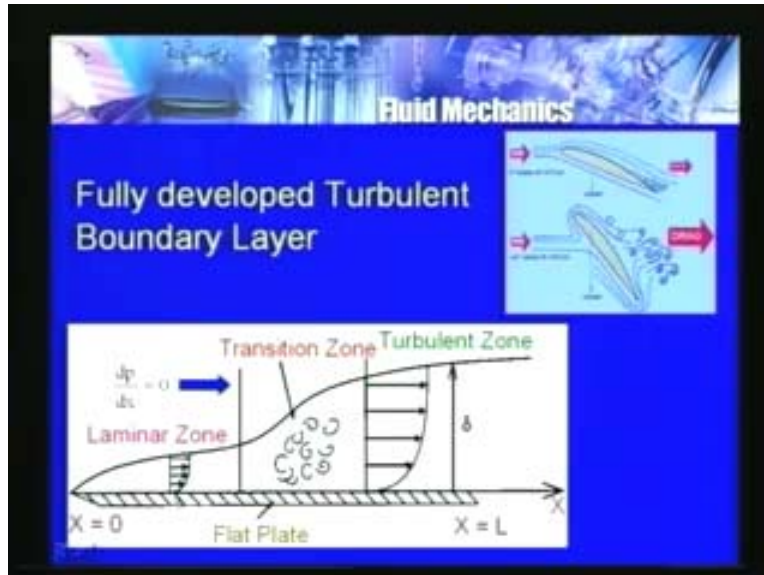


Fluid Mechanics
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Lecture - 34
Boundary layer Theory and Applications

Welcome back to the video course on fluid mechanics. In the last lecture, we were discussing about the fully developed turbulent boundary layer and the drag calculations.

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We have calculated the coefficient of drag and various other parameters as we have seen here. We were discussing about the flow over a flat plate. We have seen that initially there is a laminar zone and transition zone and then we were discussing about the turbulent zone.

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- Velocity distribution in turbulent flow over a flat plate valid for $R_x < 10^7$ $\frac{u}{V_\infty} = 8.74 \left[\frac{\sqrt{f_x/2} y}{V_\infty} \right]^{1/4}$
- From Turbulent flow analysis of pipe, $\tau_0 = 0.034 \rho V^2 \left(\frac{Vd}{\nu} \right)^{-1/4}$... (3)
- Average velocity, $V = 0.817 u_{max} = 0.817 u_\tau$
- Hence, $\tau_0 = 0.0233 \rho u_\tau^2 \left(\frac{u_\tau \delta}{\nu} \right)^{-1/4}$... (4)
- Using (1) (2) and (4) $\frac{\delta}{x} = \frac{0.379}{(R_x)^{1/5}}$... (5)
- Using (5) in (4) $\tau_0 = 0.0295 \rho u_\tau^2 (R_x)^{-1/5}$... (6)

For the turbulent zone, we have seen that starting from the blasius one seventh power law, we have shown the shear stress equations and also we have shown that this boundary layer thickness, delta by x is equal to 0.379 divided by R_x to the power 1 by 5 as in this equation.

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- Local Friction Coefficient, $C_f = \frac{\tau_0}{(1/2) \rho u_\tau^2} = \frac{0.059}{(R_x)^{1/5}}$... (7)
- The Friction drag per unit width for one side of the plate of length L is $F_{tot} = \int_0^L \tau_0 dx = 0.0368 L \rho u_\tau^2 (R_L)^{-1/5}$ and $C_f = \frac{F_{tot}}{(1/2) \rho u_\tau^2 L} = \frac{0.074}{(R_L)^{1/5}}$... (8)
- Eqn. (8) is valid for Reynolds number in the range of $5 \times 10^5 < R_L < 10^7$
- For $R_L > 10^7$, Prandtl calculated $C_f = \frac{0.455}{(\log_{10} R_L)^{2.58}}$... (9)

We have seen the local friction coefficient C_f is equal to 0.059 divided by R_x to the power 1 by 5 and then from the calculations for the Reynolds number greater than 10 to the power 7 Prandtl calculated the coefficient of drag C_f as 0.455 divided by $\log_{10} R_L$ to the power 2.58 .

Now, using these various coefficients, various equations which we have seen we can calculate various parameters like the drag force, the local drag force at various sections and log in the corresponding drag coefficient. Also, we can calculate the total drag on a flat plate. Then, we can also calculate the drag coefficient for the given plate. Before proceeding further we will briefly discuss a small example here.

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Example

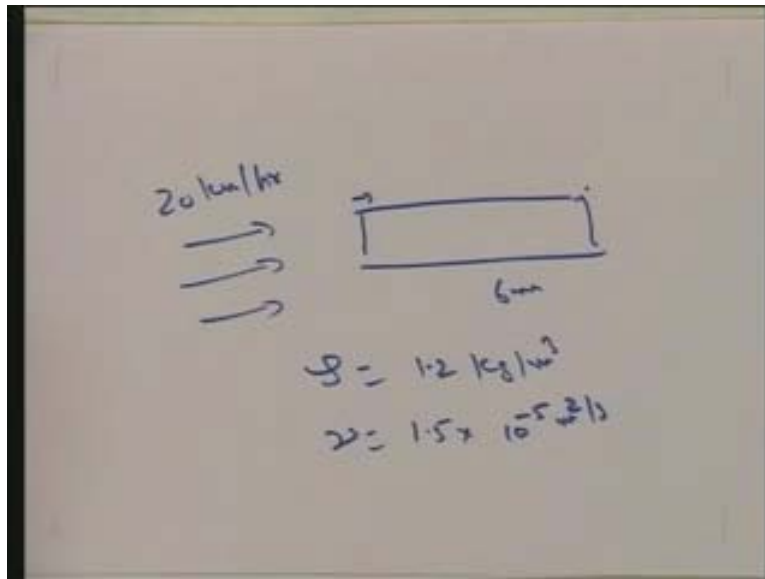
A 20 km/h wind blows over a 6 m flat plate, if the Density and Kinematic viscosity of air are 1.2 kg/m^3 and $1.5 \times 10^{-5} \text{ m}^2/\text{s}$ respectively, calculate the force per meter width of the plate, also estimate the thickness of the boundary layer at the trailing edge

Solution

- Wind velocity, $u = \frac{20 \times 1000}{3600} = 5.56 \text{ m/s}$
- Reynolds number, $R_d = \frac{uL}{\nu} = \frac{5.56 \times 6}{1.5 \times 10^{-5}} = 2.22 \times 10^5$

So, here the example problem is a 20 kilometer per hour wind blows over a 6 meter flat plate, if the density and kinematic viscosity of air are 1.2 kilogram per meter cube and 1.5 into 10 to the power minus 5 meter square per second, respectively. Calculate the force per meter width of the plate and also estimate the thickness of the boundary layer at the trailing edge.

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Here, the problem is that we have a flat plate of size 6 meter and then a 20 kilometer per hour wind is coming and then the density of the layer ρ is 1.2 kilogram per meter cube and kinematic viscosity is 1.5×10^{-5} wind square second. For this case, we have to find out the force per meter width on the plate that means if this is the plate then we have to calculate the force per meter width. Also, we have to estimate the boundary layer at the trailing edge. This is the leading edge; here is the trailing edge; here, we have to find out the boundary layer thickness.

