Fluid Mechanics

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Lecture – 41

Pipe Flow Systems

Welcome back to the video course on fluid mechanics. In the last lecturer, we were discussing about the pipe flow systems and pipe networks with respect to a typical example, we have seen how to derive the equations and the mask balance or continuity balance equation for a typical case and the generalize form of this equations. Further, we will see how the generalize equations can be written with respect to the continuity balance equation, the energy balance equation and how we can solve this equations with respect to generations or successive approximation. We further discuss various types of pipe flow and analysis of pipe flow solutions.

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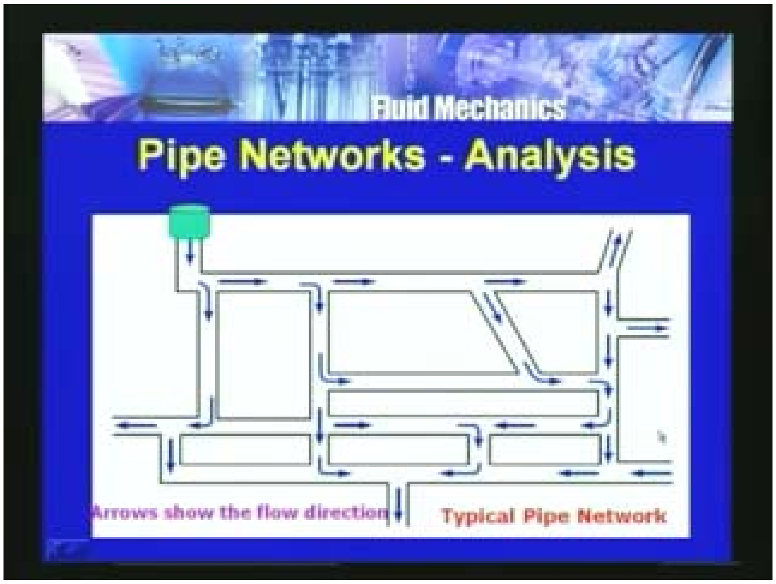

The pipe network, we have seen that, it will number of loops. Most of the time, we have to assume the flow direction and flow through each pipe, we have seen this aspect.

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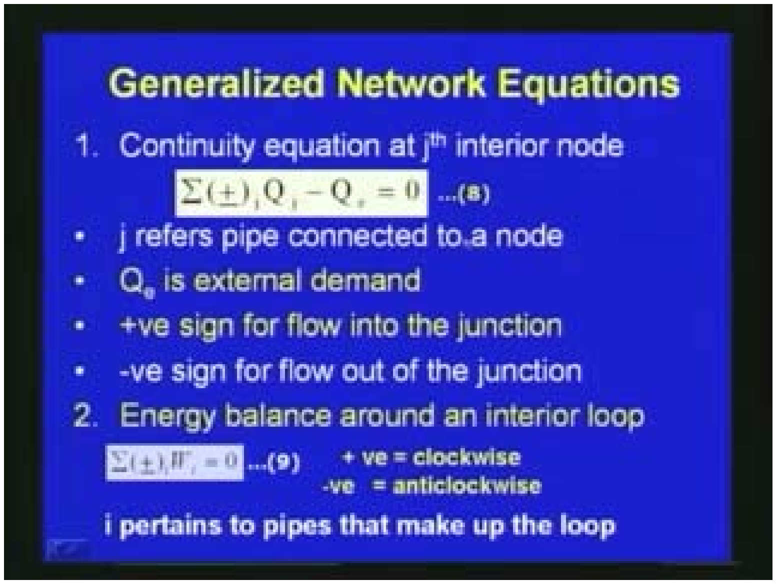

In the last lecture, we have seen the generalize network equation the continuity equation and jth interior node, we can write as sigma plus or minus$_j$ Q_j minus Q_e is equal to 0 as in this equation where, j refers pipe connected to a node and Q_e is the external demand and this here we use the positive sign for flow into the junction and negative sign for flow out of the junction. As we discussed in the last lecturer, the energy balance around an interior loop can be returned as sigma (plus or minus)$_i$ W_e equal to 0 as in this equation positive is used to for clockwise and negative is used for anti-clockwise. Here i pertain to pipes that make the loop. These are the two general form of equation one is the continuity equation and another one is the energy balance equations.

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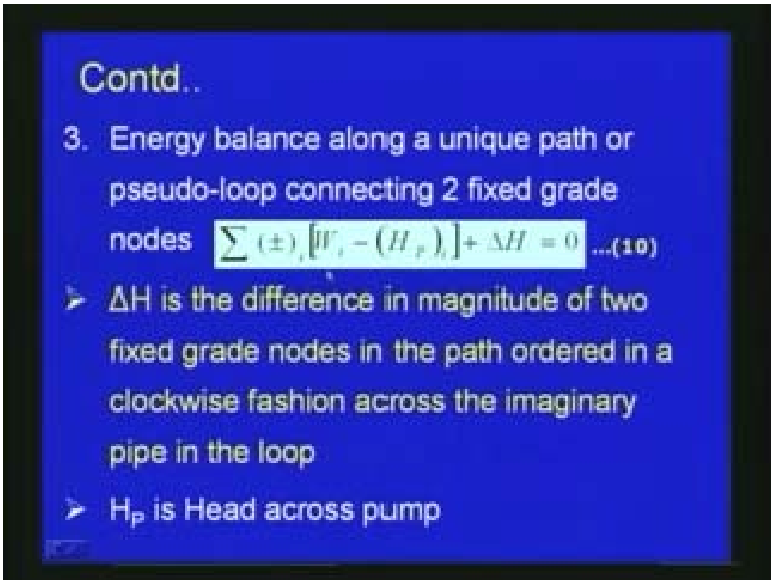

The energy balance along a unique path, we have seen that, there is possibility is pseudo loop at a fixed grade node or at the reservoir. We can have the pseudo loop that we have seen in the last example, the pseudo loop connecting to fixed grade nodes, we can write: sigma (plus or minus) $_i$ W_e minus $H_{p\,i}$ plus delta H equal to 0 where this H_p is the head closed pump and delta H is the difference in magnitude of two fixed grade nodes in the path ordered in a clockwise fashion across the imaginary pipe in the loop. This equation details also, we have seen in the last lecturer.

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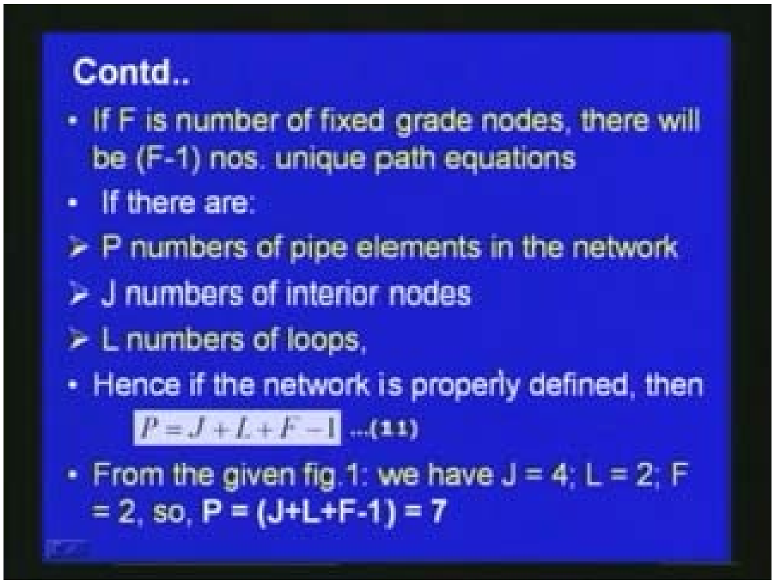

If F is the number of fixed grade nodes, there will be F minus 1 number of unique path equations. As we have seen in the last example, there were two reservoirs, we can how can unique path or depending upon the number of fixed grades nodes there will be F minus 1 number of unique path equations. If there P number of pipes elements in the network, J numbers of interior nodes and L numbers of loops then, the pipes are properly defined, the pipe network is properly defined when, P equal to J plus L plus F minus 1 as in given in equations. Where P is the number of pipe elements, J is the number of interior nodes, L is number of loops and F is the number of fixed grade nodes. If we consider the last example, we can see that, there are seven pipes to reservoir and two loops. Here, we can that, pipe network is defined properly when this equation that means P equal to J plus L plus F minus 1, here for this pipe network, we have J equal to 4 number of interior nodes L is equal to 2 that is number of loops 2 and F equal 2, P equal to J plus L plus F minus 1 that is equal to 7. So, we can say that, it is properly defined and we can get solution by using the generalize equations which we have discussed now.

