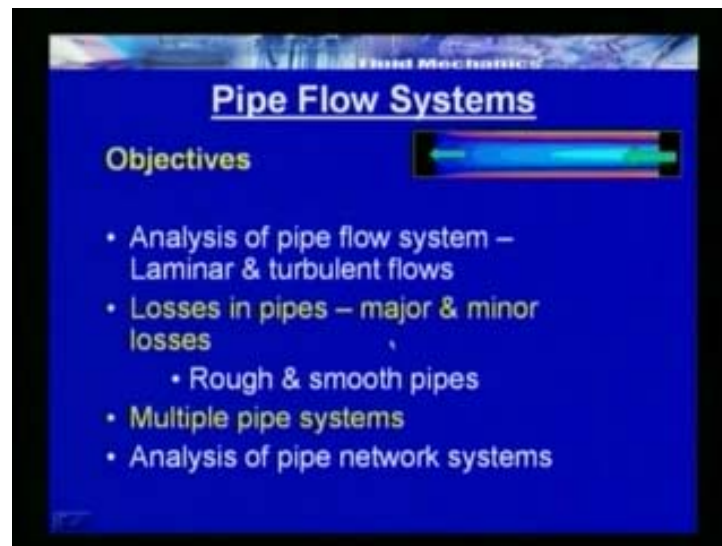


Fluid Mechanics
Prof. T. I. Eldho
Department of Civil Engineering
Indian Institute of Technology, Bombay

Lecture No. # 36
Pipe Flow Systems

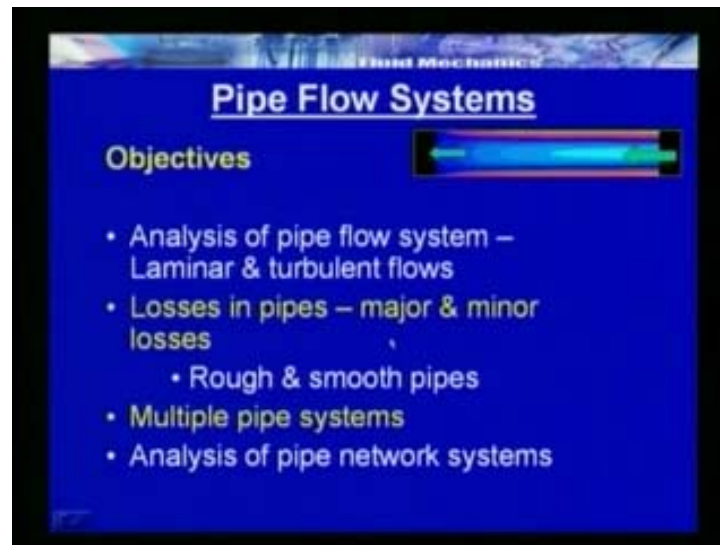
Welcome back to the video course on Fluid Mechanics. In today's lecture, we will discuss a new chapter - the Pipe Flow Systems. As all of you know, pipe flow is one of the most important, say, subjects in fluid mechanics. Since almost all, say way of life, we have to use, say some way or another way, the pipe flow; may be for water supply or may be for sewage flow or may be for say transport chemicals or a petroleum products, etcetera, number of applications are there for pipe flow systems.

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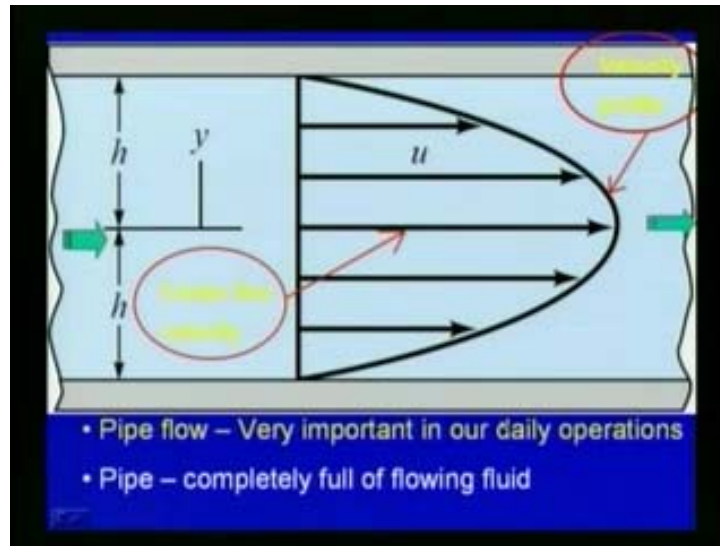
So, with this pipe flow system in mind here, so, the main objectives **in, the**, this chapter which will be discussing are - the analysis of pipe flow systems. So, we will be briefly discussing about the laminar pipe flow systems and turbulent pipe flow systems. Say earlier, we have already seen some aspect of this, but we will be discussing more about the laminar and turbulent flows in pipe systems, and then, we will discuss the losses in pipes - major and minor losses - and then, we will be discussing the various as far as type losses are concerned, the rough pipes, smooth pipes, all these things we will be discussing in details.

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And then, we will be discussing the multiple pipe systems - pipes in parallel, pipe in series and then branched pipe system, etcetera will be discussed in the... we will be discussing in the multiple pipe systems. And then, finally, we will be discussing about the analysis of pipe network systems. As we can see, that most of the water supply schemes or sewage networks schemes or whatever the other kinds of, say wherever the pipes are used, say in the system which we are discussing here, will be of full pipe flow, as in the case of water supply or the, say the pressured system, but sewage system is generally, say, not it is acting just like in the open channel system where the pressure is not coming till picture, but here, what we will be mainly discussing will be the pressure flows as far as pipe is concerned. So, with respect to that, we will be discussing about the analysis of pipe network systems.


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So now, let us see here this is a pipe flow. So, you can see that the flow is in this direction. So, say here, as we discussed the pipe is flowing full, so, it is always under pressure; so, the pipe flow is, you can see that if you plot the velocity variations, you can see that due to the nose tip condition on the wet periphery of the pipe, the velocity will be 0 here, and then, the velocity be maximum the center line as shown here. So, this is the typical velocity profile as far as pipe flow is concerned.

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Analysis of Pipe Flow

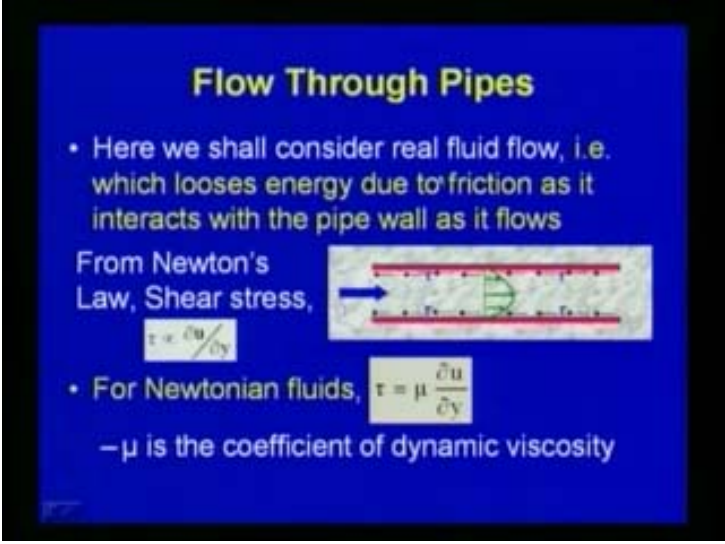


- This is the closed conduit flow of liquids under certain pressure
- Application in all phases of life
- It consists the study of
 - Different types of flow through pipes (e.g. laminar or turbulent)
 - Flow of different kind of liquids
 - Pipe networks and its components
 - Application and suitability conditions

So, as we have seen pipe flow is one of the most important system in fluid mechanics and this is essentially an internal flow system. So, the pipe flow which we discuss here is the closed conduit flow of liquids under certain pressure. So, as we discussed, there are number of applications in all phases of lives, and this pipe flow system is concerned, it consists the study of different types of flow through pipes; example, laminar or turbulent flows, and flow of different kinds of liquids, say it may be water, it may be chemical fluids or the say petroleum products or whatever it is, different kinds of fluids may be passing through the pipes.

So, depending upon the case, we have to discuss or we have to study the different kinds of liquid flows through the pipes, and then the pipe network, and these components, and then the application and suitability conditions for various cases.

