Fluid Mechanics

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Lecture No. # 35

Boundary Layer Theory and Applications

Welcome back to the video course on fluid mechanics. In the last lecture, we were discussing about the drag force, and then, lift force. We have seen various theories behind the drag force and various applications with respect to the drag force, and then we were discussing about the lift force. So, as we discussed, the lift force is the force exerted along the direction normal to the flow; that is the lift force. And, we have also seen that, say, some cases like, say air foil or the aeroplane is concerned, lift is very much essential and we are looking to get better lift. But some, say, cases like the design of a car or auto, the design of a bus, we are trying to reduce the lift.

So, depending upon the case, we will be trying to increase or decreasing the lift; so, that way, we have to see that which way we have to plan, whether, say, which we have to design, say, such a way, that lift should be to generate lift or to reduce the lift; so, that we have to see the case.

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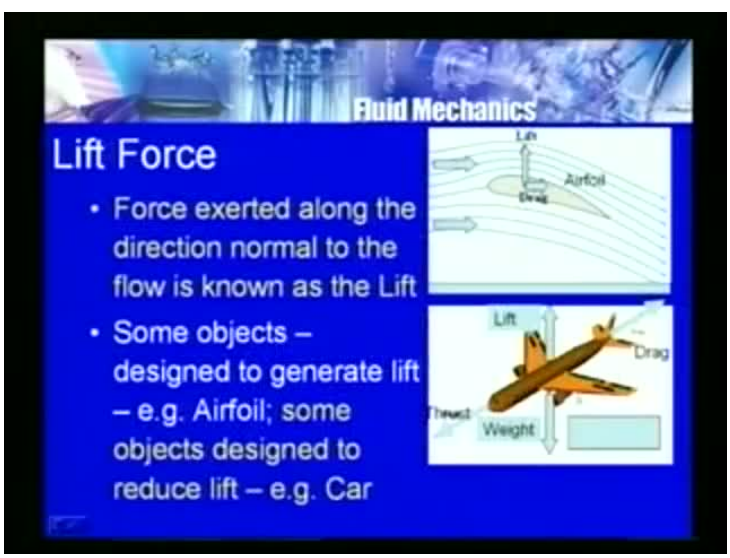

So, as I mentioned, for an aircraft is concerned, you can see that the thrust is in this direction and drag is here, and then weight of the aircraft and lift is in this direction, which is, say, here. We are generating this lift, so that the aircraft is going up and flying. But as far as car is concerned, you can see that, since with respect to the lift, the efficiency of the vehicle will be reduced; so, we want to reduce the lift.

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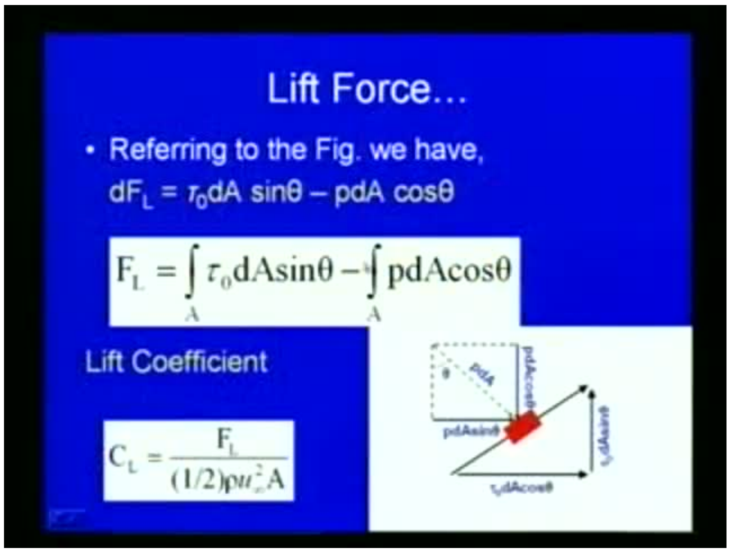

So, as we discussed earlier also, so the lift force where we can calculate, say, with respect to the sheer stress and the pressure force. So, if you consider an elemental area like this and then the pressure is acting p into dA, in this elemental area like this; and then, it has got two components - one is pdA sin theta in this the horizontal direction, pdA cos theta in the vertical direction - and the sheer stress is also constant. We have tau 0 dA cos theta in the horizontal direction and tau 0 dA sin theta in the vertical direction; so, that for this elemental area, the lift will be dF is equal to tau 0 dA sin theta minus pdA cos theta, where theta is this and to here.

So now, for the body is concerned, we can indicate with respect to the area, so that, the total lift force will be equal to integral upon area A tau 0 dA sin theta minus integral upon area A pdA cos theta. So, this is the general expression for the lift force, which we generally use in the calculations and for the design purpose.

And then, generally, the lift is concerned, we can represent in terms of a, say coefficient; it is called coefficient of lift; coefficient of lift is defined as the ratio of the lift force divided by this, half rho u infinity square into A, where this u infinity is the free, is the free stream velocity and A is the area of the, the body area which we consider; and rho is the density. So, coefficient lift is equal to lift force F L divided by half rho u infinity square into A, where u infinity is the free stream velocity.

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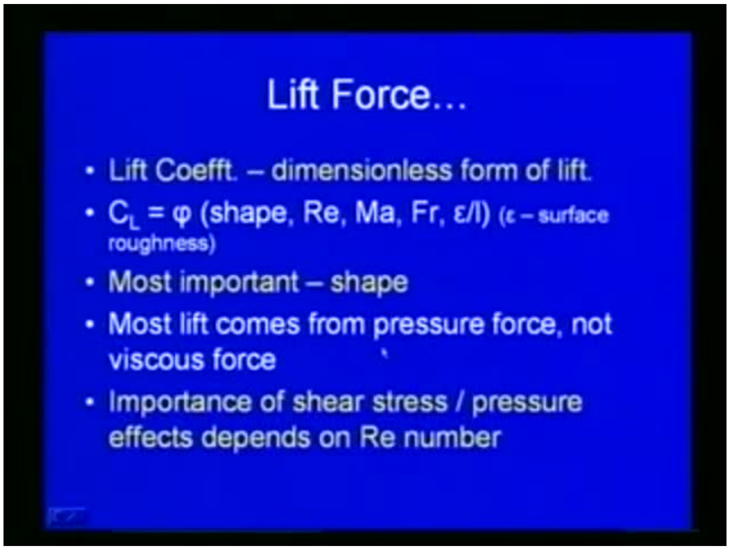

And then, we can see that this is a lift coefficient is a dimensional form of lift, and then, this is an, as we discussed in the case of drag, lift coefficient or lift force also depends upon the shape of the body, and then, various fluid flow parameters, like Reynolds number, Mach number, Fraud number, and also the surface roughness of the body, which we consider. So, the lift coefficient here is a function of shape of the body, the Reynolds number of the flow, Mach number of the flow, the Fraud number and the surface roughness ratio is epsilon by l, so, where epsilon is the surface roughness.

So, out of this, we can see that most important parameter is generally the shape; so, in the case of drag also, we have seen the effect of the shape as far as the drag coefficient or the drag force is concerned. Similarly, here, the lift force is also considered. We can see that shape is very important; so, and then, with respect to the earlier expression which we have seen for the lift force, we can see that most lift comes from the pressure force and not from the viscous force.

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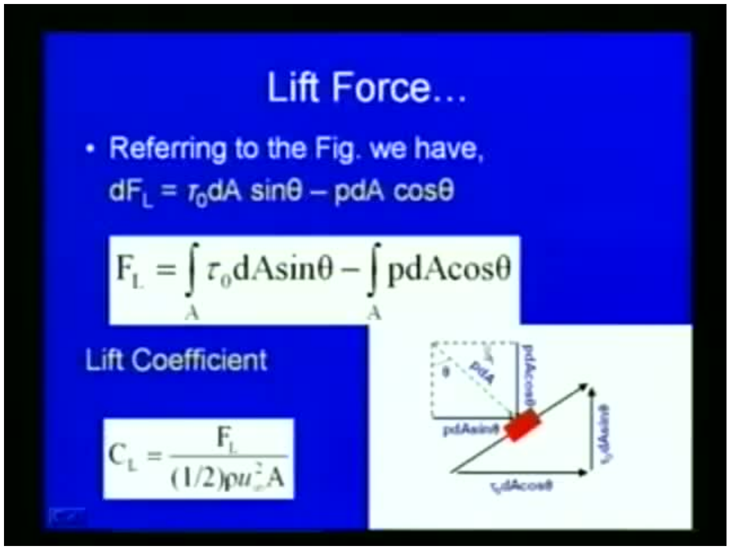

So, the major component here, we can see this, in this expression, this step pdA cos theta is the major contributor as far as the lift force is concerned, and sheer stress tau 0 dA sin theta component, this is more important as far as lift is concerned. So, we have to create or to generate lift, we have to see that there is a pressure difference between the top of the body which we consider and bottom of the body, so that the lift is generated or if we want to reduce the lift, so accordingly, we have to see.

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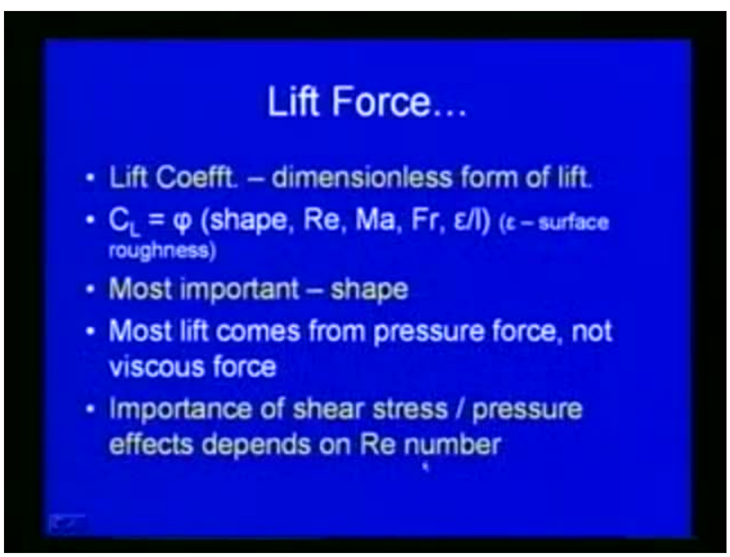

So, and then importance of sheer stress or pressure effects depends upon the Reynolds number. So, we have seen that here, the lift coefficient is a function of shape, Reynolds number, Mach number, Fraud number and epsilon. These are the first reference, but you can see that the important parameter is here is the Reynolds number; Mach number and Fraud number as we have seen in the case of drag force or drag co-efficient, the effect is small. But Reynolds is another important factor or important parameter here; so, depending upon whether the flow is a laminar turbulent or say when the Reynolds number is increasing or decreasing, we can see that the lift force changes.

