

Fluid Mechanics
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Lecture 33
Boundary Layer Theory and Applications

Welcome back to the video course on fluid mechanics. The last lecture we were discussing about the drag and lift forces coming on the immersed bodies in fluids. So we have seen the expression for the drag force and lift force and various situations where the drag and lift acts on the bodies then how it can be calculated. Also we have seen that, depending upon the shape of the body, depending upon the profile of the body and then how we place it just like in the case airfoil, we have seen the drag changes and also lift forces changes.

So also we have seen mainly, that we have the pressure drag and the friction drag with respect to the viscous effects of the fluid. Now before further discussing on how to find out the drag coefficient for laminar turbulent, with respect to the boundary layers, we will just see initially, we will discuss how this we can reduce the drag effects on bodies, the way the moving bodies or stationary bodies in fluid. So first the topic which we are going to discuss is reduction measures of pressure and friction drag.

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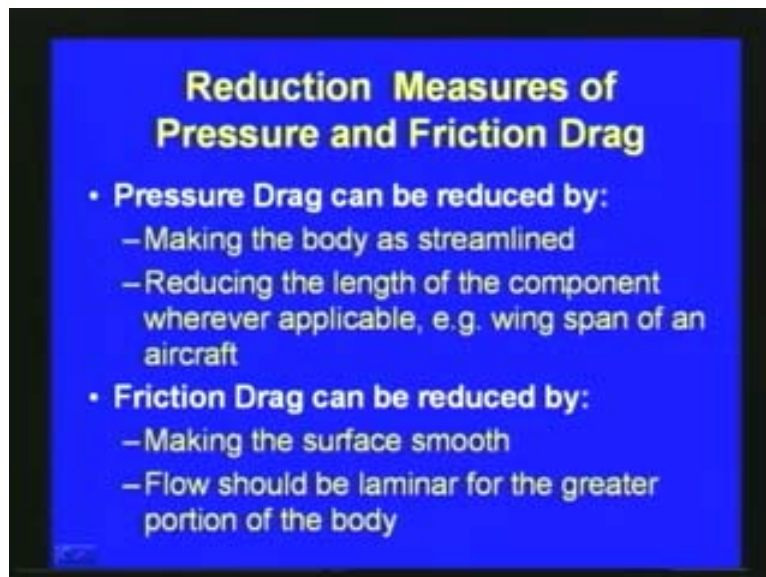
Here we can see in this slide here, we have an aircraft and then while it is flying we can see that various forces acting on the drag in this direction. Then there will be a thrust in this direction and then the weight of the aircraft and then lift. So as we have already discussed the drag is a mechanical force generated by a solid object moving through a fluid as in the case of an aircraft. The main drag effects are here, the pressure drag and friction drag. So what are the various possibilities depending upon whether it is a blunt body or it is a streamline body? Depending upon the shape of the body and profile the drag effect will be different. How we can reduce this pressure and friction drag? That is what we want to discuss here.

As I mentioned earlier, if you consider an airfoil as in this slide, an airfoil is placed just within. If the free stream flow is coming like this and the airfoil is placed in the direction of the free stream, then we can see that the drag effect due to the pressure drag effect will be much lower and only friction drag will be there, but when we slightly tilt the airfoil in such a way that for example the angle of attack is 5° as in this figure. Then we can see that the drag effect, the friction drag effect. The drag effect compared to the pressure drag will be more. So depending upon the angle of attack the second figure again, the angle of attack is 40 degrees.

That we can see again the flow phenomena surrounding the airfoil is changing. What we can observe here with respect to this slide is that, depending upon the placement of the location or how the profile of the body which is immersed in the fluid flow. Accordingly, the drag effect whether it is pressure drag or the friction drag it changes, its nature changes. The question here is how we can reduce the pressure and friction drag depending upon the shape or whether depending upon the placement of this say as in the case of this airfoil.

Now the pressure drag we assumed the drag is concerned either it can be mainly it can be pressure drag or the friction drag. The pressure drag can be reduced. Two important steps which we can use it here, one is making the body as streamlined. The pressure drag effect can be reduced by making the body as streamlined and the second possibilities that we can reduce the length of the component wherever applicable.

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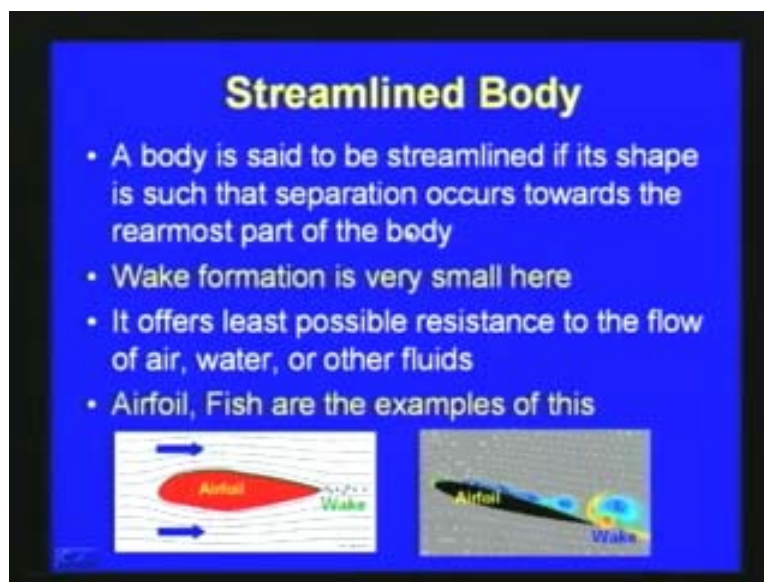


So example, the wing span of an aircraft, we can see that here is the aircraft and then the wing span. Depending upon how you place the wing and then how it is oriented and if you reduce the length component then the pressure drag can be reduced. So like this various measures we can adopt to reduce the pressure drag. Second kind of drag is the friction drag we can reduce the friction drag by making the surface smooth.

One of the step which we can adopt to reduce the friction drag is the surface is rough makes it more smooth and also the flow should be laminar for the greater portion of the body. The flow is laminar then the drag friction drag effect to be much lesser compared to the turbulent flow effects. Flow should be laminar for the later portion of the body. So these are the two measures possible with respect to the friction drag to reduce the friction drag.

Now we have seen with respect to the pressure drag if you make the body streamline, then the pressure drag is reduced. So that we have already seen here, make the body streamline. We will discuss here briefly about the streamline body. A body is said to be streamlined if its shape is such that, a separation occurs towards the rearmost part of the body.

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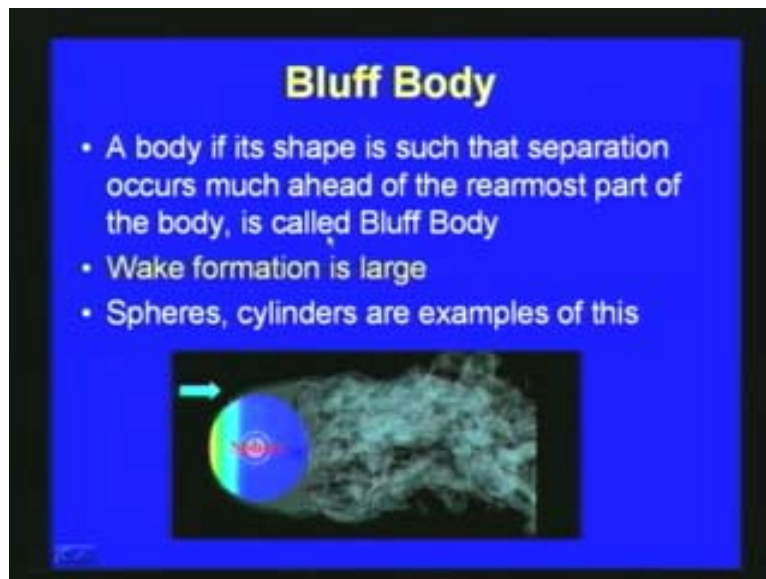


We call a body as streamlined if the shape of the body is such way that the boundary layer separation occurs towards the rearmost part of the body. In that case, we can see that the wake formation is very small. It offers least possible resistance to the flow of air water or other fluids where the body is immersed. We can see the airfoil which we have already discussed earlier, airfoil or the fish. You can see that the shape of the fish is in such a way that it is we can say that the body is streamlined, so that the wake formation is

very small and then the shape is such a way that when the fish moves the separation occurs towards the rearmost part of the body. Here we can see that whenever the airfoil is placed like this, we can see the flow is coming like this and then the wake formation is only due to the placement in such a way that the body is streamlined.

