Fluid Mechanics
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Lecture - 27
Navier-Stocks Equations and Applications

Welcome back to the video course on fluid mechanics. In the last lecture, we were discussing about the Navier-Stocks Equations and its derivations. For viscous flows, we have to consider the viscous effect using the Newton's second law motion. We have seen how to derive the Cauchy's equations first of any kind of viscous flows. From the Cauchy's equation we have derived the Navier-Stocks Equations for two-dimensions. We have seen how to extend this two-dimensional equation to the three-dimensions. In this Navier-Stocks Equations, we have seen in the formulation with the velocities and pressures. We have in three-dimension uvw three velocity components as unknown p the pressure unknown. We have three equations of motions called Navier-Stocks Equations. We have the continuity equations in three-D four equations and four unknowns to find out the velocity distribution or pressure distribution; we have to solve this equation by using the appropriate initial conditions and boundary conditions. Here, we can see that, this equation is second order and non-linear in nature. So, it is very difficult to get an exact solution for few simplified cases only we can show the exact solutions. Most of the time for practical problems, we cannot get an exact solution. We have to solve this second or non-linear defers equation partially refresh equation generally using numerical methods.

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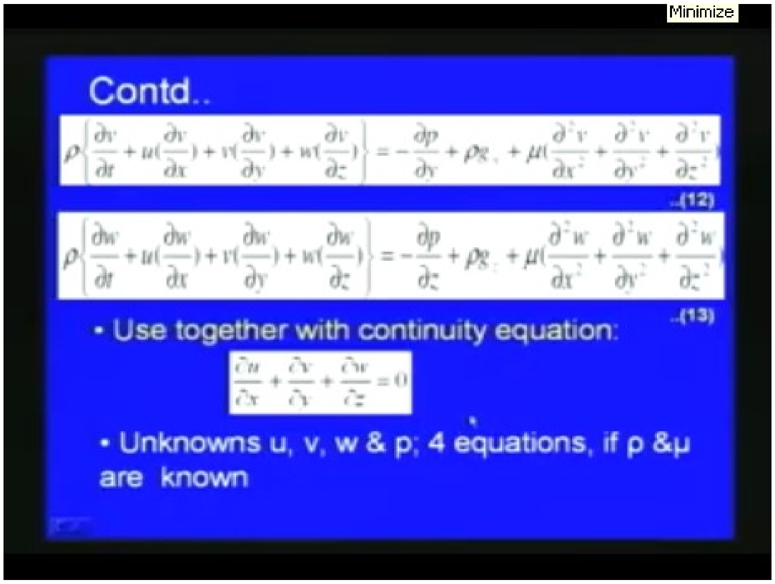

Here, these are the equations in three-D; we have three equations of motions called Navier-Stocks Equations and the continuity equation here. We assume that, the density and the coefficient viscosity mu are known we try to solve this equation. As we discussed earlier in the Navier-Stocks Equations are the generalized equation for viscous flow. We can use this equation for most of the fluid flow problems with appropriate modifications. We can use for laminar flow, turbo ran flow even use for compressible flow with appropriate changes. This is the advantage of Navier-Stocks Equations.

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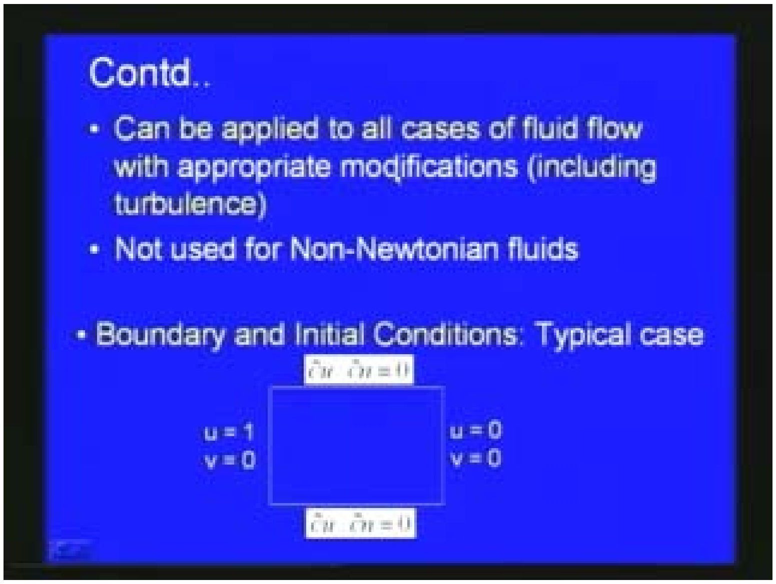

Once we make a model flow the Navier-Stocks Equations can be applied for various cases of fluid flow. Generally, we use this Navier-Stocks Equations our assumption is since such a way that, these equations are applicable for Newtonian fluids. We cannot utilize Navier-Stocks Equations for non-Newtonian fluids. With the 04:11 equations for the even problem, we have to supplement with the initial and boundary conditions. Here, as we can see for a two-D problem, the initial conditions can be either velocity or pressure the distribution will be known throughout the domain the boundary conditions. We can supply generally, in terms of velocities or in terms of depending upon the pressure or other parameters. Here, this particular two-dimensional problem. For a typical case, we can say that velocity u component is equal to 1 v is equal to 0. Here, v is equal to 0 u is equal to 0 and here the flex del u by del n is equal to 0 del u by del n is equal to 0. This way, we can supplement the problem with the appropriate boundary conditions. Similar way, if you consider a three-dimensional problem with respect to x y and z.

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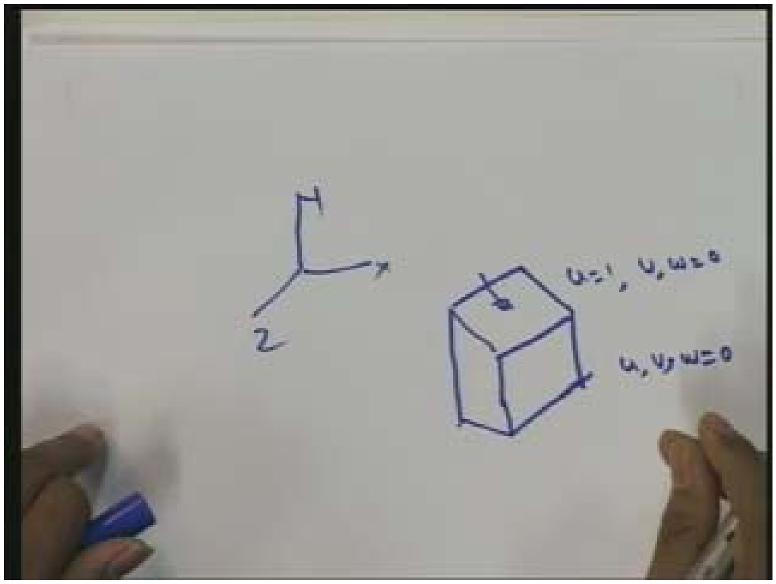

If you consider the flow in a cube cavity like this we can use the Navier-Stocks Equations with appropriate boundary conditions. For example, if you consider the incompressible viscous flow in this cavity, let us assume that, this lid this moving with velocity u is equal to 1 v and all other size also u v w equal to 0. This can be typical cavity problem which is generally used to develop the computer course using Navier-Stocks Equations for verification. But, the boundary conditions are u v w or 0 or all this sides and on the top lid u is equal to 1 and v is equal to 0 and w is equal to 0. The initial condition, we assume either the velocity as 0 or even non-values the velocities are known then initial conditions are given to solve this partially differential equation are the Navier-Stocks Equations. All the problems; once the gaven equations either as the form which we discussed or any other different forms, we have already seen in the case of turbulent flow. The Navier-Stocks Equations will be in the some other different format by considering the turbulent components velocity fluctuating components or in some other formulation like the string function formulation different kinds of formulations are available for different problems.

Accordingly, we will be supplying the initial conditions and the boundary conditions to solve the particular problem which we consider. This way once the domain is defined depending upon the problem, we decide which form of the Navier-Stocks Equations will be used continuity equations will be used. If exact solutions are not possible, we will be using some approximate or numerical methods to solve the equations by using appropriate initial conditions and boundary conditions.

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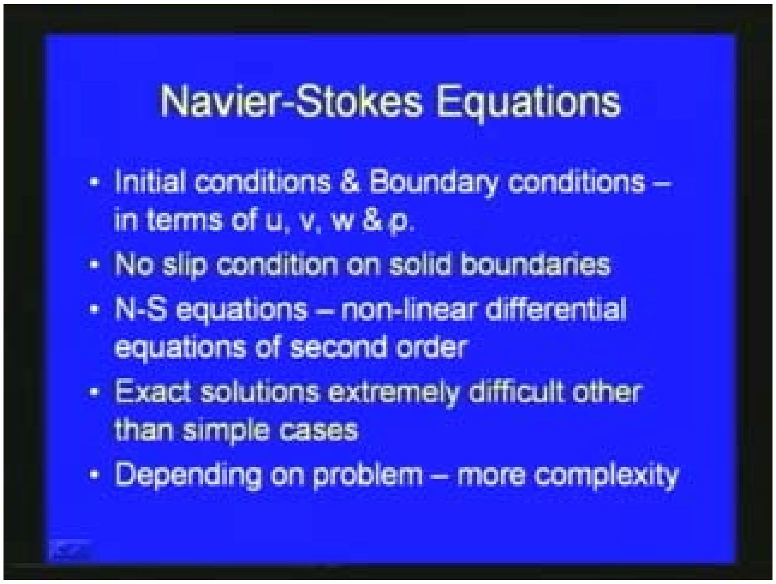

Now, in the some other aspects of this equations initial conditions and boundary conditions can be generally in terms of u v w that means the velocity components and pressure, either velocity of pressure depending upon the problems. As we have already seen, generally, whenever the solid boundaries are there, we concerned viscous flows no slip condition is used. As we have seen this Navier-Stocks Equations non-linear differential equations of second order. Exact solutions are extremely difficult other than simple cases. Depending upon the problem either turbulence is involved or the problem is even compressible it can also be approximated with respect to change. We can change the equations such whether compressibility it also can be considered. Depending upon the problem which we consider or need to solve the problem the problem may become more complex because, exact solutions extremely difficult other than the simple problems.

