

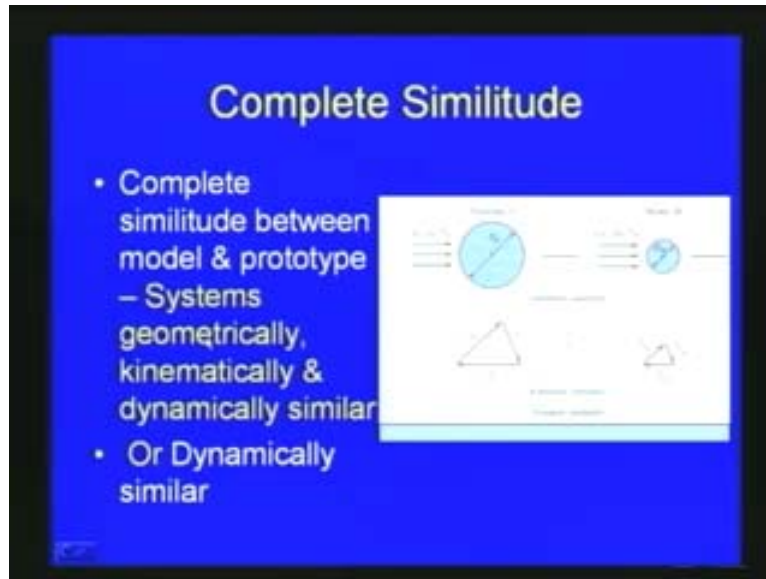
Fluid Mechanics
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Lecture - 25
Dimensional Analysis

Welcome back to the video course on fluid mechanics. The last lecture, we were discussing about the dimensional analysis and we have seen the various laws like Reynolds number law Froude number and other various dimensionless numbers. We were also discussing about the various similarity or similitude theories. We have seen basically, there are three fundamental similitudes or similarities which we have to deal while modeling. First one is geometric similarity and second one is kinematic similarity and third one dynamic similarity. In the geometric similarity, we will be looking for the similarity of similitude of the length, breadth and height is the area or the volume with respect to the modeling which we do. In the second similarity, like kinematic similarity, the similarity of the prototype and the model will be dealing with correspondingly the velocity acceleration type parameters similarity will be considered. In the third similarity is the dynamic similarity, the forces acting on the model or the prototype and ratio is considered. The forces like viscous force, shear forces the pressure all these aspects are considered in the dynamic similarity.

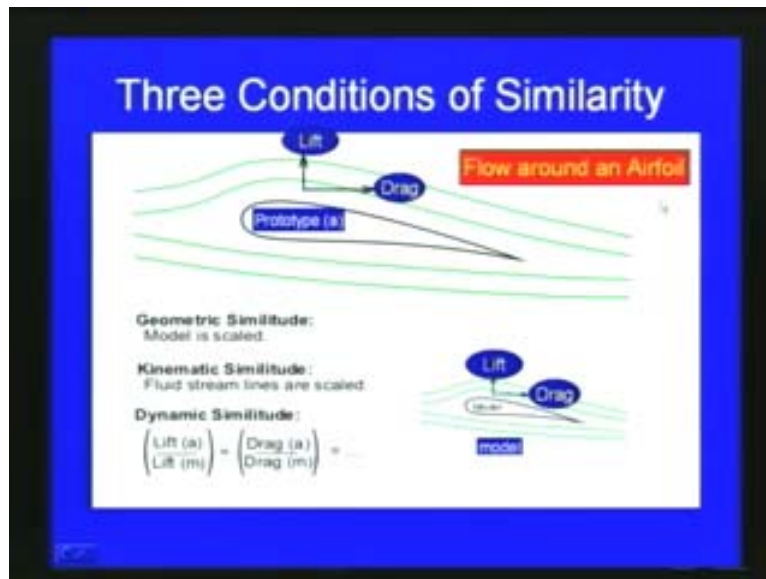
While doing the physical modeling in the laboratory with respect to the real field case or the prototype, we have to see that, this geometric or kinematic or dynamic similitude or similarities are kept. When we discuss about the similitude or similarity in the modeling, a complete similitude between the model and prototype are met whenever we considered geometric similarity kinematic similarity and dynamic similarity.

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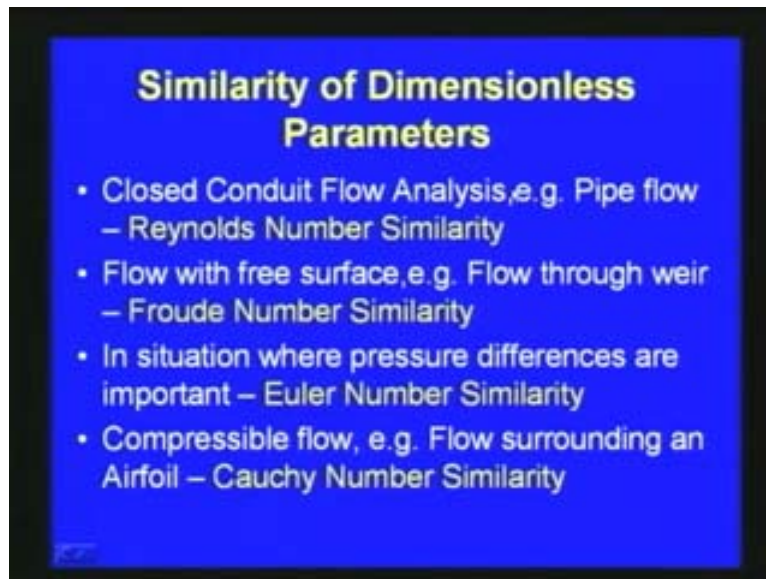
We say that, in the case of model whenever the system geometrically, kinematically and dynamically similar the complete similitude is achieved with respect to the prototype and modeling. As we have seen earlier, when dynamic similarity is met actually the kinematic as well as geometric similarity also considered. Even if we say that, the model which we do in the laboratory with respect to the real case or with respect to the prototype then, we can say that, the complete similitude is achieved. Here, in this slide you can see the prototype and the model here you can see this is the prototype and this is the model. When the dimensions are considered, its similarities meet then we said geometric similarity, here the velocity or acceleration or parameters are considered then, we say kinematic similarity, the forces are considered and the similarity are met then, we say dynamic similarity met. The complete similitude or complete similarity, the geometric and kinematic and dynamic similarity are met with respect to the physical modeling and prototype.

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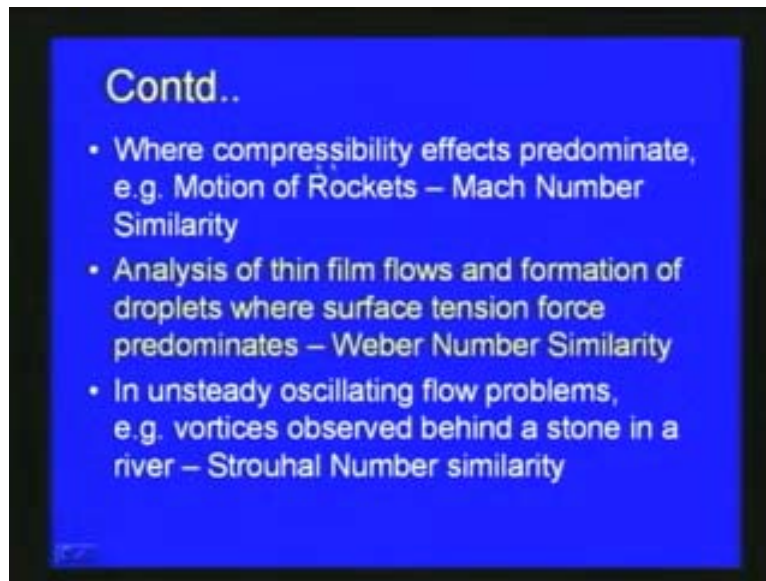
With respect to this, here we can see flow over an airfoil, here flow is coming we can see that airfoil is considered this is the prototype, this is the model. Corresponding to the prototype you can see that, geometric similarities met by considering the shape, size with respect to length width and other parameters. So, geometric similitude is model **scaled** according to the geometric similarity or geometric similitude. The second one is the kinematic similitude fluid streamlines are scaled we can see here with respect to this figure and this figure with respect to prototype and the model. Here, you can see the streamlines are also scaled accordingly, we can say that, kinematic similarity is met and third one is, you can see the various forces acting like drag force **lift** force with respect to that also considered then we say that the dynamic similarity is met. Here, the lift of the prototype divided by lift of the model that is equal to drag divided by drag of the model. Like this, we say that, dynamic similarity or dynamic similitude is met with respect to the modeling.