Here the wake is very small and then the drag effect will be much less especially the pressure drag effect will be much smaller. Similarly, the fish in the case of a fish also we have shape of the fish is in such way that we can say that the wake formation is very small and the separation the boundary layer separation occurs towards the rearmost part of the body of the fish. In the same airfoil if you place it in other way as we have seen the airfoil is concerned, it is how you are placing it also important, how is the angle of attack accordingly the drag effect changes? So this is about the streamline body and then the next kind of body which we have seen is so called Bluff body.

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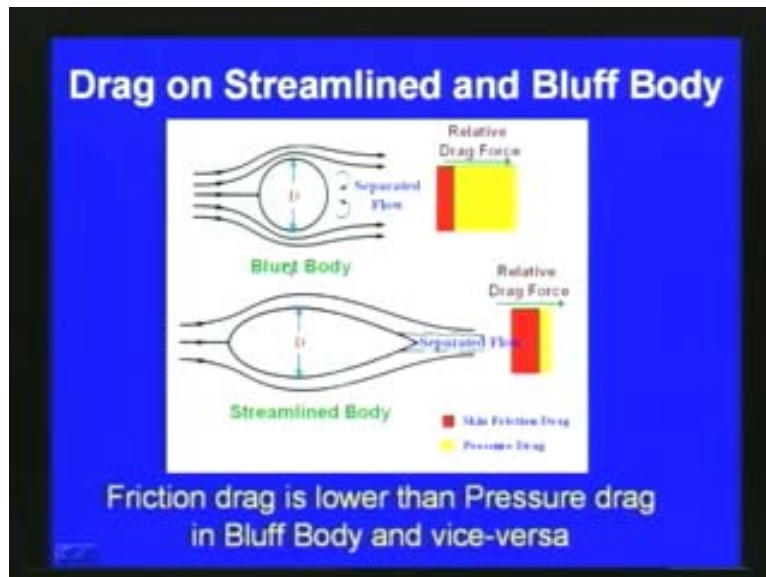


In the case Bluff body say as in the case of a sphere or the case of a circular cylinder we can see that the body shape is such that separation occurs much ahead of the rearmost part of the body. You can see that this is the sphere here, the separation occurs much ahead of the rearmost part of the body. That is why it is called as a bluff body and then you can see the wake formation is large as in the case of a sphere or a cylinder and object

as shown in this figure. The body is as far as with respect to the drag is concerned, we can say to reduce the drag effect, we consider that we should have a streamline body. So either the streamline body or bluff body is concerned, you can see the wake flow formation will be much larger and then the pressure drag effect will be much more, so if you consider the drag on streamline and bluff body.

So here you can see that, if you consider circular cylinder like this is the blunt body or the bluff body. So you can see that the separated flow is here. Here the pressure drag in the case of a bluff body, the pressure drag will be much higher. So here in this figure the friction drag is shown as red color and the pressure drag is indicated as the yellow color.

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We can see that in the case of bluff body like in the case of a circular cylinder the pressure drag will be much more and then friction drag or skin friction drag will be much smaller and also the total drag will be much higher compare to the streamline body. So the case of an airfoil which is a streamline body, we can see that the skin friction drag.

Since here we can see that skin friction drag will be more compared to the pressure drag. The pressure drag here is also indicate as yellow color and the skin friction drag is shown as red.

So you can see that the relative drag force will be much smaller for a streamline body as shown in this figure. So here the friction drag is lower than the pressure drag in the case of bluff body. So bluff body, the friction drag is smaller and pressure drag is more. Then in the case of streamline body the friction drag is higher and then the pressure drag is much smaller. These are the effect with respect to whether the body is a bluff body or whether the body is a streamline body.


So accordingly, we can say that the drag effect is the pressure drag or the friction drag which one is more in the case whether it is a bluff body or it is a streamline body. So the drag effect is concerned, this special drag and the friction drag are the two most important effect, as far as the drag is concerned other than this pressure drag and the friction drag. There can also be some other kinds of drag.

Depending upon say for example, if you consider this slide, the movement of a boat, here a boat is going through the water where also the effects are there, and then there are so many other kinds of drags are also possible. Except pressure and friction drag there are different kinds of drag. So these are first one is called induced drag and second one is called profile drag, and third one is called heel drag, and fourth one is called residuary drag, fifth one is called added wave drag.

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Other Type of Drag Forces

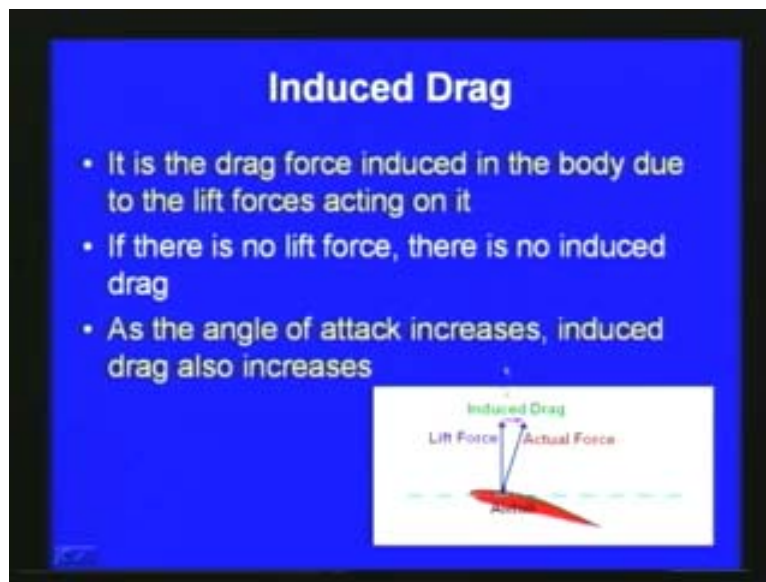
- Except pressure and friction drag there are other kind of drag forces applicable to certain cases
- Those are:
 - Induced Drag
 - Profile Drag
 - Heel Drag
 - Residuary Drag
 - Added Wave Drag



If you consider the moment of a boat then we can see that many of this drag effect will be coming over the boat, which is moving over the water like this. But most of the time the pressure drag and the friction drag are the most important. The important components of the drag force but others are also possible like induced or profile drag or the heel drag like that. Let us see the definition of this induced or these various kinds of other drags. So now the induced drag, actually it is the drag force induced in the body due to the lift forces acting on it.

We have seen when body is immersed in the fluid either in the moving fluid or depending upon whether the body is moving or the fluid it is moving. So with respect to the immersed body we have the drag and as well as lift. So this lift components also may be induced some kinds of drag that is called induced drag.