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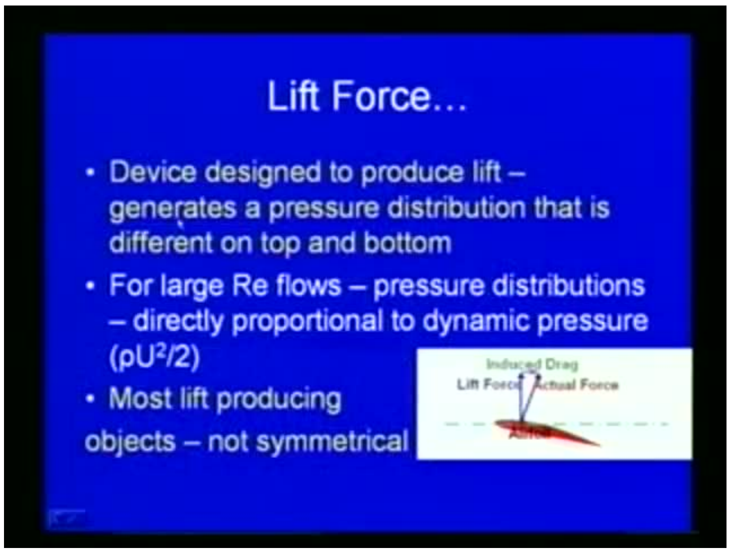

So, now, as far as lift force is concerned, we can, say, design devices to produce lift, as we have seen in the case of an aircraft, so that a lift is generated, so that the pressure distribution that is different on top and bottom. So, if you consider, say for example, an airfoil here, so, lift will be, say, we can generate lift by having a pressure difference on the top and the bottom. So, in a very similar way, we can see that aircraft is, in the wings of the aircraft there will be a pressure difference, and then, we can generate the lift. And then, for large Reynolds number of flows, the pressure distribution is directly proportional to the dynamic pressure; so, when the Reynolds number is increasing, that means, when the flow become turbulent, then we can see that the pressure distribution is generally proportional to the dynamic pressure over rho u square by 2; and then, say depending upon this factor, rho u square by 2, the lift generator or the coefficient that needs to be changing.

And then, also another important thing is that, which we can observe, say for example, with respect to this airfoil or most of the other designs, most lift producing objects will be not symmetrical or you have to design in such a way that this non-symmetry or the object is straight or it is designed in such a way that it is not symmetrical; so that way, say more lift will be generated. So, this is a typical case, is this air foil when we put it at an angle of at, we can see that the lift will be more.

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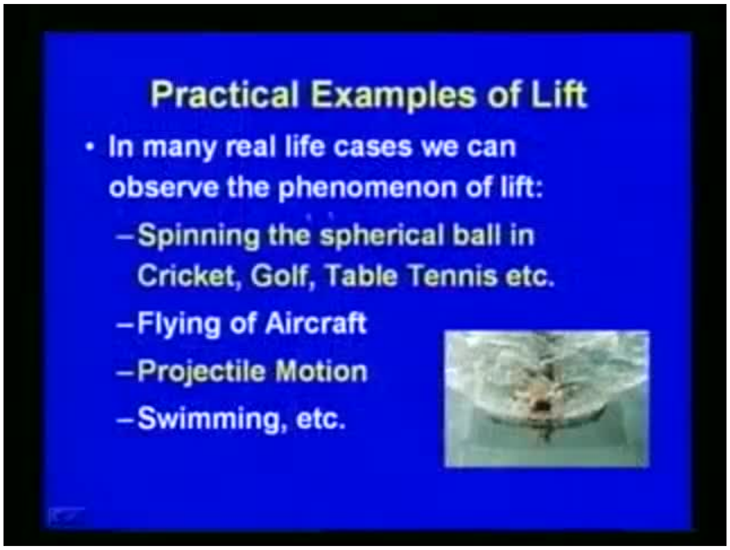

So, some of other practical examples as far as lift is concerned, we can see many real, live cases and we can observe the phenomena of lift; so, say for example, when we spin the spherical ball, in the case of cricket, golf, table tennis, etcetera, a lift will be generated; so, these we discuss later, say which is the affect, so-called Magnus effect, we will be discussing later. And then, of course, the flying of aircraft which is, we can see, observe in the case of an air foil, and then projectile a motion, and even say, in the case of swimming, so while swimming in this direction, you can see that some lift will be generated from the bottom due to the pressure differences.

So, like this, we can have number of examples, real life examples, where this lift is very important; as I mentioned in the some cases we want to generate the lift, so that it will be useful in the application of that particular type of problem; just-like in the case of aircraft or in some cases, we want to reduce the lift effect, so that we have a smooth operation, say for example, with the movement of a car. So, you can see that whenever a lift is generated, the efficiency will be reduced, so that we want to reduce the lift effect on the car; so, like this number of examples, we can have as far as lift is concerned.

So, now, before further discussing the lift effect with respect to the flow surrounding a cylinder or an airfoil, we will just briefly discuss another important parameter called Circulation. So, the Circulation effect, we have discussed earlier when we, say, discussed about the potential flow in the earlier lectures; but here, with respect to lift is concerned, we will be discussing more about the circulation.

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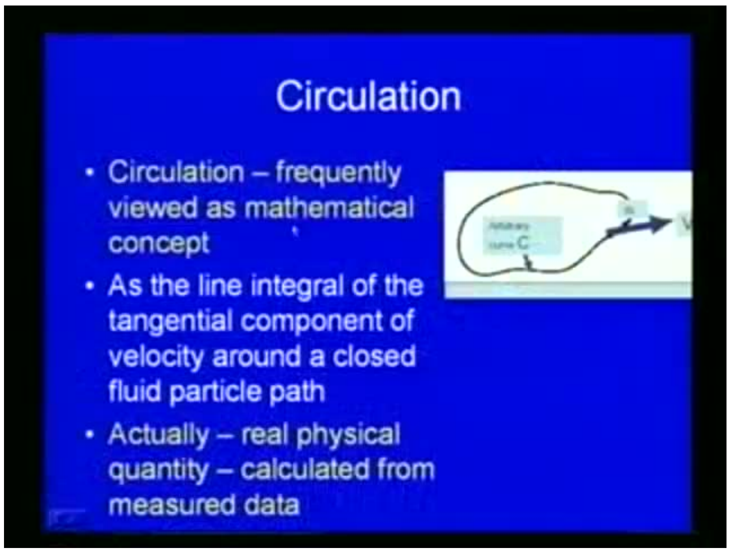

So, as far as circulation is concerned, and say it is a generally, we would have mathematical concept and the circulation is the line integral of the tangential components of velocity around a closed fluid particle path; so, you can see, that if we consider a fluid particle path like a closed fluid particle path like this, defined by this curve C, and then the circulation is the line integral of the tangential component of the velocity, so, the tangential component of the velocity V here around the closed fluid particle path. So, actually, the circulation is a real physical quantity; and then, if we can measure the velocity component or with respect to the measured data, we can calculate the circulation.

So, this circulation is, as you can see, it is a measure of the swirl of the fluid flow; so, the circulation is coming from the rotational effect or the with respect to here, you can see that we are considering a closed fluid particle path.

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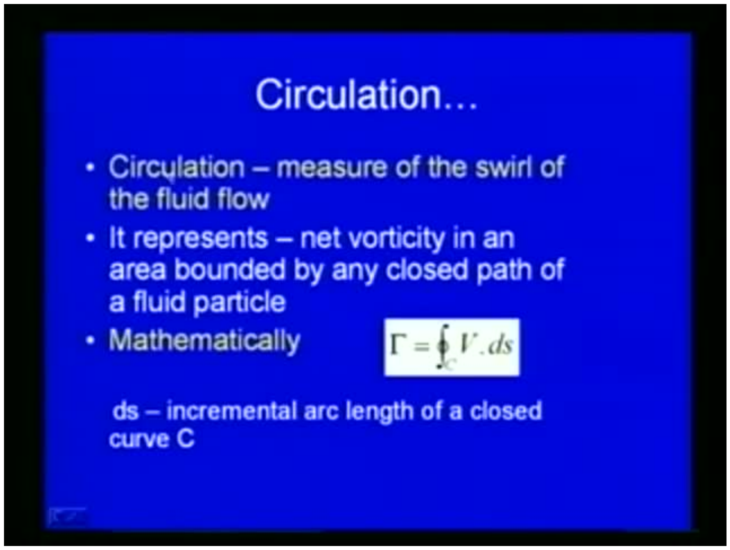

So, it is generally considered as the measure of swirl of the fluid flow; so, and it represents the net vorticity in an area bounded by any closed path of the fluid particle. So, mathematically, the circulation we can write as, mathematically the circulation can be represented as, gamma is equal to, capital gamma, is equal to the integral over the curve C V dot ds; so, where ds is incremental arc length of a closed curve C as described in this figure. So, the mathematical expression for the circulation is capital gamma is equal to the integral over the curve C V dot ds, where V is this; this as we discussed, V is the velocity, the tangential component of the velocity, as shown here in this figure.

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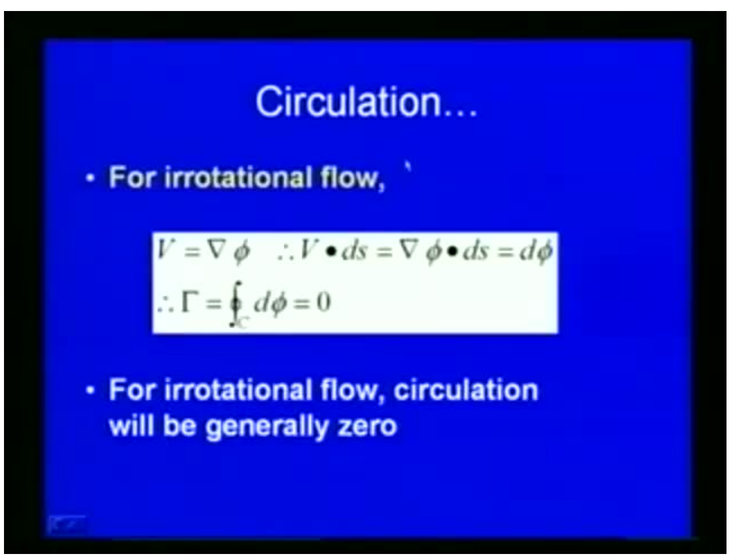

So, now, with respect to this, the definition of the circulation, so, as we have seen in the case of potential flow, when we discussed about the potential flow, so, if we consider the flow as irrotational, so, as far as irrotational flow is concerned, we can see that, we can write the velocity component V is equal to del phi; so, here, this V is equal to del phi. And therefore, V dot ds is equal to del phi dot ds and that is equal to d phi; so, that we can write. Therefore, this circulation gamma is equal to integral of the closed path curve C d phi; so, we can see that this is equal to 0 for irrotational flow.

