

**Fluid Mechanics**  
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**Lecture - 24**  
**Dimensional Analysis**

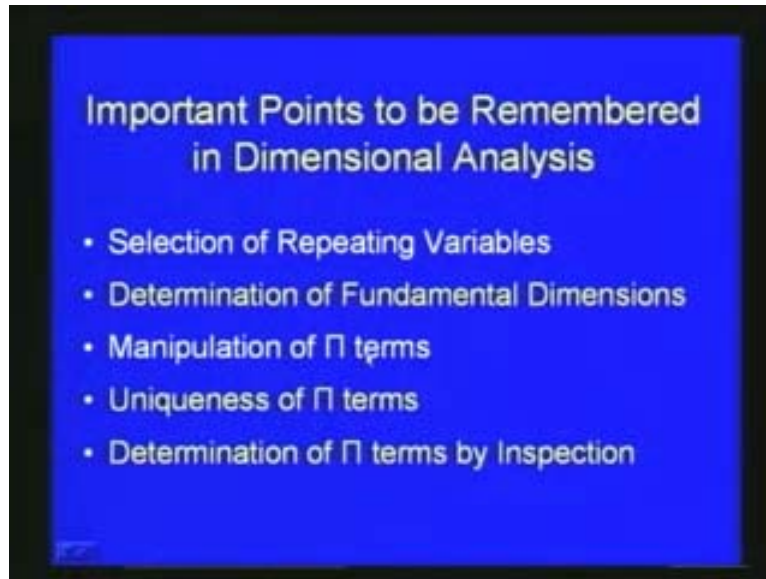
Welcome back to the video course on fluid mechanics. In the last lecture we were discussing about the dimensional analysis similarity and simply to a model studies, we have seen what are the importance of dimensional analysis and we have studied we have discussed two important methodologies as far as dimensional analysis concerned, one is Rayleigh's method and second one is Buckingham pi theorem.

We have seen, how we are doing the dimensional analysis with respect to the Buckingham pi theorem what are the importance of each pi term and how we are getting the dimensional terms with respect to the pi terms described in the Buckingham pi theorem.

Before going further, we will just see the importance points to be remembered while doing this dimensional analysis using Buckingham pi theorem which is the one of the most commonly used methodology. The importance points to be remembered in dimensional analysis while doing by using Buckingham pi theorem, first one is the selection of repeating variables. The number of variables will be repeating with respect to each pi term that, we should be how we select; second one is determination of fundamental dimensions.

We have seen that either  $mlt$  mass length time or  $flt$  force length time these are the fundamental dimensions we use, with respect to this we have to decide which the fundamental dimensions we use.

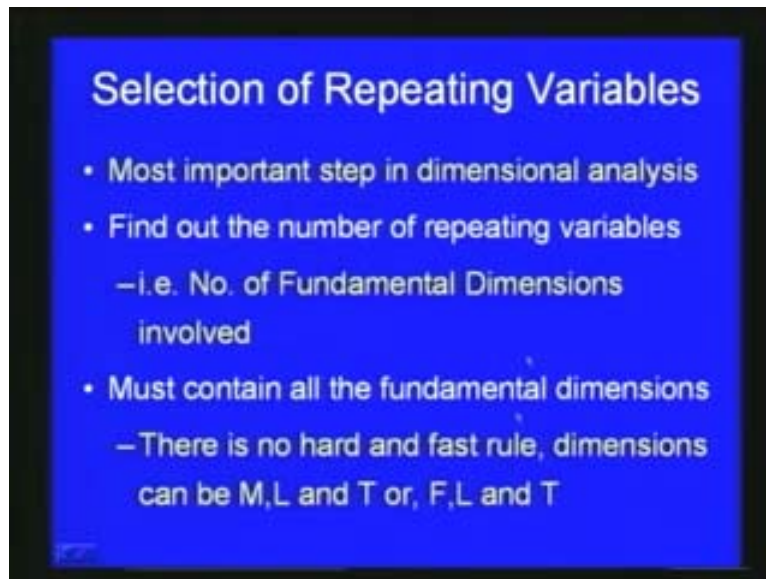
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Next one is manipulation of pi terms, we have seen how we are forming each pi term in the dimension analysis we may have to manipulate this pi terms uniqueness of pi terms.

With respect to the analysis whether the pi terms which are using unique in nature or the uniqueness, we have to test determination of pi terms by inspection, these are the important points to be remembered in dimension analysis using the Buckingham pi theorem.

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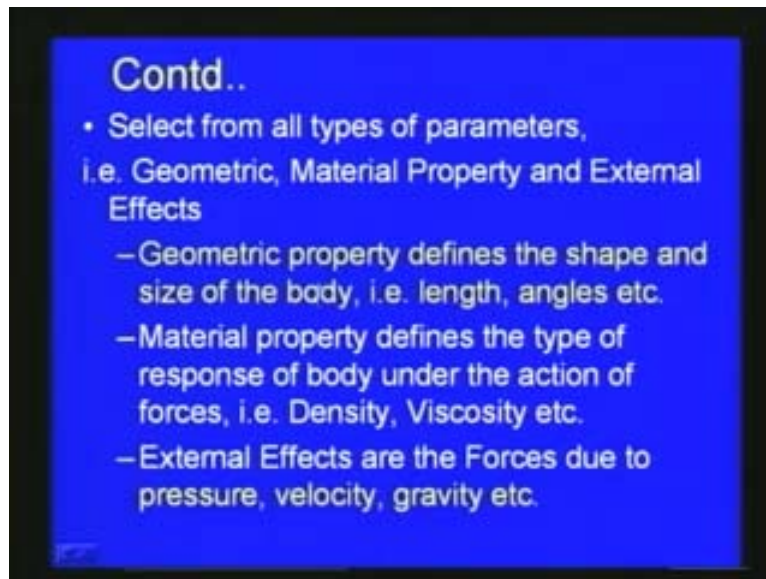


As far as the first point selection of repeating variables are concerned, this is one of the most important steps in dimensional analysis. Since we have been identifying which are the terms repeating how we will be utilizing it, here we are finding out the number of repeating variables that is number of fundamental dimensions involved?