So, to solve this problem first that wind velocity is given as 20 kilometer per hour. We will convert the wind velocity into meter per second, (Refer Slide Time: 04:36) so 20 into 1000 divided by 3600 will give 5.56 meter per second. Initially, we will find out the Reynolds number; once we find the Reynolds number we can identify whether the flow regime is in laminar or transition or turbulent so that we can use the appropriate equations. If you find the Reynolds number, (Refer Slide Time: 05:09) Reynolds number Re_L is equal to uL by ν , u is already obtained as 5.56, so 5.56 into 6 divided by the kinematic viscosity 1.5×10^{-5} ; so that will give the Reynolds number as 2.2×10^6 .

Here, we can observe that the Reynolds number obtained is more than critical Reynolds number, 5×10^5 . So, the boundary layer flow is in the turbulent flow regime. We can utilize the equations which we have seen in the last lecture for the coefficient of friction drag and also various other parameters. So, the coefficient of friction drag for the turbulent flow when the boundary layer is in a turbulent flow regime.

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Soln. (Contd.)

- Coefficient of Friction Drag, $C_{fD} = \frac{0.074}{(Re_L)^{1/5}} = 0.00398$
- Drag force on one side of the plate per unit meter width is given as:

$$F_D = \frac{C_{fD} \times \rho \times A \times u^2}{2} = \frac{0.00398 \times 6 \times 1 \times 1.2 \times 5.562^2}{2} = 0.4429 \text{ N}$$

- Turbulent Boundary layer thickness at the trailing edge ($x = 6\text{m}$) is given by

$$\delta = \frac{0.379x}{(Re_x)^{1/4}} = \frac{0.379 \times 6}{(2.22 \times 10^6)^{1/4}} = 0.1217 \text{ m}$$

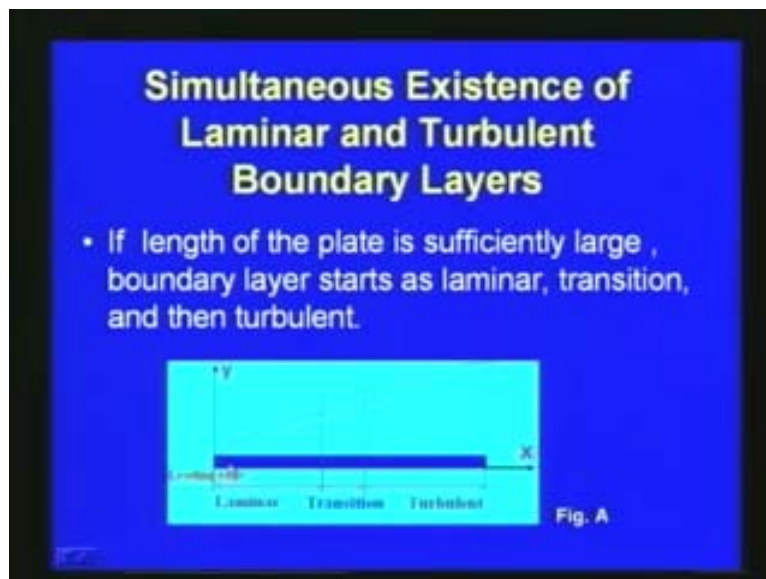
We get C_{DF} is equal to 0.074 divided by Re_L to the power 1 by 5; this if we calculate after putting this Reynolds number in the previous slide we have seen that Reynolds number is equal to 2.22×10^6 , so if you put this we will get C_{DF} - the coefficient of friction drag as 0.00398 and then the drag force on one side of the plate per unit meter width is given as the coefficient of drag multiplied by area into ρu^2 by 2. Here, C_{DF} we obtained as 0.00398 into 6 into 1, we consider 1 meter width of the plate, so 6 into 1 into 1.2 into 5.562 square ρu^2 by 2. That is equal to 0.4429 Newton. Thus, we get the drag force on one side of the plate per meter width.

Then, the second part of the question is to find out the boundary layer thickness at the trailing edge. That means end of the plate as we have already seen here x is equal to 6 meter. We can utilize this equation which we have just seen in the last lecture for

turbulent boundary layer flow, δ is equal to $0.379 x$ divided by R_{ex} to the power $1/5$, that is equal to 0.379 into 6 divided by 2.22 into 10 to the power 6 to the power 1.5 . So, we get the boundary layer thickness as 0.1217 meter. This is a simple example. So like this whatever we have seen so far is the boundary layer and its related drag force, friction drag, pressure drag and all different kinds of drag forces. Based upon what we have studied so far we can solve different kinds of problems to calculate the drag force or to find out the drag coefficient. Also, other parameters like shear stress and the force at particular location like that we can use the various equations that we have seen in the last lecture as far as turbulent boundary layer and corresponding drag is concerned. Now, we have seen the drag coefficient, the drag force and the boundary layer as far as laminar flow is concerned. Also, we are seen the turbulent flow is concerned; in the boundary layer flow is turbulent. Now, we have already seen there can be a transition from laminar flow to turbulent flow.

If the boundary layer is say, if you want to find out various flow parameters or various say, like a coefficient of drag or the drag force, wherever simultaneous existence of laminar turbulent, boundary layer or the transition range comes then we have to use either the laminar concept as well as turbulent concept and we have to find out the various parameters.

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Next topic is simultaneous existence of laminar and turbulent boundary layers. Here, we can see that this is the...(09:07). Here also we consider a flat plate which we have seen in our earlier derivations; the flat plate is placed like this and then the free stream velocity comes in this direction; this is the leading edge and here is the trailing edge. Initially, as we have seen the laminar boundary layer is developed and then we have the transition and then the turbulent. This is the ray where the transition that is where we want to see how to calculate various parameters.

The length of the plate is sufficiently large so that we can see here the boundary layer starts as laminar then we have the transition and then we have the turbulent. This depends upon the length of the plate; if the length is very small then we may get only laminar or turbulent but then here in this figure shows we have starting from laminar; then, we have transition and the turbulent boundary layer.

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- Existence of Laminar boundary layer over a small initial length decreases the total friction drag exerted by the fluid over the plate
- Ratio of the coefficients of friction drag in Laminar and Turbulent boundary layer is

$$\frac{C_f(\text{Laminar})}{C_f(\text{Turbulent})} = \frac{\frac{1.328}{\sqrt{R_L}}}{\frac{0.074}{(R_L)^{1/4}}} = \frac{17.94}{(R_L)^{1/4}}$$

For this kind of problem, the existence of laminar boundary layer over a small initial length decreases the total friction drag exerted by the fluid over the plate. As we can see initially the length of the plate in such a way that there is a laminar boundary layer exists. This existence of the laminar boundary layer over a small initial length actually decreases the total friction drag exert by the fluid over the plate. If you consider the ratio of the

coefficient of friction drag in laminar and turbulent boundary layer, we have already seen the coefficient of friction drag as far as laminar boundary layer is concerned. We have seen the coefficient of friction drag for turbulent boundary layer is concerned. If you consider the ratio of the coefficient of friction drag laminar and turbulent boundary layer we can write C_f laminar divided by C_f turbulent is equal to $1.328 \sqrt{R_L}$, where R_L is the Reynolds number. This is corresponding to the coefficient of friction drag for laminar boundary layer and then C_{ft} the coefficient of friction drag for turbulent boundary layer we have already seen which is 0.074 divided by R_L to the power $1/5$, where R_L is the Reynolds number. So, this is the ratio 1.328 divided by root R_L divided by 0.074 divided by R_L to the $1/5$. We can show that this is equal to 17.94 divided by R_L to the power $3/10$, where R_L is the Reynolds number. Now, the transition stage with reference to the earlier figure which we have seen is the friction over an initial length x over which the boundary remains laminar is obtained as:

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- Referring Fig. A, the friction over an initial length x over which the boundary remains laminar is obtained as

$$(F_{Df})_{\text{laminar}} = \frac{1.328}{(R_{x_{cr}})^{1/2}} \left(\frac{1}{2} \rho u_{\infty}^2 x \right)$$

- $(R_{x_{cr}})$ is the critical Reynolds number, signifies the change of flow in the boundary layer from laminar to turbulent (Generally 5×10^5)

If you refer to this figure where this length of the transition for laminar starts as far as this length is concerned, the friction over an initial length x over which the boundary remains laminar is obtained as $F_{Df \text{ laminar}}$ is equal to 1.328 by $R_{X \text{ critical}}$, critical Reynolds number to the power $1/2$ into $1/2 \rho u_{\infty}^2 x$, where u_{∞} is the free stream velocity, R_X is the critical Reynolds number at that location. That means at x the

$R_{X \text{ critical}}$ is the critical Reynolds number which signifies the change of flow in the boundary layer from laminar to turbulent; whenever the Reynolds number changes at this critical location then the transition starts from laminar to turbulent. Generally, various experiments conducted by researchers' shows that this critical Reynolds number can be 5 into 10 to the power 5; as far as this flat plate is concerned, the critical Reynolds number where this transition starts is 5 into 10 to the power 5.

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- The friction drag of turbulent boundary layer over the rear portion of the plate, i.e. from a distance x to L is

$$(F_{Df})_{\text{turb}} = \left[\frac{0.074L}{(R_L)^{1/5}} - \frac{0.074x}{(R_x)_{\text{crit}}^{1/5}} \right] \left(\frac{1}{2} \rho u_{\infty}^2 \right)$$

- Hence, total Friction Drag is

$$F_{Df} = (F_{Df})_{\text{lam}} + (F_{Df})_{\text{turb}}$$

$$= \left[\frac{1.328x}{(R_x)_{\text{crit}}^{1/2}} + \frac{0.074L}{(R_L)^{1/5}} - \frac{0.074x}{(R_x)_{\text{crit}}^{1/5}} \right] \left(\frac{1}{2} \rho u_{\infty}^2 \right)$$

If you use this value 5 into 10 to the power 5 then we can show that the friction drag of turbulent boundary layer over the rear portion of the plate, that is, from a distance x to l , here x to l means we can see that if this is the transition range and then this l is at this n . Now, with reference to this the friction drag of turbulent boundary layer over the rear portion of the plate, that is, from distance x to l is F_{Df} turbulent that is equal to, if you consider this as turbulent then $0.074 L$ divided by R_L to the power 1 by 5, this is as far as turbulent boundary layer is concerned minus $0.074 x$ by $R_{X \text{ critical}}$ to the power 1 by 5 into $1/2 \rho u_{\infty}^2$, where u_{∞} is the free stream velocity, ρ is the density of the fluid and x is the critical location. This expression gives the friction drag for the turbulent boundary layer; then, total friction drag as we can see we can add the F_{Df} is equal to total friction drag is equal to F_{Df} laminar plus F_{Df} turbulent. Now, under this transition state there is simultaneous we can say that between this range there is

sometimes we can consist laminar as well as turbulent. So, the total friction drag we can add this F_{DF} laminar plus the friction drag for laminar plus friction drag for turbulent. We have seen earlier this friction drag for laminar case is concerned. This Reynolds number, critically the Reynolds number with respect to $R_{X \text{ critical}}$, this F_{DF} laminar is equal to $1.328 x$ divided by R_X to the power $R_{X \text{ critical}}$ to the power 1 by 2 plus this term $0.074 L$ divided by R_L to the power 1 by 5 minus 0.074 into x divided by $R_{X \text{ critical}}$ to the power 1 by 5 into $1/2 \rho u$ infinitive square. This gives the total friction drag as far as the simultaneous existence as we have already seen a simultaneous existence of laminar and turbulent boundary layer in this range. So, we get the total friction drag as F_{DF} is equal to $1.328 x$ divided by $R_{X \text{ critical}}$ to the power 1 by 2 plus $0.074 L$ divided by R_L to the power 1 by 5 minus $0.074 x$ divided by $R_{X \text{ critical}}$ to the power 1 by 5 into $1/2 \rho u$ infinitive square, where x is that critical distance where the Reynolds number is critical for the flat plate case; as we have seen it is 5 into 10 to the power 5 and then R_L is the Reynolds number at the training edge of the plate and l is the length of the plate. This gives the total friction drag for the concern problem. Wherever the simultaneous existence of laminar boundary layer and the turbulent boundary layer case, the equation of total friction drag we put the critical Reynolds number as we have seen $R_{X \text{ critical}}$ is equal to 5 into 10 to the power 5 . If we consider this x is equal to, since if we consider a linear variation with respect to this Reynolds number that means, we can write x by $R_{X \text{ critical}}$ is equal to l by r . If you use that linear relationship and then put x is equal to l into $R_{X \text{ critical}}$ divided by R_L .

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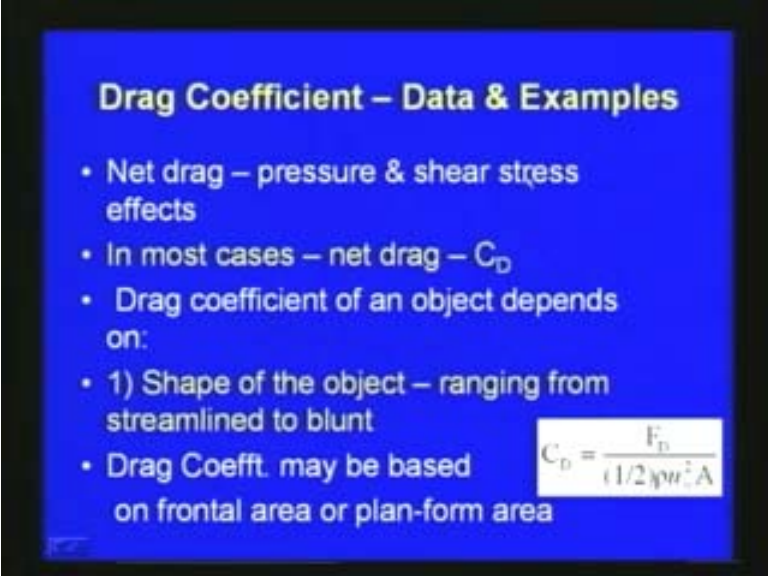
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- Now in the equation of total Friction Drag, putting Critical Reynolds Number, $(R_x)_{cr} = 5 \times 10^5$ and $x = [L(R_x)_{cr}]/R_L$
- Total friction drag $F_{1st} = \left[\frac{0.074}{(R_L)^{1/5}} - \frac{1700}{R_L} \right] \left(\frac{1}{2} \rho u_\infty^2 L \right)$
- Coefficient of Skin Friction, $C_f = \frac{0.074}{(R_L)^{1/5}} - \frac{1700}{R_L}$

We get the total friction drag, F_{DF} is equal to 0.074 divided by R_L to the power 1 by 5 minus 1700 divided by R_L into $1/2 \rho u_\infty^2 L$. So, this is after substituting this value of the critical Reynolds number R_x 5 into 10 to the power 5 we get the total friction drag as this quantity. This is the expression and now from this we can see that the friction drag is equal to, so this is per unit width which we consider, l is the length. So, from this expression we can write the coefficient of skin friction is equal to C_f is equal to 0.074 divided by R_L to the power 1 by 5 minus 1700 divided by R_L . This gives the coefficient of skin friction, wherever the case which we consider, simultaneous existence of the laminar and turbulent boundary layer.

