Fluid Mechanics

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Lecture 28

Navier-stocks equations and applications

Welcome back to the video course on fluid mechanics. In the last lecture we are discussing about the exact solutions for the navier-stocks equations. The navier-stocks equations, we have derived earlier for the viscous flow and we have seen it is second order non-linear equations only for simplified cases only we can derive the exact solutions.

We have seen few cases like plane poiseuillie flow couette flow; also we have seen a simple transient type case where an exact solution can be derived for the navier-stocks equations. In this lecture, further, we will discuss few more naviers the exact solutions which are possible for the navier-stocks equations, these problems are again some of the simplified cases first we will see the case of pie flow the steady state flow in pipes this case is called hagen-poiseuille flow.

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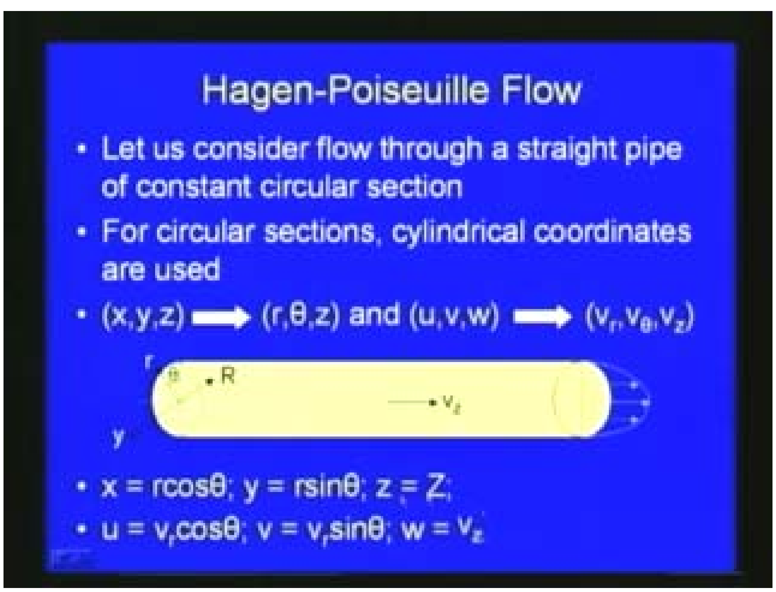

Let us consider the flow through a straight pipe of constant circular section. We can see here is a pipe section this is long pipe and the radius is r capital r for the circular section. As we have already seen we have seen in the navier-stocks equations two forms one is the cartesian coordinate forms and second one is the cylindrical coordinate forms pipe is consigned, we are dealing generally we are dealing with the radius length of the pipe and the angles with which we consider.

The cylindrical coordinate system will be better for this type problem, for the Hagen-poiseuille flow problem here r is the radius of the pipe theta is the angle which we consider. The particular location the length of the pipe which we consider here corresponding to the Cartesian coordinate x y and z, we consider t theta and z and corresponding to the velocity components u v w in x y z direction. We consider vr velocity theta and vz correspondingly with respect to this figure we can write x is equal to r cos theta and y is equal to r sine theta and z is equal to z.

We can write now this u is equal to vr cos theta v is equal to vr sine theta and w is equal to vz this is the problem structure which we are trying to apply the navier-stocks equations, try to get an exact solution for this kinds of problem the flow through pipes which is steady state flow that is hagen-poiseuille flow.

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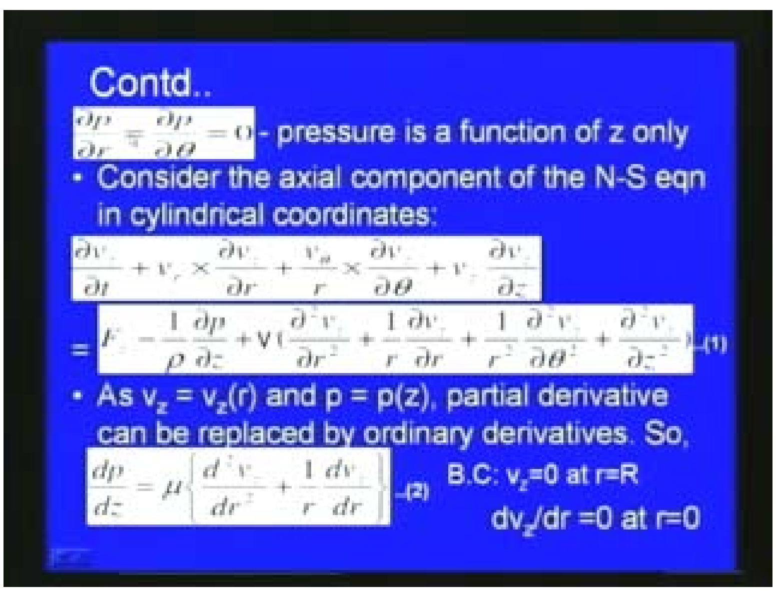

For this particular problem, since the pressure variation pressure is a function of z only that means with respect to the length of the pipe only, the pressure is varying that we can write del p by del r and del p by del theta they both are 0 that we can write del p by del r is equal to del p by del theta is equal to 0 pressure is the function.

Let us consider the axial component of the navier-stocks equations in the cylindrical coordinates. We have seen earlier the navier-stocks equations cylindrical coordinate; we can write r theta and z component here we consider the axial component of the navier-stocks equations in the cylindrical coordinate system.

As we have seen earlier, this equation the axial component can be written as den vz by del t plus v_r into del v_z by del r plus v theta by r into del v_z by del theta plus v_z into del v_z by del z is equal to f_z minus 1 by rho del p by del z plus new the kinematics viscosity into del square v_z del r square plus 1 by r into del v_z by del r plus 1 by r square plus del square v_z by del theta square plus del square v_z by del z square.

Where f_z is this the body force component for the problem considered as in the previous figure. We consider horizontal pipe where we do not have to consider the body force for this particular problem and we consider the flow to be steady state that this the del v_z by del t term but time dependent term vanishes or this 0 and as the velocity v_z the z direction is varying with respect to the radial direction vz is a function of vz r and p is a function of p_z and partial derivative can be replaced by ordinary derivatives.

This equation the time component is gone; we can just simplify this equation with respect to the various assumptions used. We can write as dp by dA is equal to mu into del square v_z by del r square plus 1 by r into dv by dry. The final equation become dp by dA is equal to mu into del square v_z by del r square plus 1 by r into dv by dry. For this pipe flow problem, we can see that pipe is consigned all boundary that means, due to no slip condition at r is equal to capital r we can write v_z is equal to 0.

That means on the pipe boundary the velocity is 0. The v_z is equal to 0 at r is equal to r that is one boundary condition we can see that for pipe flow, as describe here we can see that the velocity variation is parabolic and maximum will be at the center of the pipe. At this location at r is equal to 0, we can write dvz by dr is equal to 0 since maximum velocity at the center line at r is equal to 0, we can write at the derivative of the velocity with respect to r dvz by dr is equal to 0. For this particular problem we have simplified the navier-stocks equations we have derive the boundary conditions now for the simplified problem, we will be trying to find out and exact solution with respect to the various conditions here.

Our aim is to find out the velocity in order to find this velocity. With respect to this earlier equation which derived here, we will integrate equation number two. let us assume dvz by dr is equal to s, so that we can write equation number two in terms of this s, as ds by dr plus 1 by r into s is equal to 1 by mu dp by dz equation number three. From the mathematical test books we can assume indication factor if i is equal to equal to e to the power integral 1 by r dry this is equal to e to the power log e r that is equal to r.