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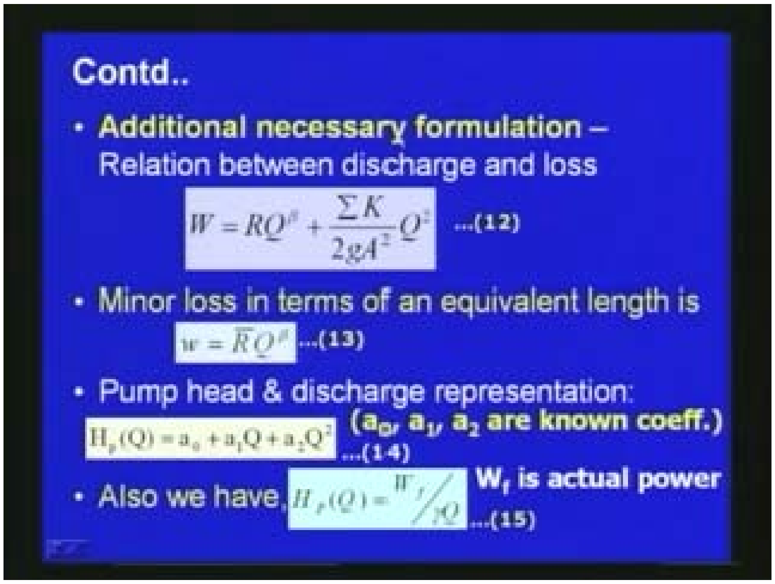

The additional necessary formulation required is the relation between the discharge and laws, as we derived earlier we can write: W equal to RQ to the power beta plus sigma K by 2gA square into Q square. So, minor laws we can generally this beta equal to 2, so minor laws in terms of an equivalent length as we discussed earlier we can write, w equal to R bar Q to the power beta.

So, we can change minor laws also we can add this and get a relationship between the discharge and losses where, r is the pipe resistance coefficient which discussed earlier and sigma K is corresponding to the minor losses. If there is a pump is involved in the network as we discussed earlier, we can have pump head and discharged representation is discharged relationship. Generally, this will be given by the pump manufacture and the general form of the equation may be: H_p Q equal to a_0 plus a_1 Q plus a_2 Q square where, a_0 a_1 and a_2 are the non-coefficients so pump head and discharged can be represented as function of Q also with actual power utilized we can write: H_pQ equal to W_f by gamma Q into W_f is the actual power, Q is the discharge through the pump and gamma is the specified to the fluid which is pump. We can utilize the generalized relationship with respect to this equation here the continuity equation and the energy balance equation. If there is pump involved in the network then, changes are to be added. With respect to this, we can this equation number twelve for addition necessary formulation relation between discharge and laws. By utilizing we can see that, the equations of the energy balance or the mass balance we can see they are non-linear nature. To solve these non-linear equations, we have to go for iterative solutions generally. It will be much easier if we can linearise energy equations so that solution will be much easier. We will briefly discuss how to linearise the energy equation.

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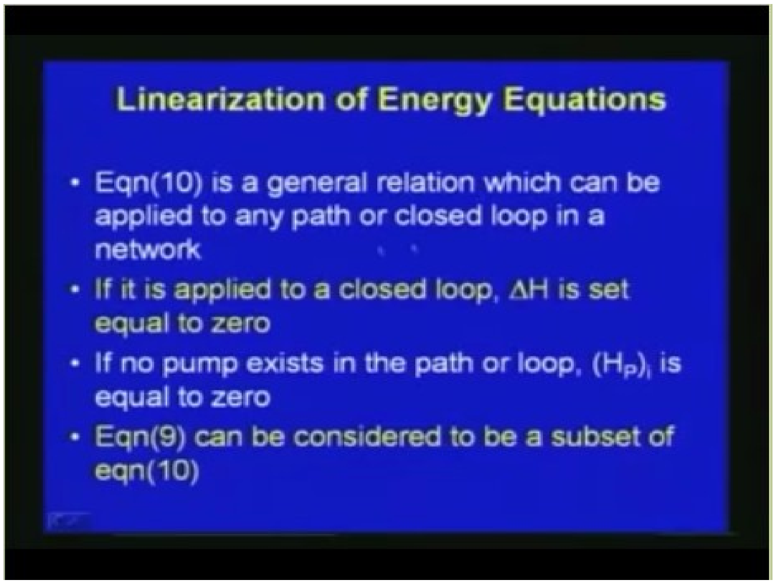

Here, this equation 10 which we have seen the general relation which can be applied to any path or closed loop in a network, if it is applied to close loop then, delta H is set equal to 0. Here, this equation delta H is set to equal to 0, it is closed loop, if it is applied to a closed loop delta H is set 0 and if no pump exist in the path or loop H_{pi} is equal to 0 and equation 9 can be considered to be a subset of equation 10. Here, equation number 9, this is actually a sub set of equation number 10 as we have seen here.

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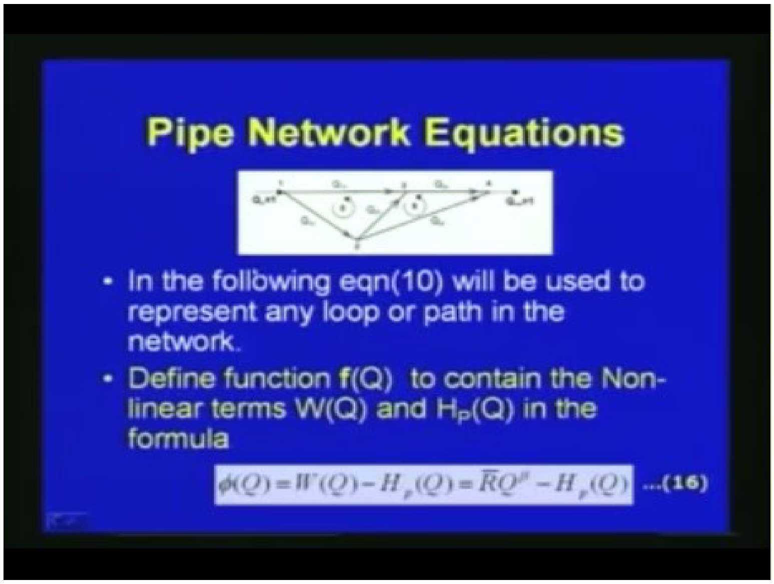

With respect to the network, our aims is since the non-linear equations are difficult solve, we discuss only the possibility of linearising it. So, in the equation number 10, we are using the loop or path in the network. If we define the function f of Q to contain the non-linear terms W of Q and H_p Q in the formula. So, if you represent this function we defined a function phi Q to contain the non-linear terms W Q and H_p Q then, we can write phi Q is equal to W Q minus H_p Q equal to R bar Q to the power beta minus H_p Q as in equation number 16.

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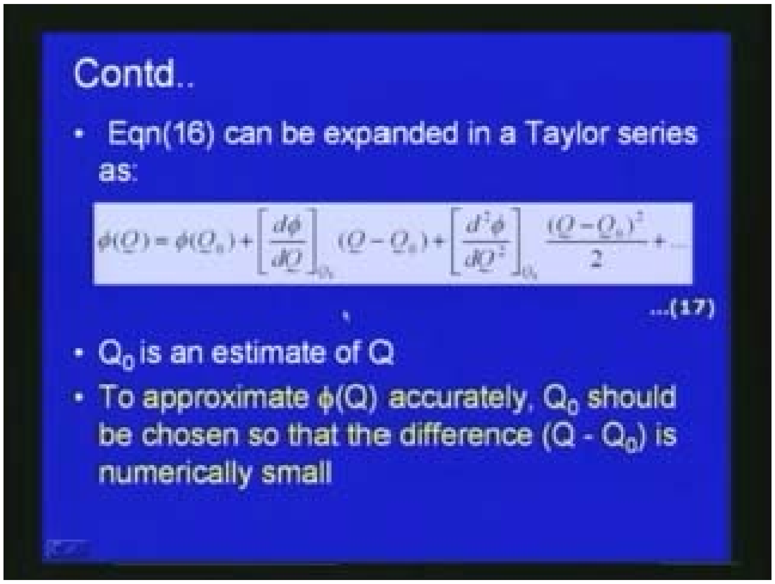

Equation 16 can be expanded in a Taylor series that we can write: phi Q equal to phi Q_0 phi as the function of Q_0 plus d phi dQ into Q minus Q_0 plus d square phi by dQ square into Q minus Q_0 whole square by 2. As in this equation number 17 this obtained from the Taylor series. Here, see that, this Q_0 is estimate to Q and to approximate to phi Q accurately Q_0 should be chosen the difference Q minus Q_0 is numerically small. So, we are trying to represent the non-linear equation we defined a function phi Q with respect to the non-linear terms W Q and H_p Q as in equation number 10.