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Flow Through Pipes

- Here we shall consider real fluid flow, i.e. which loses energy due to friction as it interacts with the pipe wall as it flows

From Newton's Law, Shear stress,

$$\tau = \mu \frac{\partial u}{\partial y}$$

• For Newtonian fluids,

$$\tau = \mu \frac{\partial u}{\partial y}$$

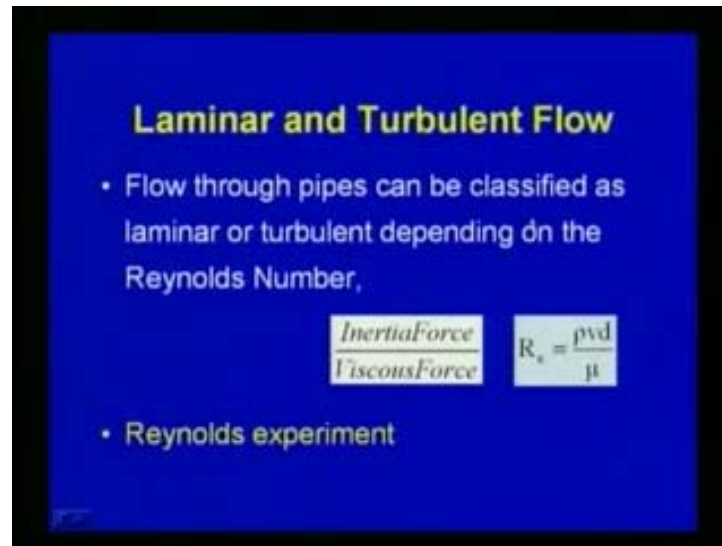
— μ is the coefficient of dynamic viscosity

So now, first, we will discuss in detail about the laminar flow systems in pipes, and then the turbulent flow systems in pipes, and then we will discuss about the losses in pipes - major loss and minor loss.

So, first let us see the pipe flow system with respect to the laminar flow conditions. So, here, we shall consider the real fluid flow; that means, when the fluid flow which loses energy due to friction, as it interacts with the pipe wall as it flows. So, this, if you consider the a Newton's law of shear stress, from the Newton's law of shear stress, we can write, we can see that the shear stress is proportional to the velocity gradient $\frac{\partial u}{\partial y}$ by

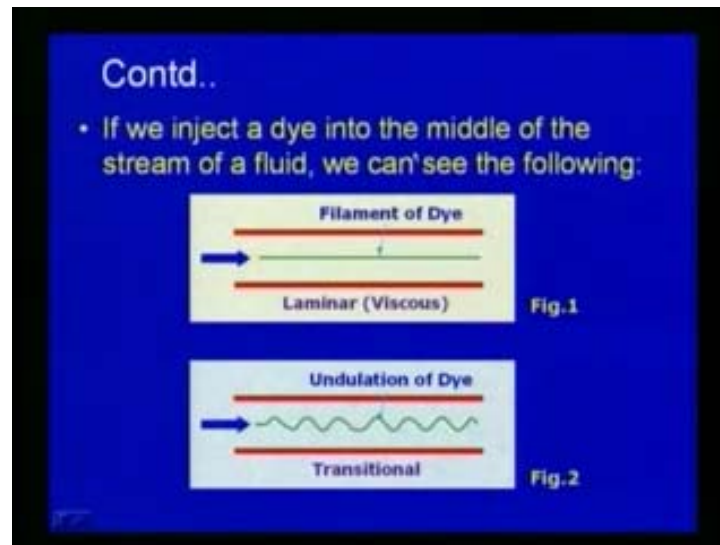
del y. So, here, this is the pipe flow, and then, if we consider say the flow in this direction, then you can see that due to the viscous effects, there effects there will be say shear stress, and in the other direction; so, this Newton showed that this **tau** is the shear stress is proportional to the velocity del u by del y and he proved that tau is equal to mu del u by del y, where mu is the co-efficient of dynamic viscosity.

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So, now, say the flow is concerned, since we are dealing with our real fluids, so, of course there will be the shear stress **is there** and then the inertia of force will be there. So, when you discuss the pipe flow with respect to the laminar and turbulent flow, as we discussed in earlier lectures, say the one of the most important parameter, the most important dimensional number which we have to, say see in each flow, is the Reynolds number. So, depending upon the Reynolds number, say Reynolds showed that the flow can be laminar or turbulent and he showed this through the Reynolds experiment. So, the Reynolds number which is defined as inertial force, the ratio of inertial force to viscous force, which we can define as far as pipe flow is concerned, R_e is equal to $\rho v d$ by μ - where v is the velocity; d is the diameter of the pipe; μ is the coefficient of dynamic viscosity. So, with respect to these, we can classify the fluid flow as in the pipe as the laminar flow or turbulent flow.

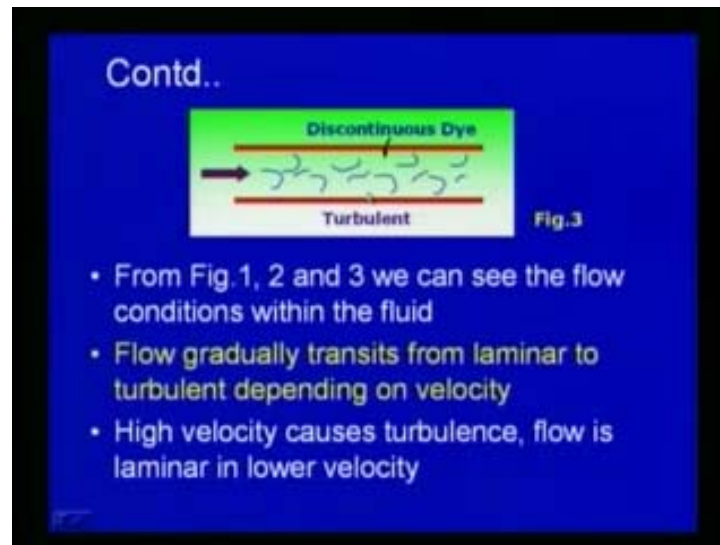
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So now, say this we can easily visualize. Say in a pipe flow, if we inject a dye in the middle of the stream of fluid, say if it is fully laminar, you can see that in the slide, so, this is the pipe and pipe the flowing fluid is there, and then, if we inject some dye at the middle, then you can see that the flow is laminar, then you can see there will be a filament of dye like this and it is say flowing like this just as symbol layers. So then, we say that the flow is laminar.

But if we increase further the velocity of the fluid flow through the pipe, then we can see that at certain stage, this the filament of dye which is vary as a thin layer, it is flowing. So, when the velocity increases at certain stages, we can see that there is disturbance takes place for these the dye and you can see that undulation of dye takes place, and then we can see that the flow changes from laminar condition to, say to finally when we keep on increasing the velocity, we can see that the flow become finally turbulent. So, this stage is so-called transitional stage as far as the flow in the pipe is concerned.

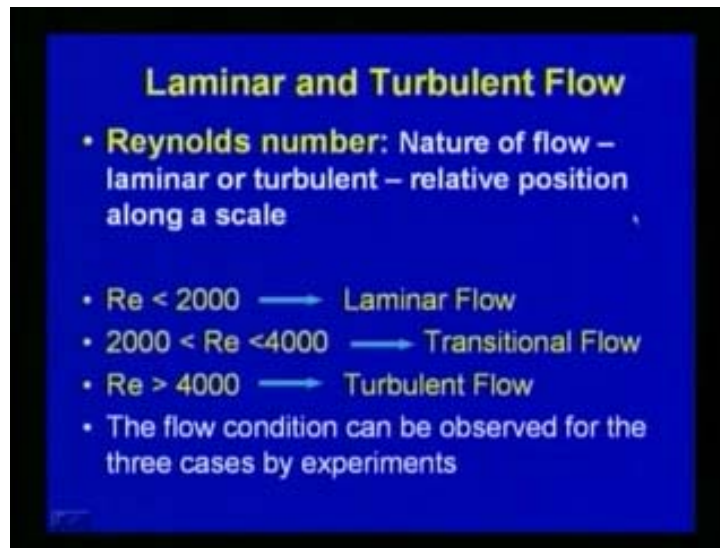
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So, finally after, the velocity is reaches at stages, we can see that the flow will be, say with respect to the dye, you can see that the dye particle mixed like this, and then there will be discontinuous here and there, and finally, total mixing takes place. So, this situation is the turbulent flow condition. So, from these figures 1 2 3, we can see that the flow condition with in the fluid. So, starting from the laminar, say there is a transitional stage and then it goes to the turbulent flow conditions. So, that means the flow gradually, so, when we keep on increasing the velocity, then we can see that the flow gradually transits from laminar to turbulent depending upon the velocity. So, if we keep on increasing the velocity, finally from the flow transits from the laminar to the turbulent stage.

So, here, you can see that when we increase the velocity, the velocity causes the turbulence or the mixing of the particles fluid particle, and then, say finally, the flow become completely develops to turbulent. So, **but in the**, as far as the laminar flow condition is concerned, the velocity is small, so that the flow is flowing in as a layered flow, and then, there is not much disturbance as far as the flow situation **is flow** in the pipe is concerned.

