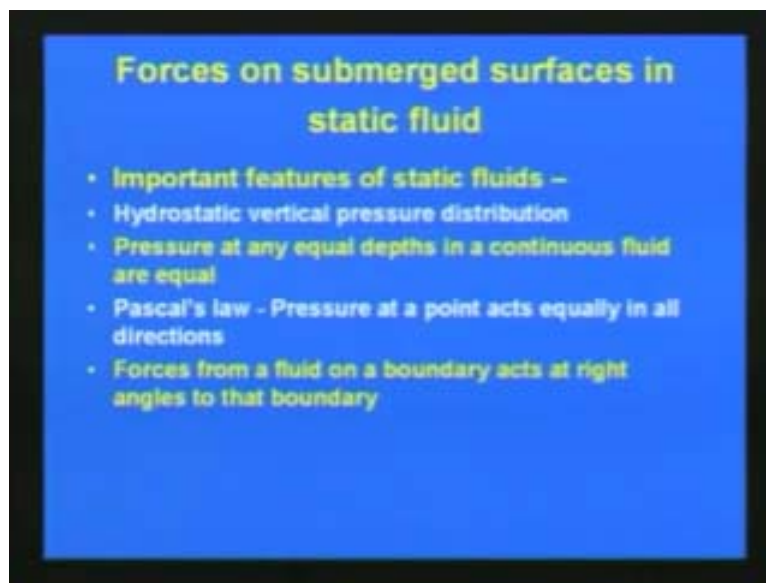


Fluid Mechanics
Prof. T.I. Eldon
Department of Civil Engineering
Indian Institute of Technology, Bombay

Lecture – 4
Fluid Statics

Welcome back to the video course on fluid mechanics. The last lecture we have discussing about the fluid statics. We have seen the various metrologies for pressure measurement, and then we are seen the manometers then automatic measurements equipments like burden gage.

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Then, we are discussing about the forces on submerged surfaces in static fluid. As we discussed in the last lecture, the important features of static fluids are: the hydrostatic vertical pressure distribution is always asset generally; second one is pressure at any equal depths in continuous fluid are equal; then as we already seen in the Pascal's law that means pressure at a point acts equally not direction that is very much valid in all the problem which we are discussing, and then the forces from a fluid on a boundary acts at right angles to that boundary as we have seen earlier. Based up on this, we have already seen; the forces on a submerged surface in static fluid.

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Forces on submerged surfaces in static fluid ..

- Fluid pressure on a surface $F = P \delta A$ (at right angle)
- $F_1 = P_1 \delta A_1$
- $F_2 = P_2 \delta A_2$
- $F_n = P_n \delta A_n$



Force At Right Angle To The Surface

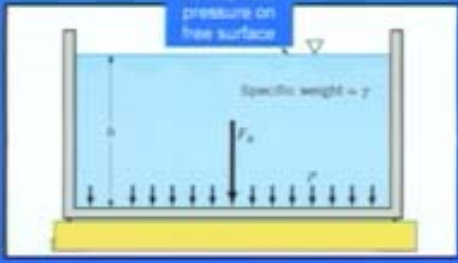
- Resultant Force
- $R = P_1 \delta A_1 + P_2 \delta A_2 \dots + P_n \delta A_n$
- Resultant Acts Through Centre of Pressure at Right Angle to Plane

Then we have seen the resultant force is equal to P_1 into δA_1 and across P_2 into δA_2 as shown in this figure. This we have already discussed in the last lecture.

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Forces on submerged surfaces in static fluid ..

- Forces on Horizontal submerged plane – pressure p equal at all points
- Resultant force $R = \text{Press.} \times \text{area of plane} = pA$



Atmospheric pressure on free surface

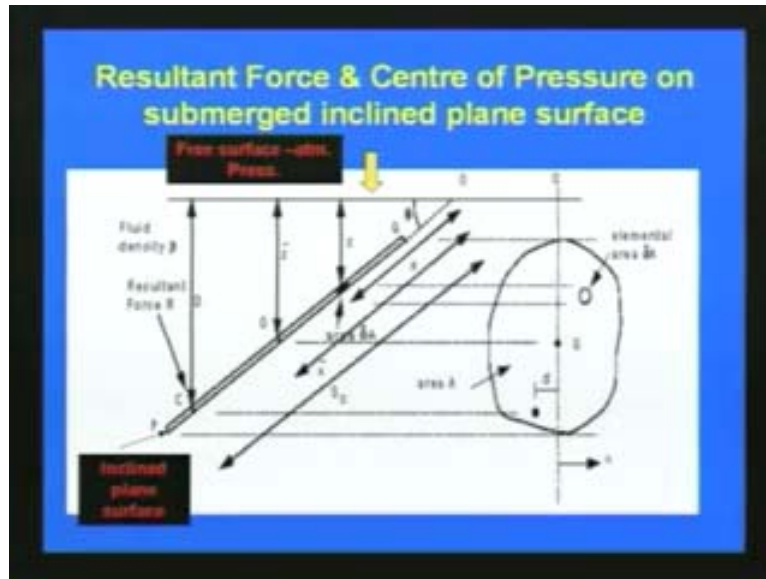
Specific weight = γ

Depth = h

Resultant force F_R

Also we have seen the forces on submerged surfaces in static fluid and the resultant forces pressure into area of plane that is equal to P into A as shown in this slide.

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In today's lecture, we will discuss the resultant force and center of pressure on submerged inclined surface, inclined plane surface.

As I mentioned in the last lecture, so we have water tank and then we are putting a small inclined plane like this, we want to find out the resultant force on this lane and then what will be the center of pressure?

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On this inclined plane so that is what we will be discussing today first.

Here in this slide we can see that if they are there is the fluid in a water tank like this, and then its free surface atmospheric pressure distribution, here you can see and then there is a inclined plane like this, and then the fluid density is ρ , and we are considering now in element of area δA like this, and then the total area the inclined plane is A and we want to find out the resultant force and the center of pressure.

Here the element which we are considering to that the depth is z , as you can see here in this slide and then due to the total areas concerned to the center of pressure from the topic is \bar{z} , and resultant force is R . So in the next slide, see how we can determine this resultant force and this plane is this inclined plane surface is , inclined at angle θ as shown in this slide.

(Refer Slide Time: 4:23)

Resultant Force & Centre of Pressure ...

- Pressure on element δA
 $p = \rho g z$
- Force on element $= p \delta A$
 $= \rho g z \delta A$
- Resultant $= R = \rho g \sum z \delta A$
- $\sum z \delta A = A \bar{z} = A \bar{x} \sin \theta$ = First moment of area
- Resultant force on plane $R = \rho g A \bar{z} = \rho g A \bar{x} \sin \theta$
- Centre of pressure \rightarrow Point through Resultant force

Now, the resultant force and center of pressure, so the pressure on element δA as we have seen in the previous slide is, P is equal to ρ into g into z where, ρ is the density of fluid, g is the acceleration due to gravity, and z is the depth at which the element δA is concerned.

So force on element A is equal to p into δA that is equal to ρ into g into z into δA and then finally, we can take the sum of all the four, different elements which we are considering the plane area and then the resultant will be R is equal to ρ into g sigma z into δA .

That is, the resultant force is equal to $\rho g \bar{z} A$ so this \bar{z} into ΔA so this \bar{z} into ΔA you can represent as $A \bar{x}$ or so, in the inclined plane that is equal to $A \bar{x} \sin \theta$ so that is equal to first moment of area.

The resultant force on plane R is equal to $\rho g A \bar{z}$ so if you substitute this \bar{z} into ΔA in this expression so that you will get the resultant force plane R is equal to $\rho g A \bar{x} \sin \theta$ or this \bar{x} can be $\bar{x} \sin \theta$. So that is equal to $\rho g A \bar{x} \sin \theta$ as shown in this, in the last slide. Here you can see it is already explained what is the \bar{z} and here this is \bar{x} this is θ and $\bar{x} \sin \theta$ is with respect to the distance measured here.

Now, we work the resultant force - Now the center of pressure is the point through the resultant point through which the resultant force acts. So here you can see the point where the resultant forces act that is called the center of pressure. How to find this center of pressure? With respect to problem which we are discussing now.

