Fluid Mechanics

Prof. T. I. Eldho

Civil Engineering

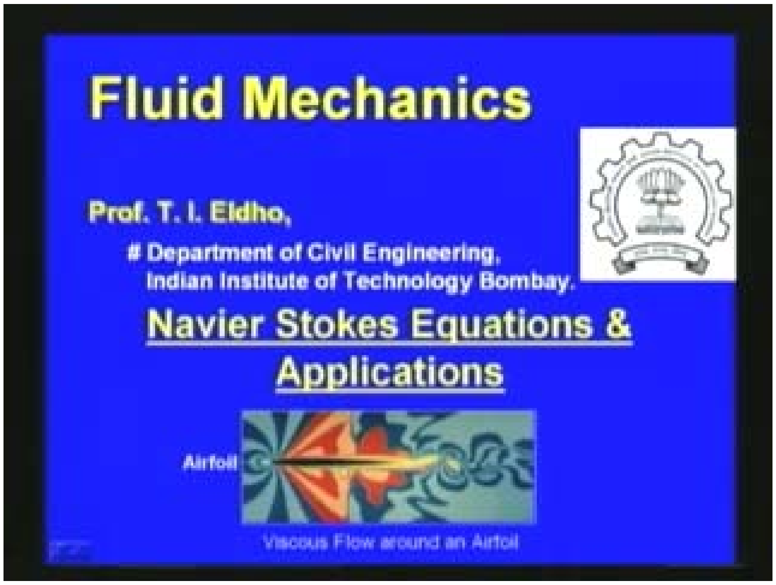
Indian Institute of Technology, Bombay

Lecture-26
Navier-Stokes Equations and Applications

Welcome back to the video course on fluid mechanics. Today we will start a new chapter which is one of the most important in fluid mechanics, Navier-Stokes Equations and Applications. We have seen various aspects of fluid mechanics is the last few lectures, starting from statics, kinematics, dynamics, full flow and then we discussed the dimension analysis and now in this chapter we will concentrate more upon the viscous flows.

As all of you know viscous flow is very important since most of the real flows we have to consider viscous flows and we have to consider the viscous effect in the derivation of the basic equations and then we have to get the solutions according to by considering the viscous effects.

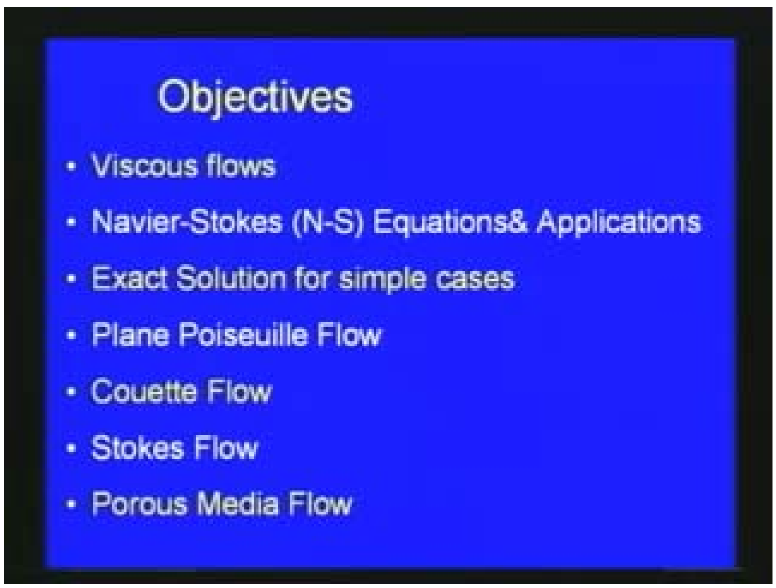
(Refer Slide Time 02:30)



As you can see here in this slide, this is flow over an airfoil, we can see that say, since the fluid passing over the airfoil is viscous then the flow pattern is totally different, what we considered earlier like when we considered the added fluid flow over the potential flow then here if the fluid flow is viscous, the effect is much more different. The way of approach is we have to consider viscous flows; we have to consider the shear stress, all these we have to consider and then we have to consider the applications accordingly.

So the objectives of this section of this chapter on Navier-Stokes equations and applications, first we will see the viscous flows.

(Refer Slide Time 03:12)

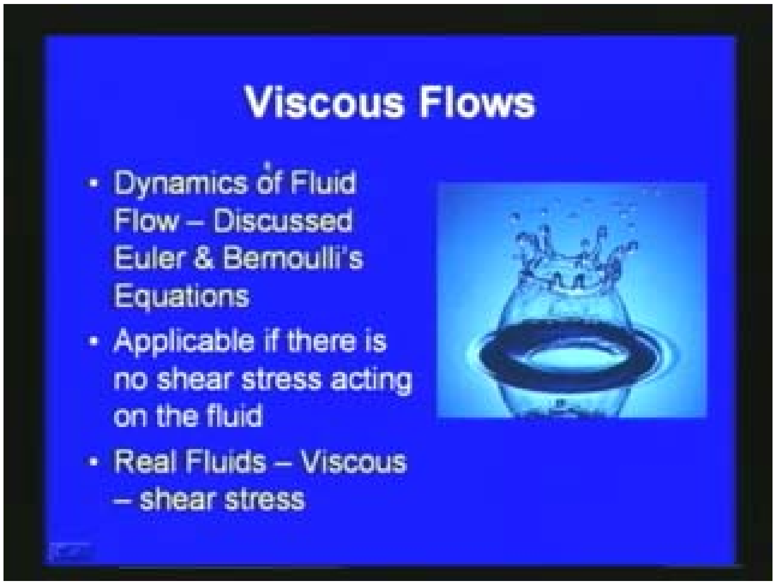


The importance of viscous flow and then how generally we will be trying to solve the viscous flows; and then we go to the Navier-Stokes equations, we will derive the Navier-Stokes equations and we will discuss briefly the applications of this Navier-Stokes equations; and then we will discuss the exact solution for some of the simple cases; and also we will be discussing the Plane Poiseuille flow; flow like Couette flow; stokes flow; and Porous media flow.

So the objectives of this chapter includes say the deliberation on the Navier-Stokes equations, its derivations and the applications; and exact solutions available and further studies on this.

Now as I mentioned, viscous flow most of the real fluid flows are viscous, so that we are to consider the shear stress. So the dynamics of fluid flow which we considered earlier we have derived the Euler equation and the Bernoulli's equations, there we neglected the effect of viscosity.

(Refer Slide Time 04:33)



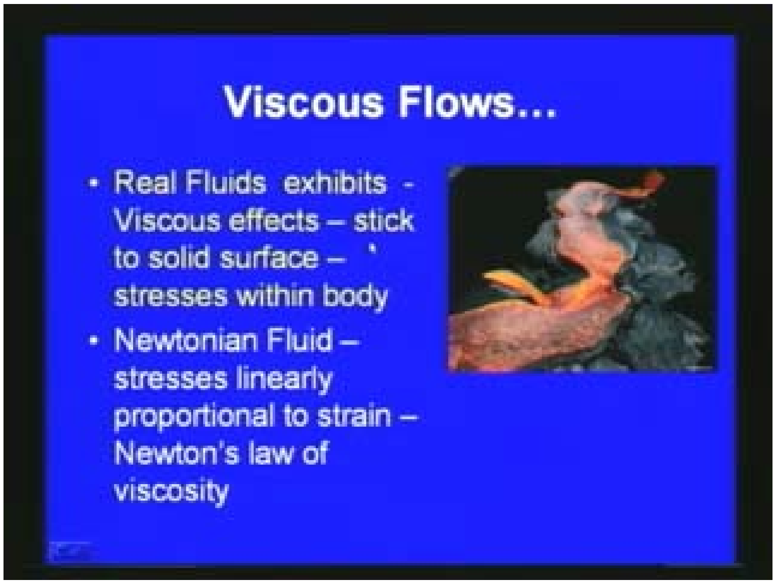
So the derivations of the equations are much simpler and the solutions we can easily get. That is what we are seen as far as the Euler's equations and the Bernoulli's equations are considered. And even though these equations are derived for the simple case of adil fluid flow or potential flows but still we try to use these equations with certain simplifications for the real cases. That is what we have seen in the lectures when we discussed the Euler and Bernoulli's equations.

But we can see that, here either we are approximating that the viscosity is negligible, so that the shear stress acting on the fluid is not much and then we derived the equations. But that weight is an approximation and the accuracy of the solution will be affected by if you use Euler equation or Bernoulli's equations, but when we deal with the real fluids the viscosity is much more important and the shear stresses play a major role.