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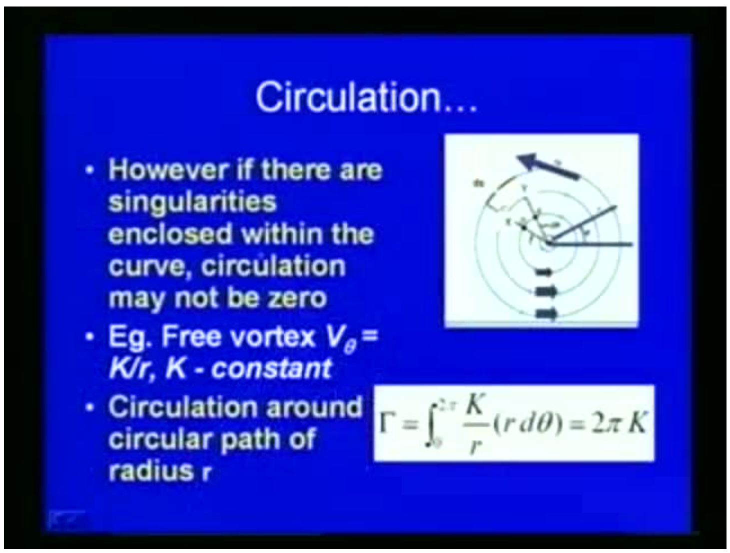

So, generally, we can say that, as far as irrotational flow is concerned, the circulation will be 0; but you can see that, as far as the circulation is concerned, within that closed curve which we consider, if there are singularities enclosed within the curve, then circulation may not be 0. So, if you consider, say, for example, the circulation around the circular path if you consider, as far as here is concerned, you can see that if you consider a closed circular path like this, then you can see that, here we will be having a free vortex V theta is equal to K by r, where K is a constant; and the circulation around the circular path of radius r, for this case, will be gamma is equal to integral 0 to 2 pi K by r r d theta, where r is this radius, defined here and theta is this angle and ds is this and here; so, gamma is equal to, you can see that it is 2 pi K, so, where K is a constant.

So, you can see that, even though here, what we consider is irrotational flow and potential flow, but whenever there is a singularity or if singularities are there, within that closed curve, then you can see that circulation will not be 0. So, we get a value for circulation gamma is equal to 2 pi K. So, this way, we can see that this, whatever we have discussed, as far as circulation is concerned , say, with respect to the lift coefficient or lift of the on the body, we will just discuss now, how the circulation effect will be coming. We will discuss with respect to the case of a rotating cylinder.

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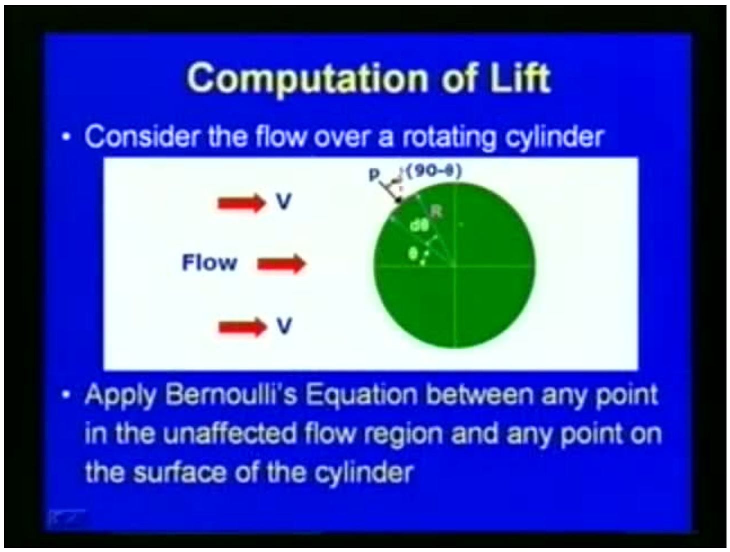

So, now let us compute the lift for the case of a flow over a rotating cylinder. So here, you can see here, the cylinder is here, the cylinder is of radius r and it is rotating, and then a free stream velocity, free stream flow is coming; free stream velocity of V is in this direction. So, now this cylinder is also rotating in this direction; so, for this case, by, let us assume the flow to be say the potential flow, so that we can apply the Bernoulli's equation between any point in the unaffected flow region, that means this region and any point on the surface of the cylinder, say particular point, we consider on the cylinder, and then between we consider the pressure difference between any point on this fluid, that means, outside unaffected flow region and any point in the surface of the cylinder.

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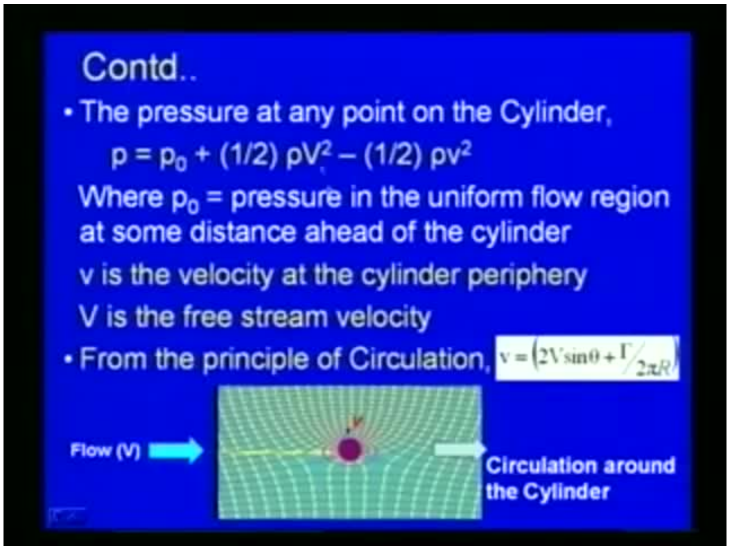

So, this case; so, the pressure at any point on cylinder, we can derive as, p is equal to p 0 plus half rho V square minus half rho small v square, where rho p 0 is the pressure in the uniform flow region at some distance ahead of the cylinder, and small v is the velocity at the cylinder periphery, and capital V is the free stream velocity.

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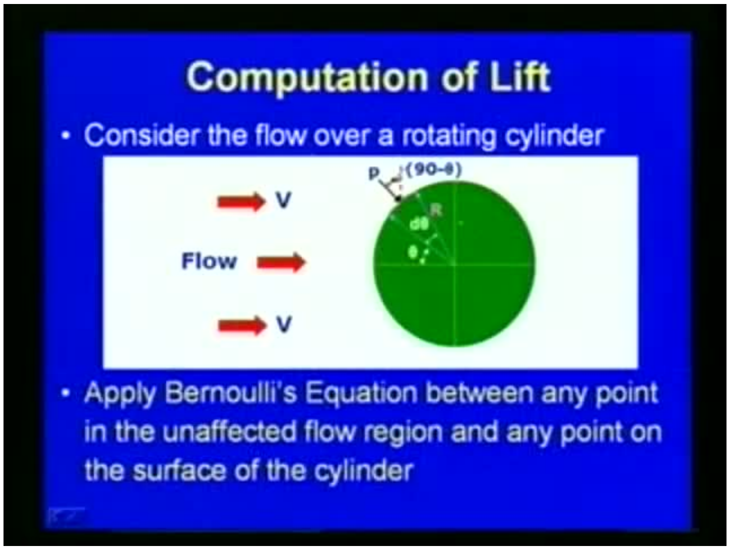

So, we can write the pressure at any point on the cylinder here, at any particular point p, so, if we consider the p 0 as the pressure outside the cylinder which is not affected, so that we can see here, p is equal to p 0 plus half rho capital V square minus half rho small v square, where capital V is the free stream velocity and small v is the velocity at the cylinder periphery.

So, now, we have seen the circulation aspect earlier, so from the principle of circulation, we can write, we can see that the velocity on the periphery, that means, the velocity on a cylinder periphery, we can write it as v is equal to 2 capital V sin theta plus this gamma circulation divided by 2 pi R; so, this we can get from the principle of circulation. So, here, this is the case we consider; this is the rotating cylinder and free stream velocity or free stream flow with the velocity capital V is this direction; so, the circulation around the cylinder when we consider, we can write the velocity at the cylinder periphery v is small v is equal to 2 V 2 capital V sin theta plus gamma by 2 pi R.

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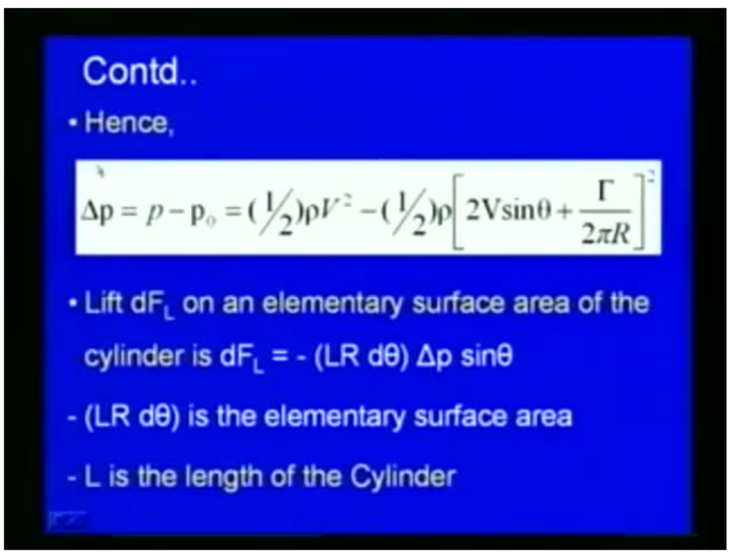

So, now, if we use this expression, in the previous expression which we have seen here (Refer Slide Time: 16:18), p is equal to p 0 plus half of V square minus half rho small v square; so, if we use this, then we get the pressure difference delta p is equal to p minus p 0 is equal to half rho V square half rho capital V square minus half rho into 2V sin theta plus gamma by 2 pi R whole square. So, where small v, we substituted for this small v here, which is equal to 2 capital V sin theta plus gamma by 2 pi r I, in this expression. So, we get the pressure difference delta p is equal to p minus p 0, that is equal to half rho V square minus half rho into 2V sin theta plus gamma 2 pi r whole square.

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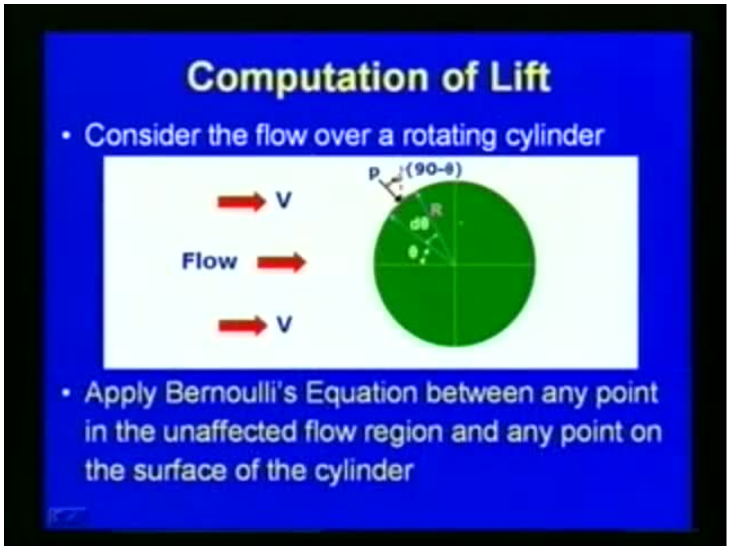

Now, we lift dF L on an elementary surface area of the cylinder, so, if you consider, in the previous figure here, if you consider the elementary surface area, then we can get this, this dF L can be written as dF L is equal to minus LR d theta delta p sin theta.