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Resultant Force & Centre of Pressure ...

- Moment of R about O = Sum of moments of forces on all elements of δA about O.
- Force on $\delta A = \rho g z \delta A = \rho g s \sin \theta \delta A$
- Moment of elemental forces about O = $\rho g \sin \theta \delta A s^2$
- Sum of moments of forces on all elements of δA about O = $\rho g \sin \theta \sum s^2 \delta A$
- Moment of R about O = $\rho g A \bar{s} \sin \theta$
- Equating $\rho g A \bar{s} \sin \theta = \rho g \sin \theta \sum s^2 \delta A$
- Centre of pressure from O $\bar{s}_c = \frac{\sum s^2 \delta A}{A \bar{s}}$

We will take, we can take the moment of R the resultant force about the point O. So that will be equal to sum of moments, or forces on element, all elements of ΔA above A so that is equal to, so that we are finding on. the center of pressure by taking moment of the resultant force about O, that is the indicating that to the sum of moments of process on all elements of the ΔA about A. So force on ΔA is equal to as we are already seen

$\rho g z$ into δA that is equal to ρg into s into $\sin \theta \delta A$ with respect to the previous slide.

Moment of elemental forces above O is equal to ρg the $\sin \theta \delta A$ into s square. So and then sum of moments of forces on all elements of δA about O is equal to ρg into $\sin \theta$ into $\sum s^2 \delta A$ and then finally the moment of R the resultant force about O is equal to ρg into $A \bar{x} \sin \theta$ into S_c .

Now we required this moment of R about O two the sum of moments of forces all elements of δA about O. So that we are equator go into $A \bar{x} \sin \theta$ into ac this is equal to $\rho g \sin \theta$ into $\sum s^2 \delta A$. So finally, we get the center of pressure from O is equal to X_c is equal to $\sum s^2 \delta A$ by $A \bar{x}$. Here in the previous slide with respect to this figure we have already seen. So we want to find this center of pressure with respect to certain force and then with respect to elemental A which we are considering.

Finally, we got the center of pressure from O as s is equal to $\sum s^2$ into δA by $A \bar{x}$.

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Resultant Force & Centre of Pressure ...

2nd moment of area about O = $I_o = \sum x^2 \delta A$

$S_y = \frac{2^{nd} \text{ Moment of area about a line through O}}{1^{st} \text{ Moment of area about a line through O}}$

Depth to Centre of pressure $D = S_y \sin \theta$

• Second moment of area from parallel axes theorem $I_o = I_{oc} + A\bar{x}^2$

$\frac{bd^3}{12}$

The diagram shows a yellow rectangular area with width b and height d . A horizontal line passes through the center of the rectangle, labeled with O at the left end and C at the right end. A small circle is at the center of the rectangle.

In the previous slide as we have seen here, this center of pressure is equal to $\sum s^2 \delta A$ by $A \bar{x}$ and then the second moment of area about this o into O which we are considering return as that is equal to $\sum s^2 \delta A$.

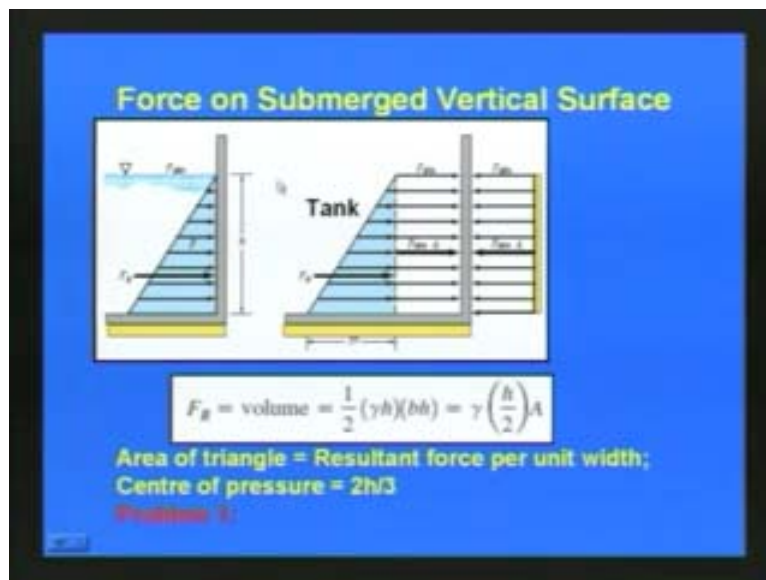
Finally, this sc can be written as a ratio, second moment of area about a line through O divided by 2. The first moment of area about a line through o so s is equal to second moment of area about A line through O divided by first moment of area about a line through O .

That is the second, this Sc so that is what we have already seen this center of pressure is Sc . So that is what we are getting here, and then depth of center of pressure D is equal to $Sc \sin \theta$ since we are considering inclined plane as in the previous slide and then how to find out the second moment of area about a parallel axes? the axes like in this figure in this slide here.

If you want to find this second moment of area about the, we can use parallel axes theorem. So with respect to that, we can write I_o is equal to IGG this axes plus $A \bar{x}^2$ square. So for this from this rectangular element like this it will be bd^3 cube by twelve.

This we can find second moment of area from parallel axes theorem and then the center of pressure will be the ratio second moment of area about a line through out the first moment of area about a line through O . So like this we can determine the so the resultant force on the inclined plane and then, we can determine the center of pressure where the resultant pressure is acting on the inline plane.

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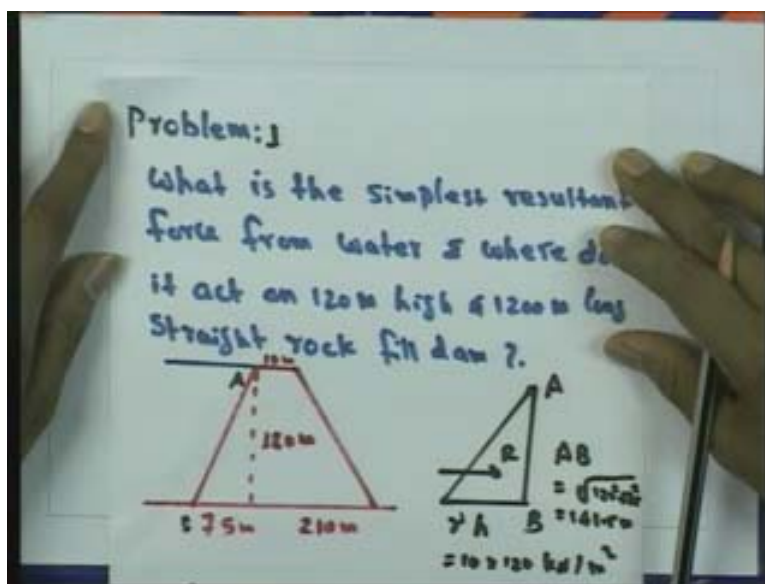


Now, we will discuss. So vertical surface instead of inclined plane if you consider the vertical surface as you can see here in this slide. Put the case of vertical surface this is a time, where we want to find what will be the resultant force acting on this particular whole here. So here you can see this height, the height of the liquid of the water here is h and here this surface of the time is affected by the atmospheric pressure.

So, with respect to this we can write the resultant force is equal to half γh into b into h so where b is the without the tank which we are considering. So that we can write this as the resultant force is equal to the vole which we are considering on this particular with respect to this elemental vole. So that can be written as γh by 2 into the cross section area b into h . So the resultant force is equal to the area of this triangle so we can write that is equal to half into h by 2 into A . So that ids the resultant force acting on the this vertical surface with respect to this tank.

The area of triangle is equal to resultant force per unit with and in this case the center of pressure is equal to $2h$ by 3. So this way, we can determine the resultant force on either inclined plane or we can determine the resultant force on a vertical surface, and we already seen the resultant force with respect to other surface, and then in the next will be discussing about the resultant force and in the center of pressure with respect to curved surface, before that we will discuss a small example problem.