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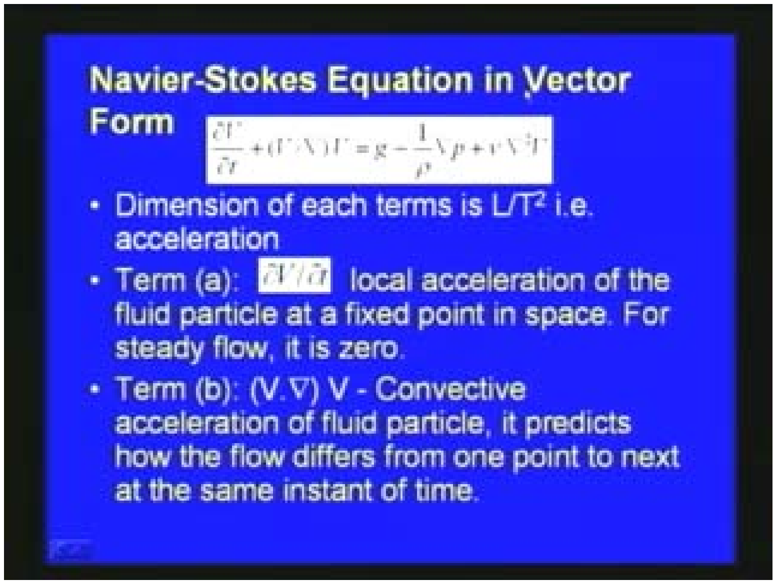

Now, Navier-Stocks Equations which we have already derived earlier, we can write in vector form as: del v by del t plus v dot del v is equal to g minus 1 by rho del p plus mu del square v where, mu is the dynamitic viscosity, v is the velocity vector and p is the pressure rho is the density and g is the acceleration gravity. Here, Navier-Stocks Equations is retuning vector form and if we analyze we can see that, each of term is actually acceleration term. The acceleration term u can see that, the unit is length divided by time square L or L by T square. Depending upon each term represents an acceleration term here, you can see the equation the first term, if we consider del v by del t here it is the change in the velocity with respect to time. That means acceleration local acceleration can be by it is local acceleration, local acceleration fluid particle at a fixed point in space for steady state flow. If you consider steady state flow this del v by del t term will be 0 otherwise we have to consider del v by del t. This term is called local acceleration. The second term here v dot del v this term is actually, the convective acceleration of fluid particle it predicts how the flow differs from one point to next at the same instant of time. We can see that, when the fluid is flowing from one place to another depending upon the pressure difference. We can see that, there is the local acceleration; we have already seen the convective acceleration.

The velocity of the fluid also take the part flow to one place to another it is called convective acceleration. It gives how the flow differs from one point to the next at the same. If we consider at a particular time from one point to another how the flow takes place is what this convective acceleration gives. The third term that means g is body force. The force due to the gravity or this shows the effect of the acceleration to gravity or gravity effects on the body and it is generally represented as body force. The term c represents the acceleration due to gravity.

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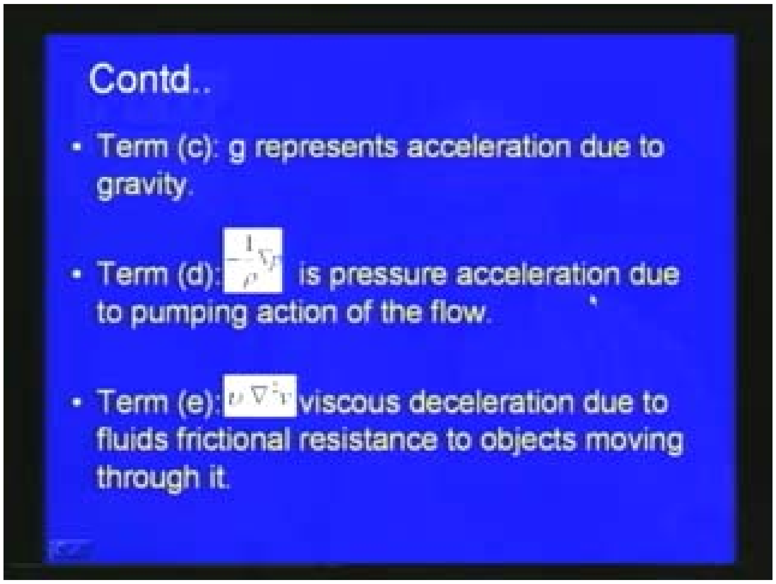

Term d that means minus 1 by rho del p in the in the previous equations. This term 1 minus 1 by rho del p this represent the pressure acceleration. Even that also we can represent as pressure acceleration due to pumping action of the flow that gives the pressure variation term. The next time here the last mu del square v that gives this term e of the fifth term mu del square v gives the viscous deceleration due to fluids frictional resistance to objects moving through it. Here, we consider the viscous flow there is always resistance for the flow. This term gives the mu del square v gives the deceleration. Actually due to the viscous effect there is effective deceleration type this term gives the viscous deceleration and due to the fluid fissional for resistance to objects moving through it.

Like this we can see that here in the Navier-Stocks Equations when we write the vector form we can represent in acceleration and deceleration term depending upon the case which we consider. Now, before further discussing how to solve or how to get some solution for this equation we will see each few of the terms the significance some of the terms.

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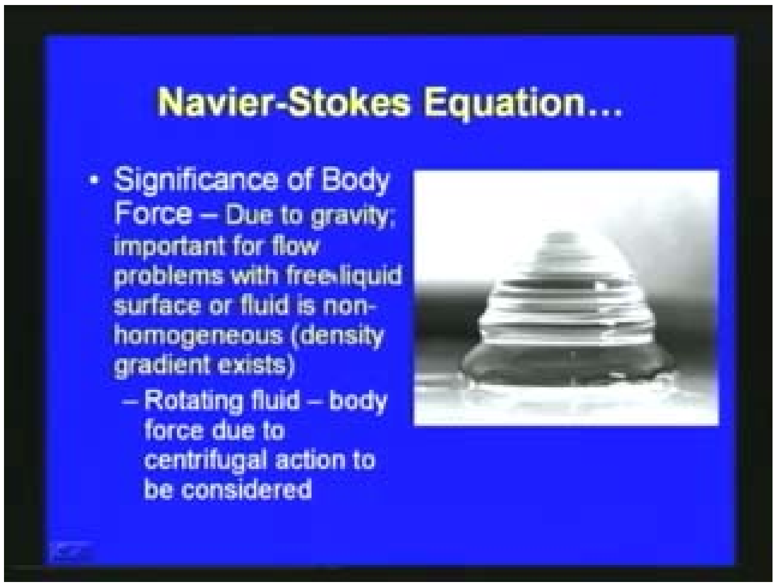

As we are seen in the significance of body force which is due to gravity this is important for flow problems with free liquid surface or liquid in non-homogeneous or with density gradient. If you consider any flow with free surface for example a flow is taking place on inclined plate. We consider a free surface flow over displayed we can see that, the access into gravity effect is there for this plate is inclined at angle theta, g component is active here. For this the body force is predominant here, we have to consider this body force.

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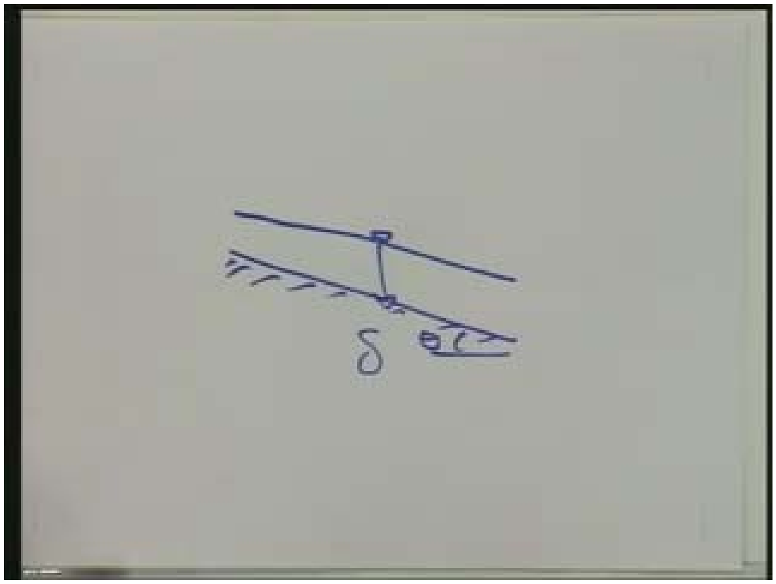

Due to gravity and it is very important and also sometimes non-homogeneous density that means in the flow fluid is non-homogeneous there is density gradient exists. In that case, we have to consider the body force also in case if there is any rotating fluid. You can see that, many times, we have to consider especially in mechanical there are number of machine parts with fluid will be rotating. If we consider the rotating fluid then, the body forces due to centrifugal action also to be considered.