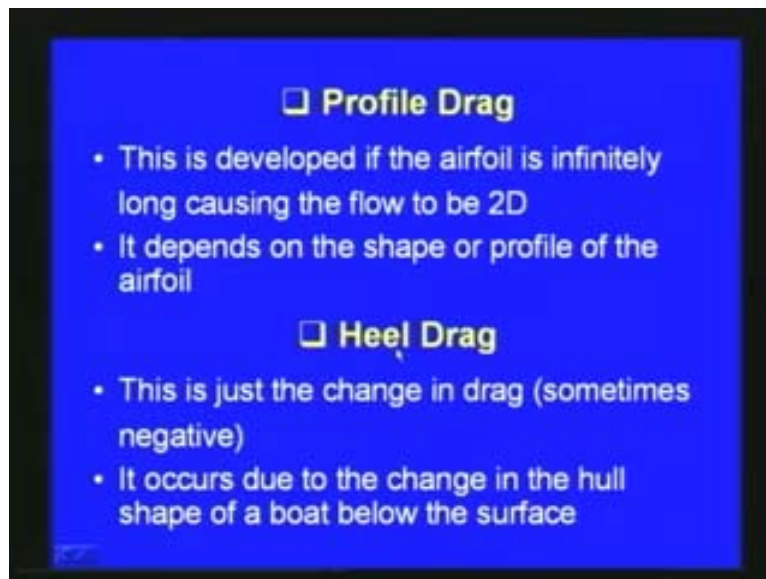
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Here we can see that if you consider an airfoil like in this figure here, then we can see that the drag force will be in this direction and then the lift force will be the normal direction of flow direction. Then we can see that since the airfoil is placed with respect to some angle of the free stream flow then we can see that an induced drag will be coming here. So this is due to the lift force.

If there is no lift force, when there is no induced drag so as in the case of an airfoil you can see that when the angle of attack increases the induced drag also increases. So actually this is this induced drag is concerns this depends upon how we are placing the body or how it is how its profile is there and how you say place the body with respect to the flow direction of the fluid. This is so called induced drag and then next one is the profile drag. This profile drag is developed say for example if you consider the airfoil if it is infinitely long then the flow it to be two dimensions, so this is coming from the profile of the of the body. That is why it is called profile drag. This depends on the shape or profile of the shape, if you consider the airfoil it depends upon the shape and how you say place it or the profile. So that is called profile drag and then third kind of drag here is called heel drag.

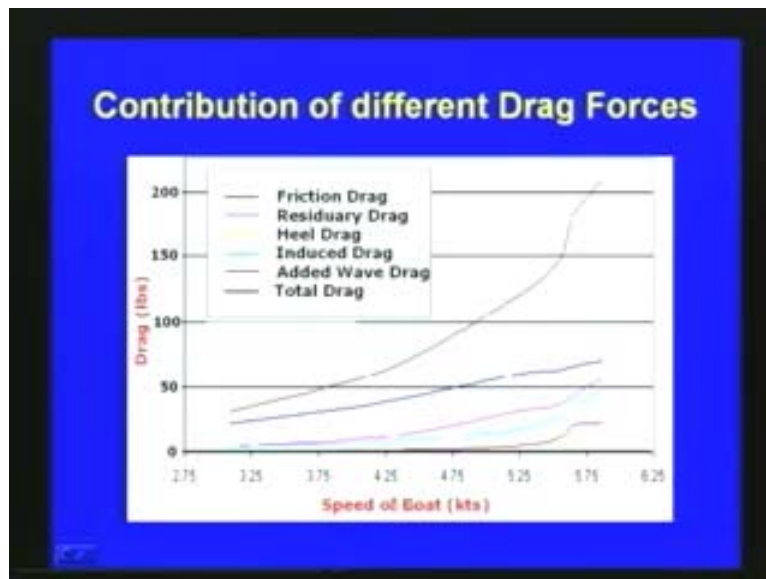
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This heel drag, this is due to the change in drag sometimes says negative. This occurs due to the change in the hull shape of a boat below the surface. We have seen this boat moment here, so if the hull shape depending upon the hull shape of the boat then you can see that there is possibility of the say there is just change in the drag effect and its occurs due to the shape which shape you gives say in the case of for a boat below the surface how it is provided and then how it is moving. So that is called the heel drag and then the next kind of drag is called residuary drag. This is actually the combination of the, we

have seen that when the boat is moving, there is also waves will be there on the water surface and then say how this waves with respect to moment of the boat. So this residuary drag is coming, it is the actually the combination of the wave making and viscous pressure drag or the form drag. This is the resistance caused by the creation of bow and stern waves; we can see that here in this figure. When this boat is moving how it behaves and how the waves are coming so with respect to the drag formation that drag is called residuary drag and also then added wave drag this is the additional drag caused by the oncoming waves. These are some of the other kinds of drag than the pressure drag or the friction drag which be consider, which are the most important kinds of drag or the fluid immersed in a body or the body moving through fluid. Now we have seen various kind of drag here, let us see how this drag effect will be there with respect to say a boat moving through say through the sea or a lake, what kind of say how this various kinds of drag effect will be coming over the boat. Here this slide show say with respect to the speed of the boat, the drag is also plotted here. So you can see that this figure shows the contribution of different drag forces. Here this line shows the total drag, you can see that this friction drag is which is also predominant here, this is the friction drag.

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And then we have this residuary drag and then this heel drag is very small and then we have this blue color here is this color here induced drag and then added wave drag, so that

the total drag is this line. This line is the total drag and then also you can see that this depends upon the speed of the boat with respect the speed of the boat is increasing you can see that the drag effect is also increasing as plotted here.

The various kinds of drag here we have seen and then say with respect to the movement of boat how the drag variations for with respect to various components, that is what is explained here in this slide. Now we have seen various kinds of drag and how the drag can be reduced with respect to shape or with respect to the placement of the of the body in the fluid. Now we will analyze how we can calculate, we can find this drag coefficient or how to find out the drag and lift with respect to the fluid flow and then with respect to the body which we consider. As we have already defined the drag force say we have already seen the equation for drag force and lift force.

If you analyze the say whenever we consider the a body immersed in fluid, with respect to the drag effect and lift effect, if you consider a dimension analysis and say we can see that with respect to the various parameters which have to be considered while dealing with a drag force are the length of the body, the dimensions, length, breath and other see the depth or dimensions and then ρ is the density of the fluid and μ is the coefficient of dynamics viscosity and E is the velocity of the velocity force or with respect to the fluid and g is the acceleration due to gravity and then the free stream velocity u infinity. So these are the important parameters we have to consider while considering, while analyzing the drag force and the lift force. Similarly, the drag force can be considered as a function of this length L of the density ρ .