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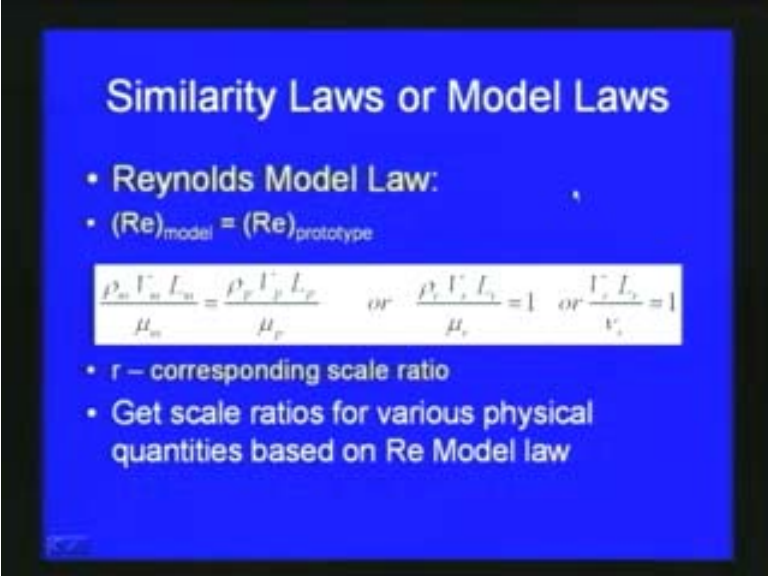
While doing all this similitude are considered with depending upon the problem, we have seen earlier number of dimensionless numbers like Reynolds number Froude number then Weber number Euler number etcetera. We consider this dimensionless numbers and we can derive similarity with respect to this dimensionless numbers depending upon the type of problem. If we considered, for example the closed conduit flow like pipe flow they are actually, we have to see that, the governing parameters, governing dimensionless number is Reynolds number and we will be choosing the Reynolds number similarity between the prototype and the model. For closed conduit analysis, we use Reynolds number similarity. Similarly, depending upon for example: open channel flow through or flow over dam or a spill way then, we consider the major force here is the gravitational force. With respect to this we can see that, there will be free surface also in these kinds of problems, we model using the Froude number similarity. Similarly, we can see that, wherever the pressure differences are considered, the pressure difference between the various sections is important. We consider the dimensionless number corresponding to the pressure; we can consider the Euler number similarity in situation where pressure difference is important. For example, compressible flow like flow over an airfoil the compressible similarity, like gas flow compressibility is considered, in that case, we will be using Cauchy number similarity.

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Where compressibility effects predominate like motion of rockets there, we consider the Mach number similarity. Mach number is related to the compressibility effect, we use the Mach number similarity. Similarly, wherever the surface tension effects are important like analysis of thin film flows and formation of droplets in that case, we can use the Weber number similarity. Like in unsteady oscillating flow problems they are the vortices observed behind a stone like a stone in a river there, we can consider the Strouhal number similarity. Like this, we have seen this dimensionless number Reynolds number, Froude number Cauchy number, Mach number Weber number, Strophe number between the prototype and the model which we can consider with similarities of this model and the prototype with respect to this dimensionless numbers. We can derive various laws based up on this dimensionless numbers to get meaningful results with respect to the physical model. Now, with this similarity the dimensionless numbers, we consider generally, most of the fluid mechanics problem the most important dimensionless numbers which will be commonly used are the Reynolds number and the Froude number. Here in this lecture, we will be considering only Reynolds number similarity law and the Froude number similarity law similar way, we can consider other laws also.

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Similarity Laws or Model Laws

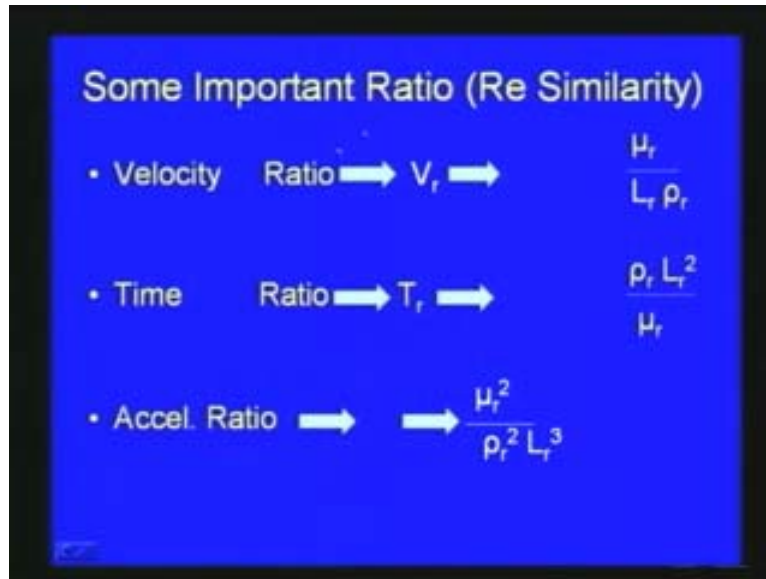
- Reynolds Model Law:
- $(Re)_{\text{model}} = (Re)_{\text{prototype}}$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p} \quad \text{or} \quad \frac{\rho_m V_m L_m}{\mu_m} = 1 \quad \text{or} \quad \frac{V_m L_m}{\nu_m} = 1$$

- r – corresponding scale ratio
- Get scale ratios for various physical quantities based on Re Model law

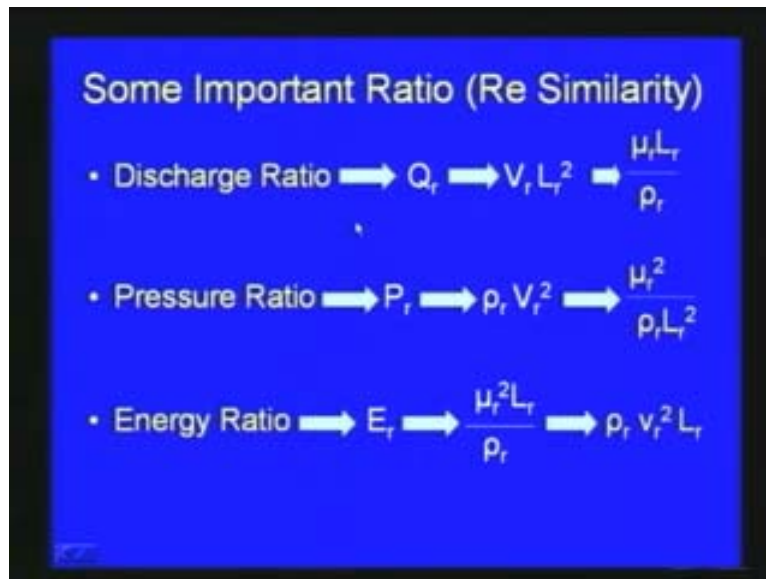
Here, the similarity laws or model laws, as we have seen first, we are considering the Reynolds model Law. As I mentioned the Reynolds number similarity between the model and the prototype are considered. We can write the Reynolds number of model is equal to Reynolds number of the prototype. Correspondingly, the Reynolds number is generally represented as ρV_L by μ where ρ is the density V is the velocity L is the characteristics length and μ is the coefficient of dynamic viscosity. With respect to the model, we can write: $\rho_m V_m L_m$ by μ_m that is equal to with respect to the prototype $\rho_p V_p L_p$ by μ_p . If we consider the ratio between the model parameters and the prototype parameters, we can write: ρ_m by ρ_p as ρ_r and V_m by V_p as V_r and L_m by L_p as L_r and μ_m by μ_p as μ_r , we can write this correspondingly $\rho_r V_r L_r$ by μ_r this should be equal to 1. Instead of dynamic viscosity we consider kinematic viscosity this we can write as $V_r L_r$ by μ_r that is equal to 1, here r corresponds to the scale ratio. As we have seen will be using particular scale ratio with respect to the geometric or with respect to parameter this r represents the corresponding scale ratio. If you use this Reynolds model law correspondingly, for various parameters like velocity, acceleration, discharge pressure, energy, power etcetera, we can use this Reynolds model law to derive various scale ratio. **We can derive the scale ratio using the Reynolds model law.**

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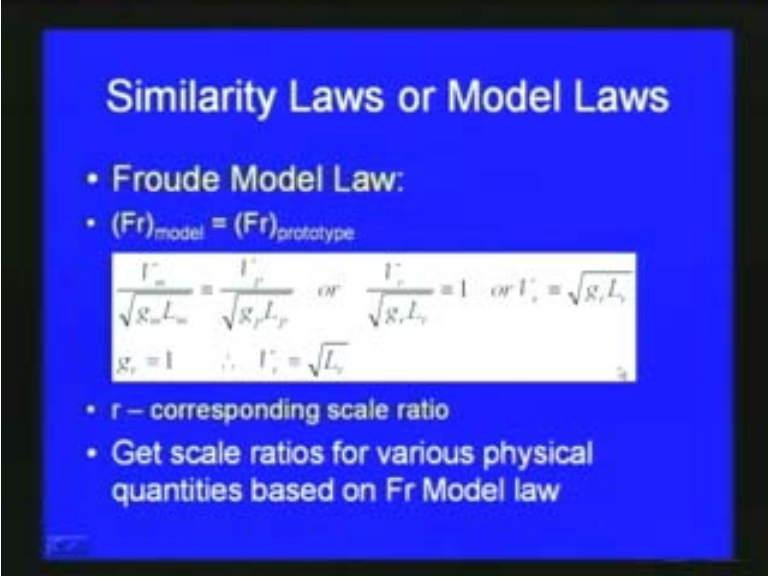
Very similar way, here as I mentioned, using the Reynolds gravity, we can derive the various scale ratio. If we consider the velocity ratio with respect to the Reynolds gravity, we can write: V_r is equal to μ_r by $L_r \rho_r$. You can see V_r is equal to corresponding to μ_r by $L_r \rho_r$ where, μ is the corresponding coefficient of dynamic viscosity L is the length and ρ is the density. Similarly, if we consider the time ratio with respect to the Reynolds similarity, we can write: $\rho_r L_r$ square by μ_r and acceleration ratio, we can write: μ_r square by ρ_r square L_r cube all these parameters, we are deriving with respect to this Reynolds model law and r corresponding to the scale ratio.