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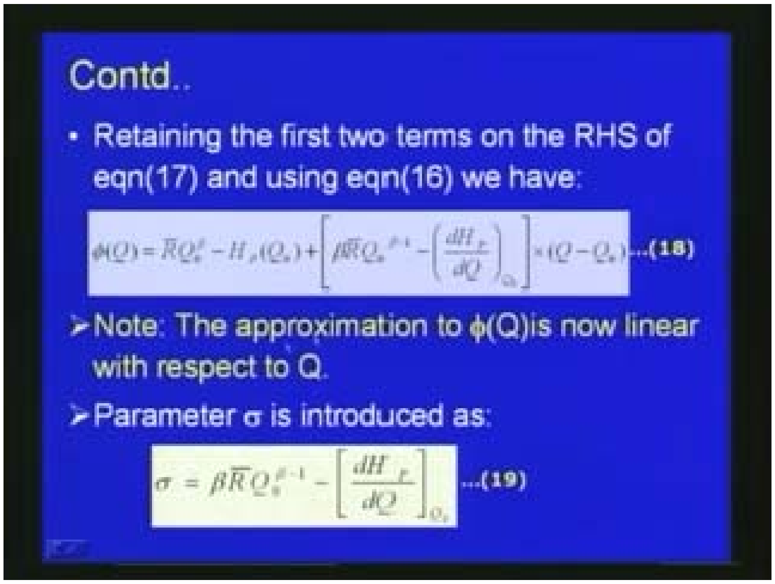

We are using the Taylor series by retaining the first two terms of the right hand side equation 17 and using equation 16. We can show phi Q equal to R bar Q_0 power beta minus H_p Q_0 plus beta R bar Q_0 to the power beta minus 1 minus d H_p by d Q whole into Q minus Q_0 as given in equation number 18. Here, this approximation of to phi Q is now linear Q.

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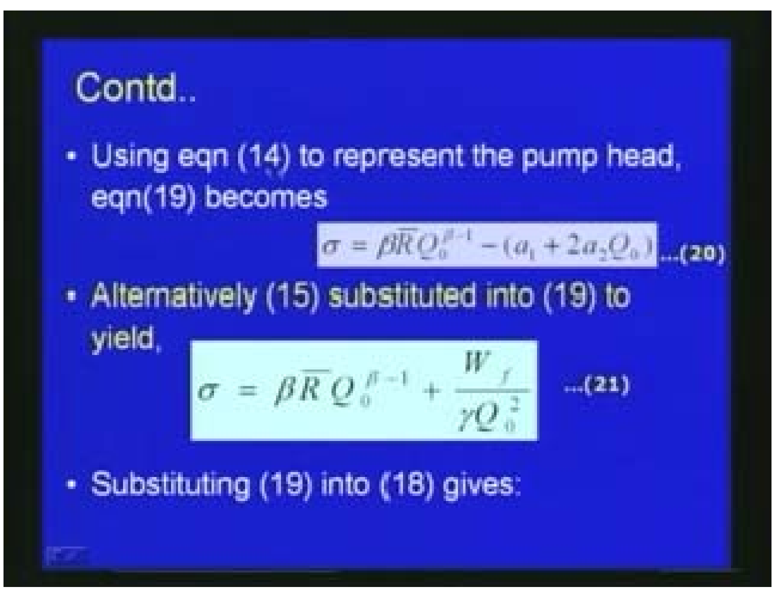

We achieved this by using Taylor series and only using these first two terms this will continue as in this equation 17. By retaining first two terms and right hand several equation seventeen here we get phi Q equal to R bar Q_0 to power beta minus H_p functional Q_0 plus beta R bar Q_0 to the power beta minus 1 minus d H_p by d Q into Q minus Q_0. Let us introduce here parameter sigma equal to beta R bar into Q_0 into the power beta minus 1 minus d H_p by dQ as in equation number 19. If you introduce this using equation 14 here, the pump equation, we can write: sigma is equal to R bar Q_0 to the power beta minus 1 minus a_1 plus 2 a_2 Q_0 as in equation number 20. Alternatively this equation number 15 here the actual power use is given by this equation 15, we can transform this equation into this: sigma is equal to beta R bar Q_0 to the power beta minus 1 plus W_f by gamma Q_0 square as in equation number 21.

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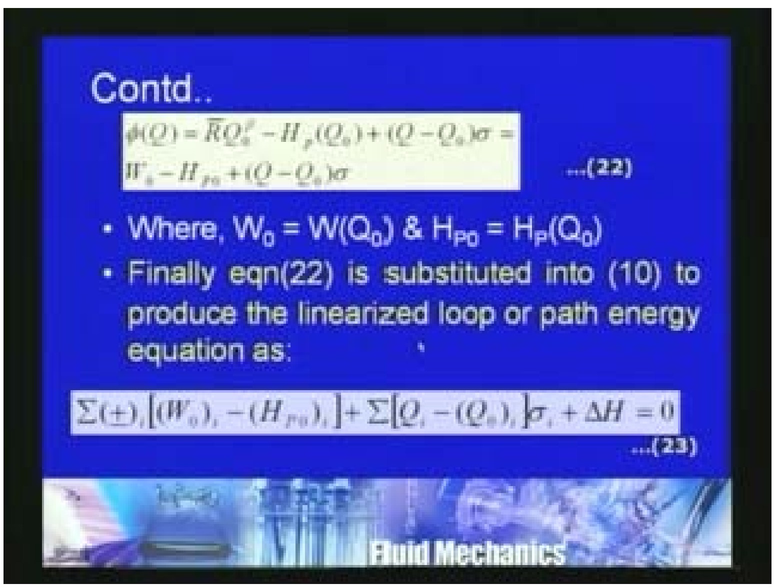

So finally, substituting 19 that means this equations for sigma into 18, we get phi Q equal to R bar Q_0 to the power beta minus H_p and Q_0 plus Q minus Q_0 sigma equal to W_0 minus H_{p0} plus Q minus Q_0 into sigma as in equation number 21 where, W_0 is the W Q_0 W as function Q_0 and H_{p0} functional Q_0 as H_p Q_0 here. Finally, the equation 22 substitute into the formal equation 10 to produce the linearised loop or path energy equation as: sigma plus or minus$_i$ W_{0i} minus $H_{p0\ i}$ plus sigma Q_i minus Q_{0i} into sigma$_i$ plus delta H equal to 0. So, here we can see that, beta generally used as to we finally get the equation 22 as a linear equation. Finally, after substituting this equation 22 into our earlier equation the energy equation 10, we get the equation 23 which can be the general form of the equation which we can utilize for the solution of the pipe flow networks. So far we have derived the equations generalize the equation for energy equation for various pipe known of the various pipes. We have seen the generalize equation continuity or continuity balance. So, these are the generalize equations which we use for the pipe network analysis and the energy equation is non-linear nature we have seen how we linearise this equation and go for the solutions. We can write the general form for any kind of pipe network and we can derive the generalize equation, the continuity balance and the energy; we can solve these equations for the unknown discharges at various node or other various locations.

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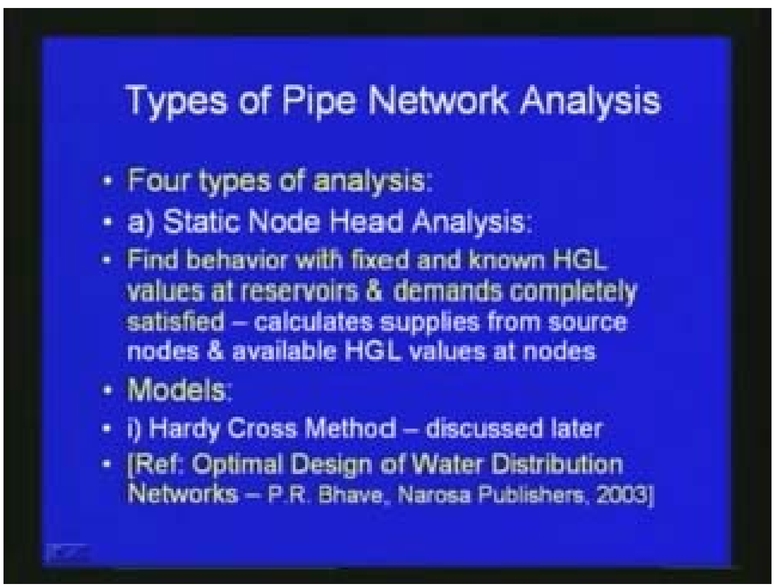

So, next question is how to solve these equations. Here, we can see different types of analyses of pipe network. We will discuss few of the important types of analysis with respect to the generalized equations. In literature, we can see four types of analysis different types of analysis are based upon whether it static or dynamic that means, the analysis which we carrying out is whether it is with time or non-varying time. So if it is non-variant time, we that kind of analysis static and if the time variation also included then, we say that it is dynamic analysis whether we analysis on the node heads we have seen that, the pipe network have number of node number of pipes are connected are junctions are there and there are also loops. So the analysis which we do with the node heads then, it is one type of analysis and whether if you doing the analysis with the each flow of junction then (17:02) this whether it is we are going for static analysis are dynamic analysis are we go for node head analysis or node flow analysis generally, we can have four types of analysis pipe network analysis.