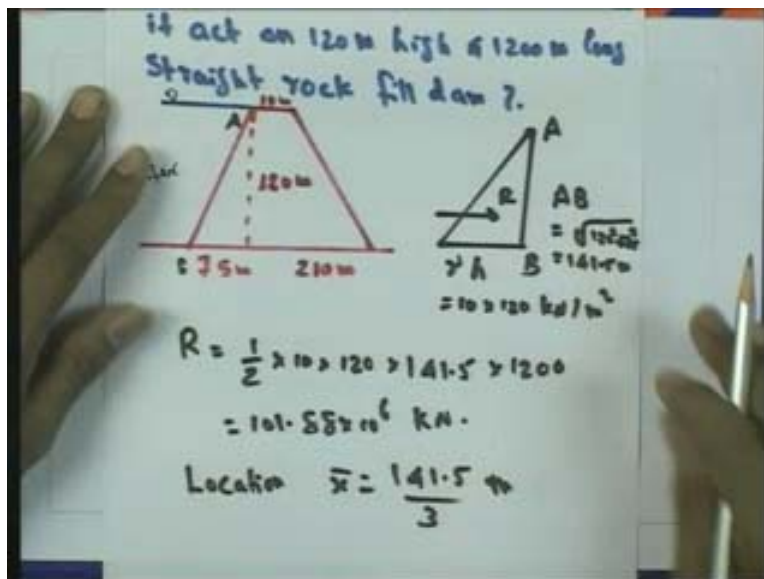
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The problem here, we are discussing is , there is rock fill dam like this, so we want to determine the what is the simplest resultant force from water and where does it act on? 120 meter height and twelve hundred meter long, straight rock fill dam that is the problem.

Here you can see, the dam is the base width is 285 meter and height is 120 meter and top width is 10 meter so, and here this is inclined plane here on AB where the water surface is acting. So here, this is the water surface is acting here, so this is water in the reserve wire, so we want to determine the the simplest resultant force and then where it is acting. So this is an inclined plane. So we can determine this inclined plane AB that distance can be determine so that it will be AB will be equal to 120 square its height square, plus 75 square this base width. So that to give AB and then we can draw A the pressure distinguishing triangle like this. So on this AB the resultant force will be acting like this R and then , the base on the base the pressure will be gamma into h. So gamma if you take, 10 Kilonewton per meter cube, so that will be 10 into 120 Kilonewton per meter square at the base and then the resultant force.

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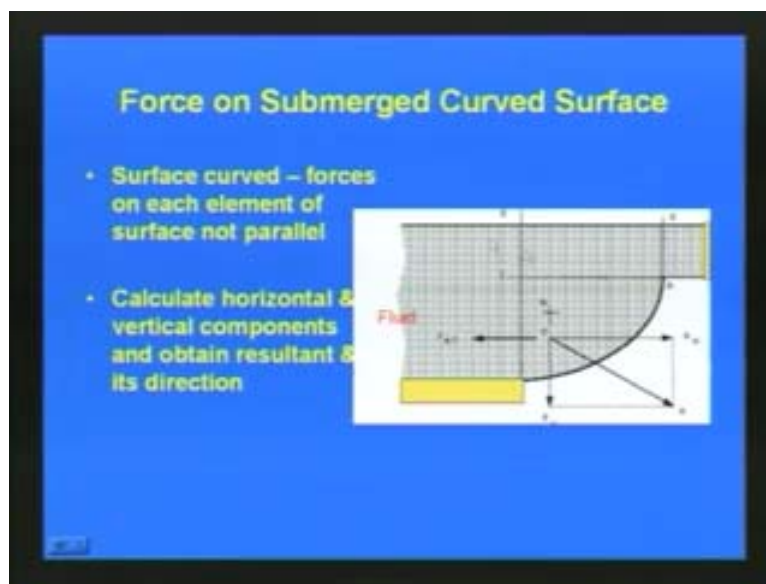
Will be r is equal to half into this base gamma into h in this length AB this height AB which we are considering in this inclined height. So that will be 141.05 which we have already determined multiplied by the width of the length of the dam that is 1200 meters.

The resultant force will be equal to half into R into 10 then is the gamma Kilonewton per meter cube 10 into 120 into 141.05 multiplied by twelve hundred so that will be equal to hundred and one point eight eight into the ten to the power of six Kilonewton. The resultant force is acting this R this particular location that will be the location will be \bar{x} will be equal to AB by 3 from the base so that will be 141.5 by 3 from the base.

This is the center of pressure, and the resultant symbol resultant force will be this much, R and it will be acting at these locations. So this is the simple problem here with respect to a rock fill dam we have calculated the simplest resultant force and then the location of the or the center of pressure where the force is the resultant force is acting.

Next, we will discuss the force on submerged curve surface. So we have seen if forces acting on a horizontal plane, we have seen the force acting on a vertical planes also, we have seen the force acting on an inclined plane, then, as for as fluid static is concern, when the force is acting on a curved surface. Then, we have to for example if you consider this bottle here. So this side of the bottle is curve, so you can see that when we have determine the force on the curved surface like this, we have to take the horizontal component and vertical component and then we have to determine the resultant force acting with respect to the curved surface. Now, we will discuss the force on a submerged curve surface.

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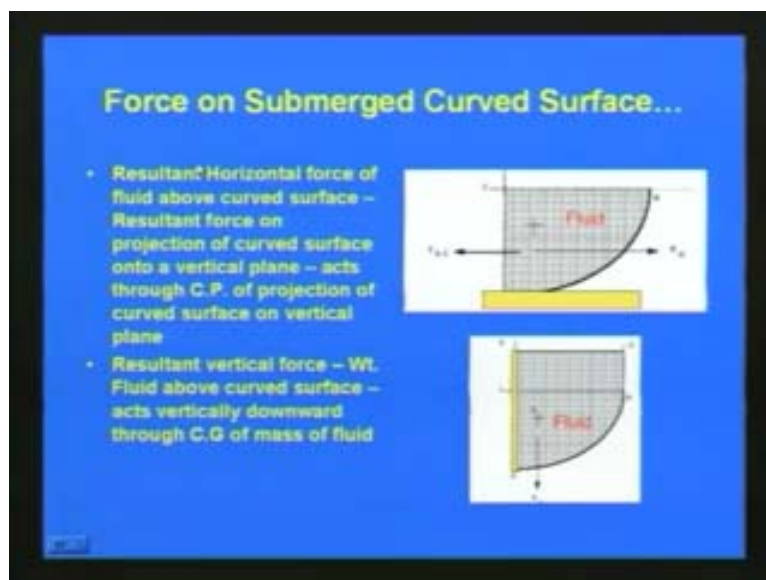


In this slide here, you can see here the fluid is at this location. Here the small tank, and then there is a curved weight like this, is the gate here the curved at the gate location here AB and then the fluid is above that gate. So we want to determine the resultant force acting on the gate with respect to the fluid above the gate and then we want to find the this location as where the force is acting the resultant force is acting.

The surface here is curved the forces on each element of surface are not parallel as we have seen in case of vertical surface or horizontal surface it is not parallel. We have to take it the horizontal component and electrical components of we will be calculating the horizontal and vertical components and finally we will obtain the resultant and then we will obtain the in its direction.

With respect to this gate curved gate here, you can see this is the RH is the resultant force acting on this gate, and then with respect to the fluid weight of the vertical force RV acting this direction and then we have to find out the resultant of the Rh and RV so that will be connecting in this direction R. We want to find out this R with respect to the curved surface so for this purpose.

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Here, the resultant force the resultant horizontal force of fluid above the curved surface is the resultant force on projection of curved surface. We can project the curved surface to a vertical plane, this and then we can find out, what will be the force acting on those vertical surfaces. So that will give the resultant horizontal force. The case of this curved

weight here, the horizontal force we can determine by projecting curved surface on a vertical plane

In this case, this is the vertical plane where the projecting this curved surface and that will be the resultant horizontal force and then it will be acting through the center of pressure projection of the curved surface on vertical plane. Here, this will be the center of pressure where it will be acting. The horizontal force the resultant horizontal force is acting in this direction and then with respect to this horizontal vertical projection here it will be the center of pressure.

