Fluid Mechanics

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Lecture - 30

Boundary Layer Theory and Applications

Welcome back to the video course on fluid mechanics. In today's lecture, we will see a new chapter, boundary layer theory and applications. The important objectives of this lecture or the different topics which we will discuss are: first one is external flows and boundary layer; then we will discuss the various concepts of boundary layer; then we will be discussing the laminar and turbulent boundary layer and related theory; next we would be discussing specifically on the boundary layer over flat plate and then boundary layer in pipe flow; then we will be discussing various topics on flow separation; we will also be discussing various topics like drag and lift on immersed bodies; finally, we will be discussing on the circulation.

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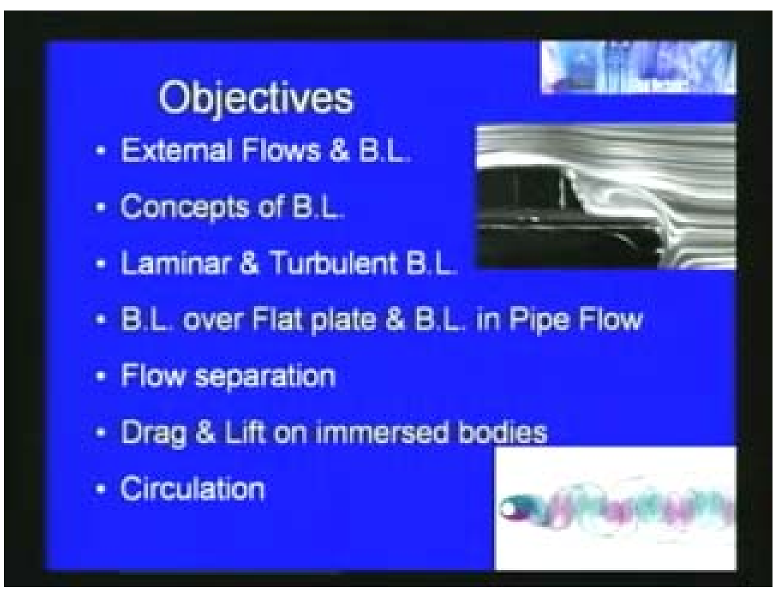

We discussed earlier in some of the other lectures that this boundary layer is one of the most important subjects in fluid mechanics. The flow, as we have discussed earlier, can be classified into external flows and internal flows. Most of the times the fluid is passing through either just in a pipe, like in the case of internal flows or it is passing through different kinds of bodies or it is passing through the channel, like in an open flow. So, in all these places, we can see that, say, the fluid is some part of, say, at the bottom or on the sides of the container through which the fluid is passing. Then, there is always the fluid, which is touching the solid boundary.

So, the fluid is moving and solid boundary, most of the time, it may not be moving. Sometimes it may be moving. So, there will be always a velocity difference. If the solid is not moving, then the velocity of the container, over which the fluid is moving, the velocity is zero. Then, fluid has greater velocity and say, sometimes like in the case of a moving vehicle such as a car or a bus or flying machine like airplane or space shuttle, then the body itself is moving and also the fluid may not be moving, but, some kinds of fluids also may be moving. So, in all these aspects, you can see that there is always some way of interaction between fluid and structure or solid body interaction fluid and solid body interaction.

So, you can see here, say, when a car is moving here, you can see that, it is passing through the air and there is a window also. Then, we can see that, if you analyze the flow passing over it, you can see the various phenomenon such as initially, a boundary layer is created. Also, the flow is transformed from laminar to turbulent. Or if you consider the flow passing through over an immersed circular cylinder or a sphere, you can see how the system is working. So, there is always a cylinder here, which is not moving; it is on the periphery consider that fluid due to the viscous forces, you can see that, there is a no slip condition.

So, this flow pastiness bodies, as we are seeing here are, in all these cases, many field problems, we can see that, there is always no slip condition and then fluid flow can be classified as external flow and internal flow. Here, this is external flow that means the flow avarine just like block like this. Otherwise, we can see that this bird is flying and you can see that, here the bird is moving and then the fluid is surrounding it. Here, we can see that this or this case are external flow problems or the fluid flow surrounding in airplane or in space shuttle or submarine, all these cases are external flows. But, here, we can see that the solid or the body, which we are considering is like this and the fluid is surrounding it.

So, that is called external flow or flow past object or immersed bodies. So, there are number of examples, as I mentioned, airplane like in the aerodynamics or flow past vehicles, buildings, and so on.

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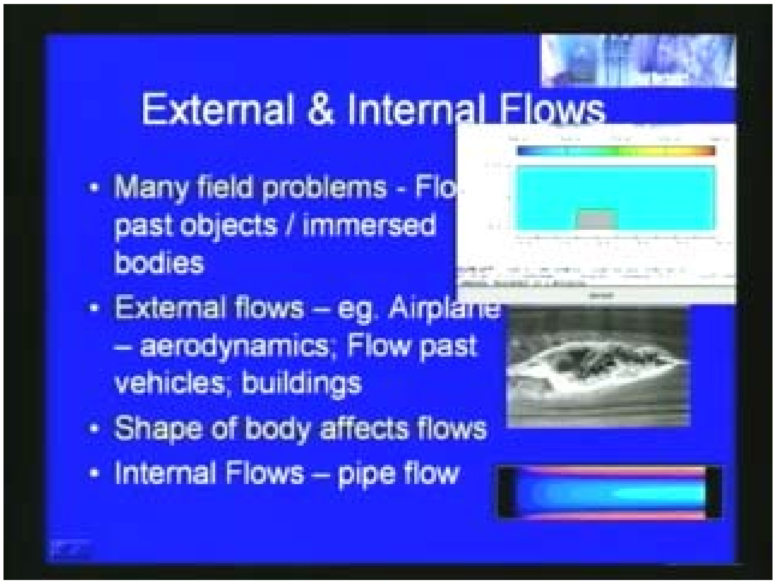

Here, the important parameter is the shape of the body. So, depending upon the shape of the body, the flow behavior changes and then we have to analyze depending upon the shape of the body, whether it is a blend body or it is a smooth body or it is a stream line body. Depending upon that, the flow behaves differently. So, external flows are important class of fluid flow problem, which we have to deal most of the time. Then, in this case, we can see here, the fluid flow surrounding a bird and you can see that, there is a boundary layer that is created. Also, here, you can see in this typical case, say, whenever the flow is taking place over the block, then we can see that fluid flow will be coming like this. Then, this solid is always set; it is not moving. So, there is a no slip condition here and boundary layer is created.

Like this, most of the external flows problems, there is always boundary layer created. Then we have to deal with respect to this boundary layer. That is, when we analyze the fluid flow, we have to deal with this boundary layer and then the remaining fluid flow. Sometimes, other than the boundary layer, we can consider as a potential flow or ideal flow. Also, in the case of other class of the flow, that is, the internal flow, where the fluid flow is contained in the solid media or in a container like in the case of pipe, we can see here, this is a pipe flow. Then we can see that, from here, the flow starts and then as the flow progresses, you can see that, here on the sides of the pipe; here on this side and this side, we can see, it is a no slip boundary condition. Then, the fluid, say, with respect to this, there is velocity change is taking place and there is also a boundary layer created for the internal flows also.

Like this, there can be number of examples, where we can show that, wherever the fluid may be flowing through an internal flow, in the case of pipe flow or the fluid passing over the immersed body is or past objects; then external flow, all these cases, boundary layers are created. Then we have to deal most of the practical fluid flow problem, this boundary layer. Like this, always, this boundary layer is the major area which we have to study in detail, how the boundary layer is behaving, whether it is laminar turbulent and then how the velocity changes, how the pressure and other parameters change in the boundary layer. This is same most of the time, say, fly aerodynamics like flying vehicle, air plane or space shuttle. So, this boundary layer analysis is very important, since based on that only, we can design this air plane or the space shuttle, since this velocity variation is very important. But, outside the boundary layer, we can approximate and depending upon the flow condition, we can even consider it as a potential flow. So, this concept of boundary layer is, say, first, we discuss and then we will see some of the equations related to the boundary layer.

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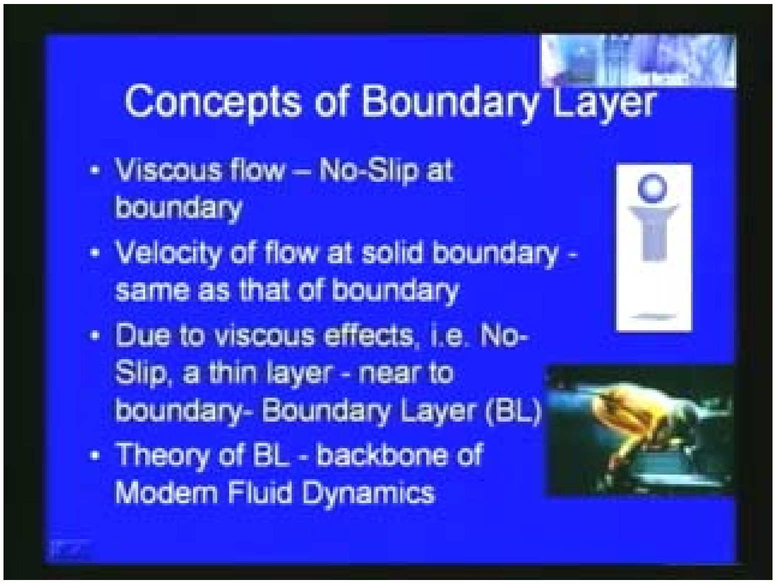

As we have already seen in many of the cases, wherever viscous flow is there, you can see that there is a no slip condition at the boundary. Say, the fluid is flowing and then due to this no slip condition at the boundary, the flowing fluid has a velocity. Then, this velocity flow of the solid boundary should be same as that of the boundary. Due to this condition and the viscous effect, a thin layer near to the boundary layer is generated. So, you can see that, even in this case also, in this figure also, you can see that the thin layer here is generated and this is called the 'boundary layer'.