So first one is static node analysis, second one is dynamic node head analysis, third one is static node flow analysis and fourth one is dynamic node flow analysis. Briefly we will discuss different types of analysis. Here, first one is the static node head analysis. So, here, we find the behavior with fixed and known hydraulic gradient line values at reservoir and demands completely satisfied such that, we calculate the supplies from the source nodes and available HGL values at each node.

In this static node head analysis our aims is find the behavior the already non-HGL hydraulic gradient line and the fixed nodes, we assume that, already demands are completely satisfied, we calculate the node heads with the supply and source nodes. In the static node head analysis the different types of model available actually most of the water supply pipes lines most of the analysis generally, we will be doing static node head analysis, it is much simpler computer the other three types analysis which we discuss earlier. The static node head analysis is the most commonly used analysis for pipe network. There is number of models available few of the models are described here one of the most commonly used is static node head analysis called Hardy-Cross method which will be discussed in details later. Difference types of analysis pipe networks, we can see in the text book optimal design the water distribution network by P R Bhave published by Narosa publishers in 2003.

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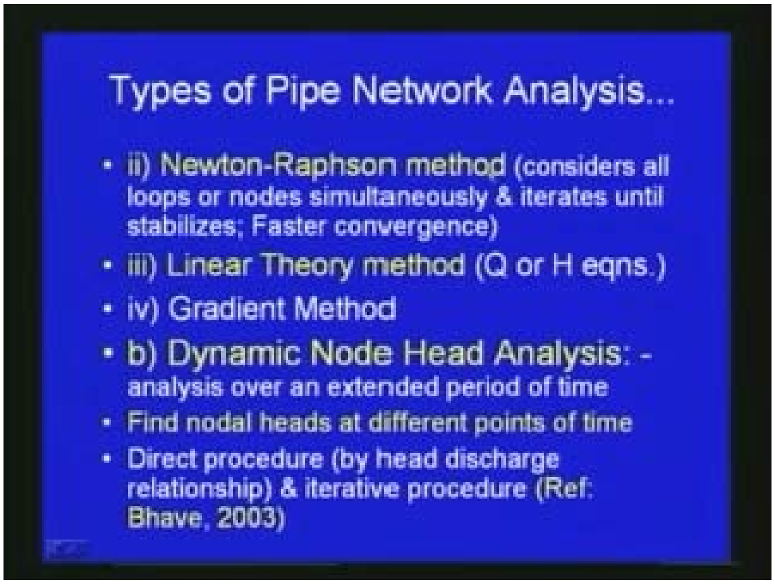

As I mentioned there are different types of static node head analysis first one is Hardy-Cross method which will be discussed later. Second type of analysis is called a Newton-Raphson method. In the Hardy-Cross method generally, consider one loop at a time; we will be trying to the successive approximation which we discussed earlier. In the Newton-Raphson method, we consider hold loops, hold nodes simultaneously and we do iteration until we get stabilised value so that between the iterations the convergences are achieved.

Hardy-Cross method is simplified form of the method of successive approach nation but Newton-Raphson method we consider whole the loops and nodes simultaneously and we do iteration such that, we get stabilised solution. The advantage of the Newton-Raphson method is that, we will be getting faster convergence. Other methodology is static node head analysis include linear theory method which we discussed early based upon the equation in terms of either discharge Q of H equations. Fourth method is gradient method. In gradient method, we will be calculating the various unknown. Like this number of methodology is available in static node head analysis. Those who are interested can refer to the book by Prof. Bhave. The second type of analysis which we will discuss briefly is dynamic node head analysis. So as a mention the dynamic node head analysis, we will be looking for the heads at different point of time. It is the analysis with time so at various time levels or various time intervals, we will be calculating the node heads at various nodes. This analysis here you can see that, it is with time. There are different methodologies so also for dynamic node head analysis. It is inverse method like direct procedure that means, by head discharge relationship and also there are methodologies like iterate procedure.

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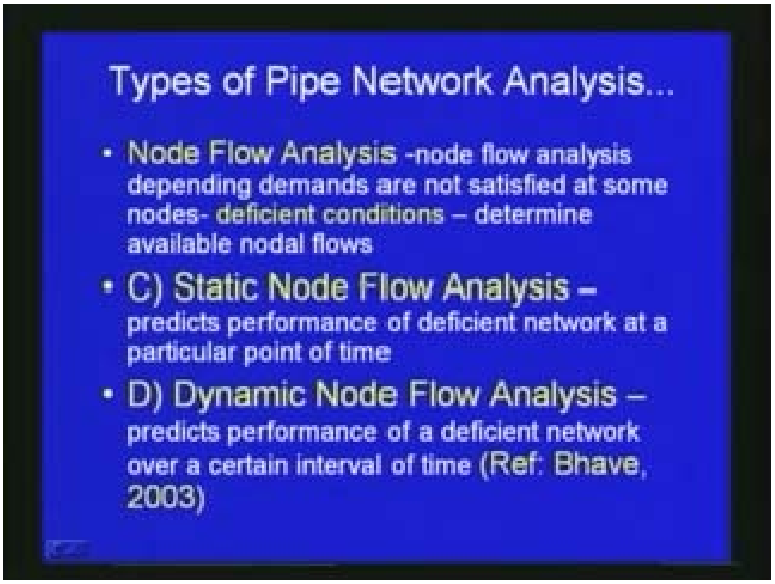

So here, again as we discussed earlier this solution is obtain through iteration. There are more interested details about this methodology you refer to the book by Bhave. We have seen dynamic node head or static node head analysis. Here, we are taking the heads at various nodes and trying to solve for that on the assumption that, all the flow demands are met at all the nodes. Based upon that assumption only we do the node head analysis but this node flow analysis all the demands may not be satisfied at some nodes. There can be diffusion conditions we have to determine the available node flows. So that will difference between node head analysis and node flow analysis. For node flow analysis this assumption of that all the demands are satisfied are not use, demands may are not satisfied at some nodes but, demands of not satisfied there can be diffusion conditions. So with this diffusion conditions only will be trying to solve the problem, we have to determined the available node flows at various location at various junctions, here also with respect node flow analysis also, we can have this static or dynamic conditions. The third methodology is called static node flow analysis. So in the static node flow analysis, we predict performance of deficient network at a particular point of time. Here, the diffusion conditions are also considered, we try to predict performance of the deficient network at the particular point of time. In a very similar for dynamic node flow analysis, we are trying to predict the performance of a deficient network over certain interval of time. At various intervals of time, we can find the flow through various nodes the efficient conditions. So these are methodologies like static node flow analysis, dynamic node flow analysis are also discussed in details in the book by Prof. Bhave. Those who are interested can refer to this test book. We will now discuss the earlier methods most commonly used type of pipe network analysis is static node head analysis.

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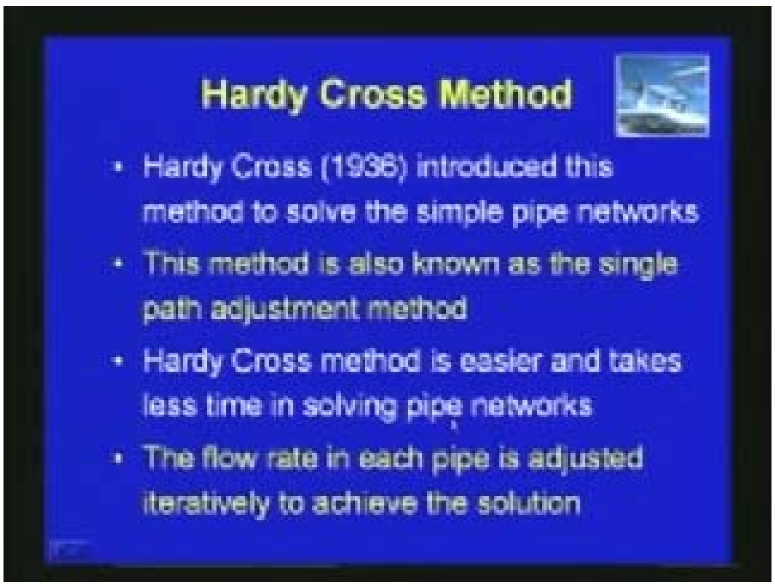

We will now discuss Hardy-Cross method in detail which is very commonly used for the pipe network analysis. So in the Hardy-Cross method actually this method was developed in 1930s Hardy-Cross 1936 introduce this methods to solve this symbols pipe network. This method as we seen the generalize equations is also based upon the generalized network equations which we have seen earlier. This method is also known as single path adjust method. In the network number of loops may be there number of paths may be there but, in the Hardy-Cross method, we consider the single path at a time or single loop at a time and we adjust the flow within that loop.