Similarly, in the case of the vertical component, so the resultant vertical force is as for the gate is concerned, the resultant vertical forces the weight of fluid above the surface here, we have the curved surface. What will be the weight of fluid above the curved surface that we can determine and then that acts through the center of gravity of the mass of fluid. So here as far as this mass of fluid where, above the curved surface so, this much fluid where we can determine the weight of the fluid above this curved surface and then what is in the position of the center gravity, you can determine and then the resultant vertical force will be this weight of the fluid above the curved surface and then it acts through the center of gravity of the mass of fluid.

We have the finding of the horizontal resultant horizontal force and then where it is acting and then we are finding on the resultant vertical force and then it acts to the center of gravity of the mass of fluid above the gate which you are considering. So like this we are splitting the resultant force to resultant horizontal force and resultant vertical force and then for example if we consider in the particular case.

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Force on Submerged Curved Surface...

- Resultant force
- Angle of resultant force makes to horizontal

$R = \sqrt{R_h^2 + R_v^2}$

$\theta = \tan^{-1} \left(\frac{R_v}{R_h} \right)$

$-V_{eff} = V_1 + V_2$

$-V_T = V_1 + V_{eff}$

The slide contains two diagrams. The top diagram shows a curved surface submerged in a fluid, with forces V_1 and V_2 acting on it. The bottom diagram shows a similar setup but with a different configuration of forces and fluid levels, illustrating the concept of effective vertical force.

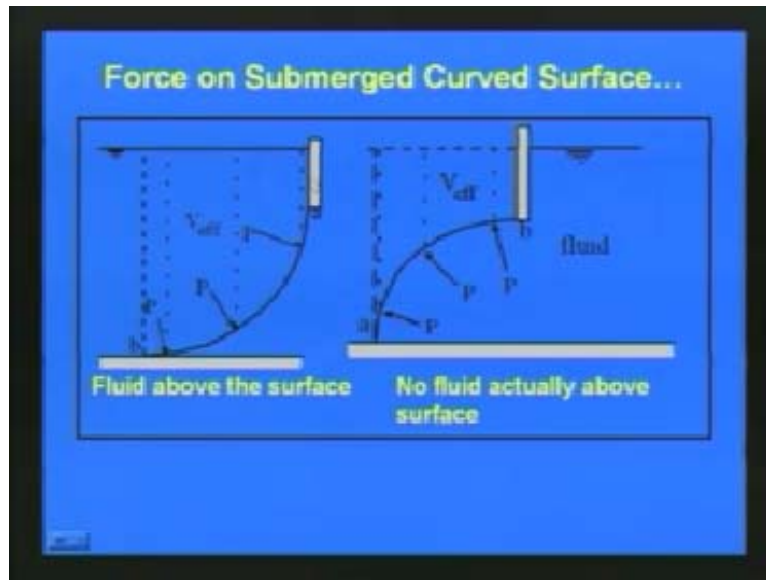
The resultant force, if you want to find out the case of vertical force here, the liquid is there and here it is empty. If you find the effective vertical force it will be V_1 here and then V_2 V_2 here and V_1 just above that, will be the effective vertical force and in the case of liquid it is kept here the bottom as in this second figure here. Here, V_1 is effectively, this is the fluid here and the total fluid is this, V_1 plus V effective just above the gate. So the total will be given plus V effective, but V effective we can find out V_T minus V_1 from this slide from this figure.

The resultant force finally, after determine the horizontal force and the resultant horizontal force and the resultant vertical force, the resultant force that we can determine R is equal to square root of R_h square, the square of resultant horizontal force plus, the square of resultant vertical force, so its square root is the total resultant force as far as the the curved surface on the submerged curved surface is concerned and then the angle of resultant force we can determine θ is equal to \tan^{-1} R_v the resultant force and the vertical direction divided by the resultant force in horizontal direction. So θ is equal to \tan^{-1} R_v divided by R_h so that is the the angle. So here you can see this is the R_h and this is the R_v .

The resultant force that is called root of R_h square plus R_v square and then this θ $\tan \theta$ will be equal to the \tan will be equal to R_v divided by R_h and then finally, we can get θ is equal to \tan^{-1} R_v by R_h . Like this, we can determine the resultant force and then where it is acting the location of where the resultant force is

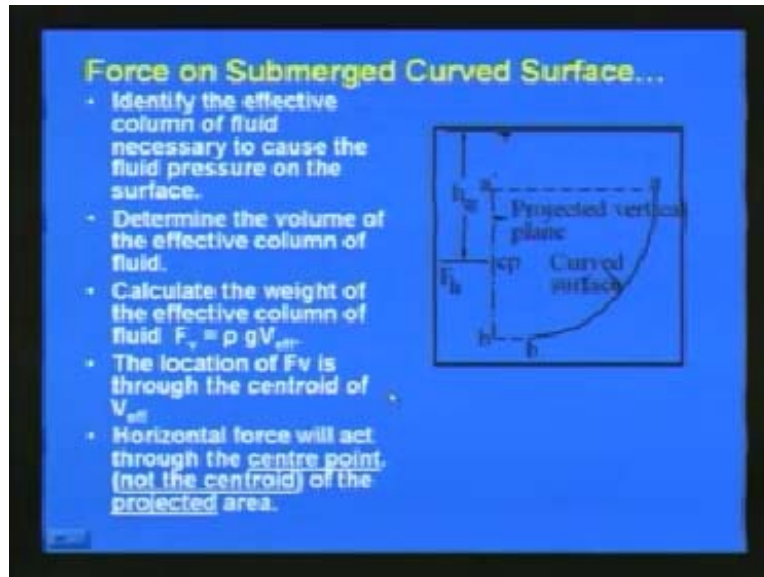
acted, that we can determine through this, by taking the horizontal component and taking the vertical component as explained here now, as we have discussed the flow.

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A gate is in this fashion and then the fluid is just above the gate. The fluid is above the surface we can see that the effective pressure is acting like this in the slide. So the effective pressure is acting just normal to the curved surface in this direction. The effective force we can determine, as we have already resultant force we can determine as, we have seen the previous slide. In the second case, here in this figure, you can see the fluid is just below the gate, so the resultant force is acting so with respect to this pressure is acting this direction. So p is acting no fluid above the surface and the effective the vertical force we have seen how we are calculating the previous slide.

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So like this we can, determine the effective vertical force, resultant force and the horizontal force and finally, here we will summarize how we are doing, as far as the curved submerged surface is concerned. First we can identify the effective column fluid, necessary to cause the fluid pressure on the surface.

Here you can see in this figure - the effective fluid pressure we can calculate and then second one is determine the volume of the effective column fluid and then calculate the weight of the effective column of fluid which is obtained ρg into v effective then we can find the location of F_v through the centroid of v effective and then we have already seen this is as far as the vertical resultant force for the various cases are concerned and then horizontal force will be just acting through the center point of the projected area and that also we can determine as we have seen in the previous case. So like this we can determine the resultant force and then where the resultant force is acting and as far as submerged curve surface is concerned.

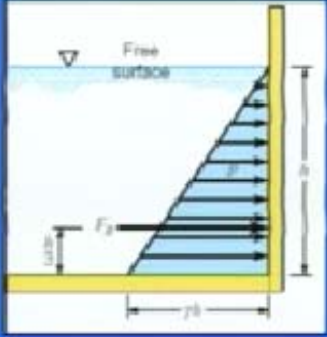
Now, we have already seen how we can determine the resultant force and then where the resultant force is acting as far as the curved surface, than inclined plane vertical plane and horizontal plane. Now, we will discuss some of the applications of this some practical applications. So how we can determine the resultant force? And then how we can determine the center of pressure? First some practical cases, we consider a water tank. So here you can see in this figure.

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Resultant Force & Centre of Pressure - Applications

- a) Tanks

Pressure at Side Walls -

$$P = \gamma(b \times H) \frac{H}{2}$$


If you want to find out the pressure at side wall, we have already discussed earlier, so P is equal to gamma into B into h into H by 2. So the area of this triangle here that we will the pressure it is a side wall. So this gamma which is the pressure at the bottom and then H is the water column height. So this is the resultant force and it acts at h by 3. This one-third of this height is from the bottom. So this is as for as how we can determine the resultant force and the center of pressure for water tanks.

