

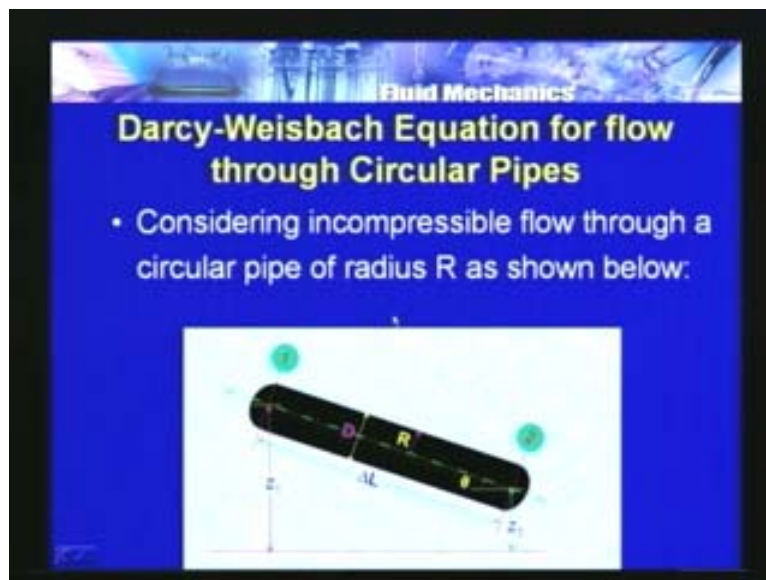
**Fluid Mechanics**  
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**Lecture – 37**  
**Pipe Flow Systems**

Welcome back to the video course on fluid mechanics. Last lecture, we were discussing about the pipe flow system; we have seen various aspects of lamina flow conditions at turbulent flow conditions with respect to the pipe flow and over with respect the various losses also with respect to pipe flow we have discussed the major losses and minor losses.

In today's lecture, further we will discuss the various pipe losses. First, we will discuss the various aspects of major pipe losses and then we will discuss the minor pipe process. As we discussed earlier, in pipe flow with respect to the real fluid we have the shear stress on the pipe wall and then shear force is there and then viscous velocity plays a major role.

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One of the most important equations as far as major pipe flow is concerned is Darcy Weisbach Equation. First, we start with the Darcy Weisbach equation the derivation of Darcy Weisbach equation.

Let us consider a pipe like this; flow is in this direction and the diameter of the pipe is  $d$  and radius is  $r$  and it is at an angle  $\theta$  as central line  $\theta$  angle  $\theta$  as one here. So let us consider two sections between 1 and 2, say, at section 1 the height of the central line the data height  $z_1$  and that section 2 the data height  $z_2$  from the central line of the section which we consider  $\Delta L$ . For such a section, let us consider the various fundamental equations are: first, we apply the Bernoulli's equation between section 1 and 2.

Here at section 1 the pressure is given as  $p_1$  and section 2 pressure is  $p_2$  and the velocity of section 1 be  $v_1$  and velocity section 2 be  $v_2$  and then due to the discuss effect and the shear stress effect there is friction loss let it be represented as  $h_f$  is the head loss due to friction.

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- Applying Bernoulli's Equation between Sec. 1 & 2
 
$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f \quad \dots(1)$$
- $h_f$  is the head loss due to friction
- From continuity,  $A_1 V_1 = A_2 V_2$  as,  $A_1 = A_2$ ,  $V_1 = V_2$ 

$$h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} + (z_1 - z_2) = \frac{\Delta p}{\rho g} + \Delta z \quad \dots(2)$$
- Momentum equation along flow direction,
 
$$\Delta p (\pi R^2) + \rho g (\pi R^2) \Delta L \sin \phi - \tau_w (2\pi R) \Delta L = \rho Q (V_2 - V_1) = 0 \quad \dots(3)$$

Let it be represented as  $h_f$ . So, by applying the Bernoulli's equation between section 1 and 2 we can write  $p_1$  by  $\rho g$  plus  $p_1$  square by  $2g$  plus  $z_1$  equal to  $p_2$  by  $\rho g$  plus  $v_2$  square by  $2g$  plus  $z_2$  plus  $h_f$ . This is obtained from the Bernoulli's equations. So here  $h_f$  is the loss with respect to the friction with respect to the viscosity effect between section

1 and 2. The  $h_f$  is what we want to find out from here in this equation from the Bernoulli's equations. So from the continuity equation we can write  $a_1 v_1$  equal to  $a_2 v_2$ .

Now we consider the pipe flow; the diameter is same as  $a_1$  is equal to  $a_2$ . So between section 1 and 2 we can see that here  $v_1$  is equal to the average velocity of section 1 and section 2  $v_1$  is equal to  $v_2$  from the continuity equation. After using the continuity equation in this equation number 1, we can write the head loss due to  $h_f$  is equal to  $p_1$  by  $\rho g$  minus  $p_2$  by  $\rho g$  plus  $z_1$  minus  $z_2$  the difference in data head. This we can write as if  $p_1$  minus  $p_2$ , the pressure difference, so  $\Delta p$  by  $\rho g$ . So  $h_f$  is equal to  $\Delta p$  by  $\rho g$  plus  $\Delta z$  the diatom difference between section 1 and 2, so as in equation number 2.

Now let us use the momentum theorem between section 1 and 2. Here, we have seen that the pressure difference between section 1 and 2 is  $\Delta p$ . By applying the momentum equation along the flow direction,  $\Delta p$  into  $\phi r^2$ , where  $r$  is the radius of the pipe plus  $\rho g$  into  $\phi r^2$  in to  $\Delta l \sin \theta$ . If forces acting are the pressure force then the weight of this fluid is obtained as  $\rho g$  into  $\phi r^2 \Delta l \sin \theta$  and then the shear force,  $\tau_w$  by  $2 \phi r$  into  $\Delta l$ . So that should be equal to the change momentum  $\rho$  is  $r \rho q$  into  $v_2$  minus  $v_1$ . Since  $v_1$  is equal to  $v_2$ , this is equal to 0 as the equation number 3, where  $q$  is the discharge through the pipe and then  $r$  is the radius of the pipe. So here the forces acting on this pipe element between section 1 and 2 are the pressure force and the weight between section 1 and 2 and then shear force. So, the net force, the arithmetic force should be equal to rate of change of the momentum. So rate of change of momentum is equal to  $\rho q$  is equal to  $v_2$  minus  $v_1$ , where  $v_2$  is the velocity section into  $v_1$  is velocity at section 1.

Here, we consider the flow as steady and fluid is incompressible. So with respect to all these assumptions we can see that here since velocity  $v_1$  is equal to  $v_2$  rate of change of momentum is 0 so that the arithmetic force is equal to 0 as in equation number 3. Now here with respect to this figure the  $\Delta$  in this length. Now,  $z_1$  minus  $z_2$  that means  $\Delta z$ , we can write as  $\Delta z$  equal to  $\Delta l \sin \theta$ .

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- Since,  $\Delta z = \Delta L \sin \theta$ , we get as:

$$\frac{\Delta p \pi R^2}{\rho g (\pi R^2)} + \Delta z = \frac{\tau_w (2\pi R) \Delta L}{\rho g (\pi R^2)} \quad \text{or,} \quad \frac{\Delta p}{\rho g} + \Delta z = \frac{2\tau_w \Delta L}{\rho g R}$$

- Hence, from (2),  $h_f = \frac{2\tau_w \Delta L}{\rho g R}$
- Now, Wall shear stress  $\tau_w$  can be expressed in functional form as:  $\tau_w = F(\rho, V, \mu, D, k)$
- $k$  is avg. roughness height,  $V$  is 'avg. velocity'
- From Buckingham's  $\Pi$ -theorem, we get

$$\frac{8\tau_w}{\rho V^2} = f = F(R_e, \frac{k}{D}) \quad \text{f is the friction coeff.}$$

After putting this delta z equal to delta sin theta in this equation number 3, we get delta p into phi r square divided by rho g phi r square plus delta z is equal to tow\_w u into 2 phi r delta l divided by rho g phi r square, where tauw is the shear stress. This we can simplify as delta p, this equation we can simplify as delta p by rho g plus delta z is equal to tau\_w into delta l by rho g r. Now if you put in the equation number 2 by considering this equation number 2 coming from the Bernoulli's equations, we get h f the head loss due to friction h f is equal to 2 tau\_w into delta l by rho g r.

Since delta p by rho g plus delta z is called, from the equation number 2 we get h f is equal to 2 into tau\_w into delta l by rho g r. Now the wall shear stress tau\_w can be expressed in functional form as, wall shear stress is functional of the density of the fluid, the average velocity, dynamic coefficients of viscosity nu, the diameter and the reference height.