Like this, we found this skin friction coefficient and then total friction drag. Similarly, we can find out other quantities as shear stress and other parameters we can determine as far as the drag force and drag coefficient as far as this flat plate case is concerned. So very similar way other problem also can be considered and we can derive various relationships for coefficient of drag coefficient, friction drag, the shear stress and like that various parameters. Now, let us consider the various cases depending upon the various parameters how the drag coefficient change and the how the effect of parameters like fluid number or the Reynolds number or the shape of the object of which the flow takes places.

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Drag Coefficient – Data & Examples

- Net drag – pressure & shear stress effects
- In most cases – net drag – C_D
- Drag coefficient of an object depends on:
 - 1) Shape of the object – ranging from streamlined to blunt
- Drag Coeff. may be based on frontal area or plan-form area

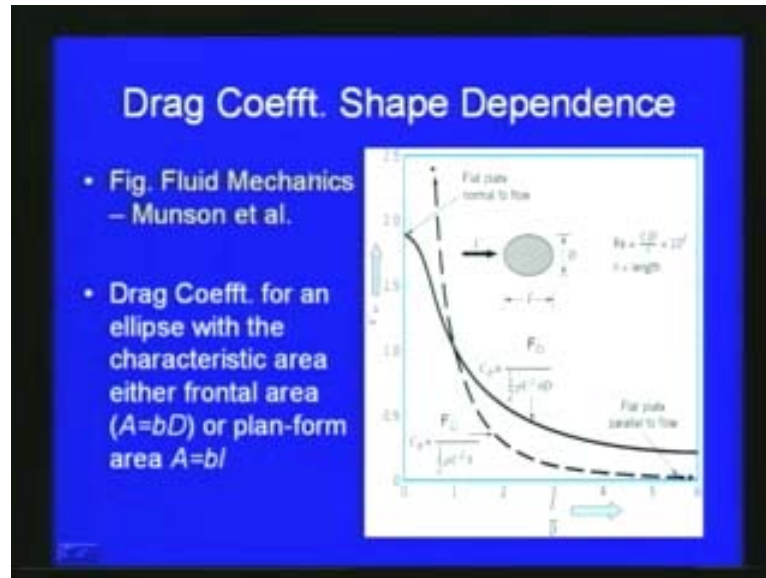
$$C_D = \frac{F_D}{(1/2)\rho u^2 A}$$

Now, here we discuss the drag coefficient data and examples. Net drag, as we have seen net drag is coming from the pressure drag and the shear stress effects. In most cases net drag we represent as C_D ; so, the drag coefficient of an object depends upon various parameters. This parameters depends even what is the shape of the object or which the flow take place or which orientation we put say, even the same body or like depends upon the fluid flow parameters.

The first case here we discuss is the drag coefficient and how the changes as far as drag coefficient takes place for various shapes of the bodies and various examples that we consider here. The various parameters which have some effect as far as the drag coefficient is concerned: first one is the shape of the object. So, the shape of the object is concerned; we can have different shapes as far as body is concerned; we can have the blend bodies like sphere or a cylinder or we can have the airfoil which is streamlined. So the shape of the object can be streamline to blend body. Accordingly, we can show experimentally that the drag coefficient also varies; the drag coefficient as we have seen is based upon the frontal area or plan-form area. So coefficient of drag is equal to drag force divided by $1/2 \rho u$ infinitive square into A , where ρ is the density of the fluid, F_D is the drag force, u infinitive is the free stream velocity and A is the area plan; area either depending upon the case which can be frontal area or the plan-form area. As i

mentioned whether the same body which may be placed, here for example, if you consider this as a flat plate, then the flat plate is, if you place it just like this or if you place the free stream velocity is this direction then whether you place this plate in the direction like this or if you place this plate like this or if you place the plate like this, depending upon this the drag force or the drag coefficient changes.

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Here, in this slide a graph taken from Munson et al fluid mechanics text book Munson et al. The drag coefficient, based upon the shape dependence of the drag coefficient is plotted here. You can see the drag coefficient for an ellipse with the characteristic area either frontal area that means A is equal to b into D , where b is the length and then D is if you consider an ellipse then this is l and here this is d and b is the length or plan form area will be a is equal to b into l .

If you plot the coefficient of drag, so through experiments, we can find out the coefficient of drag or you can first find the total drag force. Then, from which we can find the drag coefficient C_D is equal to F_D divided by $1/2 \rho u_{\infty}^2$ into A . From this, if you plot the coefficient of drag on y-axis and this typical case of ellipse that means where l is this and d is this dimension. So, l by D on the x-axis, then we can show that depending upon the shape so how by varying whether using the b into l or b into d like

that then you can show that coefficient of drag is varying like this. This case if you use this, say, corresponding to a flat plate normal to the flow you can show that the drag force will be above 1.9.

Here, it is shown flat plate normal to the flow, the drag coefficient will be drag coefficient, C_D will be 1.9 and then if the flat plate is put parallel to the flow then you can see it is very small value, the coefficient of drag is very much produce then you can see it is almost 0 or it is 0.001 or double 0.2, like that a very small value is obtained. That is why i mentioned when even the body over which the flow takes place how you place it if it is normal to the flow when the drag force as we have seen in the case of flat plate the drag coefficient will be much higher or if you place the flat plate which we consider parallel to the flow then the drag coefficient will be much lower as shown in this plot.

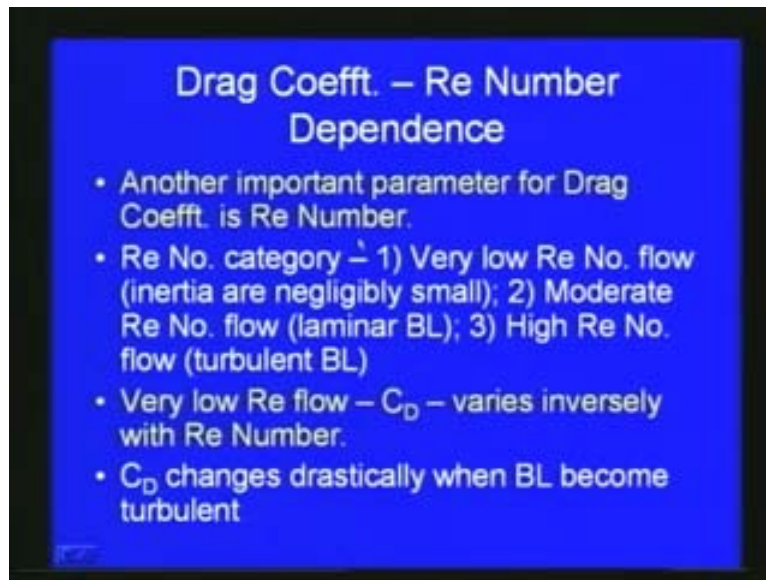
Similarly, this figure shows as far as an ellipse is concerned of this shape you can see that how the variation takes place starting like this e_1 as C_D is equal to FD by $1/2 \rho u^2 b$, corresponding to whether which way we take. Either we take a plan form area or we take the frontal area corresponding to the ellipse, that is, for this and then this thick line is concerned C_D is equal to FD by $1/2 \rho u^2$ into b into D , that is, corresponding to this. In this case, the area is d into 1 as considered. Like this, this is corresponding to the Reynolds number 10 to the power 5. Like this we can show that depending upon the shape of the body the drag coefficient changes or effectively the total drag is also changing.