Whenever we deal with viscous flow, this boundary layer is generated due to the no slip condition, all the times we have to deal with boundary layer. So, this theory of the boundary layer is actually the backbone of the modern fluid dynamics. So, you can see that, the boundary layer theory has been proposed in the beginning of the twentieth century by brand in and other side. This and then, many theories were developed related to the boundary layer. Since, we could say, distinguish the boundary layer and then say, we could derive some expression for the velocity and pressure variation boundary layer. Due to this development in the boundary layer and in the modern fluid dynamics only, most of this, say, flying airplane or space shuttle, all these developments took place in the twentieth century.

So, before going into more details on the boundary layer, I just want to show you seconds of video produced by hundraous which is available in the internet. Here, you can see how a boundary layer is generated; say a flow is going like this. Then if a solid is placed like this, then you can see that how a boundary layer is generated and how the velocity distribution takes place and then how the separation takes place.

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So, related to this, further, we will be discussing all this phenomena in detail. Then, we will be trying to derive expression for these boundary layer aspects later. So, now, as I mentioned, as we have already seen, here this boundary layer is very important phenomena, which we have to consider. Since we can differentiate, whenever fluid flow is taking place, especially in the case of very immersed body or a flow past body. If we can consider the boundary layer, then the flow outside the boundary layer can be considered as a potential flow or ideal flow. So, you can see here, in this slide, say, if this is upto the boundary layer which we consider, when you deal with viscous fluid.

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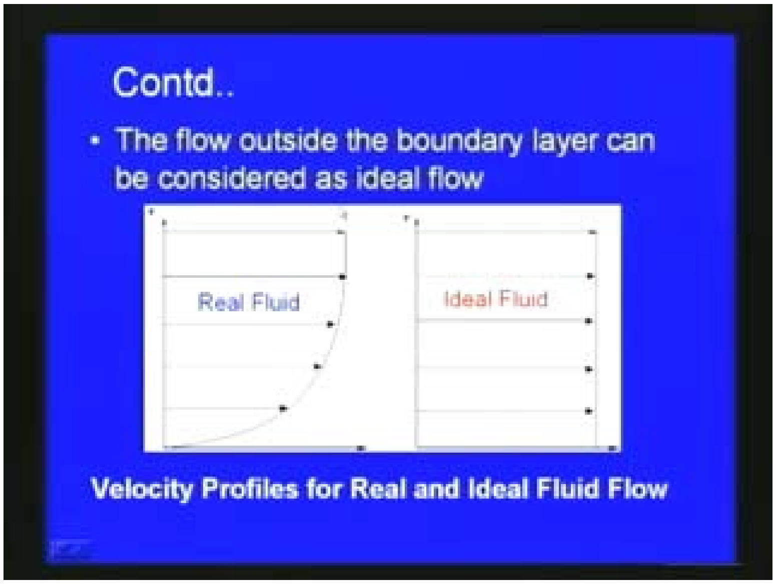

If you flow the velocity profiles for real and ideal fluid flow. So, real fluid, you can see that, due to the no slip condition here, the velocity distribution will be going parabolically like this and then depending upon the flow depth, it is coming here, the maximum. But, in the case of ideal fluid flow, we do not have to consider this no slip condition. So, it is the fluid velocity which will be like this.

If you can identify the exact location of the boundary layer and then say, after that, we can approximate, eventhough it may not be hundred percent accurate approximation. But, say, we can approximate many times after this boundary layer, we can consider as an ideal fluid flow or a potential flow. So, we can reduce the problem and then we can deal with the problem much easier in comparison, when we consider throughout the complete domain, where the flow over a plain or flow over a space shuttle when we consider.

Then, we have to go for a large domain for analysis, but if this boundary layer can be identified and then say, upto that, we have to deal very precisely. Then, beyond the boundary layer, we can either consider as an ideal fluid flow for potential flow, so that we get very simplified relation to solve the problem. So, when we consider the boundary layer, these kinds of separation always help to get a better solution. Now, you will further discuss various fundamental definitions as far as boundary layer is concerned. As we have already discussed the differences in velocity of the fluid particles between the object and the fluid surface as confined to a region is called a boundary layer.

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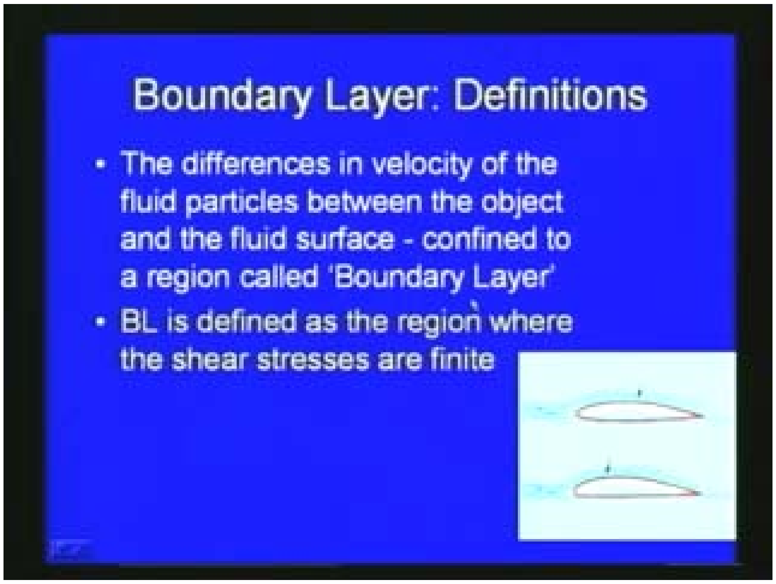

That is the definition. So, we can define the boundary layer as the region where the shear stresses are finite and then the fluid particles between the object and fluid surface is confined to a region. If you consider a body like this, then we can see that, this free stream flow is coming and free stream velocity is coming like this. These are the stream lines that are shown here. Then, say, this difference in velocity of the fluid particles between this object and the fluid surface is confined to a layer and this layer is called boundary layer. Then, we can see that, here the shear stresses are finite and then we can analyze with respect to this boundary layer. Remaining fluid flow we can consider as a potential flow or ideal fluid flow, depending upon the case. So, this way, we can simplify the problem by considering the boundary layer and then the flow beyond the boundary layer.

To define various terms in boundary layer theory, let us consider free stream flow over a flat plate. Here, in this case, the flow is taking place freely and then we introduce a flat plate. Here, we can easily visualize that how the boundary layer is generated. With respect to that, we can define various parameters or various aspects of boundary layer can be explained. So, this figure shows the boundary layer growth over a flat plate.

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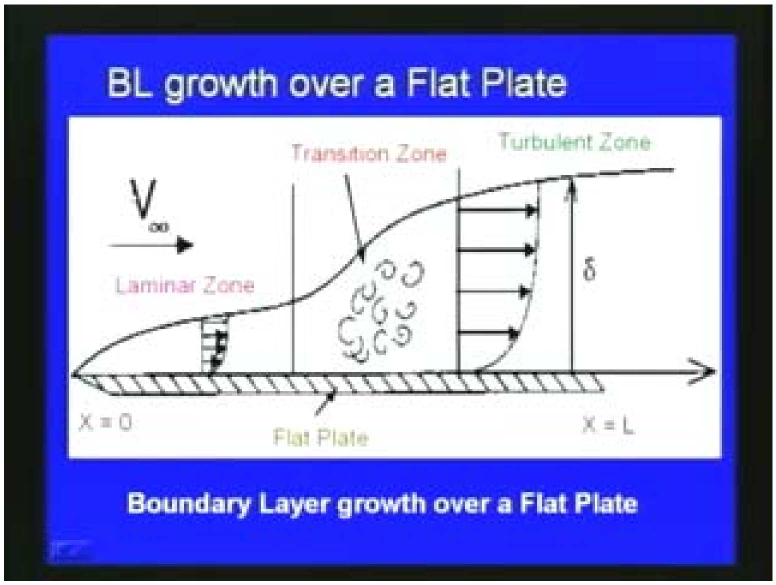

Here, the flat plate is placed like this. Here is the origin of the flat plate. So, X is equal to zero and length of the flat plate is l. Then velocity of v infinitive or here it is indicated as v infinitive, this is the free stream velocity coming. So, it is touching the flat plate placed on this. Now, here, it is flat plate which is not moving and obviously, they say, due to the no slip condition here, where the fluid is touching on the surface of the flat plate where the velocity should be zero.