Here, you can see in this slide (Refer Slide Time: 14:48) rotating fluid in this case we also have to consider problems such as the centrifugal action that is on the body force with respect to centrifugal action. The other important term is the viscous term. We have already seen Navier-Stocks Equations which we are derived is for viscous flow. Viscous term means when the viscosity increases we can see that, there is more assistant to flow depending upon the fluid which we consider. The viscosity will be changing.

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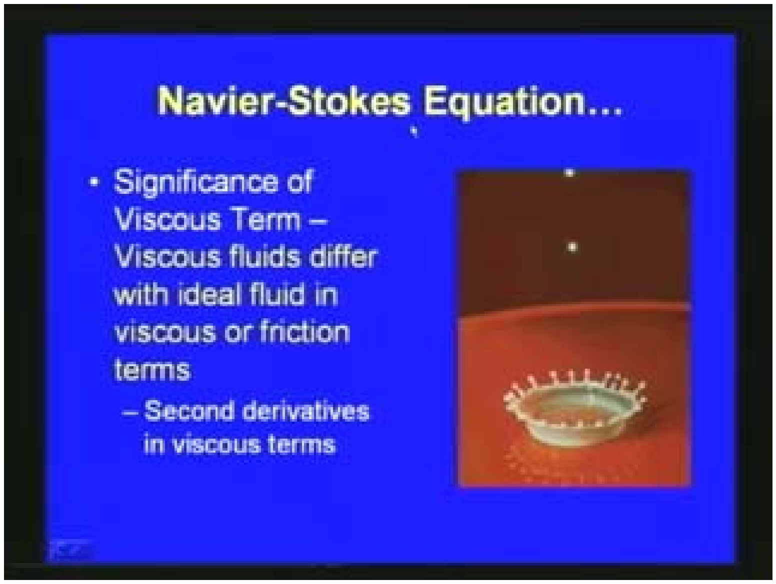

Viscous fluid differs with add l fluid the potential flow which we considering viscous or friction terms. The equation we have seen the second derivatives of viscous term. Depending upon the fluid if it is more viscous then, this effect will be much higher. They are more resistant to the flow and hence deceleration takes place depending upon the fluid which we consider the Navier-Stocks Equation (15:55). Before further going to the solution of some of this especially exact solution for cases of the Navier-Stocks Equation; we will consider some of the limiting cases. Navier-Stocks Equations here, the viscous forces tend to 0 that means the viscous effect is very less then; we can consider the flow as potential flows. The second term mu into del square u del square v or del square w that term can be neglected depending upon the problem.

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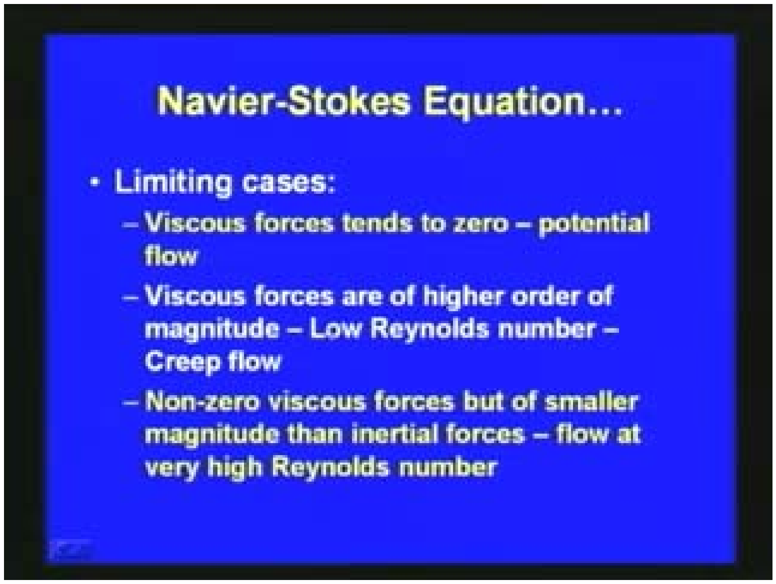

Second case is viscous forces are higher order magnitude that means in this case the second limiting case here the viscous forces are of higher order of magnitude and low Reynolds number. Here, this is creep flow so the Reynolds number is low and viscous forces are higher order magnitude. The fluid velocity is very less and we call such kinds of flows as creep flow or sometimes stock flow. We will be discussing more details of the creep flow later. Here, this is another second limiting case where the viscous flows are higher order magnitude. The third case is non-0 viscous forces but, have small magnitude than inertial forces. Especially, the air dynamics flow problems, Reynolds number will be high. These are some of the limiting cases where, we consider the Navier-Stocks Equations. Depending upon the case, as I mentioned earlier, we have to modify the equation and Navier-Stocks Equations and try to get a solution. As far as potential flow is considered, let us consider the non-viscous fluid as in the case of potential flow theory.

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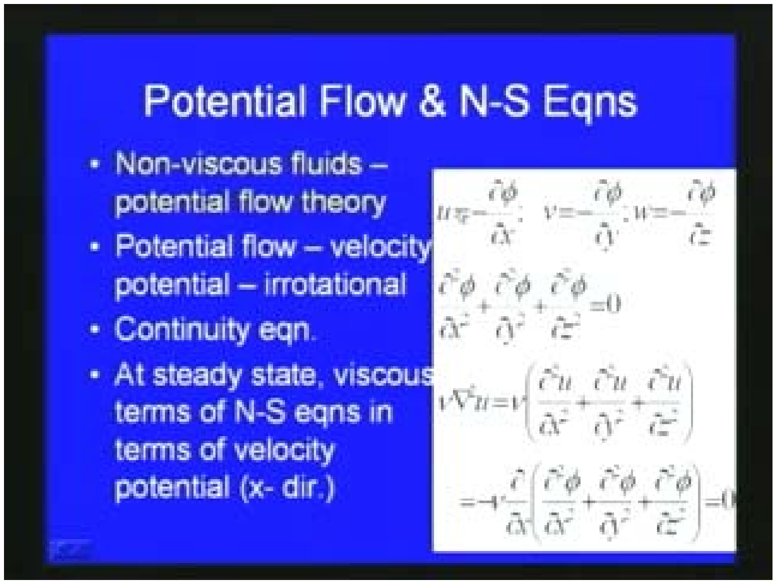

For potential flow problems we have already seen that, the velocity components u v w can be expressed that u is equal to if phi is the potential velocity potential u is equal to minus del phi by del x and v is equal to minus del phi by del y and w is equal to minus del phi by del z. We can also write for ir-rotational potential flow case, we can write del square phi by del x square plus del square phi by del y square plus del square phi by del z square is equal to 0. Now, also by using the continuity equation at steady state we can write this mu into del square u can be written us mu del square u by del x square plus del square u by del y square plus del square u by del z square. If substituting u v w and interchanging the differentiation in the parameter here. We can write: mu into del square u is equal to minus mu del by del x del square phi by del x square plus del square phi by del y square plus del square phi by del z square. This term we have here for ir-rotational flow potential flow this term is 0. We can say that, mu into del square u is equal to 0 for non-viscous fluids with respect to the potential flow theory as steady state, we can see that, the viscous terms of the Navier-Stocks Equations in terms of velocity potential once it is return. We can show this mu del square u is equal to 0. Similar way the second y direction as well as is a direction we can show that, mu del square v is equal to 0 and mu del square w is equal to 0. That means; for potential flow characterized by the velocity potential and viscous terms in the Navier-Stocks Equations are identically 0.

That means we can see that, the potential flow the Navier-Stocks Equations reduced to Euler's equations which we considered earlier. For potential flow, we have already shown that, the viscous terms mu del square u and mu del v and mu del square w or 0 and finally the Navier-Stocks Equations are transformed into the Euler's equations so we can solve the Euler's equations. In the case of inviscid flow or ideal fluids flow and Navier-Stocks Equations which we have already seen here, the Euler's equation, the general form of the equation is del v by del t plus v dot v is equal to g minus 1 by rho del p.

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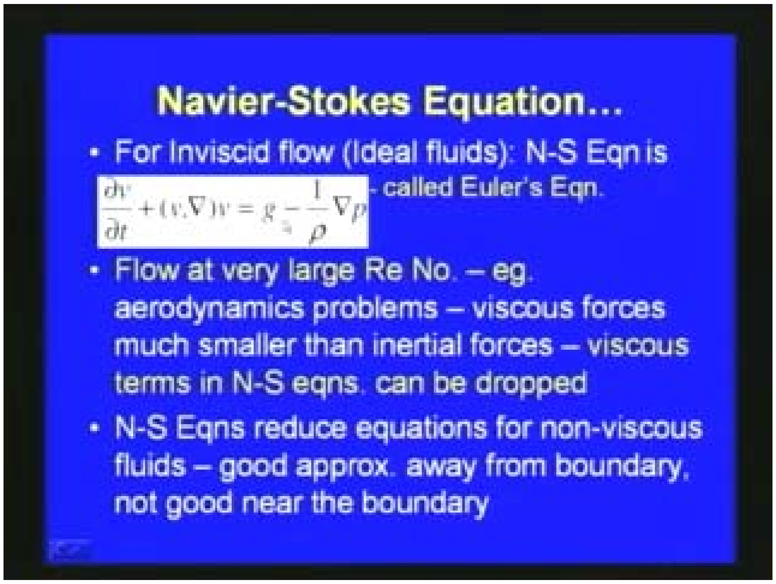

This form of the equation we have already seen in the Euler's equations del v by del t plus v dot del v is equal to g minus 1 by rho del p. In the previous slide, we have already shown that, these viscous terms become 0 for a potential flow, we get the Euler's equations and also flow at very larger Reynolds number example: aerodynamics problems viscous force is much smaller than inertial forces and viscous terms in the Navier-Stocks Equations can be dropped. For higher inertial number flow also we can drop the viscous terms since the viscous flows are much smaller than the inertial forces. So, we can drop the viscous terms and Navier-Stocks Equations reduced equations for non-viscous fluids. In the case of the aerodynamics problems the higher inertial number flow, we can show that, the viscous terms can be dropped if you modal using the case with or without the viscous terms that will be also good approximation. But away from the boundary near the boundary of course the effects will be there.