Due to these changes, we can design various automobiles like bus or car or if you are depending upon what shapes you give the drag force can be reduced and then the wake we can make it much more efficient while driving so that is the importance of this studies on this drag force.

Similarly, if you consider airfoil compared to sphere or a blend body then we can see that the drag force is we can reduce too much and then that is why we can make it stream line and then we can reduce the drag forces. That means the drag force or the coefficient of drag is the shape of the body as considerably influenced on the drag force or coefficient

of drag. So, that is the first component of first parameters which affect the drag force or drag coefficient and next one is the Reynolds number.

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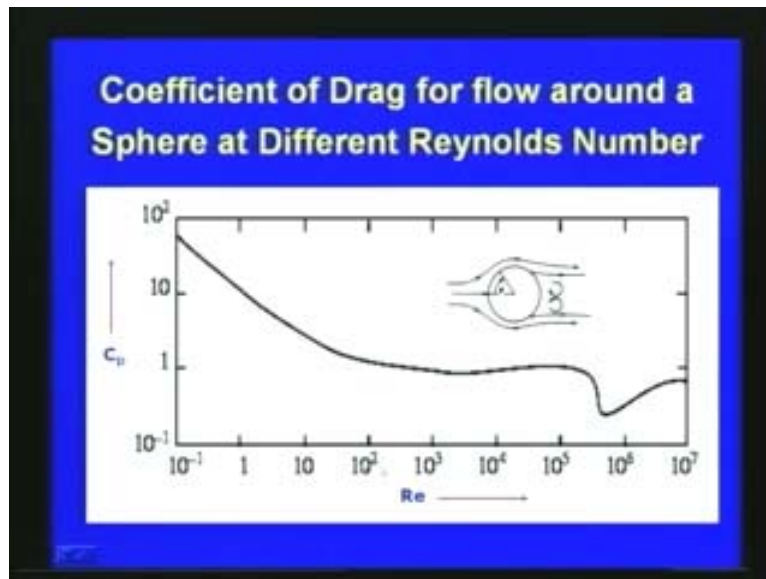


Here, we discuss the drag coefficients and its dependence on the Reynolds number. We have seen that depending upon the flow that means depending upon the velocity the flow can be either laminar or turbulent. So, in the last lecture as well as just now we have discussed dependency on this Reynolds number. This Reynolds number is also another important parameter for drag coefficient so whether the drag coefficient is increasing or decreasing depending upon the Reynolds number.

As far as Reynolds number is concerned there are three categories: one is very low Reynolds number flows that we can show inertia are negligibly small; second one is moderate Reynolds number flow where laminar boundary layer is taking place; third case is high Reynolds number flow where the boundary layer is turbulent. For very large Reynolds number, the coefficient of drag we can show experimentally that the coefficient of drag varies inversely with respect to the Reynolds number and then the coefficient of drag changes drastically when the boundary layer become turbulent. This we can show experimentally. The coefficient of drag is very much or the drag force is very much depending upon the Reynolds number of the fluid flow. If the flow is laminar then we can

show that coefficient of drag varies inversely with respect to Reynolds number. When the flow becomes turbulent there is a drastic change as far as coefficient of drag is concerned. Whenever the Reynolds number is very low the inertia are negligibly small and then also the drag force is very much changing whether it is low Reynolds number or moderate Reynolds number or high Reynolds number flow. So, this slide shows the coefficient of drag for flow around a sphere at different Reynolds number.

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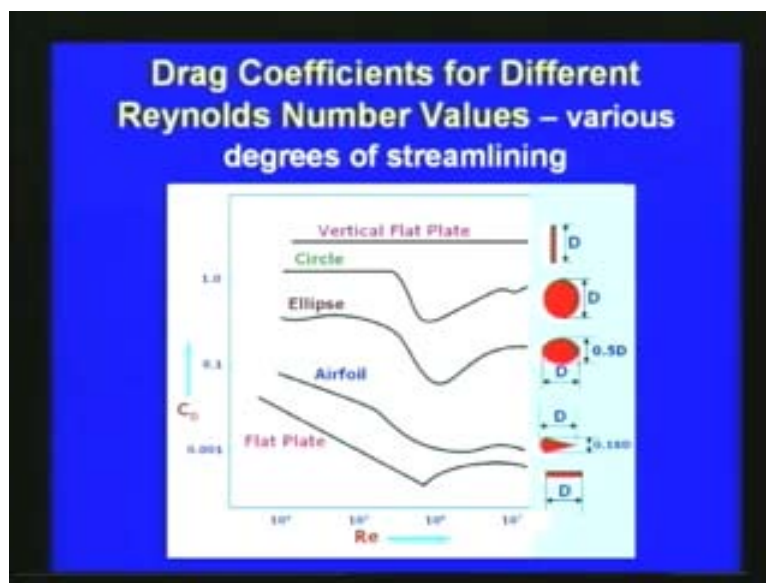


You can see that the coefficient of drag is plotted on the y-axis and Reynolds number on the x-axis. Here, this is the case of flow around a sphere, here is the sphere and then the flow is in this direction. If you plot the coefficient of drag versus Reynolds number then we can see that whenever the Reynolds number is much smaller than the coefficient of drag is much higher like this and then when the Reynolds number is increasing, say, when the flow is becoming laminar to this we can see that it is coefficient of drag is keep on reducing until this critical Reynolds number range is reach. Again, it is going down like this it is further reducing to the range of 10 to the power 6. This is the coefficient of drag versus the Reynolds number for flow around a sphere.

Then, we can see that it is coming down to the level and then further when the Reynolds number is increasing that means the flow become total turbulent. Then the coefficient of

drag is again slightly going on increasing like this. This we can experimentally show that depending upon the Reynolds number. So, for a very low Reynolds number the coefficient of drag is much larger and then when the flow become the laminar range up to 10 to the power 5. It keeps on reducing and there is again further reducing when the transition ranges. When the flow becomes further turbulent again the coefficient of drag is slightly increasing as shown in the slide. This shows the dependence of the coefficient of drag with respect to the Reynolds number of the fluid flow.

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In this slide the drag coefficient for different Reynolds number values are plotted for various shapes.

We have already seen depending upon the shape of the body the drag force change and now we consider the Reynolds number also the drag force change for the same body. Depending upon the shape as well as the Reynolds number we can see that here in this slide for various shapes starting from vertical flat plate; the same flat plate well placed in horizontal, then circle, ellipse, airfoil. Like that for various shapes here the coefficient of drag plotted with respect to the Reynolds number.