This must contain all the fundamental dimensions in the analysis all fundamental dimensions covered there is no hard. As far as we select the repeating variables which include the fundamental dimensions, generally, it can be mass length and time or force length and time.

either we can use MLT system or FLT system the selection of repeating variable is one of the most important step As far as dimension analysis the using the Buckingham pi theorem, next one we have to select from all types of parameters.

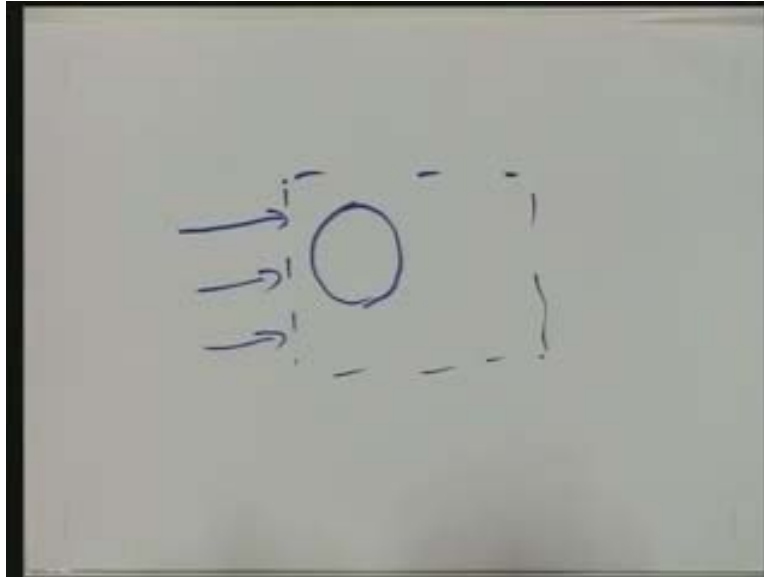
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Like whether in the dimension analysis dealing with the geometric material property and external effects, with respect to all this we have to select all the important parameters. As far as geometric properties are concerned we have to define the shape and size of the body length angles etc.

When we do the modeling we have to see that whether we are going to construct for example if you are considering the flow over cylinder the with respect to this if we are doing a model, we have to see what will be the dimensions which we are going to modeling this the size of the cylinder and all other parameters we have to define.

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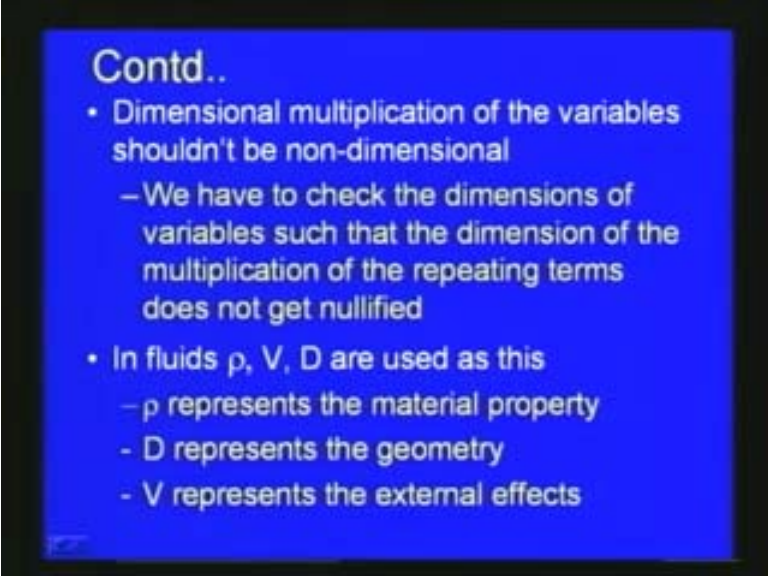
Geometric properties define the shape and size of the body that is length angles extra and second one is the material property, this material property defines the type of response of body under the action of forces, like which in a fluid mechanics we will be dealing with very this kinds of fluids .

The density of the fluid the viscosity of the fluid all these things related to material property we have to select and consider appropriately in the dimensional analysis with respect to the methodology we use. External effects are the forces due to the pressure velocity gravity extra.

When we do the dimension analysis, we have to see that how the forces are acting or how the pressure is varying or how velocity and gravity are affecting the particular model or particular study which we do, these are some of the when we select the different types of parameters, we have to take care the geometric property material property and the external effects. Then next point is the dimension multiplication of the variables should not be non-dimensional.

When we in the dimensional analysis Buckingham pi theorem we have see how we are doing the various groups will be multiplied, we have to see that we have to check the dimensions of variables such that.

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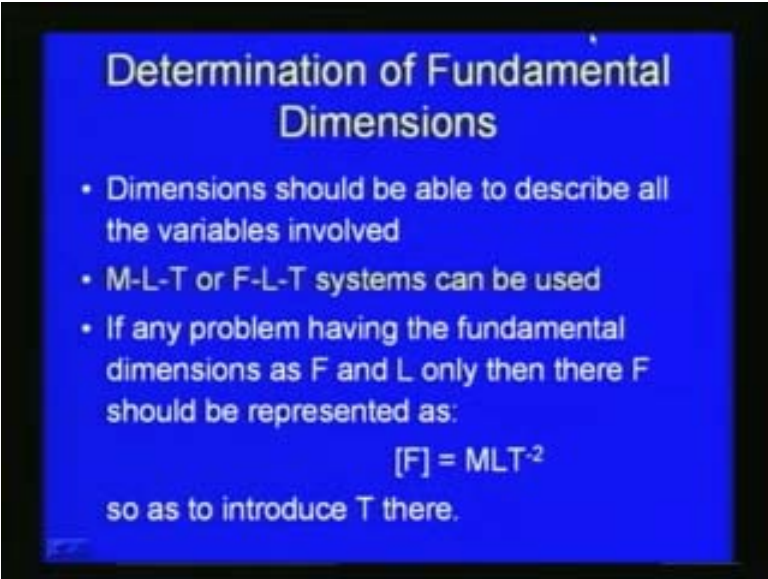


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- Dimensional multiplication of the variables shouldn't be non-dimensional
  - We have to check the dimensions of variables such that the dimension of the multiplication of the repeating terms does not get nullified
- In fluids  $\rho$ ,  $V$ ,  $D$  are used as this
  - $\rho$  represents the material property
  - $D$  represents the geometry
  - $V$  represents the external effects

Dimension of the multiplication of the repeating terms does not get nullified this is another important point the dimension of the multiplication of the variable should not be non-dimensional. The next point is in fluids the density velocity and term  $D$  which represents the geometry are used the rho represents the material property  $D$  represents the geometry and  $v$  represents the external effects, this parameter we have to take care in the case of dimension analysis using the Buckingham pi theorem.