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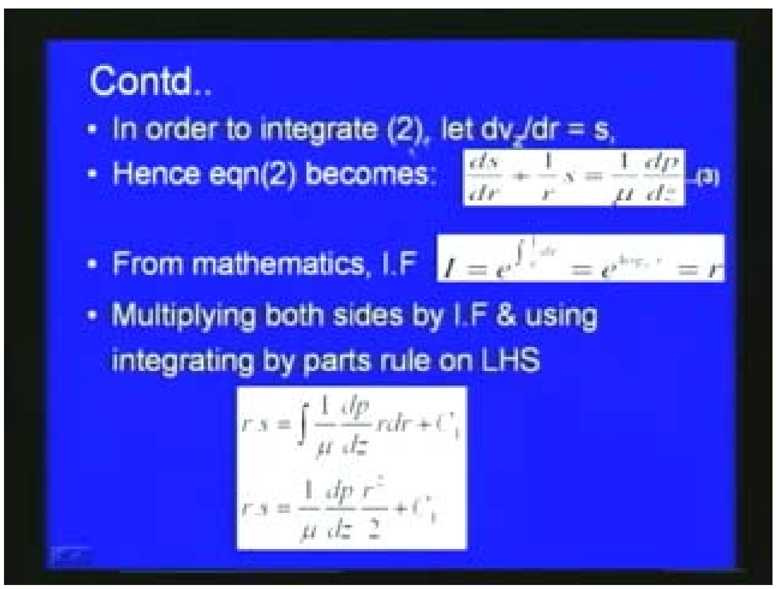

We assume and integration factor if as i is equal to r. We will multiply both sides of equation number three by this integration factor, we will use the integrating by parts rules on the left hand side of this equation finally multiplying by the integrating factor if given i is equal to r using the integration by parts, we can write this equation number three as r into s is equal to integral 1 by mu dp by dz r dr plus c_1 where c_1 is a constant of integration.

This again we can write as rs is equal to 1 by mu dp by dz r square by 2 plus c_1 finally this equation three is stands from v as this equation here. Also we can write r after substituting for this s here, we can write r into dvz by dr is equal to 1 by mu dp by dz r square by 2 plus c_1 as in equation number four. We have the boundary conditions that we can find out the constant of integration here same at the central line my velocity is maximum that dvz by dr is equal to 0 at r is equal to 0.

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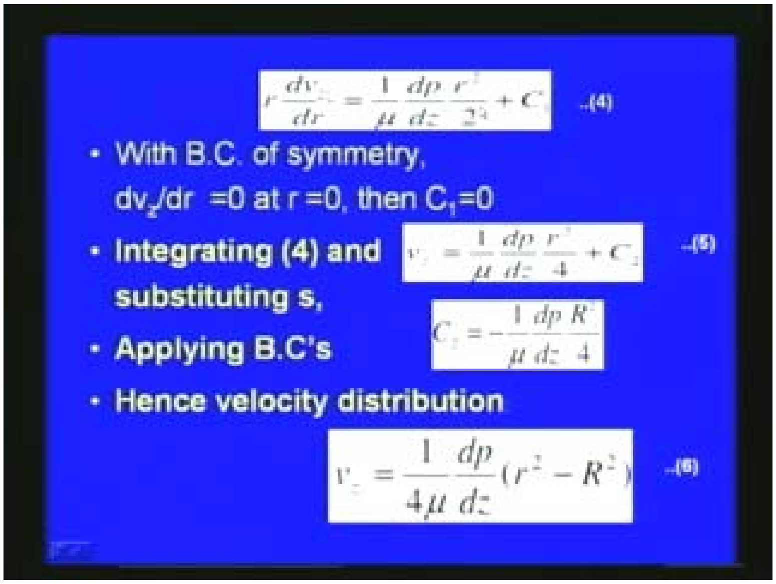

That in this from this equation number four, we will get c_1 is equal to 0 then integrating this equation again and substituting s, as we have already seen and we get vz is equal to 1 but mu dp by dz into r square by 4 plus c_2 as in equation number 5, again here the c_2 we can find out by using the boundary condition at r is equal to r we have the velocity is 0 that we will get c_2 is equal to minus 1 by mu dp by dz into r square by 4.

This r is a capital; r is a radius of the pipe. Finally we get here this constant of inherent c_2 as c_2 is equal to minus 1 by mu dp by dz r square by 4 r, after substituting finally we get the velocity distribution vz is equal to 1 by 4 mu dp by dz into s square small r square minus capital r square this expression equation six gives the velocity distribution at various location small r is the distance from the central line

At various locations we can substitute and get the velocity distribution as vz is equal to 1 by 4 mu dp by dz into r square minus capital r square where r is the radius of the pipe.

We can drive the velocity distribution as for as the pipe flow is consigned now once we get this we can derive the various other parameters like discharge and other parameters. This equation for v velocity distribution we can write v is equal to minus 1 by 4 mu dp by dz into r square minus r square equation six a, the maximum velocity at the center and this is given, we here we can substitute r is equal to 0 that gives u_{max} is equal to minus 1 by 4 mu dp by dx into r square as in equation number seven.

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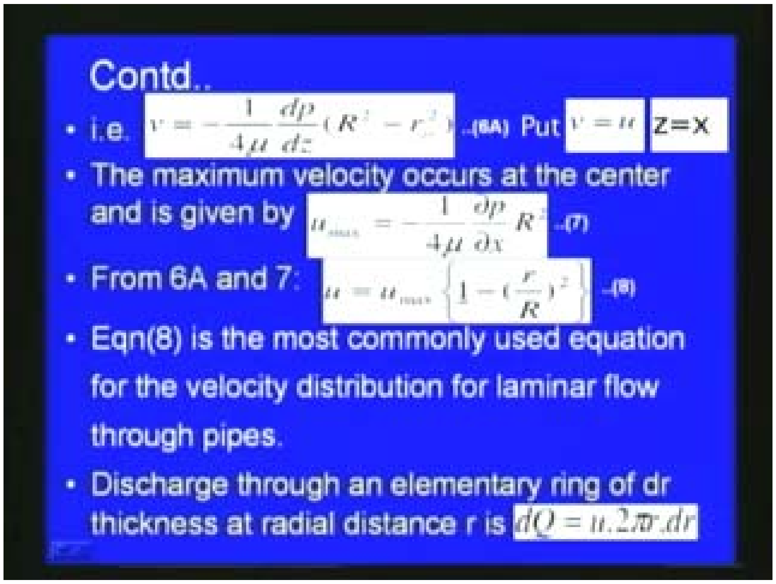

If you use equation number six and seven, here we assume the logical direction coming back in the in this direction of u is equal to we can write the velocity variation u is in terms of the maximum velocity, u is equal to u max into 1 minus r by r power square as in equation number eight. This equation eight is the most commonly used equation for the velocity for laminar flow through pipe; here our assumption is that flow is steady state laminar and same the pipe is parallel horizontal direction.

All these assumptions we applied and only that, we could use the navier-stocks you simplified from the navier-stocks equation to derive on exact solution. Once we get the velocity distribution we can find out the discharge, the discharge through an elementary ring of dr thickness at radial distance r is dq is equal to u into 2 pie r dr, we can write dq is equal to with respect to after we substitute for u dq is equal to u_{max} into 1 minus r by r whole square 2 pie r dr.

Total discharge we can integrate integral dq that is equal to integral 0 to r u_{max} 1 minus r by r whole square into 2 pie r dr that is equal to pie by 2 into u_{max} r square and average velocity once we get the discharge average velocity, we can find out by just dividing by the area of procession.

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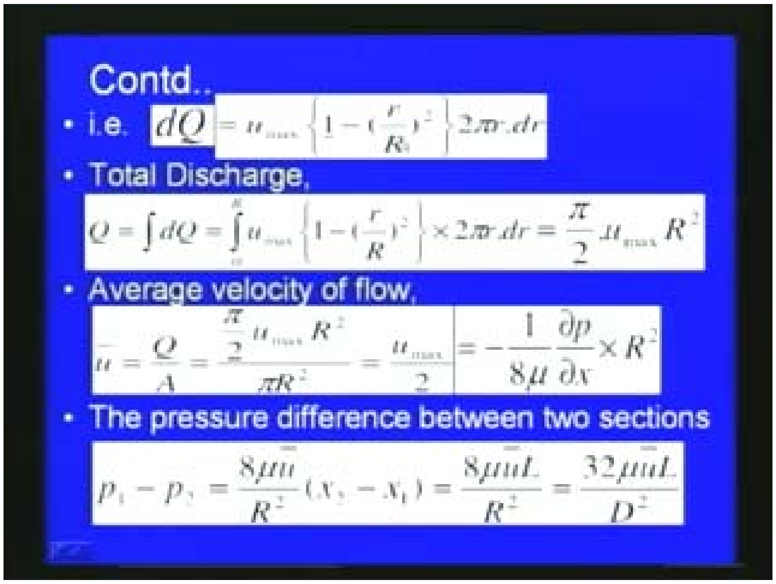

Average velocity is u bar is equal to q by a, from this expression we can write q q by a is equal to pie by 2 u_{max} r square by pie r square that is equal to u_{max} by 2 that is equal to minus 1 by 8 mu dp by dx into r square this will be the average velocity and the pressure difference between the two sections. We have found the velocity distribution we got an expression for velocity we derived an expression for the discharge if you want to find out the pressure difference between two points two sections we can just use that equation.