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Analysis of Drag and Lift

- Drag Force, $F_D = f_1(L, \rho, \mu, E, g, u_\infty)$
Lift Force, $F_L = f_2(L, \rho, \mu, E, g, u_\infty)$
- Using dimensional analysis, it can be shown that,

$$\frac{F_D}{\rho L^2 u_\infty^2} = f_3 \left[\frac{\rho u_\infty L}{\mu}, \frac{u_\infty^2}{Lg}, \frac{u_\infty}{\sqrt{E/\rho}} \right] \text{ and}$$

$$\frac{F_L}{\rho L^2 u_\infty^2} = f_4 \left[\frac{\rho u_\infty L}{\mu}, \frac{u_\infty^2}{Lg}, \frac{u_\infty}{\sqrt{E/\rho}} \right]$$

And the coefficient viscosity μ and the elasticity E and access due to gravity and the free stream velocity. Similarly, the lift force also can be considered as a function of these parameters as shown here F_L is equal to f_2 as a function of L , ρ , μ , E , g and u_∞ . So we can do an analysis with respect to these various parameters. Using the dimensional analysis we can show that especially while doing experiment if you are doing the dimension analysis through dimension analysis we can show that this drag force and then lift force say if you consider F_D the drag force F_D is divided by $\rho L^2 u_\infty^2$ where u_∞ is the free stream velocity, so that is equal to as a function of say f_3 here $\rho u_\infty L$ by μ , u_∞^2 by Lg , u_∞ by square root of E/ρ . Similarly, this lift force we can write as F_L divided by $\rho L^2 u_\infty^2$ is equal to as a function of f_4 , $\rho u_\infty L$ by μ , u_∞^2 by Lg , u_∞ by square root of E/ρ . So we can represent in the dimension analysis like this with respect to the drag force and then with respect to the lift force after writing like this, we can see that, so we can have seen that this coefficient of drag and coefficient of lift we can write in terms of this. So the coefficient of drag is equal to the drag force F_D divided by half $\rho u_\infty^2 A$ where A is the area of the body which we considered, so whether where u_∞ is the free stream velocity. Similarly, coefficient of lift we can write as C_L is equal to the lift force F_L divided by half $\rho u_\infty^2 A$. Now if you consider this with respect to the

analysis here the dimension analysis which is explained here in this slide, we can write this coefficient of drag it can be put as a function of the Reynolds number, Froude number and Mach number.

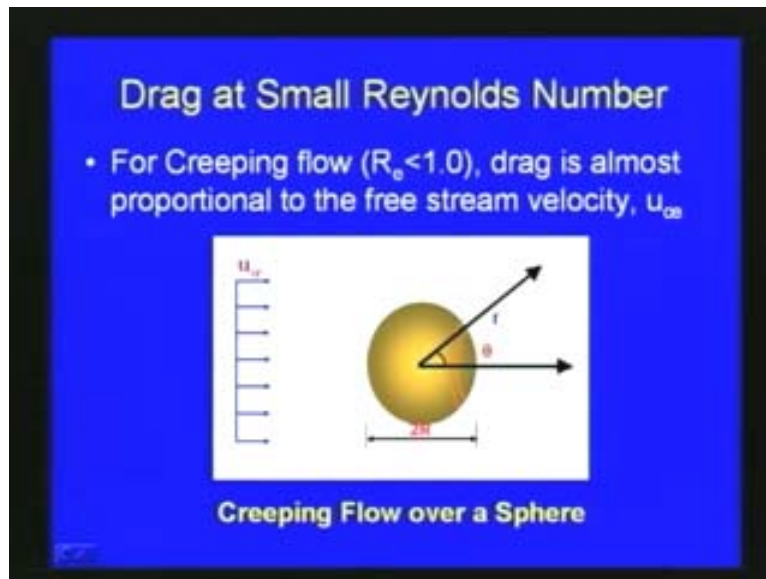
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- Coefficient of Drag, $C_D = \frac{F_D}{(1/2)\rho u_\infty^2 A}$
- Coefficient of Lift, $C_L = \frac{F_L}{(1/2)\rho u_\infty^2 A}$
- $C_D = \Phi_1(R_e, F_r, M_a)$ and $C_L = \Phi_2(R_e, F_r, M_a)$
 Where, R_e is the Reynolds Number
 F_r is the Froude Number
 M_a is the Mach Number
- Effect of F_r and M_a may be neglected

So you can see this component here $\rho u_\infty^2 L$ by μ represent the Reynolds number and the second term $u_\infty^2 L g$ representing the Froude number and this u_∞ by square root of E/ρ represent the Mach number. So we can represent the coefficient of drag and the coefficient of lift in terms of the as a function of the Reynolds number, Froude number and the Mach number. Generally, when we consider the drag and lift effect and then when we are finding the coefficient of drag and coefficient of lift we can show that, this Froude number and Mach number that term or that effect will be much smaller, so we can neglect it and generally we can represent this C_D coefficient of drag as a function of Reynolds number and also coefficient of lift as function of Reynolds number by neglecting this Froude number and Mach number which we considered here in this analysis. This way we can represent the coefficient of drag and coefficient of lift.

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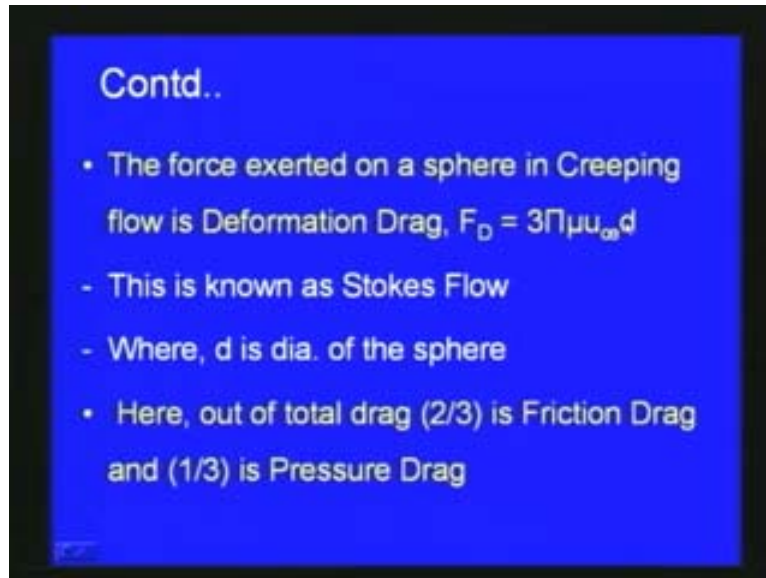


Now we will analyze the drag effect say initially with respect to the small Reynolds number and then we will see the laminar flow with especially with respect to a flat plate, we see the drag force for the coefficient of drag and its various parameters, coefficient of drag and other parameters for flat plate. Initially let us discuss about the drag at small Reynolds number, so this strokes conducted number of experiments with respect to the drag effect and then say especially in the case of Reynolds number is small Reynolds number say especially when the Reynolds number is less than 1, so this category of flow as been classified as creeping flow which we discussed earlier. Here you can see that the drag is proportional to the, so through experiments stocks shows that the drag is proportional to the free stream velocity u infinitive.

If you consider as in this figure say if you consider a sphere the creeping flow so wherever the Reynolds number is very small so that flow which is categorized as creeping flow so we can see that here the free stream is coming here and then if you consider sphere like this the creeping flow over a sphere say the radius R and if you consider an angle θ like this, so here the number of experiments conducted by stocks and then he has shown that the deformation drag for a creeping flow wherever the Reynolds number is very small as we have seen here say Reynolds number less than 1, there stocks showed that this deformation drag F_D is equal to $3 \pi \mu u$ infinitive into d ,

where μ is the dynamic coefficient viscosity; u_{∞} is the free stream velocity; and d is the diameter of the sphere, as shown here this is the diameter of the sphere; and u_{∞} is the free stream velocity. So for small Reynolds number considers the creeping flow stokes showed that this deformation drag F_D is equal to $3\pi\mu u_{\infty}d$.

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So this known as Stokes flow where this equation is applicable that kind of flow is called as stokes flow and as far as drag is concerned stokes shows that generally this after the total drag two third is the, say in the case of a creeping flow say if we consider the creeping flow over a sphere is shown that two third is friction drag and one third is generally pressure drag.