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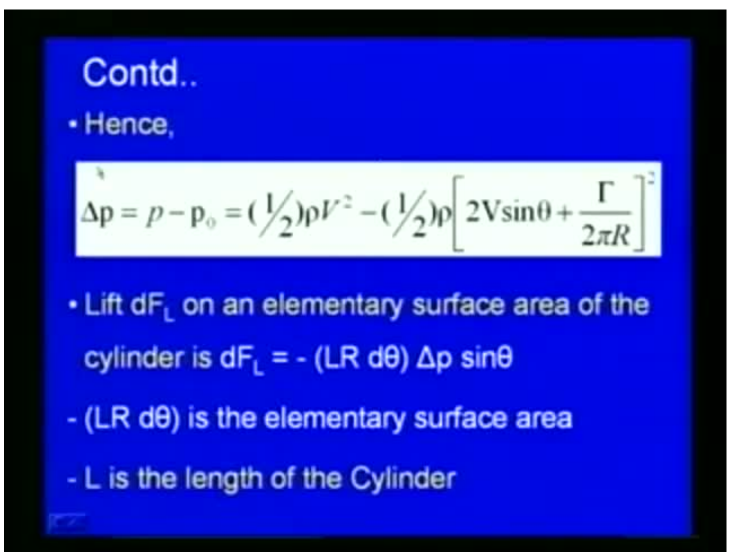

So, this LR, where L is the length of the cylinder, so, R is the radius of the cylinder, LR d theta is the elementary surface area and we will get the lift on the elementary area as dF L is equal to minus LR d theta delta p sin theta; so, we can see here, with respect to this figure, if we consider this elementary area, and we get, as far as lift is concerned we get the expression as, dF L is equal to minus LR d theta into delta p sin theta.

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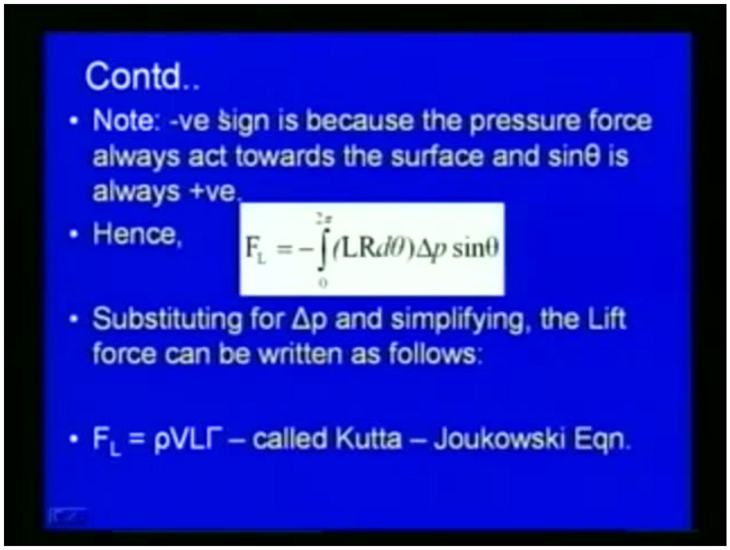

So, here this negative sign is because of the pressure force always act towards the surface and sin theta is always positive. So, finally, if we integrate the lift force F L is equal to minus integral 0 to 2 pi LRd theta delta p sin theta; so, we get the lift force on the rotating cylinder F L is equal to minus integral 0 to 2 theta LRd theta delta p sin theta. So, this delta p, we have already seen the value of delta p here; so, delta p is already defined here, and then, substituting for delta p and simplifying, the lift force can be written as follows, F L is equal to rho VL into gamma.

So, where the lift force on the cylinder, F L is equal to rho VL gamma, where rho is the density of the fluid, capital V is the free stream velocity, L is the length of the cylinder and gamma is the - capital gamma - is the circulation. So, this expression is called the Kutta-Joukowski equation; so this expression independently derived by Kutta-Joukowski, and this expression is called as the circulation, with respect to circulation here the lift force on the cylinder equal to rho VL gamma.

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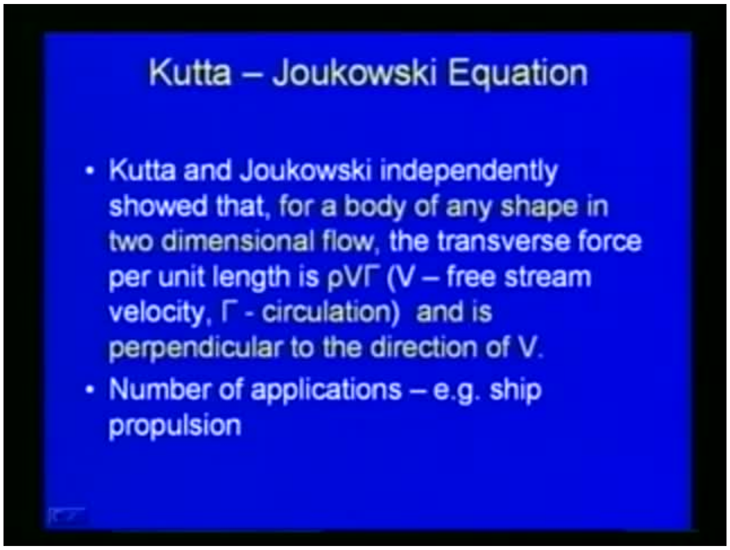

So, this Kutta-Joukowski equation is one of the important equation; and Kutta and Joukowski, independently showed that for a body of any shape in two-dimensional flow, the transverse force per unit length is rho V gamma, where V is the free stream velocity, and gamma is the circulation, and rho is the density of the fluid, and this force is perpendicular to the direction of V; so, this is one of the important expression as far as this lift is concerned. And then, we can see that this Kutta-Joukowski equation has got number of applications like, in the ship propulsion, and like that, we can see many applications in the case of mechanical engineering also. So, Kutta-Joukowski equation is one of the important equations and this gives an expression for the lift force.

So, as we have seen here, in the previous slide, lift force is equal to rho VL t; so, if you consider unit length for, we have seen here, for the case of a rotating cylinder, but they have shown that the body final shape and this equation is valid. If we consider this a two-dimensional flow, so that, a lift force is given as rho V gamma, where rho is the density of fluid, capital V is the free stream velocity and gamma is the circulation. So, this Kutta Joukowki equation is one of the important equations and a number of applications are there

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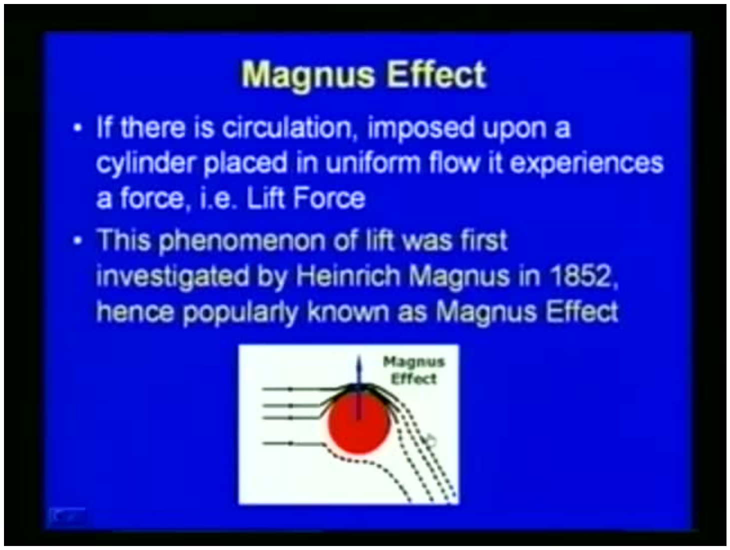

And another important aspect with respect to this circulation and lift force is called the Magnus effect. So, here, if you consider a, either a rotating ball or a rotating cylinder like this, and a free stream flow is coming like this, so, this lift effect which we have discussed earlier. So, if there is circulation imposed upon a cylinder, placed in uniform flow, it experiences a force, that is a lift force. So, this phenomenon of lift was first investigated by Heinrich Magnus in 1852 and this effect is called Magnus effect. So, the free stream flow is coming, and if there is a circulation for the circular cylinder as we can see, so, this effect is called Magnus effect, and then, this has also has got number of applications.

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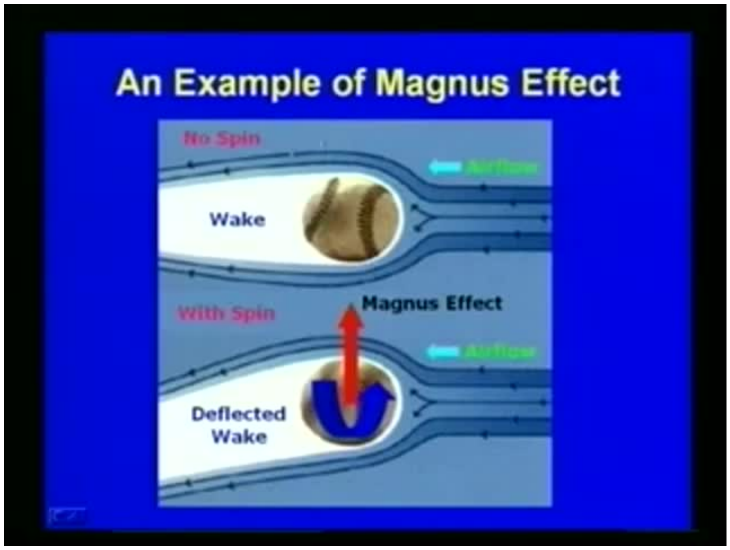

So, say for example, if you consider, say, either a cricket ball or tennis ball, and then if there is no spinning, the ball is not spinning, it is, we are just throwing like this, just throwing without spinning, then you see that the effect is shown in this figure here. There is say no spin; you can see that the body the ball is going like this, but now, you can that, if there is a spin, that means, you are also, while the, while it is throwing or if there is a flow, and then also, you are spinning the ball, then you can see that there would be a lift effect, there will be a lift effect and that is called the Magnus effect. You can see that here, it is shown in this figure here; so, this is the ball and it is also rotating or it is spinning, so that whenever it moves, you can see that this so-called Magnus effect is there.

So, this effect is very important; you can see that when we play either cricket or tennis or golf or any where we use balls like this, you can see that while throwing the ball or while batting the ball, if say spinning is also provided, then you can see that there will be lifting effect other than the movement of the ball due to the spinning. So, this so-called Magnus effect is very important. We have to consider this effect in many cases like, when we play cricket or when we play tennis, say like that, the spinning effect has got a lift effect on the ball. So, that is very important in many of the problems; so, we have to consider this Magnus effect.