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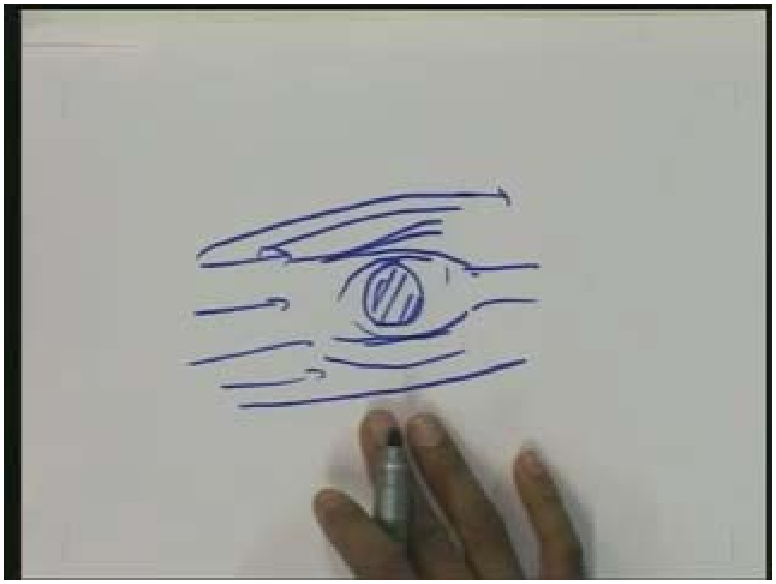

For example: let us consider a cylinder like this flow is coming in this direction. Hence see that away from this here near by the cylinder there will be definitely the effect the boundary layer. But away from the cylinder you can see that, there will not be much effect. Near the boundary we have to consider the boundary layer. By solving this kind of problem, if you consider the flow over a cylinder like this, we can consider the nearby area, we may have to solve the Navier-Stocks Equations.

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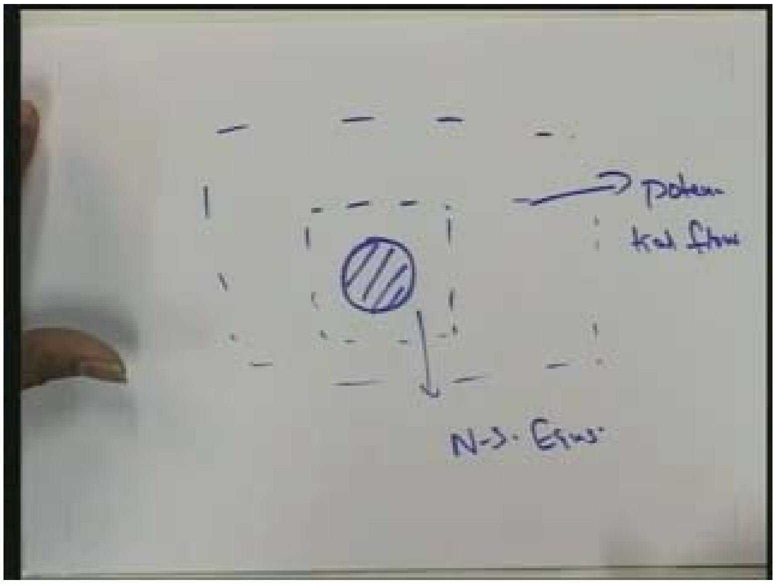

But far away from the solid boundary, we can consider and solve this. Here the Euler’s equation or even simple potential flow, we can use this area but near to the solid boundary, we have to consider. Since viscous effects will be there definitely and the boundary layer formulation takes place in this region, we can directly solve the Navier-Stocks Equations. Further away from the boundary this region, we can consider as potential flow as an approximation. This gives a better approximation especially when we solve large scale problems this gives lot of advantage since we can restrict our solution of the Navier-Stocks Equations near to the boundary. Away from the boundary we can consider the flow as potential flow. For the cases wherever the higher inertial number then we can see that, in good approximation is away from the boundary. We can drop viscous terms and near the boundary the solid there we will be solving the Navier-Stocks Equations.

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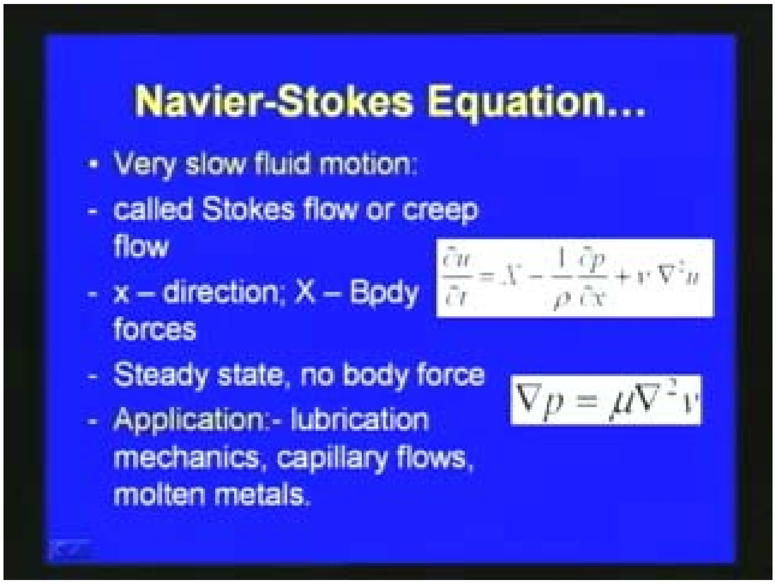

As we have already seen for very slow fluid motion called stokes flow or creep flow. In the case of on steady or transient condition we can write the equation as del u by del t is equal to x minus 1 by rho del p by del x plus mu del square u. If you consider the equation in x direction and where X is the body force and steady state. If there is no body force then we can write this equation as: del p is equal to mu del square v where v is the velocity component the velocity vector which we consider p is the pressure. For very slow fluid motion like in the case of this equations are very much useful in the case of lubrication mechanics, capillary flow and also molten metal especially, metallurgy wherever we consider the molten metal, we can utilize this form of the Navier-Stocks Equations.

We have now seen three limiting cases, one is and the creeping flow or the stock flow where the viscosity high. We will be considering the equation as for without the viscous the effect is much higher, we consider the last case here we consider. The other cases the potential flow where, we consider the viscous terms can be neglected. We can transform the case to the Euler’s equations. Third case is wherever the problems like aerodynamics where the higher inertial number case, there also as a limiting case the effect of the viscous terms sometimes, we can neglect far away from the solid surface. Since that effect will not be much of the viscous effect. These are some of the limiting cases where we consider the Navier-Stocks Equations. Now, the Navier-Stocks Equations which we derived are in the Cartesian coordinates certain problems like flow through pipes or rotating fluid cases then, if you consider cylindrical coordinates in terms of r theta and z then, the Navier-Stocks Equations, we can derive for that kind of problems.

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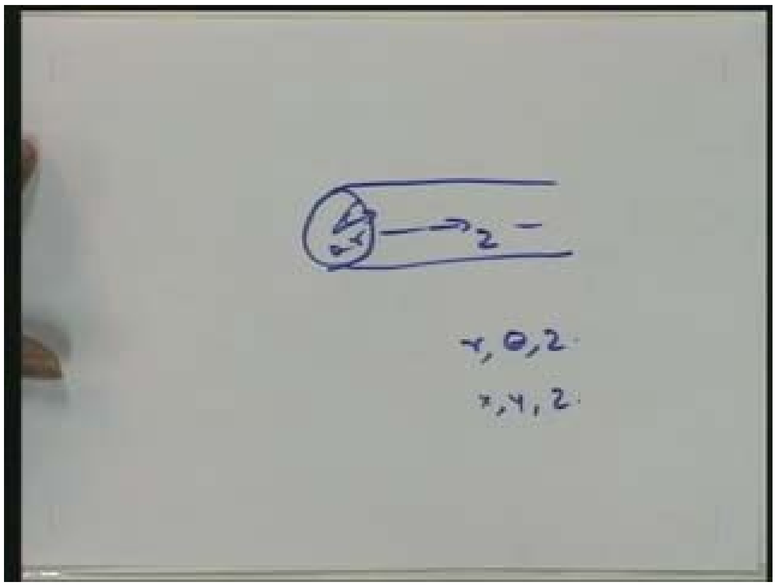

For example: either a pipe flow like this where r is the radius and theta is the angle and z is this direction r theta z here, the coordinate systems are in terms of r theta z instead of x y and z. The velocity components correspondingly, the velocity components are v_r v_{theta} and v_z. This equation what we have seen, the Cartesian coordinates equations, we can transform into the cylindrical coordinates in terms of r theta and corresponding velocity v_r v_{theta} v_z.