So this is the case where the Reynolds number is small say generally less than 1, where stokes stored this drag effect is concerned two third will be generally the friction drag and one third is the pressure drag and also say if we consider this stokes flow around a sphere then the coefficient of drag we can show that the coefficient of drag is equal to 24μ divided by $\rho u_{\infty}d$, d is the diameter of sphere, so that is equal to 24 by Re . This is valid for Reynolds number less than .1 and when the Reynolds number is between .1 to 1, we can show that this coefficient drag will be 24 by Re into one plus 3 by 16 into Re .

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- Coefficient of Drag for Stokes flow around a sphere can be given as:

$$C_D = \frac{24\mu}{\rho u_\infty d} = \frac{24}{Re} \quad \text{For } Re < 0.1$$
$$C_D = \frac{24}{Re} \left(1 + \frac{3}{16} Re\right) \quad \text{For } Re < 1.0$$
$$C_D = \frac{24}{Re} \left(1 + \frac{3}{16} Re\right)^{\frac{1}{2}} \quad \text{For } Re \text{ up to } 100$$

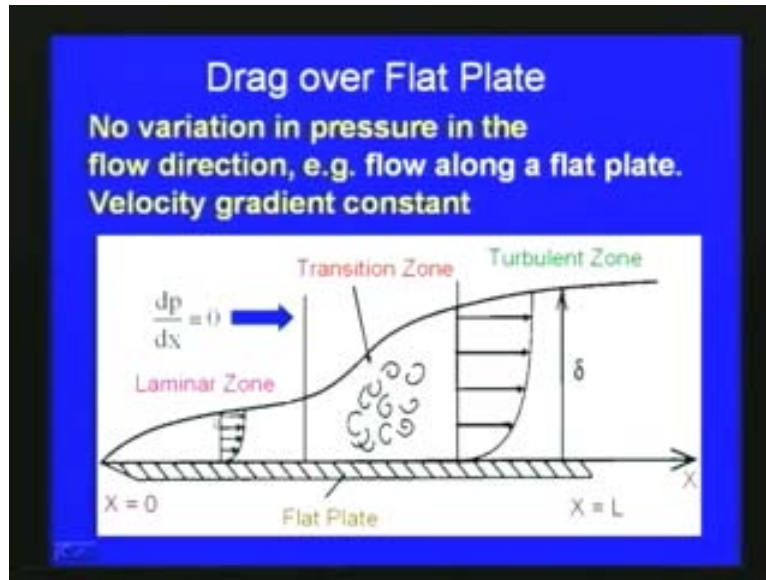
- Analysis of Sedimentation and Silting problems in water

Where Re is the Reynolds number which we considered and again say large number of experiments conducted for different Reynolds number and this coefficient of drag we can show that it will be C_D is equal to 24 by Re into 1 plus 3 by 16 Re to the power 1 by 3 for Reynolds number up to 100 . So this we can see that the coefficient of drag is varying with respect to the Reynolds number as shown in this slide. So this creeping flow which we discussed earlier also with respect to the Stokes flow, these kinds of analysis very important especially in civil engineering and sedimentation and silting problem is there in water and also the settlement of dust in the atmosphere, we can utilize this Stokes flow and then with respect to the we can calculate the coefficient of drag and also the drag force coming on the particle.

So that is the case where the Reynolds number is so small where we consider the creeping flow, we have seen the drag effect in the coefficient of drag and then the drag force. Now we will further analyze here the drag effect coefficient of drag and various other parameters with respect to the flow over a flat plate. We have already seen this flow over a flat plate case, when we consider the flow over a flat plate that can be three conditions first one is the boundary layer formation is that is laminar boundary layer; and then there is a transition states; and then there can be turbulent zone. If you consider the

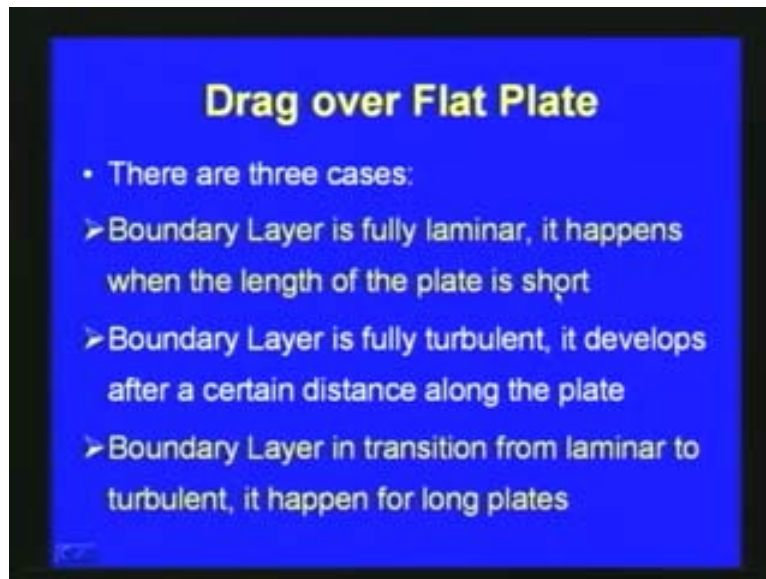
drag over a flat plate we can see that generally there is no variation pressure in the flow direction as we have discussed earlier so that velocity gradient is constant.

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So in this case we can see that here we have the laminar zone and then there is transition zone and here with respect to boundary layer formation we have the turbulent zone. Now with respect to these various cases, we will determine the drag how to find out the drag for a laminar case and then turbulent case and also the transition case.

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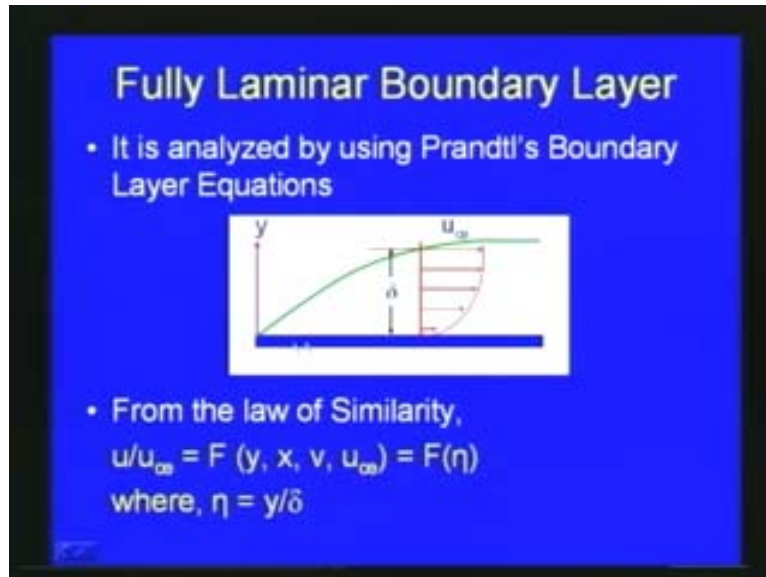
When we consider as we discussed for drag over a flat plate boundary layer is fully laminar. So generally it happens when the length of the plate is short or initially as we have seen in the previous slide initially it can be laminar and then transition and turbulent or say only laminar boundary layer you if the length of the plate is very small and then the velocity is also the free stream velocity is small and then second case is when the boundary layer is fully turbulent so it develops after a certain distance along the plate as we have seen here. So here finally here the boundary layer is totally turbulent.

Then the third case is boundary layer in transition from laminar to turbulent, so this is the case where the transition zone here we can see that this is a laminar here it is turbulent so here is the transition zone. Here we want to find out the drag force with respect to say for the flow over a flat plate since this is one of the generally used to case to analyze various parameters, so here also to analyze the drag force the coefficient of drag we use the flow over a flat plate problem.