P_1 minus p_2 since our earlier velocity equation is with respect to the pressure that we can write p_1 minus p_2 is equal to 8 mu u bar by r square into x_2 minus x_1 that is the distance from one section two. Another section that is equal to 8 mu u l by r square you well the distance between sections one to two, we can write p_1 minus p_2 is equal to 8 mu up by r square. We can write in terms of diameter as this is equal to 32 mu u bar l by d square.

What we did here is the navier-stocks equation ,we simplified for the given conditions with respect to boundary condition we derive an exact solution for the velocity and from the velocity we can determine various other parameters like pressure variations from one section to another we can find out the discharge like this. We can utilize the navier-stocks equation for the simplified case we can derive an exact solution for the problem this way two more other problems we will discuss here. This case is the hagen-poiseuille flow where we derived the expression for the velocity variation, we have seen the pipe flow, we will check some more and try to derive the exact solution for two more other problems another typical problem which is generally used mechanical engineering is flow between concentric rotating cylinders.

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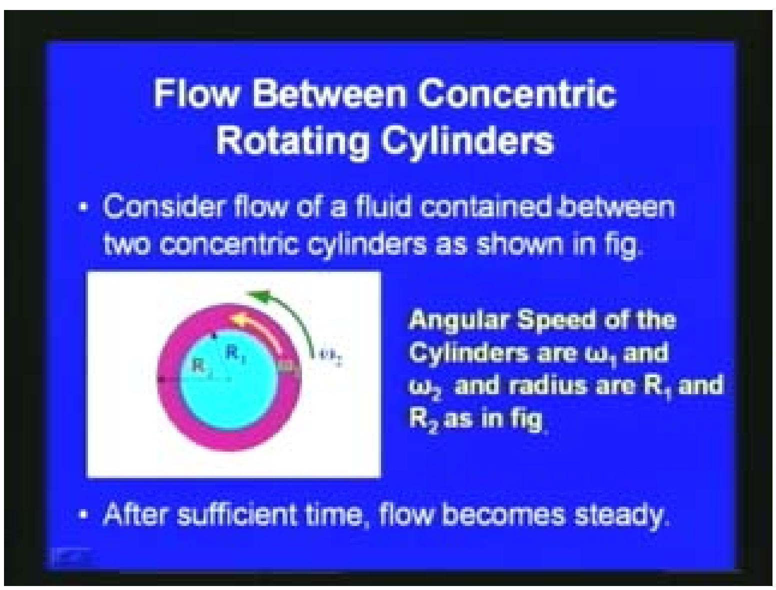

This is very common problem in mechanical engineering our problem is that there are two or thirty cylinders and the fluid is between this two rotating cylinder our aim is to derive an expression for the velocity variation for the rotating cylinder we start with navier-stocks equation we will apply all the simplifications. We want to derive an analytical solution here, the problem is we consider a flow of fluid contained between the two concentric cylinders as shown in figure here this is the first cylinder of radius r_1 and here this is the second cylinder of radius r_2 and the flow is between this two cylinders and both the cylinders are rotating like this with first one with radio the angular velocity of omega 1 and second 1 with an angular velocity of omega 2.

Angular speed of the cylinders are omega 1 and omega 2 and radius are r_1 and radius are r_1 and r_2 as in figure this kinds of problem after sufficient time is a once we start the machine it maybe transient the nature of the flow problem maybe transient nature but after me time the problem become flow become steady state that we can assume the case of steady state here for this typical the flow of rotating cylinders constant rotating cylinders now in the spacing r_2 minus r_1 is small strictly speaking and the Reynolds number re is equal to rho into omega 2 minus omega 1 into r tow minus r_1 square divided by mu if it is less than or equal to 1 and you can see that the 1 d flow.

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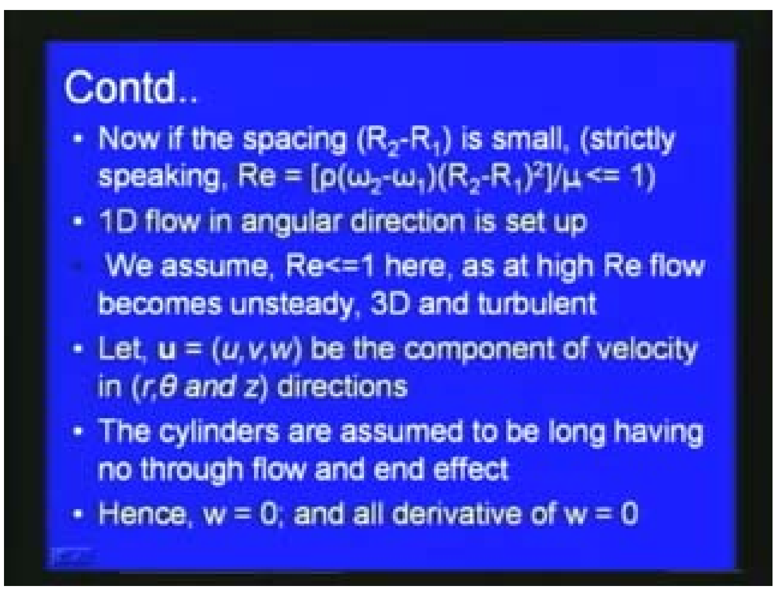

The flow is we can consider as one dimensional flow in angular direction that is typical problem we can consider the flow as 1 d flow angular direction

If you assume here if Reynolds number is less than equal to 1 as at high Reynolds number flow become unsteady and three dimensional turbulent, we are considering this case only for Reynolds number less than or equal to 1 since when we consider the Reynolds higher Reynolds number the flow is generally unsteady, we have to consider three dimensional and turbulent flow depending upon the case.

Here we consider the Reynolds number as less than or equal to 1 and here let the velocity vector u correspondingly in x y z direction will be w be the components of velocity in r theta and z directions here the velocity component in r theta and z direction, we are writing as u v and w the cylinders are assuming to be long having no flow no through flow and end effect here for this typical problem before using the navier-stocks equations and simplifying and get trying to get an exact location we assume that there are now end effect the, we are having no through flow and we can write in this typical case w is equal to 0 and all derivative w is equal to 0 where w is the velocity component in the z direction for this typical problem now due to the rotational symmetry here in the previous slide you can see that there is symmetric with respect to the rotation

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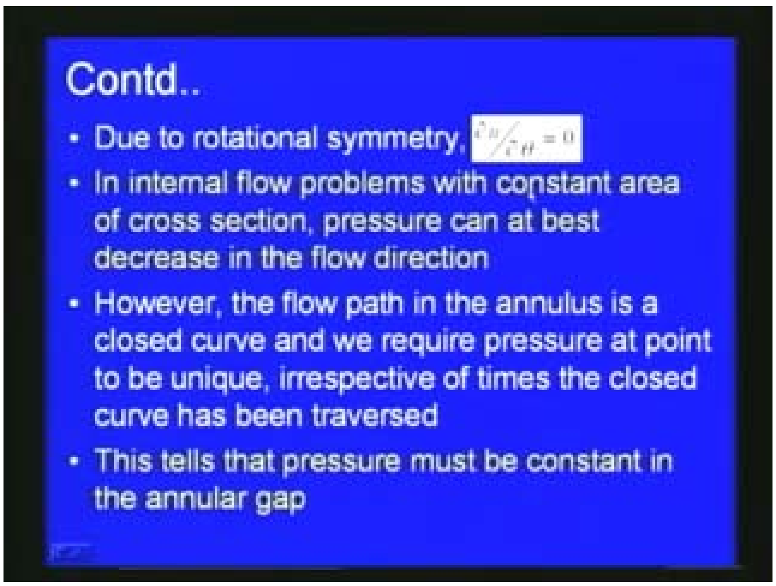

Due to the rotational symmetry we can write del u by del theta is equal to 0 in internal flow problems with constant area cross section pressure can at best decrease in the flow direction for this kinds of problem however the flow path in the annulus is a closed curve and we require pressure at point to be unique irrespective of times the closed curve has been traversed these are the assumptions which we used for this problem this tells that pressure must constant in the annular gap we are using certain assumptions like del u by del theta is equal to 0, we assume that flow path in the annulus is a closed curve we also assume that pressure must be constant in the annular gap.