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**Determination of Fundamental Dimensions**

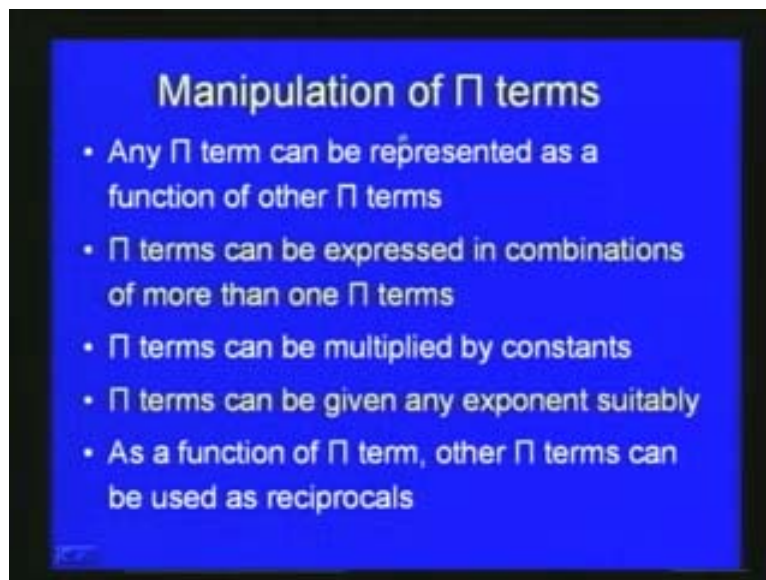
- Dimensions should be able to describe all the variables involved
- M-L-T or F-L-T systems can be used
- If any problem having the fundamental dimensions as F and L only then there F should be represented as:  
$$[F] = MLT^{-2}$$
so as to introduce T there.

The other important point is we have to decide the fundamental dimensions which will be using the dimensions in the Buckingham pi theorem or this methodology dimensions should be able to describe all the variables involved MLT mass length time or FLT force length time systems can be used, depending up on the case whether we can MLT or FLT systems and if any problem having the fundamental dimensions as F and L only then there F should be represented as F is equal to MLT to the power minus 2 that means mass into acceleration force is equal to mass into acceleration .

This we have to represent the force in this form F is equal to MLT to the power minus 2 as to introduce the time component there, these are some of the important factors when we decide the fundamental dimensions either MLT or FLT systems and next point is the manipulation of pi term.

When we discuss the Buckingham pi theorem and when earlier we have seen how to solve a particular problem with using a Buckingham pi theorem, we can we have seen that some of the pi terms we can manipulate to get appropriate form of dimensions form, with respect to this the manipulation of pi term .

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Possible in the Buckingham pi method any pi terms some of the important point. As far as manipulation of pi terms or any pi terms can be represented as a function of other pi

terms, we have represented the pi terms with respect to as a function of pi can be here we have already see pie as pie1 pie2 pie3 extra out of this number of pi terms, we can represent each with respect to other pi term.

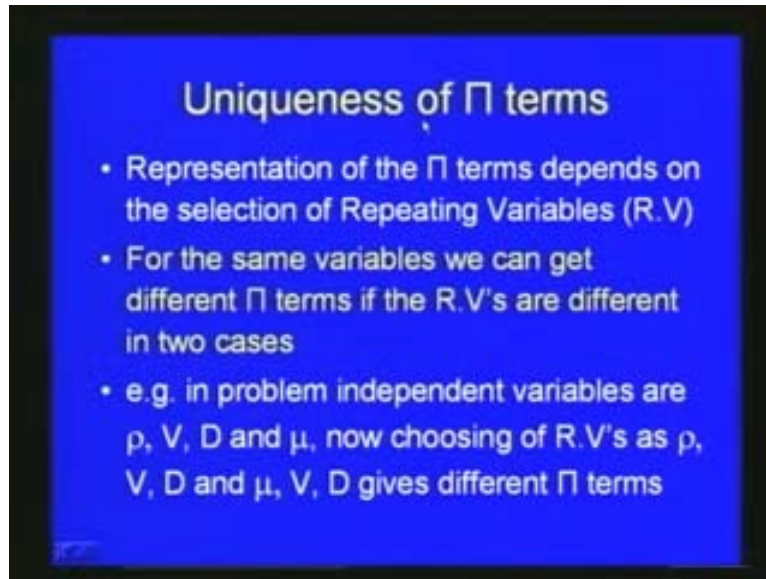
Any pi term can be represent as a function of other pi terms and pi terms can be expressed in combinations of more than one pi terms, this is also possible then third point is pi terms can be multiplied by constants.

We can represent each pi term by multiplying by another constant term next point pi terms can be given any exponent suitably, we can we represent the pi terms here you can see that, we can put with respect to abc or we can multiply with respect to by using a constant for pi terms can be given exponent suitably and as a function of pi term other pi terms can be used as reciprocals also.

While doing the dimensional analysis Buckingham pi theorem, the pi terms can be manipulated define ways like we can represent one pi term with respect to other pi terms or we can multiply with respect to constant or reciprocals can be used or we can suitably give the exponents such that, we get appropriate form of the dimensional form or the appropriate equations can be formed this importance. As far as Buckingham pi theorem is concerned here we can manipulate the pi terms at different ways that we will get suitable forms. Next point is.