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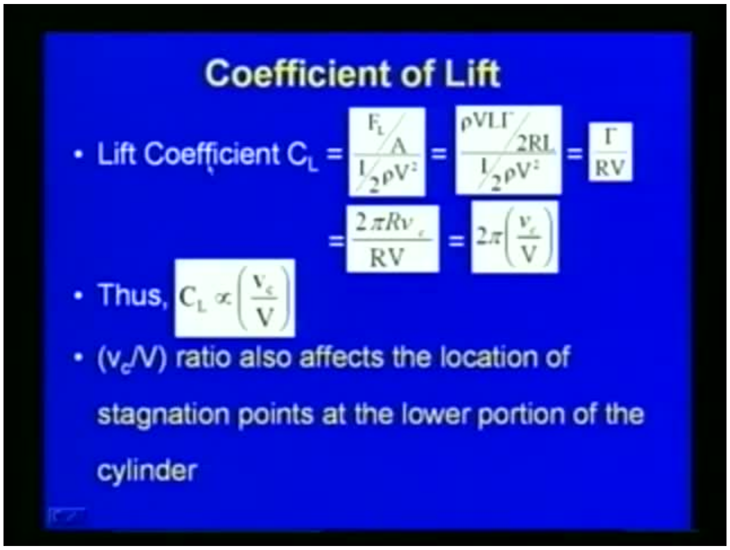

So, now, we have seen the lift force and lift coefficient with respect to the circulation, and also we have seen the Magnus effect; and now, with respect to this, the circulation effect, when we consider the lift coefficient, we can write C L is equal to the lift force by area divided by half rho V squared. So, now, if you consider the circulation also, then we can write this F L is equal to rho VL gamma, where V is the free stream velocity and L is the length of the cylinder which we consider and gamma is the circulation. So, this area is concerned, if we consider a cylinder and you can see that it will lead to 2RL, so, rho VL gamma by 2RL divided by half rho V squared; so that is equal to gamma by RV.

So, this is the lift coefficient CL, and now, this is equal to, this is also equal to, now, if you substitute for this gamma say with respect to the earlier discussion, with respect to earlier discussion we can write gamma is a also equal to 2 pi Rv c, where this small v c is the velocity on the periphery of the cylinder; so, 2 pi Rv c by RV; so, this is equal to 2 pi into v c by capital V, where capital V is the free stream velocity.

So, finally, the lift coefficient, we can show that it is a ratio; it is proportional to the velocity at the periphery of the cylinder divided by free stream velocity. So, CL is proportional to v c by capital V, so, this v c by V is the ratio, it is the ratio which affects the location or the stagnation point at the lower portion of the cylinder; so, CL is a function of v c by V or it is proportional to v c by V.

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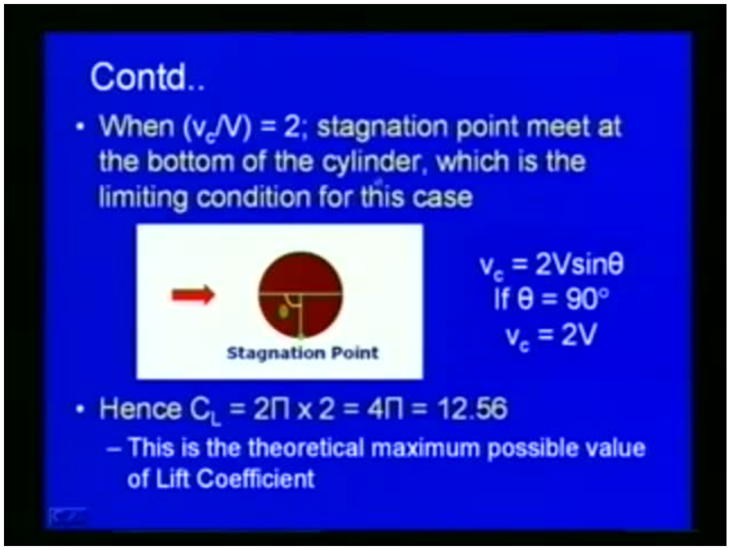

So, now, you can see that, when we analyze various cases, when this v c by V is equal to, you can see that stagnation point meet at the bottom of the cylinder, as in this case here, which is the limiting condition for this case. So, we have already seen this v c, that means, the velocity on the periphery of the cylinder, v c is equal to 2V sine theta, and this will be maximum when theta is equal to 90 degree. So, if theta is equal to 90 degree, we can write v c is equal to 2 into V.

So, hence, we can write coefficient of lift which we discussed; this CL here, we have already discussed. This CL will be equal to 2 pi, 2 pi is the… here we have seen that 2 pi into v c by V, so, v c is equal to 2V or v c by V is equal to; so, coefficient of lift is equal to 4 pi, which is equal to 12.56, and this is the theoretical maximum possible value of lift coefficient for the case, which we considered as in the case of a rotating cylinder. So, like this, for various other cases also, we can check for the coefficient of lift with respect to the circulation effect.

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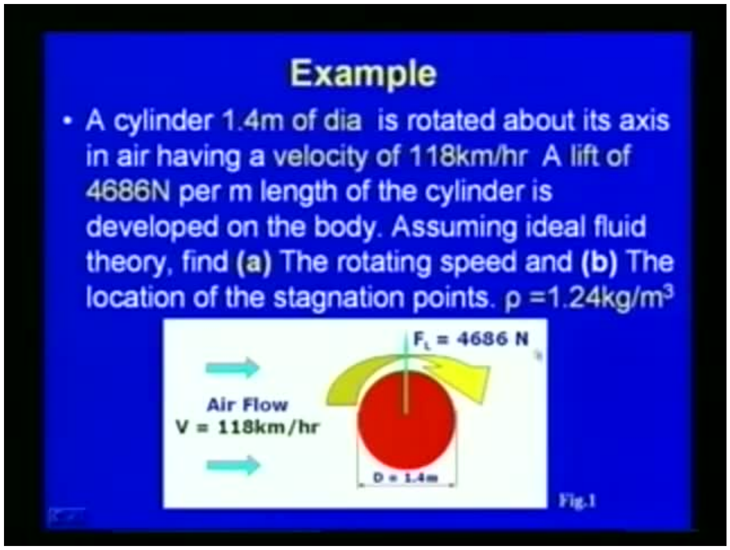

So, now, we have seen the coefficient of lift, and the circulation, and the circulation effect on the lift force is concerned. And we have the case for the circular cylinder. So, before, further we discussed the case of an airfoil and its lift effect with respect to the circulation, we just discuss a small example here.

So, the example problem is a cylinder 1.4 meter of diameter is rotated about its axis in air, having a velocity of 118 kilometer per hour, a lift of 4686 Newton per meter length of the cylinder is developed on the body, assuming ideal fluid flow theory, find (a) The rotating speed and (b) The location of the stagnation point. And the fluid density is given rho is equal to 1.24 kilogram per meter cube. So, now, the problem here is, we have rotating cylinder and the diameter of the cylinder is 1.4 meter as shown in this figure. And the free stream velocity is the, which is airflow, the free stream airflow is 118 kilometer per hour, and it is observed that the lift force is 4686 Newton in this direction. So, we have to find out rotating speed of this cylinder and the location of the stagnation point for the problem.

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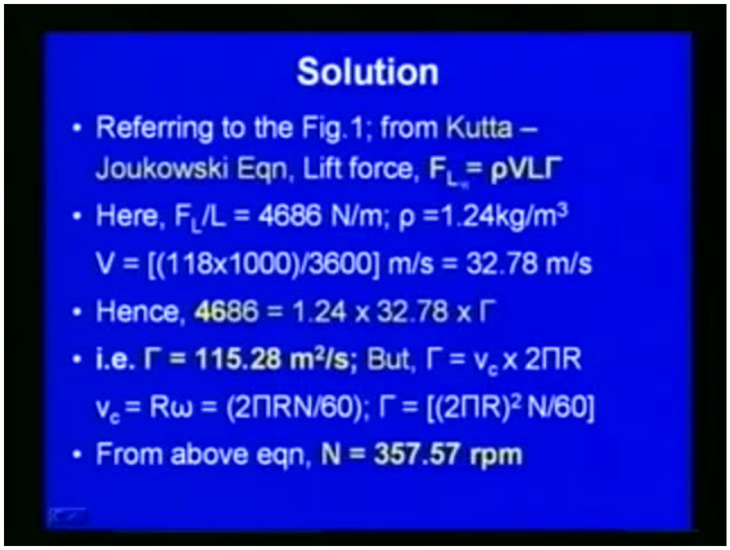

So, now, as we discussed earlier, so, with respect to the figure, we can use the Kutta-Joukowski equation here, so that, lift force is obtained as F L is equal to rho VL gamma. So, here, this already, the lift force per unit length is given as 4686 Newton per meter and rho is equal to 1.24 kilogram per meter cube, so, that we can get here; this we can find out gamma; so, before finding circulation gamma, we will convert this velocity, free stream velocity, in terms of meter per second. So, it is given as, in the previous case, the airflow is 118 kilometer per hour, so, if we convert it into meter per second, 118 into 1000 by 3600 into meter per second, that is equal to 32.78 meter per second.

So, now, if we use this relationship F L is equal to rho VL gamma, so that, we will get 4686 is equal to 1.24 into 32.78 into circulation gamma; so that, we get the circulation gamma is equal to 115.28 meter squared per second. But also, we can see that in the case of a circular cylinder like this, gamma is equal to the velocity on the periphery v c into 2 pi R, so, and also, we can see that v c is equal to R into omega, where the omega is revolutions per minute rpm of the of the, with respect to the omega is the angular velocity. So, with respect to the rpm, so, v c is equal to, we can write R omega, where omega is the angular velocity, so, that is equal to 2 pi RN by 60, where N is the rpm revolutions per minute; so that, finally, we get gamma is equal to 2 pi R whole squared into N by 60. So, from which, we can find out this number of revolutions per minute n is equal to 357.57 rpm revolutions per minute. So, this gives the speed of the cylinder.

So, now, the second part of the question is, we have to find out the stagnation point; so here, for this problem here, we want to get the location or the stagnation point, so, we have already found the rotating speed.