Now first, let us consider the laminar boundary layer formation or for flow over a flat plate in the flow is fully laminar boundary layer and then how we can get the drag coefficient and then the drag force. So with respect to the flow over a flat plate for fully laminar boundary layer Prandtl analyze this problem by using the Prandtl's boundary

layer equations which we have discussed earlier, so you can see that here the this is the flat plate here and then flow free stream velocity is coming like this and then you have the boundary layer formation.

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So this is the boundary layer thickness and here the u_{∞} is the free stream velocity. By considering the Prandtl's boundary layer equations from the law of similarity, we can write say the velocity at any location u by u_{∞} is a function of y , x , ν and u_{∞} where ν is the dynamic viscosity here, so this is equal to $F(\eta)$ so where η we can represent as this y is in this direction η is equal to y by δ the location which we consider where the velocity is u and the location is at y .

So now with respect to this say from the boundary layer flow analysis of unsteady motion Prandtl's analysis showed that this boundary layer is proportional to square root of ν into t where ν is the kinematics viscosity and t is the time required for a fluid particle to travel a distance x with velocity u_{∞} . Here you can see this figure earlier, so the flow is taking place like this and then we have this the fully laminar boundary layer, δ the boundary layer thickness is proportional to square root of ν into t where t is the time required for a fluid particle to travel distance x with velocity u_{∞} the free stream velocity.

So this we can write this is proportional square root of νx by u_{∞} or we can write δ is equal to x by square root of R_x where this R_x is the Reynolds number at that particular location where we considered. So now in the previous slide we have seen this y is represent as this η is equal to y by δ .

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- From B.L flow analysis of unsteady motion,

$$\delta = \sqrt{\nu t} = \sqrt{\frac{\nu x}{u_{\infty}}} = \frac{x}{\sqrt{R_x}}$$
 t is the time required for a fluid particle to travel a distance x with velocity u_{∞}
- Now $\eta = \frac{y}{\delta} = y \sqrt{\frac{u_{\infty}}{\nu x}}$
- The stream function, $\psi = \int u dy = u_{\infty} \delta \int F(\eta) d\eta$
- Hence, $\psi = \sqrt{u_{\infty} \nu x} f(\eta)$ where, $f(\eta) = \int F(\eta) d\eta$

This is equal to y into square root of u_{∞} by ν into x , so with respect to this equation here Prandtl's analysis shown that η is equal to y into square root of u_{∞} by ν into x . Here if you introduce the stream function which is defined as ψ is equal to integral $u dy$ so that is equal to $u_{\infty} \delta$ integral $F(\eta) d\eta$, so that is equal to u_{∞} substitute for δ we can write u_{∞} into square root of νx by u_{∞} into function of $F(\eta) d\eta$. So that we can write ψ is equal to stream function is equal to square root of u_{∞} into ν into x into $f(\eta)$ where $F(\eta)$ is equal to integral $f(\eta) d\eta$ coming from here $f(\eta) d\eta$.

Now we know that the velocity can be represent as u is equal to $\frac{\partial \psi}{\partial y}$, so that we can write u is equal to $\frac{\partial \psi}{\partial \eta} \frac{\partial \eta}{\partial y}$. This is equal to $u_{\infty} f'(\eta)$. So velocity is represented as at any location u is equal to $u_{\infty} f'(\eta)$ and then the velocity gradient we can write as $\frac{\partial u}{\partial y}$ is equal to $\frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial y}$, so this equal to u_{∞} square root of u_{∞} by νx .

into f'' , so second derivative. This f' , so here first derivative here the second derivative or f'' and then the shear stress on the surface of the flat plate from the Newton's law we can write $\tau = \mu \frac{du}{dy}$ at $y = 0$, so that is equal to $\mu u_\infty \sqrt{\frac{u_\infty}{\nu x}} f''(0)$ at zero location.

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- Now velocity, $u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta} \frac{\partial \eta}{\partial y} = u_\infty f'(\eta)$
- Velocity Gradient, $\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial y} = u_\infty \sqrt{\frac{u_\infty}{\nu x}} f''(\eta)$
- Shear stress on the surface of the flat plate, from Newton's Law $\tau = \mu \left(\frac{du}{dy} \right)_{y=0} = \mu u_\infty \sqrt{\frac{u_\infty}{\nu x}} f''(0)$ (1)
- Blasius soln. of B.L flow analysis over a flat plate gives, at $\eta = 0$, $f''(0) = 0.332$

So this is the shear stress on the surface of the flat plate. And then by using this Prandtl's equations Blasius derived this solution by the discussed steps and then the Blasius solution for the boundary layer flow analysis or flat plate at $\eta = 0$, he showed that $f''(0) = 0.332$, so that we can write say from the velocity profile obtained by Blasius solution, the boundary layer thickness δ is equal to $5 \sqrt{x}$ by square root of R_x where R_x is the Reynolds number at that particular location which we consider at say where u/u_∞ is equal to 0.992 with respect to the definition. So that δ is obtained as $5 \sqrt{x}$ by square root of R_x where R_x is the Reynolds number and also here δ^* the displacement thickness is equal to $1.73 \sqrt{x}$ Blasius showed that δ^* is equal to $1.73 \sqrt{x}$ divided by square root of R_x .

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- From the Velocity Profile obtained by Blasius Solution, B.L Thickness, $\delta^* = \frac{5x}{\sqrt{R_x}}$ at $u/u_\infty = 0.992$
- It can be shown, $\delta^* = \frac{1.73x}{\sqrt{R_x}}$ and $\theta = \frac{0.664x}{\sqrt{R_x}}$
- Now from (1) $\tau_0 = 0.332\mu u_\infty \sqrt{\frac{\rho u_\infty}{\mu x}} = 0.332 \frac{\rho u_\infty^2}{\sqrt{R_x}}$
- R_x – Local Re. Number = $(\rho u_\infty x) / \mu$

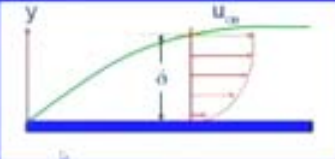
Where R_x is the Reynolds number at that particular location and the momentum thickness θ is equal to $0.664x$ by square root of R_x where R_x is the Reynolds number at that location. Now from the equation number 1 here is substitute in this equation number 1 we can show that τ_0 is equal to $0.332\mu u_\infty \sqrt{\frac{\rho u_\infty}{\mu x}}$ so that is equal to $0.332 \rho u_\infty^2$ by square root of R_x where R_x is the local Reynolds number represent as represented as $\rho u_\infty x$ by μ . So this way by using the Prandtl's equations Blasius derived this expression for δ , δ^* , θ and then also expression for τ_0 the shear stress on the flat plate.

And then now we want to see with respect to drag effect so the when we consider the local friction drag coefficient with respect to this figure here, local means at the particular location so here C_f is equal to the local friction drag coefficient C_f is equal to friction drag divided by dynamic pressure drag so that we can write C_f is equal to the friction drag is even a τ_0 into A by this dynamic pressure drag is obtained half ρu_∞^2 into A , this is equal to τ_0 by half ρu_∞^2 . So this he sort with respect to the previous expressions here for τ_0 which is obtained here, so Blasius showed that C_f is equal to 0.664 by square root of R_x where R_x is the local Reynolds number.