So, the velocity if you plot, we can see that from zero, it will be keep on increasing like this. Now, we can identify a boundary layer like this. Depending upon the case, here, initially it will be laminar zone boundary layer. Then, after sometime, turbulent starts and there is a transition zone. Finally, the boundary layer become turbulent and then beyond this, as we discussed, it can be considered as a potential flow or ideal flow depending upon the problem. So, this figure shows the boundary layer growth over a flat plate. Now, in this typical case, we will be considering here to define various parameters. As we have already seen, as the fluid passes over the plate, the action of the viscous shear retards, such that the fluid velocity is reduced in the lateral direction. Then, the thickness of the boundary layer increases.

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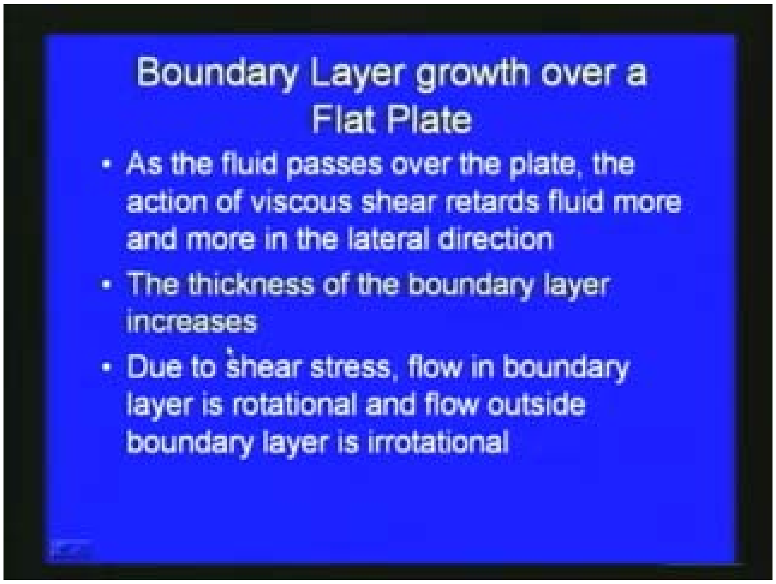

Due to the shear stresses, flow in the boundary layer is rotational and flow outside boundary layer is irrotational. Here, in the boundary layer region, this flow is rotational and then we can consider outside the boundary layer, the flow as irrotational. Depending upon the cases due to shear stress, flow in the boundary layer is rotational and flow outside boundary layer is irrotational.

As the fluid passes over the flat plate, then the boundary layer thickness is increasing depending upon the problem. Like this, we can define the various aspects of the boundary layer growth and how the system is behaving? So, at the beginning of the twentieth century, the famous scientist, Prandtl tried to define boundary layer and then he tried to establish certain relationship, so that, various parameters like velocity and the pressure can be calculated, with respect to the boundary layer. Prandtl considered the boundary layer as a surface dividing between the rotational and irrotational flow, just like in the case of a flow over a flat plate as we have already seen. He considered this boundary layer as the layer, which differentiate between the flow, whether it is rotational or irrotational.

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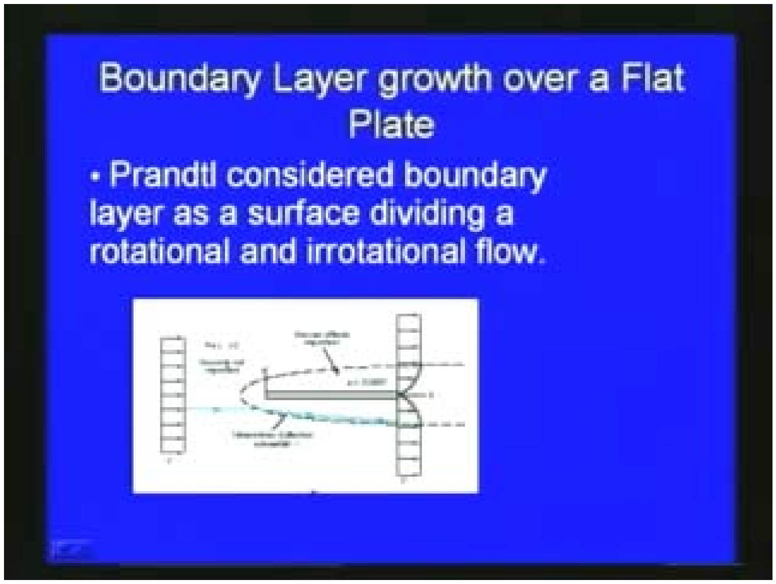

Within the boundary layer, Prandtl considered the fluid flow as rotational and beyond the boundary layer; he considered the flow as irrotational flow. Here, you can see, when free stream velocity is coming over the flat plate, both sides it is shown. So, the boundary layer is developing and then this is the rotational flow free within the boundary layer, both sides of the plate. Beyond this, prandtl considered as irrotational flow.

So, this is plotted for Reynolds number, 10. At ten, how the fluid flow is behaving with respect to flow over a flat plate? Now, we will be defining various parameters of the boundary layer such as boundary layer thickness, the displacement thickness, moment thickness, energy thickness like that, so that, we can differentiate various parameters with respect to boundary layer. Further, we can try to quantify or we can try to derive or we can try to get an expression for the various parameters like velocity and pressure. In the beginning of the twentieth century, when prandtl considered this boundary layer theories and then when he developed the boundary layer theory.

As we have already seen, he differentiated the boundary layer between the rotational flow and irrotational flow. When we can plot the velocity distribution, you can see that, due to the no slip condition at the bottom of the plate, at that location, the velocity is zero. Then, it keeps increasing. So, most of the definitions, we can see, the normal boundary layer thickness is generally defined as, at that location, where they say, 99% of the free stream velocity is achieved. So, the boundary layer thickness, delta is defined as the small distance from the boundary, where velocity attains a value which is very close, say about, 99% to free stream velocity, u infinity.

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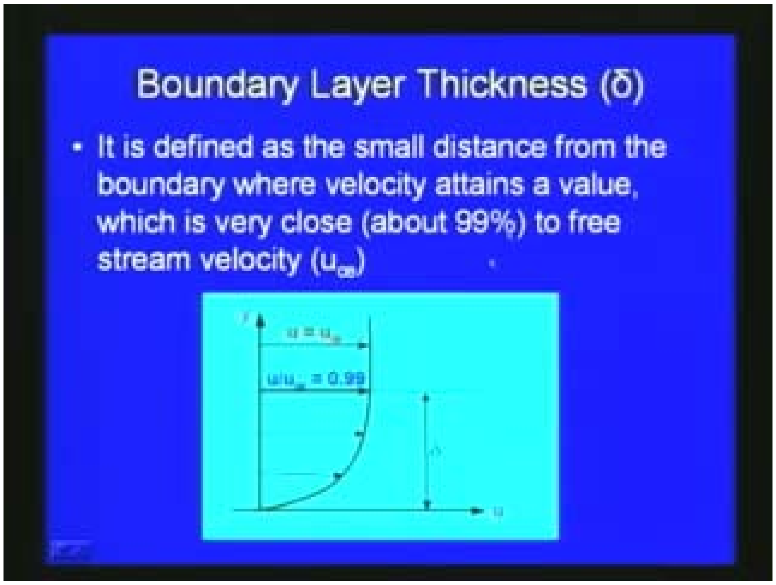

So, if u infinity is the free stream velocity at this location, the boundary layer thickness delta is defined as this distance delta at which u by u infinity, that means, the velocity at any location is u in the boundary layer, then u by u infinity is equal to 0.99. That distance is defined as a boundary layer thickness. Through this, we are able to differentiate whether the fluid flow is within the boundary layer or it is beyond the boundary layer. So that location, wherever the velocity within the boundary layer u by u infinity is 99 % or u by u infinity is 0.99. So, that location is generally defined as the boundary layer thickness. Depending upon various fluid parameters, flow parameters, this boundary layer thickness can vary.

So, it depends upon very parameters like the viscosity and density of fluid and Reynolds number. The boundary layer thickness depends upon the many parameters like, if the Reynolds number is very high, then the boundary layer thickness may be much smaller. However, anyway depending upon the fluid flow, just like, in the case of flat plate or other kinds of fluid flow, so we have differentiated the boundary layer thickness, wherever the u by u infinity, that velocity ratio is equal to 0.99. So, this is the definition of the boundary layer thickness.

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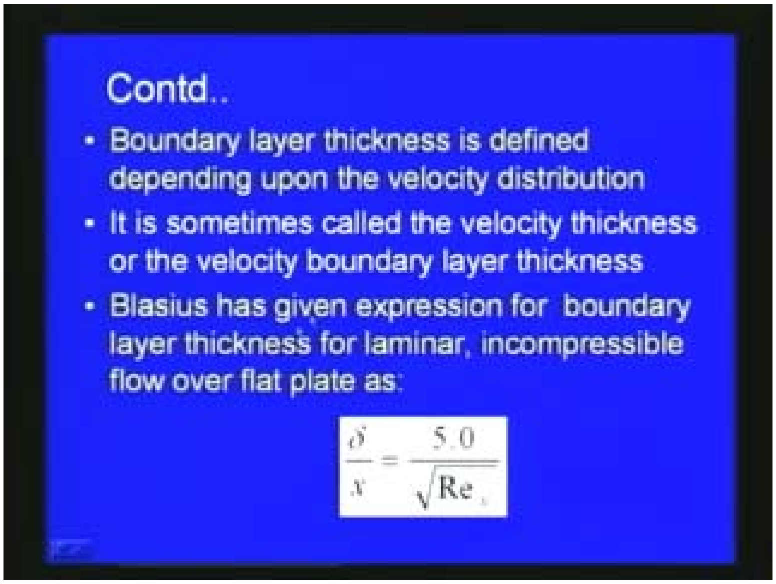

So, boundary layer thickness, as I mentioned, it is defined depending upon the velocity distribution. Hence, it is sometimes called the velocity thickness or the velocity boundary layer thickness. We have already seen, with respect to the velocity only, we have defined the boundary layer. Sometimes, due to this fact, it is called velocity thickness or the velocity boundary layer thickness. By a large number of experiments conducted in the controlled facility, Blasius has given an expression for boundary layer thickness for laminar flow case, for incompressible fluid flow over flat plate.