So tau\_w is functional of rho v nu d and k is the average roughness height v is the average velocity. If you do dimension analysis using Buckingham phi theorem which we discussed earlier we can write this, we can drive an expression for this tau of u in terms of this parameter rho v mu d k over in terms of r is number r u d and k by d. So from the Buckingham phi theorem we get 8 tau\_w by rho v square is equal to as a functional of f

functional Reynolds number and  $k$  by  $d$ , where  $k$  is the average reference height and  $d$  is the diameter in the pipe and this is equal to  $f$ , where  $f$  is the friction coefficient.

We can show this  $8\tau_w$  by  $\rho v$  square is the friction coefficient as per the pipe flow is concerned. Now this is what we got from the Buckingham phi theorem. We can put it back here with respect to  $\tau_w$  and then we can get an expression for the head loss due to friction  $h_f$  we can obtain.

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- Hence,  $\tau_w = \frac{f\rho V^2}{8} = \frac{f}{4} \cdot \frac{1}{2} \rho V^2$
- so  $h_f = \frac{2\tau_w \Delta L}{\rho g R} = \frac{2 \cdot \frac{f\rho V^2}{8} \cdot \Delta L}{\rho g R} = \frac{2 \cdot \frac{f\rho V^2}{8} \cdot \frac{2\Delta L}{D}}{\rho g}$
- or,  $h_f = \frac{fL V^2}{2gD}$   $\Delta L$  is replaced by  $L$

—This is Darcy-Weisbach Equation valid for duct flow for both laminar and turbulent conditions.

Hence,  $\tau_w$  is equal to  $f\rho v$  square by 8 that can be written as  $f$  by 4 into half  $\rho v$  square. We can write  $h_f$  is equal to  $2\tau_w \Delta l$  by  $\rho g r$ . That is equal to up to substituting for  $\tau_w$   $h_f$  is equal to  $2f\rho v$  square and  $\Delta l$  by  $8\rho g$  into  $r$ .

So that is equal to  $2f\rho v$  square to  $\Delta l$  by  $8\rho g d$  where  $d$  is the diameter. So  $d$  into 2 times,  $r$  is equal to  $d$  by 2 when we write this in terms of  $d$  by 2 into  $\Delta l$  by  $d$ .

So finally we get the  $h_f$  as  $h_f$  is equal to  $f l v$  square by  $2g d$ . So here  $\Delta l$  is replaced by the length which we consider. Now we have derived the Darcy Weisbach equation by considering the momentum principle and the Bernoulli equation between section 1 and 2 and then this equation is applicable for most of the fluid flow for both laminar and turbulent conditions and in the case of deep flow it is essential that flow should be the

circular in cross section it can be any kind of pipe flow and the Buckingham phi equation is applicable to both laminar and turbulent conditions.

So this Darcy Weisbach equation as in the basic equation and the fundamental equation under very much use decoration as for as the head loss calculation with respect to friction loss between two sections in any kind of the pipe line problems or pipe flow systems.

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- For Hagen-Poiseuille flow, we have,

$$V = \frac{\Delta P D^2}{32 \mu l}$$

- Head loss,

$$h_f = \frac{p_1 - p_2}{\rho g} = \frac{\Delta P}{\rho g} = \frac{32 \mu V L}{D^2 \rho g} = \frac{f L V^2}{2 g D}$$

- Hence,

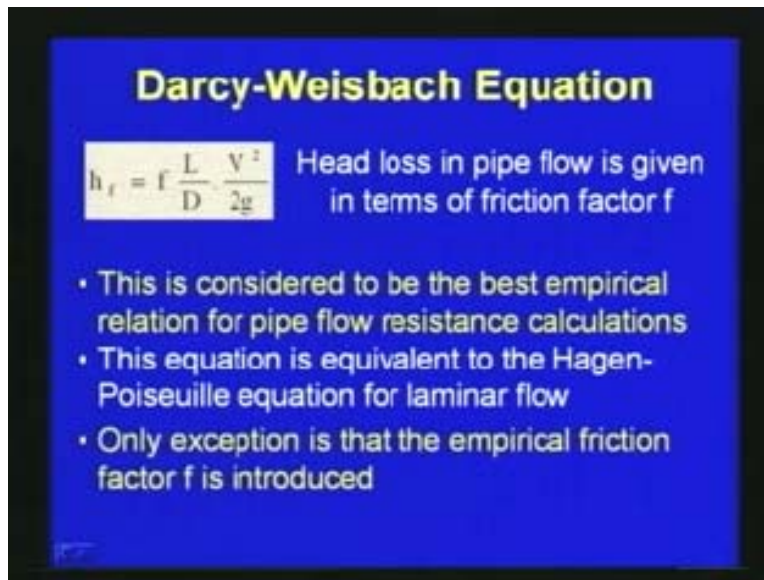
$$f = \frac{64}{Re}$$

So now the equation which is considered for Hagen-Poiseuille flow as per as laminar flow condition which we considered earlier, for Hagen-Poiseuille flow we can write  $v$  is equal to  $\Delta p d^2$  by  $32 \mu l$ , where  $\Delta p$  is the pressure difference  $d$  is the diameter  $\mu$  is the coefficient of dynamic viscosity and  $l$  is the length. So  $v$  is equal to  $\Delta p d^2$  by  $32 \mu l$  and head loss, we can represent as  $h_f$  equal to  $p_1$  minus  $p_2$  by  $\rho g$  that is equal to  $\Delta p$  by  $\rho g$ . That is equal to by using these equations here  $32 \mu$  here by  $d^2$  into  $\rho g$  that is equal to  $f l v^2$  by  $2 g d$ .

As far as laminar flow condition is concerned we can say that friction factor  $f$  is equal to  $64$  by  $Re$ , where  $Re$  is the known number say  $\rho d$  by  $\mu$  as per as the pipe flow constraints. Finally, we get the coefficient  $f$  is equal to  $64$  by  $Re$  for known number considering the Hagen-Poiseuille flow as in the case of laminar flow conditions. That way we can show that the loss is Buckingham equations which we have derived is valid

for laminar flow as well as less treble flow conditions. Starting from this for the earlier case, consider the Bernoulli's equations and the momentum theorem we got the Darcy Weisbach equation. Similarly the by consent the Hagen-Poiseuille flow also we get the same expression  $h_f$  is equal to  $f l v^2$  by  $2 g d$ .

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**Darcy-Weisbach Equation**

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

Head loss in pipe flow is given in terms of friction factor  $f$

- This is considered to be the best empirical relation for pipe flow resistance calculations
- This equation is equivalent to the Hagen-Poiseuille equation for laminar flow
- Only exception is that the empirical friction factor  $f$  is introduced

Now, let us discuss more details about this Darcy Weisbach equation and then the coefficient friction, since in most of the pipe flow systems one of the most important aspect as far as flow is considered is the coefficient of friction. So this  $h_f$  is equal to  $f l v^2$  by  $2 g d$  the head loss equation is given as Darcy Weisbach equation. Here we can see that the head loss is given in terms of the friction factor  $f$ . You can see that the head loss is directly proportional to the friction factor  $f$  and directly proportional to the length of the pipe and that is proportional to the square of the velocity average velocity and inverse proportional to the diameter  $d$ .

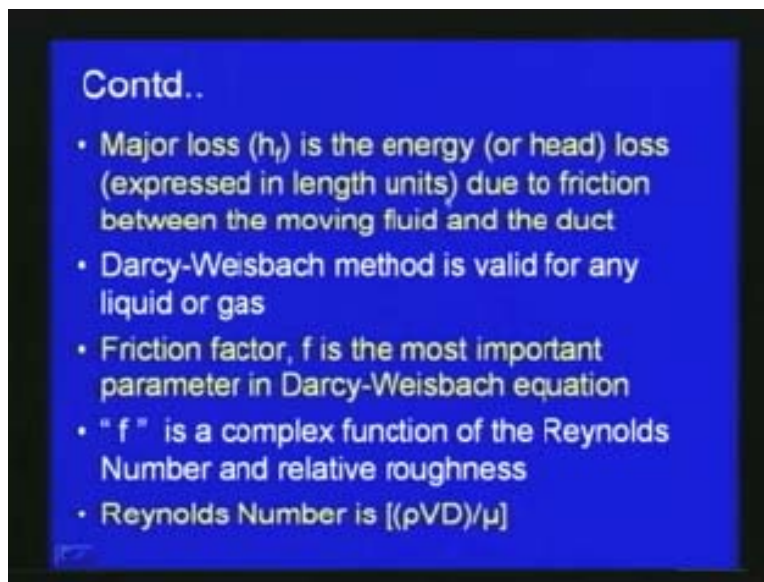
With respect to this Darcy Weisbach equation we can calculate the head loss, for any kind of pipe process systems. We can calculate the head loss by using the Darcy Weisbach equation in terms of the friction factor  $f$  in terms of the length and velocity square and with respect to the diameter of the pipe. This Darcy Weisbach equation is considered to be the best empirical relationship for pipe flow resistant calculation. In



most of the pipe flows which we considered, we have to calculate the head loss with respect to the friction. So this equation the Darcy Weisbach equation is one of the most poiseuille equations as far as head loss is considered and this equation is equivalent to the Hagen poiseuille equation for laminar flow as we are shown.