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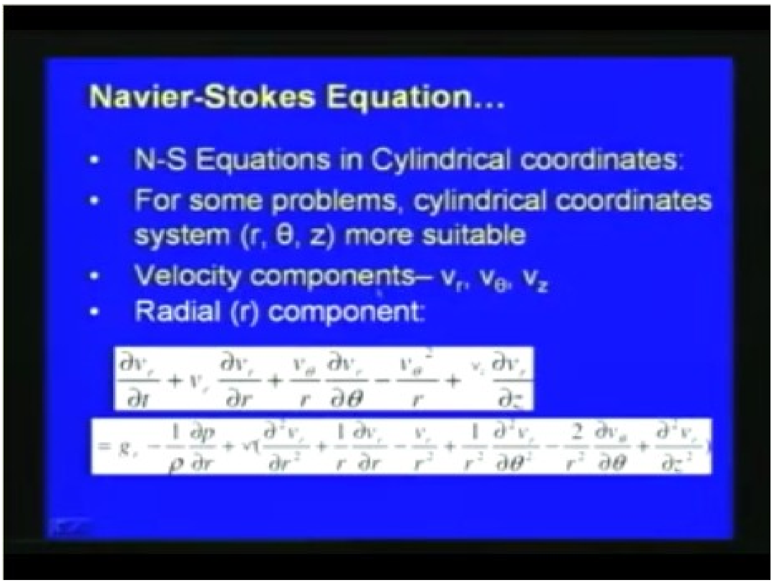

Here, we are not going for the derivation but, the equations here are written directly. The radial component r component is: del v_r by del t plus v_r del v_r del r plus v_{theta} by r del v_r by del theta minus v_{theta} square by r plus v_z del v_r by del z is equal to g_r minus 1 by rho del p by del r plus mu into del square v_r by del r square plus 1 by r del v_r by del r minus v_r by r square plus 1 by r square del square v_r by del theta square minus 2 by r square del v_{theta} by del theta plus del square v_r by del z square. This is the radial component in cylindrical coordinates here v_r is the radial velocity v_{theta} is a tangent of velocity and v_z is the axial velocity and r is the radius which we consider y is the density g_r is the acceleration to gravity in the radio direction.

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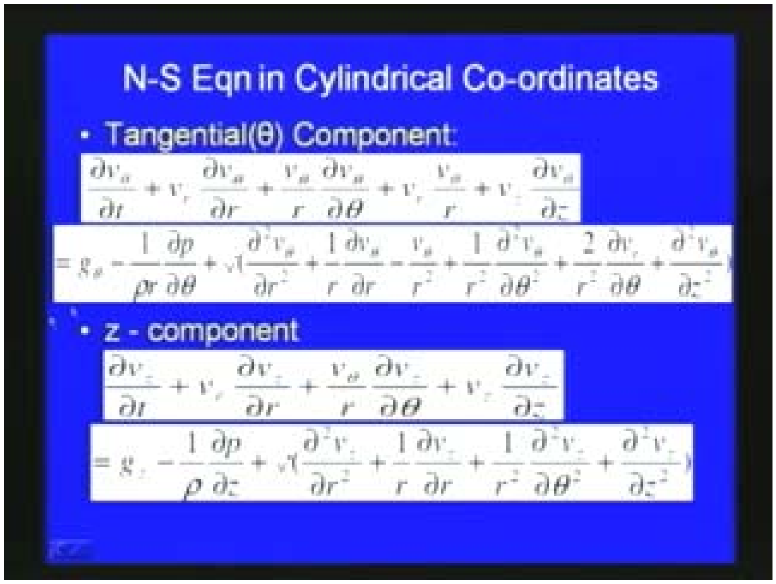

This gives the radio component of the Navier-Stocks Equations. The tangential component theta component can be written like this: del v_{theta} by del t plus v_r into del v_{theta} by del r plus v_{theta} by r into del v_{theta} by del theta plus v_r into v_{theta} by r plus v_z into del v_{theta} by del z is equal to g_{theta} minus 1 by rho r del p by del theta plus mu into del square v_{theta} by del r del r square plus 1 by r del v_{theta} by del r minus v_{theta} by r square plus 1 by r square del square v_{theta} by del theta square plus 2 by r square del v_r by del theta plus del square v_{theta} by del z square here mu is a kinematic viscosity and g_{theta} is the acceleration in the direction of the theta component and z component third component for the Navier-Stocks Equations can be written as: del v_z by del t plus v_r into del v_z by del r plus v_{theta} by r del v_z by del theta plus v_z del v_z by del z is equal to g_z minus 1 by rho del p by del z plus mu into del square v_z by del r square plus 1 by r del v_z by del r plus 1 by r square del square v_z by del theta square plus del square v_z by del z square this is the z component. This the cylindrical coordinate system the Navier-Stocks Equations which we can apply in the case of wherever r_{theta} and z and v_r v_z v_{theta} components are problem like rotating fluids better to use this cylindrical coordinate equation than the Cartesian coordinate equation. Before going for the exact solution, numerical solution of the Navier-Stocks Equations and the application of Navier-Stocks Equations, as I mentioned earlier this Navier-Stocks Equations some of the fundamental equation of the fluid mechanics basic partial differential equations as far as fluid viscous fluid is concerned. These equations are written especially for Newtonian fluids. We can apply for most of the problems including turbo ran flow, laminar flow or compressible or incompressible flow all the varieties. We can modify the equation and get the appropriate equation depending upon the problem.

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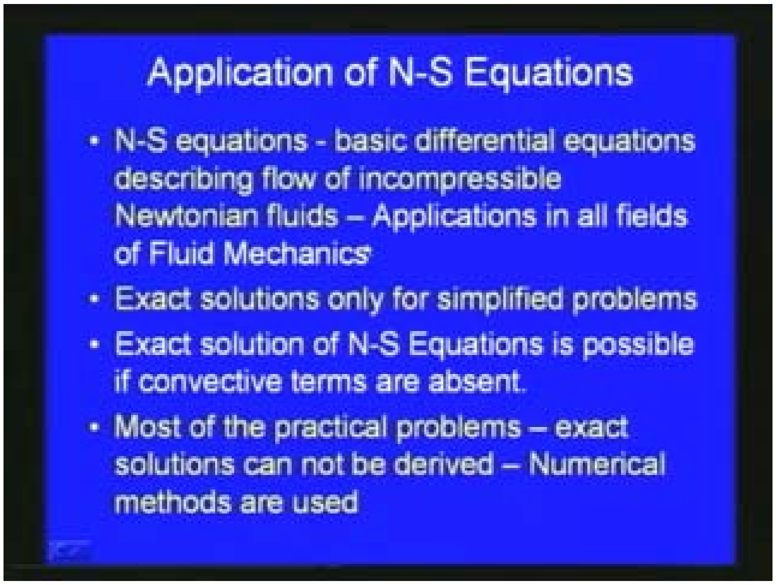

As far as solutions are concerned exact solutions for this equation are extremely difficult only simplified cases especially one-dimensional, two-dimensions also steady state conditions, we can have some simple analytical or exact solutions. But, otherwise since second order, non-linear partially refresh equations, we cannot get exact solutions and most of the practical problems are also the boundary conditions will be complex and geometry will be complex. For most of the practical problems, we cannot get any exact solutions we have to go for numerical solutions. Exact solutions of Navier-Stocks Equations are only possible if convective terms are absent. But, as I mentioned for most of the practical problems, we cannot derive exact solutions but, for simplified boundaries and simple cases where, the convective terms are absent we can derive some of the exact solutions. Here we will be discussing the exact solutions before we discuss briefly on the numerical solutions for the Navier-Stocks Equations. As far as exact solutions are concerned, the solutions are exact that the equations satisfied at every point of the flow domain. If we consider the flow in a two-D cavity or any kind of flow the exact solution if you derive that solution should satisfy throughout the domain.

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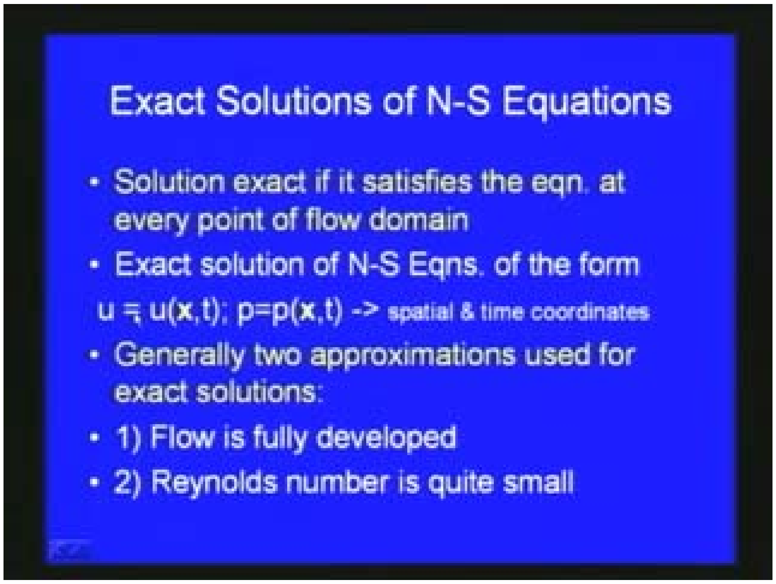

The exact solutions we can generally write it can be in terms of velocity. We can write u as a function of u x t this x represent the coordinate system x y z corresponding two-D or three-D or one-D problem and p is in terms of p x t that means spatial and time coordinates. As far as we get the velocity variations and the spatial variations in terms of the spatial and time coordinates. Generally, whatever analytical solution derived for basically, the analytical solution there are two approximations, one is flow is fully developed most of the analytical solution which are derived in literature for fully developed flow. The Reynolds number is quite small; it is extremely difficult to derive any of the analytical solution when Reynolds number is large. Generally, in literature the analytical solutions which are available for fully developed flow and the Reynolds number for the flow is small. Here, we will be discussing few of the analytical solutions.