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Similar way, if you consider the discharge with respect to the model and prototype, we can derive the scale ratio as: $\mu_r L_r$ by ρ_r since discharge is equal to V_r in to L_r square this gives Q_r discharge ratio as: $\mu_r L_r$ by ρ_r similarly, pressure ratio P_r we can write as: ρ_r into V_r square. We can write as: μ_r square by $\rho_r L_r$ square similarly energy ratio, we can write: μ_r square L_r by μ_r or $\rho_r \mu_r$ square L_r this gives the energy ratio, these different parameters which we consider, while doing the physical modeling with respect to the prototype. The various scale ratios, we can for various parameters like velocity, discharge or pressure we can derive using this equation. Correspondingly, we can do the experiment in the model or we can develop the model run the experiments correspondingly, the Reynolds similarity will be met. With respect to the Reynolds similarities here, only few parameters are listed but, in very similar way other parameter also can be derived and can be expressed as ratio with respect to the Reynolds similar.

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Similarity Laws or Model Laws

- **Froude Model Law:**
- $(Fr)_{model} = (Fr)_{prototype}$

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}} \quad \text{or} \quad \frac{V_r}{\sqrt{g_r L_r}} = 1 \quad \text{or} \quad V_r = \sqrt{g_r L_r}$$

$g_r = 1 \quad \therefore \quad V_r = \sqrt{L_r}$

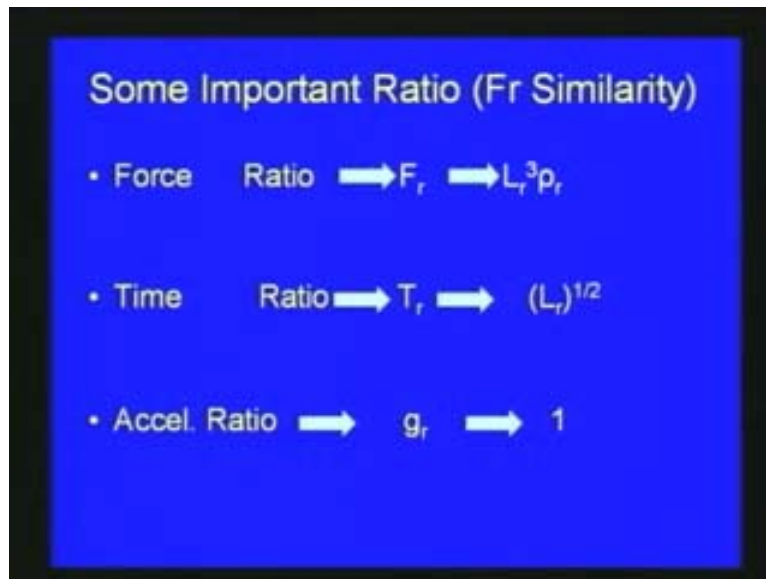
- r – corresponding scale ratio
- Get scale ratios for various physical quantities based on Fr Model law

The second one the other important similarity law or model law is the Froude model law. As I mentioned Froude number is, with respect to the effect of gravity indicates the effect of gravitational forces, the cases like open channel flow various hydraulic structure we can consider the Froude model law. As per this Froude model law, we can write, the Froude model is equal to Froude prototype with respect to the Froude number. We can write this Froude number is generally, expressed as V by root g v is the velocity g L is acceleration due to the gravity L is the characteristics length. We can write corresponding to this model we can write: V_m by square root of $g_m L_m$ that is equal to corresponding prototype V_p by square root of $g_p L_p$. In a very similar way, as we have seen the Reynolds model law here also, if we consider the ratio between the velocity length and acceleration, we can write here: V_r by root $g_r L_r$ that will be equal to 1.

We can write the velocity now can be written as: square root of $g_r L_r$ where, r stands for the corresponding scale ratio. Now, most of the cases corresponding to the prototype as well as the model generally, you do in the laboratory with the gravitational scale will be always 1. Since, if we are not going for **certifies** modeling like with high gravity 10g or 50g then, the case is different.

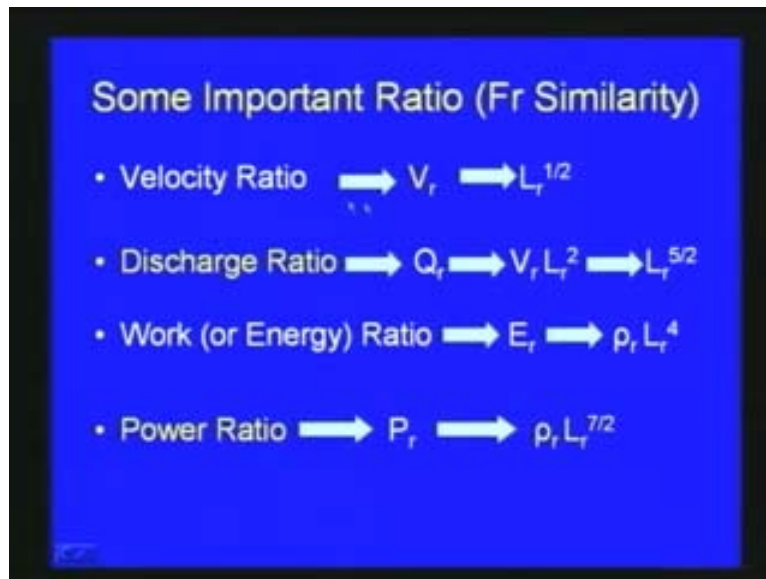
Most of the laboratory models we develop will be in the prototype as well as the laboratory model will be with respect to same gravity acceleration due to gravity. We can write this g_r is equal to 1, we can write now V_r is equal to square root of L_r with respect to the Froude model law. This gives the Froude model law. Now, using this, we can derive various scale ratios for various parameters as we have seen in the case of Reynolds model law. Now, the same the Froude model law, we can use with respect to this parameter.

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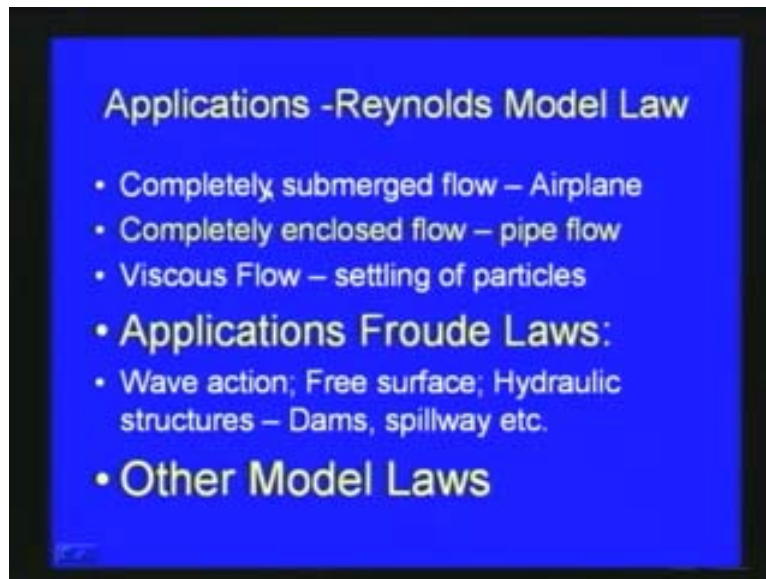
In the next slide shows how you can utilize it here using this Froude model similarity. Force ratio we can write as: F_r is corresponding to L_r cube ρ_r and L is the characteristics length ρ is the density and r is the scale ratio. With respect to the Froude similarity law, we can write: T_r is corresponding to L_r to the power 1 by 2 square root of L_r acceleration ratio you can see that, here Froude law is considered the acceleration ratio here is g_r is corresponding to 1. Since as I mentioned, we are experimenting the same acceleration due to gravity corresponding to prototype acceleration ratio g_r is equal to 1.

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Then, the velocity ratio as we have already seen V_r is corresponding square root of l_r or L_r to the power 1 by 2 and discharge ratio is corresponding Q_r is V_r into L_r square or L_r to the power 5 by 2. That gives the discharge ratio with respect to the Froude number similarity. We can write work ratio E_r is corresponding to $\rho_r L_r$ to the power 4 and power ratio we can write: $\rho_r L_r$ to the power 7 by 2. Here, only important parameters are listed very similar way, we can derive the scale ratio with respect to the Froude number similarity, we can derive the scale ratio; we can correspondingly model in the laboratory with respect to the prototype. As I mentioned Reynolds number and Froude number are most commonly used dimensionless number or similarity principle similarity laws. Let us see some of the important applications if we consider the Reynolds model law.