For this conditions now we simplify the navier-stocks equation, as we have already seen the curve flow is we are assuming the flow is to be steady state various assumption, we have seen if you utilize all these assumption then we can write the navier-stocks equations cylindrical coordinate with the mentioned the simplification in the r direction. We can write rho v square by r is equal to dp by dr in theta direction, we can write the second derivative of the velocity d square v by dr square plus 1 by r v r minus v by r square is equal to 0 this two substitute store the second derivative and this is the first derivative.

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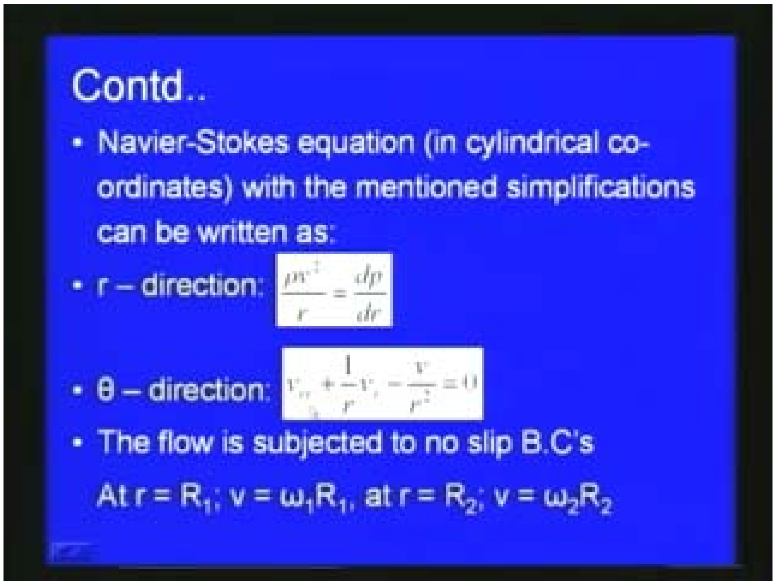

Vary plus 1 by r vr minus v by r square is equal to 0 the flow is subjected to no slip boundary condition that we can write at r is equal to r_1 v is equal to omega 1 r_1 and at r is equal to r_2 v writ V is equal to omega 2, these are the boundary conditions for this problem and the navier-stocks equation r simplified in this four. We have already seen as in the previous case here we have this equation theta direction we have got a second derivative the velocity.

We can integrate to solve is this equation you there be 2 constant c_1 and c_2 finally we can get a solution vr is equal to 1 by r into c_1 into r square by 2 plus c_2 this is the expression for the velocity variation for this typical problem here the problem is like this, we have already seen this r is the difference between this r_2 to r_1 that is what we have small r is between the difference between r_2 to r_1 this the solution.

Finally, the solution becomes vr is equal to 1 by r c_1 into r square by 2 plus c_2 and c_1 we can obtain two boundary conditions and use this to get the constant c_1 and c_2, c_1 is equal to 2 into omega 2 r_2 square minus omega 1 r_1 square divided by r_2 square minus r_1 square and c_2 is equal to minus omega 2 minus omega 1 r_1 square r_2 square divided by r_2 square minus r_1 square like this, we can derive the expression drive an expression for the velocity variation for the flow between the to constant a cylinders.

Here what we did is we applied various simplifications for the problem and we got a simplified form of the navier-stocks equations we integrated the equations that we can get a solution. Finally we got a solution for the velocity variation that can be used find out various other parameters, these kinds of problem there is application for to a viscometers here how we apply this solution to viscometer application to viscometer which measures the viscosity of liquid.

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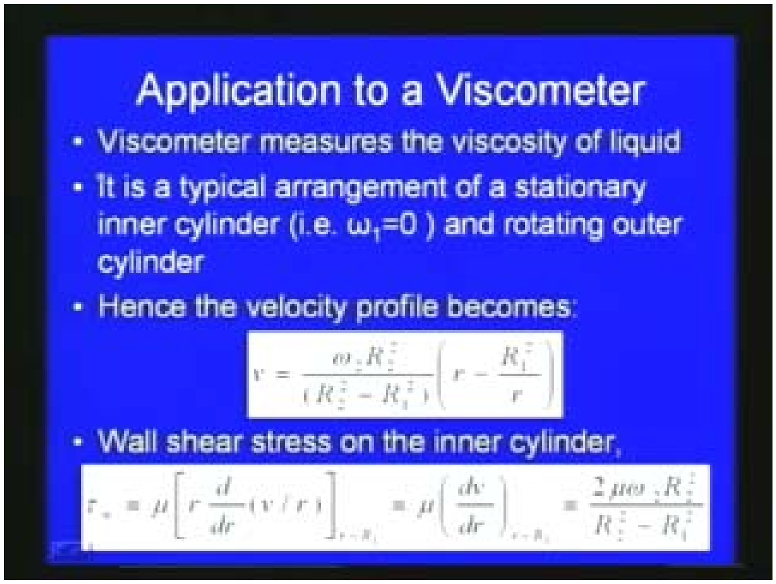

Here the next slide shows how a schematic sketch of viscometer, this is schematic sketch of the viscometer here now it is a typical arrangement of a stationary inner cylinder that we can assume omega 1 is equal to 0 the ambient velocity omega 1 is equal to 0 and rotating outer cylinder hence the velocity profile, with respect to our earlier equation which we derived here the velocity profile we can write v is equal to omega 2 r 2 square divided by r 2 square minus r 1 square into small r minus r 1 square by r, we can get the wall shear stress on the inner cylinder can be derived as by using newton's second law Newton law viscosity, here we have got v we can use the newton's law viscosity to get the wall shear stress on the inner cylinder.

Toe w is equal to mu into r d by dr of v by r that is equal to mu dv by dr at r is equal to r 1 we get an expression for the wall shear stress on the inner cylinder as 2 mu omega 2 r 2 square divided by r 2 square minus r 1 square.

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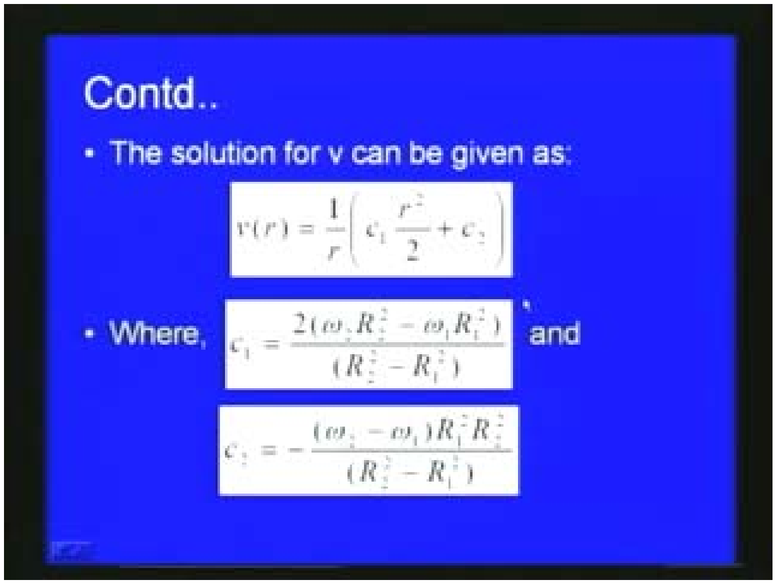

Once we get the velocity we can find out various parameters like shear stress on the wall and other parameters, what we derive for the concentric cylinder problem, the velocity expression mu directly utilized for the viscometer which we have already seen here for the torque can be measured by a torsion wire.