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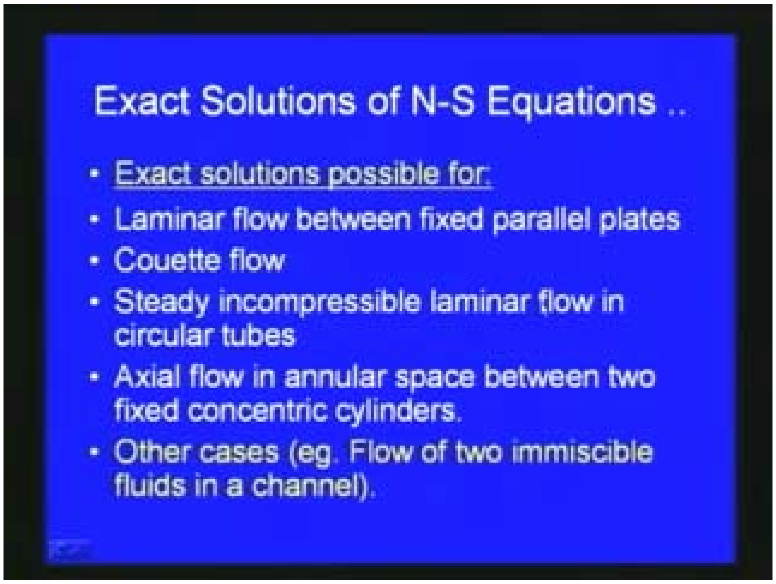

Some of the exact solutions or analytical solutions possible for Navier-Stocks Equations are listed here: First one is laminar flow between fixed parallel plates and this we will be discussing detail. Next one is Couette flow then steady incompressible laminar flow in circular tubes axial flow in annular space between two fixed concentric cylinders. Other cases like wherever the flow of two immiscible fluids in a channel steady state conditions. These are some of the cases where, we can derive the exact solutions for the Navier-Stocks Equations which we have seen. In all these cases we assume that, the flow is fully developed Reynolds number is low that, we can consider the flow as laminar. Few of exact solutions here, we will be discussing in detail. First case which we want to discuss here is called plane Poiseuillie flow.

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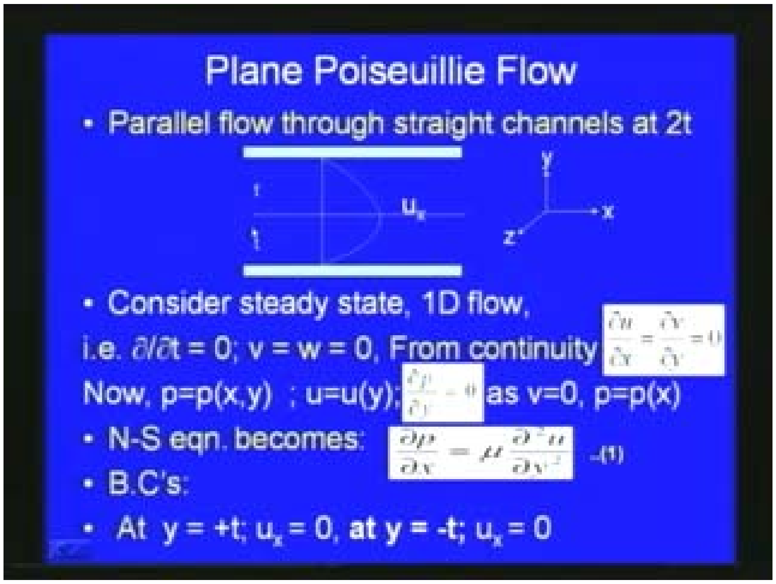

This flow flow through two parallel straight channels placed at a distance and rho is parallel flow through between two plates. Here, we can see two place are placed at a distance 2 t and here we consider the access in this directions, we consider here the flow as steady state flow is one-dimensional. Since steady state means now to get an exact solution generally what we do is we know the generalized Navier-Stocks Equations. Depending upon the problem, we use the various assumptions and very simplification which we used for the particular problem for which we are trying to derive the exact solution. Here in this particular case we assume that, flow is fully developed. The flow is steady state which means this del by del t is equal to 0 and here this particular case actually it is one-dimensional flow we can see that, v is equal to w is equal to 0. Now since v is equal to 0 w is equal to 0. We can see that, on the continuity equation we can write: del u by del x is equal to del v by del u is equal to 0. Now, the pressure variation p is function of x and y we can see that, now the u is varying only the in the direction of y so u is a function of y and also we can see this particular case del p by del y is equal to 0 as v is equal to 0. The pressure is varying with respect to x velocity is varying with respect to y. Finally, with respect to this assumption here the assumptions for this plane Poiseuillie flow are: its flow is steady state for its one-dimensions and parallel flow. Now by using all this assumptions, we can simplify the Navier-Stocks Equations which we are seen earlier as u single equation as del p by del x is equal to mu del square u by del y square as in equation number one. After putting all this simplification finally we get del p by del x is equal to mu del square u by del y square. Now, we got the plane Poiseuillie flow equation our aim here is to get the velocity variation or the pressure variation as we have discussed earlier. Now the boundary conditions since here no initial conditions since we consider the flow steady state. The boundary conditions here we what we are using is the noslip condition that means with respect to this figure on the wherever the flow is here on the plate that means on this location and on this location at y is equal to plus t and y is equal to minus t we can see that, velocity is 0 that actually the velocity variations will be parabolic like this. The boundary conditions are at y is equal to plus t u_x is equal to 0 at y is equal to minus t u_x is equal to 0. Now, we got the equations here our aim is to get an expression for the velocity variations. Now, the equation which we have is mu del square u be del y square is equal to del p by del x the velocity is second or differential equation what we works. Here to find the velocity, we can integrate twice this equations we can apply the boundary condition.

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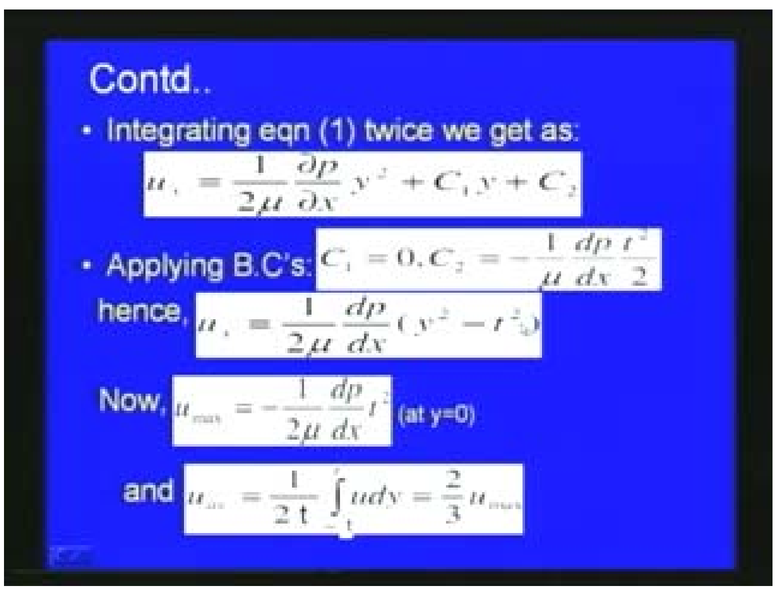

If you integrate twice, we get u_x is equal to 1 by 2 mu del p by del x y square plus c_1 y plus c_2. This is the expression for the velocity. Now, c_1 c_2 are the constants. We can apply the boundary conditions so we have two boundary conditions here at t is equal to t velocity u_x is equal to 0 t is equal to minus t also velocity u_x is equal to 0.

We can use this boundary conditions that we can see c_1 is equal to 0 and c_2 is equal to minus 1 by mu dp by dx into t square by 2. Now, we can substitute for this c_1 and c_2 in this equations that the expression for velocity is obtained as u_x is equal to 1 by 2 mu dp by dx into y square minus t square. If you want to find out the max velocity in this particular case a maximum velocity at the center line at y is equal to 0 the maximum velocity u max is equal to minus 1 by 2 mu dp by dx into t square this gives the maximum velocity. If you want to find out the average velocity u average is equal to 1 by 2 t we can integrate between the limits minus t to t udy this will be equal to two third of the maximum velocity. The average velocity for this case is two third of the maximum velocity like this. This is a typical problem the plane Poiseuillie problem where the flow if between two parallel plates to fixed parallel plates. As we have seen we use the various assumptions for this particular problem simplify the general Navier-Stocks Equations to get an expression for either velocity or pressure the unknown parameters. We use the simplified form of the Navier-Stocks Equations integrate to get the velocity. Here we have indicated twice to get an expression for the velocity we use the boundary conditions to obtain the expression for velocity. This is the way where, we consider the Navier-Stocks Equations to some of the analytical solution or exact solutions. So the case which we have seen is the plane Poiseuillie flow problem.