This is the first case, and then second case if you want to determine the resultant force and center of pressure application case of a gate.

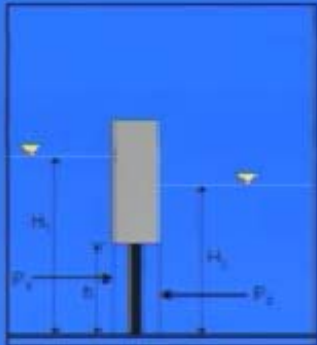
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Resultant Force & Centre of Pressure - Applications...

- b) Gates

$$P_1 = \gamma A \left(H_1 - \frac{h}{2} \right)$$

$$P_2 = \gamma A \left(H_2 - \frac{h}{2} \right)$$

$$P = P_1 - P_2$$


So [] gate are very much used in the case of many of the hydraulic structures as far as canals or dams are concerned. So how we can determine the resultant force?

Here this slide shows, here, there is a [] gate and here the water column of the upstream side is H_1 and downstream is column height is depth of water is H_2 . So we can determine if you want to find out the pressure acting on this gate. We can determine p_1 is equal to p_1 is acting what is acting on this gate here this is the gate location p_1 is equal to γA into H_1 minus h by 2. Where h is the height of the gate so p_1 , what is acting this gate is equate only effectively acting on the gate is equal to γA into h_1 minus h by 2 and the force acting from the downstream side p_2 is obtained as γA into H_2 minus h by 2. So the effective pressure or the effective force acting on this the gate is equal to p is equal to p_1 minus p_2 .

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Resultant Force & Centre of Pressure - Applications...

- c) Dams
- Pressure at Upstream Face

$$P = \gamma A \bar{x}$$

- Acting at from Top

$$\bar{h} = \bar{x} + \frac{I_G}{A \bar{x}}$$

This is the case of a [] gate and the next case is if you want to determine the resultant force and send a pressure as far as dam is concerned. This we have already discussed earlier as in the case of problem.

So that you talk to pressure and we have the pressure will be acting at h by 3 very similar to that the case of the tank which we have discussed. So this gives the pressure on a dam, like this we can see so many practical application. In the case like [27:36] or with respect to dam water tanks many cases we have determined the resultant force and then we have to determine the center of pressure. So that we can properly design the structures we can see how much pressure is acting and then properly manage the design the section so like this various other applications also we can find as we have seen here, the pressure acting from the top is equal to \bar{x} plus $\frac{IG}{A \bar{x}}$ as we have already seen in the previous case and then finally, the resultant force is equal to P is equal to $\gamma A \bar{x}$ in the case of a dam.

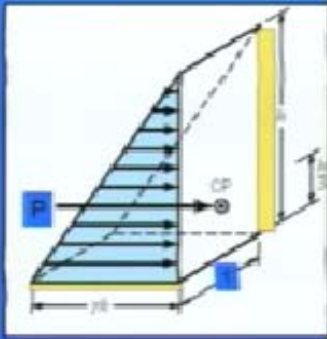
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Resultant Force & Centre of Pressure - Applications...

- So Resultant Force is

$$P = \gamma A \bar{x}$$
$$P = \gamma (1 \times H) \frac{H}{2}$$
$$P = \frac{\gamma H^2}{2}$$

- At ' $\frac{2h}{3}$ ' from Top



Here, P is equal to gamma H square by 2 for unit to, so then it is acting 2h by 3 from the top and h by 3 from the bottom, so this also we have seen.

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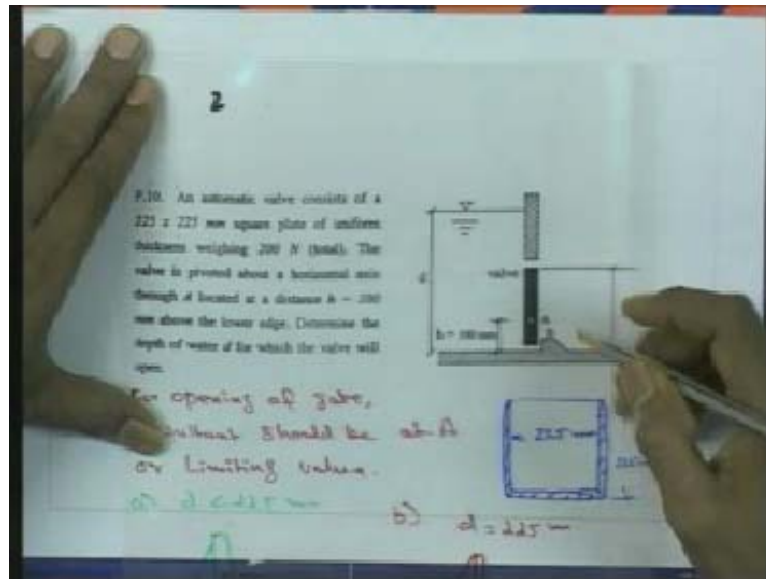
Resultant Force & Centre of Pressure - Applications...

- Examples

- Problem 2
- Problem 3
- Problem 4

Now, with respect to the resultant force and the center of pressure applications we will discuss few example problems. First example is we will discuss a problem where we want to find out, there is an opening? [] gate liking this, here you can see this figure, gate and then there is an upstream water, we want to find out this depth of water with respect to for which the valve will open.

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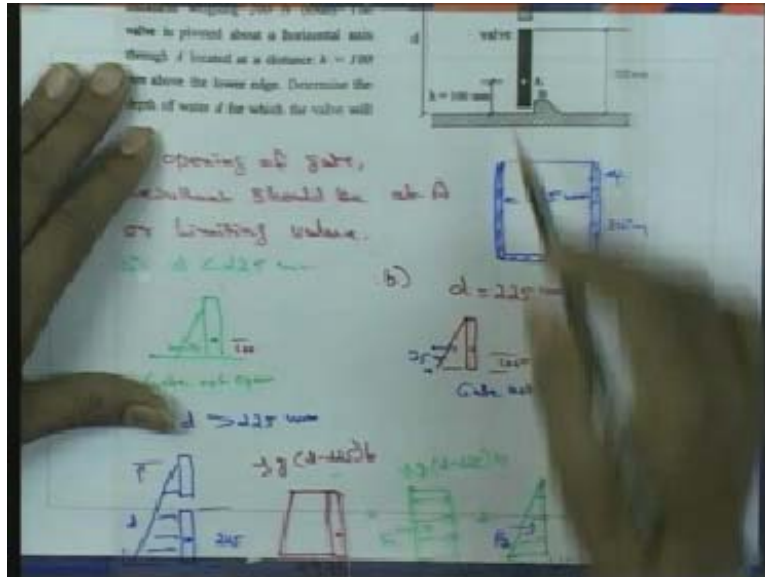
There is a valve here which at this point and below the downstream this [] the water depth is 225 millimeter and then this; the gate is the pipe pivoted at h is equal to 100 millimeter like this.

The problem statement here is so we want to determine, there is an automatic valve there is an automatic valve which consist of the 225 into 225 millimeter square plate of uniform thickness weighing 200 Newton as shown in this figure. The valve is pivoted about a horizontal axis through A this is A located at a distance h is equal to 100 millimeter above the lower edge determine the depth of water d this depth of water d for which the valve will open.

Here the problem statement is, what a [] gate we want to find out this depth b for which the valve will open. So the minim depth for which the valve will open. So here already there is water depth 225 millimeter in the downstream and here the valve is the pivoted at h is equal to 100 millimeter from the bottom.

So we want to find this depth d for which the valve will minim depth for which the valve will open so for this problem now we will consider three cases.

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So first case is we can see here for opening of gate the resultant should be at A. So if we consider the various forces acting and then if you determine the resultant force, so that the force opens the minim if you want to find out the minim depth of water in this case we want to find that the resultant should be the resultant of this water column on the upstream side of this gate that should pass through this hint at A. So that will be limiting value so to solve this problem here you consider three cases. First case is when if the depth of water Column and the upstream served if these less than 225 millimeter here the downstream water column is 225 millimeter.

