation and dynamic equation is obtained by here this relationship.

Here this dynamic equation, continuity equations are valued for the tailrise in the surge tank here. We can see that both equations are ordinary differential equations; so we can get some closed form solutions for this also, since these are ordinary differential equations, but generally, here also we use numerical methods like a finite defined method and Runga Kutta method we can utilize to get solution for surge tank.

Finally to summarize so in this section of pipe flow systems we have seen in the basic principle of pipe flow, the governing equations, Darcy-Weisbach equations, then we consider the laminar and turbine flow systems and we have seen the losses in pipe flow and also we have discussed the multiple pipe flow systems like in pipe in series, pipe in parallel, branching pipes, etc., we have discussed.

Also, we have seen the pipe networks and solutions we have seen with respect to the pipe flow system, which we discussed here. Finally, in this chapter we have discussed the unsteady flow in pipes, the hydraulic transients and the surge tank.

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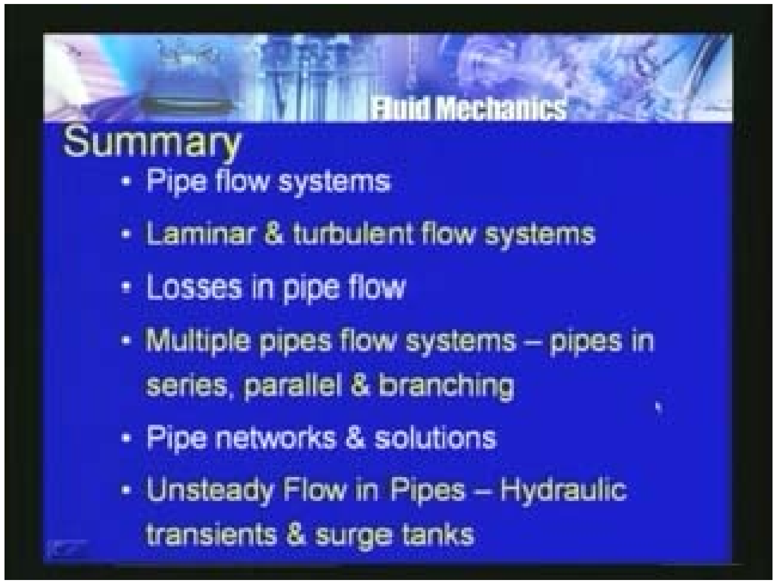

This is the last lecture of this fluid mechanics course. Here we have seen the various aspects of fluid mechanics and also we have seen some of the advance course on fluid mechanics including the boundary layer theory and the unsteady flows and also we have seen various fundamentals and also some of the applications as far as fluid mechanics course is considered.

In this 42 lectures we have covered most important aspects of the fluid mechanics, but still say fluid mechanics is very large subject, other aspects are also there, like we have not covered the compressible flow or we have not gone through the details of computational flow dynamics; other number of topics are there, but the main purpose of this course was to go through the fundamentals and some of the advance topics which are generally used for bachelor level as well as master level courses in IIT, Bombay.

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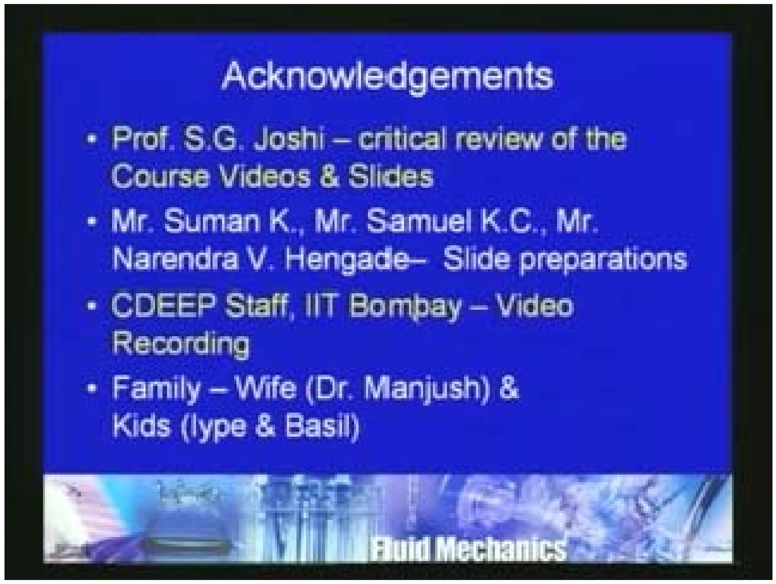

Finally, to conclude this video course, here the acknowledgements: I am very thankful to professor S. G. Joshi who has reviewed this video course with critical review of the course videos, as well as slides; so I am very thankful to Professor Joshi for the critical review given on the video as well as slides.

I am also very much thankful to my students Mr. Suman K, Mr Samuel, Mr Narendra who prepared most of the slides; very beautiful slides were prepared by the students; I am very thankful to them.

I am very thankful to the CDEEP staff who has done a wonderful job in this video of this course and very good quality and all this video recording I am very thankful to them.

Finally I am thankful to them my family my wife Dr Manjush and my kids Iype and Basil. Since, put I lot of efforts to develop to course to this form, so I am also thankful to them.

Finally I hope all these 42 lecture which has been given in this fluid mechanics course will be useful to the teachers as well as students, bachelor level as well as master level, whose taking fluid mechanics course.

Thank you very much. I hope you will enjoy these lectures.