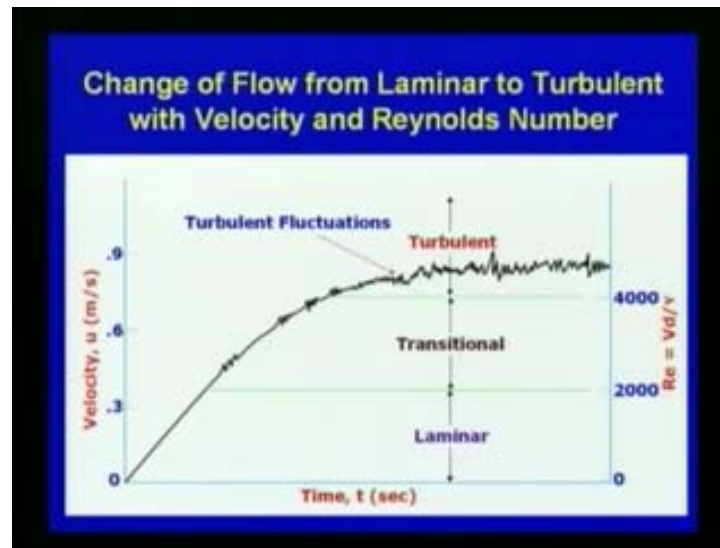
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So, Reynolds through his experiment showed that by using the Reynolds number, we can classify whether the flow in the pipe is whether it is laminar or turbulent. So, the Reynolds number shows the nature of flow in the pipe; so, we can classify according to a scale. So, generally, say when the Reynolds number is less than 2000, we call the pipe flow as laminar flow, and then, if the velocity is more than 2000, it is in the transitional flow; the flow is in the transition, and say, generally, in literature, you can see that this transitional flow is kept between the range of Reynolds number 2000 to 4000 and when the Reynolds number is more than 4000, the flow is fully turbulent.

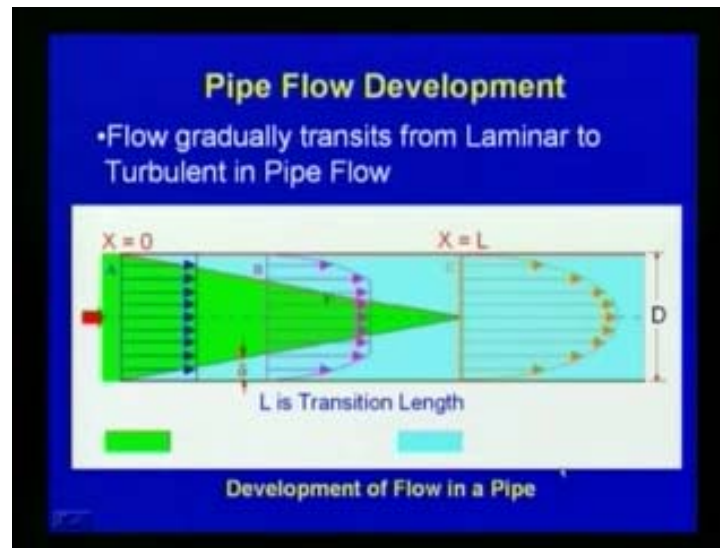
So, the flow of condition can be observed for the three cases through experiments, as we have already seen through a pipe flow. If we introduce some dye, we can see depending upon the velocity whether it is laminar or the transition flow or whether the flow is turbulent.

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So, if we plot the velocity versus time, and then, with respect to if you increase the depending upon the velocity, we can see that say at this laminar stage, you can see the velocity variation will be like this. So, change of flow from laminar to turbulent, and say if we, the Reynolds number is in this axis, and here the velocity, and with respect to time, we can see that say this range, this is the laminar flow range, and say between these Reynolds number 2000 to 4000, we have the transitional stage and then say 4000, you can see that here the velocity is fluctuating with respect to a mean value. So, due to the fluctuations, the flow is turbulent and you can see that the flow condition is like this. So, with respect to time and if you plot velocity, and with respect to Reynolds number, we can see that how the flow situation is whether the laminar, transitional and the turbulent flow conditions.

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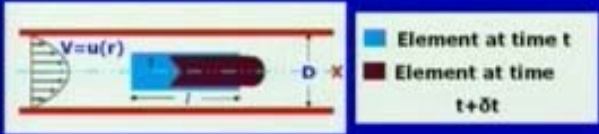
So, we further analyze the pipe flow. Then, we can see that, say depending upon the flow condition velocity and then other parameters, say we can see here, say if the flow starting the flow is laminar and then depending upon other parameters, parameters flow gradually transits laminar to turbulent. So, here, you can see with respect to the boundary layer developments and the flow is say at this stage, we can say that flow is fully developed into turbulent. So, you can see, and as far as turbulent flow is concerned, velocity variation is like this. So, here, this flow is fully laminar, and here, the flow is turbulent, and so, this shows the transition length. This L is the 0 to L is the transition length with respect to the boundary layer for motion; and then, finally, the flow becomes fully developed turbulence flow. So, this way, we can show experimentally how the flow turns from laminar to turbulent flow.

So, now, initially say we have already seen the flow pipe wise concerned, whether it is laminar, laminar or turbulent or say between the states, in the transitional stage. So, now, say we will discuss briefly some of the important relationship as far as laminar flow and turbulent flow in pipes. Some of these relationships we have already discussed earlier in the earlier topics, but as far as the pipe flow system, as an internal flow system which we discuss now. We will briefly review the various relationships for laminar flow; fully developed laminar flow in pipe system; and then, fully developed turbulent flow in the pipe systems.

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Fully Developed Laminar Flow

- It develops for flow through long, straight and constant diameter Pipe
- Velocity Profile for the flow is of prime importance which indicates the condition of flow
- Consider the fig. below, r - radius of fluid element

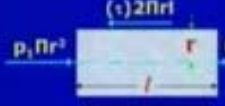


So, first, we will see the fully developed laminar flow. So, the fully developed laminar flow as we can see flow develops say at a distance. So, if we consider say a long straight and constant diameter pipe like this, so, if we consider a long straight and constant diameter pipe here, the velocity profile for the flow is of prime importance which indicates the condition of flow. So, we can see that the velocity variation is like this. So, velocity is maximum here and a parabolic variation. So, if we consider fluid elements at time t , so, here, this blue color is the fluid element at time t , and then, if we consider fluid elements at time t plus delta t , so, here, the diameter of the pipe is d and we consider a fluid element of length l and, the, for the fluid element the radius is small r .

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- From the FBD of the fluid element as shown,



from force balance,

$$(p_1 \pi r^2) - (p_1 - \Delta p) \pi r^2 - (\tau) 2 \pi r l = 0$$
- So, $\frac{\Delta p}{l} = \frac{2\tau}{r} \dots (A)$
- Shear stress distribution throughout the pipe is a linear function of the radial coordinate, i.e. $\tau = \frac{2\tau_w r}{D} \dots (B)$ where, τ_w is the wall shear, D - dia.

So, now, with respect to, since we now consider the flow as a fully developed laminar flow. So, from the free body diagram of the fluid elements, you can see that say the flow is taking place from this direction to this direction like this. So, here, on this face of the fluid element which we consider, the pressure is p_1 . So, the total pressure is p_1 into πr^2 square, and on this side, the pressure is total pressure is p_1 minus Δp into πr^2 square - where r is the radius of the fluid element, and then, the as far as shear force is concerned, so, τ into $2 \pi r l$ is the shear force acting in the opposite direction; τ is the shear stress. So, the pressure force and to the shear force you can see.

So, from the free body diagram of the fluid element, from the force balance since we consider the flow as steady state condition and laminar condition so that we can write the force on this fluid element from the free body diagram, we can write p_1 into πr^2 square minus p_1 minus Δp into πr^2 square minus this other direction the shear force minus τ into $2 \pi r l$ so that should be equal to 0.

So, this we can write from the Newton's second law, say we are equating the algebraic sum of the force. So, here, this is in the steady state condition. So, the algebraic sum of the force should be equal to 0. Since here, the forces here which we consider the pressure force and the shear force, so, here, by this relationship, we can see that the pressure difference is Δp between these two sections. So, we can write Δp by l is equal to 2 into τ by r , so, where r is the radius of the fluid element and τ is the shear stress.

So, now, the shear stress distribution throughout the pipe, you can see that it is a linear relation or the linear function of the radial coordinate. So, we can write this say τ by, say if we consider this fluid element, say then the shear stress, say at this location is τ . So, τ by r is equal to the τ_w which is this wall shear τ_w by d by 2 or we can write τ is equal to 2 into τ_w into small r by D - where D is the diameter of the pipe and r is the radius of the fluid element. Now, from this, we get τ is equal to 2 into τ_w into r by D - where τ_w is the wall shear.