Here shown that, this delta by x, where delta is the boundary layer thickness, x is the distance from the beginning of the flat plate. Delta by x is is equal to 5 by square root of Re_x, where Re_x is the Reynolds number at that particular location. That means, here, depending upon this flow over a flat plate, if this is x, which is the distance, consider if the flow is laminar Blasius zone, that this delta by x. So, delta is the boundary layer thickness, x is the distance from here, blasius zone is equal to delta by x, which is equal to 5 by square root of Re_x, where Re_x is the Reynolds number at that particular location, where we considered the section or to where we find the boundary layer thickness.

Like this, from various problems, large number of experiments conducted, in the case of laminar flow and turbulent flow and a number of relationships are available in literature. So, now, we have seen the boundary layer thickness and next important definition in boundary layer is called 'displacement thickness'. It is generally identified as delta star.

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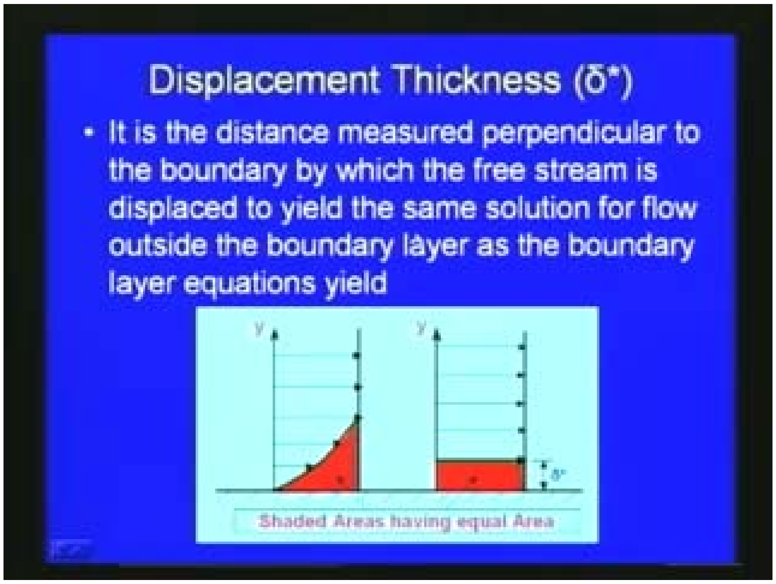

The displacement thickness is defined as a distance measured perpendicular to the boundary, by which the free stream is displaced to yield the same solution for flow outside the boundary layer as the boundary layer equations yield.

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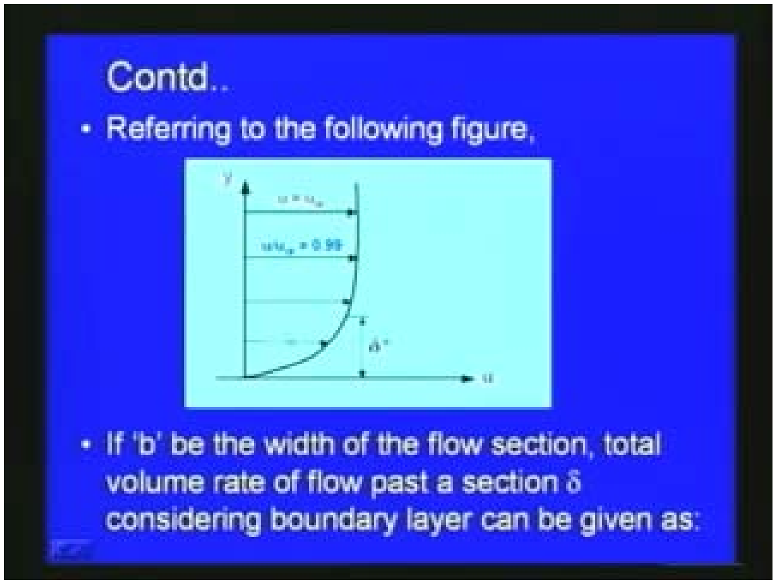

Due to the presence of the plate or presence of the solid, all the free stream velocity is there and then the boundary layer is introduced. Then, there will be shifting of the flow behavior. So, this displacement thickness is defined as the distance measured perpendicular to the boundary, by which this stream is displaced to yield the same solution for outside the boundary layer. Here, it is shown in this figure. This is the boundary layer. So, this delta star is defined as the distance by which it is shifted, so that the free stream is displaced to yield the same solution. That is defined as the displacement thickness. Now, if you consider this figure, here, this is the velocity variation with respect to flow over a flat plate.

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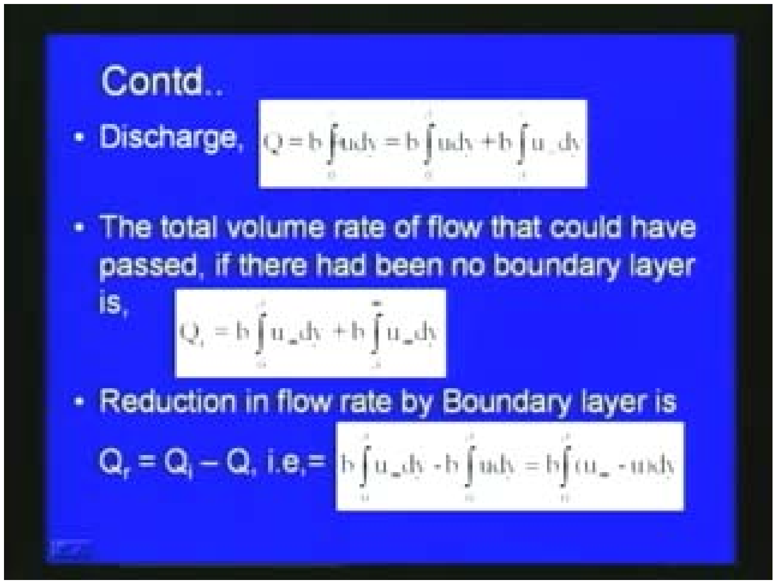

So, u infinity is the free stream velocity and u by u infinity plotted here, with u versus y. If b is the width of the flow section and total volume rate of flow past a section delta, which we consider as the boundary layer thickness. With respect to this figure, we can write the discharge which is passing over, we can write as Q is equal to b integral zero to infinity u dy, so that, we can write as b into integral zero to delta u dy plus b integral zero to delta t infinity u infinity dy. This will be the discharge, which we consider over the flat plate case. So, the total volume rate of flow that could have passed if there had been no boundary layer; the first expression here is, whenever there is boundary layer.

That is, with respect to this velocity variation, since the presence of this solid and the no slip condition, the velocity is reduced, so that the velocity variation in the boundary layer is expressed as u. If there had been no boundary layer, then the same expression discharge, we can write as this is equal to b into integral zero to delta u infinity dy plus b into integral delta two infinity u infinity dy.

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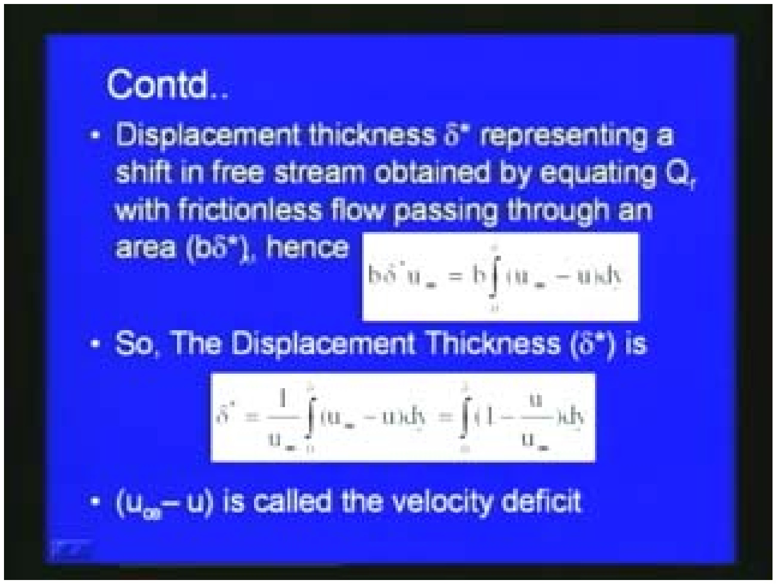

So, this one is with the boundary layer and this is the one without the boundary layer. So, with this, we can see that reduction flow rate by formation of the boundary layer is equal to Q_r is equal to Q_i minus Q, so that, we can write this is equal to b into integral zero to delta u infinity dy minus b integral zero to delta u dy. So, that is equal to b integral zero to delta u infinity minus u dy. This is the reduction flow rate by the boundary layer formation.