Now when we consider this, the viscous flows, for all most of the real flows which we deal in all the aspects of life is actually the viscosity effect is there, we have to consider the viscosity.

So the real fluids exhibit viscous effects and then most of the cases this stick to the solid surface.

(Refer Slide Time 06:03)



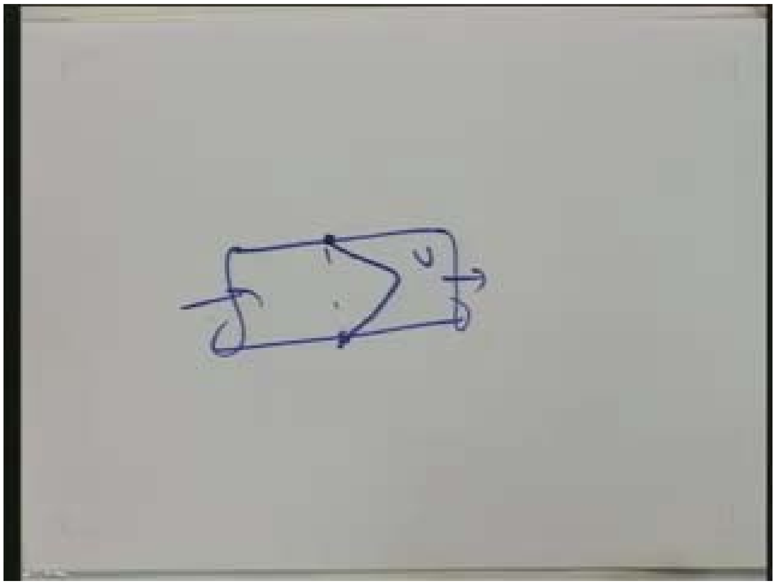
When we put it say in water or in say oil, when we put the spoon and then take it out you can see that some fluid stick to the solid surface, so there is stresses within the body. So we have to consider this effect.

Here also you can see that in volcano when the lava is flowing, we can see that it is highly viscous fluid is flowing and then we have to consider the viscosity effect and then lot of parameters change. When we consider the Newtonian fluid, we can consider most of the time the stresses vary linearly with respect to strain so we can derive the Newton's law of viscosity. Newton's law viscosity is very much used in all the derivations of the viscous flows.

And then of course when we consider say when a fluid is contained in a container or in pipe or whatever it is, then the fluid will be sticking to the solid surface so that we have to consider the nocil condition. So we can see that if you consider here say the fluid is, if you consider the pipe flow, as far as the pipe flow is concerned, say the fluid it is coming like this then we can see that here the pipe is solid surface and then so here the velocity of this 0 on this surface and on this surface. So we can see that the velocity variation is like this, so this is the velocity. Here due to the mostly condition on the boundary, so here the pipe flow is concerned this side and this side or throughout the periphery of the pipe we can see that the velocity is 0 and the velocity be maximum.

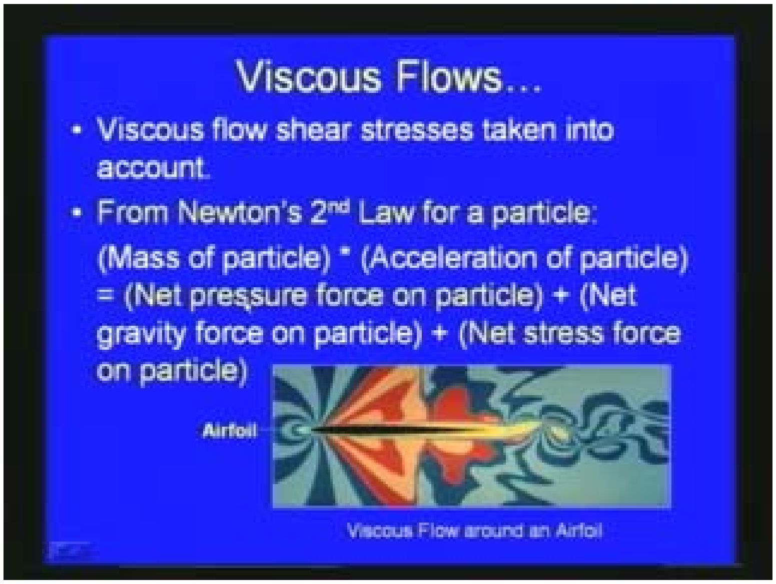
So viscous through it concerned we have to consider this nosilic condition as far as the with respect to when it is contained in the in a container or in a pipe as we have seen.

(Refer Slide Time 08:03)



So viscous effect is very important and the corresponding shear stress we have to consider. Viscous flows the shear stress are taken into account and then generally as we have seen in most of the other cases we consider Newton’s second law so that we can write the mass of the particle multiplied by the acceleration of particle that will be equal to, we can equate the adequate sum of the forces.

(Refer Slide Time 08:39)



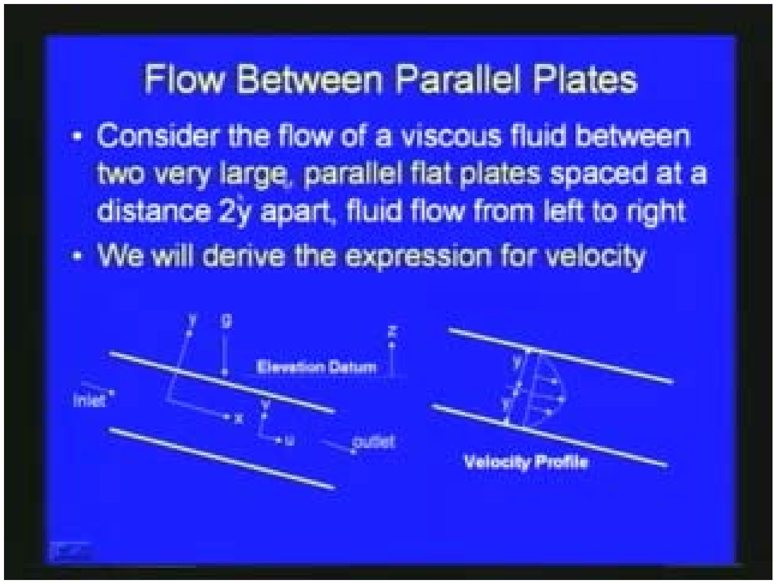
So here if you consider any particular case for viscous flow is concerned, the various forces will be net pressure force on particle, net gravity force on the particle, net stress force on particle. So if you consider the viscous flows and then using the Newton's second law with respective to a particle we can write, the mass of the particle multiplied by the acceleration of the particle that will be equal to net pressure force on particle plus net gravity force on particle plus net stress force on particle. So this stress force can be shear stress or normal stress for the pressure force depending upon the case.

This generally for simple cases we consider like this the Newton's second law for a particle and then we try to drive remaining equation for that particular problem. It is difficult to drive for each individual case like this so we can drive a generalized equation. These generalized equations are called Cauchy's equations and the Navier-Stokes Equations.

So before discussing the generalized equation for viscous flows, we will see a particular case how we are deriving Gavinian equation with respect to simple case by considering the Newton's second law.

So let us consider the flow between two parallel plates. This earlier also we have discussed but with respect to the viscous flows now what we have considered earlier in the laminar flow, now again here we are discussing when we deal with viscous flow how we can apply the Newton's second law and directly derive the relationship for velocity for various other parameters.

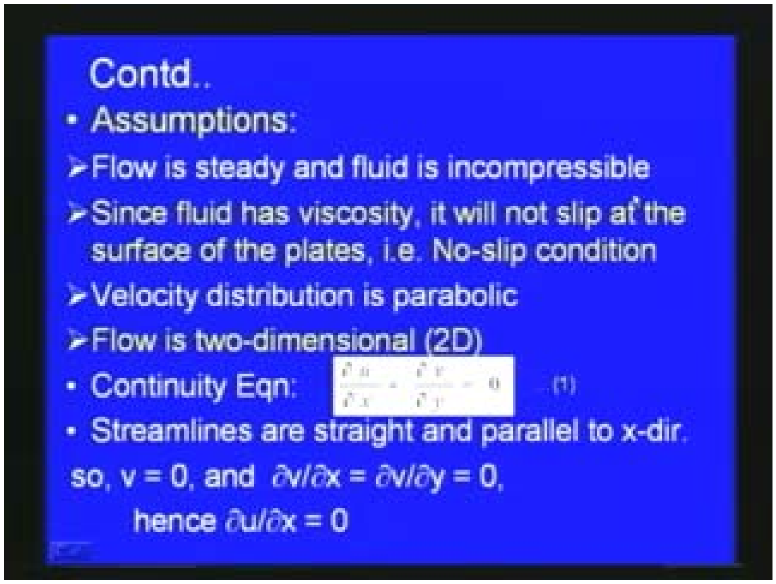
(Refer Slide Time 10:52)



So here in this slide we can see that we consider the flow of viscous between two very large parallel flat plates placed at a distance 2y apart. So we can see here these two plates this plates at 2y at a distance of 2y between them and then the flow is taking place from left to right like this so this is a inlet of a flow here is the outlet of the flow. So we consider the axis at the centre of the of the plate between the plate, so here it is origin is here X direction is here Y direction is here and then acceleration due to gravity we have to be consider.