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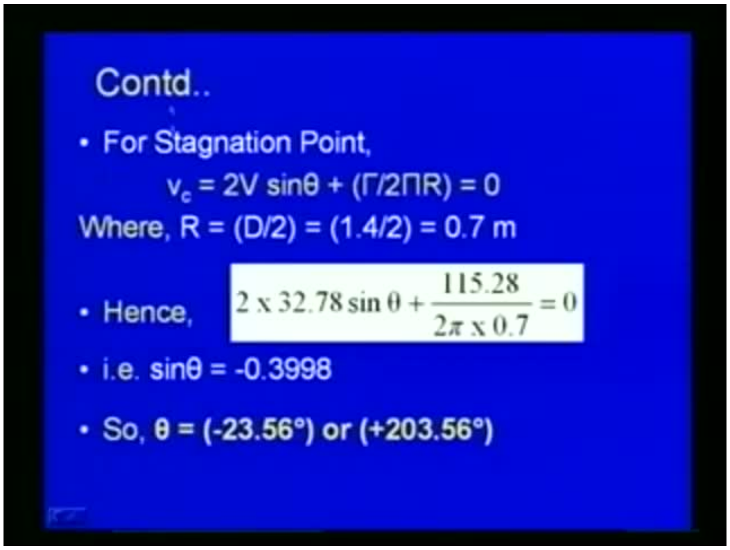

So, if we put the, to find out the stagnation point, we can see that the velocity on the periphery should be equal to 0; so, we can write v c is equal to 2 V sin theta plus gamma by 2 pi R, the circulation by 2 pi R, that should be equal to 0; so here, this R is equal to D by 2, that is equal to 0.7 meter, so that, we can write this expression, we can equate to 0 so 2 into 32.78 sin theta plus 115.28, which is the circulation we have already calculated, divided by 2 pi into 0.7, that is equal to 0; so that, finally, we get sin theta value as minus 0.3998; so, theta is equal to minus 23.56 degree or 203.56

So, like this, this is a simple example. Like this, various other problems we can solve. So, by considering the problem with the circulation, say we have to get consider, the circulation effect, and then we have to use the Kutta-Joukowki equation, which we have derived earlier. So, this is the case, the lift force and the effect on of circulation on the lift force, so, we have seen with respect to this example.

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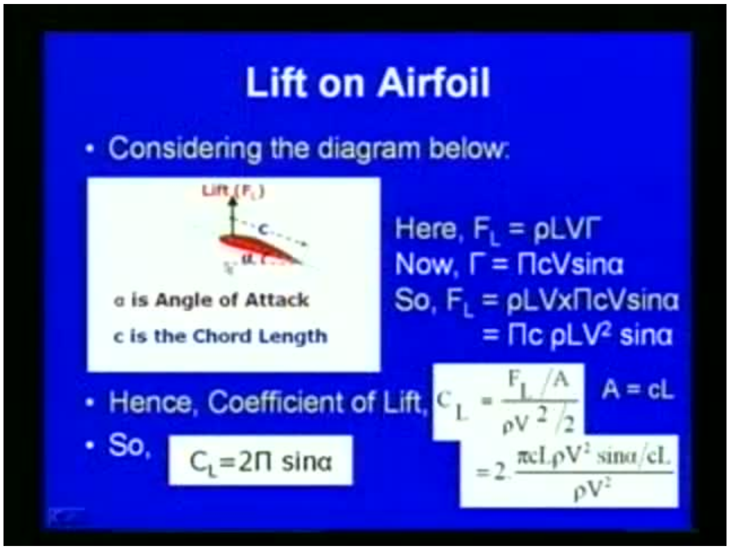

So, now, we will discuss in detail about the lift effect on an airfoil. So, let us consider the lift on an airfoil. So, if you consider an airfoil like this in this slide, you can see that the airfoil is oriented like this. So, there the alpha is the angle of attack, and if you consider c as the chord length, and you can see that the lift force effect is coming like this. So, if we use the Kutta-Joukowski equation, we can write, say if circulation is also considered, we can write F L is equal to rho LV gamma, where gamma is the circulation, that is equal to pi c into V sin alpha, where alpha is the angle of attack, and c is the chord length, and V is the free stream velocity; so that, finally, we can write the lift force, as far as this airfoil is concerned, we can write F L is equal to rho LV into pi cV sin alpha; so, that is equal to pi c into rho LV squared sin alpha, where c is the chord length for the considered airfoil.

So, now, we generally express the lift effect with respect to the coefficient of lift. So, we will write the lift effect on the airfoil as the coefficient of lift, so that, we can write C L is equal to, coefficient of lift, is equal to F L, the lift force by area divided by rho V squared by 2; so, here, for this airfoil is concerned, A is equal to c into L, where c is the chord length, and then we can write, this can be put as, so, we have already derived F L is equal pi c rho LV squared sin alpha; so, we can substitute that here, and a is here, so, we can put it back, and finally, we will get the coefficient of lift for the airfoil as, 2 pi sin alpha. So, here, we have derived the coefficient of lift for the airfoil, so, by considering the lift.

So, here, you can see that, this only the coefficient of lift is only a function, 2 pi is, 2 and pi are constants, so, only it is a function of the angle of attack. So, whenever you can see that this airfoil is placed in such a way, that this alpha is equal to 0, then you can see that the coefficient of lift will be 0; so, lift will be also 0. So, to generate lift, we have to keep this at some angle; so, this angle of attack is the most important parameter, as far as the airfoil is considered here.

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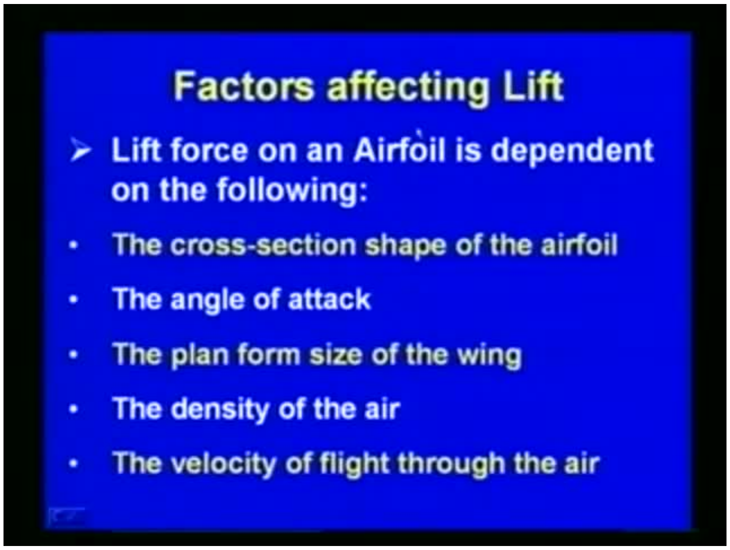

So, with respect to the airfoil which we analyzed just now, we can see the important parameters or the important factors affecting the lift or lift force on an airfoil is dependent upon the cross-section shape of the airfoil; so, what is the shape of the airfoil, which way we designed the airfoil, and then of course, the most important parameter is the angle of attack, and then, the plan form size of the wing. So, if we consider the aircraft, then with respect to the airfoil which we discussed, so, how we put the wing, such way that the plan form size of the wing, so, that was an another important effect with respect to the lift. And then, now, of course, the density of the air, so, since we have seen that Kutta-Joukowki equation which we have seen earlier, you can see that F L is equal to rho LV gamma. So, this depends upon the density of the air and the velocity of the tflight through the air.

So, these are some of the important parameters of, which affects the lift. So, if you want to optimize the lift, say for the maximal lift, as far as the aircraft is concerned, we have to design it in such a way that, the shape of the airfoil should be optimized. And then, the angle of attack whenever the aircraft is flying, then the angle of attack should be in such a way that it will get a maximum lift, and then the plan form size; then, other important parameters are the density of the air, and then of course, the velocity of the air. So, based upon these parameters, we can optimize the design in such a way that we get the best effect, as far as lift is concerned.

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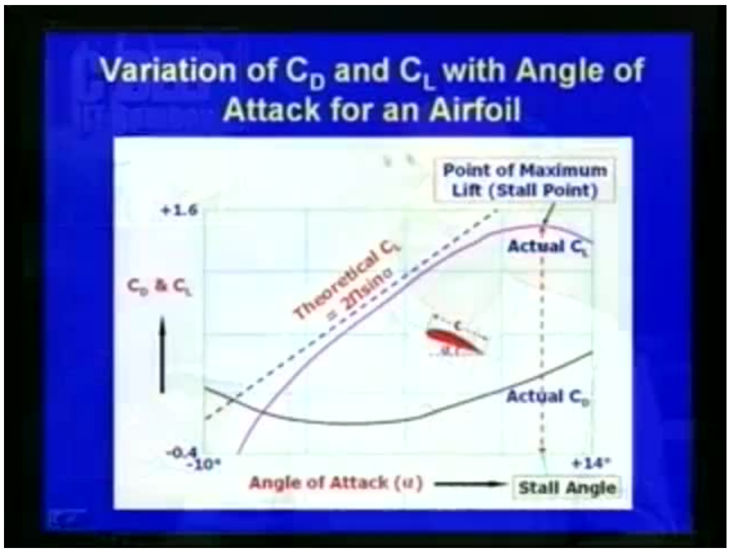

So, now, through experiments, we can conduct various experiments in the laboratory, and then we can observe the effect of this angle of attack on the, for example, in the case of an airfoil. So, if you plot the angle of attack on the x-axis, and then, if you plot the coefficient of drag and coefficient of lift on the y-axis, then you see that, this as far as coefficient of lift is concerned, you can see that it is going like this. Whenever, say here, starting from this, where it is going? Like this, it is increasing, keep on increasing, so actually, even the theoretical coefficient of lift is 2 pi sin R, for which we represent this line. But reality will be, we can see that instead, it will be coming like this; and then, we reach a maximum point or the point of maximum lift, where we will define a point called stall point and then it reduces.

So, similarly, the drag coefficient, we can see that it is going like this; so this, with respect to these various, this plot, we can obtain through various experiments by placing the airfoil at different angles. And then, we can get the, we can plot the coefficient of drag and coefficient of lift.

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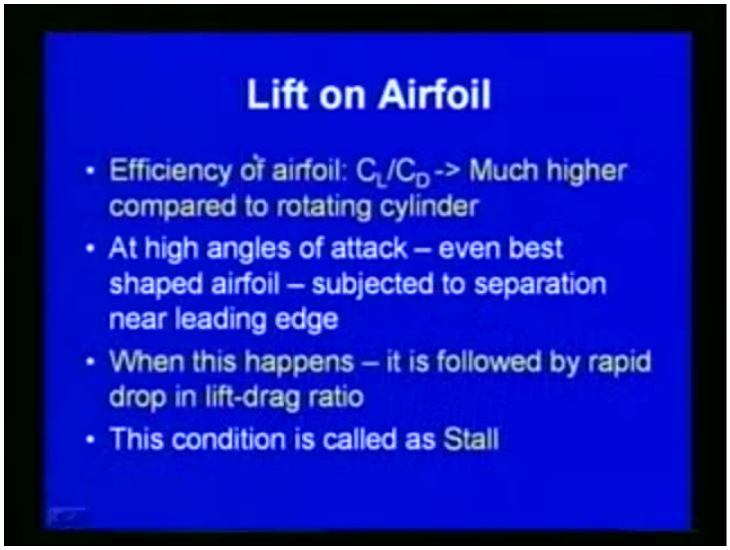

So, now, as far as the lift on the airfoil is concerned, say, the efficiency of the airfoil, you can see that it is obtained as a ratio of coefficient of lift to coefficient of drag, so, you can see, that it is much higher compared to the rotating cylinder. So, the rotating cylinder which we have discussed earlier, if we compare with respect to the airfoil which we discussed now, you can see that here C L by C D will be much higher; and, at high angles of attack, in the case of airfoil, we can see that even best shaped airfoil is subjected to separation near leading edge. So, due to the, when the angle of attack is higher, then you can see that there will be, say separation near the leading edge, and when this happens, it is followed by rapid drop in the lift-drag ratio and this condition is called as stall.