Now, the displacement thickness, we can write as the displacement thickness delta star representing a shift in free stream is obtained by equating this Q_r with frictionless flow passing through an area b delta star. So, with respect to this figure, this is delta star and b is the width of flow which we consider. We got the reduction in flow. So, this we can equate this reduction of flow with respect to this displacement thickness, b into delta star into u infinity is equal to b into integral zero to delta u infinity minus u dy. Finally, we can get an expression for the displacement thickness delta star, as delta star is equal to one by u infinity integral zero to del u infinity minus u dy. So, that is equal to integral zero to delta one minus u by u infinity dy.

This is the expression for the displacement thickness. So, this is actually an expression connecting with respect to the free stream velocity and the velocity in the boundary layer and the displacement thickness. So, this difference in velocity of the free stream and the velocity in the boundary layer, u infinity minus u is the ‘velocity deficit’ or ‘velocity defect’ or velocity deficit with respect to the boundary layer formation. So, the difference between the free stream velocity and the boundary layer velocity is called velocity deficit or the reduction in the velocity. Now, we have first seen the boundary layer thickness and then the displacement thickness. Now, you can see that, when we consider most of the fluid flow behavior, we will be dealing with the momentum and then we will be dealing with the energy.

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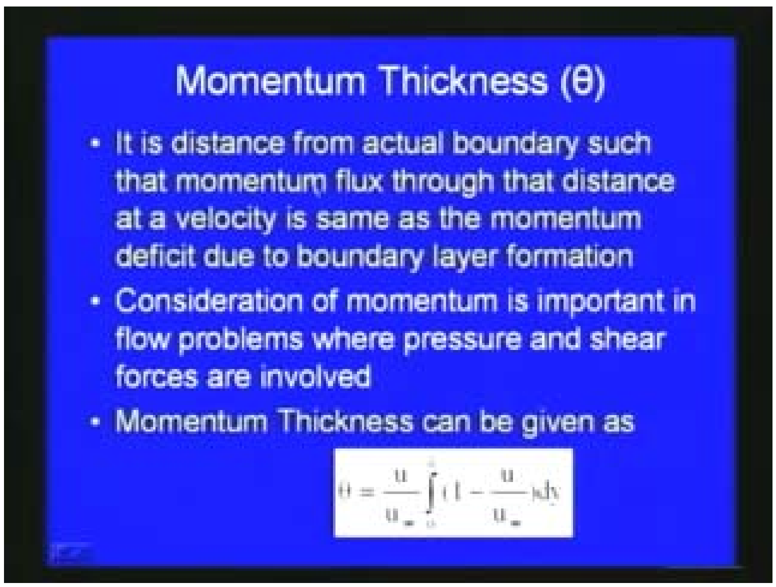

So, what happens to the momentum, say, in the free stream flow, if flat plate is introduced or with respect to the formation of boundary layer? So, we can see that, there will be also a change in momentum due to the formation of the boundary layer. Here, we can define an expression called momentum thickness and it is put as theta. So, this momentum thickness can be defined as the distance from actual boundary such that momentum flux through that distance at a velocity is same as the momentum deficit due to boundary layer formation. Since the boundary layer is formed, you can see that the velocity changes. When the velocity changes, then the momentum also changes. So, there will be deficit in momentum also with respect to the velocity change. Hence, the momentum thickness can be defined as the distance from actual boundary such that momentum flex through that distance at velocity is same as the momentum deficit due to the boundary layer formation.

If you consider the momentum, since we have already seen most of the fluid flow is concerned, momentum is very important. So, consideration of momentum is important in flow problems, where pressure and shear forces are involved. We can see that, in the case of viscous flow, this is the shear force we are considering. Hence, this momentum is very important. So, we derive an expression here from momentum thickness. Actually, the expression is written as theta is equal to u by u infinity integral zero to delta one minus u by u infinity dy. Also, we can derive as in the earlier case, where we discussed the displacement thickness.

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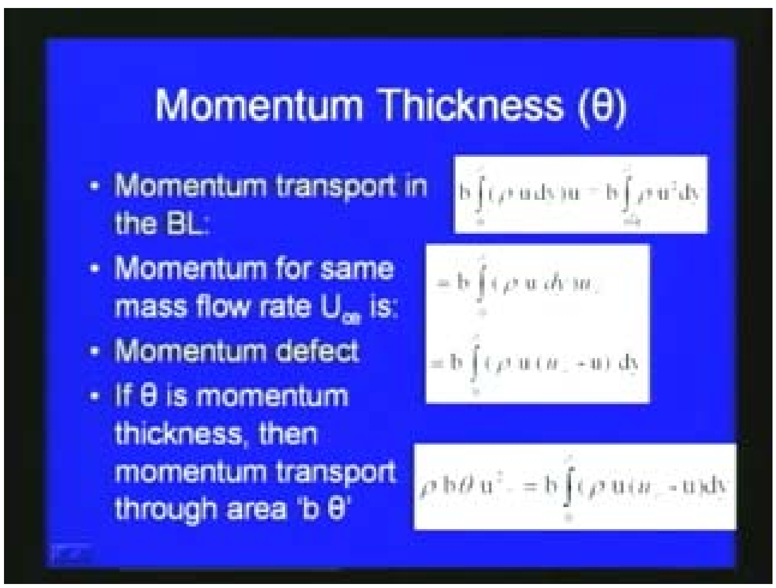

Here, with respect to the flow over a flat plate, which we have seen earlier, say without fluid flow which we consider is b. Then, the momentum transport in the boundary layer we can write as b into integral zero to delta rho u dy into u, where delta is the boundary layer thickness, u is the velocity in the boundary layer, and rho is the density of fluid flow.

That is equal to b into integral zero to delta rho u square dy. Now, momentum for this same mass of flow, when we consider the u infinity, that means, with respect to the free stream velocity, we can consider this is equal to b into integral zero to delta rho u dy u infinity. Now, if you consider between these two expressions, we can see that there will be a momentum defect or detection in momentum. So, we can write as momentum defect or deficit. We can write it as b into integral zero to delta rho u infinity minus u dy.

So, we consider the momentum, if there is no boundary layer, then this is the expression and there is boundary layer, then this is the expression. The momentum defect or momentum deficit, we can write as b into integral zero delta rho u infinity minus u dy. Then, we have defined momentum thickness, theta. If theta is the momentum thickness, then moment transport through area b into theta you can written as rho b theta u infinity square. That we can equate with respect to this momentum defect and derive here, that is equal to b into integral zero to delta rho u infinity minus u dy.

From this, we can get an expression which we have already seen in the last slide. From the earlier derived expression, from this relation, we get theta is equal to u by u infinity integral zero to delta and one minus u by u infinity dy. So, this is the expression for the momentum thickness which we can derive from this expression. Like this, we can derive the expression of the momentum thickness. For flow past flat plate, if you consider the displacement thickness delta star and momentum thickness theta, then experimentally or various cases, this delta, the boundary layer thickness is equal to three times delta star the displacement thickness, that is equal to 7.5 times the momentum thickness which we have considered. So, for flow past flat plate, we can show that delta is three times delta star is equal to 7.5 times theta. Also, one factor called shape factor, in general, define literature h is equal to delta star by theta, that means, the ratio of the displacement thickness to momentum thickness is defined as the shape factor.

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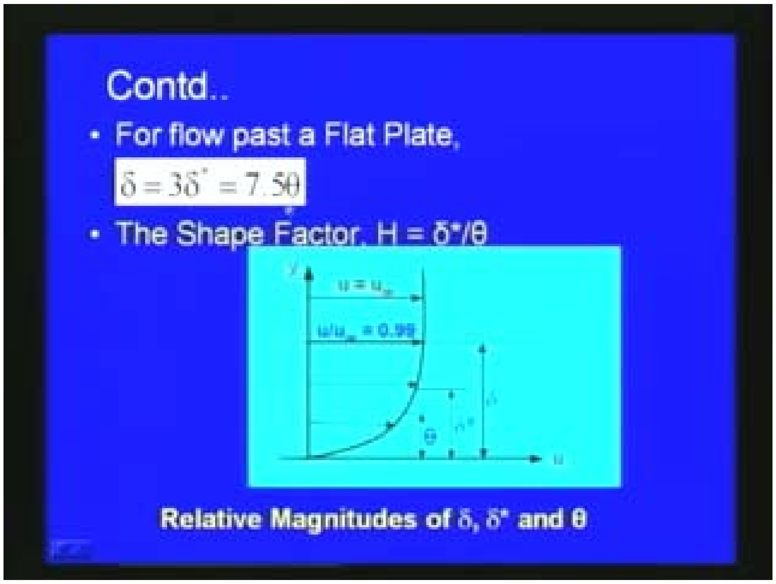

So if you plot the velocity variation with respect to depth y, we can show this theta here, then delta star. Then, this is the delta boundary layer thickness, this is the displacement thickness, this is the momentum thickness. So, relative magnitudes of the delta, delat star, and theta are plotted here for a flow over a flat rate. This is about the parameters of momentum thickness, displacement thickness, and the boundary layer thickness delta. Another important aspect is, what happens whenever in a free stream flow we introduce a flat plate and then what happens to the energy variations with respect to the boundary layer formation.