So the elevation datum is here, as we have already seen in this case also due to the nocilic condition the velocity variation will be like this, the parabolic variations to maximum here at the center line and minimum of 0 on the plates. So here what we are trying to do? Here we consider the viscous flow; we derive the expression for velocity for this particular case for laminar flow. Before considering before deriving the equations we will be using number of assumptions so that we can use the Newton's second law.

(Refer Slide Time 12:17)



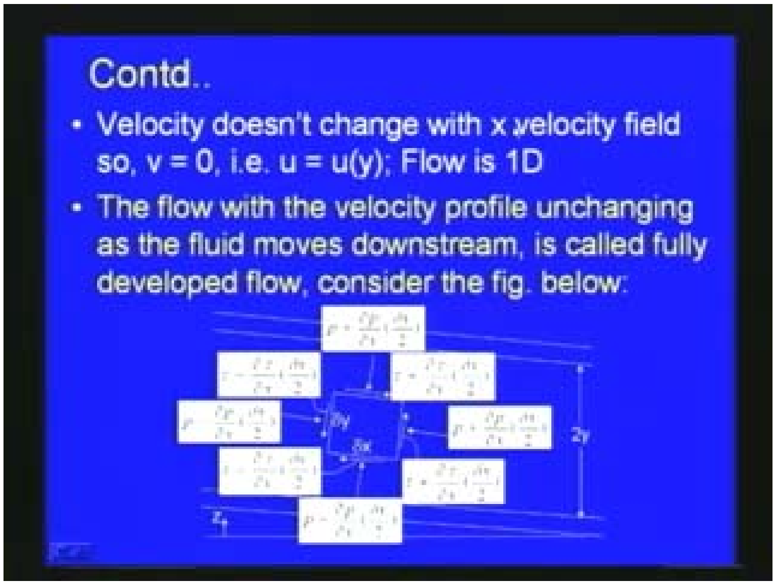
Here the assumptions used are the flow is steady and fluid is incompressible where as the time dependent variables you can neglect since flow is steady and the density variation is not there since flow is fluid is incompressible. Second assumption is fluid has viscosity it will not slip at the surface of the plates that means we consider the no-slip condition. And as we have seen for this particular case velocity distribution is parabolic.

In this case the flow is two dimensional. So we can use the continuity equation which we have derived earlier in the differential form, in the continuity equation u and v are the velocity components in X and Y direction, for this case we can write del u by del x plus del u by del y is equal to 0 as in equation number 1.

Due to the special nature of this particular problem, we can say that streamlines are straight and parallel to the X direction as you can see in this figure. Now here this particular problem is concerned, this velocity v is equal to 0, so that del v by del x is equal to 0 and del v by del y is equal to 0. So by using this condition here in the continuity equation we can write del v by del x is also equal to 0. That means this is a parallel flow which we consider here with respect to this plate so flow between the parallel plates, so v component is 0, so we consider only the velocity in the X direction u.

So now our aim is to derive the Gavinian equation for velocity or other parameters for this particular problem. As we were using earlier we can consider a small fluid mass and then we will consider what are the forces acting on that and then we will be applying the Newton's second law.

(Refer Slide Time 14:33)



Here since velocity does not change with X velocity field so we got v is equal to 0, that means we can see here the velocity is varying with respect to y only, so effectively this case is a one dimensional flow. Now we will consider the flow with the velocity profile unchanging as the fluid moves downstream. This we can say this particular case the flow is fully developed flow and then we consider a small flow particle of size delta X by delta Y like this and then what are the various forces active on this particle or this small volume of the fluid?

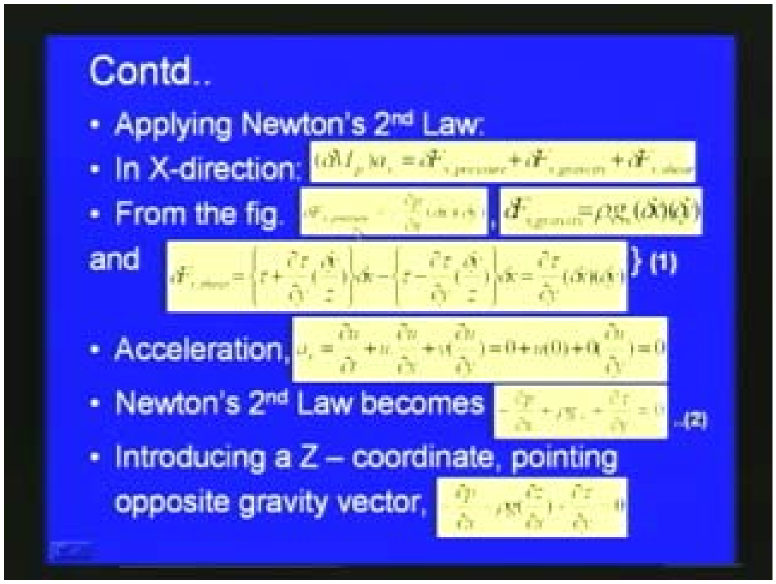
So here this particular case is concerned, we can see that the various forces acting here are mainly the pressure force, so there is a pressure variations that is why the flow is from left to right and then since we consider the viscous flow, so the shear stress we have to consider here. Here if you consider the pressure flow on this plates of this fluids mass, so here it p is the pressure then this face we can write p minus del p by del s into delta x by 2, and on this face this p plus del p by del s into delta x by 2, and on this face it is p plus del p by del y into delta y by 2, and the corresponding other side it is p minus del p by del y into delta y by 2. So these are the pressure forces acting on this fluid mass.

And then shear forces are due to the shear to force we can write correspondingly, we can write on this face it will be tow the shear stress tow minus del tow by del x into delta x by 2 and correspondingly, this side we can see it is tow plus del tow. So this is on this face actually here tow minus delta by delta x delta x by 2, this face and this face it is tow plus delta by del x into delta x by 2 and similar way on this face the shear stress is tow minus delta by del y into delta y by 2 and here tow plus delta by del y into delta y by 2.

So for this particular case the forces acting are the pressure forces on the fluid mass for the fluid the say delta y by delta is the mass of fluid which is consider and then the shear forces for the corresponding shear stresses on the size of the fluid mass which we consider. So now here our aim is to drive the expression for the velocity variation. Here we are using the Newton's second law.

In x direction if we consider, so here the mass into acceleration, so delta M_p if that is the mass for this particular fluid element which we consider so delta M_p into $a_{x,}$ a_x is the acceleration in the x direction, so that is equal to delta $F_{x,pressure}$ the pressure component the x direction plus delta $F_{x,gravity}$ the gravity force components and delta $F_{x,shear}$ the shear force component.

(Refer Slide Time 18:14)



So from the figure the earlier figure so here we can see that, the pressure component in x direction we can get us delta $F_{x,pressure}$ is equal to minus del p by del x into delta x into delta y and similarly the gravity force is concerned so here the shear force as well as the pressure force and then of course the gravity force also we have to consider for this problem.

So gravity force is concerned delta $F_{x,gravity}$ we can write test rho into gx into delta x into delta y where gx is the acceleration due to gravity in the x direction and then the shear force is concerned we can write this as in the x direction, we can write as the difference between tow plus del tow by del y into delta y by 2 into delta x minus tow minus delta y by del y in delta y by 2 into delta x, so that we can write as delta y delta by into delta x into delta y. So this gives the shear force.