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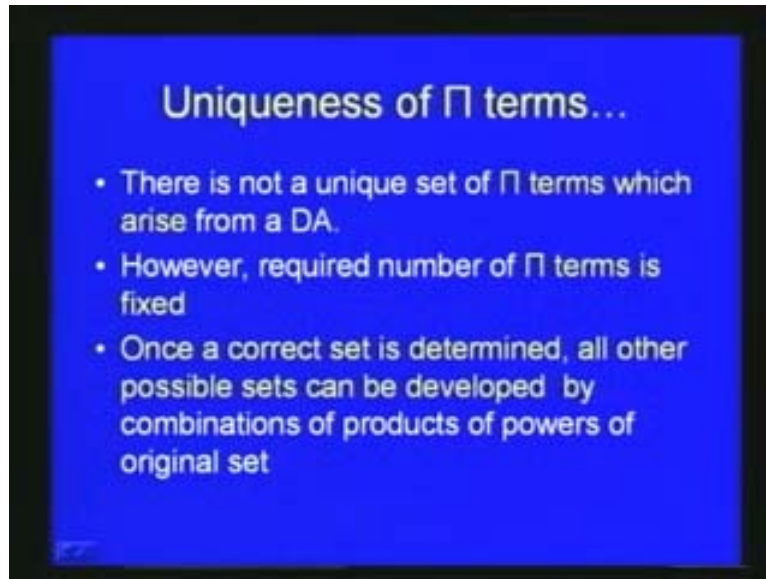


The uniqueness of pie terms, we have seen in the Buckingham pi theorem analysis, each pie term depends up on the selection of the repeating variable and we have seen there are certain repeating variables.

In this the pie terms depends up on the selection of which have the repeating variables for the same variables, we can get different pie terms if the repeating variables are different in two cases, for example, if two cases the repeating variables are different then the same variable can get a different pi terms, for example here you can see problem independent variable are rho the density v is the velocity D is the geometric like diameter length or breath and mu is the viscosity. Now choosing the repeating variables we can use as rho VD or rho and rho VD gives different pie terms, here if you choose rho VD we can choose in different way that it will give different pi terms.

As we can see there is not a unique set of pie terms which arise from dimensional analysis in the dimension analysis, we can see that there is no unique set of pi terms however required number of pi terms is fixed.

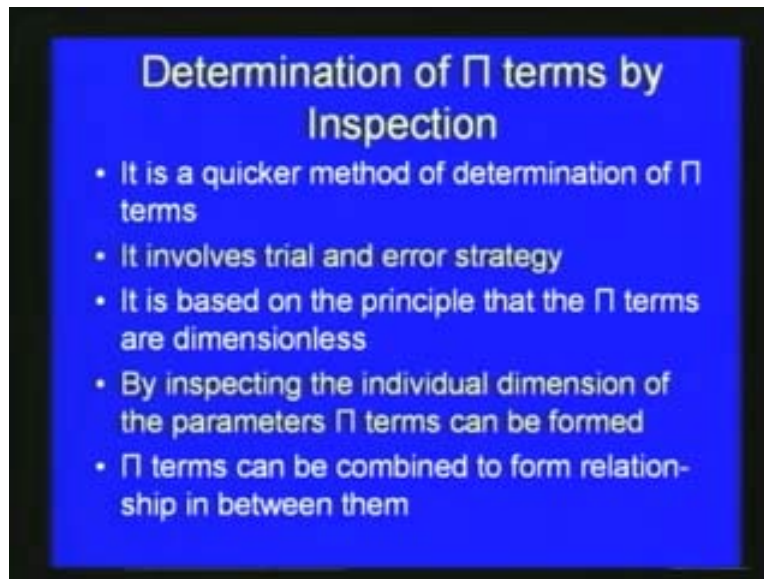
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In the analysis which we have seen the basic steps, we have seen earlier the number of pie terms is fixed, there is not a unique set of pie terms which is coming from the dimensional analysis, once a correct set is determined all other possible sets can be developed by a combination of products of powers of the original set, what we do once the different pie term are defined and required number of pie term is fixed.

What we can do with the correct set which we required we can determine by a combination of the products of the powers of original set, this is the uniqueness of the pie terms.

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This uniqueness is also should meet while doing the dimensional analysis using Buckingham pi theorem. Next one is the determination of the pie terms by inspections, while doing the dimensional analysis was using the Buckingham pi theorem, we can dome inspection or we can check and we can determine the pie terms which way it is coming here it is quicker method of determination of pie terms by inspection.

It involves trial and error strategy. It is based on the principle that pie terms are dimensionless by inspecting the individual dimensions of the parameters pie terms can be formed and pie terms can be combined to form relationship between them. Once we decide the repeating variables the fundamental dimensions while inspecting the particular while checking the particular problem.

We can through inspection through trial and error we can decide how the pie terms formed or which way we can combine each pie term, these are some of the important points which we should take care while doing the dimensional analysis using the Buckingham pi theorem.

Even though theme times we use a releighs methods as we have discussed earlier Rayleigh's method has got some limitations if the number of parameters are variables increases then realize method is not suitable, we have to go for the Buckingham pi

theorem. We have seen how we are doing dimensional analysis using the Buckingham pi theorem, different possibilities are there, we can manipulate the pie terms and we can check the uniqueness with respect to the pie terms. We have to see that the appropriate the groups are formed and we can use the Buckingham pi theorem for the dimensional analysis. These two important methodologies are used in the dimension analysis further. There are number of dimensionless numbers or groups in fluid mechanics which is formed by various aspects and various theories. Now we will discuss the common dimensionless groups in fluid mechanics.