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- Local Friction Drag Coefficient,
- $C_f = \frac{\text{Friction Drag}}{\text{Dyn. Pr. Drag}}$



- i.e. we have

$$C_f = \frac{\tau_w A}{\frac{1}{2} \rho u_{\infty}^2 A} = \frac{\tau_w}{\frac{1}{2} \rho u_{\infty}^2} = \frac{0.664}{\sqrt{R_x}}$$

So this C_f is the local friction drag coefficient for the laminar boundary layer formation over a flat plate. Now the friction drag say over one side of the plate of length l per unit width we can write as F_{Df} is equal to integral 0 to L of $\tau_w dx$, so this is equal to integral zero to L of $0.332 \frac{\rho u_{\infty}^2}{\sqrt{R_x}} dx$.

So this is equal to $0.664 \rho u_{\infty}^2 \sqrt{L}$ by u_{∞} , this is the friction drag and then from this F_{Df} the friction drag over one side of the plate, so the average coefficient of friction drag we can get from this expression this C_f is equal to F_{Df} by half $\rho u_{\infty}^2 L$, so this is equal to $0.664 \rho u_{\infty}^2 \sqrt{L}$ by half $\rho u_{\infty}^2 L$, so that is equal to 1.328 divided by square root of R_L where R_L is the Reynolds number at the trailing edge of the drag plate. Finally, here we got the coefficient of friction drag for the plate C_f as C_f is equal to 1.328 divided by square root of R_L . In the previous space here what we considered here is the local friction drag coefficient which we got as 0.664 divided by square root of R_x , so here we got as C_f is equal to 1.328 divided by square root of R_L where R_L is the Reynolds number at the end of the plate which we consider.

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- Friction drag over one side of the plate of length L per unit width, can be given as:

$$F_{\text{Df}} = \int_0^L \tau_w dx = \int_0^L \frac{0.332}{\sqrt{x}} \rho u_\infty^2 dx = 0.664 \rho u_\infty^2 \sqrt{\frac{L}{\nu}}$$

- The average coefficient of Friction Drag can be given as:

$$C_f = \frac{F_{\text{Df}}}{\left(\frac{1}{2} \rho u_\infty^2 L\right)} = \frac{0.664 u_\infty^2 \sqrt{\frac{L}{\nu}}}{\left(\frac{1}{2} \rho u_\infty^2 L\right)} = \frac{1.328}{\sqrt{R_L}}$$

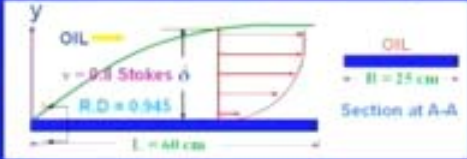
Now like this here and various parameters have been derived for the coefficient of drag whether it is local or at the end of the plate, so with respect to this we have seen say by starting from the Prandtl's boundary layer equations the parameters like delta, delta star, and then momentum thickness etcetera are derived by Blasius in the earlier slides which we have discussed. Now we will discuss say a small example here, so the example here is say we have to calculate the friction drag on a flat plate so here this a flat plate problem.

So calculate the friction drag on a flat plate of 25 centimeter wide and 60 centimeter long placed longitudinally in a stream of oil of relative density 0.945 and kinematic viscosity .8 stokes flowing with a free stream velocity of 3 meter per second and also find the thickness of the boundary layer and shear stress at the trailing edge. So here the problem is this is the free stream velocity we have free stream velocity of 3 meter per second and then the rate density of the fluid is .945 and length of the plate is 60 centimeter and width is 25 centimeter and kinematic viscosity is .8 stokes.

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Example

- Calculate the Friction drag on a flat plate of 25 cm wide (B) and 60 cm long (L) placed longitudinally in a stream of oil of Relative Density 0.945 and Kinematic Viscosity 0.8 stokes, flowing with a free stream velocity of 3 m/s also find the thickness of the boundary layer and shear stress at the trailing edge



This is the section of the plate, so for this problem we have to find out the thickness of the boundary layer and the shear stress at the trailing edge. We have already seen for these kinds of problem enough for the we can first find out the at the trailing edge what will be the Reynolds number, so that we can see that with respect to the whether the flow is laminar or not and then we can use the equations which we have discussed earlier to get the boundary layer thickness and the shear stress.

Here for this problem the Reynolds number at the trailing edge Re_l is equal to uL by ν , so 3 into .6 by .8 into 10 to the power minus 4, since here .8 stokes is the kinematic viscosity. So this is equal to 2.25 into 10 to the power 4, so Reynolds number at the trailing edge of the plate is 2.25 into 10 to the power 4. So here we can see that 1 stoke is equal to 10 to the power minus 4 meter square per second, so here say we have when we discussed the initially about the boundary layer we have seen say the especially for the case of a for flow over a flat plate then the Reynolds number is less than 5 into 10 to the power 5 we are categorize the boundary layer as laminar, so here we got 2.25 into 10 to the power 4 as a Reynolds number.

So you can see that the boundary layer is laminar so we can use the equations which we have seen earlier since that boundary layer is laminar in nature, so the Blasius solution we

can use so delta is equal to 5 into x by square root of R_x , that is the in the boundary layer thickness equation E 1 by Blasius.

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Solution

- R_e at the trailing edge $R_{eL} = \frac{u_{\infty} L}{\nu} = \frac{3 \times 0.60}{0.8 \times 10^{-4}} = 2.25 \times 10^4$
- Note: 1 Stoke = $10^{-4} \text{ m}^2/\text{s}$
- As $R_{eL} < 5 \times 10^5$, Boundary layer is Laminar
- Hence from Blasius solution, $\delta = \frac{5x}{\sqrt{R_x}}$ at the trailing edge $x = L = 0.60 \text{ m}$
- so $\delta_L = \frac{5 \times 0.60}{\sqrt{2.25 \times 10^4}} = 0.02 \text{ m}$
- Shear stress at the trailing edge

$$\tau_L = \frac{1}{2} \rho u_{\infty}^2 C_f(L) = \frac{1}{2} \rho u_{\infty}^2 \left(\frac{0.664}{\sqrt{R_{eL}}} \right) = \frac{0.945 \times 1000 \times 3^2 \times 0.664}{2 \times \sqrt{2.25 \times 10^4}} = 18.82 \text{ N/m}^2$$

So at the trailing edge means at the end of the plate, so here at this location, so that means the boundary layer thickness here so we can see that L is equal to .6 so delta L is equal to 5 into .6 divide by and Reynolds number square root of 2.25 into 10 to the power 4 which is Reynolds number so we get the boundary layer thickness at the trailing edges 0.02 meter and now the second problem here is the find out the shear stress at the trailing edge.

So to find out the shear stress we have already seen this equation here so this equation is here τ_{wL} is equal to half rho u infinitive square $C_f L$ so that is equal to half rho u infinitive square so here this $C_f L$ is seen 0.664 square root of R_{eL} , so this is equal to rho is .945 into 1000 into say, free stream velocity is 3, so 3 square into .664 divided by 2 into square root of 2.25 into 10 to the power 4, so this will give the shear stress at the trailing edge as 18.82 Newton per meter square. And then the drag on one side of the plate F_D is equal to C_{Df} into L into B into rho infinitive square by 2 and then the C_{Df} that means the when we consider say here the say total drag so we have seen the drag is concerned the local drag coefficient is concerned local drag coefficient as well as the full

drag coefficient as we have seen here the average coefficient of friction drag C_{Df} 1.328 root Re_L so this equation we can utilize here so that C_{Df} is equal to 1.328 divided by square root of Re_L so that is equal to 1.328 divide by square root of 2.25×10^4 so this is C_{Df} is equal to 8.85×10^{-3} .