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- Pressure drop Δp and wall shear stress are related by, $\Delta p = [(4\tau_w)/D] \dots (C)$
- A small shear stress can produce a large pressure difference if the pipe is relatively long, i.e. $(l/D \gg 1)$
- Now, shear stress can be given by (for pipe), $\tau = -\mu(du/dr) \dots (D)$
- From eqn. (A) and (D): $\frac{du}{dr} = -\left(\frac{\Delta p}{2\mu l}\right)r$
- On Integration, $\int du = -\left(\frac{\Delta p}{2\mu l}\right) \int r dr$

So, hence, this pressure drop Δp between the section 1 and 2 we can write the in terms of wall shear stress as Δp is equal to 4 into l into τ_w by D as in this equation number c. So, a small shear stress can produce a large pressure difference in the pipe is relatively long, so, depending up on this l by D ratio. If l by D ratio is much larger, then we can see that the pressure difference will be large depending on this l by D ratio. So, now, as we discussed earlier, the shear stress in the pipe we can write with respect to the pipe flow; we can write τ is equal to from the Newton's law of viscosity; we can write τ is equal to minus μ du by dr as in this equation number D.

So, if you use this equation number D and our earlier equation A here, equation A here, this equation Δp by l is equal to τ by r . So, using A and D, we can write du by dr the velocity gradient with respect to r du by dr is equal to minus Δp by 2 μ l into r , so that if you want to find out the velocity or discharge, we can just use this general

relationship. So, for as far as laminar flow in the pipe, so, integral du is equal to minus delta p by 2 mu l integral r dr. So, we will get, if we integrate say to find the velocity relationship, we can get integral from this relationship; we get integral du is equal to minus delta p 2 mu l which are constant here into integral r dr.

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- Hence, $u = -\left(\frac{\Delta p}{4\mu l}\right)r^2 + C_1$
- Now, B.C:
- at $r = D/2$; $u = 0$, so, $C_1 = \left(\frac{\Delta p}{16\mu l}\right)D^2$
- Hence the Velocity Profile can be given as:

$$u(r) = \left(\frac{\Delta p D^2}{16\mu l}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] = V_c \left[1 - \left(\frac{2r}{D}\right)^2\right] \quad \text{where, } V_c = \left(\frac{\Delta p D^2}{16\mu l}\right) \text{ ~ centerline velocity}$$

- From eqn (C):
- R – Radius of pipe $u(r) = \left(\frac{r_u D}{4\mu l}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$

So, hence, we can derive a relationship for the velocity as u is equal to minus delta p by 4 mu l into r square plus C 1. So, here, this C 1 is the constant of integration. So, finally, we got the relationship for the velocity.

So, here, this constant we can just apply the boundary condition. So, the boundary conditions here at r is equal to D by 2; that means on the pipe wall surface at r is equal to D by 2, we can see that due to nose tip condition, u is equal to 0. So, we get this constant C 1 as delta p by 16 mu l D square. So, finally, by using this C 1 in this equation, we get the velocity profile as u r is equal to delta p D square by 16 mu l into 1 minus 2 r by D whole square. So, this is the general relationship. So, with respect to the fluid element radius r and the diameter D of the pipe, we can write the velocity at any location u r is equal to delta p D square by 16 mu l into 1 minus 2 r by d whole square, so, where delta p is the pressure difference; D is the diameter of the pipe; mu is the dynamic viscosity; l is the length we consider.

So, this, we can write as u_r is equal to V_c into $1 - 2r$ by D whole square, where V_c is the central line velocity. So, V_c is represented as $\Delta p D^2$ by $16 \mu l$, which is the central line velocity for the pipe.

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- Pressure drop Δp and wall shear stress are related by, $\Delta p = [(4\tau_w)/D] \dots (C)$
- A small shear stress can produce a large pressure difference if the pipe is relatively long, i.e. $(l/D \gg 1)$
- Now, shear stress can be given by (for pipe), $\tau = -\mu(du/dr) \dots (D)$
- From eqn. (A) and (D): $\frac{du}{dr} = -\left(\frac{\Delta p}{2\mu l}\right)r$
- On Integration, $\int du = -\left(\frac{\Delta p}{2\mu l}\right) \int r dr$

And then, from our early equation c which is here, say this equation Δp is equal to $4 l \tau_w$ by D . We can write u_r is equal to, if you substitute for τ_w with respect to the τ_w , we can write u_r is equal to τ_w into D by 4μ into $1 - r$ by R whole square, where capital R is the radius of the pipe. So, here D is the diameter of the pipe; capital R is radius of the pipe. So, we get the velocity as a relation between the wall shear stress and the diameter or the radius of the pipe.

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- Discharge through the pipeline is

$$Q = \int u \, dA = \int_{r=0}^{r=R} u(r) 2\pi r \, dr = 2\pi V_c \int_0^R \left[1 - \left(\frac{r}{R}\right)^2\right] r \, dr$$

- Hence,

$$Q = \frac{\pi R^2 V_c}{2}$$

- now, $V = (Q/A) = Q/\pi R^2$
- So,

$$V_c = \frac{\pi R^2 V_c}{2\pi R^2} = \frac{V_c}{2} = \frac{\Delta p D^2}{32 \mu l} \quad Q = \frac{\pi D^4 \Delta p}{128 \mu l}$$

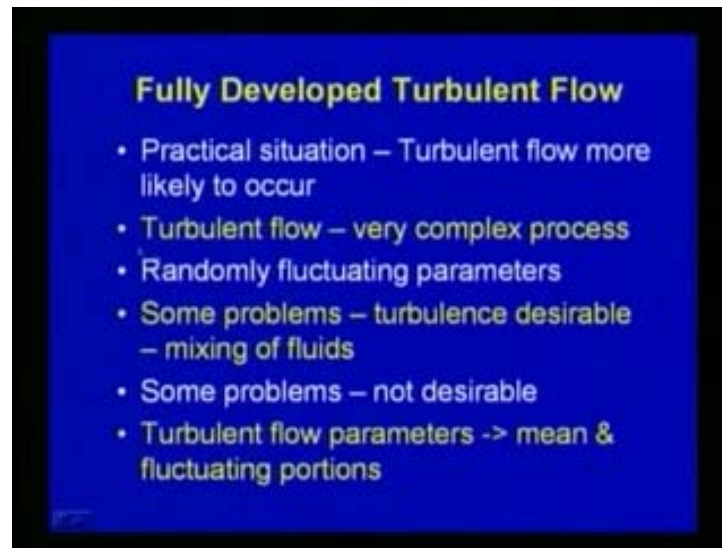
- Hagen – Poiseuille flow

So, now, if you want to find out the discharge, we can just integrate the Q is equal to integral $u \, dA$. So, that is equal to integral r is equal to 0 to capital R ; so, $u \, r \, 2\pi \, dr$; so, that is equal to $2\pi V_c$ integral 0 to r $1 - \frac{r}{R}$ whole square into $r \, dr$. So, finally, we will get discharge Q is equal to $\pi R^2 V_c$ by 2, where V_c is the central line velocity. And now, if we introduce this V is equal to Q by A , which is the average velocity, so, we get V is equal to Q by πR^2 . So, here, V is the average velocity; so, V is equal to $\pi R^2 V_c$ by $2\pi R^2$ so that we get V is equal to V_c by 2. So, the average velocity will be the half of the maximum velocity as far as pipe flow is concerned. So, that is equal to $\Delta p \, D^2$ by $32 \mu l$ or we can write Q is equal to π into D to the power four Δp by 128μ into l .

So, this relationship is the Hagen Poiseuille equation, and now, say this laminar flow condition, this flow condition in the pipe flow is called Hagen Poiseuille flow condition, and like this, we can get the various parameters like the velocity variation or the discharge through the pipe, and also using this relationship, you can find out the shear stress variation as far as the pipe flow is concerned. So, this is basic relationship as far as the fully developed laminar flow in a pipe system.

So, now, we will briefly discuss the turbulent flow condition or the fully developed turbulent flow in pipe systems.