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**Common Dimensionless Groups in Fluid Mechanics**

- Reynolds Number Ratio of Inertia Force to the Viscous Force, expressed as:  
$$Re = \frac{\rho V \ell}{\mu}$$
- Froude Number Ratio of Inertia Force to the Gravity Force,  
expressed as:  $Fr = \frac{V}{\sqrt{g \ell}}$

Here this slide we can see the here we will discuss the various dimensional group. first one is the Reynolds number, we have already seen the importance of Reynolds number in fluid mechanics by desiring by determine Reynolds number we can see that the flow is whether laminar or turbulent or how various fluid flow parameters. We can classify the flow according to the Reynolds number Reynolds number is one of the most important dimensionless groups in a fluid mechanics.

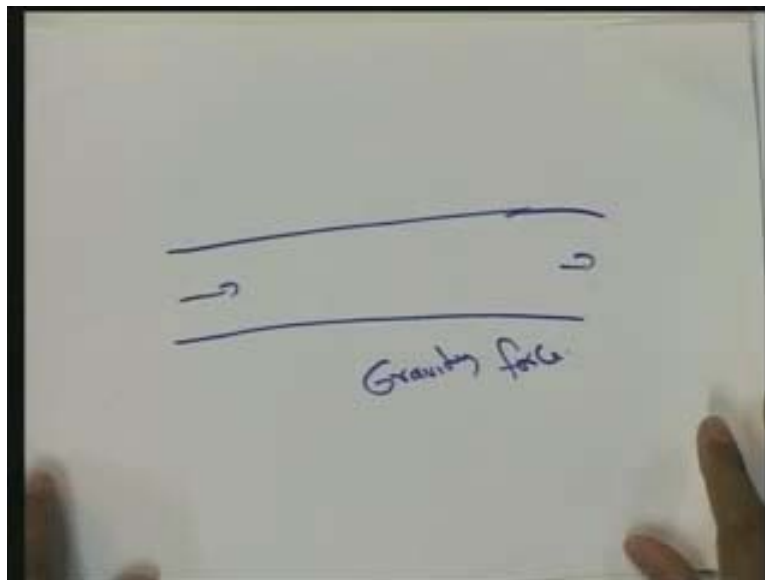
This Reynolds number is ratio of inertia force to the viscous force, two important forces in most of the fluid flow is inertia force and second one is viscous force, the Reynolds number gives a ratio of inertia force to viscous force and Reynolds number is expressed

as  $Re$  is equal to  $\rho V l$  by  $\mu$ . Where  $\rho$  is the fluid density  $v$  is the velocity  $l$  is the characteristics length depending up on the case whether it is length or it is for example pipe flow  $l$  is represent as  $D$  the diameter of the pipe.

For open channel flow it is length dimension,  $\rho V l$   $u$  is the coefficient of dynamic viscosity the dimensional as group Reynolds number find as ratio of the inertia force to viscous force and expressed as  $\rho V l$  by  $\mu$  second number is froude number froude number is the ratio of inertia force to the gravity force.

Especially in the case of open channel flow we can see that the fluid flow is governed by the gravity force, for example if we consider a flow through channel like this you can see that the with the inertia force the another important aspect is the gravity force here the Froude number is another important dimension as loop especially used in open channel flow.

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It is the ratio of inertia force to gravity force and it is expressed as the froude number  $Fr$  is equal to  $V$  by  $\sqrt{g l}$  where,  $l$  is the characteristics length it can be depth or it can be various other parameter, depending up on which may be other dealing the problem it is the froude number  $FR$  is equal to  $V$  by  $\sqrt{g l}$  or  $l$  can be in the case of open channel flow  $t$  can be depth of flow.

The dimensional number is second one is froude number this is also very important where ever the flow is governed by especially gravitational effects are there that this based up on this dimensionless number will be making models in fluid mechanics. Third number is called Euler number.

Euler number it is the ratio of pressure force to inertial force. Eu is equal to  $p$  by  $\rho V$  square where  $p$  is the pressure  $\rho$  is the mass density  $v$  is the velocity, wherever we deal with the fluid flow where the pressure is we have to consider effectively there we define this dimensional number Euler number.

Next important dimensions number is called a mach number, Mach number is the ratio of inertial force to the compressibility force, here this is defined as  $m$  is equal to  $V$  by  $c$  where  $v$  is the velocity of flow  $c$  is the speed of sound. This number Mach number is used wherever the compressibility effect is very important especially in aerodynamics we use Mach number.

Where  $c$  is the speed of sound which is with respect to this speed of sound only define the Mach number. Another important number is called a dimensionless number is called Weber number. Weber number is the ratio of inertial force to the surface tension force.

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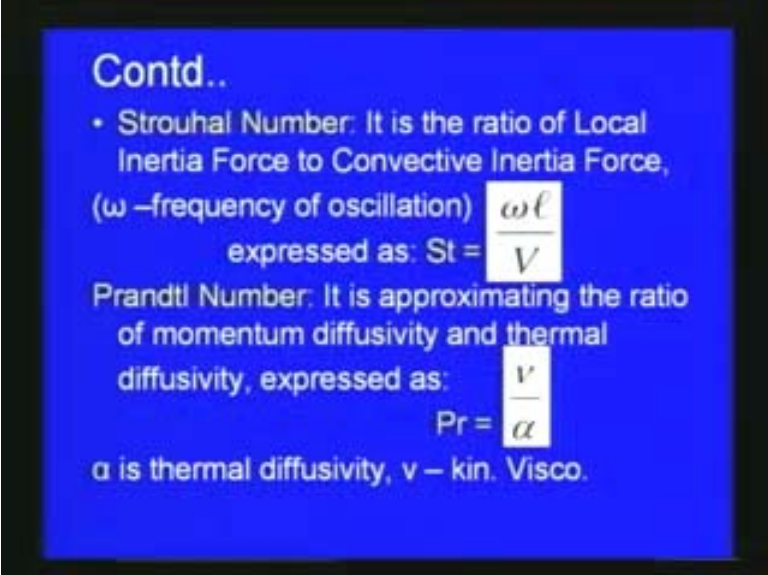
- Weber Number: Ratio of Inertia Force to the Surface Tension Force, expressed as:  
$$We = \frac{\rho V^2 \ell}{\sigma}$$
- Cauchy Number: It is the square of Mach Number, expressed as:  
$$Ca = \frac{\rho V^2}{E_v}$$
  
-  $E_v$  - bulk modulus

Many fluid mechanics problem surface tension is important here we can use this weber number the dimensional number weber number. O this Weber number is defined as  $\rho v^2 l$  by  $\sigma$  where  $\rho$  is the density of the fluid  $v$  is the velocity  $l$  is the characteristic length and  $\sigma$  is the surface tension force.