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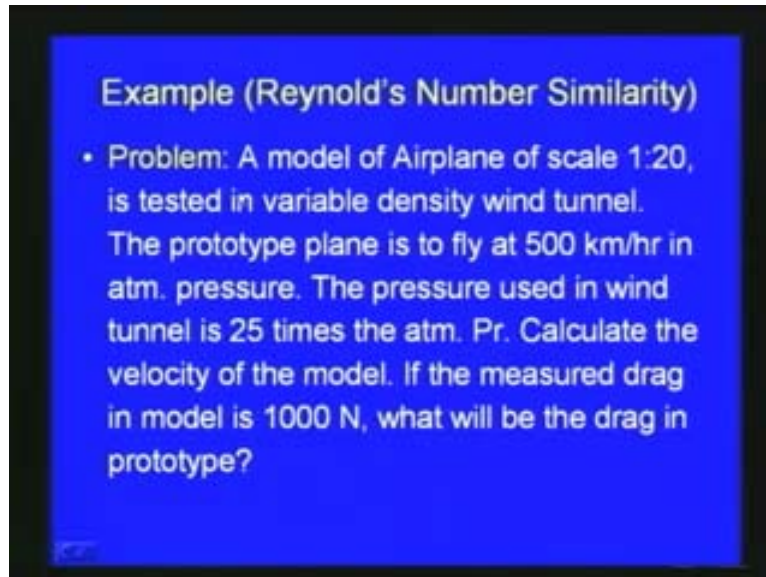


Reynolds model law, we can see that, cases like completely submerged flow if you consider if a modeling the airplane with respect to the laboratories, we will generally considering the Reynolds model law. It is completely submerged flow airplane or **top** all this kind of modeling, we will be using the Reynolds model law. Second one is completely enclosed flow. For enclosed flow like pipe flow Reynolds number is more important, we will be using the Reynolds model law. Third case is viscous flow like settling of particle like **reservoir sedimentation** or the dust settling, Reynolds model law is very important and we can utilize it for all these cases. Similarly, as I mentioned Froude law or Froude number is considered, we can see that, wherever the gravitational effect is more important, Froude is considered.

Cases like wave actions like estuary models or like proton halberd models; we can use the Froude law. Problems with free surface like flow over a spillway, we can use Froude model law and also hydraulic structures like dams, etcetera in modeling of these problems, we can use the Froude model law. As I mentioned since Reynolds model law in Froude model law are most important model laws in physical modeling generally used physical modeling. We have described only Reynolds model law and the Froude model law. Very similar way, like we can derive the other model laws, we can correspond scale ratios and depending upon the problem, we can utilize the other model law. Before

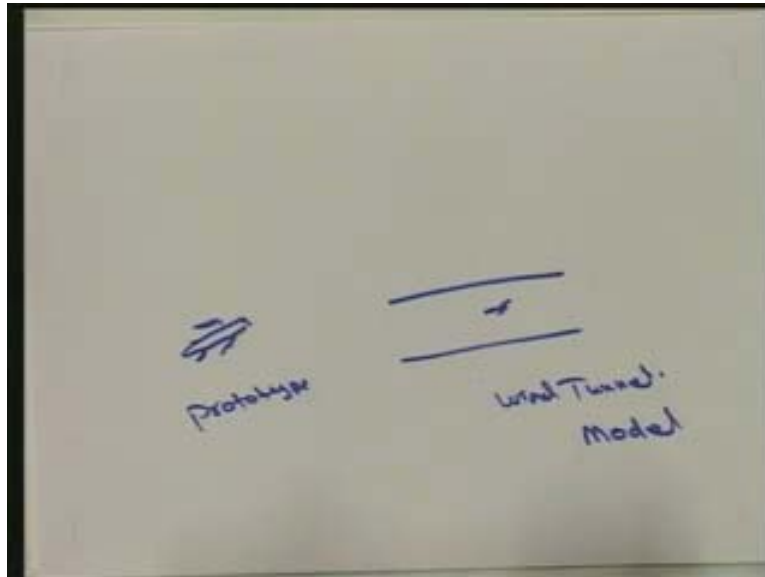
proceeding to further aspects of physical modeling, let us consider two examples with respect to the Reynolds model law and the Froude model law. First case is example on Reynolds number similarity.

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We consider a problem a model of airplane of scale 1:20 is tested in variable density wind tunnel the prototype plane is to fly at 500 kilometer per hour in atmospheric pressure. The pressure used in wind tunnel is 25 times the atmospheric pressure, we have to find the velocity of the model and if the velocity to be used in the model. If the measured drag in the model is thousand Newton what will be the corresponding drag in the prototype. This is the problem; let us consider an aeroplane. We can see that, we are considering a wind tunnel for the wind tunnel we have developing a model with respect to this is wind tunnel and aeroplane model. If you consider the prototype and this is the wind tunnel model with respect to this here, we are modeling the areoplane which is flying at the rate of 500 kilometer per hour with respect to this, we have to consider the modeling here.

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Now for this problem,

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Soln:

- $(Re)_m = \frac{\rho_m V_m L_m}{\mu_m} = (Re)_p = \frac{\rho_p V_p L_p}{\mu_p}$

Here, $L_m/L_p = L_r = 1/20$
Viscosity will remain constant, i.e. $\mu_m = \mu_p$
Now, at constant temperature, $p/p = \text{Const.}$
Hence, Pressure Ratio, $p_m/p_p = 25 = \rho_m/\rho_p$
So, from Reynold's No. similarity,
 $V_m = V_p \cdot (\mu_r / \rho_r L_r) = 500 \cdot (20/25) = 400.00 \text{ km/h}$

As I mentioned the Reynolds model law is important, if we consider the Reynolds model law $\rho_m V_m L_m$ by μ_m is equal to corresponding to the prototype $\rho_p V_p L_p$ by μ_p . Here the scale ratio with respect to length is considered L_m which is given as 1:20 L_m by L_p is L_r is equal to 1 by 20 and viscosity, we consider in a both cases ν is remain constant.

If we assume the viscosity remains constant μ_m is equal to μ_p and if you assume constant temperature also can write p by ρ is constant that is pressure by density is constant. Shown that assumption, we can write the pressure ratio P_m by P_p that is here it is given that the pressure used in the wind tunnel is 25, the atmospheric pressure that means P_m by P_p the model and the prototype that is equal to 25. We can write that is equal to ρ_m by ρ_p . From the Reynolds number similarity, we have already seen that, this V_m can be represented as V_p into μ_r by $\rho_r L_r$ this gives the velocity of air in the model. U_p is the velocity of the prototype, μ_r is the viscosity scale ratio, ρ_r is the density scale ratio, L_r is the length scale ratio. V_p the prototype velocity that is 500 kilometer per hour then μ_r is already constant ρ_r here, you can see it is here 25 and L_r is 1 by 20 we can write 500 into 20 by 25 that gives 400 kilometer per hour.

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- Here the ratio of model velocity to the prototype velocity is:

$$V_r = V_m/V_p = (20/25)$$

Force Ratio, $F_r = L_r^2 \rho_r V_r^2$

$$= (1/20)^2 \cdot 25 \cdot (20/25)^2$$

$$= (1/25)$$

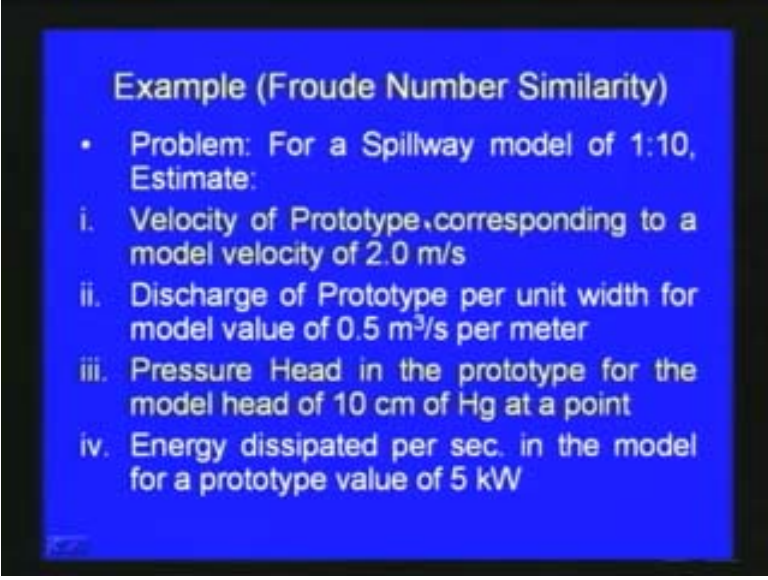
Hence the Drag at Prototype is:

$$F_p = F_m/F_r = 1000 \cdot 25 = 25000 \text{ N}$$

The ratio of the model velocity to the prototype velocities V_r is equal to V_m by V_p that we get as 20 by 25 and the force ratio, as we have seen force ratio, we have to consider drag force here force ratio is equal to $L_r^2 \rho_r V_r^2$ the length ratio is consider 1 by 20. 1 by 20 square ρ_r is 25 V_r is 20 by 25 20 by 25 square this, we can get as 1 by 25 force ratio with respect to the model and prototype correspond by 25 hence the drag it is given as the model the drag is given as 1000 Newton. F_p is equal to F_m by F_r that will give 1000 into 25; we get the drag as 25,000 Newton. This is the simple problem where

the Reynolds model law is applied very similar way different kinds of problems wherever Reynolds number the predominant or is important, we can use the Reynolds model similarity to solve this kinds of problems. Second one is we consider an example here Froude number similarity.