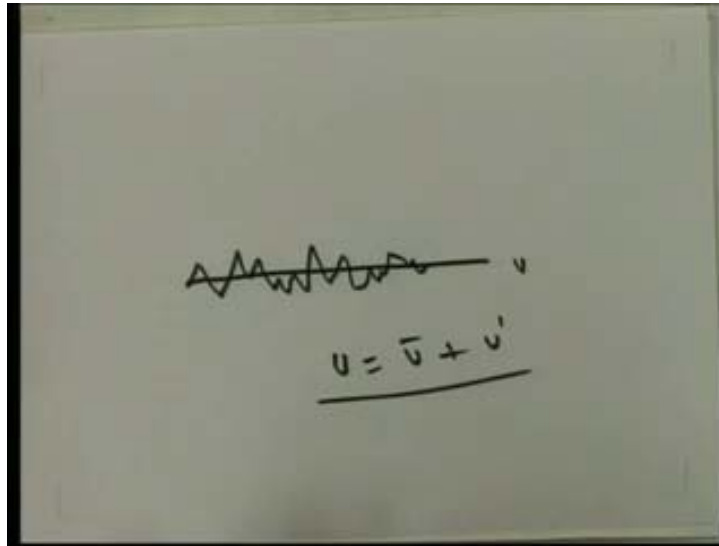
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So, as we can see most of the flow condition practically most of situation depending up on the velocity. As far as pipe system is concerned, turbulent flow are more likely to occur. So, we have to consider the relationship for the turbulent conditions when the when we deal with the pipe flow. So, turbulent flow, as we discussed in our early chapter on turbulence, we have discussed the details about the turbulent flow. So, as far as turbulent flow, we have seen that it is very complex process, and then, we have to take care, the flow situation with respect to a mean value and a fluctuating component.

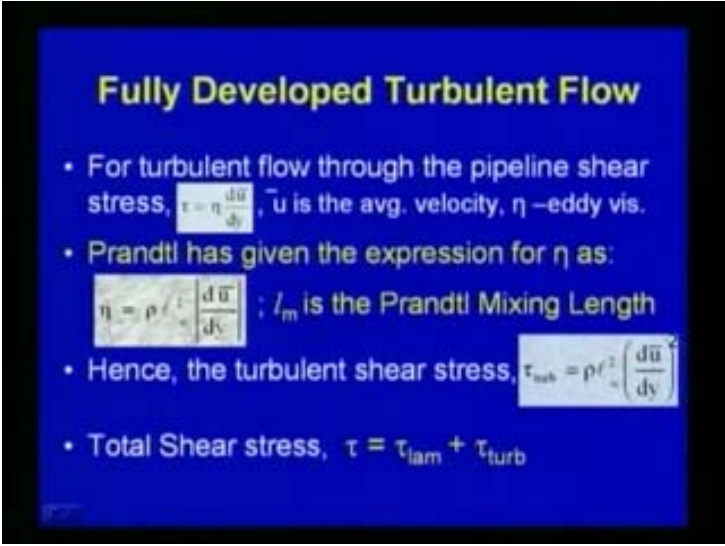
So, most of the parameters are randomly fluctuating parameters, and then, we have also seen the turbulence is concerned, sometimes it is desirable and many times it is not desirable depending up on the condition. Say for example, if you consider the mixing of chemicals or mixing fluids, then the turbulence is desirable, but many other situations, the turbulent flow condition is not desirable, but we cannot avoid since it depends upon the velocity of flow. So, and the turbine flow parameters when we say discuss the turbine flow parameters generally as we have seen earlier, we put a mean situation mean condition and then the fluctuating portions.

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So, here, say we have already seen earlier condition, say for example, if the velocity is plotted with respect to the mean component, then the turbulence is concerned, there will be variations like this. So, there will be, as far as the velocity is concerned, we can have a mean component and the fluctuating component. So, similar way, say if you consider the x component, y component or z component in the velocity, we will be having a mean component and then corresponding fluctuating component. So, similar way, the pressure is also concerned and we have seen that we can put it times of the mean component and the fluctuating component. So, generally, as far as turbulent flow is concerned, we consider the for various flow parameters; we consider a mean parameter, mean value and then the fluctuating portions.

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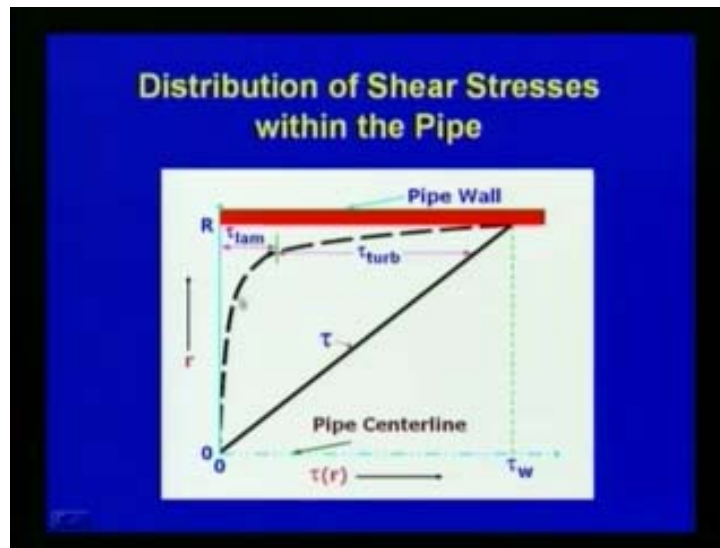


Fully Developed Turbulent Flow

- For turbulent flow through the pipeline shear stress, $\tau = \eta \frac{d\bar{u}}{dy}$, \bar{u} is the avg. velocity, η –eddy vis.
- Prandtl has given the expression for η as:
 $\eta = \rho l_m^2 \left| \frac{d\bar{u}}{dy} \right|$; l_m is the Prandtl Mixing Length
- Hence, the turbulent shear stress, $\tau_{turb} = \rho l_m^2 \left(\frac{d\bar{u}}{dy} \right)^2$
- Total Shear stress, $\tau = \tau_{lam} + \tau_{turb}$

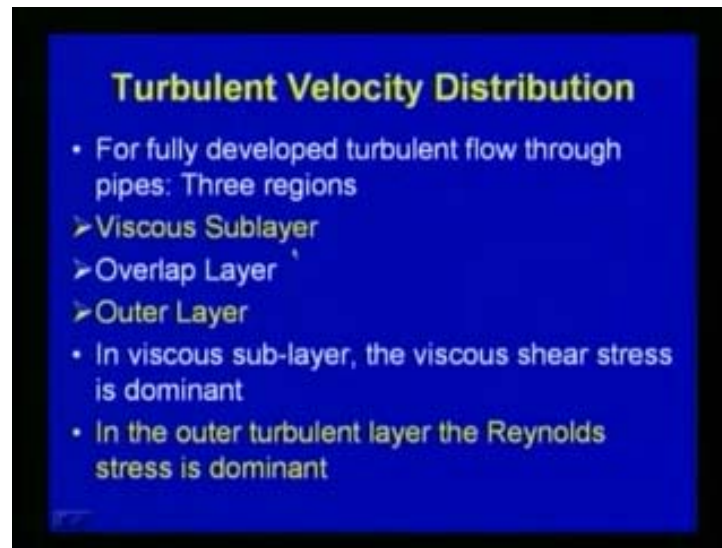
So, if we consider the turbulent flow through the pipeline through the pipe systems and as for shear stress is concerned, we have already seen earlier; we can represent the shear stress τ is equal to $\eta \frac{d\bar{u}}{dy}$, so, where \bar{u} is the average velocity and η is the eddy viscosity. So, we have discussed these details earlier. So, Prandtl has shown that this η is the eddy viscosity can be written as $\rho l_m^2 \left| \frac{d\bar{u}}{dy} \right|$ the modulus value $\frac{d\bar{u}}{dy}$, where l_m is the Prandtl's Mixing Length, and hence, this the turbulent shear stress, τ_{turb} can be written as $\rho l_m^2 \left(\frac{d\bar{u}}{dy} \right)^2$. If we substitute for η , it will be ρl_m^2 into $\frac{d\bar{u}}{dy}$ square. And then, depending upon the case, depending up on the problem and the total shear stress will be τ is equal to τ_{lam} and τ plus τ_{turb} . So, this is for the case of fully developed turbulent flow.

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And then, if you plot this shear stress variation as far as pipe flow is concerned, so, you can see that say if the shear stress τ is on this axis, and if the pipe bar is here and then with respect to if this gives the central line pipe center line, and say with respect to, if you plot the shear stress, then we can see that it will be 0 at the center line and then it will be varying like this, but if you split into the laminar and turbulent condition, then we can see that the shear stress variation as far as laminar conditions will be like this. And then, as far as turbulent condition is concerned, the term shear stress variation will be like this, but generally, we **consider**, so, for pipe flow, the shear stress variation is τ variation is considered like this. So, this shows the distribution of the shear stress within the pipe.