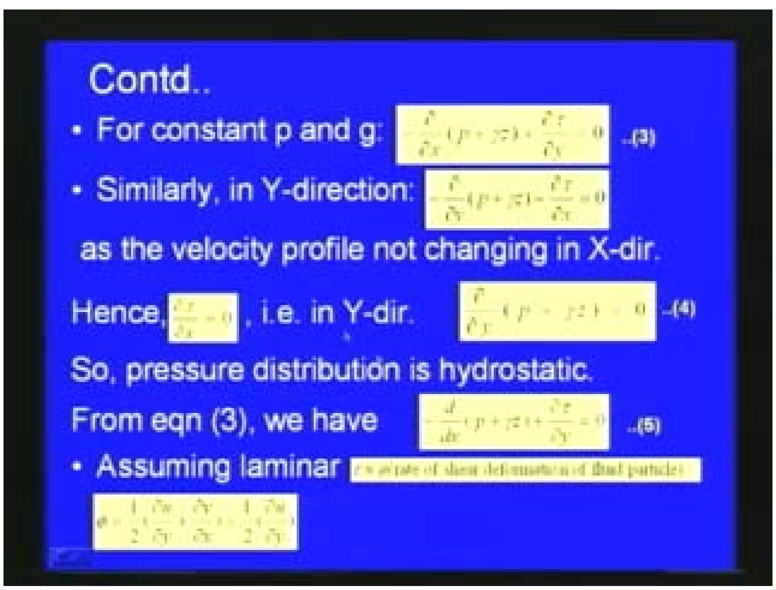
And then the acceleration is concerned this as far as the fluid element this concerned acceleration x is equal to del u by del t plus u into del u by del x plus v into del u by del y, so here since we consider as steady state flow del u by del t is 0 and this del u by del x we are already seen for this parallel flow it is 0 and then v component v is 0. So you can see that finally the acceleration is equal to 0.

Now if you substitute in these equations since the left hand side here mass into acceleration, acceleration is already 0, so we can see this becomes 0 and then we can just equate the other by some of the forcers, so now the pressure force we can write as del p by del x into delta x into delta y; and gravity force rho into gx into delta x into delta y plus the shear force delta by del y into delta x by delta y. So we can add all this and then if you use the Newton's second law we can write minus del p by del x plus rho gx plus delta by del y is equal to 0. So this is coming from the Newton's second law.

Now here we introduce, we assume the z coordinates pointing opposite to gravity vector so that we can just approximate this rho gx in this fashion. Here we can write equation number 2 as del p by del x minus rho g this x direction with respect to z we can write us del z by del x so minus rho g del z by del x plus del tow by del y is equal to 0. This equation 2 is transformed this way and then here for constant p and g, so the pressure and acceleration due to gravity we can write equation number 2 as minus del by del x p plus gamma z plus delta y del y is equal to 0 as in equation number 3.

So in a very similar way in by direction we can write us minus del by del y p plus gamma z plus delta y by del z is equal to 0.

(Refer Slide Time 21:11)



Now here we get an expression with respect to the shear stress, so we can use Newton's law viscosity to convert to get in terms of velocity. So here now the velocity profile is not changing in x direction as we are seen, so hence del tow by del x is equal to 0 that means in y direction is concerned, this equation becomes del by del y of p plus gamma z is equal to 0. So pressure distribution is from this equation number 4, we get the pressure distribution is hydrostatic.

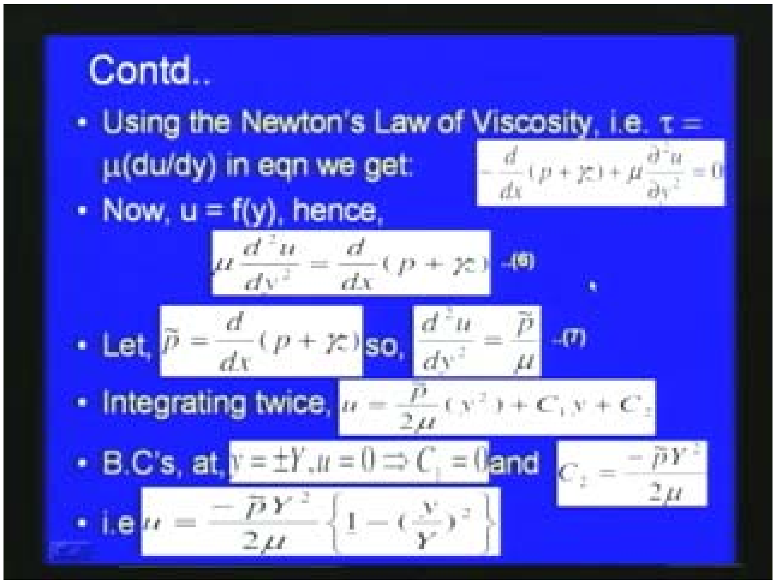
So now as far as equation 3 is concerned this we can write as so now this pressure is varying only with respect to x so we can write minus d by dx of p plus gamma z and then plus del tow by del y is equal to 0 as in equation number 5.

So here if you are assuming the flow is laminar that we can write the rate of shear deformation of plate particle, tow is approximately equal to rate of shear deformation of fluid particle that means the shear stress is approximately equal to rate of shear deformations, so that this phi dash that is equal to half del u by del x plus del v by del x so here we can see that this becomes del u by del x is 0, so that this become phi dash is equal to half del u by del y, so that we can write tow is equal to mu du by dy which is also coming from the Newton's law of viscosity.

So now directly we can write this from the Newton's law of viscosity and substitute here so that we will get from this equation if you substitute equation number 5, if you substitute for the Newton's law of viscosity tow is equal to mu du by dy finally we get minus d by dx plus, p plus gamma z plus mu del square u by del y square is equal to 0.

So now finally we get the velocity variation as a function of y so that we can write mu d square u by dy square is equal to d by dx of p plus gamma z. So here in this particular case if you write p tilta is equal to d by dx of p plus gamma z, so that we can write d square u by dy square is equal to p tilta by mu, as in equation number 7.

(Refer Slide Time 23:28)



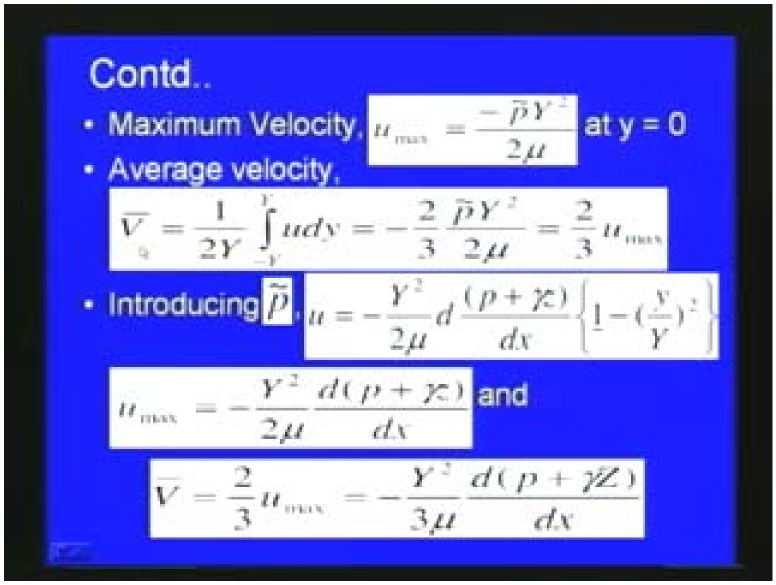
So we finally get an expression for velocity in terms of pressure, so we can integrate twice this expression d square u by dy square, so that we will get a velocity u is equal to p tilta by 2 mu y square plus C_1 y plus C_2. So integrate twice we get this expression. So now for this particular case we have a boundary condition so the flow is between two parallel plates, on the top as well as bottom where the fluid touches the plates, there we can assume the velocity is 0, so that we will get the boundary condition to boundary condition. So the boundary conditions here are y is equal to plus or minus y u is equal to 0, so we get C_1 is equal to 0 and finally we get C_2 is equal to minus p delta y square by 2 mu.

So finally we get an expression mu is equal to minus p delta y square by 2 mu into 1 minus y by y whole square. So here now with respect to this figure (Refer Slide Time 10:52) we can see so here we get a velocity variation is parabolic as now we have derived. So we have applied the boundary conditions I am using the Newton's law of viscosity to get an expression for these velocity variations, so that is what we did here.

So here in this particular case we consider the origin as on the central line of the between the plates, so we get an expression for u as minus p tilta y square by 2 mu into 1 minus small y by y, so this y gives the variation.

So whether between the plates, so the y is starting from the central line between the plates. So this is the expression for the velocity. Now for this particular case we know that the velocity is maximum at the center so y is equal to 0 we can get the maximum velocity u_{max} is equal to minus p tilta y square by 2 mu.

(Refer Slide Time 26:53)



So this is the maximum velocity and then if you want to find out the average velocity V bar is equal to 1 by 2 Y integral minus Y to Y within the limit u dy that is equal to we integrate we can write minus 2 by 3 p tilta Y square by 2 mu, so that we can show that this will be an average velocity will be two third of the maximum velocity.