Here Weber number is defined as  $\rho v^2 l$  by  $\sigma$  another important dimensional number is Cauchy number. Cauchy number is the square of mach number we have already seen the Mach number.

Cauchy number is defined as the square number and it is defined as  $Ca$  is equal to  $\rho V^2$  square by  $E_v$  where  $v$  is the velocity  $\rho$  is the density and  $E_v$  is the bulk modulus, here also wherever the cases of compressibility effect is much more important with Cauchy the dimension is Cauchy number is utilized some other numbers like dimensional number. Another important number is called Strouhal number. Strouhal number is, the ratio of local inertia force to convective inertia force where ever the convective forces are important here we use the strouhal number.

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- Strouhal Number: It is the ratio of Local Inertia Force to Convective Inertia Force, ( $\omega$  –frequency of oscillation) expressed as:  $St = \frac{\omega l}{V}$
- Prandtl Number: It is approximating the ratio of momentum diffusivity and thermal diffusivity, expressed as:  $Pr = \frac{\nu}{\alpha}$   
 $\alpha$  is thermal diffusivity,  $\nu$  – kin. Visco.

Strouhal number  $St$  is equal to  $\omega l$  by  $V$  where  $\omega$  is the frequency of relation  $l$  is the characteristics length and  $v$  is the velocity, Strouhal number is the ratio of local inertia force to convective inertia force. Another important number is prandtl number.

Prandtl number is it approximates the ratio of momentum diffusivity and thermal diffusivity wherever the thermal effect also considering fluid mechanics here we use the Prandtl number, Prandtl number is defined as  $Pr$  is equal to  $\frac{\mu}{\rho \alpha}$  where,  $\mu$  is the kinematics viscosity and  $\alpha$  is the thermal diffusivity this number is very important wherever the thermal effects also consider in fluid mechanics this is the prandtl number  $pr$  is equal to  $\nu$  by  $\alpha$ .

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- Fourier Number: It is the ratio of kinematic viscosity or diffusivity to advection, expressed as:  $Fo = \nu / UL$

Peclet Number: It is the ratio of advection to diffusion (product of Reynolds & Prandtl number), expressed as:

$$Pe = \frac{UL}{\alpha}$$

$\alpha$  is coefft. diffusion

Another important number is called Fourier number dimensions number Fourier number is the ratio of kinematics viscosity of diffusivity to advectons, here the Fourier number is defined as  $\frac{\nu}{UL}$  where  $\nu$  is the kinematics viscosity  $U$  is the velocity and  $l$  is the characteristic length which we consider.

Fourier number is defined as  $Fo$  is equal to  $\frac{\nu}{UL}$  another important number is called Peclet number. Peclet number is dealing with the wherever the advection is also important peclet number is the ratio of advection to diffusion effect this is actually the product of Reynolds and prandtl number.

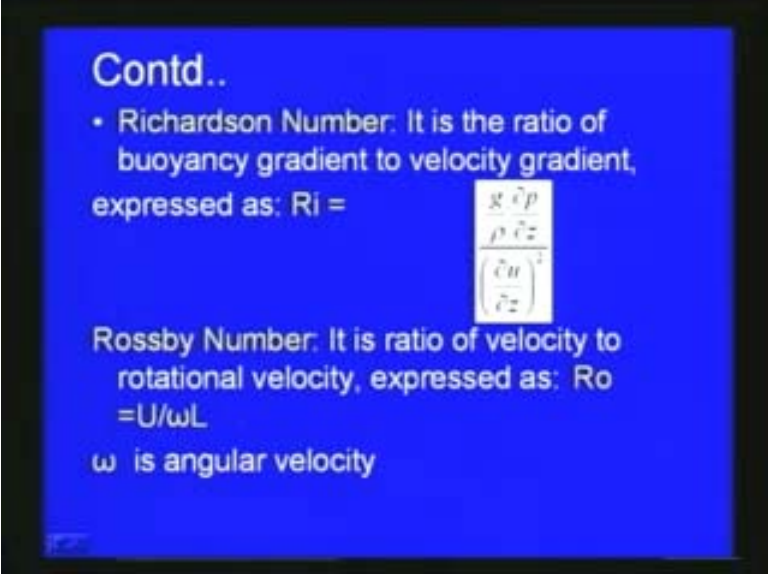
The peclet number is defined here  $Pe$  is equal to  $\frac{UL}{\alpha}$  where  $u$  is the velocity  $l$  is the characteristic length and  $\alpha$  is the coefficient of diffusion, this Peclet number is very important especially when we do numerical modeling wherever especially transport



equation. Transport of various particle or contaminant or concentration with respect to this when we numerical modeling we will be checking the Peclet number.

That we can see that the numerical modeling which developed will be stable with respect to the various other parameters, this is called a peclet number and another important dimensional is number is called Richardson number.