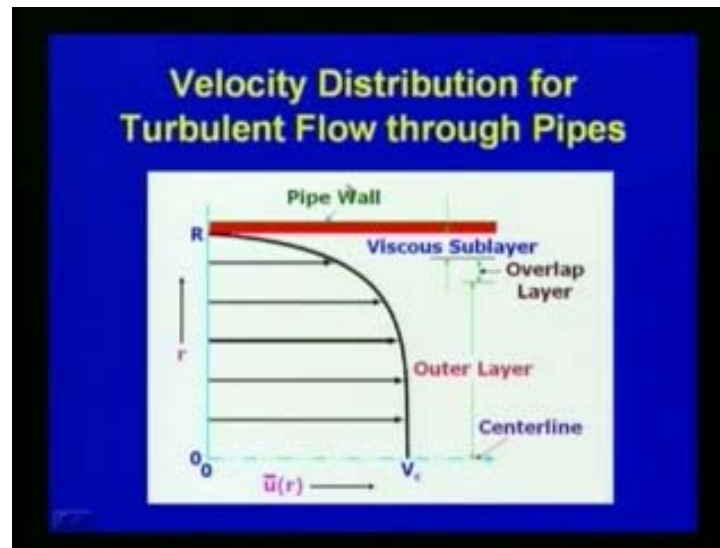
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So, now, say we can see that say when we analyze the turbine flow situation the pipe system, then for fully developed turbine flow through the pipes, we can observe three regions - one is the viscous sub layer and then second one is the overlap layer and then we are having an outer layer. So, these three regions, with respect to this three region, in viscous sub layer, the viscous shear stress is dominant, and the outer turbine layer, the Reynolds stress is dominant.

So, since this is the case of fully developed turbulent flow, so, we can see that the when we critically analyze the say at various location, we can see that there will be a viscous sub layer and then there will be an overlap layer and an outer layer. So, in the viscous sub layer concerned, viscous shear stress will be dominant, and then, the outer layer, we can see that the Reynolds stress is dominant.

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So now, if you plot the velocity distribution for turbulent flow pipes, say here, if this is the pipe wall and this is central line, and then, we can see that say with respect to r , we capital R is the radius and small r is the radius at various locations from the central line. So, this velocity variation you can see that it will be velocity distribution as far as turbulent flow will be parabolic like this. So, here, it will be maxima at the center line which is the V_c center line velocity, and here, with respect to these three layer which we discussed viscous sublayer, we can see that velocity variation will be like this and then this is an overlap layer as far as with respect to the fully developed turbulent flow.

And then, this is the outer layer where the Reynolds stress is dominant, and here, the viscous sub layer we can see that the viscous shear stress is dominant. So, this way, the velocity distribution changes with respect to the fully developed turbulent flow through the pipe system.

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- In the viscous sublayer the velocity profile can be written in dimensionless form as: $\frac{\bar{u}}{u^*} = \frac{y u^*}{\nu}$
 - $[y=(R-r)]$ is the distance measured from the wall
 - \bar{u} is the time-averaged x-component of velocity
 - u^* is termed the *Friction velocity* = $\left(\frac{\tau_w}{\rho} \right)^{1/2}$
- Note that u^* is not the actual velocity of fluid, only it is having the dimensions of velocity

And then, also we have discussed earlier in detail about the various aspect of turbulent flow through the pipe system. So, here, we just discuss the important relationships we derived at that time. So here, as far as in the viscous sub layer is concerned, the viscous sub layer is concerned, the velocity profile can be written in dimensionless form as \bar{u} by u^* is equal to $y u^*$ by ν , where y is the r minus capital R minus r ; where r is the radius of the pipe and r is the distance of the fluid element which is considered and \bar{u} is the time averaged x component of the velocity and this u^* is the friction velocity or the shear velocity and this u^* is equal to square root of τ_w by ρ .

So, here, with respect to this the shear velocity or friction velocity, we can show that in the viscous layer, the ratio of the mean velocity with respect to shear velocity can be written $y u^*$ by ν , and this u^* is the actually you can see that this is not an actual velocity of the fluid, but it is only having dimensions of the velocity and it is called friction velocity or the shear velocity.

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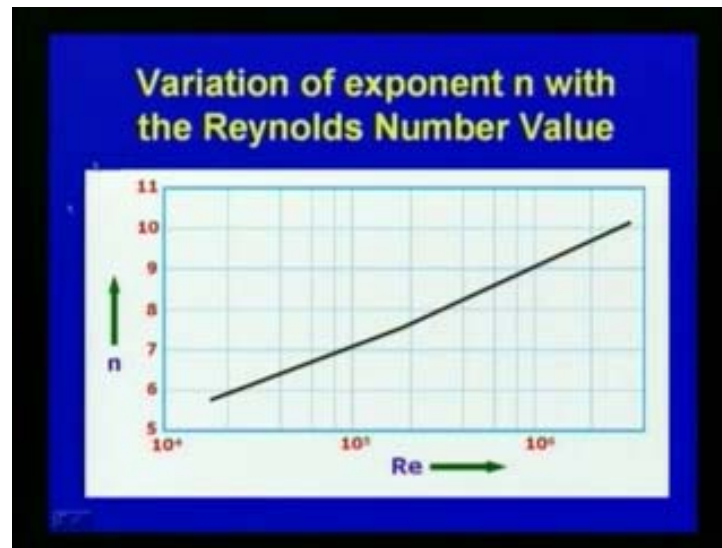
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- In the overlap region the velocity should vary as the logarithm of y
- So, it can be given as: $\frac{\bar{u}}{u^*} = 2.5 \ln \left(\frac{yu^*}{\nu} \right) + 5.0$
- In the outer layer,
- Velocity profile is given by empirical Power Law, as: $\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R} \right)^{1/n}$ n depends on Re and reasonably taken as 7
- For $n=7$, the law becomes "One-seventh Power Law"
- Where V_c – centerline velocity

So, this is the relationship, as far as the velocity variation with respect to the previous figure in this viscous sub layer, this is the relationship for the velocity variation \bar{u} by u^* is equal to y into u^* by ν , and then, in the overlap region, the velocity should vary as the, logarithmic, logarithm of y . So, you can see that with respect to this is the overlap layer. So, in this layer, you can see the velocity variation can be expressed as discussed earlier \bar{u} by u^* is equal to $2.5 \ln y u^* / \nu$ plus 5. So, this gives the velocity variation in the overlap region, and then, as far as the outer layer is concerned, you can see that velocity profile is given by we can express, there are different formula are available, but one of the commonly used formula is coming from the one-seventh power law. So, we can write as \bar{u} by V_c is equal to $1 - r/R$ to the power $1/n$, where n depends on the Reynolds number, and reasonably, generally it is taken as n is equal to 7 for the turbulent flow.

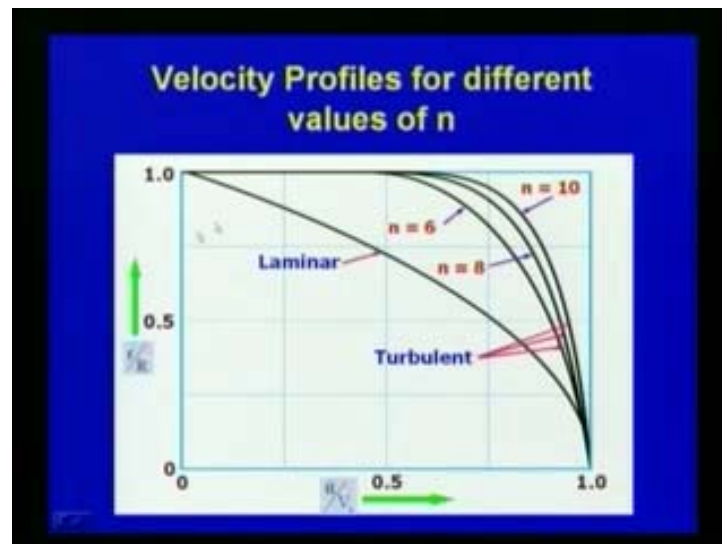
So, this is coming from the power law. So, with respect to the velocity variation in this range here, this outer layer is represented as \bar{u} by V_c is equal to $1 - r/R$ to the power $1/n$ - where V_c is the center line velocity and n is the co-efficient; n is equal to 7 as far as, say then we call this as one-seventh power law.

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So, now, this as far as the variation of n is concerned with respect to Reynolds number, we can see that this the value of n is changing like this the Reynolds number is plotted on the x axis and, this value, this n is plotted on the y axis. You can see that here say for the conditions of say whether laminar range or the transition or the turbulent range, we can see that the variation takes place. So, here onwards you can see that the turbulence starts; so, that is generally n is equal to 7. So, variation of the exponent n with the Reynolds number value is shown in this right here.