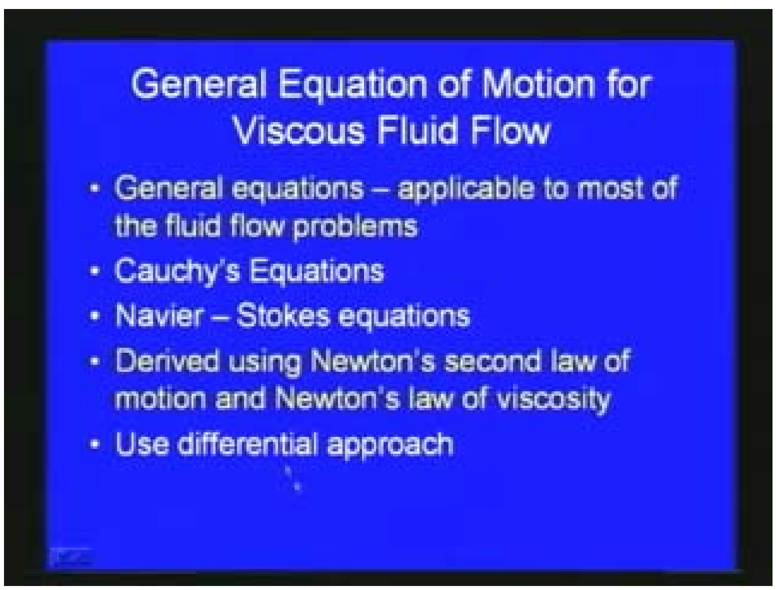
So now in this expression we can put back with respect to the pressure variations so p tilta we can write as with respect to d by dx of p plus gamma z so that is equal to u is equal to minus y square by 2 mu d of p plus gamma z by dx into 1 minus y by whole square. So this is the final expression and u_{max} it can be written as minus Y square by 2 mu of d p plus gamma z by dx and b bar is two third u_{max} as we have already shown that is equal to minus y square 3 mu d of p plus gamma z by dx.

So like this we can derive various parameters so here the aim of this deliberation here is for viscous flow we consider the viscous effect that means the shear force and the pressure force and also the gravity force are considered and for this simple case of flow between parallel plates which is one dimensional to parallel flow there we have derived the equation for velocity variation. So by here we use the Newton's second law to derive the equation.

In a very similar way for various simple cases as demonstrated earlier in the case when we discuss the laminar flow problems we can derive various expressions for simple simplified cases. Now in the very similar way our aim here is to derive the fundamental equations for the viscous flow.

Here what we are going to discuss further is we are deriving the fundamental equations which we called Navier-Stokes Equations or the first form of the equation Cauchy's equation and then say later Navier and Strokes there modified this equations and called as Navier-Stokes Equations. So our aim here is to derive the equations which are generally applicable to any kinds of problems, applicable to most of the fluid flow problems.

(Refer Slide Time 29:37)



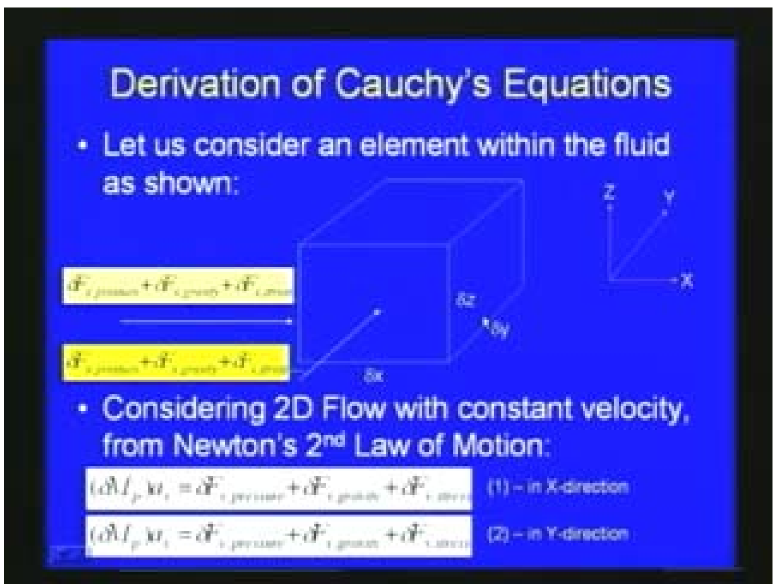
First, we will derive Cauchy's equation the Cauchy's equations are applicable for all kinds of problems compressible and incompressible or work varieties of fluid flow problems and then we will discuss especially we consider the incompressible flow, from the Cauchy's equations we will derive the Navier-Stokes Equation. Here as I mentioned as we have already discussed for the flow between parallel plates here we will use the

Newton's second law and the Newton's law of viscosity by using the differential approach we will derive the basic equations.

So first we will derive the Cauchy's equations and then based upon the Cauchy's equation we will derive the Navier-Stokes Equations. So here the fluid flow problems which we generally consider are three dimensional in nature, but here to derive the equations we are using two dimensional flow so that it will be much easier to understand.

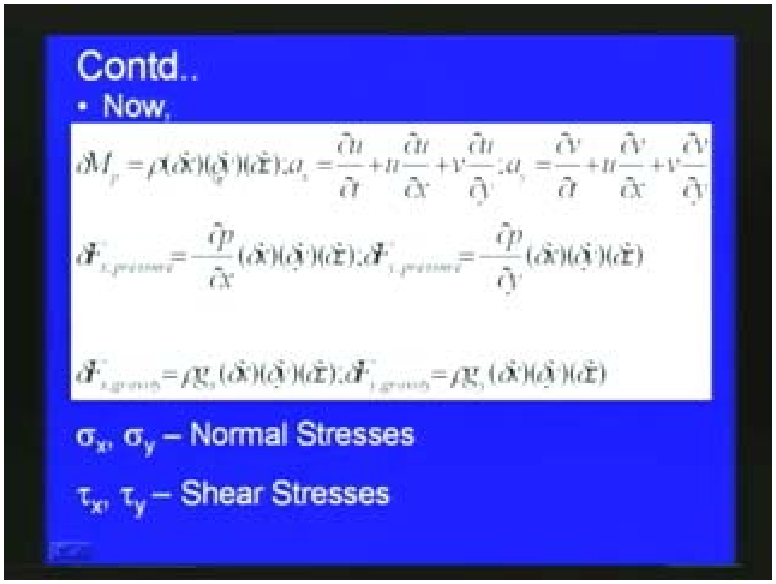
The steps involve for three dimensional flow equations are also same but here we explain as far as 2-D flow, two dimensional flow is concerned and then we will discuss the three dimension equations but the derivation is for two dimensional flow.

(Refer Slide Time 31:40)



Now let us consider here we derive the Cauchy's equation, so let us consider the element within the fluid as shown here. Here a fluid element of size delta X delta Y and delta Z we are considering. The various forces as we are seen earlier the forces are the pressure force. If you consider this fluid element delta $F_{x,pressure}$ and delta $F_{x,gravity}$ the gravity force and delta $F_{x,stress}$ forces, the stress can be normal stress or shear stresses.

(Refer Slide Time 33:16)



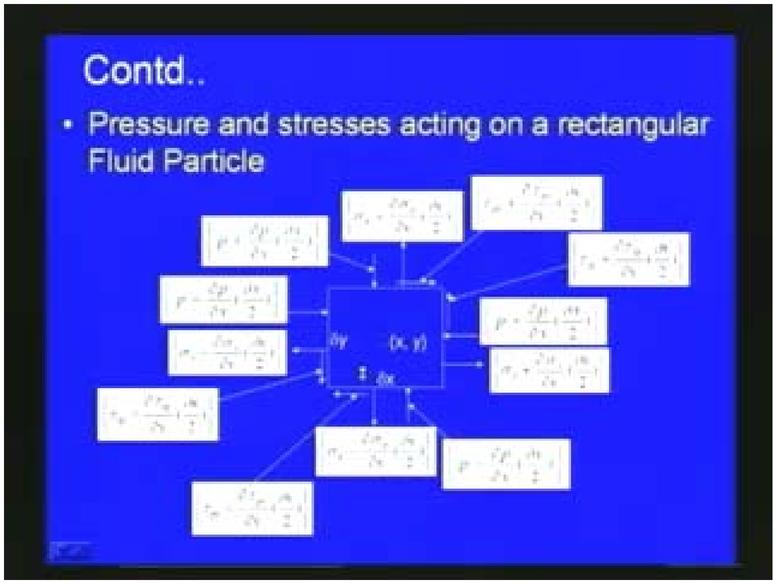
Now either this face or the with respect to the X direction or Y direction we have to consider the pressure force gravity force and the stress force. If we consider two dimensional flow with constant velocity if we use Newton's second law of motion as we have as discussed earlier, here for this flow problem we can write the Newton's second law mass into acceleration is equal to the average of sum of the forces.