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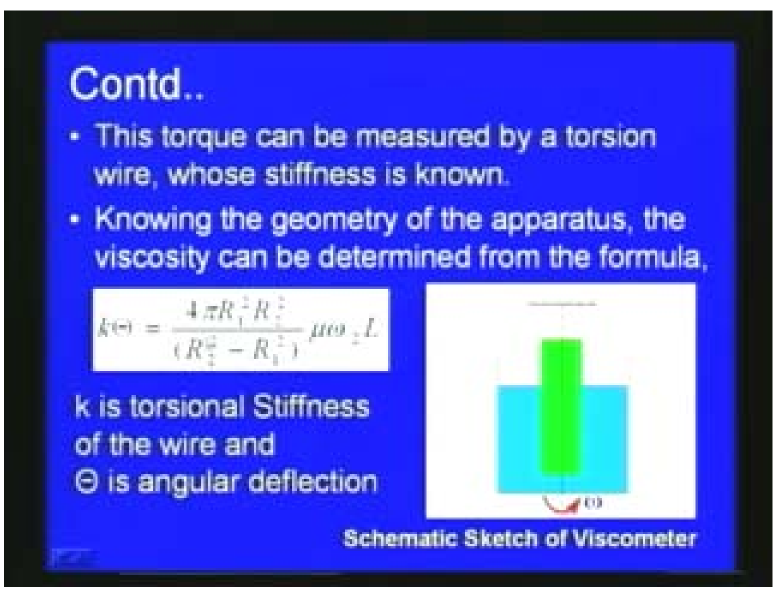

Here the problem is the viscometer problem, we have it is this cylinder is suspended with respect to a wire here, this torque can be measured by a torsion wire whose stiffness is knowing the geometry of the apparatus the viscosity, can be determined from the formula K theta is equal to 4 pie r 1 square r 2 square divided by r 2 square minus r 1 square into mu omega 2 into l Where theta is the angular deflection and k is the torsional stiffness of the wire like this, we can find out the other parameters for these kinds of problem.

Here we applied the concentric the equations derived for the flow between the concentric cylinders; we work expression for the velocity and the shear stress and other parameters for these kinds of problem. This is a typical problem which we started with the navier-stocks equation simplified the equations we derive the expression for velocity and other parameters. Finally, before closing this exact solution for navier-stocks equation we will consider one more example. The example problem is the statements like this a liquid flow down a wide inclined plate at an angle theta with horizontall.

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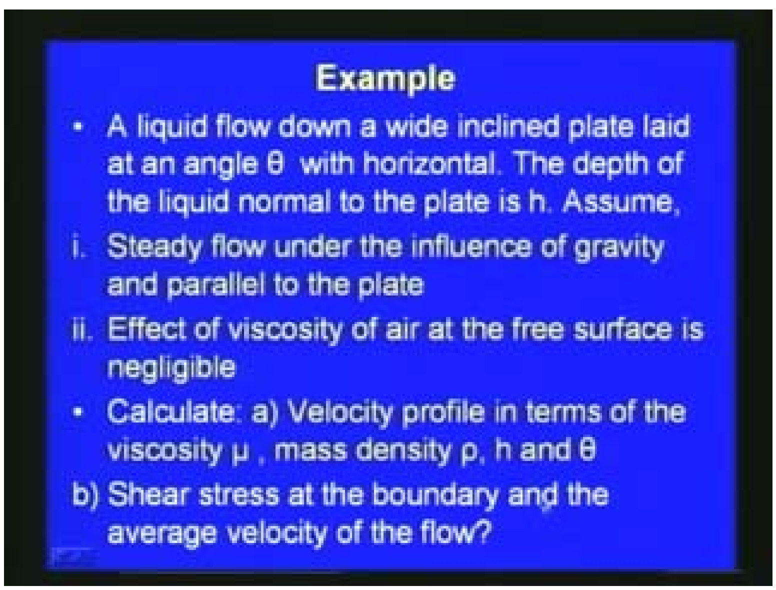

The depth of the liquid normal to the plate is h asses steady flow under the influence of gravity and parallel to the plate the effect of viscosity of air at the free surface is negligible. We have to find out the velocity profile in terms of the viscosity mu mass density rho the h and theta and also we have to find out the shear stress at the boundary and the average velocity of the flow this is the problem is defined in this figure.

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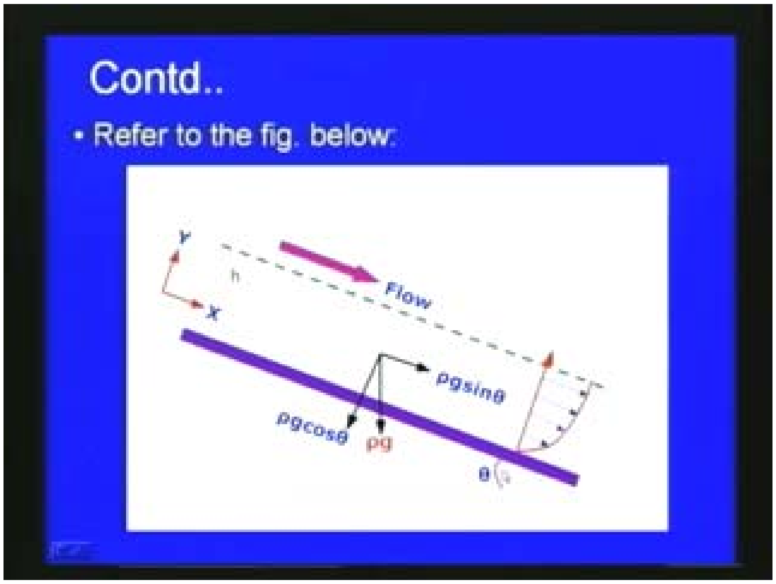

Here we have what an inclined plate and the flow is taking place above the plate the thickness of the flow is h and the angular inclined of the plate is theta, here comparing to the earlier problem like plane poiseuille flow or the other kinds of the just flow plate, here the plate inclined, we have to consider the body force. This typical the problem is we have to consider the body force this kinds of problem here our aim is to use the navier-stocks equations then we simplify the navier-stocks equation, we want to get an exact solution for this kind of problem.

The solution as the plate is wide the z direction need not to be considered, we can see that we consider wide plate this z direction need not be consider and here we assume the flow to be steady del by del t 10 is equal to 0 and flow is steady the terms in 10 become 0, we assume that the flow is parallel to the plate that is x axis.

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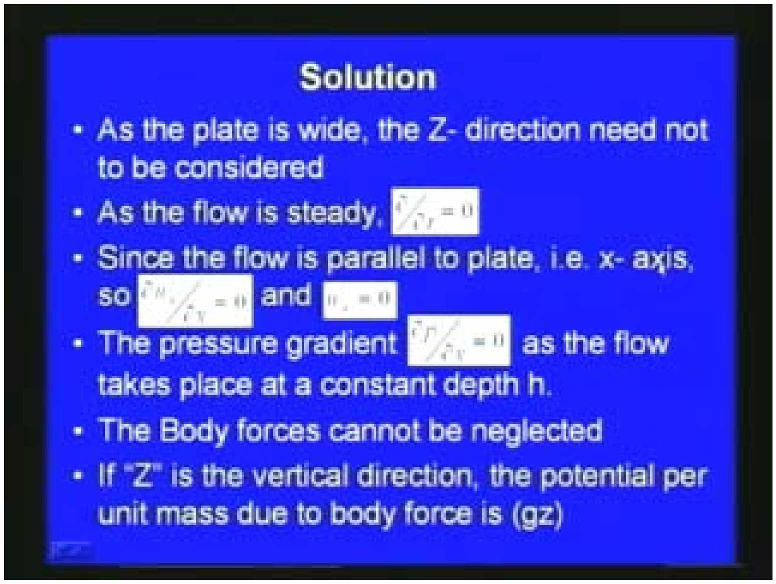

We can write del u by del x is equal to 0, that up is equal to 0 this is the third assumption also the pressure gradient dp by dx is equal to 0. As the flow takes place at a constant depth h these are the assumptions for the problem once the problem statement is known, we are trying to see that what are the assumptions we can forward such that we can simplify the equations with respect to the various assumptions here for this typical problem. We assuming the flows to be steady flow is parallel to plate the pressure gradient is del p by del x is equal to 0.