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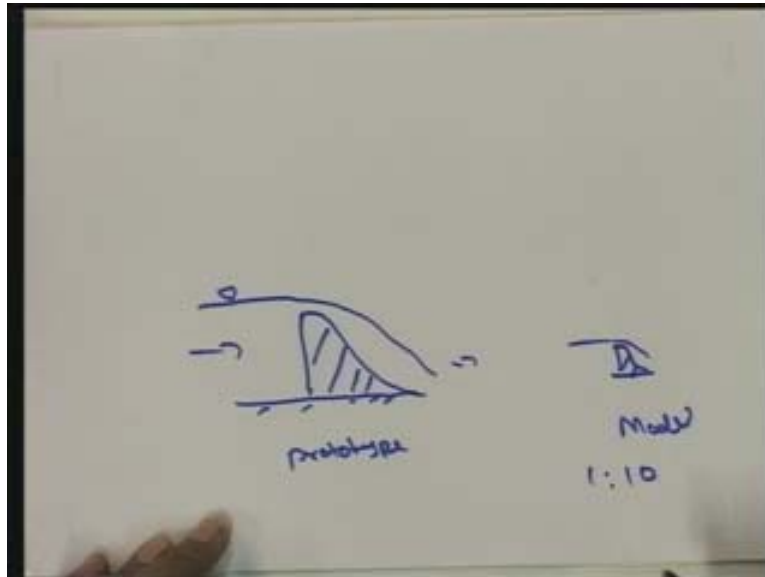


Example (Froude Number Similarity)

- Problem: For a Spillway model of 1:10, Estimate:
 - i. Velocity of Prototype corresponding to a model velocity of 2.0 m/s
 - ii. Discharge of Prototype per unit width for model value of 0.5 m³/s per meter
 - iii. Pressure Head in the prototype for the model head of 10 cm of Hg at a point
 - iv. Energy dissipated per sec. in the model for a prototype value of 5 kW

Here, the problem is a spillway model of 1: 10. Estimate the velocity of prototype corresponding to a model velocity of 2 meter per second and discharge of prototype and unit width of model values 0.5 cubic meters per second meter. Pressure head in the prototype for the model head of 10 centimeter of mercury and at a point energy dissipated per second in the model for a prototype value of 5 kilo watt here what we consider is, a spillway. If you consider a spillway like this, this is the prototype the flow is taking place like this. If you consider a small model of scale 1: 10 corresponding to this is prototype here, we have the model we have to consider the problem like this the scale is given 1:10

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For this problem, we have to determine the velocity of prototype then discharge of prototype, pressure head in prototype and energy dissipated per second. Here, as I mentioned this particular problem is considered, this gravitational effect is more importance of Froude number similarity we can directly use. If you use here the Froude numbers similarity.

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- Soln:
- At present condition Froude Number similarity will be satisfied, hence;
 - i. $V_r = L_r^{1/2}$; Length Ratio, $L_r = 1/10$; so the velocity of prototype, $V_p = V_m / L_r^{1/2}$;
i.e. $V_p = 2 \cdot (10)^{1/2} = 6.32 \text{ m/s}$
 - ii. Ratio of discharge per unit width = $q_r = V_r L_r$
Now, $q_r = q_m / q_p = V_r L_r = L_r^{3/2}$
 $q_p = q_m / L_r^{3/2} = 0.5 \cdot (10)^{3/2} = 15.81 \text{ m}^3/\text{s/m}$

We have already seen, the velocity ratio is length scale ratio to the power 1 by 2 or square root of L_r length ratio is given as L_r is equal to 1 by 10. Velocity of prototype, we can write: V_p is equal to V_m by L_r to the power 1 by 2 U_p is equal to V_m is 2 meter per second L_r is 1 by 10, 10 to the power 1 by 2. This gives 6.32 meter per second this is the first part of the question. Second part of the question is ratio of discharge per unit width that is q_r is equal to $V_r L_r$. Now q_r is equal to q_m this m is corresponding to model P correspond to prototype r corresponds to scale ratio q_r is equal to q_m by q_p that is equal to $V_r L_r$ that is equal to L_r to the power 3 by 2. Correspondingly, q_m is already given as 0.5 q_p get as 0.5 into 10 to the power 3 by 2 that is 15.81 cubic meter per second per meter that is the second part of the question.

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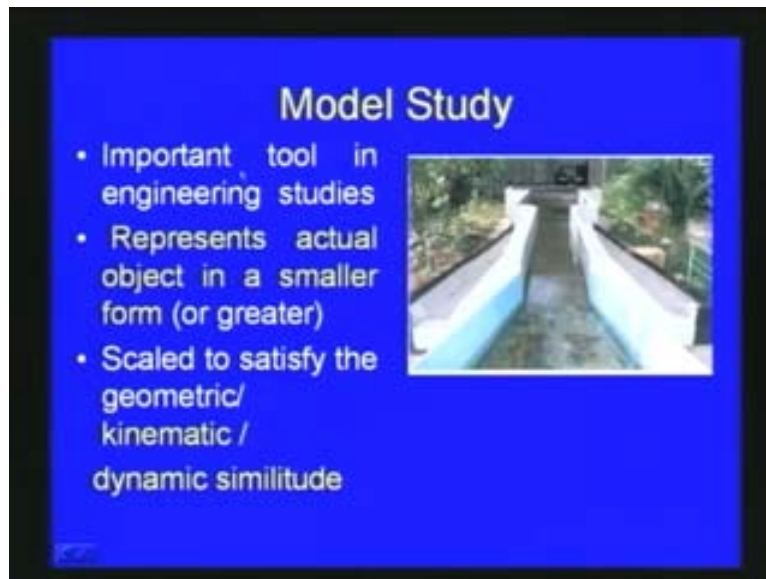
iii. Pressure Ratio, $P_r = P_m / P_p = L_r \rho_r$;
 Assuming density as constant, $\rho_m = \rho_p$;
 $\rho_r = 1.0$, i.e. $P_r = L_r$
 Hence, $P_p = P_m / L_r = 10 \times 10 = \underline{100 \text{ cm of Hg}}$

iv. Power Ratio = (Energy Loss/Second),
 $P_r = \rho_r L_r^{7/2}$; $\rho_r = 1.0$
 Hence $P_r = L_r^{7/2}$; $P_m = P_p \times L_r^{7/2}$;
 i.e. $P_m = 5000 \times (1/10)^{7/2} = \underline{1.58 \text{ watt}}$

Third part is, we have to find out the pressure ratio pressure ratio is P_m by P_p so that we can write as $L_r \rho_r$ we assume the density as constant, if we assume water is consider in both cases, we can assume density as constant. ρ_m is equal to ρ_p ρ_r is equal to 1, correspondingly, the pressure ratio will be P_r is equal to $L_r P_p$ the corresponding the prototype P_m will be divided by L_r here P_m is given as 10, 10 into L_r is 1 by 10, 10 into 10 is 100 centimeter of mercury.

This is the third part of the problem. Fourth case is we have to determine the power ratio, power ratio is equal to energy loss by second we can write as: P_r is equal to $\rho_r L_r$ to the power 7 by 2 ρ_r density equal to 1, we can write the pressure ratio P_r is equal to L_r to the power 7 by 2 so P_m is equal to P_p into L_r to the power 7 by 2, we can see here, the model the power used will be 5000 is given here problem 5 kilo watt is given in the prototype. Correspondingly, 5000 into 1 by 10 to the power 7 by 2 and that will above 1.58 watts. This shows how we are utilizing the Froude model law corresponding to all this kinds of problem. Here, we consider the flow over a expiry way, this problem is governed by the Froude number similarity, we have derived various parameters and we get with corresponding either prototype or the model we can find out the various parameter. This is one of the model similarity laws. Before closing this chapter, we will discuss various models aspect all this dimension analysis are the dimensionless numbers or the similitude or the similarity principles, which we considered, we use for the purpose of model study.

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As I mentioned in the beginning this model study is very important. Modeling is an important tool in engineering studies, you can see the model of a sump pump which is in our hydraulics laboratory, a channel open channel, we have done a model corresponding to an open channel and here we provide pump sump you can see this case must done for a


an industry. You see that what happens in the pump sump, when a large quantity of water is pumped out, how the behavior it is very difficult to use mathematical model or with a theoretical it is very difficult told. We construct a model for these kinds of problem this particular model, we have constructed with respect to 1:8 scales with respect to the prototype, what we are studying is what happens if this amount of water is pumped various pumps. How the disturbance the lot of turbulent have generated the vortices. What we studied here, this modeling is an important tool and it represents the actual object in a smaller form or sometime greater scale, depends up on the case it can be a smaller scale or it can be larger scale depending up on. We generally have seen, scale it to satisfy the various conditions various similarity principles like a geometric kinematic and dynamic similitude. For an example, this particular model, we have used the Froude number law and the used corresponding similarity of geometric kinematic and like that, we can either choose kinematic or geometric or kinematic or dynamic together depending up on various combinations of problems you are trying to model, we can have different concepts combination.