So here for this fluid element which we consider the mass is delta M_p and then if acceleration x direction is a_x then we can write delta M_p into a_x is equal to delta $F_{x,pressure}$ plus delta $F_{x,gravity}$ plus $F_{x,stress.}$ It is written in this equation number 1.

So in x direction delta M_p a_x is equal to delta $F_{x,pressure}$ plus delta $F_{x,gravity}$ plus delta $F_{x,stress}$ So in the similar way in y direction we can write delta Mp into a_y acceleration into y direction that is equal to delta $F_{y,pressure}$ delta $F_{y,gravity}$ plus delta $F_{x,stress}$. So now as far as the mass of the fluid is concerned, fluid element is concerned, if rho is the density and here we consider fluid element as size delta X and delta Y and delta Z size.

So mass is equal to delta Mp is equal to rho into delta X into delta Y into delta Z and then acceleration is concerned the local acceleration and convergent acceleration we consider. So in two dimensional problem which we consider here a_x is equal to del u by del t plus u into del u by del x plus v into del u by del y and a by is equal to del u by del t plus u into del u by del x plus v into del v by del y. And then the pressure force here for this typical problem here we can write the pressure force here it is explained further here this fluid element is delta X by delta Y.

(Refer Slide time 33:57)



The various pressure and stresses acting on a rectangular fluid particle are written here, so here the pressure force is in this direction p minus del p by del x into delta X by 2, and the other direction p plus del p by del x into delta X by 2 and here this direction the pressure force is p plus del p by del y into delta y by 2 and here this direction pressure forces p minus del p by del y into delta y by 2.

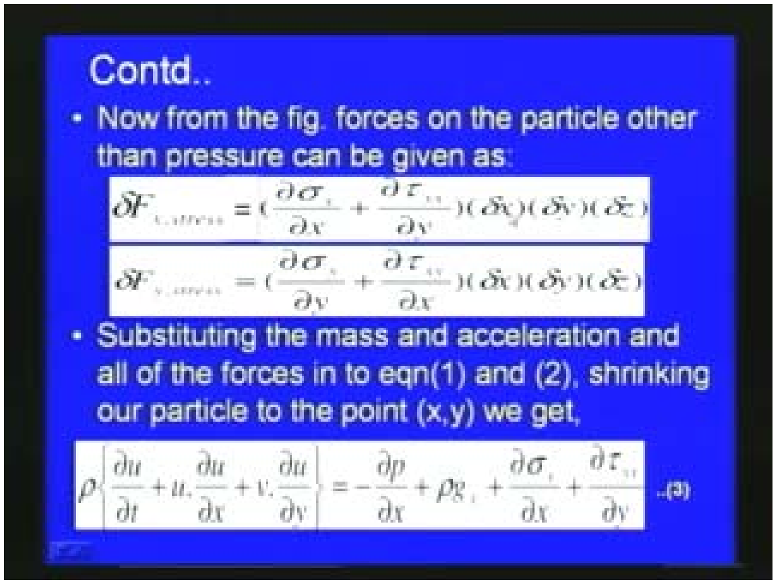
And then if you consider the normal stress force, so here this face it is sigma x minus del sigma x by del x into delta x by 2 and here this phase it is sigma plus del sigma x into delta x so delta x by 2 and on this face sigma y minus del sigma y by del x into del y by 2 and on this face sigma y plus del sigma y by del x into del y by 2 and similar way these shear stress components, so here in this direction it is tow_{yx} plus del tow_{yx} by del y into delta y by 2 and on this face tow_{yx} minus delta yx by del y into delta y by 2 and on this face it is tow_{xy} minus del tow_{xy} by del y delta x by 2 and on this face tow_{x} by plus del tow_{xy} by del y into delta x by 2. These are the pressure and stresses acting on the rectangular fluid element.

Now if we consider the net pressure force we can write here del $F_{x,pressure}$ is equal to del p by minus del p by del x so delta x into delta y into delta z we consider the fluid element thickness have delta z and delta fy in the y direction pressure force is concerned minus del p by del y into delta x into del y del z and similarly the gravity force on the fluid element del $F_{x,gravity}$ is rho gx delta x delta y delta z and del $F_{y,gravity}$ is rho gy delta x delta y delta z.

So here now the stresses are the normal stresses and the shear stresses. So depending upon the cases for the fluid which we considered definitely shear stresses are there, normal stress whether depending upon the problem we consider. So now all these stresses are shown here. By considering the forces on the particle other than the pressure, so del $F_{x,stress}$ with respect to the normal stress and shear stress we can write as with respect to the fluid element here we can write, del sigma x by del x plus del tow_{yx} del y del y into delta x delta y delta z.

So that is the net stress force in the x direction and similarly the net stress force in y direction can make as del sigma y by del y plus del tow_{xy} by del x into delta x delta y delta z. So now whole the force components the stress forces regarding normal stresses and the shear stresses and the pressure forces and the gravity forces are known and we have already seen the mass of the fluid element and the acceleration. So now we can use Newton's second law to substitute all these components to get the final form of the equations.

(Refer Slide Time 37:08)

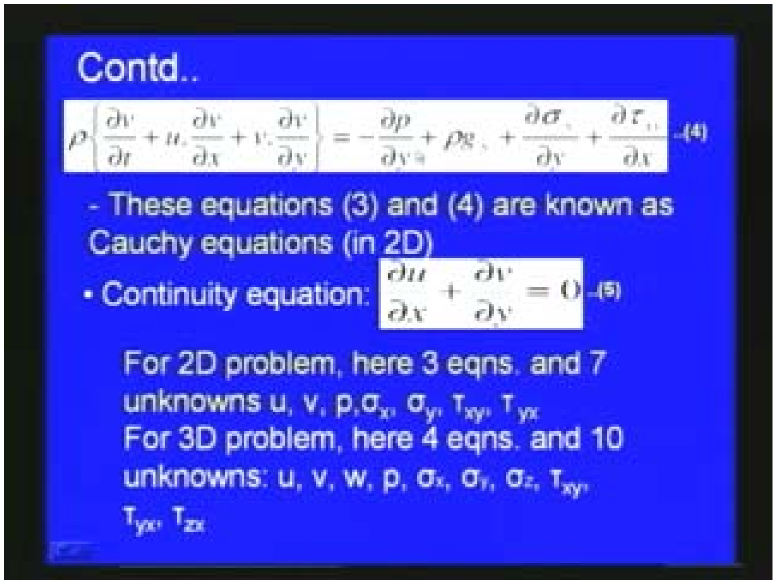


Now substituting the mass and acceleration and all of the forces into equation 1 and 2 that means in two dimensions equations we are writing the equations for two dimension cases, so equation 1 and 2 and then shrinking our particle to the point x y here, (Refer Slide Time 31:40) this figure shows we consider this equation 1 and 2 and then we consist particular point in the differential approach so we can get rho into del u by del t plus u into del u by del x plus v into del u by del y is equal to minus del p by del x plus rho gx plus del sigma x by del x plus del tow$_x$ by del y equation number 3.

So this is the final form of the equation and then in y direction we get rho into del v by del t plus u into del v by del x plus v into del v by del y is equal to minus del p by del y plus rho gy plus del sigma y by del y plus del tow$_x$ by del x equation number 4.

So this equation number 3 and 4 are the basic equations as far as viscous flows are concerned and these equations are called Cauchy's equations. So these equations are two dimensional equations, we got two equations from the Newton's second law of motion. So we got two equations. So with these two equations in two dimensional case we can supplement with respect to the continuity equation, the continuity equation gives del u by del x plus del u by del y is equal to 0.