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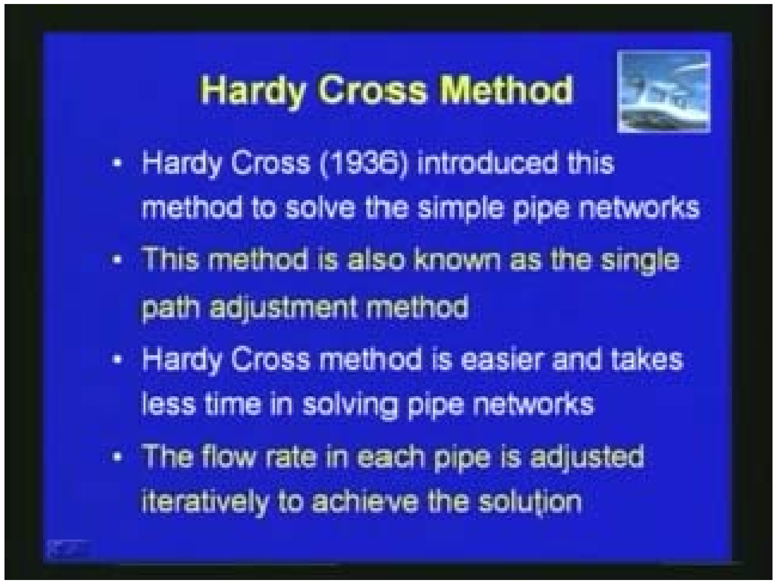

Hardy-Cross method is also known as single path adjustment method and is easier, takes less time solving pipe networks. Since at a time, we are dealing only single path so this is much easier and depending upon the problem, we can easily solve the network problem. Here, the Hardy-Cross method the flow rate in each pipe is adjusted iteratively to achieve the solutions. So here this Hardy-Cross method also we assume the flows through the network which we consider. The flow rate in each pipe is adjusted iteratively and we achieve the solution.

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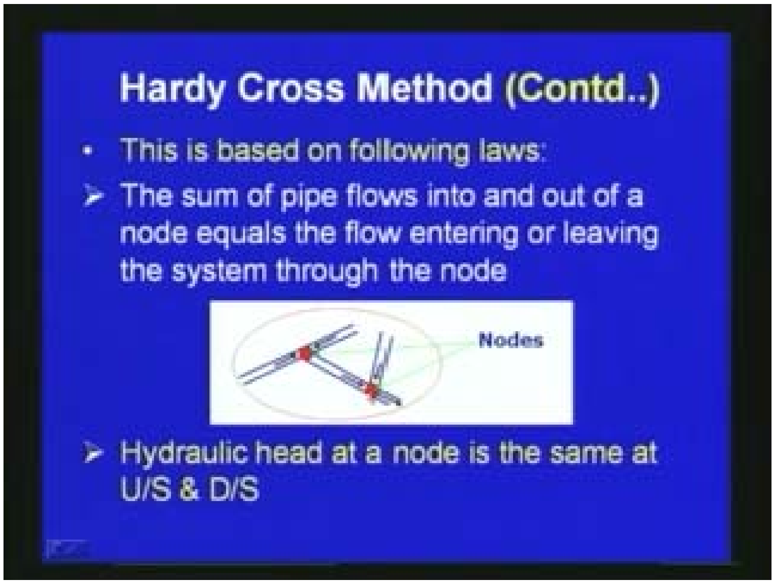

So, here this Hardy-Cross method is based upon the following laws: here if we consider a network like this and here we can see a path of the network is shown with network various pipes and here nodes are there. The Hardy-Cross method is based upon the assumption that sum of pipe flows into and out of node equals the flow entering or leaving the system through the node. So here, basically it means, sum of pipe flows into and out of a node equals the flow entering or leaving the systems. If the flow is coming from the direction then it should balance with respect to the flow going out of the node.

So based upon this law only this Hardy-Cross method is working and also the hydraulic head at a node is the same at upstream and downstream of the node. So, if we consider node then whether up stream that means here or here the hydraulic head is same. Based upon this assumption, these are the basic laws only, actually these are fundamental laws. The Hardy-Cross method is based upon these two basic laws. As we discuss it is simplified version of method of successive approximation. We have seen that, the basic equations the generalize equations, we can solve through successive approximation. Actually Hardy-Cross method is also type of the method of successive approximation but it is applied to a set up linearised equations so that, there is no inversion of matrix involved.

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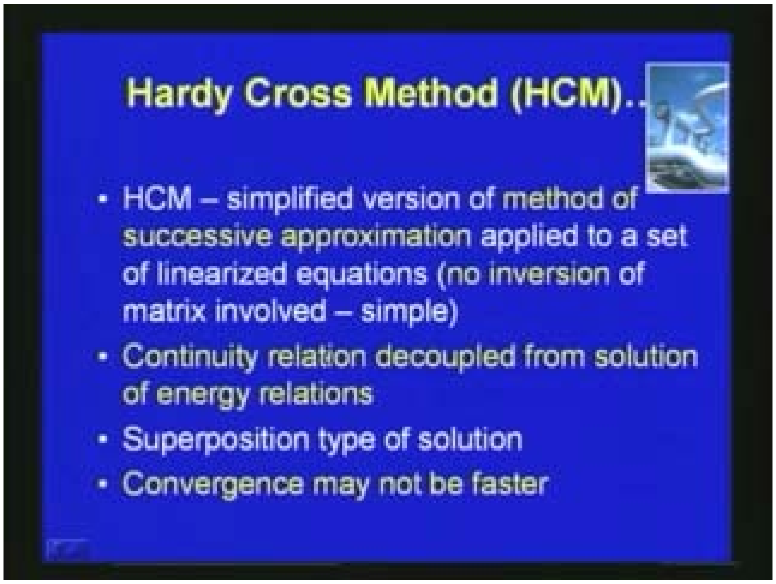

Here, we use the linearised equations which we discussed earlier. Continuity relation is decoupled from the solution of energy relations. Also, we can superposition the type of solution which we do with respect to the loop which we do and only the problem here is the convergence may not be faster. Compared to the Newton-Raphson method as we discussed we consider all loop or all the nodes at a time. But in the Hardy-Cross method, we use only one loop at a time so the convergence may not be faster compare to the Newton-Raphson method.

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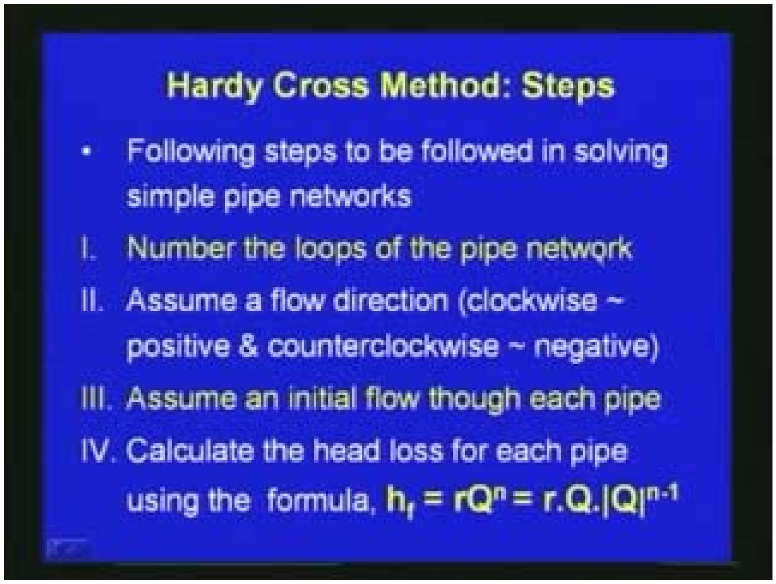

Now, we will discuss here the various steps involved in the Hardy-Cross method. Here first, we assume for the given network, we will put the node numbers and we will indentify the fixed node also we can arrange pipe the which are the loops. So the Hardy-Cross method steps include: the first number of loops of the pipe network, we can identify pipe. Step number II, we can assume a flow direction generally we consider clockwise flow direction positive and counterclockwise flow directions is negative. Step number III; assume an initial flow through each pipe. So, for the given network, we assume a flow direction, since we do not know which direction the flow is going, we assume a flow direction. Also, we assume some initial flow through the each pipe. After the assumption for the given network, we can consider the each loop and for the given loop. We can calculate the head loss for each pipe using the formula h_f equal to gamma or h_f equal to r Q to the power n where, r is the functional the fluid flow through a pipe and pipe material, h_f equal to r into Q to the power n that is equal to this generally, we write as r into Q into modules of Q to the power n minus 1. So, this is generally represented to take care of the direction which is discussed here, we write this as: h_f equal to r into Q into Q to the power n minus 1.