The last light only exception is that the empirical friction factor  $f$  is introduced here. So with respect to this here, we can see this empirical friction factor is  $f$  is one of the important aspects as far as Darcy Weisbach equation is considered. So we have say if  $f$  depend upon various parameter like material of the pipe and then flow conditions and then over the pipe the various other parameters with respect to whether the material at the pipe etc. Here by considering Darcy Weisbach equation, we have to see the friction coefficients or friction factor as far as the considered pipe flow.

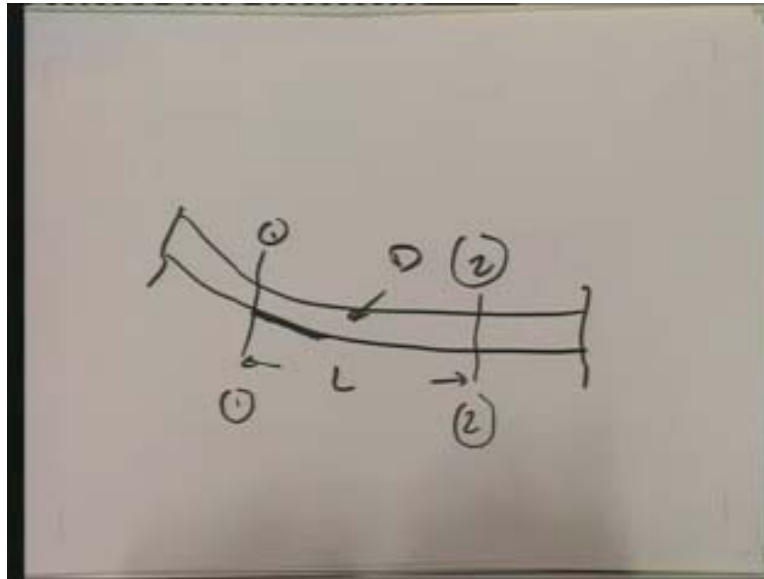
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Here, with respect to the Darcy Weisbach equation  $h_f$ , the major loss is the energy or head loss  $h$  first the length units due to the friction between the moving fluid and the duct. So here what we consider is same with respect to the pipe flow which we considered. When we consider any kind of pipe flow like this, so here same between 2 sections so here the head loss between if  $d$  is the diameter of the pipe and  $l$  is the length between this. Here by considering you can see that the head loss between say section 1 1 2, 2 2 2.



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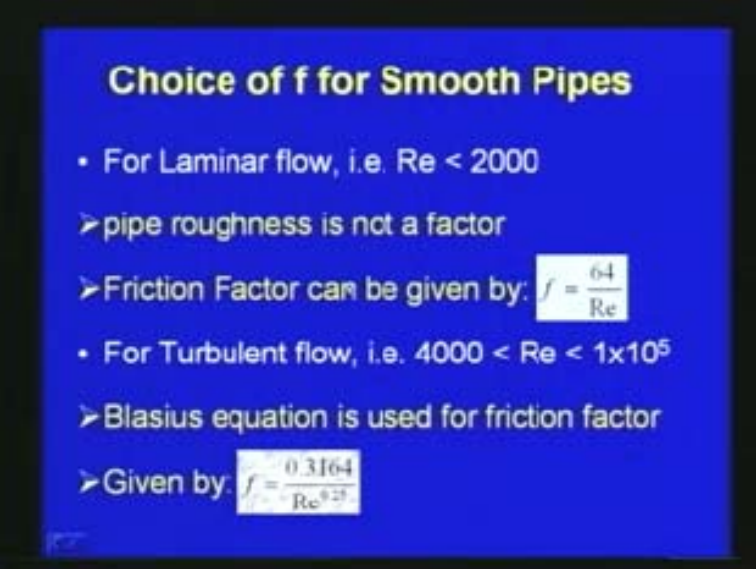


We can see that here the friction factor is most of the important parameter and then we get the energy loss, due to the friction between in  $\mu$  in fluid and then deduct in terms of as in the length unit. So friction factor  $f$  is the most important parameter in the Darcy Weisbach equation and  $f$  is complex functional of the Reynolds number and relative roughness. So, various experiments are conducted with respect to various pipe flow systems. It was shown that this friction factors  $f$  is depend upon the relative roughness of the pipe that is pipe material and it whether it is smooth or rough. So  $f$  is the friction factor depends upon the pipe materials roughness and also the Reynolds number with respect to the flow conditions. So  $f$  is functional of Reynolds number and relative roughness and Reynolds number for pipe flow is  $\rho u \text{ due by } \mu$ .

So now depending upon the case whether the flow condition and also the pipe material then whether the pipe is smooth or rough, we have to find this friction factor.

For example, now let us consider the various choices of  $f$  for smooth pipe for laminar flow that means the Reynolds number is less than 2000 is a view in the calculation we will not have to worry about the friction factors; pipe reference is not a factor.

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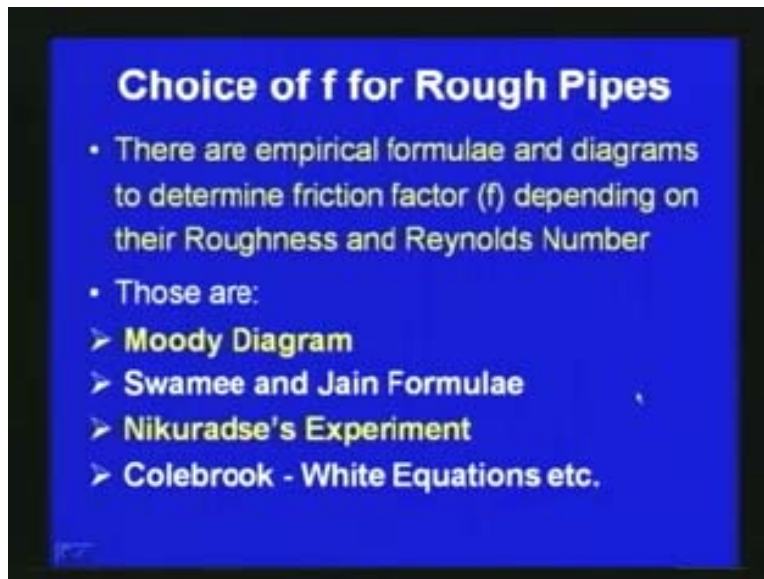
**Choice of f for Smooth Pipes**

- For Laminar flow, i.e.  $Re < 2000$ 
  - pipe roughness is not a factor
  - Friction Factor can be given by:  $f = \frac{64}{Re}$
- For Turbulent flow, i.e.  $4000 < Re < 1 \times 10^5$ 
  - Blasius equation is used for friction factor
  - Given by:  $f = \frac{0.3164}{Re^{0.25}}$

We can directly obtain the pipe reference the friction factor, directly calculate physical to 64 by Reynolds number and for turbulent flow, up to this between four thousands to the into between the range of 10 to the power of 5 to 4 four thousands Reynolds number range, blasius calculated this friction factors equal to 0.3164 divide by  $Re$  to the power 0.25.

So like this various conditions is based upon various experiment and some empirical relationship, this friction factor has been calculated for various kinds of pipe flows for various flow, various pipe material and reference factor. So as far as laminar flow is considered we can directly get with respect of Reynolds number friction factor  $f$  is equal to 64 by  $Re$  and then as for turbulent flow is considered say number  $r$  is shown through experiment that is  $f$  is equal to 0.364 by  $r$  into the Reynolds number to the power 0.25.

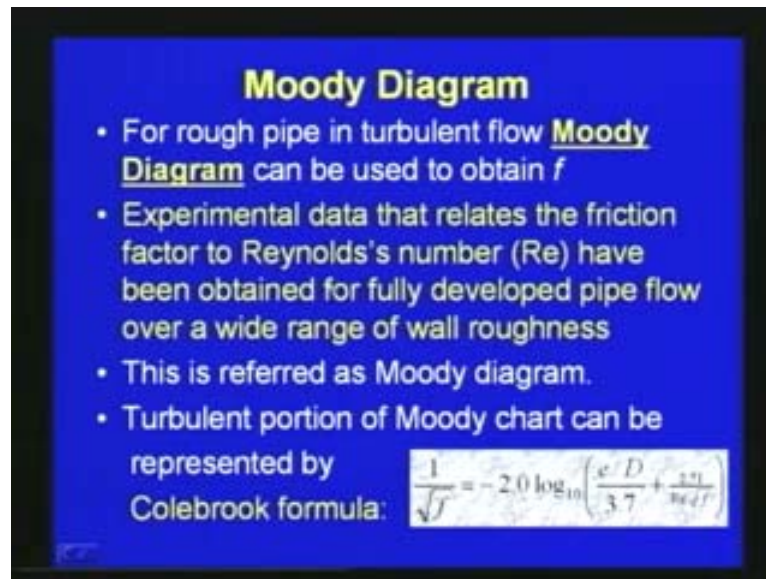
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But in other ranges also this equation is available; we will be discussing details about this friction factor and various relationships available in literature.