If it is the upstream depth is less than 225 millimeter then, what will happen? We have already seen the center of pressure will be h . This h d by 3 so d is less than 225 millimeter then it will be less than 100 millimeter or this will be less than 75 as far as the center of pressure is acting. You can see, when I draw a center of the resultant force of the pressure distribution diagram here, you can see this is will be this resultant force will be less than this 100 millimeter, which is the height of the valve hint which we are concerned.

When it is less than 100 then you can see that it will not open. So first case, when this depth of the water Column on the upstream is less than the downstream depth of the 225 millimeter gate will not open.

Then the second case, we will consider when the upstream depth is equal to the downstream depth. So d is equal to 225 millimeter so in this case here, you can see d is equal to 225 millimeter then, you can see that the pressure distribution with respect to the gate here will be, here it is 225 millimeter pressure distribution will be like this and then we know that the center of pressure will be this is the vertical surface.

So the center of pressure will be d by 3 from the bottom. So here with respect to this it will be 225 for, the limiting case 225 by 3, from the bottom surface it will be identify millimeter.

So this 75 millimeter is again less than this depth h where the A is located where the pivot is located. Since, this d by 3 is 75 millimeter less than 100 millimeters in this case also it will not open.

So the third case what will be considering is when the d the depth of upstream water level is greater than 225 millimeter. So in this case the pressure distribution can be drawn like this, here with respect to the gate this downstream this 225 millimeter and then above that also with respect to the water there would be pressure distribution will be extended like this, and then if we consider this particular location, then you can see where the downstream height is coincide with upstream height this location, if you consider we can see here, with respect to this, you can see here, this is the particular location where the gate is located.

So for that we can draw the pressure distribution. So this will be a trapezoid pressure distribution like this and here it will be ρg into d into b so this we can split into one rectangle type pressure distribution, and one triangle type pressure distribution. So this will be the rectangle type and so this we can split into one triangle distribution and one rectangle distribution. Finally, we will get this will be equal to F_1 plus F_2 for F_1 stands for the rectangle pressure distribution with respect to this and this F_2 is with respect to the triangle pressure distribution.

So from this we can write, we can solve this problem by equating F_1 is equal to we can calculate the pressure the force F_1 .

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$$F_1 = \rho g (d - 225) b \times 225 \text{ N @ } \frac{225}{2} \text{ from bottom}$$

$$F_2 = \frac{1}{2} [\rho g (225) b] \times 225 \text{ @ } \frac{225}{3} \text{ from bottom.}$$

$$F_R = F_1 + F_2$$

$$F_R \bar{x} = F_1 \times \frac{225}{2} + F_2 \times \frac{225}{3}$$

For opening gate $\bar{x} \geq 100$

$$x_{\min} = 100 \text{ mm} = \bar{x}$$

$$100 \geq \frac{\rho g b}{\rho g b} [(d - 225) 225 \times \frac{225}{2} + (\frac{225}{2} \times \frac{225}{3})]$$

So that is F_1 is equal to ρ into g into d minus 225 into b into 225 Newton acting as 225 by 2 from the bottom. So that is correspond to, this force distribution F_1 and then we can determine F_2 for this triangular pressure distribution is concerned F_2 is equal to half into ρ into g into 225 into b multiplied by 225 that will be acting this 225 by 3 from bottom.

So here we are assign d is the limiting case of 225 . So if it is in case up to this this 225 by 3 times we are considering only up to this.

So this is F_1 calculated F_2 is calculated and now the resultant force now you can see this F_1 plus F_2 so F_R is equal to F_1 plus F_2 and then we will take a moment F_R into \bar{x} . So here this total as for as a total force is concerned F_R take a moment F_R into \bar{x} and that will be with respect to this F_1 into 225 by 2 plus F_2 into 225 by 3 so this is from the bottom.

So for opening gate we know that, the with respect to this this should be either 100 millimeter just one 100 millimeter. So we put this \bar{x} is equal to the minim of 100 millimeter then we will get \bar{x} is equal to if you substitute \bar{x} is equal to 100 millimeter \times minim then we will get finally you can equate this F_R is equal to F_1 plus F_R \bar{x} is equal to F_R into \bar{x} is equal to F_1 into 225 by 2 plus F_2 into 225 by 3 . Finally, we will equate.

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$$F_R = F_1 + F_2$$

$$F_R \bar{x} = F_1 \times \frac{225}{2} + F_2 \times \frac{225}{3}$$

For opening gate $\bar{x} \geq 100$

$$x_{min} = 100 = \bar{x}$$

$$100 \times 986 \left[(d - 225) \times 225 + \left(\frac{225}{2} \times 225 \right) \right]$$

$$= 986 \left[(d - 225) \times 225 \times \frac{225}{2} + \frac{1}{2} \times 225 \times 225 \times \frac{225}{2} \right]$$

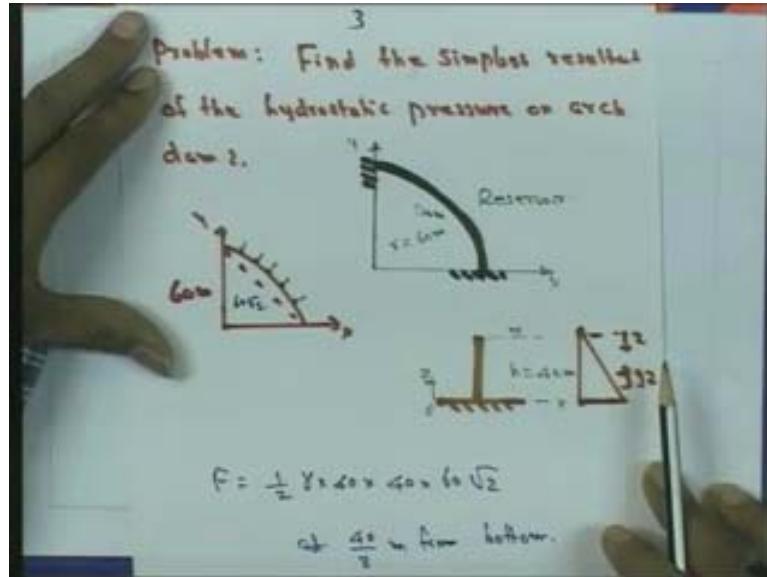
$$d_{min} = 450 \text{ mm}$$

All this, finally you will see that the d_{min} that means, here the minimum depth of water to open this gate [] gate will be four hundred and fifty millimeter. So in that case the gate will be opening. So finally, we can determine the depth of water d for which the valve will open will be four fifty millimeter

This way we can... so this problem of where we discussed the pressure distribution and the force as for as the [] gate which we consider here F_1 and F_2 and then we have determine with respect to, after taking a moment, with respect pivot we want to see the depth the minimum depth which it will start to open. So that we have calculated thus minimum is equal to 450 millimeter.

Next problem, which will be considering is we will consider an arch dam and we want to determine the simplest resultant of hydrostatic here in this figure.

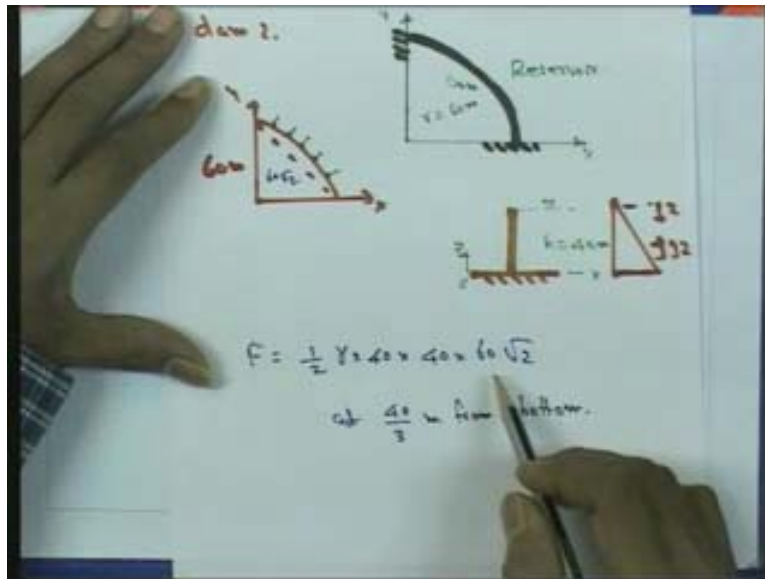
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We want to determine, there is an arch dam here and here, there is a reserve wire and the radius of this arch dam is 60 meter and we want to determine with respect to this reserve wire. We want to the height of water level is 40 meter; we want to find the simplest resultant of the hydrostatic pressure on the arch dam. So this is the dam and here is the reserve wire and depth of water is 40 meter. So here with respect to this figure, first we will find what will be the pressure distribution and the bottom of the dam? So here if we consider like this in this here 40 meter so it will be $\rho g z$ at the bottom.