We have already seen the purposes of hydraulic model study. As I mentioned many cases of complexities it is very difficult to represent theoretically or to do mathematical modeling. By this model studies, hydraulic model studies are used to understand the phenomena with respect to the realistic approach. We can see that, we are using scale ratio and we are trying to replicate the real case of the prototype in the laboratory. This is somewhat realistic approach so we can understand the various phenomena. The second one is here you can see the problem complexity as I mentioned this particular sump pump model which we did in our hydraulic laboratory.

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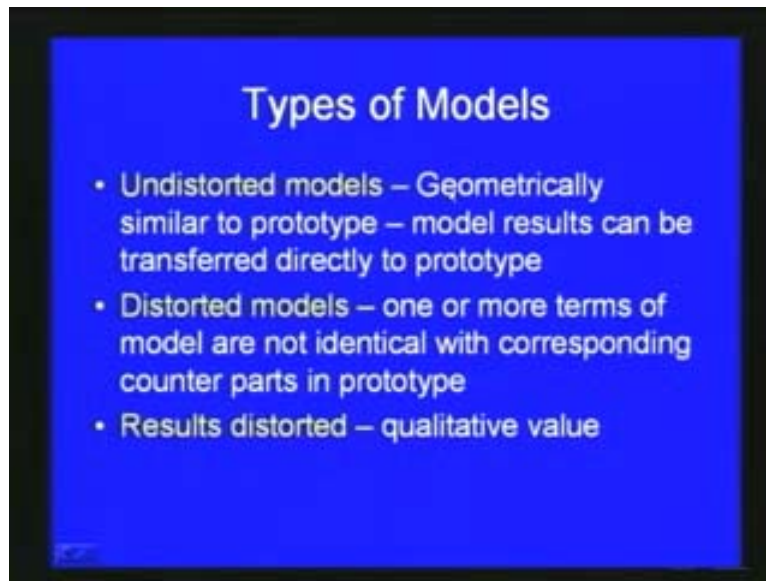
Purpose of Hydraulic Model Study

- Understand phenomenon with a realistic approach
- Problem Complexity - mathematical model difficult, hence the models are built
- **Models of dams, spillways, etc are tested in laboratories to get idea about parameters**



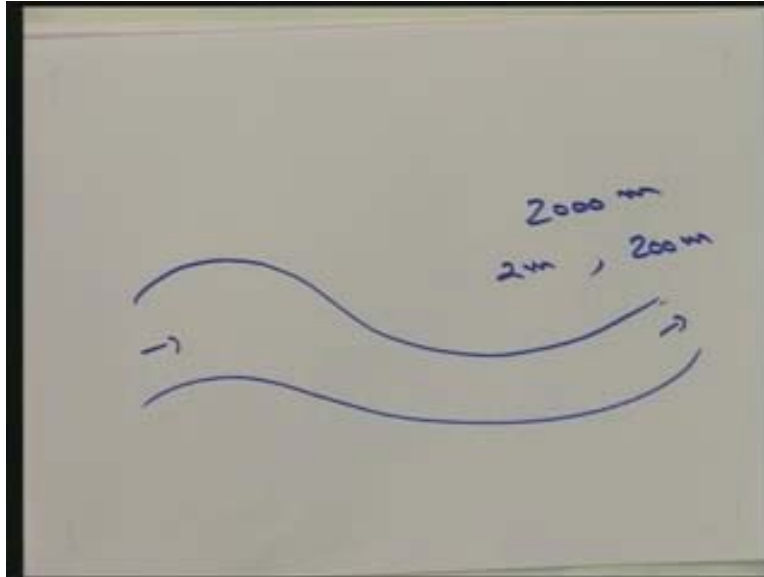
It is very difficult to mathematically represent this turbulents or the vertices formation with respect to this pumping heavy amount of pumping. The mathematical model is difficult but, the modeling if you do appropriately with respect to scale, we can observe whether the turbulent are general and are related whether the system is or the pumps are affected or system is stable. Some case wherever theory cannot directly solve the problem or mathematical modeling is difficult then, this physical modeling or hydraulic modeling study helps. Models of dams, spillway etcetera, generally a large investment is done while we construct a dam other than the theoretical study of the mathematical study, we generally do a hydraulic model study. If we go to central water power station in Pune, you can see number of model studies for the various dams, spillways etcetera. First we test in the laboratory idle about various parameters and how the system will be behaving then only we will be going for the real construction. This hydraulic model study is very important. While doing these kinds of model study; we have already seen the similarity principles like geometric dynamatic and dynamic similitude or similarity principle. As far as geometric is concerned, these different types of modeling are possible. First one is the undistorted model the modeling can be either undistorted model or distorted model undistorted models means the model is geometrically similar to the prototype.

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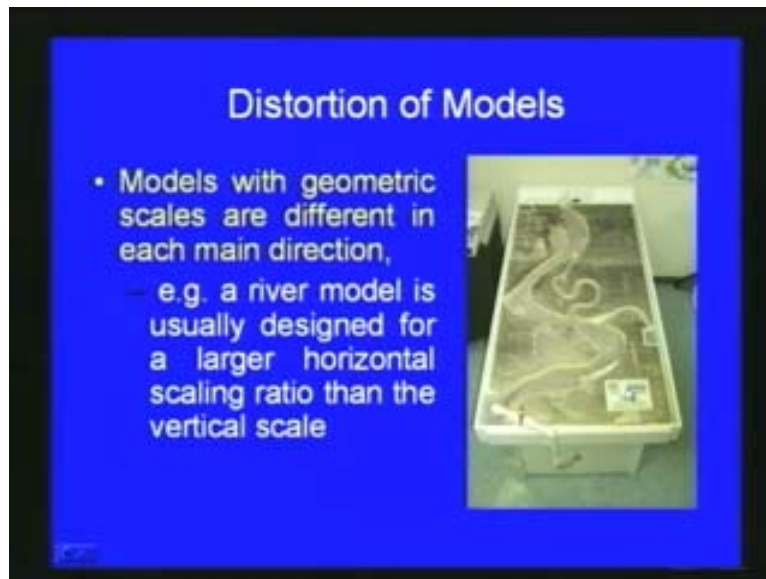
Similarly, the length wise, the breadth wise, depth wise all the geometric is concerned, the model replicate the prototype so the geometrical similarity is achieved and model results can be geometrically similar. Many time most of the cases; we can directly transfer the results with respect to the prototype. These kinds of models are called undistorted model but, sometimes if you do a modeling of a river this particular case, we can see if you are doing a modeling of a river like this then, you can see that the length parameter, we can replicate width parameters but, depth is considered then, we cannot use the same scale. If you use the same scale the depth may be 2 meter but, width may be 200 meter and length considering may be 2000 meter, we can see that, if you use the same scale for the length breadth and the depth.

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Then, you can see that, it is difficult to interrupt modeling will be difficult where we use cases for distorted model. Here, one or more terms the model or not identical with corresponding counter parts in prototype. These kinds of models are called as distorted models. Actually, whatever the results which we will be getting all distorted, we have to correspondingly do some rectification corrections after the results are obtained but, generally, all these distorted models, we give qualitative values and we can interrupt accordingly.

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The models can be either distorted models or undistorted models. You can see the distortion of a model of a river channel where the depth is very difficult to represent. Models with geometric scales are different in each direction as a river model is usually designed for a large horizontal scale ratio than the vertical scale. Distortion is many times the practical case of modeling; we may have to go for the distorted model rather than an undistorted model. It is difficult to do in the laboratory with respect to a prototype; we may have to go for a distorted model depending upon the case. Here again in the case of a distorted model, generally, you can see length and width are generally scaled to available space as we consider a river or an estuary model.

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Distorted Models

- Length & width – scaled to suit available space
- Vertical flow dimension (depth) – Froude law
- Distorted scale – no geometrical similarity



You can see that, length and width we can generally represent scaled to suit will be available space but, vertical flow dimensions especially depth is considered. Generally the gravitational effect is more important and fluid flow similarity is met. We will be making the distortion or we may not be able to meet the vertical flow scale, distorted scale finally what happens is no geometric similarities and then it becomes a distorted model.