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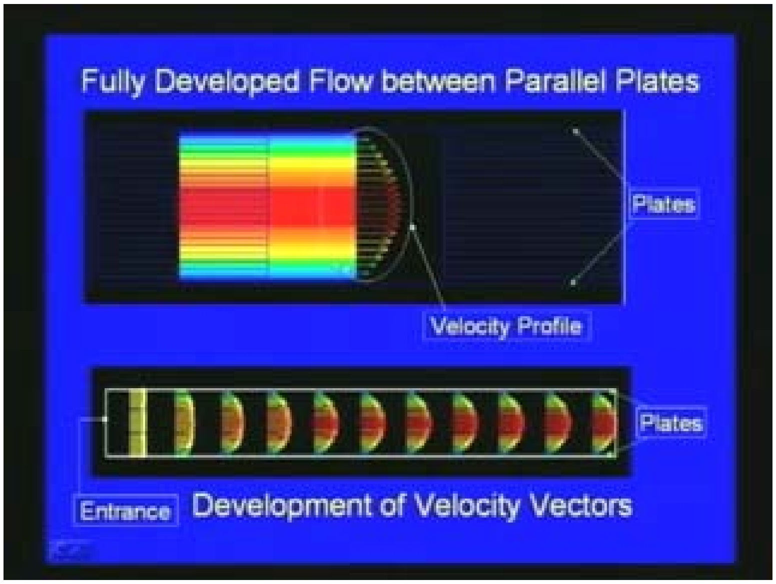

For this typical problem here to fixed plates and the flow is between them we can see that, the flow develop for the fully developed flow, the flow developed like this and the entrance is here and the flow; we can see that, the flow develops like this. The velocity vectors are portrayed here the maximum velocity is here and the velocity develops in this pattern. This is the case of flow between parallel plates; we have analyzed using the Navier-Stocks Equations. Now, the second case what we have seen is the flow between two fixed parallel plates the second case what we consider here is called the plane Couette flow.

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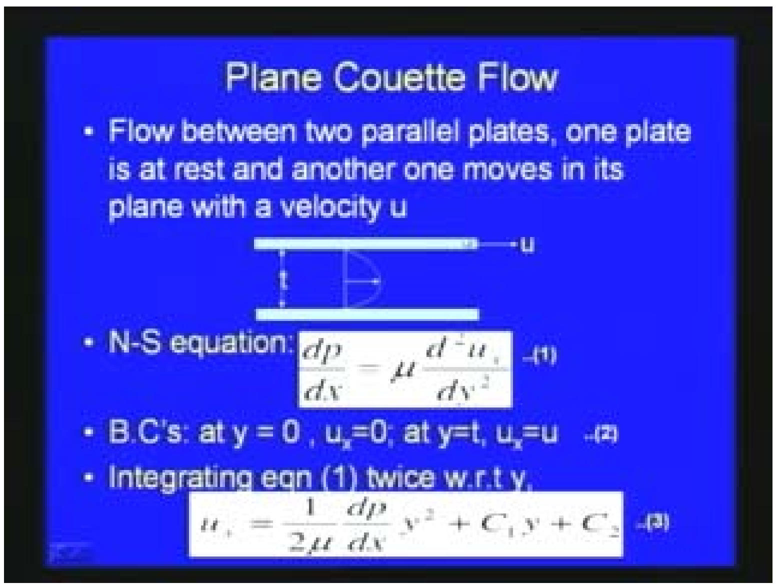

Here in this second case, the coquette, the two plates, one plate is bottom. This bottom plate is fixed and the top plate is moving with velocity u for example if we consider here this is two parallel fixed plates. The bottom plate is fixed and the top plate is moving like this with velocity u. Here, the Navier-Stocks Equations almost all the assumptions which we used earlier or valid here also only difference is that the top plate moving with velocity u. Here, we consider the distance between the plates as t. The velocity with which the top plate is moving is u. The Navier-Stocks Equations becomes dp by dx is equal to mu into d square u_x by dy square as in equation number one. Here, the difference between the planes Poiseuillie flow is here the top plate is moving. The boundary conditions change. The boundary conditions which are used here for this case where, the plates are at a distance t at y is equal to 0 that means bottom here the bottom plate is fixed that the velocity is equal to 0, u_x is equal to 0. The top light is moving with velocity u at y is equal t u is equal to u_x. These are the boundary conditions. In a very similar way as we have seen for the plane Poiseuillie flow. Here also we can integrate this expression the Navier-Stocks the simplified form of the Navier-Stocks Equations twice to get an expression for velocity. We can write u_x is equal to if you integrate twice, we get the velocity u_x is equal to 1 by 2 mu dp by dx y square plus c_1 y plus c_2 where c_1 and c_2 are the constants of integration. Now we can we have two boundary conditions here u_x is equal to 0 at y is equal to 0 and y is equal to t u_x is equal to u.

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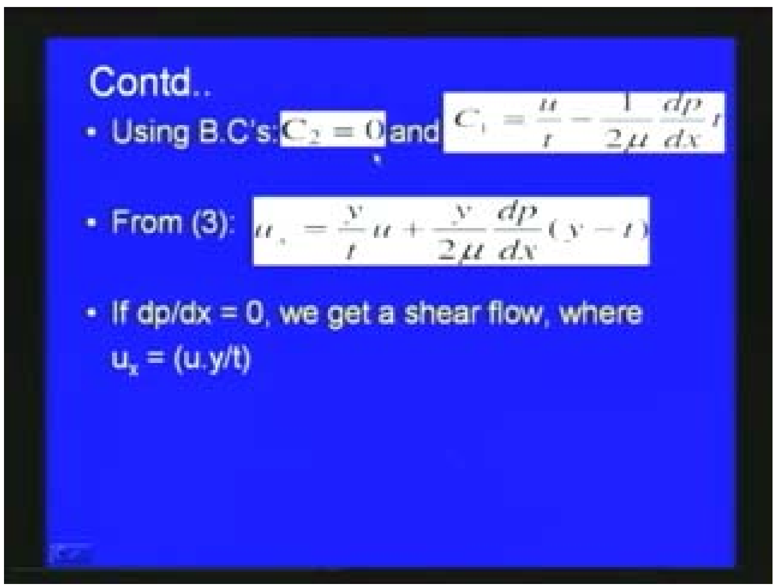

We can use two boundary conditions to get the unknown c_1 and c_2 by using the boundary conditions here we can see c_2 is equal to 0 and we get $c_1$is equal to u by t minus 1 by 2 mu dp by dx into t where t is the distance between these two plates. We get c_1 is equal to u by t minus 1 by 2 mu dp by dx into t. Now, we can substitute this c_2 and c_1 in the equation for velocity. We get: u_x is equal to y by t into u plus y by 2 mu dp by dx into y minus t this is the expression for velocity. If dp by dx this particular case in dp by dx is equal to 0 we get shear flow where we can just write u_x is equal to u into y by t. This typical case where the pressure gradient 0 is called shear flow this is a special case of Couette flow which we consider has got number of applications.

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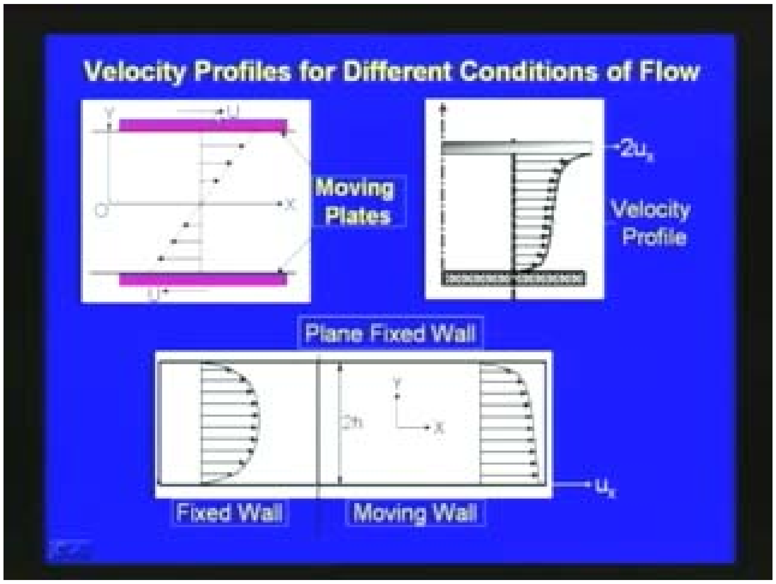

Depending upon the problem for this Couette flow you can see that, in this case we have already seen the plane Poiseuillie flow both plates are fixed. Some typical cases both places plates will be moving in opposite direction for example if we consider (47:35) the two plates like this. The plane Poiseuillie flow is two both the plates are fixed. Couette flow we consider one plate is moving like this another case which we consider is same. Both plates are moving opposite direction like this. This is another case which we can consider both plates are moving in opposite direction. Here again the assumptions are same the only boundary conditions, the top plate is will be having one velocity in one direction and the bottom plate will be having another velocity in the opposite direction.

Depending upon the case then here for various cases the velocities are profiles. Here if you consider this plate is moving with top this plate moving with velocity u and velocity bottom plate is move into opposite direction with the velocity. Then, we may get depending upon the case the velocity variations like this the bottom plate is fixed and the top plate is moving with velocity two times $2u_x$. Then you can see the velocity variations can be like this. This is the case where the plane Poiseuillie flows where both plates are fixed.

The bottom plate it is moving like this then, the velocity variations can be like this. Depending upon the problem either it can be plane Poiseuillie flow or Couette flow (49:06) both plates moving in the opposite directions. These are the various cases.