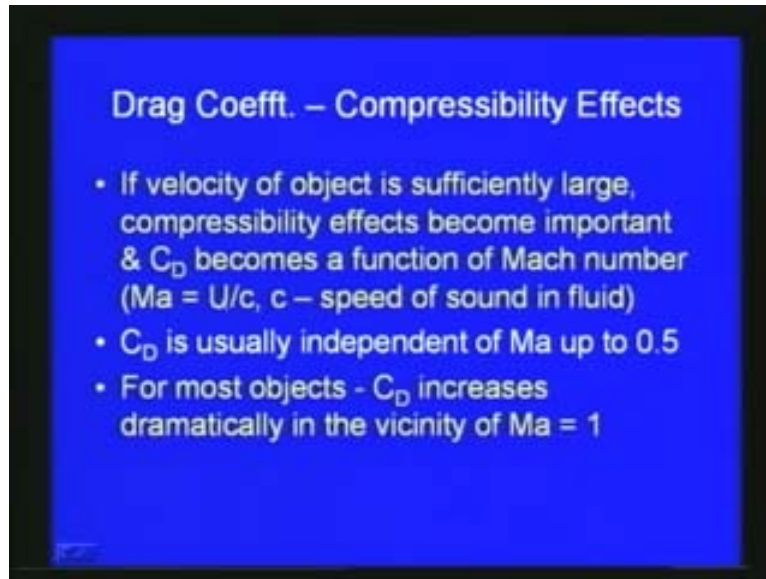
We can see for vertical flat plate, we have maximum coefficient of drag. Since we can see that the vertical plate is placed like this vertically; so the flat plate is placed vertically.

So we have the maximum coefficient of drag and then this is almost constant with respect to the Reynolds number whether it is laminar or turbulent for vertical flat plate we can see that the coefficient of drag is maximum and it is like this. If you take a cylinder or circular shape and we can see that with respect to the Reynolds number the drag force is in the laminar range; it is almost same and then the flow becomes turbulent. Then, we can see that further the Reynolds number increase then for circular shape we can see that the coefficient of drag decreases to certain range here up to 10 to the power 6 something like that it is slight increasing as shown in this figure.

If you consider an ellipse shape then you can see here the coefficient of drag the laminar range is initially slightly increases and then the keeps on decreases. You can see that in the transition stage it keeps on decreasing and further when the flow become turbulent then we can see that again the coefficient of drag is increasing like this in the case for ellipse.

Then, stream line bodies like an airfoil, we can see that when the Reynolds number increases we can see that the coefficient of drag is decreasing, keep on decreasing like this, only a small change whenever further increasing Reynolds number in this range for airfoil of this shape . The flat plate place parallel to the flow is concerned you can see that this is with respect to the Reynolds number; laminar range is decreasing shortly and then whenever it is turbulent we can see that it is slightly going on increasing like this. So, this slide shows how the coefficient of drag or drag force changes with respect to the shape of the body and then with respect to the Reynolds number. Then some other important parameters where the drag forces will be changed or coefficient of drag changes the compressibility effect.

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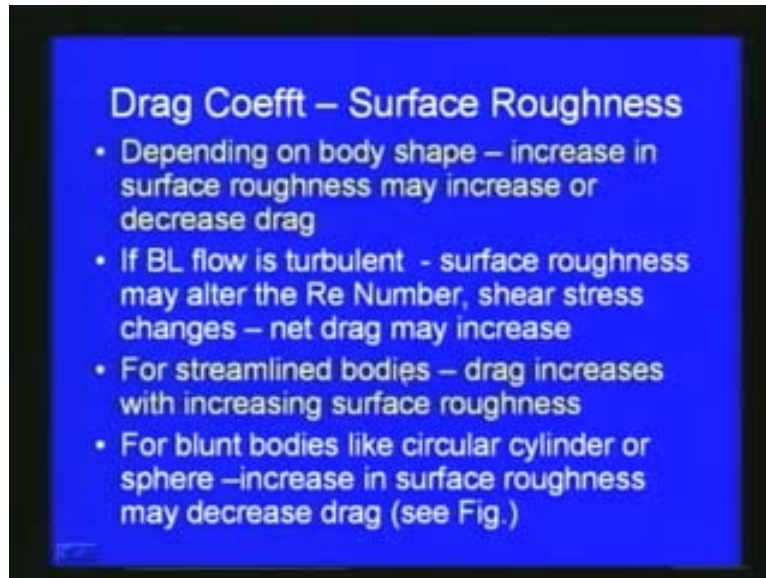


If the velocity of the object is sufficiently large, then compressibility effects become important and coefficient of drag becomes a function of the Mach number. You can see that here the Mach number is the important parameter, dimensionless number. Mach number is equal to the U by c , where u is the free stream velocity and c is the speed of the sound in the fluid.

The coefficient of drag is usually independent of the various experiments conducted shows that the coefficient drag is independent of Mach number up to about 0.5. For most objects the coefficients of drag increases dramatically in the vicinity of whenever the Mach number increases from 0.5 it is up to 1; it is the coefficient drag also increases so this is the effect of compressibility. So compressibility of the fluid has got effect on the drag coefficient or the drag force especially the flow which we consider; if it is fluid like water then the effect will be very negligible but if we consider airflow then we can see that the compressibility effect will be much higher. When the Mach number increases to 1 there is coefficient of drag which dramatically increase but up to about 0.5 we can show experimentally that there is not much effect on the coefficient of drag with respect to the compressibility of the fluid. Another important parameter as far as drag force and coefficient drag is the surface roughness. We can see that the same body if it is smooth

then the coefficient of drag or the drag force will be different and if it is rough the coefficient of drag will be different.

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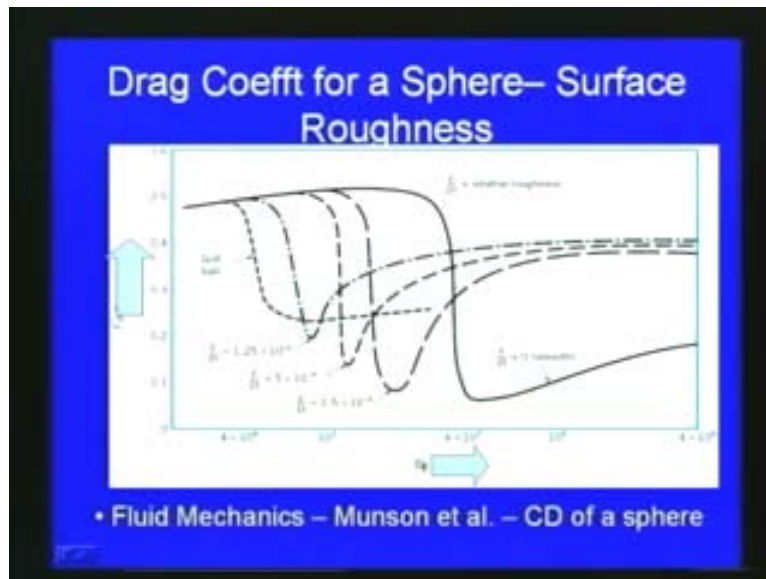


Depending on body shape the increasing surface roughness may increase or decrease the drags. This depends upon the shape of the body also. So, if the boundary layer flow is turbulent then experiment shows that the surface roughness may alter the Reynolds number and the shear stress changes and the net drag may increase. This we can show experimentally.

For streamlined bodies the drag increases with increasing surface roughness. So, free the streamlined bodies like the airfoil we can show that the drag increases with increasing surface roughness. So to reduce the drag we have to keep the surface of the body like airfoil. We have to keep it much smooth so that the drag can be decrease.