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- Drag on one side of the plate, $F_D = C_{Df}(LB) \frac{\rho u^2}{2}$
- Now, $C_{Df} = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{2.25 \times 10^4}} = 8.85 \times 10^{-3}$
- Hence, Force due to Drag,

$$F_D = C_{Df}(LB) \frac{\rho u^2}{2} = \frac{8.85 \times 10^{-3} \times 0.6 \times 0.25 \times 945 \times 1000 \times 3^2}{2}$$

$$= 5.695 \text{ N}$$
- Total Drag force on both side of the plate =
 $2 \times F_D = 2 \times 5.695 = \underline{11.29 \text{ N}}$

And then the force due to drag we can calculate F_D is equal to C_{Df} into area L into B into ρu infinitive square by 2, so this is equal to 8.85×10^{-3} which is the C_{Df} into $.6$ L is $.6$ and B is $.25$ here into $.945$ so here B is 25 centimeter so that is why $.25$ into $.945$ into thousand into 3 square by 2, so this gives 5.695 Newton. So total drag force on both sides of the plate is equal to say here this calculation is for one side both sides to be two times the drag force calculated. So 2 into 5.695 is equal to that gives 11.29 Newton. So like this we can solve the drag force with respect to the laminar boundary layer formation in the case of a flat plate and now the next case is drag over flat plate where the boundary layer is turbulent.

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Fully developed Turbulent Boundary Layer

- For turbulent boundary layer flow over a flat plate, momentum integral eqn., may be written in slightly different form as:

$$\frac{d}{dx} \int_0^{\delta} u(u_{\infty} - u) dy = \frac{\tau_w}{\rho} \quad \dots(1)$$
- Prandtl experimentally showed that for turbulent boundary layer,

$$\frac{u}{u_{\infty}} = \left(\frac{y}{\delta} \right)^{1/7} \quad \dots(2)$$
- This is also known as Blasius One-seventh Power Law

So we have seen so now the laminar we have seen now if the boundary layer is fully turbulent then say the various equations here we consider for turbulent boundary layer. Now here say we have seen when we discuss the boundary layer earlier, we have seen the momentum integral equation, with respect to this for fully developed turbulent boundary layer we can see over a flat plate the momentum integral equation maybe written in slightly different form as $\frac{d}{dx}$ of integral 0 to δ $u(u_{\infty} - u) dy$ that is equal to τ_w by ρ as in equation number 1.

Where u_{∞} is the free stream velocity and u is the velocity at any location τ_w is the boundary shear stress and ρ is the density and then Prandtl's experimentally showed that for turbulent boundary layer we can write this u by u_{∞} is equal to y by δ to the power $1/7$ so this is coming from the experimental observation by Prandtl's, so this is u by u_{∞} is equal to y by δ to the power $1/7$ as in equation number 2 this is known as Blasius one seventh power law, so this is through experimental observations. And then also the velocity distribution turbulent flow over a flat plate say when the Reynolds number is less than 10^7 through experiments it was be shown that u by square root of τ_w by ρ is equal to 8.74 into square root of τ_w by ρ into y by ν to the power $1/7$.

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- Velocity distribution in turbulent flow over a flat plate valid for $R_e < 10^7$ $\frac{u}{V_\infty} = 8.74 \left[\frac{\sqrt{\tau_w} \cdot y}{\nu} \right]^{1/4}$
- From Turbulent flow analysis of pipe, $\tau_w = 0.034 \rho V^2 \left(\frac{\nu d}{V} \right)^{1/4} \dots (3)$
- Average velocity, $V = 0.817 u_{max} = 0.817 u_\infty$
- Hence, $\tau_w = 0.0233 \rho u_\infty^2 \left(\frac{\nu \delta}{V} \right)^{1/4} \dots (4)$
- Using (1) (2) and (4) $\frac{\delta}{x} = \frac{0.379}{(R_x)^{1/5}} \dots (5)$
- Using (5) in (4) $\tau_w = 0.0295 \rho u_\infty^2 (R_x)^{-1/5} \dots (6)$

And also here this say with respect to the Blasius analysis and using the momentum integral equations same with respect to the pipe flow for turbulent flow in pipes it is shows that τ_{w0} is equal to $0.034 \rho v^2 \nu d$ by ν to the power minus one by 4 as in equation number 3 and the average velocity for with respect to pipe flow we can show that v is equal to .817 the maximum velocity u_{max} , so here the maximum velocity of the free stream velocity so that is equal to .817 $u_{infinite}$. So by using this here we can show that τ_{w0} is equal to zero .0233 $\rho u_{infinite}^2 \nu$ by ν to the power minus 1 by 4 as in equation number 4.

So now if you use the previous equation, equation number 1 and then equation number 2, equation number 2 here and then equation number 4, we can show that δ by x is equal to for turbulent boundary layer δ by x is equal to .379 divide by R_x to the power 1 by 5 as in equation number 5. So this is to find out the boundary layer at any location with respect to the turbulent boundary layer formation and now if you use this equation number 5 and 4 together, we can show that τ_{w0} is equal to .0295 $\rho u_{infinite}^2 R_x$ to the power minus 1 by 5 as in equation number 6. So now for turbulent boundary layer local friction coefficient we can find out C_f is equal to τ_{w0} by half $\rho u_{infinite}^2$ square as we have discussed earlier so this for turbulent boundary layer the local friction

coefficient we can show that C_f is equal to .059 divided by R_x to the power 1 by 5 where R_x is the Reynolds number local Reynolds number as in equation number 7.

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- Local Friction Coefficient, $C_f = \frac{\tau_0}{(1/2)\rho u_\infty^2} = \frac{0.059}{(R_x)^{1/5}} \dots (7)$
- The Friction drag per unit width for one side of the plate of length L is

$$F_{Df} = \int_0^L \tau_0 dx = 0.0368 L \rho u_\infty^2 (R_L)^{-1/5} \text{ and } C_f = \frac{F_{Df}}{(1/2)\rho u_\infty^2 L} = \frac{0.074}{(R_L)^{1/5}} \dots (8)$$

- Eqn. (8) is valid for Reynolds number in the range of $5 \times 10^5 < R_L < 10^7$
- For $R_L > 10^7$, Prandtl calculated

$$C_f = \frac{0.455}{(\log_{10} R_L)^{2.58}} \dots (9)$$

In the friction drag per unit width for one side of the plate is we can integrate F_{Df} is equal to integral zero to L $\tau_0 dx$ so this is equal to $0.0638 L \rho u_\infty^2 R_L^{-1/5}$ and hence C_f is equal to F_{Df} by half $\rho u_\infty^2 L$. This is we can show that C_f is equal to 0.074 divided by R_L to the power 1 by 5 as given equation number 8. So this equation number 8 through experiment shows that this equation number 8 is valid for Reynolds number in the range of 5 into 10 to the power 5 and between 10 to the power 7 the Reynolds number 5 into 10 to the power 5 to 10 to the power 7 and when again Prandtl calculated and also shown through experiment starts when the Reynolds number is greater than 10 to the power 7 he showed that this the friction coefficient C_f is equal to 0.455 divided by $\log_{10} R_L$ to the power 2.58.

So this is thoroughly seen by Prandtl's and verified through experiment as shown in this equation 9. So this gives for turbulent boundary layer this gives the various equation friction coefficient and the coefficient say the average friction drag coefficient for the turbulent boundary layer.

Next say we will be discussing further on the drag force and the drag coefficient in the transition case and also further with respect to various shapes and with respect to various body parts how the drag coefficient changes will be discussing in the next lecture.