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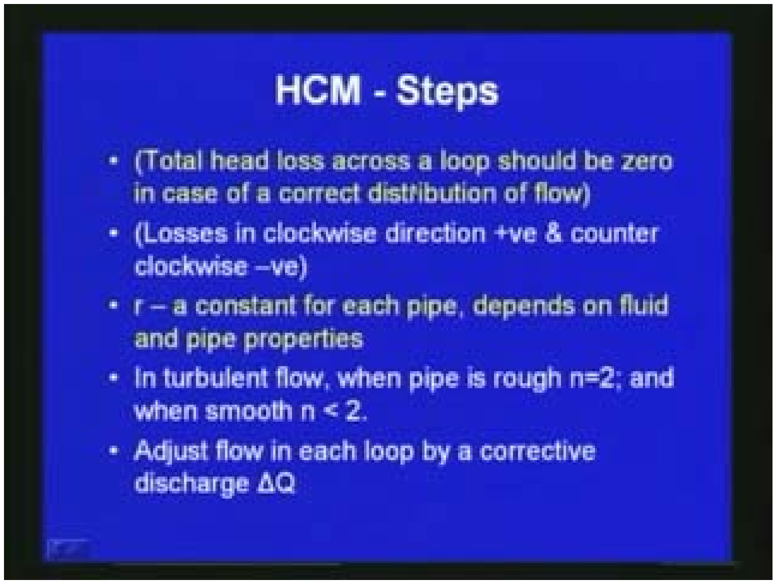

In these steps, we consider here the total head loss across a loop should be 0 in case of a correct distribution of flow. For the given network, we assumed the flow direction also the flow through each pipe. If the assumption is right then, for the given loop the head loss should be 0, as per the basic assumption used in this analysis. So if the distribution is not right then, there will be some error. We have to check and redistribute that. Also losses in clockwise direction is positive and counter clockwise is negative and here this r which we consider h_f is equal to r into Q to the power n, this r is a constant for each pipe which depends upon the fluid and the pipe properties. We can find out for the given network. We assume that, this r value is none for the given pipe network and also in this equation this n is a constant. So, here, this n is when we assume that, flow is fully turbulent and pipe is rough this n equal to 2, smooth pipe generally less than 2 may be 1.8 something like that depending upon the type of pipe and the other fluid properties.

For the turbulent rough pipe n equal to 2 here, as we discussed, we assume the direction and also assumed the initial flow. Since the assumption may not be right due to that, there will be a correction that we can find out. We can adjust the flow in each loop by corrective discharge delta Q.

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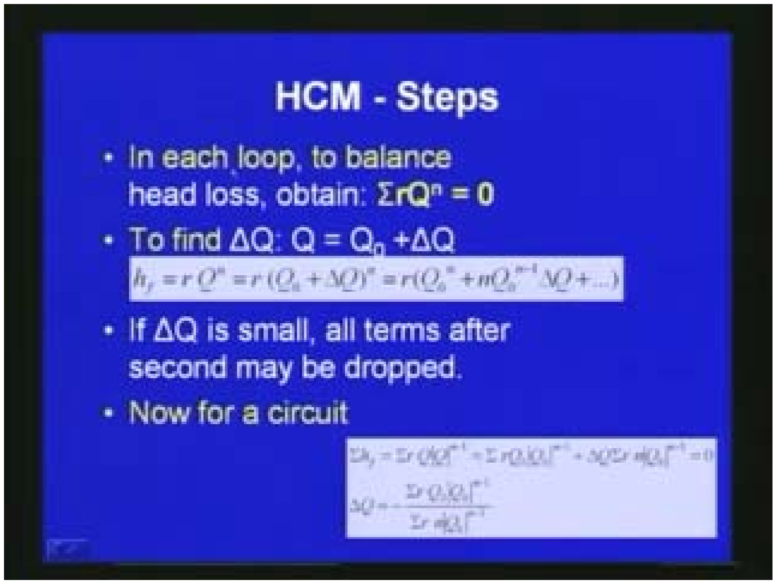

Then, in each loop to balance the head loss, as we discussed for each pipe we can write: sigma r into Q to the power n equal to 0. To find delta Q we can write: Q equal to the initial assumed value plus delta Q equal to Q_0 plus delta Q that h_f equal to r into Q to the power n that is equal to r into Q_0 plus delta Q to the power n, that is equal to r into Q_0 to power n plus n into Q_0 to power n minus one delta Q plus etc., if you consider only first two terms then, delta Q is small and all terms after second can be dropped. For the given loop or circuit we can write: sigma h_f equal to sigma r into Q into modulus Q to the power n minus 1 that is equal to sigma r Q_0 modulus Q_0 to the power n minus 1 plus delta Q sigma r n modulus Q_0 power n minus 1. So this should be the equal to 0 from which we can get this discrepancy in flow that means delta Q we can obtain from this equation delta Q equal to minus sigma r Q_0 into modulus Q_0 to the power n minus 1 divide by sigma r n modulus Q_0 to the power n minus 1. Using this equation, we can get delta Q, we can adjust with this delta Q which is discussed in the next step.

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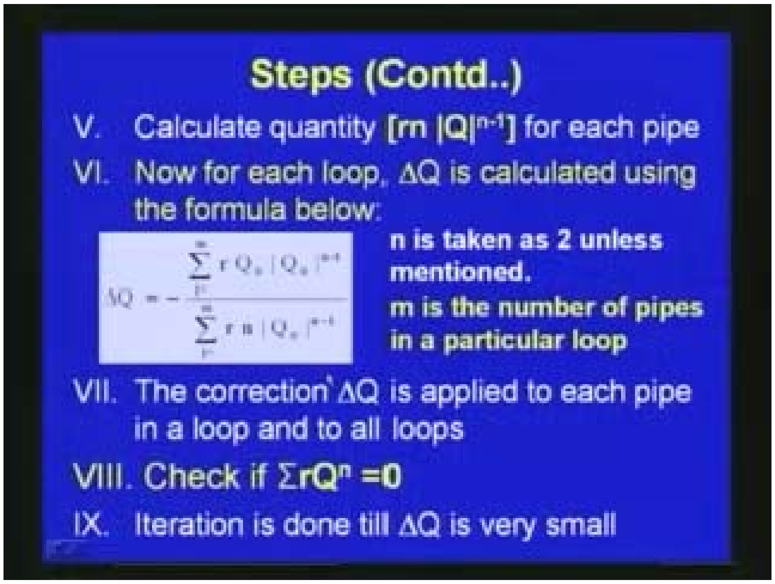

We calculated the head loss for each pipe, we can also calculate the quantity r n into Q modulus to the power n minus 1 which we discussed in the earlier equation. For each loop this delta Q is calculated how you got this delta Q is derived here. This delta Q is calculated that is equal to minus sigma j equal to 1 to m into r Q_0 into Q_0 to the power n minus 1 divide by a equal to 1 to m r n modulus Q_0 to the power n minus 1. Here, we can take n is 2 unless it is not mentioned and m is the number of pipes in a particular loop. So, j equal to 1 to m and here also j equal to 1 to n. Once, we get the correction delta Q then, we can apply in next step to each pipe in a loop. This procedure is continued for all the loops. Finally, we can check whether sigma r into Q to the power n equal to 0 for the given loop. We will continue this since we started with assumed value of delta Q the flow through each pipe. So this relationship generally sigma r into Q to the power n will not be 0. Until we put convergent limit a desired limit like 0.0001 something like that or you if you want exist 0 a number of iteration will be required. Otherwise, we can put a limit depending upon that we can find out the flow distribution through each pipe. This iteration is done till delta Q is very small. We can find out the flow distribution through each pipe. All these steps we can utilize in various cases same depending upon the network. These are the fundamental steps used in the Hardy-Cross method.