Now here the choice of  $f$  for rough pipes as I mentioned, there are empirical formula and diagrams to determine the friction factor  $f$  depending upon the roughness the Renolds number. So this is commonly used methodology include the moody's diagram, swamee and Jain formulae and nikuradses experiment results and Colebrook white equations. So these are some of the commonly used methodologies to estimate the friction factor  $f$ . We will discuss each of this methodology in details as far as the friction factor for roughness for considered.

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**Moody Diagram**

- For rough pipe in turbulent flow **Moody Diagram** can be used to obtain  $f$
- Experimental data that relates the friction factor to Reynolds's number (Re) have been obtained for fully developed pipe flow over a wide range of wall roughness
- This is referred as Moody diagram.
- Turbulent portion of Moody chart can be represented by Colebrook formula:

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10} \left( \frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right)$$

First let us see the moody diagram. This moody diagram has been derived by conducting large number of experiments at various flow conditions of fluid, various Renolds number and also fluid through various dimension diameter pipes and various smooth, rough and different kinds of roughness gives a large number of experiments were conducted and this moody diagram has been derived.

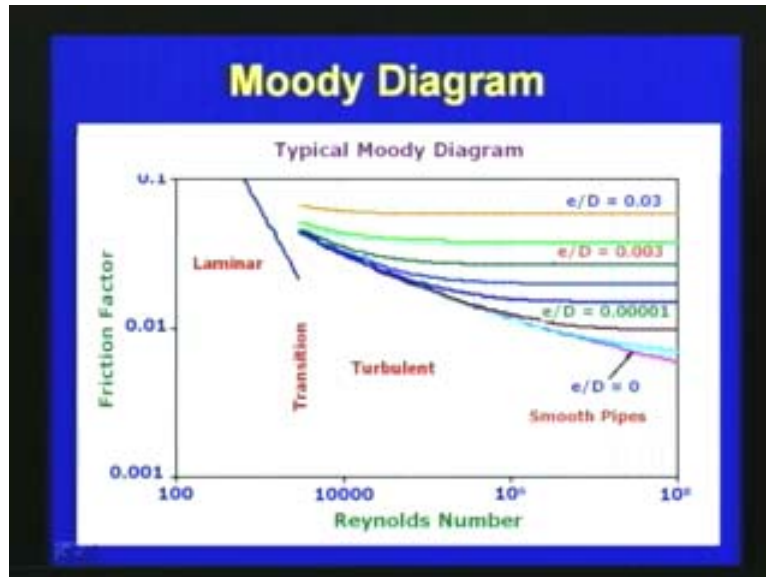
So this moody diagram is used for rough pipe in turbulent flow condition. We use the moody's diagram to obtain the re friction factor  $f$ . This moody's diagram is obtained through experimental data that relates the friction factor to Renolds number and then it is obtained for fully developed pipe flow over a wide range of wall roughness.

So from the moody's diagram depending upon the Renolds number what kind of same the turbulent say it whether it is transmission is turbulent, deferent ranges of turbulent flow we can obtain the friction factor with respect to the wall roughness friction factor is we can directly obtain to do the other calculation.

The turbulent portion of moody chart we can represent by the Colebrook formula even as  $\frac{1}{\sqrt{f}}$  is equal to minus 2 log 10  $\epsilon$  by  $d$  divide by 3.7 plus 2.51  $Re \sqrt{f}$ , where  $f$  is friction factor,  $r$  is the Renolds number is the reference height  $d$  is the diameter  $\phi$ .

So this turbulent portion of moody chart experimentally shown these values and also it can be verified that fluid values in the moody chart are very similar to the Colebrook formula even by these equations.

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So here this line shows the moody's diagram here the x axes the Renolds number is represented and on this access the friction factor is given and here the curves are given for various e by d ratio where e is the represent height the relative reference e by d is the diameter e by d is equal to 0.03 like that for various values, the roughness for various roughness are e by d issues and the Renolds number the friction factories even here.

So there is more pipe range whenever it is almost the e is almost 0 that means smooth pipes so here e by d is 0 that is the rearmost here and then we can consider that depending upon the Renolds number we can obtain the friction factor with respect to various ratios of e by d. We can obtain this pair wall this line for lamina flow and this range is for the transition. Actually we can see that moody diagram is commonly used for to find out the friction factor for turbulent for region is that is where we use this f factor friction factor commonly. So it is given for various scales are given for various e by d ratio as shown in this slide and then as a mentioned the various methodologies are available.

First one which we discussed is the moody's diagram and then log number of other relationship same to obtain the friction factor are available in the tracker. So few of this relationship we will discuss here.

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**Swamee and Jain Formulae**

- Empirical formulae (Jr. of Hydraulic Div., Proc. ASCE, pp 657-664, May, 1976)

$$f = 1.325 \left[ \ln \left[ 0.27 \left( \frac{e}{D} \right) + 5.74 \left( \frac{1}{Re} \right)^{0.9} \right] \right]^2 \quad \begin{matrix} 10^{-6} < e/D < 0.01; \\ 5000 < Re < 3 \times 10^8 \end{matrix}$$

$$h_L = 1.07 \frac{Q^2 L}{g D^5} \left[ \ln \left[ \frac{e}{3.7 D} + 4.62 \left( \frac{\nu D}{Q} \right)^{0.9} \right] \right]^2 \quad \begin{matrix} 10^{-6} < e/D < 0.01; \\ 5000 < Re < 3 \times 10^8 \end{matrix}$$

$$Q = 0.965 \left[ \frac{g D^5 h_L}{L} \right]^{0.54} \ln \left[ \frac{e}{3.7 D} + \frac{3.17 \nu^{0.4} L}{g D^5 h_L} \right] \quad Re > 2000$$

$$D = 0.66 \left[ e^{1.47} \left( \frac{L Q^2}{g h_L} \right)^{0.75} + \nu^{0.4} Q^{0.4} \left( \frac{L}{g h_L} \right)^{0.54} \right]^{0.33} \quad \begin{matrix} 10^{-6} < e/D < 0.01; \\ 5000 < Re < 3 \times 10^8 \end{matrix}$$

Next one is swamee and jain formula, these are the some of the empirical relationship derived by this swamee and jain by depositing the large number of experiments .Here swamee and jain drawn that f is the friction factor is f is equal to 1.325 natural log 0.27 in the e by d plus 5.74 into 1 by re to the power 0.9 over to the power minus 2 as shown this equation and this is valid friction factor this is wide in the range of e by d ratio 10 to the power minus 6 to 0.01 and the Renolds number range of five thousands to 3 into 10 to the power 8.

For this range this equation friction factor is valid as derived by the swamee and jain and then head loss directly we can obtain hl is equal to 1.07 into fuel square l divide by g in to d the power 5 and natural log e by 3.7 d plus 4.62 into new d by q to the power 0.9 what the power minus 2 where v is the discharge are the pipe and diameter and new is the coefficient sky metric and viscosity is the reference height, the pipe line thereon and l is the length which is considered.

So this equation is the head loss here is the same range with respect to a factor here. So this equation is valid at the range of  $e$  by  $d$  ratio of 10 to the power minus 6 to 0.01 and inverse number range of five thousands into three into 10 to the power 8 and then we can observe Jain derived the equation for discharge.

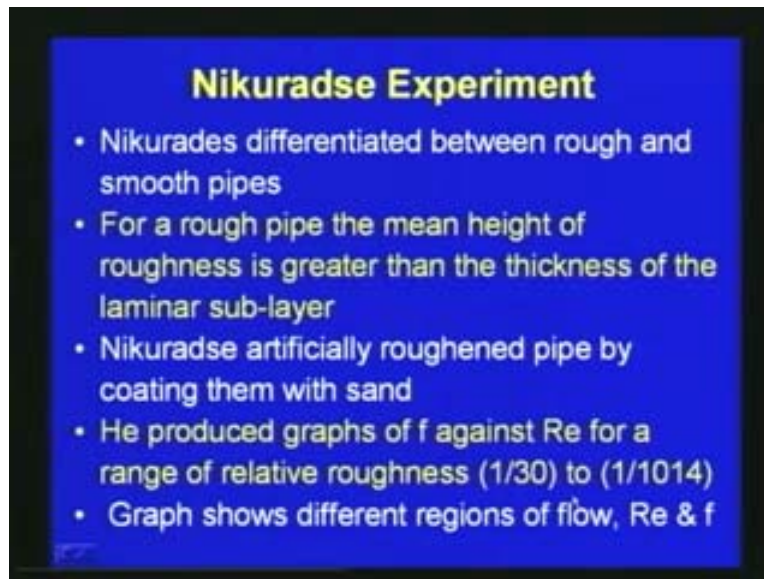
So discharge  $q$  is equal to minus 0.965  $g$  into  $d$  to the power 5  $h_l$  divided by 1 to the power 0.5 natural log  $e$  by 3.7  $d$  plus 3.17 new square into 1 divide by  $g d$  queue into  $h_l$  over to the power 0.5. So this equation for the discharge is valid for inverse number greater than 2 thousands as given by swamee and jain and here is the reference site and new is the kina metric viscosity and then they also got to design a pipe, the derived you can see that while designing the pipe for the given discharge it depends upon various parameters.