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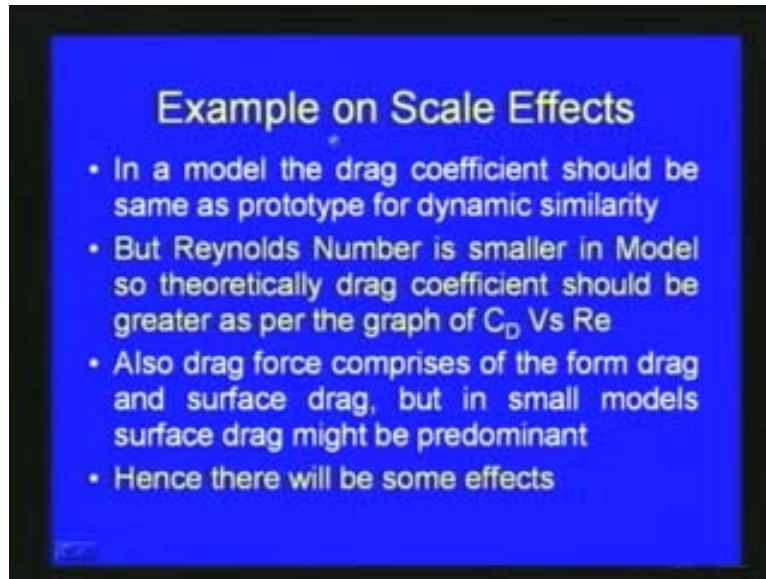
Scale Effects

- Distorted Models justified – If Re . No criterion can be neglected & pressure is nearly hydrostatic
- Distortions by effects other than dominant one, such as viscosity, surface tension - gravity is the dominant force
- Amplification in vertical scales – problem to simulate fluid resistance
- Soln. artificial roughness or turbulence

When we use this distorted models there will be scale effects, we consider the scale effect in many cases as I mentioned we can not do all the geometrical similarities. That is why, we go for distorted model. Before going for a distorted model, we have to justify whether these kinds of modeling is investing money and time and our efforts are being justified or not. Some of the cases here I have mentioned distorted model are justified for example, if Reynolds number criteria can be neglected and pressure is nearly hydrostatic. What we have seen is this shallow water flow problems are estuary problem are sometimes river problems here, we assume that, the pressure variation is nearly hydrostatic and the Reynolds number criteria can be neglected and distortion effects other than the dominant one such as viscosity surface tension, we can neglect that gravity is the dominant force.

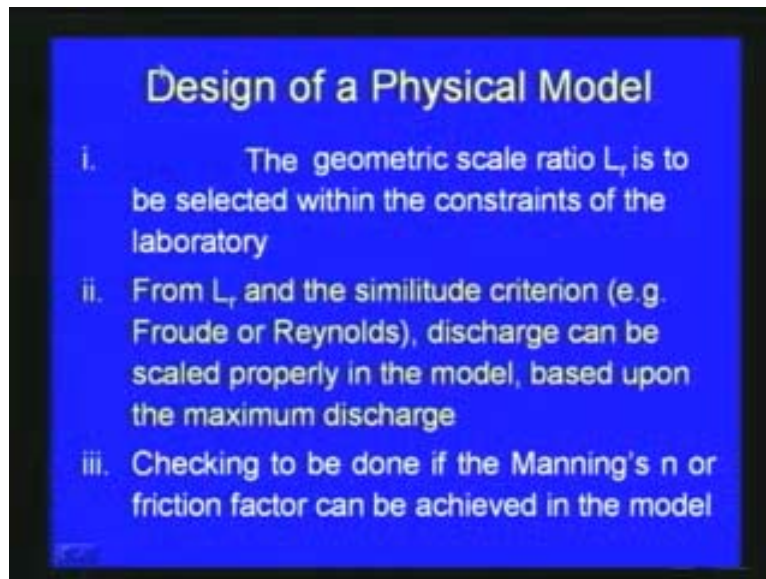
This is another case where, we can justify distortion model. Third case is amplification in vertical scales problem to simulate fluid resistance most of the difficulties are kinds of river or the estuary models what happens is vertical scales are not met. The fluid resistance we have to artificially simulate the roughness of the bottom or boundaries may not be met this we have to dome artificial reference or turbulence we may have to generate. Amplification vertical scales are a major issue. While doing these kinds of models, we should be very careful the scale effects to be considered corresponding rectification, we have to do before the model results are transfer to the prototype for either for design or for construction purposes.

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As I mentioned scale effect is major problem some of the example shows on scale effect are: In a model the drag coefficient should be same as prototype is dynamic similarity is met but, you can see that, the Reynolds number is smaller in model theoretically drag coefficients should be greater if you draw a graph of C_D the coefficients of drag Vs Reynolds number then, you can see that, Reynolds number is smaller in model so drag coefficients should be greater. Also the drag force comprises of the form drag and surface drag but, in many cases the small models surface drag might be predominant form drag may not be predominant. We can see that, there will be lot of the scale effects. These are the problems while typically if you do the drag effects, if we consider for particular problems like the surface drag may be predominant and form drag may not become like the drag coefficient with respect to Reynolds number is smaller. In that case the coefficient of drag may not be to the representing whatever this should be. These are some of the scale effect which will be generally we have to deal while doing the physical modeling, for large structures or large rivers estuaries etcetera.

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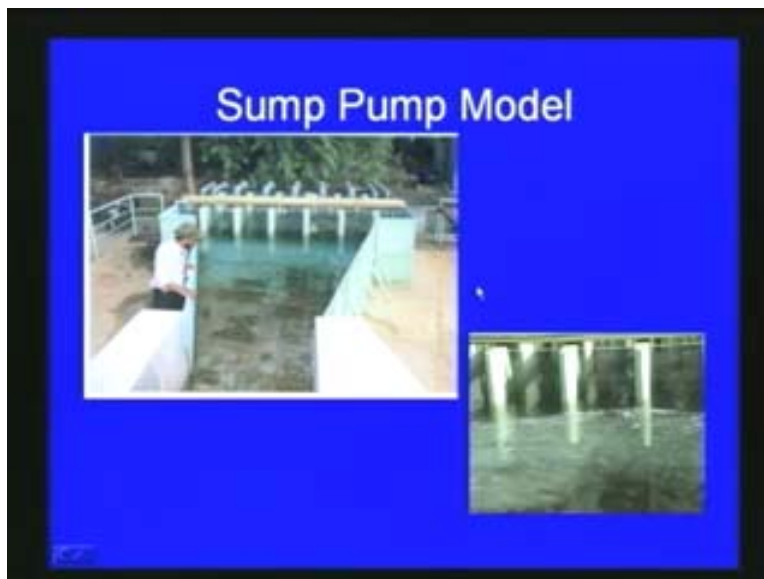
Before closing this chapter, let us see some of the important steps as far as the design of physical model is concerned. Some of the important steps are listed here, the first one is geometric scale ratio is to be selected within the constraints of the laboratory. This is important since when we are modeling a river or hydraulic structures like dam or spillway. We have to see how much space available, what can be the scale ratio that is the first point. Second point is from the scale ratio and the similitude criteria in example of Froude or Reynolds discharge can be scaled properly in the model based up on the maximum discharge. The discharge in the prototype can be very high correspondingly, we have to scale down to the model either we can use the Reynolds model law or the Froude model law to do this discharge scaling. This is the second step the discharge parameter, first step is the length parameter second is the scale the geometry is concerned; second one is the discharge parameter.

Third one is check to be done if the manning or friction factor can be achieved in the model. As we have seen in the case of a river model or in an estuary model then, it is very difficult to achieve the turbulence or the friction effect. The third important step which we have to see whether we can achieve the important reference coefficient like manning or the friction factor whether we can achieve in the modeling this is the third step. Forth

step is Reynolds number or Froude number wherever applicable should be checked for minimum flow condition.

This is very important when we do modeling with respect to minimum flow conditions whether the Reynolds number or Froude number which ever is applicable we have to check whether it is achieved this is the step number four. Step number five is finally a convenient scale is chosen to satisfy the entire similarity criterion whichever a possible like geometric kinematic or dynamic similarity that is the fifth step. Sixth step is advanced modeling such as 2D model or a distorted scale model like river model to be done in situations where simple physical models are not feasible. As I mentioned like in the river or estuary problems normal undistorted model be difficult, we can consider distorted scale model and the seventh important point to be considered for distorted model horizontal vertical scale to chosen separately and appropriately. We have to consider the horizontal scales and vertical scale appropriately like as I mentioned the river flow or the estuary flow, if the depth wise the vertical scale is different and horizontal scale will be different.

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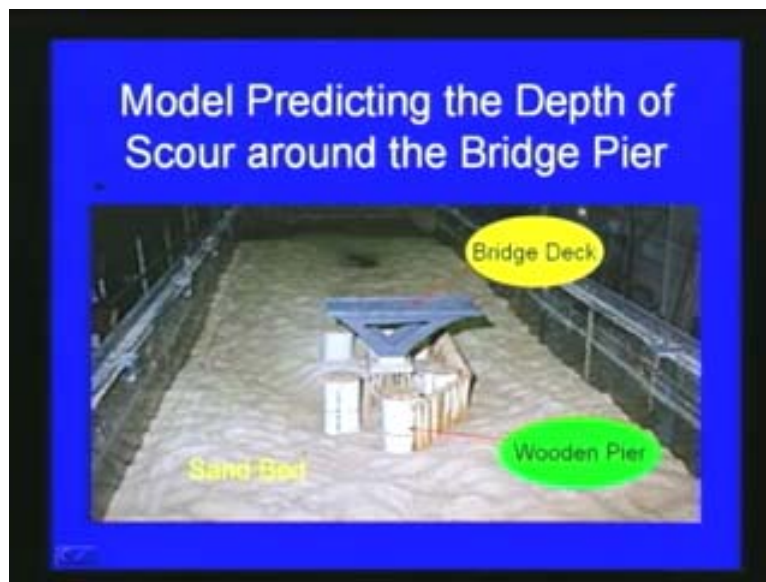


Before closing this sections some of the models we were discussing first one as I mentioned earlier one is the sump pump model. This modeling has been in our hydraulics

laboratory this model has been built to 1:8 scale here, you can see the pumps are located here and channel the formal condition to similar the format the conditions this is a flow coming. Our aim here is to see the disturbance whether the format given is ok or the turbuance generation is what happens in this sump where the pumps are taking water.