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And then, with respect to various values of the exponent n , we plot the velocity profile. Then if r by R is plotted on the y-axis and then \bar{u} by V_c is plotted on the x-axis, where \bar{u} is the mean velocity and V_c is the center line velocity, this is the center line velocity. Then, you can see that for various values of n , this is the laminar range, and for turbulent range, you can see that say here for n is equal to 6; n is equal to 8; n is equal to 10 like that we can plot the value of this velocity ratio with respect to r by R as shown in this with respect to this figure here. So, we can use the power law - one-seventh power law or the power law - as given by \bar{u} by V_c is equal to $1 - (r/R)^{1/n}$.

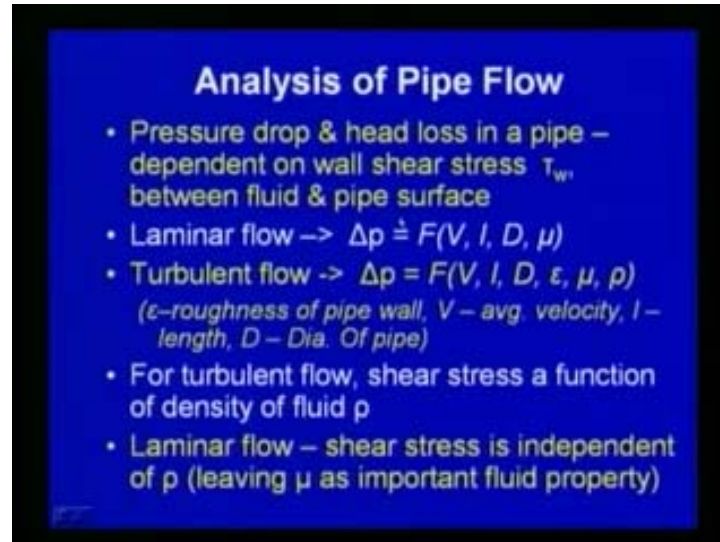
So now, say these are some of the fundamental relationships for velocity variation origin of shear stress variation as far as fully developed turbulent flow in pipe system is concerned. So now, say as far as turbulent flow in pipe is concerned, as we discussed earlier to model the turbines, it is a very difficult; the flow is so complex; so, we have seen the fundamental governing equations like Reynolds equations or the transformer navier stokes equations. We can solve these equations say for a complete information of the velocity or the various parameter variations, we had to solve the whole systems equations, but complexity of the equations and difficulties we have seen that say even the nowadays very good computer packages are available CFD - Computer Fluid Dynamics packages - are available for the solution of these equations, but for common purpose, we can use some relationships based upon the experimental data or the semi empirical formulas as we discussed earlier.

So, the solution, the, for the turbulent flow is so complex, say to solve the Reynolds equation or Navier Stokes forms of equations, we have to put large efforts, and even though nowadays very sophisticated computer and sophisticated packages are available, still to get all the parameters as with respect to turbulence flow modeling is still too difficult. Some of the approaches as we discussed earlier like zero equation models which we already discussed with respect to various equations. For the direct numerical solutions, DNS approach or the large dissimulations as far as turbine flow by using the Navier Stokes equations.

So, we are having number of approaches as far as turbulent modeling or turbulent flow simulation in pipes, but due to the difficulties, still for, say to simplify the solution, we generally use some of the empirical formulas or some of the equations based up on the

experimental data, in the, as far as practical use is concerned for the various parameter determination in the case of turbulent pipe flow analysis.

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So now, we have seen the say various relationships, important relationships as far as laminar flow through pipes and the turbulent flow through pipes. So now, based up on these basics or the basic theories, now we will discuss in detail about we will critically analyze the pipe flow, and then, we will discuss the various losses like major losses and minor losses as far as pipe flow is concerned.

So now, the pressure drop, and now, we will analyze the pipe flow; the analysis of pipe flow pressure drop and head loss in a pipe. So, we have already seen this pressure drop and head losses in the pipe depends on the wall shear stress τ_w and say the shear stress between the fluid and the pipe surface.

So, we can do some dimensional analysis to see how the pressure drop and head loss develops as far as pipe flow is concerned. So, if you do dimensional analysis for the laminar flow or turbulent flow, we can see that the various parameters as far as the pressure drop or the **head loss** is concerned. We can see that for laminar flow, this Δp , the pressure drop will be generally a function of the velocity V ; then length velocity, the average velocity V ; length of the pipe l ; the diameter of the pipe D and the coefficient of viscosity μ .

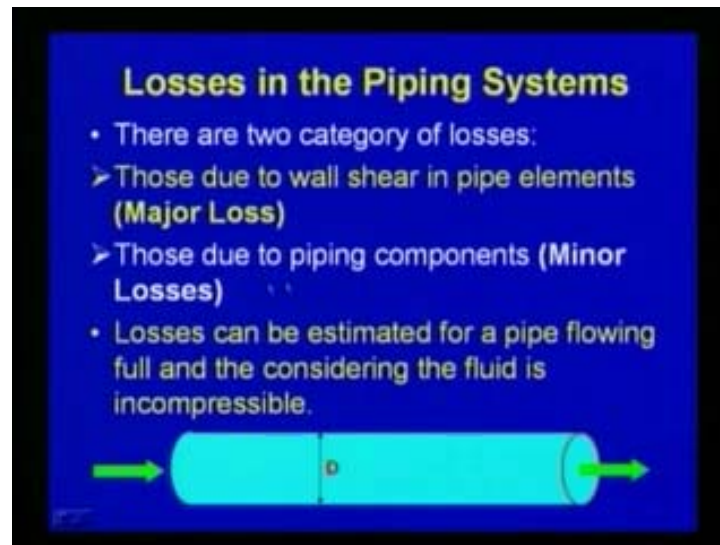
So, similar way, if you analyze the fully developed turbulent flow in pipe, we can see that Δp will be the pressure drop or the head loss will be function of say the average velocity V ; the length of the pipe L ; diameter D and then the roughness of the pipe wall and the coefficient of viscosity μ and the density. So, when we analyze the laminar flow through pipe fully developed laminar flow and fully developed turbulent flow, we can see that say the as far turbulent flow is concerned, we have to say these, an important parameter. The roughness of the pipe force is applies a major role, so, the as far as the pressure drop and head loss is concerned.

So, but as far as laminar case is concerned, this roughness pipe wall is not playing much role; it is not so important. It is generally this pressure loss, pressure drop or the head loss is head loss function so only the average velocity, length the pipe and the diameter of the pipe and the coefficient of dynamic viscosity.

And also we can see that the shear stress is a function of the density of fluid ρ . So, the laminar flow as far as laminar flow is concerned, if you analyze the laminar flow, we can see that the shear stress is independent of ρ and leaving μ as important fluid property. So, this is the difference. When we deal with pipe flow whether it is laminar or turbulent flow, we have to see that, so, whether we have to consider the roughness of the pipe wall say especially if it is turbulent flow and also we have to see the, say with respect to laminar flow, say there is the shear stress is independent of the density ρ , but as far as the turbulent flow is concerned, shear stress is a function of density of fluid ρ .

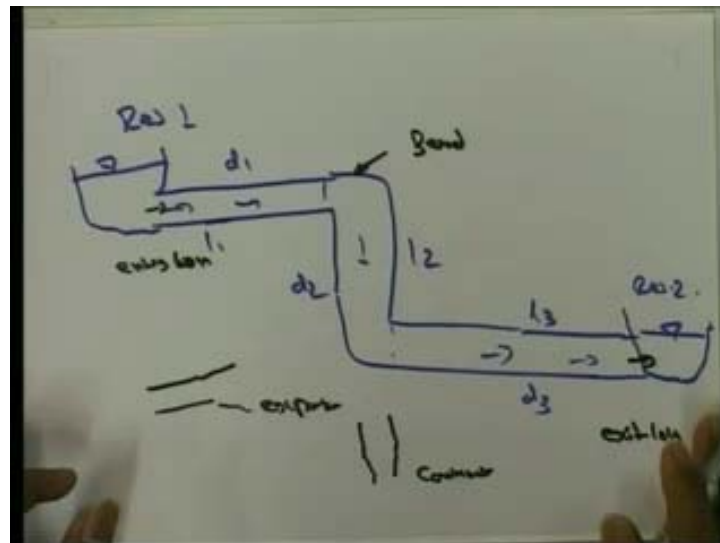
So, as far as pipe flow is concerned, when we analyze say we have to see whether the flow is laminar or turbulent. So, as we discuss them with respect to the Reynolds number, we differentiate whether the flow is fully laminar flow or flow is fully developed turbulent flow.

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So now, say as we have seen the losses are concerned, so, we have to see whether the flow is laminar or turbulent, and then, when we say one of the important aspect as far as pipe flow is concerned is the losses in the pipes. So, we can see that when the flow is taking place from one location to another location.