So some the equations which we swamee and jain derived based upon this equation they got an expression friction directly derived and expression for diameter for the pipe so  $d$  is equal to 0.6  $d$  to the power 1.25  $l q$  square by  $g h l$  to the power 4.75 plus new  $q$  to the power 9.4  $l$  by  $g h l$  in to the power 0.2 to the power 0.04. Here,  $h_l$  represent to head loss this  $h_l$  represent the head loss, so long as I mentioned this relationship derived by based upon log number experiment and then using some of the available empirical relationship already available relationship, various hagnes derived from number equations as far as the frictions factors, head loss and also discharge over the design a pipe find out the diameter of the pipe various relationship are derived.

So this equations shows the relationship as for as friction factor, head loss discharge and diameter of the pipe is concerned derived by swamee and jain and then in the literature some of the other important relationships as given by nikuradse's through his experiments. He also produced some charts with respect to the friction factor by relating to the Renolds inverse number.



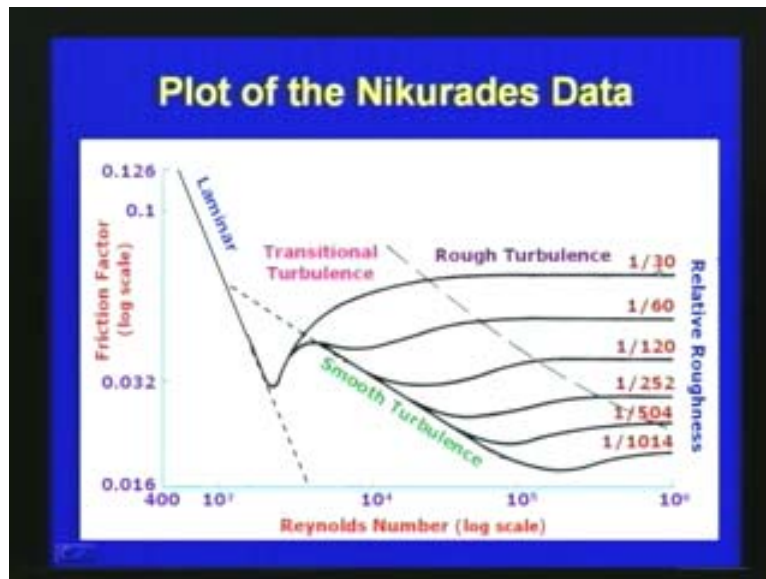
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So Nikuradse's conducted large number of experiments for rough and smooth pipes, number of difference, kinds of pipes and different materials are shown but for a rough pipe, the mean height of the roughness is greater than the thickness of the laminar sub layer. So Nikuradse's through his experiment showed that for rough pipe, mean height of roughness is greater than the thickness of the laminar sub layer and he conducted by Nikuradse's conducted all these experiments by artificially referring the pipe by coating them with respect design. So the laboratory he produced number of section the different kinds of pipes by sand coating, artificially roughening the pipe by sand coating and then he produced graph for  $f$  versus Reynolds number for  $f$  against the friction factor against Reynolds number for range of relative roughness 1 by 30 to 1 by thousand and fourteen.

So various ranges of relative roughness Nikuradse's produced the graph for  $f$  versus that means friction factor  $f$  is the Reynolds number stating from 1 by 30 relative reference 1 by 30 to 1 by 1000 and 14. So this graph shows different regions for various flows. So here this is the graph derived by Nikuradse's with respect to large number of experiments he carried out in the laboratory. Here this is very similar to the Moody's diagram which we have seen earlier. So here friction factor on log scale is put on this is on log scale and the y axes and Reynolds number, on log scale is new on the x axes and then relative roughness given here like  $\frac{e}{d}$  and  $\frac{e}{Re}$  by  $d$  in  $e$  is the roughness height.

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So 1 by 31 by 60 like that for various relative roughness and Nikuradse's produced this graph and here this is for rough turbulence on and here is this region is for transactional turbulence on and here this is smooth turbulence smooth by smooth by and this is the laminar array. So after conducting large number of experiments with respect to various flow conditions, with respect to various pipe materials, with respect to various roughnesses nikuradse's produced this graph. So from this graph also we can obtain then friction factor for various Renolds number and also various relative roughness.

So first we discussed moody diagram we have seen that swamee jain formula and now third one is the Nikuradse's plot and next one is the Colebrook equations. Colebrook derived some of the relationship with respect to the friction factor and Renolds number.

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**Colebrook Equations for  $f$  ( $Re > 4000$ )**

- Smooth pipe flow:  $\frac{1}{\sqrt{f}} = 0.86 \ln(Re \sqrt{f}) - 0.8$
- Transition zone:  $\frac{1}{\sqrt{f}} = -0.86 \ln\left(\frac{e}{3.7D} + \frac{2.51}{Re \sqrt{f}}\right)$
- Completely turbulent zone:  $\frac{1}{\sqrt{f}} = -0.86 \ln\left(\frac{e}{3.7D}\right)$

**Hazen-William formula for  $f$**

- This is given by:  $f = \frac{1059}{C^{1.85} D^{0.02} Re^{0.15}}$
- $C$  is Hazen-William's coefficient (given later)

Here Colebrook showed that smooth pipe flow  $1/\sqrt{f}$  is equal to  $0.86 \ln(Re \sqrt{f}) - 0.8$ . So this is for smooth pipe flow and in transition shown that means between the smooth to rough that transition is shown that  $1/\sqrt{f}$  is equal to  $-0.86 \ln\left(\frac{e}{3.7D} + \frac{2.51}{Re \sqrt{f}}\right)$ , where  $e$  is the height of roughness and  $D$  is the diameter. And then for completely turbulent zone, this can be reduced to the condition for fully turbulent and rough zone. Here this is  $1/\sqrt{f}$  is equal to  $-0.86 \ln\left(\frac{e}{3.7D}\right)$  this can be approximately from this equation.

So this equation is called Colebrook equations for friction factor for smooth transition for completely rough and smooth type flow. As we have seen the moody's diagram is very similar to what is given by Colebrook equation. So Colebrook equation is also the most accurate kind of equation as far as friction factors is considered and another kind of equation is could hazen William formula for friction factor here.

This is given as  $f$  is equal to  $1059 / (C^{1.85} D^{0.02} Re^{0.15})$  where  $C$  is the hazen William coefficient, we will discuss about the coefficient later  $D$  is diameter  $Re$  is the Renolds number, diameter in millimeter here.

This  $f$  is given by the friction factor is given like this by Hazen-William formula and then another important formula using literature is Barr formula.

So here the Barr formula friction factor is given as  $\frac{1}{\sqrt{f}} = -4 \log_{10} \left[ \frac{e}{3.71D} + \frac{5.1286}{Re^{0.89}} \right]$ . Here also this is the relationship between the friction factors and relative roughness and the Reynolds number

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**Barr Formula for Friction Factor**

- This is given as:  $\frac{1}{\sqrt{f}} = -4 \log_{10} \left[ \frac{e}{3.71D} + \frac{5.1286}{Re^{0.89}} \right]$
- Out of the charts and equations the Moody's Diagram is proved to be the most accurate in determining the friction factor
- It is studied that the approximations by the Colebrook's and Barr's equations are very close to as given by Moody's diagram.

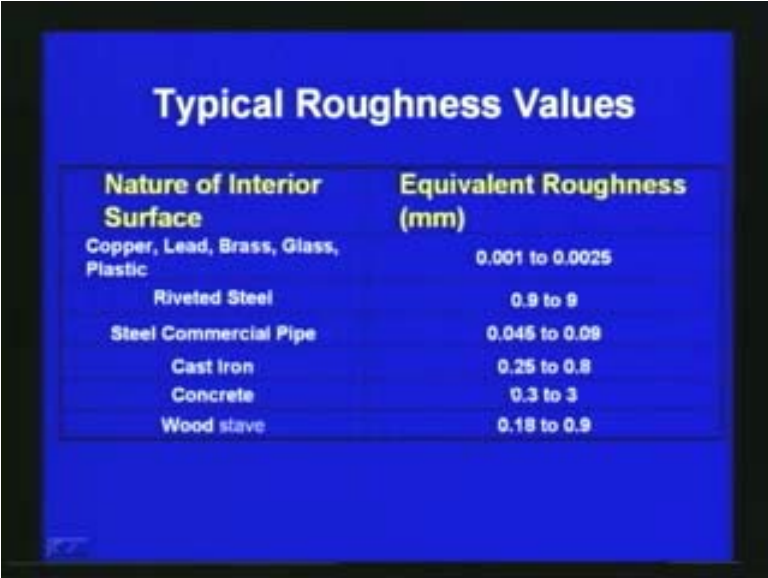
Like this there are number of few more formula relationships available for friction factor in turbulent flow conditions. Most of the equation is relates friction factors with respect to relative roughness and the array Reynolds number as we have seen Colebrook equation are embolic equation and also as soon by Nikuradse's. So out of the chart and equations and like moody diagram and the Nikuradse's charts, we can see that moody diagram is one of the most accurate in determining the friction factor so also we can that the approximation given by Colebrook and barr's equation are very close to the what is given in moody's diagram.