These are the assumptions essential assumptions for this problem that we can simplify our equation but here, compare to the earlier problem which we discussed here the body force cannot be neglected, as I mentioned the plate is inclined for the inclined plate you can see that there will be always the body force effect z is the vertical direction the potential per unit mass due to the body force we can write as go.

With respect to this figure, we can write the potential per unit mass due to body force is go finally the components of the body force in x and y direction we can write as in the body force in x direction this minus del gz by del x

That can be written as minus del g the access into gravity del z by del x is equal to g sine theta and y is equal to minus del gz by del y that is equal to minus g cos theta.

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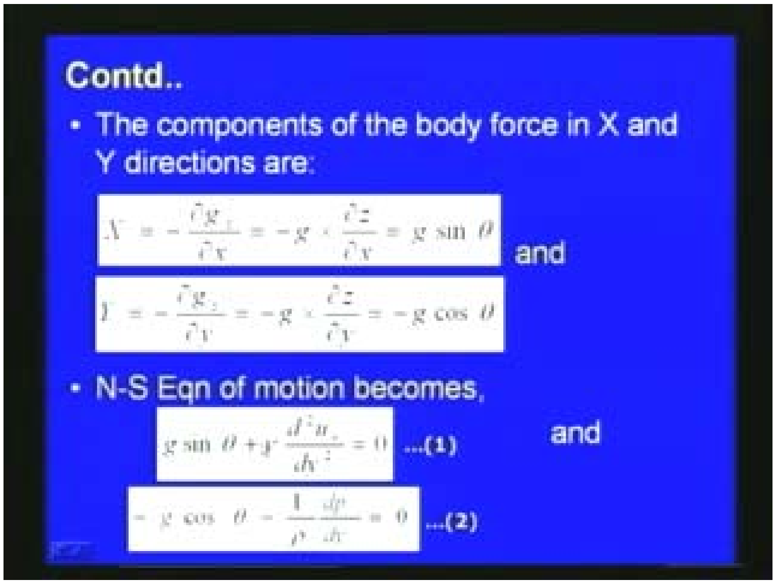

Now we use all these assumptions and finally we can get the navier-stocks equations in the following form: the navier-stocks equations become: g sine theta plus new d square us by dy square where mu is the kinematics viscosity, equation number one and second equation is minus g cos theta minus 1 by rho dp by dy is equal to 0, equation number two here due to our assumptions all other assumptions like this steady state and flow is parallel to the plate and the flow with is larger, we can consider the z direction need not to be consider.

Due to all this assumptions, all other times we can neglect and finally we have simplified the navier-stocks equation in the form of equation number one and two. Our aim for this problem is to find out the velocity distribution u_x here you can see that equation is d square u_x by in terms of d square u_x by dy square, as we did in the previous case we would see what are the boundary conditions, we would work for integration such that we will get the expression for the pressure hand velocity

For steady uniform flow parallel to the plate at a constant depth h we already seen del by del t is equal to 0 del u by del x is equal to 0 and uy is equal to 0 and dp by dx is equal to 0 del p by del x is equal to 0.

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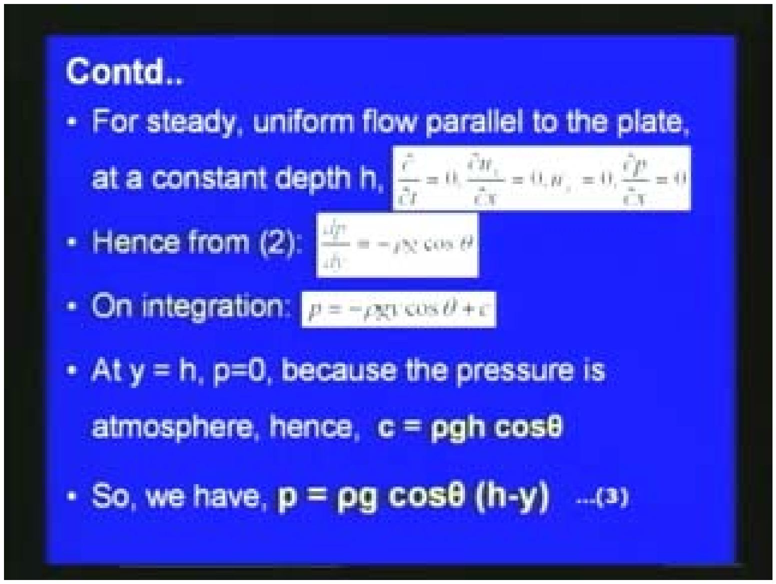

From equation number two, this equation number two become dp by dy is equal to minus rho g cos theta this equation number two, we can integrate such that we get p is equal to minus rho gy cos theta plus c the constant of integration, we get an expression for the pressure in terms of the that y p is equal to minus rho the rho is the density of the fluid minus rho g into y into cos theta plus c here we to find out this c we will use the boundary condition.

At y is equal to h p is equal to 0 because the pressure is atmosphere, that we can write c is equal to rho g h cos theta, now if you sub apply this c here in this equation, we get p is equal to rho g cos theta into h minus y. This is the expression for this flow problem. We got an expression for the pressure as p is equal to rho g cos theta into h minus y the standard procedure of integration used we applied the boundary condition to get the expression for p. This expression three is the pressure distribution for the flow our aim is to get the velocity distribution also from equation one, we can integrate twice with respect to y that gives first integration gives du_x by dy is equal to minus g sine theta by new into y plus c_1 equation number four.

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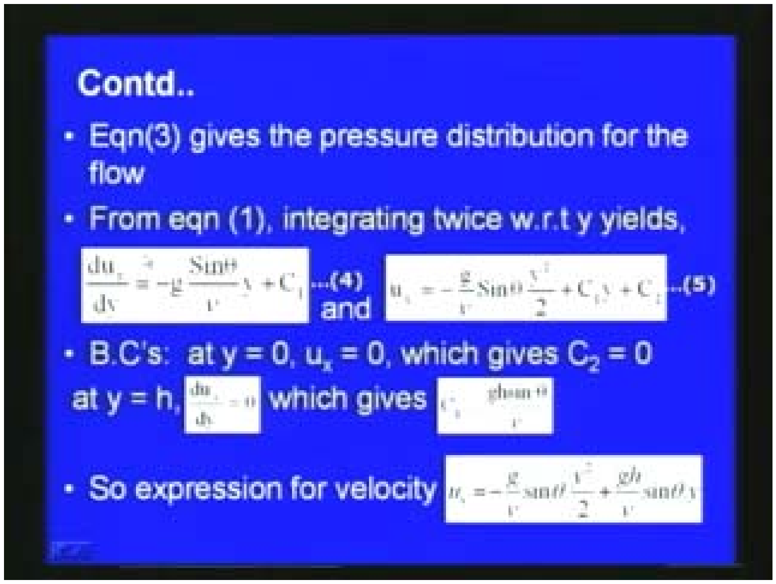

Once again in the derivative we get u_x is equal to minus g by new into sine theta into y square 2 plus c_1 y plus c_2 finally we got equation number five, here we have got constant of integration c_1 and c_2 we can utilize the boundary conditions the boundary conditions are at y is equal to 0 u_x is equal to 0 which gives c_2 is equal to 0 and that y is equal to h du_x by dy is equal to 0 that which gives c_1 is equal to gh sine theta by new.

Finally we can get an expression for velocity as u_x is equal to minus g by new sine theta into y square by 2 plus gh by new sine theta into y after substituting the boundary condition, we got c_1 and c_2 the integration constant, we work finally the expression for the velocity s u_x is equal to minus g by new sine theta y square by 2 plus gh by new sine theta into y now this can be simplified as u_x is equal to g sine theta by 2 new into 2 hy minus y square or we can write u_x is equal to rho g sine theta by 2 mu 2 hy minus y square. This is the final expression for the velocity for this particular problem, we got the velocity variation as rho g sine theta by 2 mu into 2 hy minus y square, once we get the velocity we can use Newton's law viscosity to get the shear stress shear stress at the boundary we can write tau 0 is equal to mu into du_x by dy at y is equal to 0.