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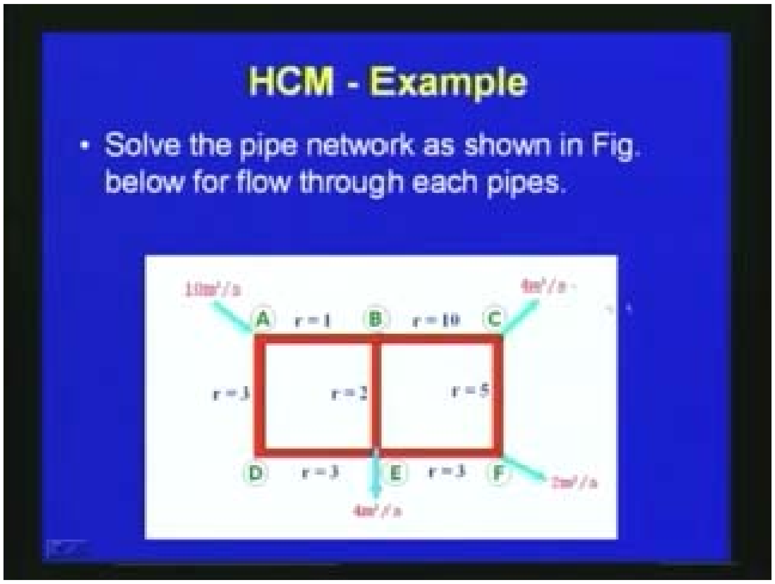

To demonstrate this Hardy-Cross method, we will solve one symbol example in pipe network including in two loops. Here, we solve the pattern network shown in this slide. In this pipe network there are two loops and flow is coming in this direction 10 cubic per seconds and for various pipes the r values are given and also there is an out flow at this node C 4 cubic meter per seconds and an out flow at F 2 cubic meter per seconds and out flow at E 4 cubic meter per second. Here our problem is you want to find out the flow through each pipe at these inflows here. We can identify the pipe junction or pipe nodes as A B C D E and F and number of pipes here 1 2 3 4 5 6 7. We have seven pipes and six nodes for this given problem 1 inflow to the system 3 out flows from the systems. We want to find out the flow distribution in each pipe for this given network. Here, we can see that, there are two loops in this network. We assume the flow direction like this. Here, the flow direction A to B to C and A to D B to E and B to E, E to F and F to C, we assume the flow initially like this. We will assume some initial flow also. As we discussed earlier for Hardy-Cross method, we will use step by step solution here. Already the r values are given, inflow is 10 here, we assume five flows through the pipe AD and five flows are through pipe AB. B to E 2 cubic x and B to E 5 cubic x and E to F 3 cubic x cubic meter per seconds and B to C 3 cubic meter per seconds and F to C 1 cubic meter per seconds.

This is our initial assumed value flow through each pipe. Now, we will apply Hardy-Cross to see that, we get the correct value. These are only assumed values. Now, we will proceed the iterative procedure step by step we will do the calculation here by using the Hardy-Cross method.

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- Iteration I: Loop 1

Pipe	r	Q_j	$rQ_j\|Q_j\|$	$2r\|Q_j\|$	ΔQ	Corrected Q_j
AB	1	5.00	25.00	10.00		6.50
BE	2	2.00	8.00	8.00		3.50
ED	3	-5.00	-75.00	30.00	1.50	-3.50
DA	3	-5.00	-75.00	30.00		-3.50
	Total		-117.00	78.00		

- Iteration I: Loop 2

Pipe	r	Q_j	$rQ_j\|Q_j\|$	$2r\|Q_j\|$	ΔQ	Corrected Q_j
BC	10	3.00	90.00	60.00		2.67
CF	5	-1.00	-5.00	10.00		-1.33
FE	3	-3.00	-27.00	18.00	-0.33	-3.33
EB	2	-3.50	-24.50	14.00		-3.83
	Total		33.50	102.00		

Shown in this table first we consider the loops. One this is loop one and this is second loop. For the loop one for pipe AB BE ED DA r values are 1 2 3, Q_j we assumed already the assumed flow is five Q max 2 Q max and minus 5, minus 5. So, here this direction if it is clockwise it will be positive if it is anti-clockwise it will be negative. That is why the plus and minus here. We find this rQ_j modulus Q_j. Here, we assume that, in this equation n is equal to 2 since no other special values we assumed so n is equal to 2 and so we write 2 r modulus Q_j. For each various values, we can now do the calculation, we can submit for this given loop. The summation will give minus 117. The summation of this value will give 78. We can find out for delta Q using this equation for all the pipes here. So we get this delta Q as 1.5. We will correct the assumed value of the flow by using this 1.5. It can add pipe plus 1.5, 6.5. In the iteration number one, we get 6.5 for pipe AB and pipe BE it is 2 plus 1.5 plus 3.5 and ED it is minus 5 plus 1.5 it is minus 3.5 and pipe DA it will be minus 5 plus 1.5 minus 3.5. Similar way we can do for loop number two iteration number one loop number two, we can see this includes BC CF EF and BE. B already the calculated same values you will use from here. The assumed values here, r values given for BC as ten CF 5, F is 3 and BE is 2. Q_j the assumed value is 3 and CF it is minus 1 FE it is minus 3, BE we already calculated. So we can write as minus 3.5 the directions in this particular loop. Now we will find out r Q_j modulus Q_j 2 r Q_j modulus 2 r modulus Q_j we will summit up and get 33.5 here under this column 102. We will find out delta Q, (Refer Slide Time: 41:02) here delta Q is minus 0.3 we will correct it. So the correction will be 3 minus 0.3 for pipe BC is 2.67 and for pipe CF it is minus 1 minus 0.33 it will be minus 1.3 and pipe FE it is minus 3 minus 0.33 so minus 3.3. Similarly, BE to be minus 3.83. Now, the first iteration is done for loop one and loop two. We will go for the second iteration for loop one. Similarly, we have already the corrected value here and also this BE considered this we will be using new value for the loop number two. (Refer Slide Time: 41:45) All these values we will be substituting here in this iteration number two. We will find out the rQ_j modulus Q_j 2 r Q_j we will summit up and find out delta Q as we discussed earlier. So, here in this iteration will get a small value. Now, the solution is improved it is 0.03 again will correct the flow through each pipe for loop one to 6.53, 3.86, 6 minus 3.47 and m minus 3.47.

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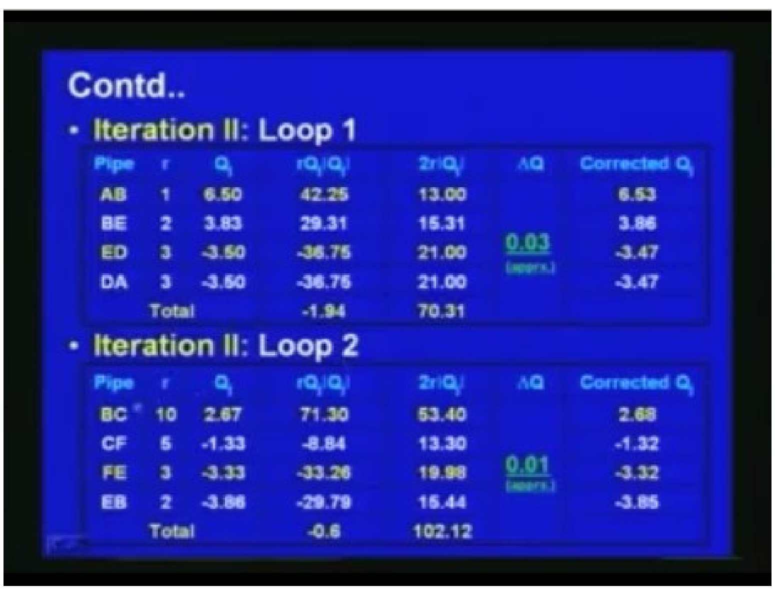
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- Iteration II: Loop 1

Pipe	r	Q_j	$rQ_j\|Q_j\|$	$2r\|Q_j\|$	ΔQ	Corrected Q_j
AB	1	6.50	42.25	13.00		6.53
BE	2	3.83	29.31	15.31	0.03 (approx.)	3.86
ED	3	-3.50	-36.75	21.00		-3.47
DA	3	-3.50	-36.75	21.00		-3.47
	Total		-1.94	70.31		

- Iteration II: Loop 2

Pipe	r	Q_j	$rQ_j\|Q_j\|$	$2r\|Q_j\|$	ΔQ	Corrected Q_j
BC	10	2.67	71.30	53.40		2.68
CF	5	-1.33	-8.84	13.30		-1.32
FE	3	-3.33	-33.26	19.98	0.01 (approx.)	-3.32
EB	2	-3.86	-29.79	15.44		-3.85
	Total		-0.6	102.12		

Now, iteration number two for loop number two. Similarly, what we did for BC CF FE and BE, Q_j is already obtained from the earlier and here again this BE is 3.86 that is now minus 3.86 in this loop and we can find out r Q_j modulus Q_j 2 r Q_j and summit up here it will be minus 0.6 it will be 102.12 here. We can find out delta Q which will be 0.01 approximately. The corrected value can be 2.68 minus 1.38 minus 3.32 and minus 3.85.

With this iteration number two is over.

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Now we will go to iteration number three. For iteration number three, again from the iteration number two the corrected Q_j is 6.53 through pipe AB pipe BE 3.85 obtained from here, pipe ED minus 3.47, pipe DA minus 3.47, very similar way we can find out r Q_j modulus Q_j and 2 r Q_j and summit up. So r Qj will be 0.05 2 r Q_j total will be summation will be 70.13. This will almost be equal to 0. You can see that it will be 0.00; you can take it almost as converge. So the corrected Q_j will be 6.53, 3.85 same value minus 3.47 minus 3.47 iteration three for loop number two, 2.68 minus 1.32 and minus 1.32, minus 3.385 from here, we will be writing here Q_j modulus Q_j 2 r Q_j and summit up to can get the delta Q here also it is almost 0, it will be 0.00. If you take only upto desired accuracy then, we can now get the corrected value of 2.68 minus 1.32 minus 3.32 and minus 3.85. In this second iteration, we have almost reached the solution further accuracy is required, we can continue the solution or we can take this as the solution. Here, we finally we can write the flow through the pipes like this, we can see that, all the mass balance and continuity is achieved.