In most of the design pipe design analysis we can use this moody diagram to get the friction factor with respect to the Reynolds number and the relative reference since the moody diagram is prompt to be another most accurate relationship power chart of the

variable, the results are very similar to what we get from the Colebrook equations over the barr's equation.

In most of the designed pipes design is considered either we can use the moody diagram or we can use the Colebrook equation. Now we have seen this when we discussed about the major loss as far as pipe friction is considered. One of important factor is the roughness height which is inside in the pipe wall. So we can see that with respect to various materials we have seen that with respect to various materials this roughness height is changing. With respect to natural or interior surface and equivalent roughness in terms of millimeter is given here for various materials.

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Nature of Interior Surface	Equivalent Roughness (mm)
Copper, Lead, Brass, Glass, Plastic	0.001 to 0.0025
Riveted Steel	0.9 to 9
Steel Commercial Pipe	0.045 to 0.09
Cast Iron	0.25 to 0.8
Concrete	0.3 to 3
Wood stave	0.18 to 0.9

Typical roughness values are given, for example, copper, lead, brass, glass, and plastic. So **this** (34:55) of this material and the roughness vary from 0.001 to 0.0025 mm and then if the pipe is riveted steel it goes from large value like 0.929 mm and steel commercial pipe this roughness equivalent roughness varies from 0.045 to 0.09, cast iron pipe is concerned then we can see that increment reference high varies from 0.25 to 0.8 mille meter and concrete pipe depending upon the finishes.

So we can see that concrete or wood is considered, the roughness side or whether the pipe is smooth or pipe is rough depends upon the finishes given to the material. Here for

concrete pipe this equivalent roughness vary from 0.323 millimeter and wood stave we considered this varies from 0.18 to 0.9 .

So depending upon the material considered and the finishes is given and we can see that the roughness height changes. So equivalent roughness height changes and accordingly we can see that friction factor all those changes as given in the various relationships various methodology which we discussed like moody diagram is Colebrook equations like depending upon the relative roughness the friction factors changes and according to friction factor changes the head loss changes.

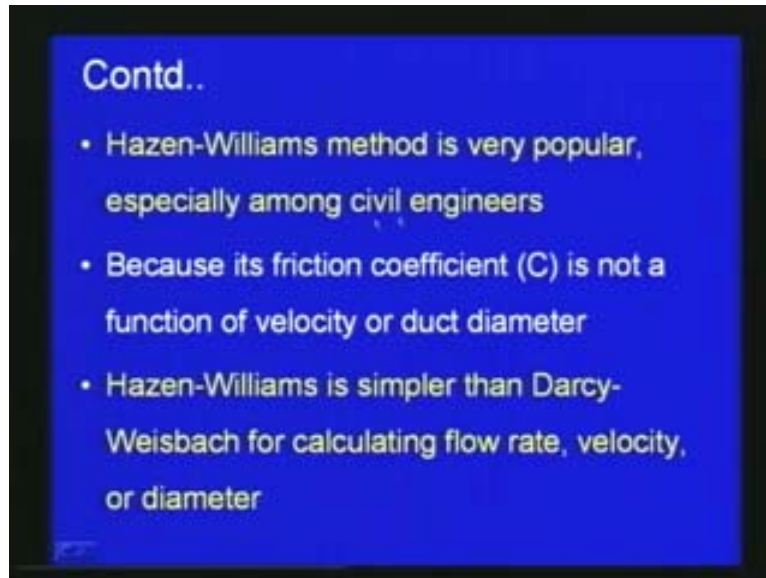
All these important aspects we have to consider in the design of pipe flow system design of the pipes considering the considered typical system and now, we can see that one of the most commonly used equation for the pipe design other than the Colebrook equation which is obtain the re friction factor of ten by Colebrook other than this one of the commonly use this equation hazen Williams equation. So hazen William equation for velocity is given as  $v$  is equal to  $k$  in to  $c$  in to  $r$  s to the power 0.63 s to the power 0.54. So here this equation is valid for water at temperatures typical city water supply system range from 4 to 25 degree centigrade.

So here  $s$  is equal to  $h_f$  by  $l$   $h$  is with respect to head loss  $h_f$  by  $l$  and  $q$  is then the discharge is given as  $V$  velocity is  $v$  into  $a$  and  $r_j$  is the if  $d$  by four that is with respect to the  $a$  by  $p$  that means we have to  $d$   $s$  obtain by  $r_h$  is equal to  $d$  by 4 for circular pipe and  $k$  is a unit conversion factor as far as hazen Williams formula is considered. so in the ps system  $k$  is equal to 1.318 and si system it is 0.85 and  $r_h$  is the hydraulic radius as discussed. So hazen William's equation is one of the formula equations as for as pipe flow design is considered other than the Darcy Weisbach equation and Colebrook equations which we discussed initially.

But Darcy Weisbach equation is much more accurate then the hazen William equation. So hazen Williams method is one the popular methodology for pipe design among civil engineers because its friction coefficient this  $c$  here this coefficients  $c$  is not a functional velocity or duct diameter, so in literature we can see that various values for  $c$  are given.

So and Hazen Williams equation is similar to that is the Weisbach equation for calculating the flow rate velocity or diameter depending upon the which parameter we are calculating for which we are designing, so Darcy Weisbach equation is one of the most commonly equation other than that Hazen Williams equations is also using in the pipe design.

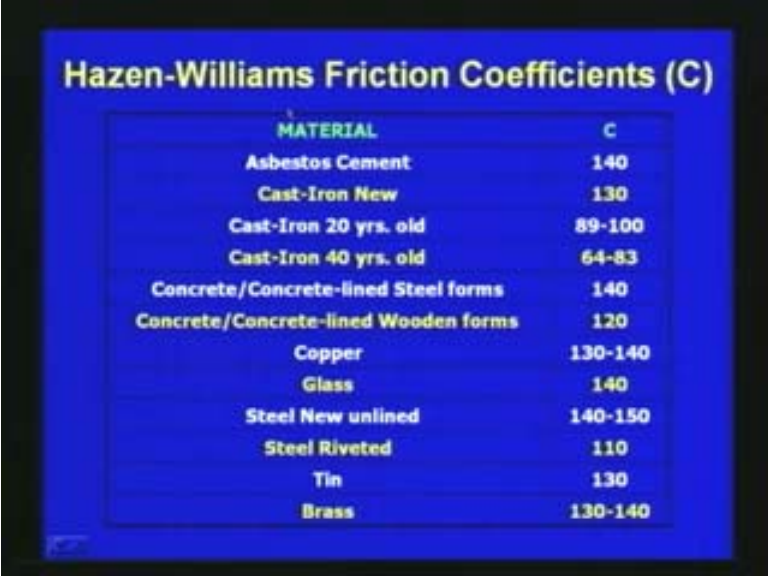
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So, the Hazen Williams friction factor which we discussed depends on the material, we can see in literature various values are given for C like asbestos cement C is equal to 140, cast iron 130, whether it is new old say twenty years forty years old C changes like this concrete lined steel pipe system it would be about 140 or lined wooden forms C to 120, or a copper pipe, copper material is concerned 130 to 140. Like this we can see this factor C as far as Hazen Williams equation is.