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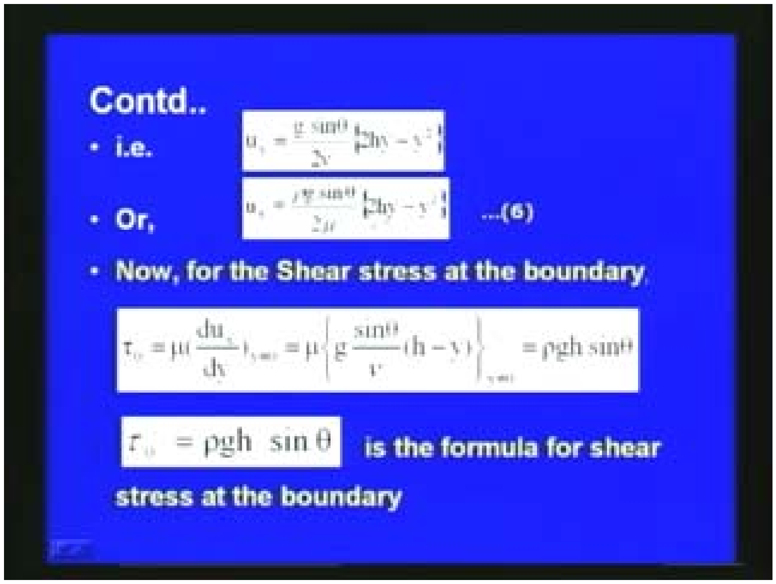

That gives mu into g sine theta by mu into h minus y at y is equal to 0, that gives rho g h sine theta. Finally, we got the expression for the shear stress at the boundary tau 0 is equal to rho gh sine theta tau 0 this is the formula for the tau 0 is rho gh sine theta this is the formula for the shear stress at the boundary.

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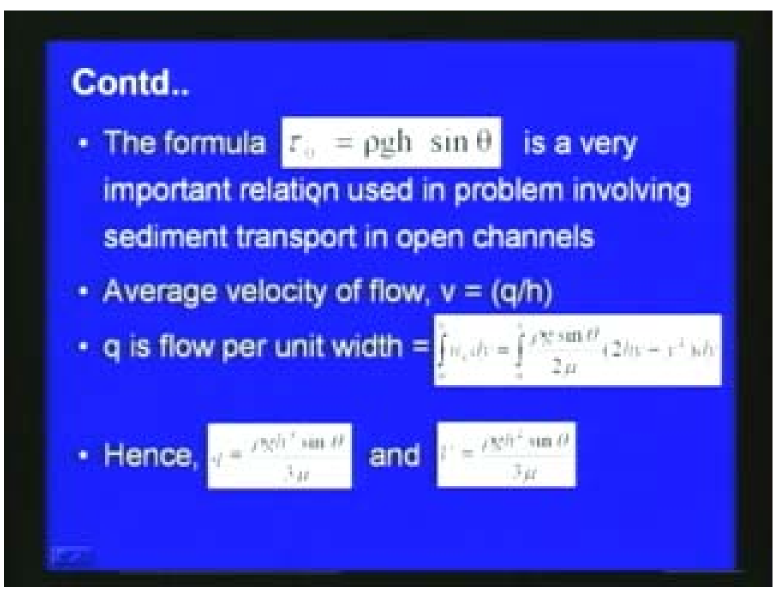

This is a very important relation used in your problem like sediment transport in open channel. Using this expression, we can see how settlement of the sediment takes place sediment transport in open channel we can use this expression if you want to find out the average velocity of flow for this particular problem v is equal to q by h where q is the flow per unit width flow per unit width q is equal to integral 0 to h u_x dy.

That is equal to integral 0 to h rho g sine theta by 2 mu 2 hy minus y square dy by from this expression we will get q is equal to rho gh q sine theta by 3 mu and the average velocity v capital v is equal to rho gh square sine theta by 3 mu these are the various expressions. Finally, by using various assumptions, we simplified the navier-stocks equation, we got the simplified the navier-stocks equation then we integrated to get expression for the velocity. We found the other parameters like a shear stress discharge per unit width by the various constant of integrations we are found by using the boundary conditions.

Like this various problems can be attempted wherever possible, wherever we can simplify in the navier-stocks equations and the problem is simple that we can try for an analytical solutions by putting forward various assumptions using the boundary condition like this few more exact solutions are available for the navier-stocks equations. But we will not discuss further since we have already seen few typical cases. Before further going to the applications navier-stocks equations we will discuss different formulations for navier-stocks equations.

We have seen the most commonly used formulation of the navier-stocks equations which we have derive earlier in two-dimensional and we ascended the equations two, three dimensions. In literature if you go through various research papers and various literatures on navier-stocks equations we can see few more other kinds of formulations for the navier-stocks equations these different formulations have been derived by using various actions and we can see that commonly three typical formulations are used in literature. First one is the primitive variable formulations or it is velocity pressure formulations.

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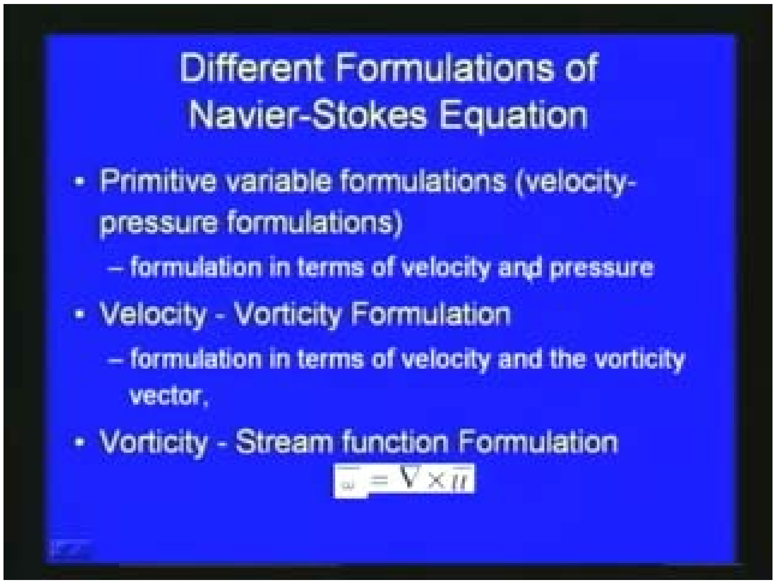

That is what we have discussed and we have derived earlier here the formulation is in terms of velocity and pressure the equations the navier-stocks equations are in terms of the velocity and pressure. These primitive variables are used like velocity and pressure that is why the formulations are called primitive variable formulations. Second kind of generally used the formulation is called velocity vorticity formulations.

Here the formulation for the navier-stocks equations in terms of velocity and the vortices that commonly used navier-stocks equations in terms of vorticity and stream functions it is called vorticity stream function. Formulation here the vorticity e is defined as del cross u bar where u bar is the velocity vector like this three typical formulations, the common three formulations are used in literature. We will see the various equations used for all this three formulations and the first one is the velocity pressure formulations the already we have derive the equations the equations. For example if you consider in two-dimensional case the equations are the continuity equations even the del u_x by del y plus del v by del y is equal to 0.

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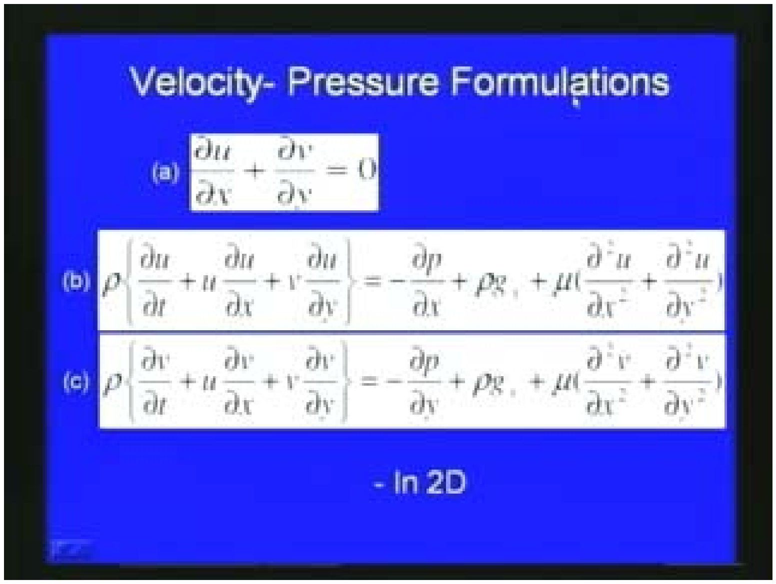

Then we have two moment equations, which we have all to derive here the equations are: rho into del u by del t plus u into del u by del x plus v into del u by del y is equal to minus del p by del x plus rho g_x plus mu into del square u by del x square plus del square u by del y square and third equation is rho into del u by del t plus u into del v by del x plus v into del v by del y is equal to minus del p by del y plus rho gy plus mu into del square v by del x square plus del square v by del y square.