Hardy-Cross method is one of the easiest methods for pipe network analysis. Actually, we are using the generalize equation method of successive approximation. We are considering one loop at a time and we are iterating to get a solution. So, this one of the most commonly used methods also we can use methods like Newton-Raphson method or numerical method like finite and final defend methods depending upon the solution which we have looking for or if you are having some packages, we can use directly to find out the solution. The hardy equations methods, we consider the static node head analysis if you are looking for dynamic node head analysis or static node flow analysis or dynamic node flow analysis, we can prove the analysis. We can solve the pipe network flow problems. With this we finished the pipe networks flows and the various aspects in the multiple pipe flow systems. As the last topic in this video course, we will now consider the unsteady flow in pipes or unsteady flow in closed conduits. We have seen earlier when we discussed the different types of flow, the flow can be steady state or unsteady state depending upon the whether there are various parameters changing such as time change can be unsteady or steady. We can the flow as uniform or non-uniform flow for parameters as same throughout from one distance to another then it will be uniform and if it is changing in distance it will be non-uniform flow. Regarding the pipes or close conduits, we will briefly discuss the unsteady flow, its communication and various aspects in unsteady flow.

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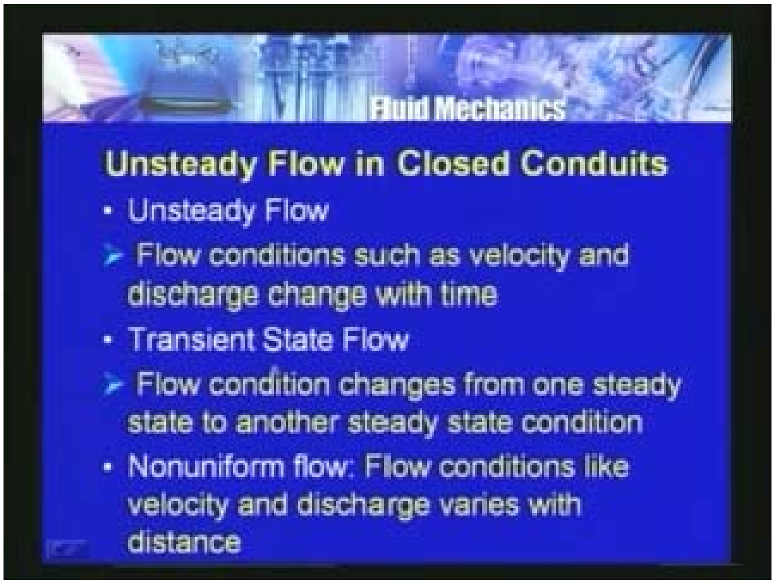

The unsteady flow as we may have discussed earlier, flow conditions such as; velocity and discharges change time. Sometimes this flow is set to be transient state flow.

So generally, transient state flow also sometimes used as unsteady flow but, in transient state flow, we take it in such way flow that, the flow conditions changes from one steady state and another steady state condition. Actually, this time dependent flow, we can consider in such a way that; at particular time this steady state, next another steady state like that. When we consider the flow in this way it is the transient state flow.

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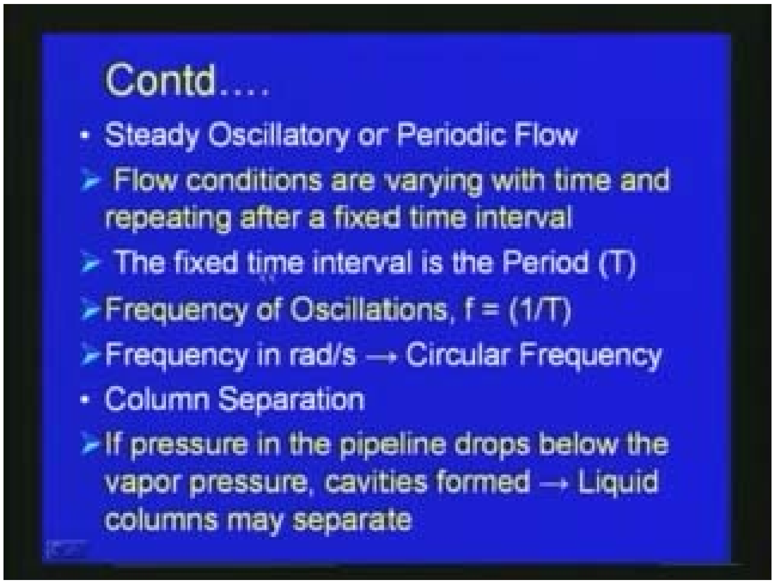

Similar way, we can have unique flow and non-uniform flow and also depending upon the flow, we can say the flow to be steady oscillatory or periodic flow when flow conditions vary with time. Repeating after a fixed time interval then, we say it is steady oscillatory or periodic flow. Here, what happens is the flow conditions vary with time but, it is repeating after a fixed time interval, after few seconds or few minutes the same flow conditions are repeating. Here, we have fixed time interval that is called the periods for the steady oscillatory or periodic flow, we have fixed time interval and is represented generally as T. We can find out the frequency oscillations. For example, when we consider the flow coming from a reservoir like this, (Refer Slide Time: 48:57) if you are having a pipe connected from the reservoir here is a flow and here is a valve. If you operate this valve when we close the valve then, we can see that oscillation taking place. We can see that it is also periodic or steady oscillatory flow. If the fixed time interval is the period, we can find out the frequency of oscillation the flow is going from one place to another and find out the frequency of oscillations. The frequency of oscillation is f equal to 1 by T where, T is the period of the flow. Frequency can be represented in radians per seconds, it is called circular frequency. Frequency can be either in terms of cycle or it can be in term of radiant per seconds. When represented in terms of radiant per seconds we call it as circular frequency. When we deal with unsteady flow conditions as we discussed here one of the common phenomena is column separation. If pressure in the pipe line drops below the vapor pressure then, we can see that, cavities are formed and liquid columns may separate. Depending upon the conditions in the pipe, pipe length or the pressure in the pipe line there can be conditions such that, the pressure in the pipe line from the drop below the vapor pressure then, we say that cavities are formed and liquid column is separate.

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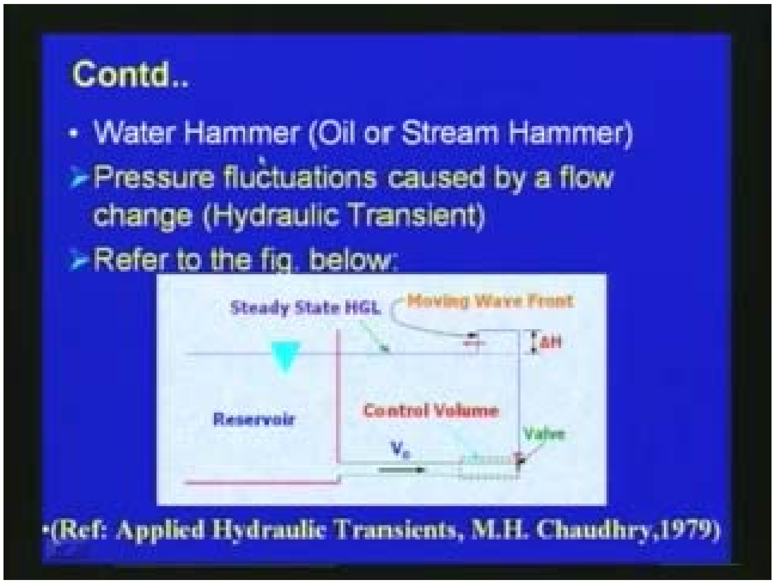

With respect to this we can define water hammer. If you are using water then call it as water hammer, if oil or stream we call it is stream hammer or oil hammer. This water hammer is the pressure fluctuations caused by a flow change or sometimes it is called hydraulic transient. The water hammer is the pressure fluctuations caused by a flow change. If there is a flow change or certain closure of valve then, there will be a flow change and pressure fluctuations with in the pipe. That kind of pressure fluctuations are termed as water hammer. This water hammer is also called as hydraulic transient. Here, we can see that, there is reservoir and there is an pipe, if we close this valve then, we can see that a wave front moving this direction will reach the reservoir and it goes back again to this direction. This way there will be pressure fluctuations or flow change which is called water hammer. In the next lecturer, we will discuss more details about water hammer and the unsteady flow equations. We can also see the text book by M.H. Chaudhry Applied Hydraulic Transients for more details. We will be discussing more about the water hammer aspect, the hydraulic transient and unsteady flow equations in the next lecturer.