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- Richardson Number: It is the ratio of buoyancy gradient to velocity gradient, expressed as:  $Ri = \frac{g \frac{\partial \rho}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2}$

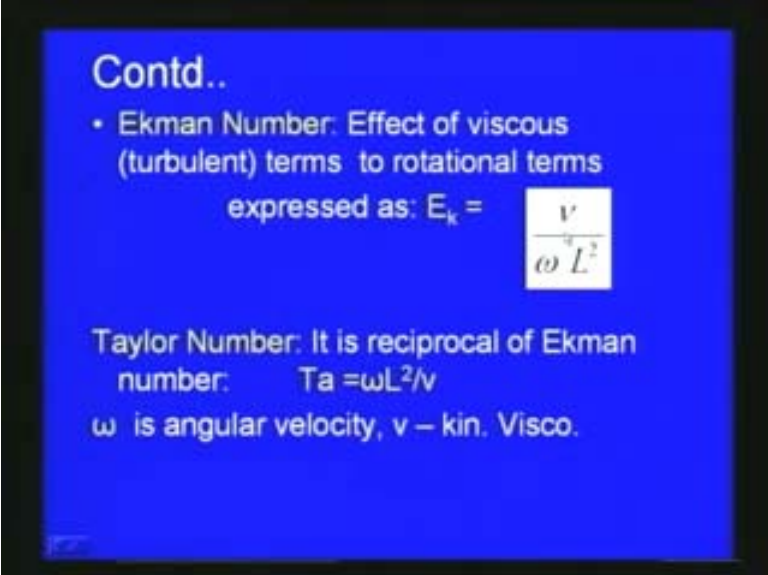
Rossby Number: It is ratio of velocity to rotational velocity, expressed as:  $Ro = U/\omega L$   
 $\omega$  is angular velocity

Wherever the buoyancy effect is very important this number is used, Richardson number is the ratio of buoyancy gradient to velocity gradient and this is expressed as Ri is equal to g by rho del p by del z divided by del u by del z whole square, this is the Richardson number. Another important number is called rossby number.

Rossby number is wherever the rotational force is when we deal with rotational flow will be using this dimensionless number Rossby number, here, this is the ratio of velocity to rotational velocity, this is expressed as Ro is equal to U by omega L where U is the velocity omega is the angular velocity and l is the characteristic length which be considered two more important numbers which we generally using in fluid mechanics is Ekman number.

Wherever the rotational effects are considered here Ekman number is the effect of this terms to rotational terms at it is defined as  $E_k$  is equal to  $\mu$  by  $\omega L^2$  where,  $\mu$  is the kinematics viscosity  $\omega$  is the angular velocity and  $L$  is the characteristic length and wherever this rotational flow considered the reciprocal of Ekman number is defined as Taylor number it is  $\omega L^2$  by  $\mu$  it is  $Ta$  is equal to  $\omega L^2$  by  $\mu$ .

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- Ekman Number: Effect of viscous (turbulent) terms to rotational terms expressed as:  $E_k = \frac{\nu}{\omega L^2}$

Taylor Number: It is reciprocal of Ekman number:  $Ta = \omega L^2 / \nu$   
 $\omega$  is angular velocity,  $\nu$  – kin. Visco.

This is also use wherever deal with the rotational flows, there are few more numbers which are using depending up on whether various other branches of fluid mechanics. but this are the few numbers like Reynolds number, the Froude number Weber number, Cauchy number, Prandtl number, Fourier number, Peclet number, Richardn number Rossby number, Ekman number and Taylor number, these are some of the important dimensions numbers which we use in fluid mechanics but few more dimensional is numbers are there.

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Dimensionless Terms	Application
Reynolds Number	All types of flow
Froude Number	Flow with Free Surface
Euler Number	Pressurized Flow
Mach Number	Compressible Flow
Weber Number	When Surface Tension is important
Cauchy Number	Compressible Flow
Strouhal Number	Unsteady Flow Analysis

Let us see with respect to this dimensional numbers where are the applications we have seen each dimensionless groups of dimensions numbers, it is the ratio of some force terms to other terms like that it is a ratio between two terms depending up on the problem we have to choose particular set of dimensionless number.

That we can further critically analyzing that particular problem with respect to this dimensionless numbers, let us here discuss the applications the corresponding dimension table the dimensionless number these terms are return and corresponding application is return.

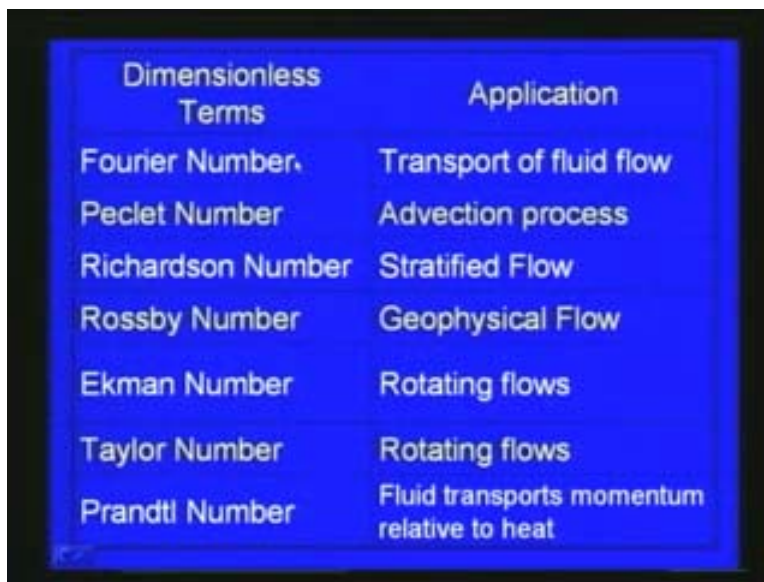
First one is the Reynolds number Reynolds number is as we have seen that this numbers gives a classification with respect to laminate of general flow and it is used in all fluid mechanics problems. But especially when we deal with the close flow like a pipe flow we directly defined the Reynolds number with respect to that classify and also open channel flow also we can utilize the Reynolds number. Second one is the Froude number Froude number the application is wherever the flow with free surface is involved.

Like we have seen the open channel flow open channel flow wherever the free surface is important there we use the Froude number. The third number is the Euler number, Euler number we have seen it is for wherever the pressure flow is considered in the case of

pressurized flow Euler number is important and there will be considering Euler number and Mach number is considered especially around a dynamics flows where compressible flows are important there we consider the Mach number. Weber number is concerned especially whenever surface tension is important for the flows for the fluid mechanics problems wherever the surface tension is important we consider the Weber number

The Cauchy number also wherever the compressible flow is considered the Cauchy number and strouhal number wherever the unsteady effect is we have to critically analyze that case we use the Strouhal number for unsteady flow analysis few other dimensionless terms of numbers which we have already discussed, for example Fourier number wherever.