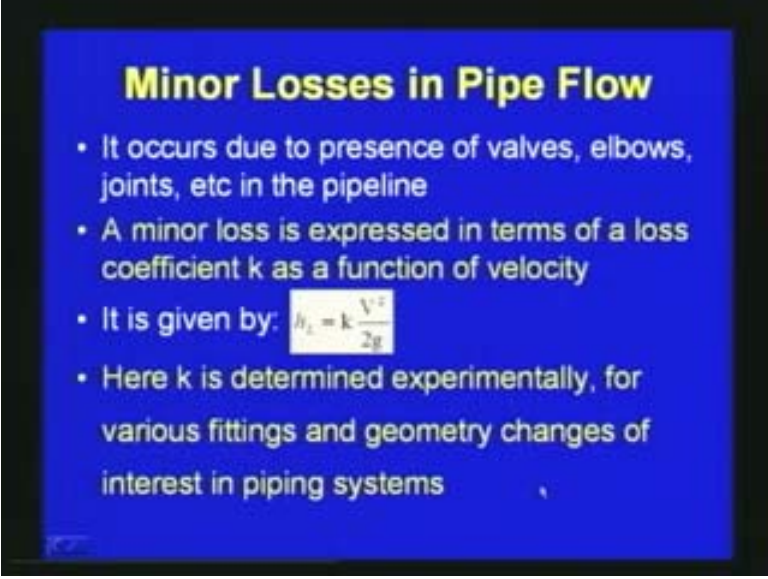
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MATERIAL	C
Asbestos Cement	140
Cast-Iron New	130
Cast-Iron 20 yrs. old	89-100
Cast-Iron 40 yrs. old	64-83
Concrete/Concrete-lined Steel forms	140
Concrete/Concrete-lined Wooden forms	120
Copper	130-140
Glass	140
Steel New unlined	140-150
Steel Riveted	110
Tin	130
Brass	130-140

So far we have discussed the major loss in a pipe systems is considered is mainly with friction loss  $h_f$  for  $h_l$ , what we considered the head loss for the friction with respect to head long friction. One of the most commonly used equation is the Darcy weisbach equation which we discussed only things is that in Darcy Weisbach equation we have to find out the friction factor and other commonly used equation is the hazen williams equations for the pipe design. So for pipe design is considered we can you say the Darcy Weisbach equation as hazen Williams equations or some of the other methodology is other equation available in literature.

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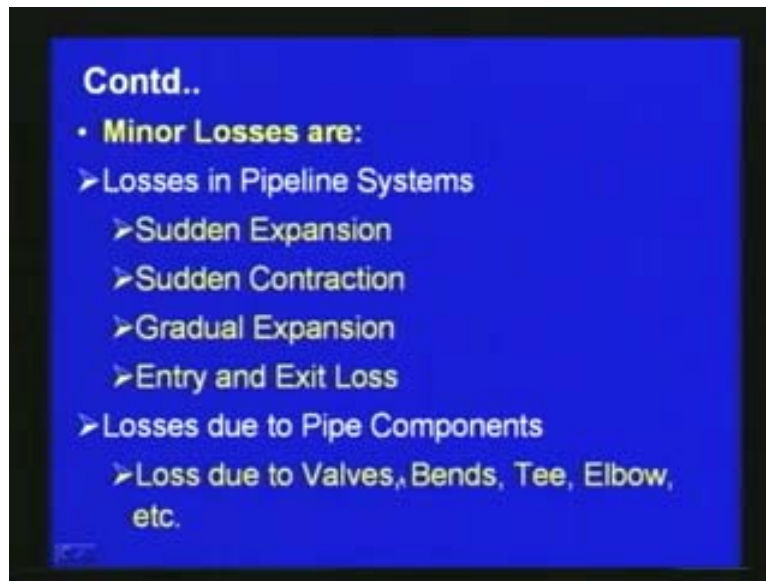
**Minor Losses in Pipe Flow**

- It occurs due to presence of valves, elbows, joints, etc in the pipeline
- A minor loss is expressed in terms of a loss coefficient  $k$  as a function of velocity
- It is given by: 
$$h_L = k \frac{V^2}{2g}$$
- Here  $k$  is determined experimentally, for various fittings and geometry changes of interest in piping systems

So that is as far as the major loss is considered what we discussed so far. Now you will discuss in detail about the minor process in pipe flow, we have seen earlier that pipe flow process are considered there are major loss due to the friction loss and minor loss, losses due to its condition like expansion contraction or due to various is pipe fitting s like bans than t junction harm, various connections there will be minor loss.

Now we will discuss in details about the minor loss in pipe flow as we discussed in the last lecture the minor loss occur due to the presence of valves, elbows, joints are contracts functions etc in the pipeline. Generally, a minor loss is especially in terms of a loss coefficient  $k$  as a function of the velocity. So here seen this major loss already so like that in minor loss also we represents the minor loss in time of the velocity as a functional of velocity a multiply by a coefficient  $k$  so generally  $h_L$  the head loss for minor loss is concern  $h_L$  is equal to  $k$  in to  $v$  square by  $2g$ .

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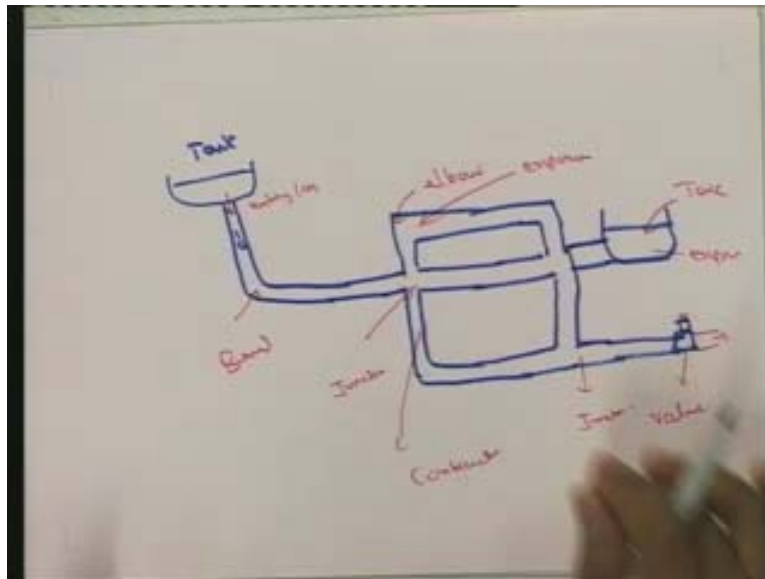


Where  $v$  is the average velocity,  $k$  is the coefficient for minor loss is the acceleration due to gravity when  $k$  is determined experimentally for various fittings and geometric changes of interest in piping system. As we discussed earlier depending upon a problem there would be different kinds of joint are different kinds of contract expansion (42.12) junction etc.

So depending upon the condition same for various depend upon the material which is use for pipe construction we can have varies of this  $k$ , so either it can determine through experiment. Sometimes depend upon the conditions manufacturing a give the values source also.

So now here we discussed various aspects of minor loss with respect to equation  $h_l$  is equal to  $k$  in to  $v$  square by  $2g$ . As we discussed the minor losses are in pipe line are consider minor losses can be due to sudden expansion, sudden contraction, gradual expansion, entry and loss exit loss and also losses the due to pipe components like loss due to valves, bends, tee, elbow etc.

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So as we have seen, for example if a connecting here the vary say time here and now if we are connecting, putting a pipe line like this with various branches and then various joints, you can see now in a such a system we may be distributing either for water supply or may be for various other kinds of network.

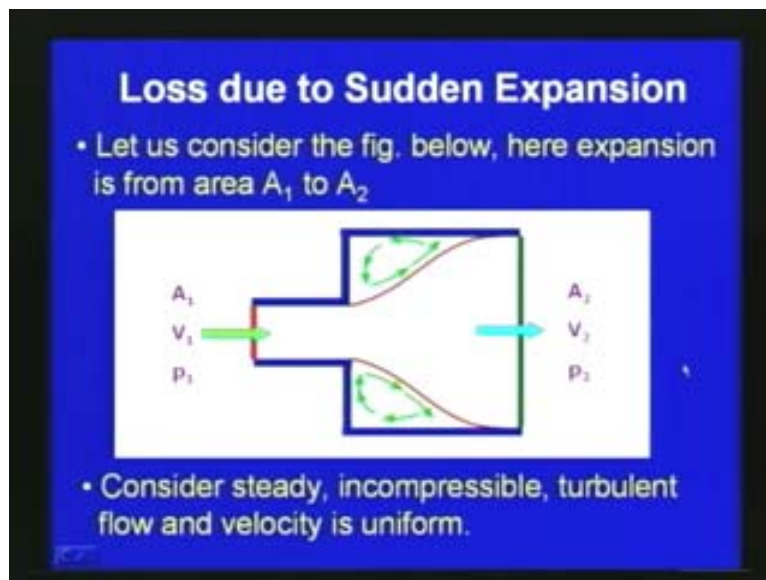
We can seen that we may be continuing the flow like this, so such a system in the pipe network like this we can see that here, may be connecting to another tank, so and here this flow may be continuing so we can see here the various connections and here if this is the tank or reservoir. So from here the flow takes place. So in such a system we can see that there can be water enters from the tank to the pipe so there is an entry loss, so here there is an entry loss and then here we provide that bench so there can be loss to be bench and then here we can see that there is a junction here. There we may give a junction, so junction loss and then we may provide an l board here over the can depend upon the pipe line diameter changes there can be expansions, so here also we can have expansions over here we can have contraction.

So like that and then now finally when now this pipe joins here then this tank then we consider that again expansion, so and here we provide there is wall and then flow continue here so this is a wall. So like this here there is a again a junction so there are

number of components there is such a flow system in there is number of components is including the expansion, contraction etc., exit then entry loss exit loss, so we have to consider all these losses as far as pipe flow is considered.