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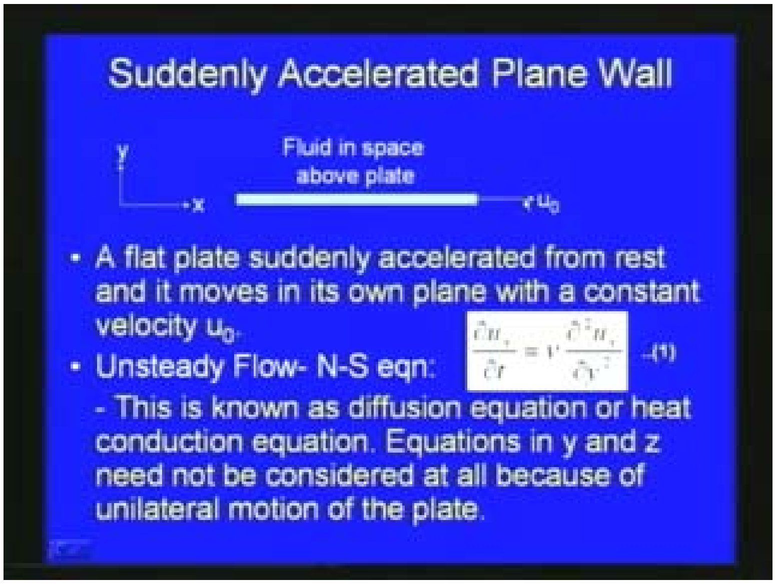

Now, you will consider another case where we can have an exact solution. It is called suddenly accelerator plane one here the Couette flow and plane Poiseuillie flow we consider the flow as the steady state. Here, we consider the case where unsteady problem. Here, the problem statement is like this: The plate is here. A flat plate suddenly accelerated from rest and it moves its own plane with constant velocity u 0. Here a plate is placed in fluid. The plate is suddenly accelerated from rest and it moves in its own plane with a constant velocity u_0. This is the problem. Compare to earlier case which we discuss here the change is from the rest the plate is suddenly moving with velocity u_0. This is an unsteady problem for this typical case, we can write the Navier-Stocks Equations which can be transformed and written like this del u_x by del t is equal to mu into del square u_x by del y square. Where, mu is kinematics velocity this gives for the typical problem this is the transformed form of the Navier-Stocks Equations. This equation is actually the diffusion equation or heat conduction equation equations in y and z need not be considered at all because of the unilateral motion of the plate.

Here we consider the equation only the plate is moving in this direction the fluid. We do not have to consider the y and z component and the Navier-Stocks Equations is the form of equation number one here. Now, we have to derive in analytical solution we are trying to derive an analytical solution with respect to this simplified form of this unsteady case of this particular problem.

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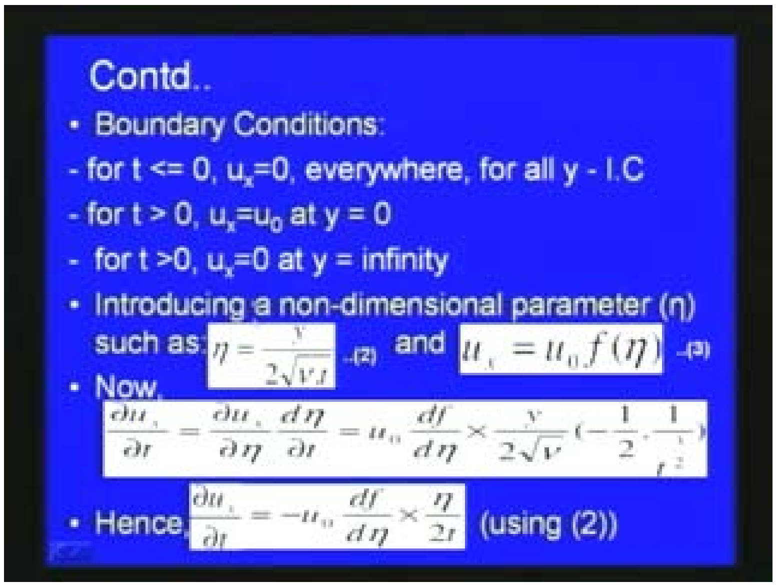

The boundary conditions are for t is less than or equal to 0 that means before the times start that means t is equal to 0 u_x is equal to 0. Since plate is addressed everywhere and also initial to condition also in the velocity component u_x is equal to 0. For t greater than 0 that means when this plate is starting to move t is greater than 0. We can write: u_x is equal to u_0 at y is equal to 0 and for t is greater than 0, we get u_x is equal to 0 at y is equal to infinitive that means here this problem at a large distance away from the plate you can see that velocity will be 0. Only at this location we have to consider t is greater than 0 u_x equal to u_0. Otherwise at large at infinitive at large distance at y is equal to infinitive u_x is equal to 0. Now, we use these boundary conditions to derive in a solution for this problem. This problem is we can see that here to parameter the independent variant t and y and the velocity is u_x. For this to derive an expression we will use some same techniques mathematical techniques here.

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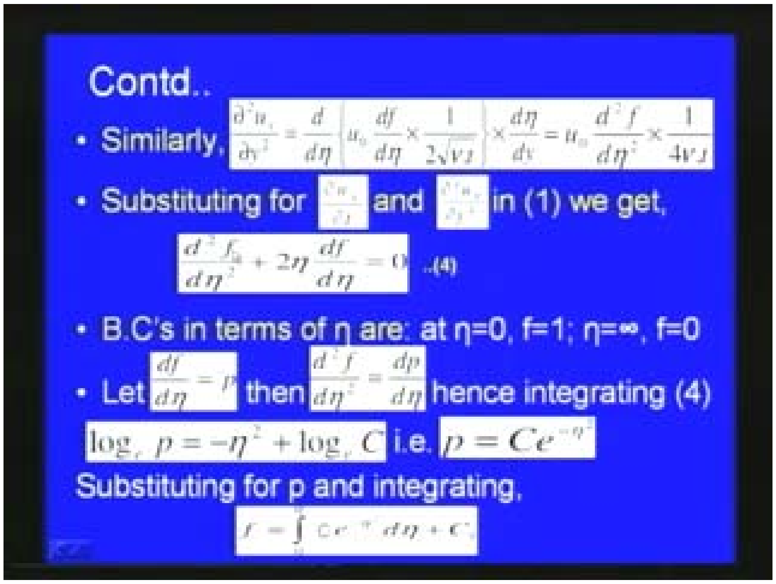

Here, we introduce a non-dimensional parameter eta such as eta is equal to y by 2 square root of mu into t where, t here is the time and mu is the kinematics viscosity. We can write the velocity variation u_x is equal to u_0 into a function f eta as in equation number three. We can write: del u_x by del t is equal to del u_x del eta into d eta by dt or d eta by dt. That we can write u_0 this is equal to u_0 into dm by d eta into y by two square root mu into minus half into 1 by t to the power 3 by 2. Hence we can write: del u_x by del t is equal to minus u_0 df by d eta into eta by 2 t and by using this as equation number two. Here, we are transforming the system in such way that, we are trying to get a solution for this simplified Navier-Stocks Equations by using these boundary conditions. Similarly, we can write: del square u_x by del y square is equal to d by d eta of into u_0 df by d eta into 1 by 1 square root of mu t into d eta by dy that is equal to u_0 d square f by d eta square into 1 by 4 mu t. We can now substitute for the approximations with respect to this introduction of this eta. We have written for del u_x by del t and del square u_x by del y square. We can put it back in equation number one, we get d square f by d eta square plus 2 eta df by d eta is equal to 0 as in equation number four. Correspondingly, with the boundary conditions in terms of eta are also we have to change at eta is equal to 0 we get f is equal to 1 and eta is equal to infinitive we get f is equal to 0. Here, in this equation number four we put df by d eta is equal to p then we get d square f by d eta square is equal to dp by d eta. Hence integrating this equation number four we can write $\log_e$ p is equal to minus eta square plus $\log_e$ C or we can write plus is equal to constant C. Here the C is a constant p is equal to c into e to the power minus eta square. Finally, we can substitute for p and we can integrate such that, we get f which will be equal to integral 0 to eta C into e to the power minus eta square d eta c_1. In this equation we have to find this c and c_1 we can use the boundary conditions

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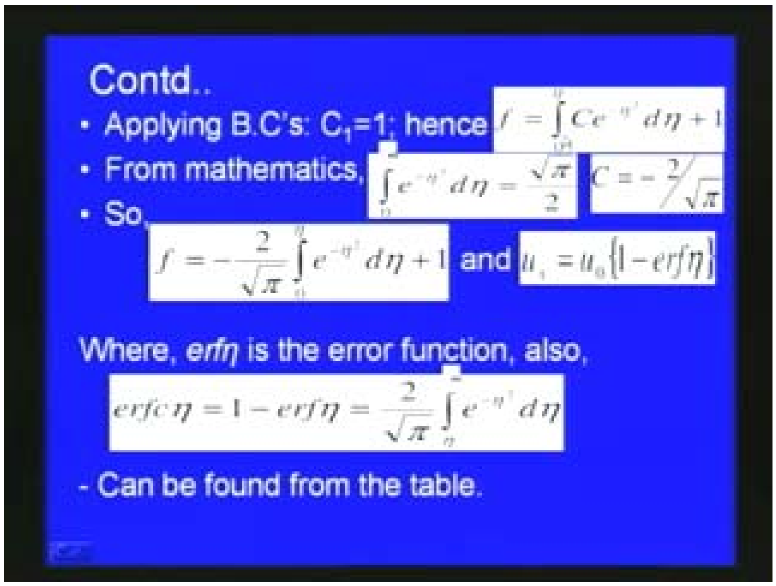

Applying the boundary conditions we will get C_1 is equal to 1 and hence f is equal to integral 0 to eta C into e to the power eta square d eta plus 1. From the mathematics, we can get integral 0 to infinitive e to the power eta square d eta can be written as root phi by 2 and C is equal to minus 2 root pi. Finally, we get f is equal to minus 2 by root pi integral 0 to eta e to the power eta square d eta plus 1 and u_x is equal to we get u 0 into one minus error function eta. This can be represent with respect to error function where erf eta is the error function and we can also write erfc eta the complement error function is equal to 1 minus erf eta that is equal to 2 by root pi integral eta to the power infinitive e to the power minus eta square d eta. Now, we can use some tables to use get this error function, we get a solution for the velocity like this. This is even a transient problem. We transform the Navier-Stocks Equations and finally we can use some mathematical techniques to get an expression for the velocity. Further we will discuss more cases on the exact solution of the Navier-Stocks Equations.