With respect to this, we can define a term called ‘energy thickness’ delta e. Here, the energy thickness, delta e is the thickness of a layer of fluid moving with velocity u infinity that represent the loss of energy transport rate. This is a definition of the energy thickness, delta e. It is the thickness of layer fluid moving with velocity u infinity that represent the loss of energy transport rate. So, decrease in energy transport rate is caused by the boundary friction. We can write it as, say, if we consider with fluid b, half b integral zero to delta rho u infinity square minus u square dy. If delta e is the energy thickness, then the energy transport through the flow area b into delta e, referred to the free stream velocity u infinity is half rho b delta e u infinity q.

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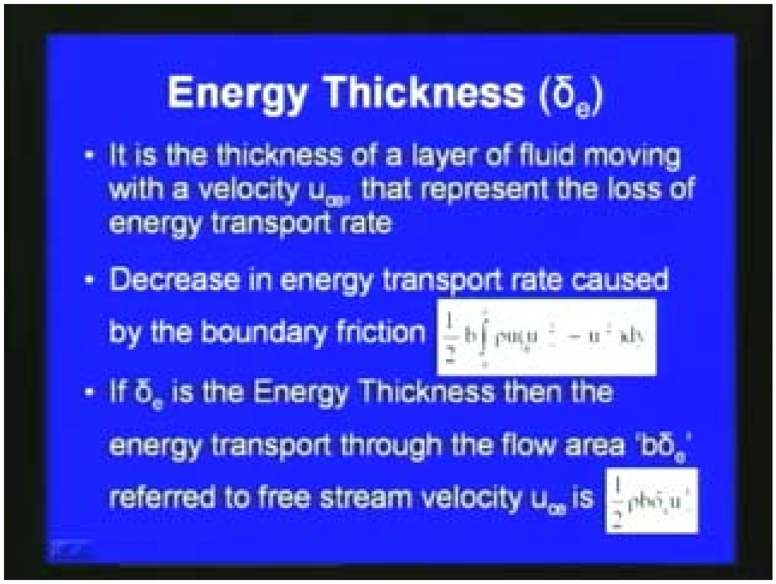

This is with respect to the energy thickness for the flow through area b delta e. Now, we can equate this to the earlier expression and this one, here, so that we can write as half rho b delta e u infinity cube is equal to half b integral zero to delta rho u infinity square minus u square dy.

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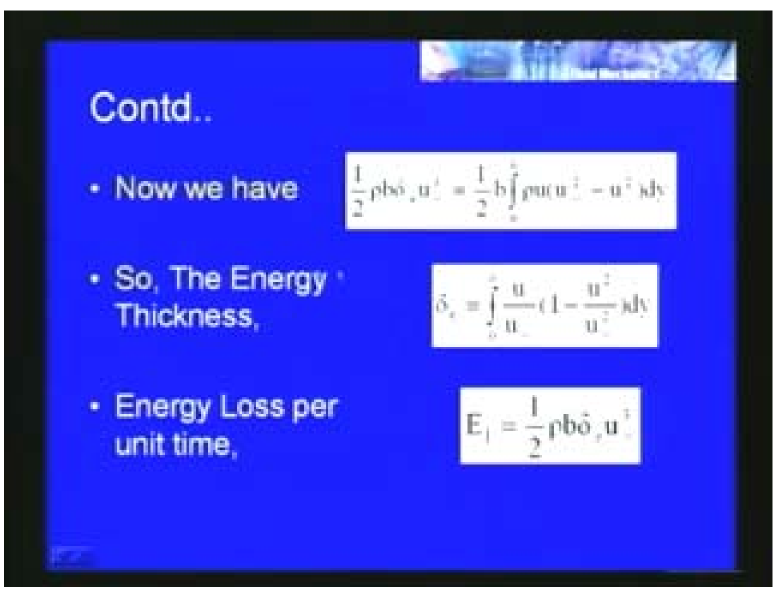

Finally, we can get an expression for the energy thickness as delta e is equal to integral zero to delta u by u infinity into one minus u square by u infinity square dy. This is defined as the energy thickness and also we can calculate the energy loss per unit time as e_l is equal to half rho b delta e u infinity cube. So, this is the energy loss per unit time and energy thickness delta is defined here.

These are some of the important parameters in the boundary layer flow which we will be considering, while estimating various other parameters like velocity, pressure, or shear stress in the boundary layer. So, the important parameters we have seen, one is the boundary layer thickness, second one is the displacement thickness, third one is momentum thickness, and fourth one is the energy thickness with respect to the boundary layer formation.

Before going further, we will just briefly discuss a small example, where we will check how the variations of these displacement thickness, energy thickness, and momentum thickness take place with respect to the boundary layer thickness. Here, we consider a small example problem, assume the velocity distribution in the boundary layer is given by u by u infinity is equal to y by delta to the power one by 7.

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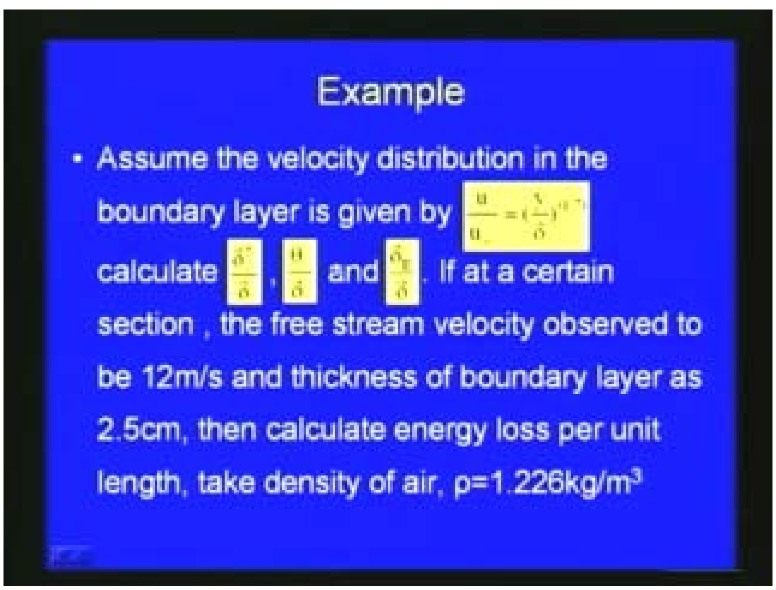

We have to calculate the ratio of displacement thickness to boundary layer thickness, delta star by delta and momentum thickness to delta, and energy thickness delta e to delta. Also, if at a certain section, the free stream velocity observed to be 12 meter per second and thickness of boundary layer as 2.5 centimeter, then calculate energy loss per unit length by taking the density of air as rho is equal to 1.226 kilogram per meter cube.

So, an expression for the ratio of the velocity in the boundary layer with respect to the free stream is even. With respect to that, we have to find out the ratio between the displacement thickness to boundary layer thickness and momentum thickness to boundary layer thickness and the energy thickness to the boundary layer thickness. So, here, the solution is simple, since we have already derived various expressions for the displacement thickness, momentum thickness, and energy thickness. We have already given the ratio between the velocity in the boundary layer to the free stream velocity, u by u infinity. So, we can just substitute in the various relationships. So, here, the displacement thickness delta star is equal to integral zero to delta one minus u by u infinity dy. That is equal to integral zero to delta one minus y by delta to the power one by seven dy. This is after substituting for this u by u infinity is equal to y by delta to the power one by seven. In this, we can approximate and then we can show that this delta star, the displacement thickness is equal to delta by eight.

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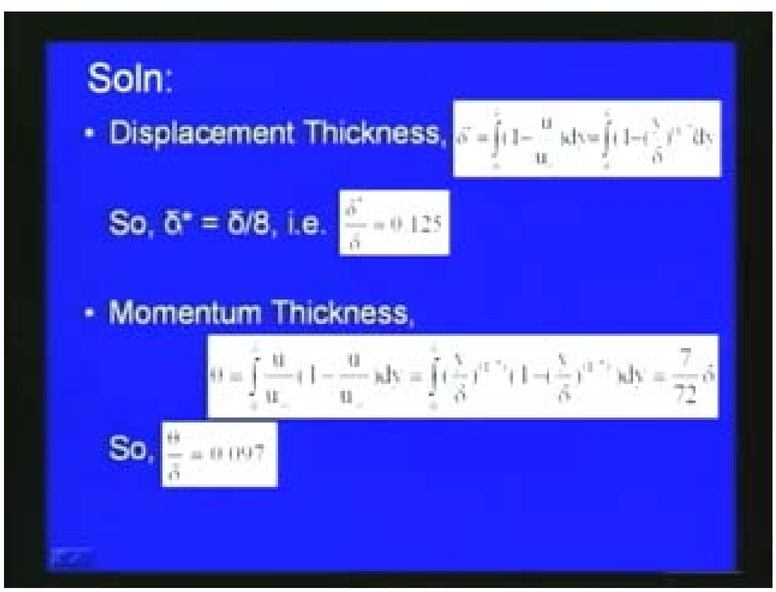

So, we can write the ratio between the displacement thickness to boundary layer thickness delta star by delta is equal to 0.125. Similarly, we can write for the momentum thickness. So, theta is equal to integral zero to delta u by u infinity into one minus u by u infinity dy. This ratio is given as y by delta to the power one by seven. We can just substitute here, that is, equal to integral zero to delta y by delta to the power one by seven into one minus y by delta to the power one by seven dy.