Now discussing in details, first let us consider the loss due to the sudden expansion. Let us consider the figure below, here you can see that a pipe is flowing, so here the pipe width is more diameters and then it is connected to pipe with log diameter. We can see that will be a sudden exponential like this.

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Flow is coming like this and then expands. Here let us consider 2 sections here section 1 and section 2 and section 1 area flow section is  $A_1$  and velocity average velocity  $V_1$  and pressure  $P_1$  and section 2 area flow section  $A_2$  and velocity is  $V_2$  and pressure is  $P_2$ . Let us consider the flow to be steady and fluid incompressible turbulent flow and velocity is uniform with respect to the assumption.

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Contd..

- Assuming shear force on pipe wall of short length between (1) and (2) is negligible and mean pressure  $p'$  of the eddying fluid in the expansion is almost equal to the pressure  $p_1$ ;  
From the Newton's equation of motion,  

$$p_1 A_1 - p_2 A_2 + p'(A_2 - A_1) = (p_1 - p_2) A_2 \quad \dots(1a)$$

$$= \rho V_2 (A_2 V_2) + \rho V_1 (-A_1 V_1)$$
 ie  $p_1 - p_2 = \rho V_2 (V_2 - V_1)$  (using Continuity Eqn)  

$$\dots(1b)$$

If you assume shear force on pipe wall of short length between 1 and 2 is negligible and mean pressure  $p$  dash of the eddying fluid in the expansion is almost equal to the pressure  $p_1$ . From Newton's equations motion we can write  $p_1$  into  $a_1$  minus  $p_2$  into  $a_2$  plus  $p$  dash into  $a_2$  minus  $a_1$  is equal to  $p_1$  minus  $p_2$  into  $a_2$  as in equation number 1.

So this we can simplify as this  $p_1 a_1$  minus  $p_2 a_2$  plus  $p$  dash  $a_2$  minus  $a_1$  equal to  $p_1$  minus  $p_2$  into  $a_2$ . So here we assumed that  $p$  dash equal to  $p_1$  that is why, we got  $p_1$  minus  $p_2$  into  $a_2$  this equation and then this with respect to change of momentum. We can write  $\rho v_2$  this is equal to  $\rho v_2$  into  $a_2 v_2$  plus  $\rho v_1$  into minus  $\rho v_1 a_1 b_1$ . So this is the momentum. Finally we get  $v$  is the continuity equation we can obtain  $p_1$  minus  $p_2$  is equal to  $\rho$  into  $v_2$  into  $v_2$  minus  $v_1$  as in equation number 1 b.

So here we consider the flow like this here one section and another is here. With respect to this equation Newton's equation of motion we got  $p_1$  minus  $p_2$  is equal to  $\rho v_2$  into  $v_2$  minus  $v_1$ . Now the energy equation if applied between section 1 and 2 we get  $v_1$  square by 2  $g$  plus  $p_1$  by  $g$  is equal to  $v_2$  square by 2  $g$  plus  $p_2$  by  $g$  plus  $h_l$ , where  $h_l$  is the head loss.

So here is an between this section to this section if  $h_l$  is the head loss we get  $v_1$  square by 2  $g$  plus  $p_1$  by  $g$  is equal to  $v_2$  square by 2  $g$  plus  $p_2$  by  $g$  plus  $h_l$ .

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**Contd..**

- The energy equation applied to section (1) and (2) with the head loss gives:

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + h_l \quad \dots(2)$$

- Solving for  $[(p_1 - p_2)/\gamma]$  from eqn (1) & (2) and from (2):

$$\frac{V_1^2 - V_2^2}{2g} = \frac{p_2 - p_1}{\gamma} + h_l$$

So now solving by  $p_1$  minus  $p_2$  by  $\gamma$  from this equation and if we use this equation 1 a and 1 b and we can write  $v_2$  square minus  $v_1$   $v_2$  by  $\gamma$  is equal to  $v_2$  square minus  $v_1$  square by  $2g$  plus  $h_l$ .

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**Contd..**

- So, head loss is,

$$h_l = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g} \left(1 - \frac{A_1}{A_2}\right)^2$$

- Loss coefficient,

$$k = \left(1 - \frac{A_1}{A_2}\right)^2$$

- It is obvious that head loss varies as the square of the velocity and it is true for all minor losses
- If  $A_2$  is extremely large (e.g., pipe opens to a reservoir),  $(A_1/A_2)=0$ , then  $k = 1$
- Here the complete K.E of flow is dissipated.

We obtain from equation using equation 1 a and 1 b and equation number 2. So finally we get the head loss  $h_l$  is equal to  $v_1$  minus  $v_2$  whole square by  $2g$ . That can be




represented as  $v_1^2$  by  $2g$  into  $1 - a_1/a_2$  whole square. So here this loss coefficient here this term this already in this head loss with represent is  $k$  into  $v$  square by  $2g$ . So if consider this  $v_1^2$  by  $2g$  has a function so we obtain here this loss coefficients  $k$  is equal to  $1 - a_1/a_2$  whole square. So here it is obvious that head loss various as this square of the velocity and it is true for all minor losses.

So here if  $a_2$  is extremely large like pipe opens to a reservoir we can see that is  $a_1/a_2$  if  $a_2$  is argue that 0 then by obtained  $k$  is equal to 1. So like that we can find out for various cases of exponential so with respect this  $k$  is equal to  $1 - a_1/a_2$  whole square. So where  $a_1$  is the rho section here at this section the smaller section and here expansion section rho section is  $a_2$ . So if  $a_2$  is extremely large, we have seen the complete kinetic energy of flow is dissipated as here the same when the flow is going from here this pipe flow is released to tank receiver so we can see that it is a completed expansion so total kinetic energy of the flow is decapitated here. So it is complete expansion. So this is the expression for the expansion. Similarly, if we consider the contraction loss due to sudden contraction let us consider the following figure.

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### Loss due to Sudden Contraction

- Let us consider the following fig.



Here, Head loss due to contraction is

$$h_c = \frac{(V_1 - V_2)^2}{2g}$$

- From continuity,  $V_0 C_c A_2 = V_2 A_2$ 
  - $C_c$  is the coefficient of contraction
  - $V_0$  is the velocity at section '0', i.e at "Vena Contracta"

Here this section 1, this is the large diameter then smaller diameter, so here the velocity is  $v_1$  at section 1 and here the velocity section 2 is  $v_2$  and then we can see that in the case of

contraction there will be a jet is formed then we can see that here as smaller section will be there which is called in contractor. So for loss due to contraction we can write  $h_c$  is equal to  $v_0^2 \text{ minus } v_2^2 \text{ r square by } 2g$  where  $v_0$  is velocity at this in a contractor and  $v$  is the velocity here at this junction. So we can show that  $h_c$  is equal to head loss due to contact is equal to  $v_0^2 \text{ minus } v_2^2 \text{ r square by } 2g$ , from continuity equation, you can write  $v_0$  the velocity here into  $C_c$  into coefficient contraction into  $a_2$  is equal to  $v_2 a_2$ , so  $C_c$  is the coefficient of contraction and  $v_0$  is the velocity at section here at in a contractor. So the head loss due to contract obtained is  $h_c$  is equal to  $v_0^2 \text{ minus } v_2^2 \text{ r square by } 2g$ .

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**Contd..**

- The Vena Contracta is the section of greatest contraction of the jet
- Now Head loss can be given as: 
$$h_c = \left( \frac{1}{C_c} - 1 \right)^2 \frac{V_2^2}{2g}$$
- **$C_c$  values determined by Weisbach:**

$(A_2/A_1)$	$C_c$	$(A_2/A_1)$	$C_c$
0.1	0.624	0.6	0.712
0.2	0.632	0.7	0.755
0.3	0.643	0.8	0.813
0.4	0.659	0.9	0.892
0.5	0.681	1.0	1.000

With respect to the vena contractor it is the section of the greatest contraction of the jet here. We can see that the jet here, the greatest contraction and hence the head loss can be written as  $h_c$  is equal to  $1 \text{ by } C_c \text{ minus } 1 \text{ r square into } v_2^2 \text{ square by } 2g$ . With respect to this appropriation here the head loss with respect to this we can write in terms of this coefficients of contraction  $h_c$  is equal to  $1 \text{ by } C_c$ , where  $C_c$  is the coefficients of contraction;  $h_c$  is equal to  $1 \text{ by } C_c \text{ minus } 1 \text{ whole squared into } v_2^2 \text{ square } 2g$ . Here you can see that the expression is in times of the velocity  $v_2$ . So Weisbach is calculated by with respect various ratios are  $a_2 \text{ by } a_1$  and here is calculation of the coefficient of contraction, so that is given the tracer here you can see here the 0.1  $C_c$  is point  $a_2 \text{ by } a_1$  is

0.1, 0.6. Like that various values are ambient calculated through experiments by Weisbach.

In the next lecture, we will be discussing more about the various losses; then we would discuss various aspects of the pipe flow design and then pipe flow system for various conditions.