For blunt bodies like circular cylinder or sphere we can show that the increase in surface roughness increases. So, depending upon the increasing surface roughness may decrease drag this depends upon the shape of the body whether it is circular cylinder or sphere or depending upon the shape of the bodies. Sometimes depending upon the case increasing surface roughness may decrease the drag.

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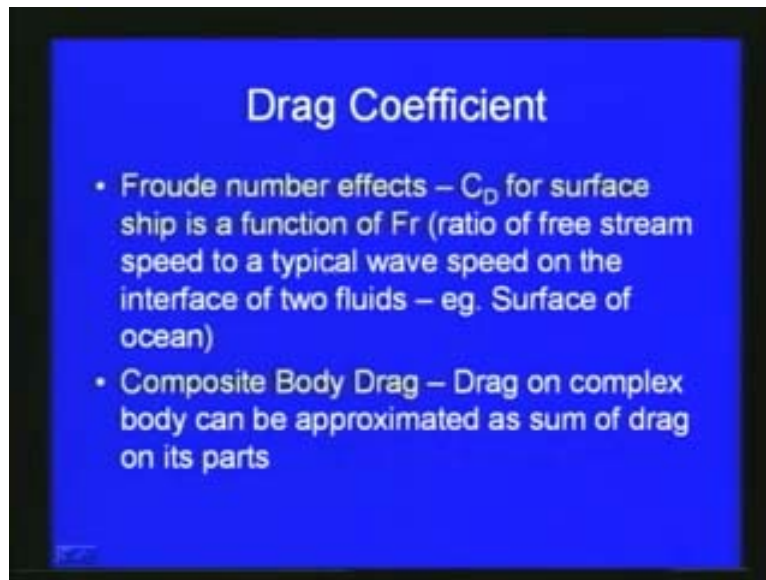


Here, a flow taken from Munson et al fluid mechanism, CD of a sphere by Munson et al. Here drag force coefficient for a sphere with respect to surface roughness is shown; Reynolds number is shown in the x axis and the coefficient drag is on the y axis. So this is the case for a sphere. You can see that with respect to the Reynolds number and then the relative roughness of this figure the drag coefficient is varying depending upon the roughness. If you consider as for an example a golf ball depending upon the Reynolds number we can see that this is the pattern and if it the excellent by d, if excellent is the roughness when excellent by d is the diameter of the sphere. So you can show that with respect to Reynolds number if the surface is very smooth then initially the coefficient of drag is slightly increasing with respect Reynolds number. Then, after the critical range when the flow become turbulent we can show that there is a sudden decrease in the coefficient of drag and then after a certain range there is again the coefficient of drag is increasing in like this.

For various parameters of the x_1 by d, x_1 is the roughness and d is the diameter of the sphere. Then, we can show for various cases, with respect to Reynolds number, coefficient of drag is changing like this. So, x_1 by d is equal to 1.25×10^{-4} gives this curve and the x_1 by d is equal to 5×10^{-4} gives this curve and x_1 by d is equal to 1.5×10^{-3} like this and this gives

x_1 by v , x_1 by d is smooth, that is, X_1 by d is almost 0. That means, x_1 is very less so the surface is very smooth then you can show this is the case.

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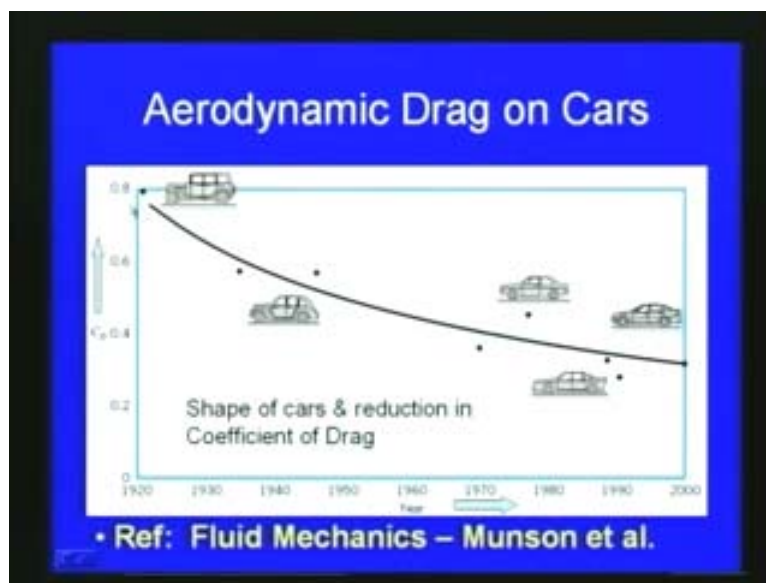
Here, we can see that other than the surface roughness also with respect to the Reynolds number the coefficient of drag or the drag force is changing. Now another important parameter as far as the drag force or the drag coefficient is the fluid number. For example, if you consider a ship a coefficient of drag for surface ship is a function of fluid number that means the ratio of free streams speed to a typical wave speed on the interface of two fluids, an example is the surface of ocean.

Then, we can show that depending upon the Froude number effects, the coefficients of drag changes. So, this depends upon the various other parameters of the wave effects and then we can show that fluid number also has got an effect as far as drag coefficient or drag force is concerned. Wherever the composite body that means if the body is a complex body where different shapes are included, some part is spherical and some part is plane. Like that different shapes are included for the concerned body that means composite body, the drag on such a body we can approximate it as sum of drag on its various parts.

That way we can find out the drag force on the coefficient of drag with respect to composite body. So far with respect to our discussions we have seen the drag force or the coefficient of drag changes with respect to the shape of the body, with respect to the Reynolds number, with respect to the surface roughness and also the fluid number or the compressibility of the fluid. So, the drag force changes or the drag coefficient is changing. So, all these aspects are considered while we design various shapes for example, say, for automobiles or for ships or while we design aero plane or aircraft or the space shuttle etc.

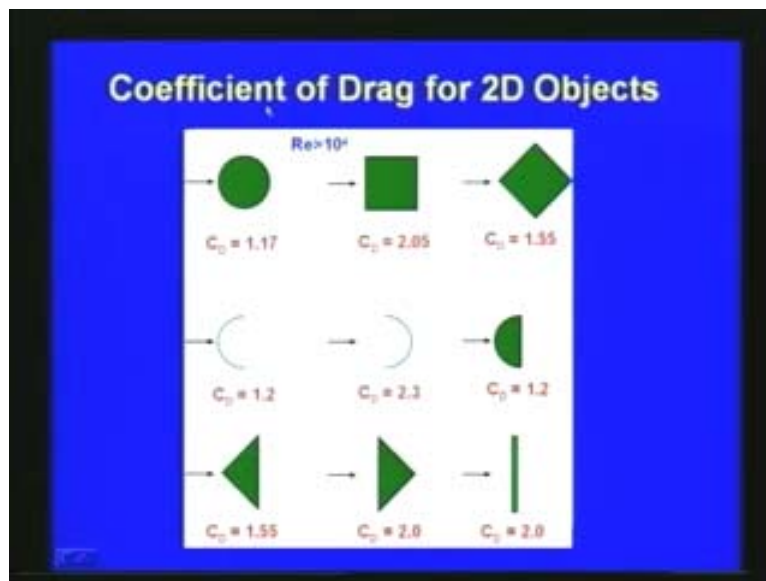
All kinds of design we can consider so these various parameters which effect the drag force and then which influences the moment of the particular body either through water or through air. So, if you consider for example, automobile industry. Whatever the design is, the automobile industry which has given in 1950's or 1960's now in 1980's or 1990's or in 2000, the shape of the car or shape of the bus or shape of the truck you can see that when there are some changes. So, the design is understood; the effect of this drag force or the coefficient of the drag and then they gives such a shape that the drag force effect or the vehicle, for example- car is reduced.