The distorted, as I mentioned the theoretical development or application theory or mathematical modeling is very difficult in the kinds of power like sump pump but, if you do correspondingly a good model is developed then, you can see the all the turbulent or all the problems with respect to sump we can easily add in the file. This model has been based up on the Froude number similarity, we have a scaling of 1:8, we have pumped that the existing design is okay, with respect to the pumping not much disturbing are generated, that is what we proved through this sump pump model.

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Second case is model predicting the depth of scour around the bridge pier. Here you can see a bridge, a bridge pier is here, this is the channel is there and this sand bed. This modeling is also done based upon the Froude number similarity, you can measure the various parameters with respect to the scour effect on the pier and this is the model with respect to bridge pier covering.

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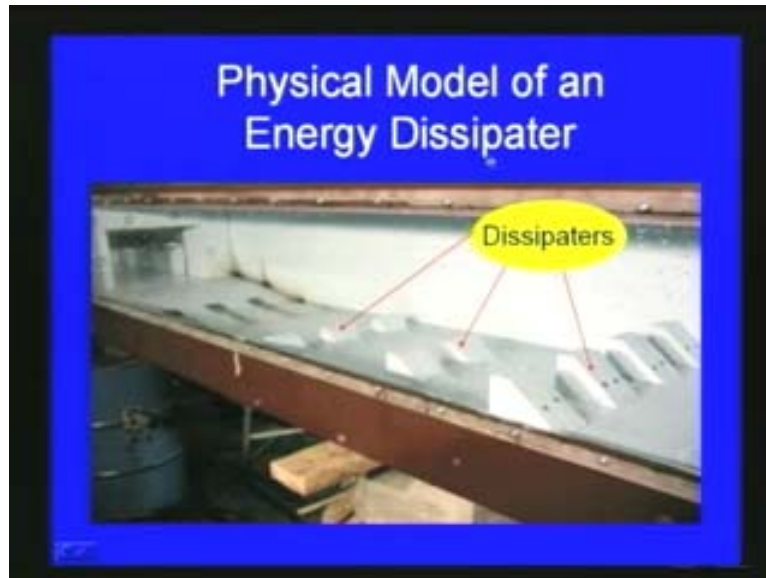
Then, as I mentioned most of the time we will be doing, before constructing a dam spillway we will do physical modeling that is very important since that gives lot of inputs before construction of the dam or spillways. Here, you can see, we use the Froude number similarity here is the reservoir, the dam with respect to spillway with can construct model going to the appropriate scale this shows a dam model with spillway.

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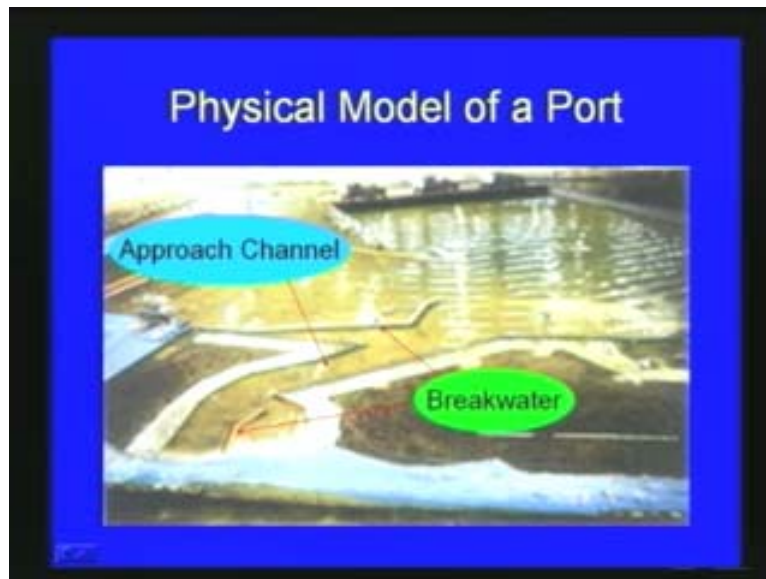
Third case: here you can see physical model of a river either a straight channel, we can construct like this in the laboratory using the Froude number similarity and you can see this here you can use a distorted model. Since the depth will be different, other parameters length and width will be different, this is the meandering channel of river here you can see physical model distorted model with respect to river.

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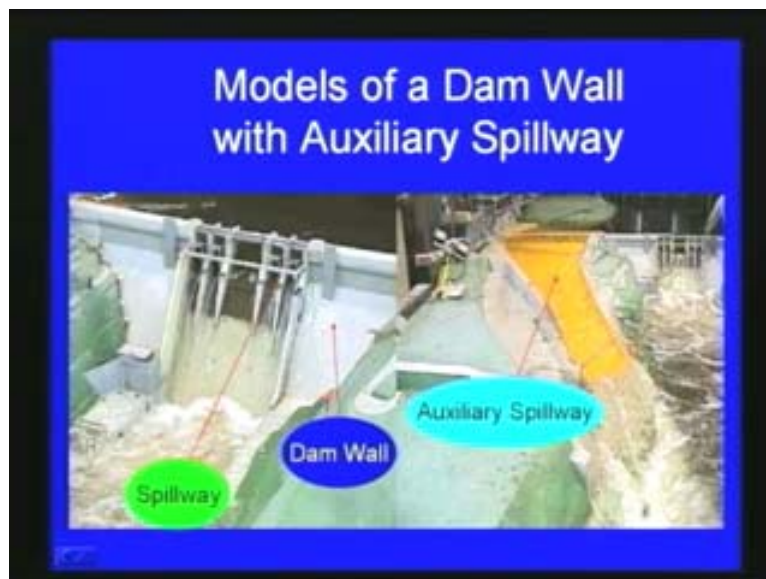
This shows the physical model of energy dissipater energy dissipation, we use the down shade of the dam and you can see all the energy dissipater placed in the model, we will do the simulation.

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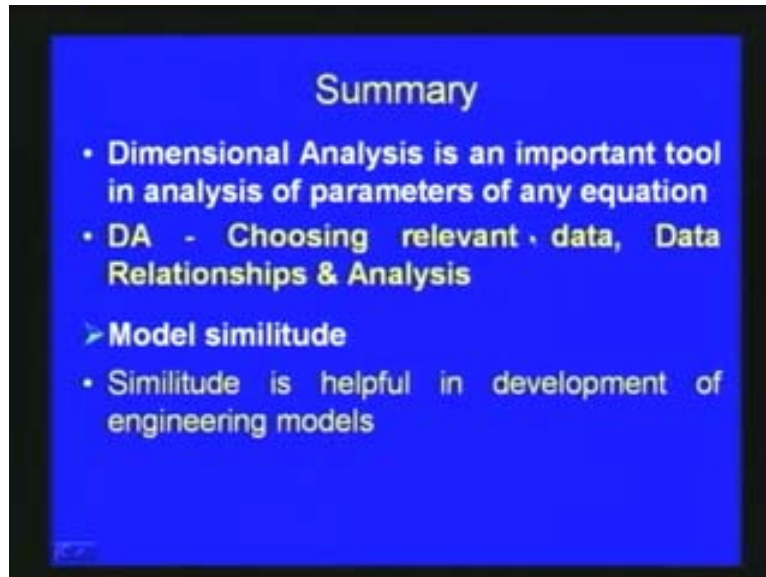
Then, next case is physical model of a port you can see that, the various channels are replicated and breakwater is replicated here in the model appropriate scaling, we can see with respect to a ship comes and if a new construction is made then what happens is waves also will be similar to here and in case tidal effect are there that also simulated in this kinds of model.

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Here this shows the model of the dam wall with auxiliary spillways this also as I mentioned very commonly **used**, we will do while before constructing spillways, we will be doing the modeling force spillway this shows how we do for a spillway.

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Finally to summarize this chapter, we have seen the dimension analysis, the important of dimension analysis, we have seen the methodologies like Raleigh's method Buckingham pi theorem also we have seen the importance of similarity or similitude. As we have seen here the dimension analysis is an important tool in analyze of parameters of any equation and using dimensional analysis, we can choose the relevant data and data relationships and analysis. The model similarity or model similitude with respect to geometrically or kinematically or dynamically it is very important similitude is very helpful in the development of engineering models as we have already seen in this chapter. Finally, the model scales, we have to choose the model scale appropriately either a Reynolds model law or Froudes model law or other kinds of model law, we can utilize and also model sometimes model can be distorted and sometimes depending up on the case model can be distorted model. Where in the case especially in the case of distorted model, we have to consider the scale effects and we have seen the theory and application of this and finally, as a final word the model studies helps in understanding a physical system, we can analyze the system in a very simple way the prototype the real case. We are representing

replicating the laboratory scale, we are studying it finally results we are getting are transferred with respect to the prototype designer construction or the operation of the prototype. So, this is about the dimensional analysis and theory. Further in this course, will be discussing **Neiverstocks'** equations, applications and the drag lift or the boundary layer theories and the pipe flow.