(Refer Slide Time 38:58)

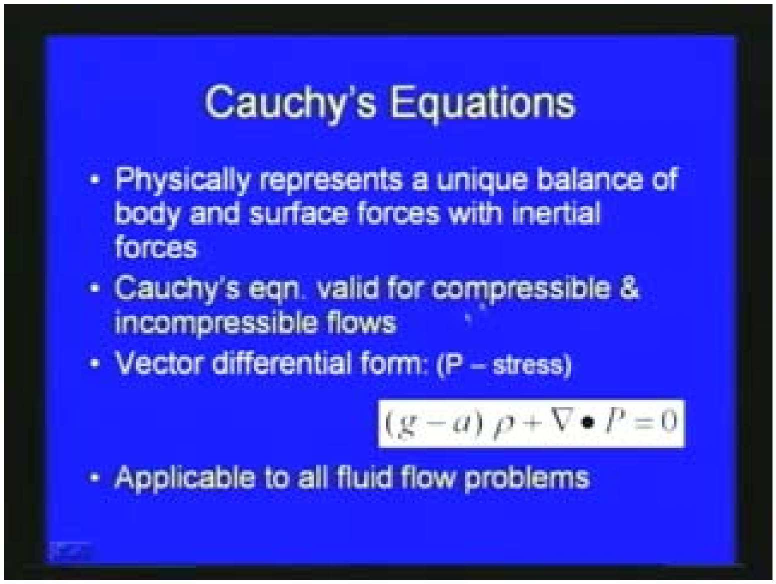


Here for two dimensional problem if you inspire if you check this equation 3, 4 and the continuity equation we can we have for 2-D problem we have three equations but we can see we have seven unknowns. The unknowns here, unknowns can be u v the velocity component, p the pressure component, t sigma x and sigma y the normal stress component and tow_{xy}, tow_{yx} and tow_{zx} the shear stress component.

So this is a case for two dimensional problem. So for three dimensional problem in the very similar way we can write the equations so there will be three equations the Cauchy's equations three equations will be there, and then we can supplement with respect to the continuity equation for three dimensional flow. So here the unknowns so if you consider these three equations of motion derive using the Newton's second law and the continuity equation we define four equations and then we can see that there will be ten unknowns u v w with component and the pressure p and three stress component sigma x sigma y sigma z and three shear stress components. So three plus three plus three velocities so nine plus one ten unknowns for general cases and here we have four equations.

So depending upon the problem we can supplement with various other equations like equations of state and then we can try to solve the problem. So this Cauchy's equation which we derived here are the basis equations for viscous flows, so here this Cauchy's equations represent a unique balance of body and surface forces with inertial forces. So here the Cauchy's equations which we derived here are valid for compressible and incompressible fluid flow.

(Refer Slide Time 42:23)



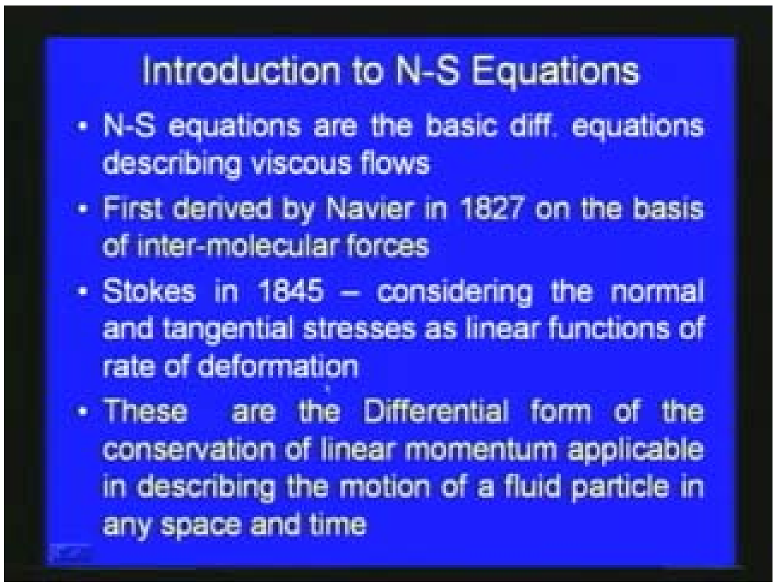
The advantage is depending upon most of the cases which we discussing this fluid mechanism course will be incompressible fluid flow but the Cauchy's equations we can also use for compressible flow also by considering the changing the density also and then here say using the Cauchy's equations only we will be deriving the Navier-Stokes Equation.

So the vector differential form we can write this the Cauchy's equations which we derived here in a vector form or vector differential form. If you consider P as this capital P as a stress component then we can write g minus a into rho plus del dot P is equal to 0, where P is the stress forces; where g is the acceleration; a is the body force component; so g is the acceleration due to gravity; and this P is the stress and this comes from the various other components. So the vector differential form of the Cauchy's equation can be written like this.

As I mention here this equation the Cauchy's equations are applicable to all fluid flow problems either compressible incompressible or any kind. So only thing is that depending upon the problem we have to change certain parameters and then we can derive the equations. So here now for the incompressible fluid flow problem we are going to use the Cauchy's equations and then we would derive the Navier-Stokes Equations.

So now, we will see introduction to Navier-Stokes Equations.

(Refer Slide Time 43:37)



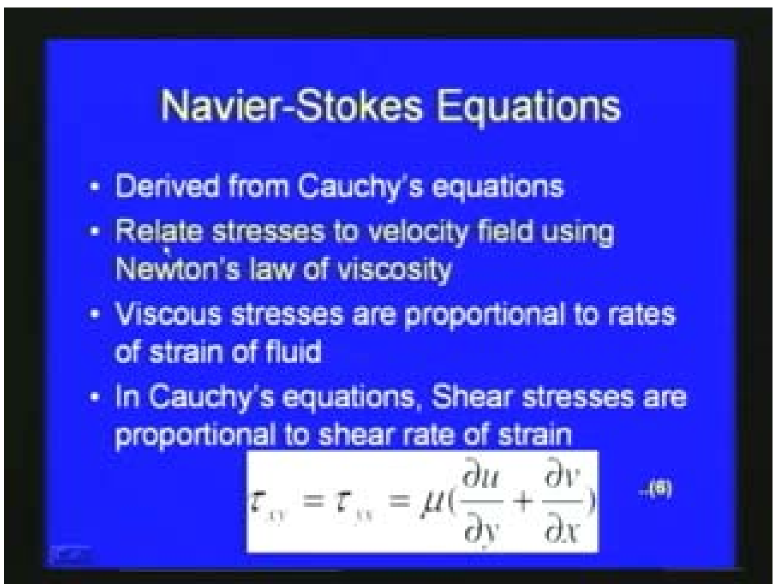
So Navier-Stokes Equations as I mentioned, for the incompressible viscous flows or even the viscous flows these are the basic different equations. So we have already seen the form for Cauchy's equation and based upon the Cauchy's equation, this Navier-Stokes Equations are derived in 1827 by Navier on the basis of inter-molecular forces and in 1845 in the nineteen century by Stokes I consider the normal and tangential stresses as linear functions of the rate of deformations.

As discussed these are the differential form of the conservation of linear momentum applicable in describing the motion of a fluid particle in any space and time. Here the time component this compressible viscous time is considered and then for the variation of any parameters either the velocity in xyz direction uvw or the pressure we can utilize this equation. So that is the beauty of the Navier-Stokes Equations.

So here we derived the Navier-Stokes Equations for the Cauchy's equation. We have already seen there are four 2-D problems as we have discussed here seven unknowns in times of u v p sigma x sigma y tow$_{xy}$ and tow$_{yx}$ and 3-D problems ten unknowns. So what we will do here for Cauchy's equation we try to represent this stresses either normal stress or shear stress in terms of say the velocity components by utilizing the Newton's law of viscosity or the stresses and strain relationship.

So here we try to relate the stresses to velocity field using the Newton's law of viscosity and viscous stresses are proportional to rates of strain of fluid. So in Cauchy's equations the shear stresses are proportional to shear rate of strain.

(Refer Slide Time 44:53)



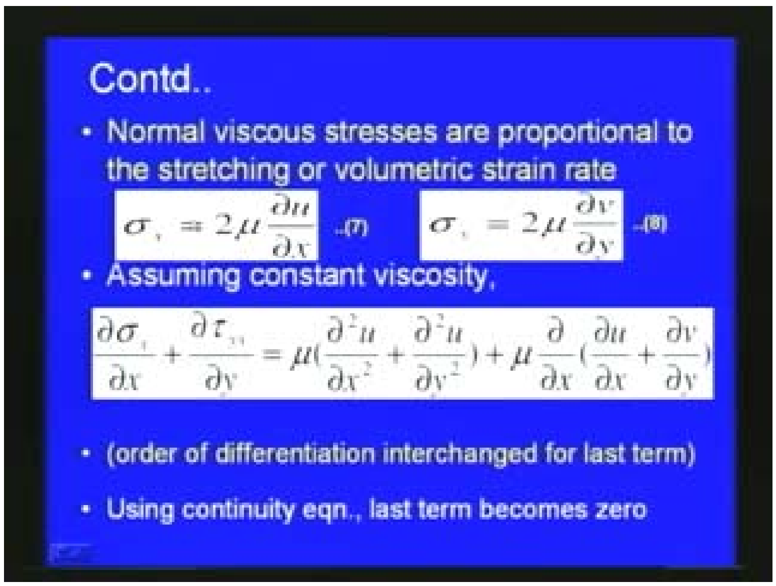
So that we can write tow$_{xy}$ is equal to tow$_{yx}$ is equal to mu del u by del y plus del v by del x and also if we consider the normal viscous stresses we can write or we can assume in this particular case normal viscous stresses are proportional to the stretching or volumetric strain rate, so that we can write the normal stress component in next direction sigma x is equal to 2 mu del u by del x as in equation number 7, and sigma y is equal to 2 mu del v by del y as in equation number 8.