Now, we can show that theta is equal to 7 by 72 delta or theta by delta is equal to theta by delta is equal to 0.097. Then, the energy thickness is concerned again, we can substitute for the expression which we have derived. So, delta e is equal to integral zero to delta u by u infinity into one minus u square by u infinity square dy. That is equal to integral zero to delta y by delta to the power one by 7 into one minus y by delta 2 by 7 dy, so that, we can show delta e is equal to 7 by 40 delta or delta e by delta energy thickness ratio with respect to the boundary layer thickness is equal to 0.175.

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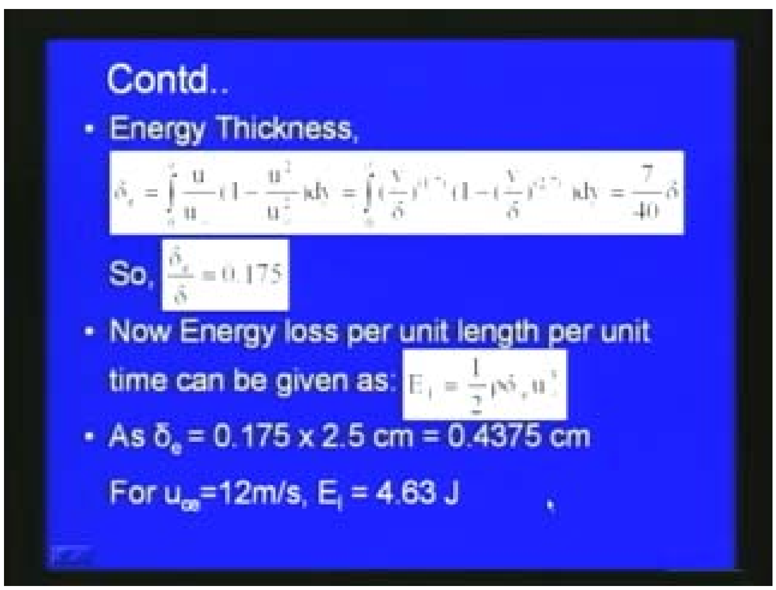

Now, the energy loss per unit length per unit time can be obtained as E_l is equal to half rho delta e u infinity square. So, delta e, we can substitute as 0.175 into 2.5 centimeter, where delta is 2.5 for the even section. So, this is equal to 0.4375 centimeter, from which the free stream velocities called e infinity as 12 meter per second. So, we can calculate this E_l the energy loss per unit length is equal to 4.63 joules.

Like this, we can calculate various parameters as we have already seen, the fundamental definitions with respect to the boundary layer as boundary layer thickness, displacement thickness, momentum thickness, and energy thickness. So, these parameters we can use to determine other parameters like velocity, shear or other expressions. Now, we have defined what is the boundary layer? Then, we have seen the various concepts behind the boundary layer theories. We have also seen with respect to the flow over a flat plate, we have seen initially, the boundary layer is laminar and then transition takes place, as the boundary layer grows and the turbulent boundary layer is generated. Now, with respect to the fundamental concepts, the boundary layer we have seen. Then we will see and the laminar boundary layer and the turbulent boundary layer.

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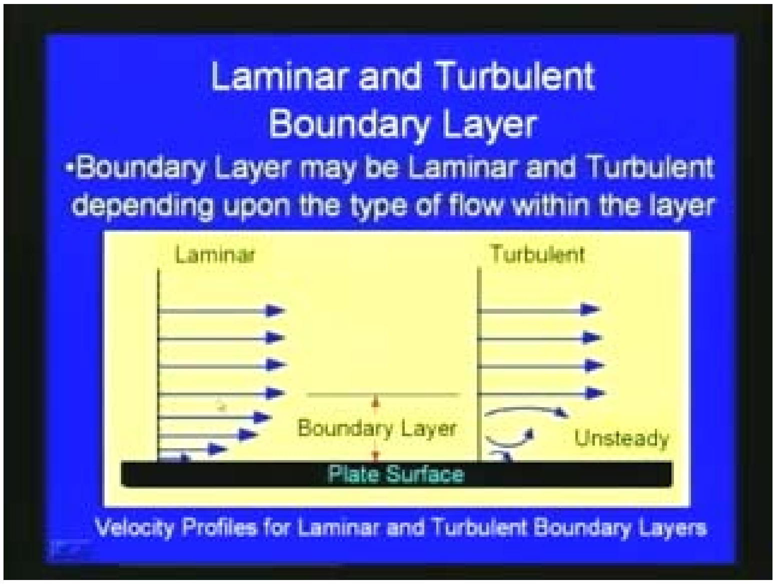

Initially, we will discuss differences between the laminar and turbulent boundary layer. Then, we will see the flow over a flat plate again. Here, in this figure, you can see that this is the free stream velocity coming and then we introduce a flat plate like this. If you plot the velocity profiles for laminar and turbulent boundary layers, we can see that, with respect to boundary layer thickness we have already shown in the previous figure, for a flow over a flat plate how the boundary layer is generated. We can see that, now upto this location, it is free stream velocity and then the boundary layer is generated. So, initial distance for some distance, we can see that the flow in the boundary layer is totally laminar in nature. Again, after sometime, a mixing starts and when transition takes place, finally, you can see that the flow becomes turbulent after distance.

In the same way, for the flow over a flat plate, we can see that laminar boundary layer, then a transition and the turbulent boundary layer. So, within this boundary layer, we can see that flow takes in layers. In the case of laminar boundary layer, here, you can see that, within this boundary layer, the flow takes place in layers; each layer gliding over the adjacent layer. Initially, for the flow over flat plate if you measure or if you observe the laboratory, we can see that flow is layers and gliding over the adjacent layers.

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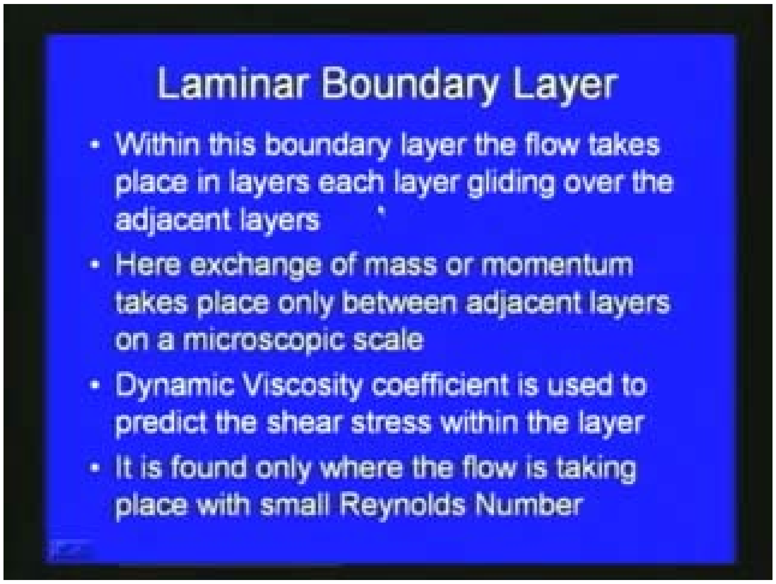

That is why, we can say that area as laminar boundary layer. Here, with respect to the laminar flow theories, which we have already seen, the exchange of mass or momentum takes place only between the adjacent layers. On a microscope scale, you can see that between the layers only the mass or momentum exchange takes place. Then, you can see that the dynamic viscosity of co-efficient is used to predict the shear stress within the layer.

Here, this mew or dynamic viscosity co-efficient is used to predict the shear stress. It is found only where the flow is taking place with a small Reynolds number. In the free stream velocity, when we say, for example, here is the free stream velocity is coming like this. Then we introduce the flat plate like this. You can see that, here, initially, in some region, the Reynolds number will be low. At that location only, the flow will be laminar. When the Reynolds number increases, the flow becomes obviously turbulent. So, the initial stages of the boundary layer development exhibit characteristics of the laminar motion.

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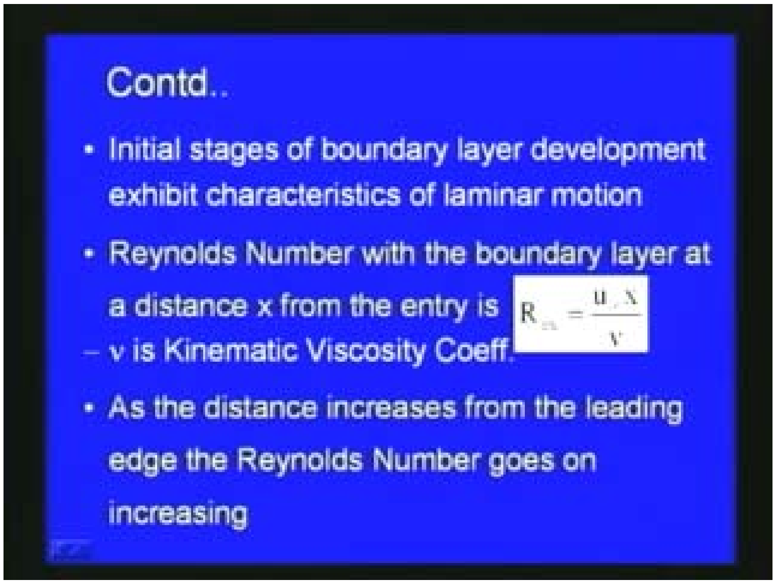

If you define the Reynolds number with the boundary layer at a distance x as Re_x is equal to u infinity into x by v. So, u infinity is the free stream velocity, x is the distance which we considered here and new is the kinematics viscosity co-efficient. So, with respect to the Reynolds number, we can say that the boundary layer keeps on growing. At different locations, if you plot the boundary layer, you can see that as we have already seen in the earlier figure, here is the flat plate which we introduced and this is the free stream velocity, which is u infinity. When it heats, you can see that a boundary layer is generated. Then after sometime, it will become, say, transition and finally, it becomes turbulent.