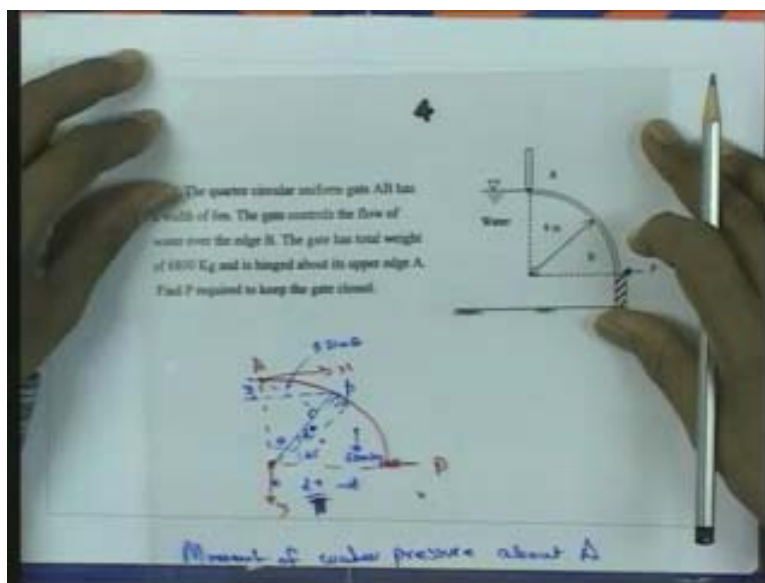
This figure you can see that from the effect of the reserve wire the force will be acting normal to the surface and then what we can do is in a simple way, we can just take what will be the this inclined plane. We can just make it as an approximation. You can just find what will be the length of the inclined plane. Here this is 60 meter radius. You can find that this will be $60 \sin \theta$. Finally we can determine the resultant force as so this with respect to this half gamma into this 40 is the height and then multiplied again 40 into $60 \sin \theta$.

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Which this case is acting? So the total the simplest resultant which in this case for this arch dam will be $\frac{1}{2} \gamma \cdot 40 \cdot 40 \cdot 60 \sqrt{2}$. Where γ is the specific weight of water and this will be acting at $\frac{40}{3}$ from the bottom. So this is the case how to find out this simplest resultant force of the hydrostatic pressure in an arch dam. So one more problem we will consider so here the last problem here.

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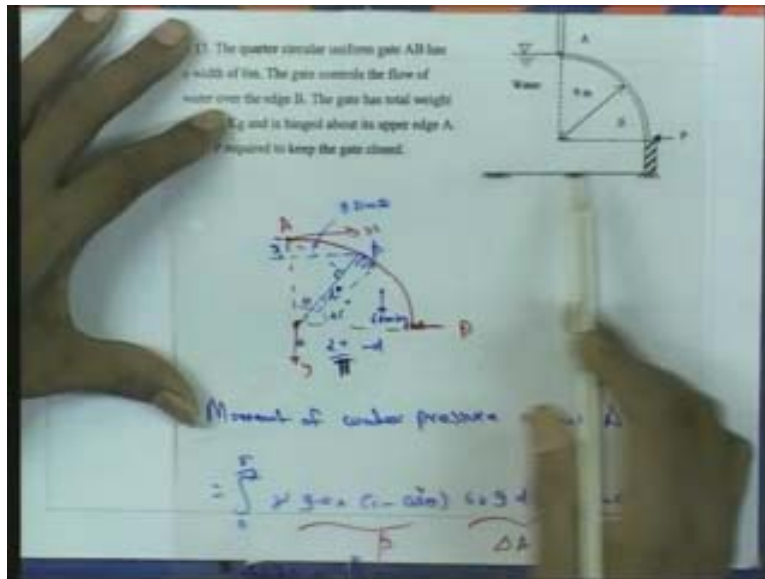


There is a problem statement is the quarter circular uniform gate. There is a gate AB has a width of 6 meter and the gate controls the flow of water over the edge B the gate has total weight of 6800 kilo gram and its hinged about it upper edge A, find P the force

required to keep the gate was, here we have got a gate and then on this side we have water and then the gate radius is nine meter and it has got a width of 6 meter.

We want to find the force P which should apply from this other side so that the gate will be kept as close. So gate if you want gate as course what should be the pressure P applied from this side so that is the problem. So the gate, the total weight of the gate is 6800 kilo gram.

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Here, we can consider the free body diagram or the pressure and forces acting as far as the problem is concerned, here p is the force which you want to determine and here with respect to the water column, there would be force acting like this and then, the weight of the gate 6800 kilogram. Also we have to consider and if you consider this as y axis and then x axis and now to solve this problem, we can take the moment of water pressure about this location A. So that we can determine we can take an integral zero to π by 2 since this angle is ninety degree.

We can find the moment of water pressure about A by taking integral zero to π by 2 gamma the specific weight multiplied by the radius of the gate nine into one minus cos theta we have consider this as angle theta. So this will be one minus cos theta so that will be the pressure intensity acting up on this gate multiplied by area delta A since we are integrating so delta will be 6 into 9 into $d\theta$.

So d we are taking as small theta here and then we are finding the area as six into nine into d theta and then that will be multiplied by the perpendicular distance area in 9 sin theta. So this will be 9 sine theta. So that is equal to, if you integrate, it will be and applying the taking the multiplying the 6 into 9 and all this value to be 4374 gamma.

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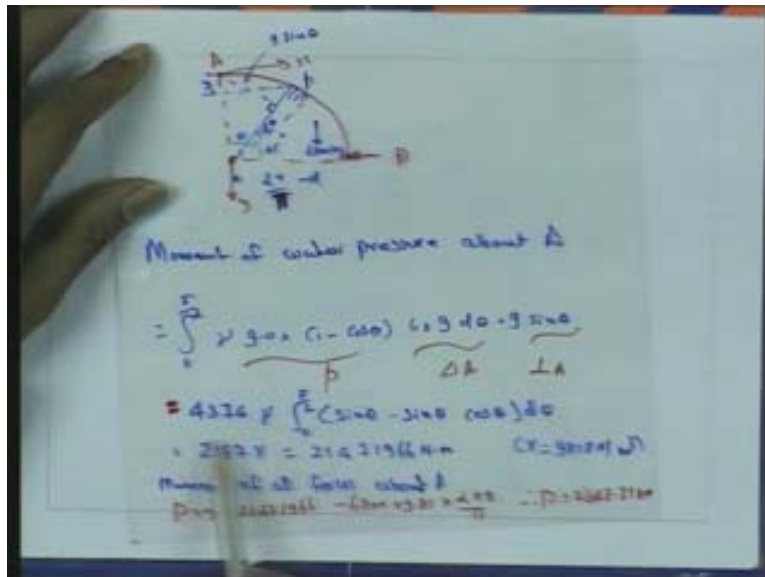


Diagram showing a curved gate segment of radius 6m and length 9m, subtending an angle of $\pi/2$ radians. A point A is marked at the center of curvature. The gate is subjected to a water pressure p acting perpendicular to its surface. The distance from A to the gate is 6m. The gate is divided into small elements of length $9 \sin \theta d\theta$.