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If you consider these changes are shown in this slide here that means aerodynamic drag on cars then the various shapes given or various shapes adopted by the car industries you can see here. In 1920's or 1930's you can see the shape of the car was like this and then 1940's and 1950's when we understood that this coefficient of drag or the drag force can be reduced then the shape of the car has been changed to this. You can see that the drag coefficient earlier for this shape was about 0.8 and then for this shape of the car you can see that the coefficient of drag is about 0.55 or about approximately equal to 0.6. Again, in 1950's or 1960's or 70's various experiments were conducted. Then, again differentiates were designed as far as the shape of the car itself is differentiate. You can see the drag force is or the coefficient drag is reduced. You can see that starting from 0.8 in 1920's. Then in 2000 the drag force as far as aerodynamic drag on car is concerned now the drag is about 0.35 or point for inch depending upon the shape of the car. So, this aspect of the shape or the orientation we can use in the automobile design, for example car, bus or truck or whichever automobile is concerned. Similarly, also this aspect is considered while we design aero plane or while we design ships depending upon the various parameters. Finally, here as far as drag is concerned we discussed the coefficient of drag.

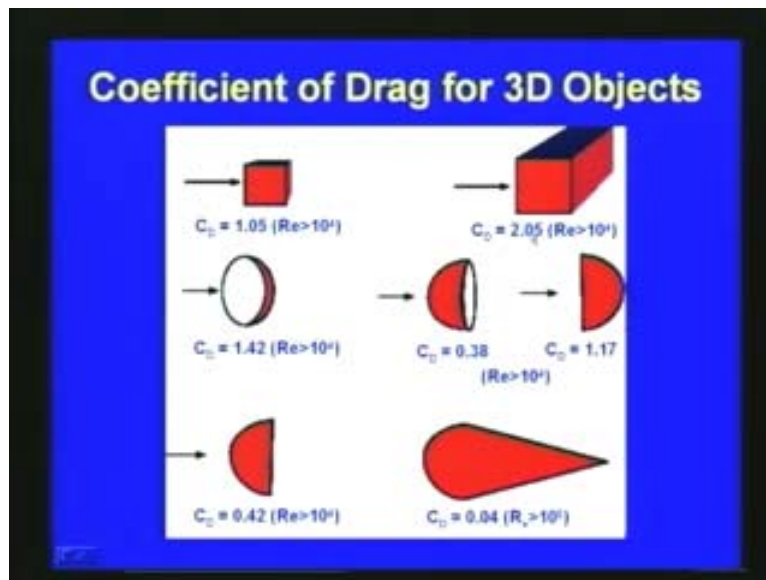
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If we consider two dimensional objects we can see here for example when the Reynolds number is about 10 to the power 4. If you consider a circular shape here you can see the coefficient of drag is about 1.17 and for rectangular shape or square shape we can see that coefficient of drag is 2.05 and same if we put in this orientation we can see that coefficient of drag is reduced that is equal to about 1.5. Similar way, if you consider an arc like this, if you place in the convex side or concave side you can see if you place the arch like this then the coefficient of drag is 1.2 but if you place it in the other way then the coefficient drag is about 2.3.

Also, if you put it as a semi spherical or semi circular shape we can see the coefficient of drag is 1.2 and then a triangular shape we can try experimentally that the coefficient of drag if you put it in this fashion it will be only 1.5 and the same triangle shape if you put in the other way, opposite way you can see that coefficient of drag is increasing to 2. Similarly, the flat plate which we put normal to the flow then you can see that coefficient of drag is about 2.

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Similarly, if you consider three dimensional objects so if you consider for example, in the Reynolds number is more than 10 to the power 4, the coefficient of drag for a cube or a

prism is about C_D is equal to 1.05 and prism is concerned; this is for cube 1.05 prism is concerned C_D coefficient of drag is 2.05.


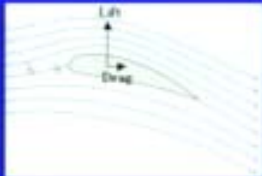
Then, we can see that a hemispherical coefficient of drag is 1.42 and it ranges from 0.38 to 1.17, which way you place if you place this way or this way, the coefficient of drag is here it is much lower 0.38 and here it is much higher, 1.17 for the Reynolds number more than 10 to the power 4 and then very similar way if we keep the body, hemisphere like this then we can see that it 0.42. If it is streamline body like this in three dimensions we can show that even in the Reynolds number is about 10 to the power 5, the coefficient of drag is much lower; it is about 0.04.

Like this, depending upon the shape, depending upon the orientation and of course depending upon the fluid flow Reynolds number we can show that the drag force or the coefficient of drag changes considerably. We can utilize this concept or the reduction coefficient of drag or the drag force we can utilize in the design of many engineering problems, either car or aero plane or ship or many other designs we can utilize this coefficient of drag aspects. Now, we have seen in the coefficient of drag, then the drag force and then also various parameters related to that.

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Lift Force

- Force exerted along the direction normal to the flow is known as the Lift
- Some objects – designed to generate lift – e.g. Airfoil; some objects designed to reduce lift – e.g. Car



The slide features two diagrams. The top diagram shows a cross-section of an airfoil with streamlines curving around it. A vertical arrow pointing upwards is labeled 'Lift', and a horizontal arrow pointing to the right is labeled 'Drag'. The bottom diagram shows a side view of an airplane. A vertical arrow pointing upwards from the fuselage is labeled 'Lift', and a horizontal arrow pointing to the right from the tail is labeled 'Drag'.

Now, next topic is very important which we want to discuss here is the lift force. We have seen the drag force whenever, especially for the flow over a flat plate or for an object in the fluid whether it is moving or whether it is placed stationary. So, depending upon the case we will be having drag force and also we will be having lift forces. If you consider for example, an airfoil like this we can see that if the free stream velocity is like this then the drag force is in this direction and then we have the lift force. So the force exerted along the direction normal to the flow is known as the lift as we discussed earlier. So this is the lift force and drag force which we consider like this. Some objects are designed to generate lift. For example, aero plane design is concerned we can see that we have to design in the shape such a way that we want to create lift so that we can lift the aero plane with respect to when the plane is moving. So that some designs like airfoil or aero plane wings we are designing such a way that we want to generate lift but some designs like for example, car or bus or truck we are designing such a way that we want to reduce the lift effect. If you consider the design of a car, we want to reduce the lift since if lift effect also comes in the picture. Then the moment of the car will be when we accelerate then we can see that there will be more friction; the lift effect also will be affecting the moment of the vehicle on the road.

So, depending upon the case for some designs like airfoil or the wing of a plane or the aero plane they design is such a way that we want to increase the lift and some designs we want to decrease the lift. So, now in the next lecture, we will be discussing further on these various aspects on the lift force and the coefficient of lift. Then, another important aspect is circulation with respect to whenever the body is moving then what are the effects as far as lift is concerned. This we will be discussing in the next lecture.