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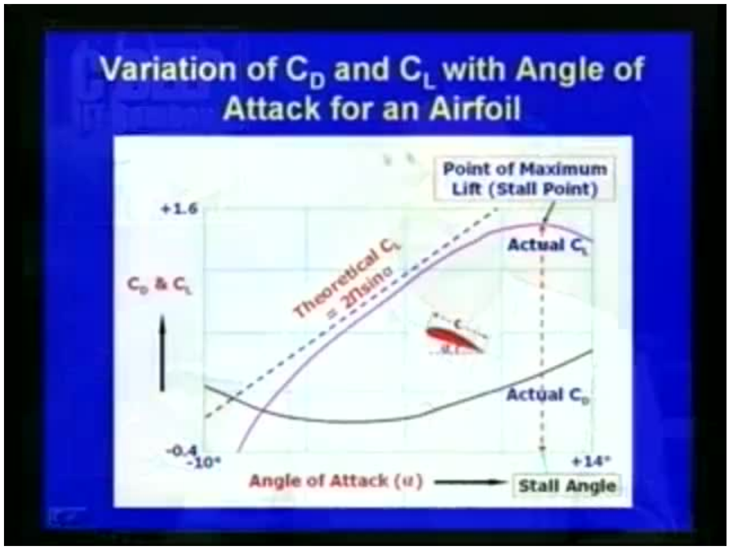

So, here, you can see that, whenever we keep on increasing the angle of attack, so initially, the lift is, coefficient of lift is increasing, and then it reaches a maximum point, and then you can see that after this, there is a sudden drop as far as coefficient of lift is concerned; and this location, this condition is called as stall, so, where the rapid drop in lift-drag ratio takes place. So, now, say with respect to this whatever we have discussed is, say, how the angle of attack is important, as far as the design of the airfoil, and now, we have seen that when it reaches, say, we cannot keep the angle of attack to, say, after a certain limit, it the coefficient of lift is decreasing, which we have seen as the condition as stall.

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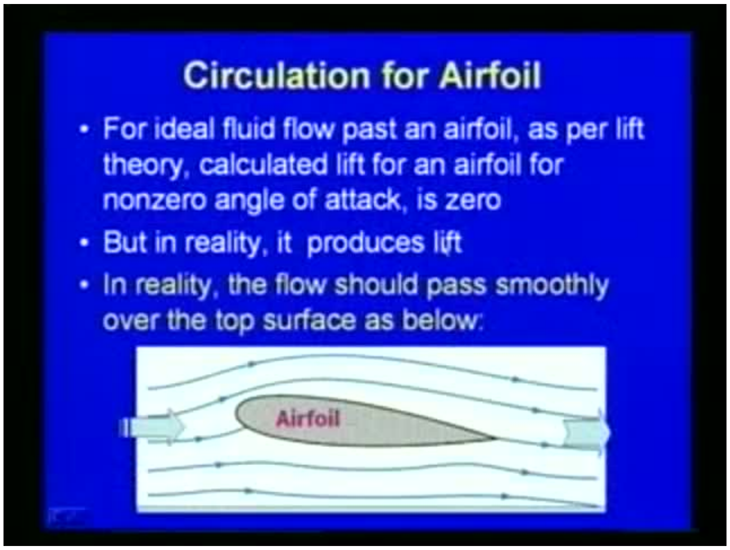

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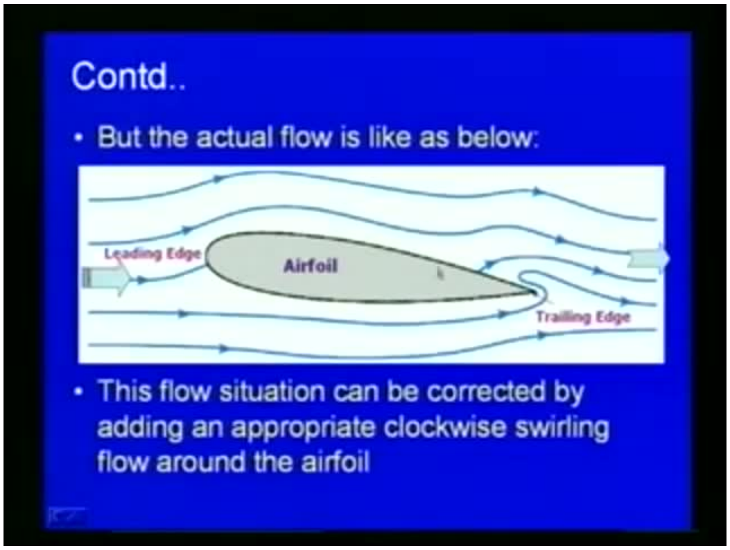

Now, let us discuss more about the circulation effect as far as airfoil is concerned. So, we have already seen the angle of attack effect, as far as the airfoil is concerned; so, but from the theories, we can see that for ideal fluid flow past an airfoil, as per lift theory, the calculated lift for an airfoil for non-zero angle of attack, actually, theory says the theory gives as 0, the say calculated lift will be 0, but, in reality, you can see that lift is produced. So, in reality, the flow should pass smoothly over the top surface as below.

So, you can see that airfoil is placed here, even in a at an angle of attack, there should be smooth flow over the top surface like this; but, say, this should be the condition, but you can see that, due to the angle of say when we keep it as at an angle or the non-zero angle of attack, you can see here, but the actual flow is like this; so, you can see that, if this is the leading edge and airfoil placed at an angle of attack, and then, you see that here, the streamlines are plotted; so, here this trailing edge is concerned, there is, say, there is disturbance takes place at the trailing edge, and then, say, the actually due to the angle of attack here, actually what happens is, there is a circulations produced. So, we can explain this lift effect, as we have seen, the theoretically there should not be any lift, but actually, lift is happening for the case of an, airfoil placed at an angle of attack, so this, we can explain with respect to the swirling flow around the airfoil.

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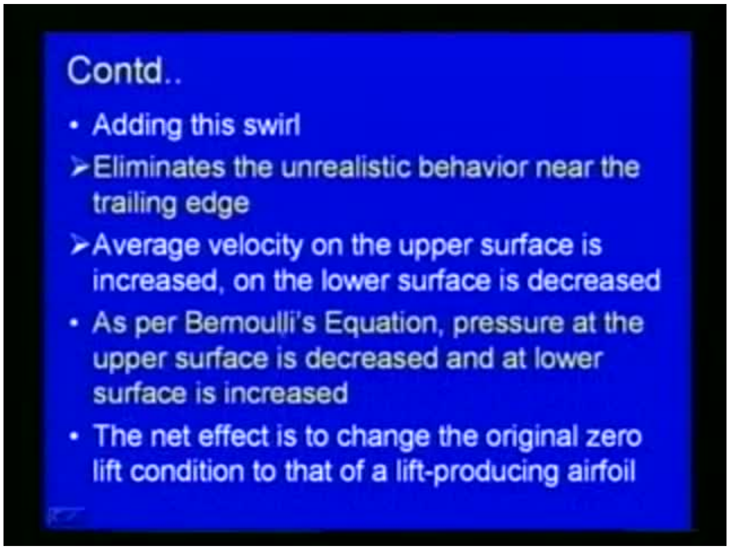

So, you can see that, since the airfoil is placed at an angle of attack, then, say, as described in the figure here, this and the trailing edge, then, the stream, if you plot the stream; now, then you can see the disturbance takes place and then a swirl takes place with respect to the airfoil; so, adding of this swirl, eliminates the unrealistic behavior near the trailing edge. So, this phenomenon of the lift with respect to the airfoil, we can explain with respect to this swirl effect or the circulation effect. So, the average velocity on the upper surface is increased, on the lower surface, if average velocity on the upper surface is increased, then in the lower surface, say, the effect is taking place and, as per the Bernoulli's equation, we can see that the pressure at the upper surface is decreased and at lower surface is increased.

So, the average velocity on the upper surface is increased and lower surface, it is decreased. And similarly, a pressure at the upper surface is decreased, and at the lower surface it is increased; so, that is, with respect to this swirl effect and the net effect is to change the original 0 lift condition to that of a lift-producing airfoil; so, this is the way which we can explain. And, what is really happening with respect to, say when the airfoil is placed with respect to an angle of attack.

So, if you consider the swirling effect with respect to the angle of attack, then we can see that, say, the velocity on the upper surface will be increased, and then the lower surface will be decreased. And similarly, the pressure at the upper surface is decreased, and at the lower surface is increased. So, finally, this effect, the net effect is a change in the original 0 lift condition, and then lift is produced as far as the airfoil is concerned.

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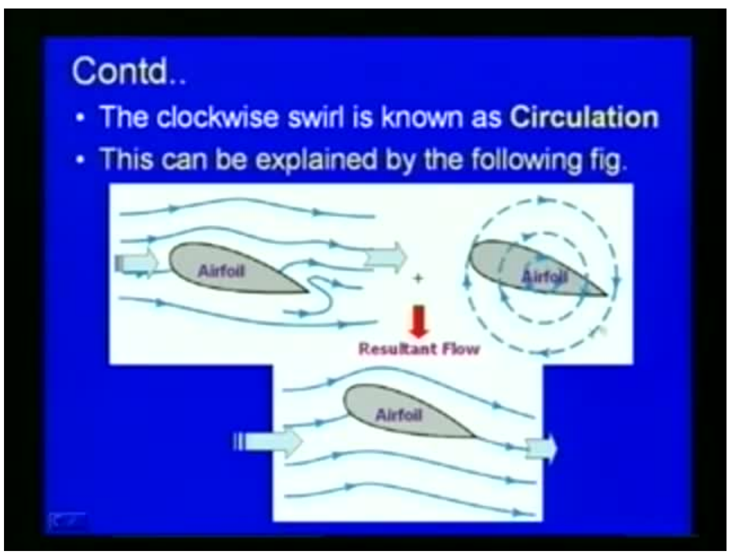

So, this effect we can explain with respect to this figure here. So, this clockwise swirl is actually the circulation which we discussed earlier; so, if you consider, say, here, the airfoil at an angle of attack, and the free stream velocity is coming, so, here, this disturbance takes place at the trailing edge; and then, if you add this, the circulation effect or the swirl effect, which we have discussed here, so here, this is the airfoil with respect to the flow and the disturbance, and then plus, say, here, this effect of circulation takes place, and finally the resultant flow is, say, with respect to the finally, if you add together, then the resultant flow will be like this; and then, finally, we can see that lift is produced.

So, here, say even the theoretically, as far as airfoil is concerned, there may not be lift, but practically, what happens is that, this, at the trailing edge, the disturbance takes place; and finally, we will be considering the circulation effect; and then, finally, the flow will be like this. So, this way we can explain the lift effect coming on an airfoil.