So let us assume for the case we consider the incompressible fluid flow we assume the viscosity as constant, so for this case so this case, this term del sigma x by del x plus del

tow_{yx} by del y we can write by using this equation 6, this equation 7 and 8 we can write this is equal to mu into del square u by del x square plus del square u by del y square plus mu del by del x of del u by del x plus del v by del y.

So by using the equation 6, 7, 8 these terms the del sigma x by del x plus del tow_{yx} by del y can be written in this form here the order of differentiation interchanged for this last term.

(Refer Slide Time 46:15)



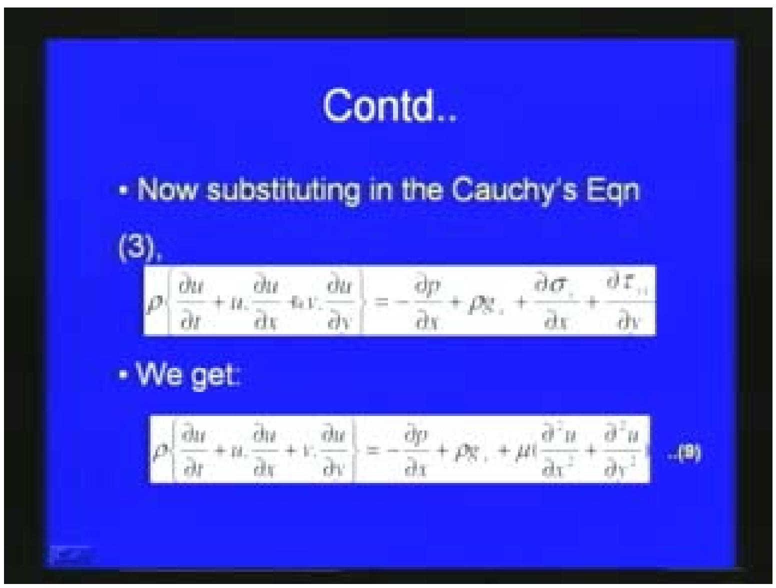
That is why this form is obtained, with respect to x and y the order of differentiation is interchanged and then this term is concerned here we can see that it for two dimensional problems is del u by del x plus del u by del y for incompressible viscous flow this term will be equal to 0. So that finally we get this del sigma x by del x plus del tow_{yx} by del y is equal to mu into del square u by del x square plus del square u by del y square.

So by using continuity equation this term becomes 0 and finally now we can use this approximation the Cauchy's equation 1 here rho del u by del t plus u into del u by del x plus v del u by del y is equal to minus del p by del x plus rho gx plus del sigma x by del x plus del tow_{yx} by del y. So here for these terms we substitute from the earlier this approximation, so finally we get rho del u by del t plus u into del u by del x plus v into del u by del y is equal to minus del p by del x plus rho gx plus mu into del square u by del y square plus del square u by del y square equation number 9.

So this is the basic equation of Navier-Stokes Equation for two dimensions problems in x direction. So similar way for equation 4 in the y direction we can write rho into del v by del t plus u into del v by del x plus v into del v by del y is equal to minus del p by del y plus rho gy plus mu del square v by del x square plus del square v by del y square as in equation number 10.

So for two dimensional problems this equation 9 and 10 which we derived based upon the Newton's second law, these equations are called the Navier-Stokes Equations for incompressible viscous flow.

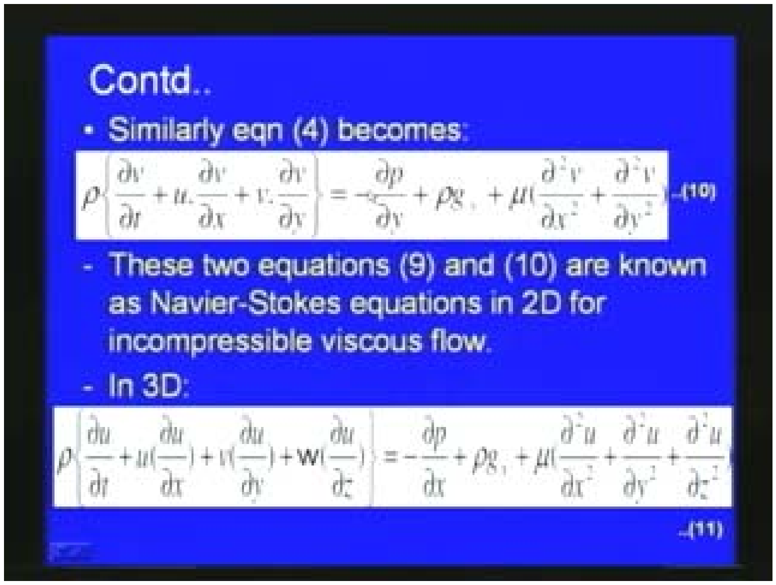
(Refer Slide Time 48:05)



So here we have demonstrated the case for two dimensions. In a very similar way we can derive for three dimensional problem and here I have just written for three dimensional problem the equations are rho in del u by del t plus u into del u by del x plus v into del u by del y plus w into del u by del z is equal to minus del p by del x plus rho gx plus mu into del square u by del x square plus del square u by del y square plus del square u by del z square this is the equation in x direction for 3-D problem, three dimensional problem by considering the velocity components u v and w.

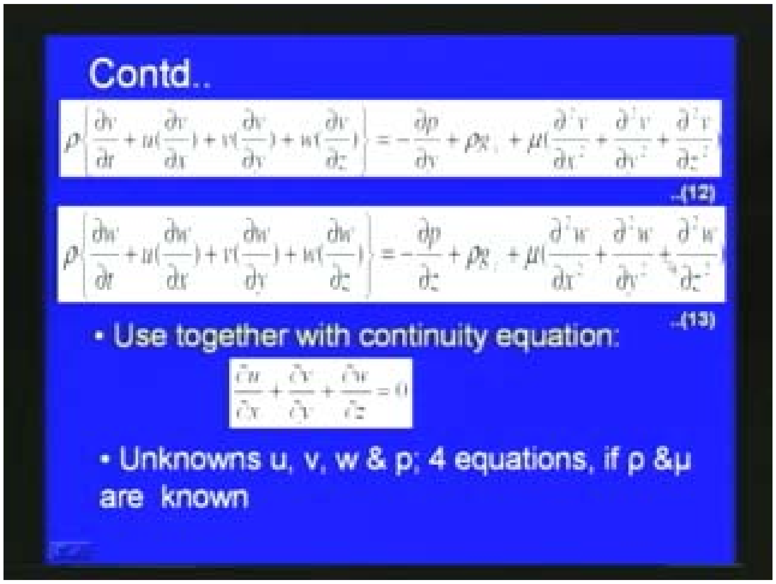
So very similar way in y direction we can write rho into del v by del t plus u into del v by del x plus v into del v by del y plus w into del v by del z is equal to minus del p by del y plus rho gy plus mu into del square v by del x square plus del square v by del y square plus del square v by del z square equation number 12.

(Refer Slide Time 48:59)



So similar way is z direction rho into del w by del t plus u into del w by del x plus v into del w by del y plus w into del w by del z is equal to minus del p by del z plus rho gz plus mu into del square w by del x square plus del square w by del y square plus del square w by del z square.

(Refer Slide Time 50:49)



So three equations in xyz direction and then we can use the continuity equation 3-D del u by del x plus del v by del y plus del w by del z is equal to 0, so here we have got four equations and then we have got four unknowns u, v, w the velocity component and the pressure component, if the density and the viscosity are known for the problem.

So these are the Navier-Stokes Equation in 3-D and 2-D also we have seen, so these are fundamental equations for incompressible viscous fluid flow.