These three equations including the continuity equations and two boundary equations are the equations using the velocity pressure formulations for the navier-stocks equations and this equation are called primitive formulations and most commonly used navier-stocks equations for most of the fluid flow problem. The second one the velocity vorticity formulations here, we transform as we have already derive the primitive very formulation that we can transform in terms of another important variable for vorticity. Finally we get the equations in terms of velocity and vorticity, if you consider three-dimensional formulations then, if the velocity vectors are u v and w and the vorticity vectors are psi eta and psi as written here.

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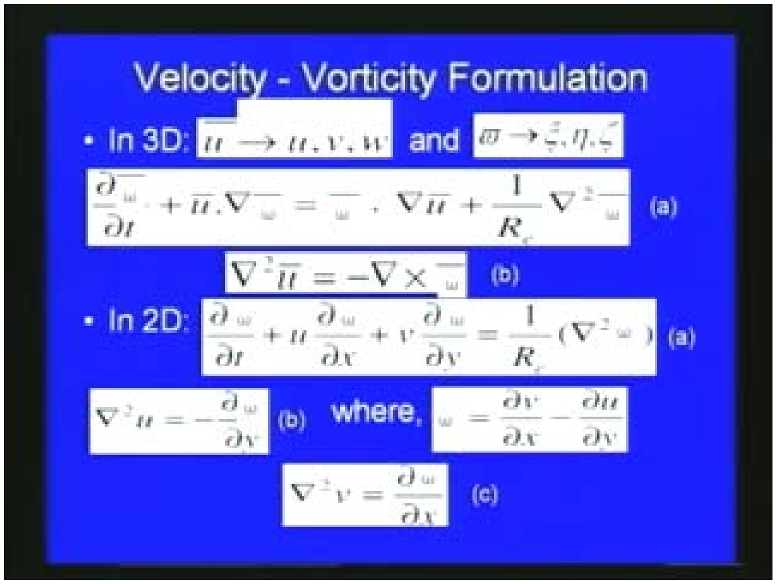

Then the equations I am not going to the derivations of this equations the derivation of equation we can see various fundamental books, where the navier-stocks equations are derive and other research papers finally the equations are generally express like this.

This first one is the vorticity transport equation del omega by del t here omega bar is the vorticity vector del omega bar by del t plus u bar dot del omega bar is equal to omega bar dot del u bar plus 1 by re del square omega bar, here re is the Reynolds number omega bar is the vorticity vector and u bar is the velocity vector.

This is one equation using the velocity vorticity formulation and second equation we use del square u bar is equal to minus del cross omega bar, as given in this equation b there are two equations in the vector, you are formed as given in equations a and b. This equation, for example if you consider two dimension problems that we can this equations we can write as for two dimensions omega is the vorticity del omega by del t plus u into del omega by del x plus v into del omega by del y that is equal to 1 by r e del square omega as in this equation this corresponding to this equation, here in two dimensions we can write del square u is equal to minus del omega by del y and same del square v is equal to minus del omega by del x as in the equation and where omega is equal to del v by del x minus del u by del y.

This equations use the velocity vorticity formulation for the navier-stocks equations in three dimensions and two dimensions, here we have some advantages like, the pressure is no directly coming the expression the equations are in terms of velocity and the vorticity. Wherever the vorticity time is important then if you use this equation, this form of the navier-stocks equations it will be much easier here generally for this formulation velocity vorticity formulations the boundary conditions will be in terms of the velocity.

Once we solve the vorticity transport and the poiseuille equation, corresponding to this equation b, if you solve this equation we get velocity distribution and the vorticity distribution throughout the domain. Once we get the velocity and vorticity then we can find out other parameters like pressure, we can find out from the obtained values of the velocity and vorticity problems. The velocity vorticity formulation is also used in many problems wherever we can express, we can use the parameter vorticity and further these equations are much easier to solve compare to the primitive variables formulation which we have seen earlier.

That is why, wherever possible we proper to use the velocity vorticity formulation of the navier-stocks equations. In the third formulation is vorticity stream function formulation depending upon the problem, wherever we can write we can express the stream function for the typical problem where we can express the stream function then we use this formulation vorticity stream function formulation.

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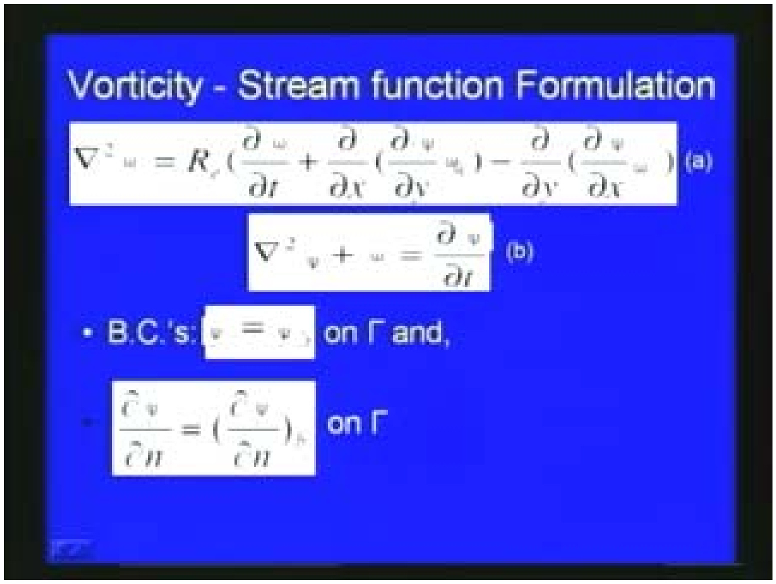

If we consider, for example two-dimensional problem here we have two equations, the first equation is written like this same del square omega is equal to r_e into del omega by del t plus del by del x plus del psi by del y into omega minus del by del y del psi del x into omega. This is first equation. Second equation is del square psi plus omega is equal to del psi by del t, here we have a two expressions, first one is equation a del square omega is equal to r e into del omega by del t plus del by del x del psi by del y of omega minus del by del y of del psi by del x omega and second equation is e 1 as del square psi plus omega is equal to del psi by del t.

Here the boundary condition are in terms of psi, psi is equal to psi v on gamma of the boundary, normal derivative del psi by del n is equal to del psi by del n at d these are the boundary conditions and the equations a and b which is in terms of the Reynolds number re and the omega and the sie.

We can solve these two equations, we get the distribution of the stream function and the what is it omega, depending upon the problem this formulation also, we can get a solution and finally once we get the omega and the psi that means, the vorticity and the string function then, we can get other parameters like velocity and the pressure. Typically, same depending upon the problem, we can solve the problem it very much easier to use the primitive very formulations. Some other case the velocity vorticity formulation will be easier and me other case depending upon the problem, we can utilize the vorticity stream function formation. Depending on the problem we have to choose the formulation then navier-stocks formulation, we have to try to solve the problem once the variables whether the primitive variable formation, directly we get the pressure velocity but in the velocity what it is formulation we get in terms of the velocity and vorticity.

Then we may have to find out the pressure, the third formulation we get the in terms of vorticity in stream function after getting the vorticity stream function, we may have to find out the velocity and the pressure distribution for the problem consider. Depending upon the problem, we can choose the particular formulation try to solve the problem, we have seen the navier-stocks equations derivation the various analytical solutions for the simplified cases and also we have seen three kinds of formulations for the navier-stocks equations. As we have already discuss the navier-stocks equations for the real fluid problem is very difficult to get the exact solution we may have to go for the numerical solutions.