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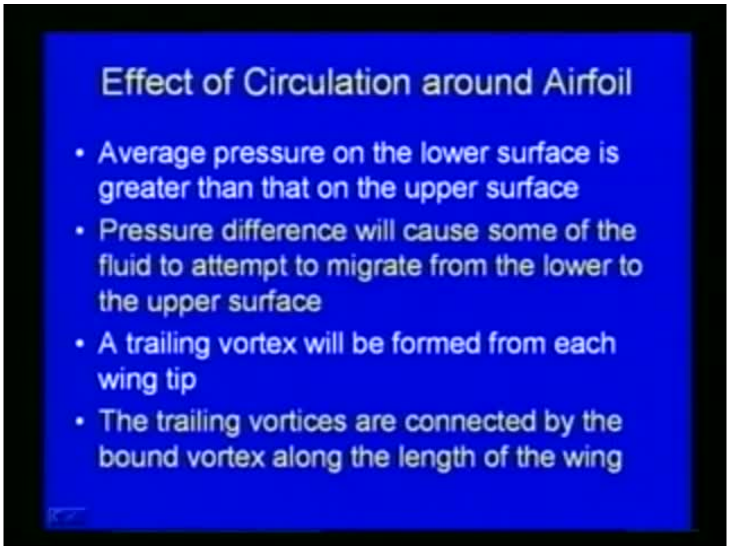

So, this effect of circulation around an airfoil, so, you can see that the average pressure on the lower surface is greater than that on the upper surface. And, this pressure difference will cause some of the fluid to attempt to migrate from the lower to the upper surface. And finally, a trailing vortex will be formed from each wing tip. So, if we consider the aircraft wing, then what happens is that, so, this, due to this pressure difference, then some of the fluid will be migrating to from the lower to the upper surface; and then, finally, trailing vortex will be formed from each wing tip. And then, the trailing vortices are connected by the bound vortex along the length of the wing.

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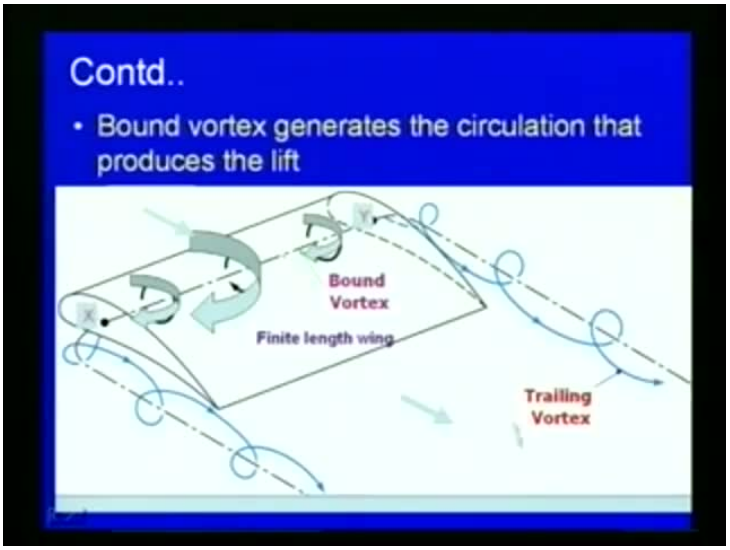

So, here, you can see that the vortex, how it is generated with respect to circulation. So, here, this is the wing of the aircraft, then you can see that the free stream, the airflow is in this direction; and here, you can see the formation of the bound vortex, and then, also you can see that trailing vortex will be formed at the, at the... This is the trailing vortex here, and this location, this is the trailing vortex; and in similar way, this, this side also, and finally, the bound vortex generates a circulation that produces the lift effect as far as the aircraft wing is concerned. So, this we can show with respect to the airfoil theory, airfoil, the lift on the airfoil and circulation effect, which we have discussed earlier.

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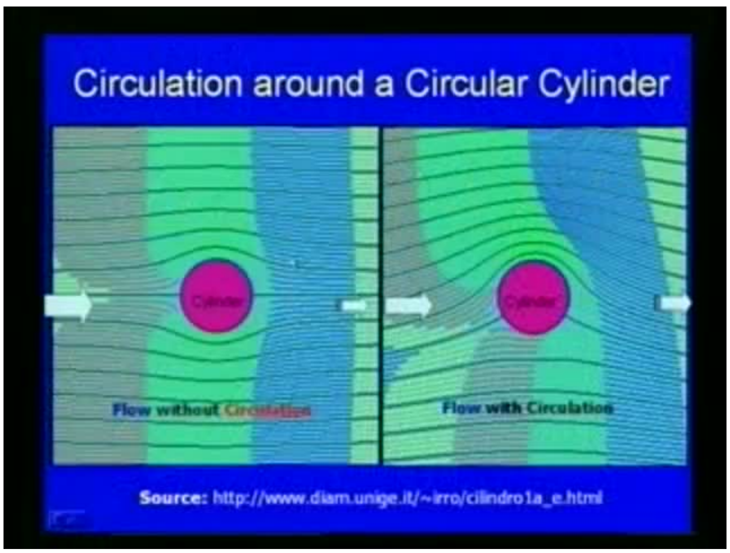

So, now, if we consider, instead of the airfoil, if you consider the circulation around a circular cylinder, which we discussed earlier, then you can see that this is the cylinder here, and then, the free stream velocity is coming and if you plot the streamlines; so, flow without circulation will be like this; that means, the cylinder is not rotating. So, here, if you the flow without circulation is going like this. But, if the cylinder is also rotating, that means, flow with circulation is, you can see that, what is the effect happening here; you can see the changes in the pattern of the streamlines, and this figure is taken from this website here, put here. So, the circulation around a circular cylinder without circulation, that means, here, there is no circulation effect and flow with circulation.

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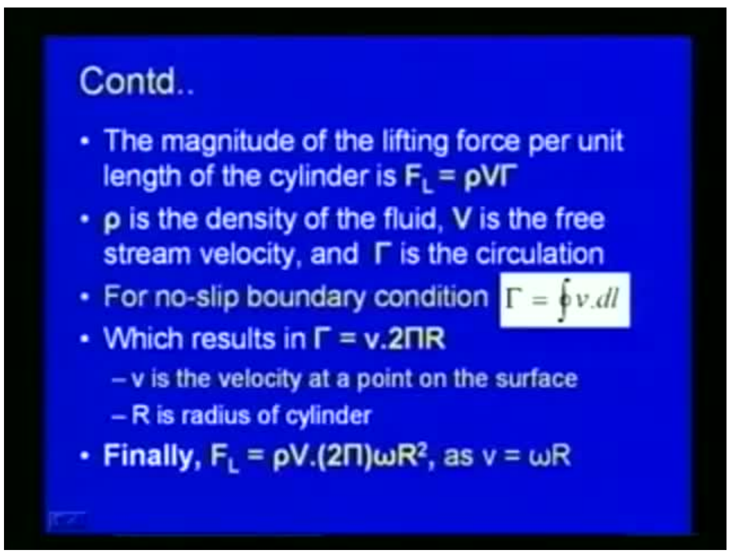

So, magnitude of the lifting force per unit length of the cylinder, as we have discussed F L is equal to rho V gamma, where gamma is the circulation, and rho is the density of the fluid, V is the free stream velocity; and say, for no-slip boundary condition, as we have discussed, gamma is equal to integral of v dl which results in gamma is equal to v into 2 pi R, where v is the velocity at a point on the surface and R is the radius of cylinder. And finally, we can write F L is equal to rho v 2 pi omega R squared as v is equal to omega R, where omega is the angular velocity and R is the radius of the cylinder. So, this effect we can see here.

So far, we have seen the lift force and coefficient of lift, and then the importance of circulation. So, we have seen the case where, if a cylinder is rotating or if not rotating, what are the effects? That means, with respect to circulation, the flow and with respect to circulation, and also we have seen an airfoil with say 0 angle of attack; there is no lift effect, but, when there is an angle of attack is there, then there is lift effect, is there, so that, what is the theory behind, that we have discussed.

And also, we have seen the various aspects like, you know, Magnus effect, that means, when we, say spin a ball, say for example, in the case of cricket, when we spin the ball, and then, or tennis, when we spin the ball, then what will be the effect? There will be a lift effect, and then it will be due to the spinning effect, and the lift effect it will be very difficult to predict which direction the ball will be moving. So, that is the effect of this, the rotating or spinning or so-called circulation effect, as far as lift is concerned.

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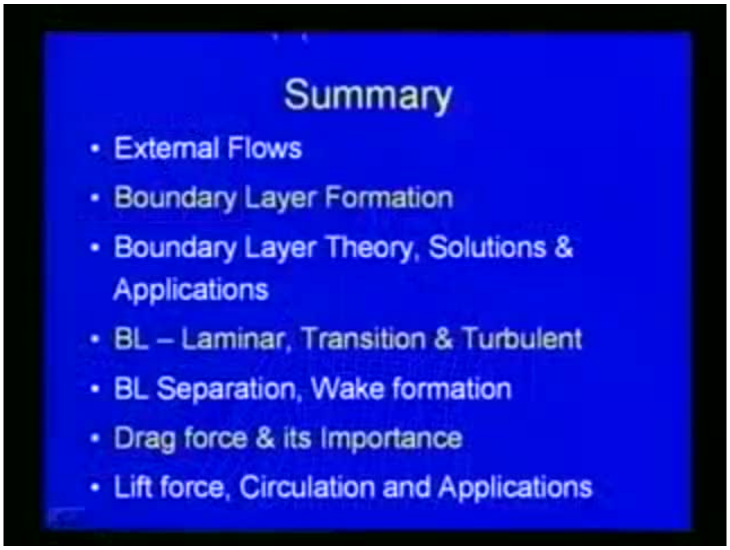

So, with this chapter, say, we are closing this chapter on the boundary layer theories and drag and lift effect. And to summarize…So, in this chapter, we have initially discussed the various kinds of external flows, and then we have differentiated with respect to the internal flow, and then also, we have seen the boundary layer formation, boundary layer theories, Prandtl’s and Karman’s theory, and then, also, various solutions with respect to the boundary layer theories, say, for various parameters like the shear, stress, and then the velocities at various locations, and then, say we have also saw that the boundary layer is concerned, say with respect to the flow over a flat plate, we have seen initially, it may be laminar, and then there will be a transition, and then finally turbulent stage is achieved. So, all these with respect to the boundary layer theory, we have discussed; and also, the boundary layer separation and wake formation also we have discussed in this chapter.

And also, the theory behind the drag and lift we have discussed in detail; and the drag force, and then coefficient of drag, and then to calculate this drag force and coefficient of drag for various cases, whenever the flow is laminar or boundary layer is laminar or boundary layer is turbulent. And also we have seen its importance, the importance of drag as far as, say, in the design of various, say, various problems like automobile industry, in the design of cars or in the design of, say, bus or various other kinds of design. And then in this lecture, we have discussed the lift force, and then the coefficient of lift, and the circulation effect as far as lift is concerned.

So, with this, the external flow here we have discussed. And now in the final chapter, we will be discussing the internal flows or the pipe flow. That we will be discussing in the next lecture.