So, this region where it is laminar, at variation locations, we can say this Reynolds number Re_x as u infinity into x by kinematics viscosity (new). If you define x from here and various locations, we can define the Reynolds number and then with respect to this we can identify where the transition starts from laminar to turbulent. Now, with respect to this, if you consider the 2d laminar boundary layer, it can be shown that referring to the figure, say, here the boundary layer thickness, this is the leading edge and this is flowing in this direction.

So, laminar region of the boundary layer, if delta is the thickness of the boundary layer at distance x from the leading edge, then you can show that delta by x is equal to 5 into square root of new by u infinity into x. So, that is equal to 5 by square root of Re_x, the Reynolds number at that particular location. We can write that, delta is equal to 5 into square root of new into x by u infinity.

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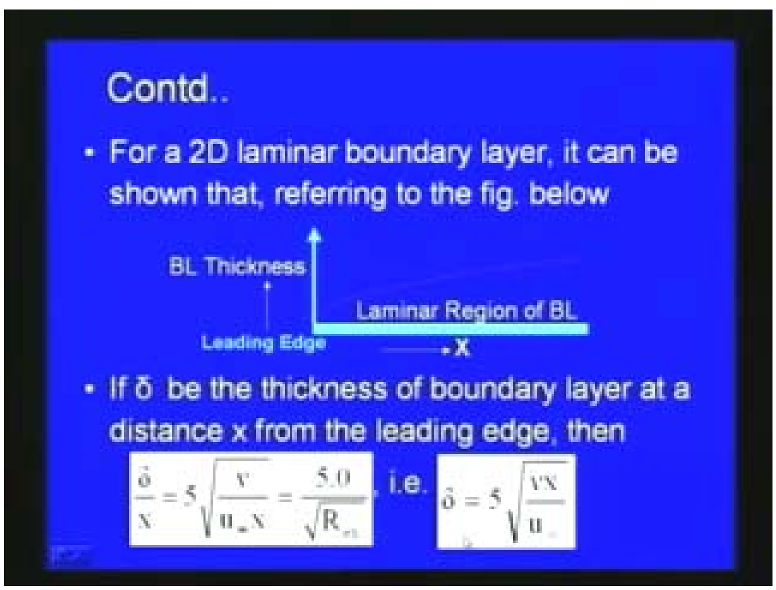

Here, as we have already shown here in this figure, at various locations, this boundary layer for this laminar region, this is x at this distance. So, we can define delta as 5 into square root of new into x by u infinity, which is the free stream velocity and x is at various locations which we considered. This relationship was derived by Blasius through various experiments we can utilize and then if you define a critical Reynolds number, from which onwards the flow is transition or turbulent. Then, we can put that condition.

So, the boundary layer thickness, delta with respect to the figure here, as we have already shown in this figure, we can say with respect to the boundary layer, various characteristics are defined for this laminar boundary layer. Here, the boundary layer thickness delta decreases with increasing Reynolds number.

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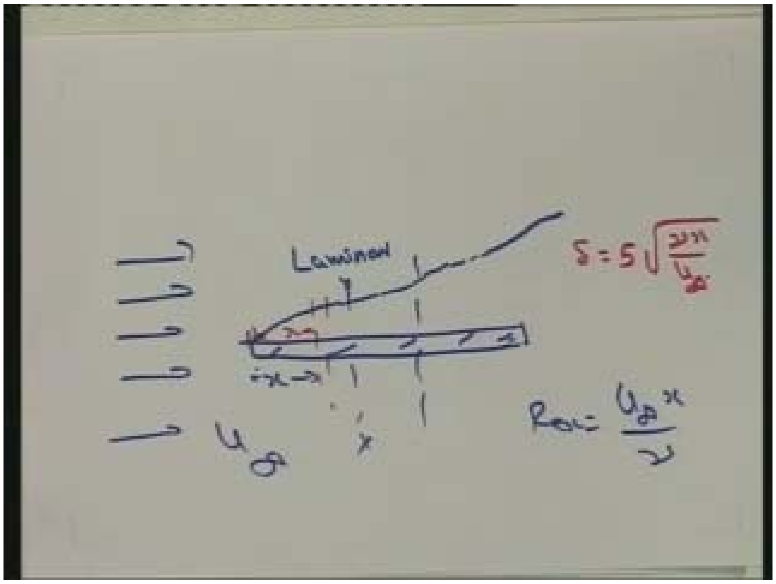

You can see that this boundary layer thickness, same as I mentioned, when the flow velocity is very large, then the Reynolds number keeps on increasing. When the Reynolds number is increasing, the boundary layer thickness reduces and then it is confining very near to the solid surface. So, one of the important observations, you can see the boundary layer thickness, say the formation of the boundary layer. When we discuss the formation of the boundary layer, it depends upon the Reynolds number, when the Reynolds number is very large, then a small boundary layer will be formed. Then the thickness of the boundary decreases with increasing Reynolds number. For high velocity flow, the boundary layer will be very thin; so the Reynolds number is increasing with respect to the flow velocity. So, we can say that when the velocity is very high, we will be having a thin boundary layer. Another important observation is, for ideal flow, the Reynolds number tends to be infinity. So, the boundary layer thickness vanishes. We have already seen earlier, if you consider the real fluid to ideal fluid, so that the Reynolds number in the case of ideal fluid, it tends to be infinity and the boundary layer finally vanishes

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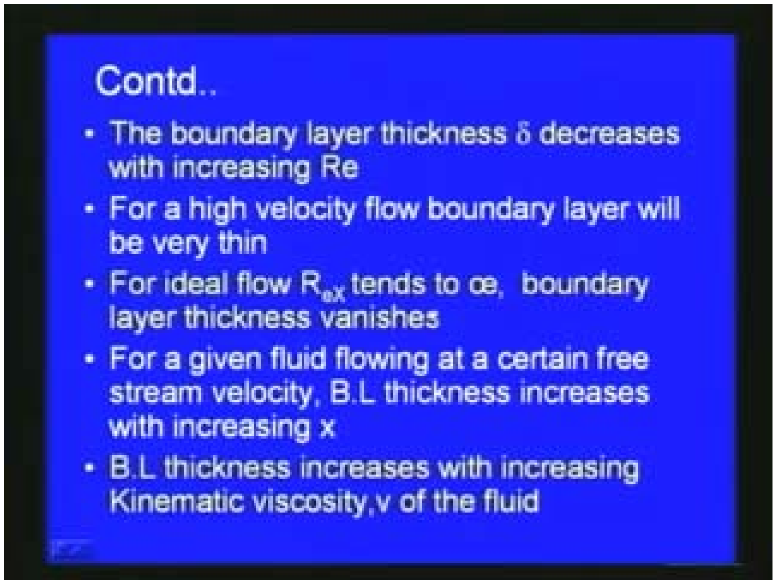

That is what we have observed in the case, for ideal fluid, there is no boundary layer. For a given fluid flowing at a certain free stream velocity, the boundary layer thickness increases with increasing x. That is what we have seen here. When the distance is increasing from this point to this point, the boundary layer thickness keeps on increasing. This is for a given fluid flow condition, when we introduce a flat plate in a free stream velocity, then starting from the leading edge of the plate and then this is a trailing edge.

From here onwards, the boundary layer thickness is increasing. So, this is for a given fluid flow at certain free stream velocity, the boundary layer thickness is increasing with increasing x. Then the boundary layer thickness increases with increasing kinematic viscosity new of the fluid.

So, we have already seen with respect to the previous expression, delta is equal to 5 into square root of new x by u infinity in the case of a laminar boundary layer. From that, we can see that the boundary layer thickness increases with increasing kinematic viscosity, new of the fluid. From this, we can see that, we can finally summarize that, as far as the boundary layer formation is concerned, the boundary layer thickness or how much is the boundary layer effect to be considered? It depends upon the kinematic viscosity, depends upon the shape of the body over which the flow is taking place, and then the Reynolds number or the fluid flow velocity. So, depending upon all these parameters, the nature of the boundary layer changes. Then, we have to consider the boundary layer accordingly for various solutions.

We have already discussed here, the fundamental concepts behind the boundary layer and then how a boundary layer is developing with respect to flow over a flat plate. Also, we have seen various definitions with respect to the boundary layer. We have also seen, say, how the boundary layer variations takes place with respect to laminar flow or the laminar boundary layer. Further, we will be discussing the various aspects of turbulent boundary layer and then we will be trying to get various expressions for velocity, shear, and so on in the next lecture.