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Say for example, if we consider a pipe flow like this say from say one reservoir to another reservoir or one tank to another tank like this, so, here, one of the important aspect is say when we discuss the pipe flow, the flow is taking this direction. So, we can

see that if this is the 1 1 pipe with diameter d_1 and here 1 2 d_2 and this is 1 3 d_3 , so depending upon the problem, so, here, this is the reservoir 1 and here reservoir 2. So, when we discuss the flow, taking place, say between this reservoir from this tank to another tank. So, for pipe flow considering this, the, we have to see the losses as far as pipe flow is concerned. You can see that the losses of pipe flow are we can classify into one as major loss and then another as minor loss.

So, here, with respect to the viscous force or the shear stress, we can see that there will be losses. So, with respect to the systems of the pipe wall, there will be losses; so, that loss is defined as the major loss. So, here, you can see that say with respect to the flow direction, there will be major loss and that we have to calculate separately, and then, other kind of loss is called minor losses.


So, that minor loss means here you can see that when the pipe enter from this reservoirs. So, this is there will be an entry loss here, and then, if there is a bend here, then there will bend loss and then another say again it is entering to a large diameter, say here small diameter to large diameter, so, there is some way of expansion, and then, here, you can see that now, say again after reaching here, it is a large diameter to smaller diameter. So, you can see that with respect to a bend, we place a there is a contraction and then this is entry loss here, and then, finally, when it exceed through the reservoir 2 here, you can see that there will be exit loss. So, all these losses we have to consider when we discuss the head loss or the pressure loss as far as the pipe is concerned.

So, there are, so, with respect to this figure here, generally we can classify the pipe losses as major loss and the minor losses. So, the major losses here, as we discussed, the losses due to the shear stress or the viscous effects of the fluid and flow resistance with respect to the pipe. So, that is the major loss and minor losses like entry loss, exit loss, then bend loss, junction losses, expansion losses, contraction losses. So, like that, number of losses will be there. So, these are called the minor losses.

(Refer Slide Time: 44:46)

Losses in the Piping Systems

- There are two category of losses:
 - Those due to wall shear in pipe elements (**Major Loss**)
 - Those due to piping components (**Minor Losses**)
- Losses can be estimated for a pipe flowing full and the considering the fluid is incompressible.



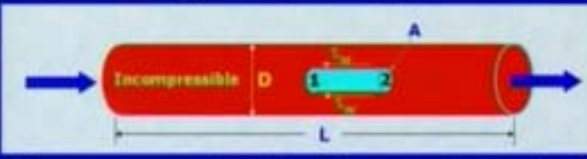
A horizontal blue pipe is shown with green arrows at both ends pointing outwards, indicating the direction of fluid flow.

So, like this, we can classify the losses in to major losses due to the wall shear, and then, the due to the minor losses, due to the piping components like bends or the entrance or exit or the junction or the expansion or contraction. So, losses can be estimated for a pipe flowing full, and consider the fluidity is incompressible, so, most of the fluid which we consider in all our discussion is incompressible fluid flow.

(Refer Slide Time: 45:11)

Pressure Loss due to Friction

- Consider a cylindrical element (dia. d , length l) of fluid flowing through the pipe, as shown



A diagram shows a red cylindrical fluid element of length L and diameter D inside a pipe. Blue arrows indicate flow from left to right. Section 1 is at the left face and section 2 is at the right face. The area of the pipe is labeled A . The word 'Incompressible' is written inside the cylinder.

- The pressure at section1 is p , and at section2 it is $(p - \Delta p)$
- Now driving force is $= [pA - (p - \Delta p)A] = \Delta p A$

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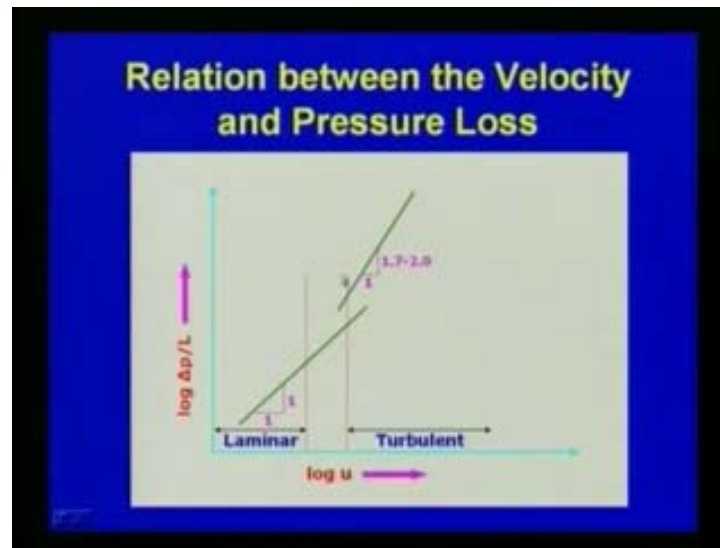
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- Resisting force due to wall shear stress is
= $[\tau_w \times \text{Area of Pipe Wall}] = \tau_w \pi D L$
- At equilibrium, Driving force = Resisting force
- i.e. $\Delta p A = \Delta p \pi D^2 / 4 = \tau_w \pi D L$
- Hence, $\Delta p = \frac{4 \tau_w L}{D}$
 - This is the expression for pressure loss in pipeline in terms of dia. and length of pipe
- The shear stress will vary with velocity of flow and hence with Re so the pressure loss will vary, as shown in Fig.

So, if you consider a cylindrical element say of diameter d and length l of flowing fluid through the pipe as shown in this figure, so, here, this is the pipe flow in this direction and we consider the incompressible flow. So, D is the diameter of the pipe, and say, the shear stress is τ . So, the pressure at section 1 is p and at section 2 is p minus Δp . So now, you can see that driving force is p into A minus p minus Δp into A ; so, that is equal to Δp into A ; so, that means the pressure difference between section 1 and 2. So, this is the pressure loss say from section 1 and 2, and then, the resisting force due to the wall shear stress we can see that here is wall shear stress so that we can write as τ into area of pipe wall. So, that is equal to τ_w is the shear stress on the wall τ_w into π into $D L$ - where D is the diameter of the pipe and L is the length we consider here.

So, at equilibrium, as we have seen the driving force is equal resisting force, so, p minus p into A is equal to Δp into πD^2 by 4 that is equal to τ_w into $\pi D L$. So, finally we can write Δp is equal to 4 into τ_w into L by D . So, this is the expression for pressure loss in a pipeline in terms of diameter and length of the pipe. So, this we have already seen the Hagen Poiseuille equation earlier. So, that way, here we get the pressure difference in terms of the wall shear stress.

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So, the shear stress will vary with velocity of flow, and hence, with respect to the Reynolds number so that we can see that the shear stress varies with the pressure loss. You can say depending upon the laminar or turbulent, we can plot like this. This gives the relationship between velocity and pressure loss. So, if you plot the logarithm of delta p by L, this ratio delta p by L on the y-axis and the velocity on the x-axis. We can see that as far as laminar flow is concerned, it will give a line like this, and for turbulent flow, it will be flowing say like this. So, this gives the relation between the velocity and the pressure loss.

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Pressure Loss during Laminar Pipe Flow

- The wall shear stress, τ_w is difficult to measure.
- For laminar flow, pressure loss can be given in terms of avg. velocity (V), pipe dimensions (L, D) and fluid property (μ) as:
$$\Delta p = \frac{32 \mu L V}{D^2}$$
 - (Hagen-Poiseuille equation) ($p = \rho g h_f$)
- The head loss due to friction is,
$$h_f = \frac{32 \mu L V}{\rho g D^2}$$

So now, the pressure loss during this, so, we have already seen earlier, the pressure loss during the laminar pipe flow of the fully developed laminar flow. The wall shear stress τ_w is difficult to measure, but say with respect to the say if we can find out what is the head loss or the pressure loss, from that we can find out. So, for laminar flow, the pressure loss can be given in terms of the average velocity V ; the pipe dimensions L and D , and the flow property μ as Δp is equal to $32 \mu L V$ by D square. So, this is the Hagen Poiseuille equation which we derived earlier, and if we put the pressure as ρg into h_f the head loss, so, the head loss with respect to the changes in pressure, we can write as h_f is equal to $32 \mu l V$ by $\rho g D$ square - where V is the average velocity of fluid and D is the diameter and μ is the coefficient dynamic viscosity and L is the length and g is the acceleration due to gravity; ρ is the density of the flow.

So, this gives the Hagen Poiseuille equation and this gives the equation for the pressure loss or the head loss with respect to the laminar flow pipe. So, similar way, the next lecture we will be discussing about the pressure loss and head loss with respect to the turbulent flow, and then, various other flow parameters with respect to the roughness coefficient and Moody's diagram, we will be discussing in the next lecture.