Moment of water pressure about A

$$= \int_0^{\pi/2} p \times 9 \times (1 - \cos \theta) \times 9 \sin \theta d\theta$$

$$= 4374 p \int_0^{\pi/2} (1 - \cos \theta) \sin \theta d\theta$$

$$= 2187 p = 2187 p \text{ (N-m)}$$

Moment of all forces about A

$$p \times 2187 \times 6 = 6800 \times 9.81 \times 2 \therefore p = 2367.21 \text{ N/m}^2$$

Gamma is the specific weight of the water so four thousand three hundred and seventy four gamma integral zero to phi by 2 sin theta minus sine theta cos theta d theta. This is with respect to this integral and then after the integration and applying the limits from 0 to phi by 2. We will get this as 2187 gamma and if you take gamma is 9880 Newton per meter cube. Finally, we will get the moment of water pressure about A and then finally, to find this P we will be taking a moment of all the forces. So this integral we have consider only as far as the water acting on this the gate is concerned.

If you want to find the pressure p, now we will be considering all the forces as far as the gate is concerned, the forces here are the water pressure acting from this side and the force which we want to keep force this p and then the weight of the gate 6800 kilo gram. If you want to find p require to keep the gate close. Find we will be taking a moment of all the forces above is location this point A. So that p is concern p into 9 so p into 9 is equal to. We have already determined what will be the moment of water pressure about, so that minus the gate is concern there will be an another force acting. This moment 6800 into 9.81 into 2, it will be acting this weight of the gate will be acting 2 R by phi from this side so it wills 2 into 9 by phi.

From this equation you will take, the moment of your forces above this location A. We can finally determine p so in this case we can determine p as 2343.31 Kilonewton. So like this we can consider all the forces acting on the gate, the water pressure from this side and the force which we have to keep the gate towards the p and then the weight of the gate. Like this we can solve various problems. Find the certain force center of pressure and its various applications as we have demonstrated in all these examples discussed here.

Now, the next topic we will discuss the buoyant force. The buoyant force, you can define as it is the resultant fluid force acting on a body that is completely submerged or floating in a fluid is called the buoyant force. Now, if we consider some water here in the container. So if you want to find, what is buoyant force so now if you just put like this, if you want to submerge, if you floating body like this just submerge partly submerged in this water. So you can see here it is the buoyant force is the resultant fluid force acting on the body.

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Like this partly submerged, so here this body floating here, so we can see that there is some force is acting on its surface that is why it is floating. So either it can be submerged so it can be floating. So this force is called the buoyant force. So the buoyant force could be generally, equal to the specific weight of the fluid multiplied of the volume of the body either which is partly submerged or fully submerged, based upon this buoyant force Archimedes is format is formally Archimedes principle.

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Buoyant Forces

- The resultant fluid force acting on a body that is completely submerged or floating in a fluid is called the buoyant force
- $F_B = \gamma V$
- γ – Sp. Weight of fluid, V – Volume of body
- **Archimedes' principle**

When a body is immersed in a fluid either wholly or partially, it is buoyed or lifted up by a force which is equal to the weight of the fluid displaced by the body.

So this is defined as the when a body is inverse in a fluid or wholly fluid either wholly and partially or is buoyed or lifted up by a force, which is equal to the weight of the fluid displaced by a body. So this is called the Archimedes principle. So you can see here in this case, what is the floating body? How much the partially submerged body? How much is the fluid which is displaced with respect to each that should be equal to? What is acting up on the body? That is the Archimedes principle. So with respect to this here, you can see in this slide if we consider the Archimedes principle.

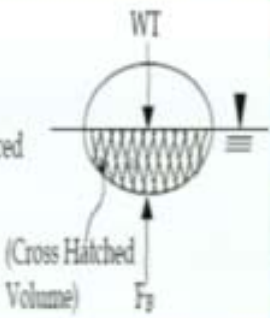
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Buoyant Forces....

- Buoyancy

Archimedes Principle
 $F_B = WT$ where $F_B = WT$ of Liquid Displaced
 $y' = \text{Centroid of Displaced Liquid}$

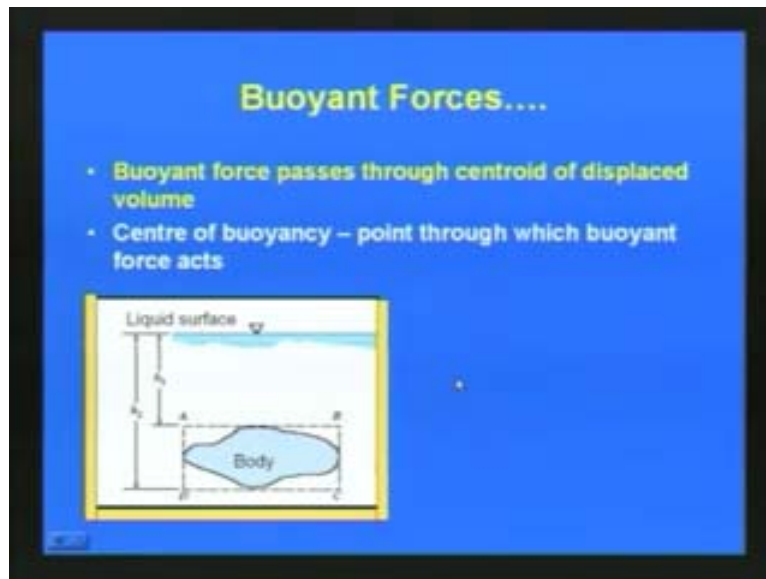
- Flotation Stability



The diagram shows a circular body partially submerged in a fluid. A downward arrow labeled 'WT' represents the weight of the body. An upward arrow labeled 'F_B' represents the buoyant force. The submerged portion of the body is shaded with cross-hatching and labeled '(Cross Hatched Volume)'. A horizontal line indicates the fluid surface.

Here this one, in a fluid so the hatched volume, we can calculate so this the buoyant force will be the WT where FB is acting so WT of liquid displaced with respect to this buoyant some liquid will be displaced. So that we can calculate the weight of the liquid that means WT is weight of the liquid displaced. That is, gives the buoyant force. Then we can determine the centroid of the displaced liquid so that we can, see where it is acting. So like this buoyant force is very important in as far as especially as far as the static fluid is concerned as we have discussed here.

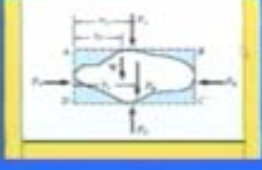
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And buoyant forces pass through the centroid of displaced volume as we have already discussed and the center of buoyancy is defined as the point through which the buoyant force acts so this buoyant force is passing for centroid of the space volume so from which we can determine the center of buoyancy. So here this you can see this is the body where it is wholly submerged with respect to that how much liquid is displaced and then with respect to that we can determine the buoyant force and then we can determine the center of buoyancy.

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Buoyant Forces....

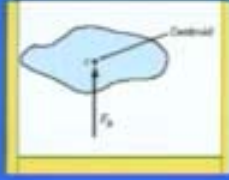


$$F_B = F_2 - F_1 - W$$

$$F_2 - F_1 = \gamma(h_2 - h_1)A$$

$$F_B = \gamma V$$

- Hydrometer uses the principle of buoyant force to determine specific gravities of liquids



Here you can see this buoyancy force is equal to F_2 minus F_1 so F_2 is the force acting up on the body which is submerged fully or submerged partially. So the key is the F_2 and then with respect to this F_1 is the force which is acting the weight of the fluid which is acting up on this. So F_B the buoyant force is equal to F_2 minus F_1 and minus W where the W is the weight of the body and F_2 minus F_1 is equal to γ into h_2 minus h_1 into A , and finally buoyant force is equal to γ into the volume of the fluid which has displaced.

This is the way which we determine the buoyant force and then we can determine the principle for many other applications like if you want to determine the specific gravity of liquid we can use hydrometer.

So hydrometer is a equipment which is just like a cylindrical form and equipment some fluid is and then it will be putting into the liquid and then this principle of buoyant force is used to determine the specific gravity of liquid. So the buoyant the Archimedes principle and the buoyant force and center of buoyancy are very important many of the fluid flow problem. This will be discussing further , the with respect to buoyant force or center of gravity how we can whether I submerged body or the floating body is stable or unstable that will be discussing in the next lecture.

Buoyant force, with respect to buoyant force, we can determine the center of buoyancy, we can determine then we can find out the center of gravity and then we can see whether the body is the floating body , stable or unstable or in an equilibrium condition.

So that you will be discusses ng in the next lecture.