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Dimensionless Terms	Application
Fourier Number	Transport of fluid flow
Peclet Number	Advection process
Richardson Number	Stratified Flow
Rossby Number	Geophysical Flow
Ekman Number	Rotating flows
Taylor Number	Rotating flows
Prandtl Number	Fluid transports momentum relative to heat

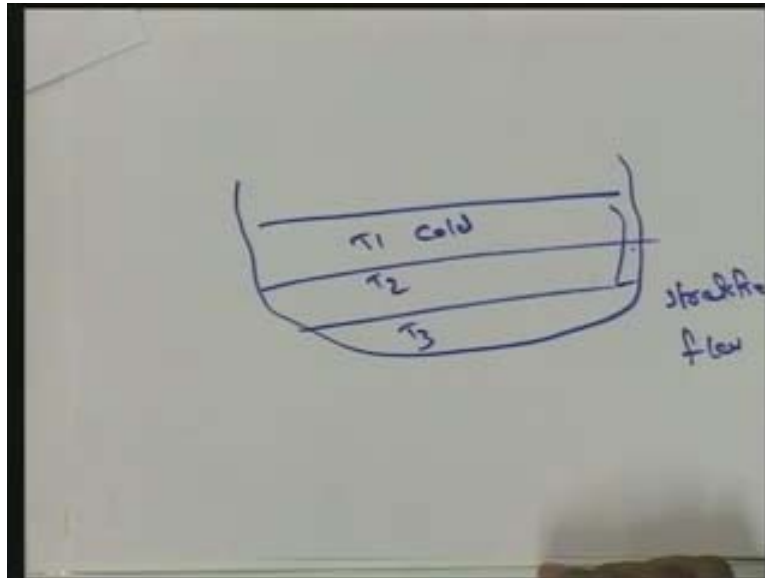
With the transport of fluid flow there in that case Fourier number is important and we use the Fourier number peclet number. As I mentioned this shows the advection defuses process especially advection process is important there we use the peclet number.

Also used in numerical analysis numerical modeling like fantail method checking the peclet number, see that the numerical method is stable. Other numbers like Richardson number we use for wherever the fluid is stratified flow like same due to temperature

effects, for example you can consider fluid flow process in a lake you can see that here we have like with respect to the temperature that can be stratification.

For stratified flow is concerned we this Richardson number, is very important one will be cold water or with respect to the  $T_1$   $T_2$  in the temperature variation is there.

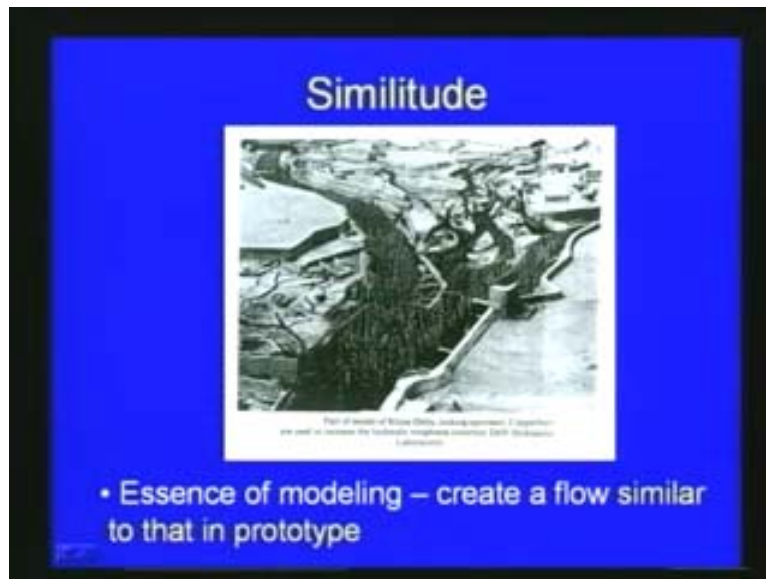
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More cold or less cold that stratification is the in that case the Richardson number is very important. Then the Rossby number the application is geophysical flow like when we consider the flow in a lechery flow or like wherever the geophysical respect to the rotations is important. That kind of flow geophysical flow is concerned rossby number is important and Ekman number we can see that wherever Ekman number layer formation takes place rotating flows this is also important flow the costal flow analysis Ekman number is al used.

Wherever rotating flow is important we use the Ekman number and Taylor number also wherever rotating flow is considered and Prandtl number wherever flow the efficiency of flow transport with respect to momentum relating to heat is considered there, we used the prandtl number. This are some of the important dimensionless groups of number dimensionless numbers are terms which be used in fluid mechanics its applications important applications here we have discussed.

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That is about this dimensionless group of numbers further when we go to the models scales and model studies. Further we will be utilizing theme of this dimensionless numbers we will be finding out the scale ratio from the model scale with respect to this dimensionless numbers, before going to the model scales and scale ratios we will discuss about the similarity similitude which we utilized in dimensional analysis or in fluid mechanics.

As I mentioned most over time much analysis in fluid mechanics, we develop models in the laboratories, there is the prototype or actual field type problem is there with respect to that field problem we will be developing scaled models in the laboratory

While doing this modeling in laboratory fields is physical modeling, we have to see that what kind of similarities is possible with respect to the real field problem and the model which we develop in the laboratory.

We can see that As far as the real field problem is concerned the dimensions length breath width or the depth of flow all this parameters, we have deal with the velocity then discharge then acceleration over time component also we have to deal with the various force coming on that particular problem like inertial forces viscous forces gravity forces

or surface tension forces pressure etcetera, all this parameter when we develop a physical model of this real problem in the field.

We have to see that what kinds of similarities are possible most of the time excess kinds of modeling will be difficult, we may be developing the physical model with respect to certain similarities and other similarities, we may be neglecting while doing this modeling exercise we have to understand what are the important parameter and consider when we develop the model with respect to the real physical problem of with respect to prototype. We have to see which are the dominating parameter and those parameters we have to consider in the scaled model which we developed in the laboratory.

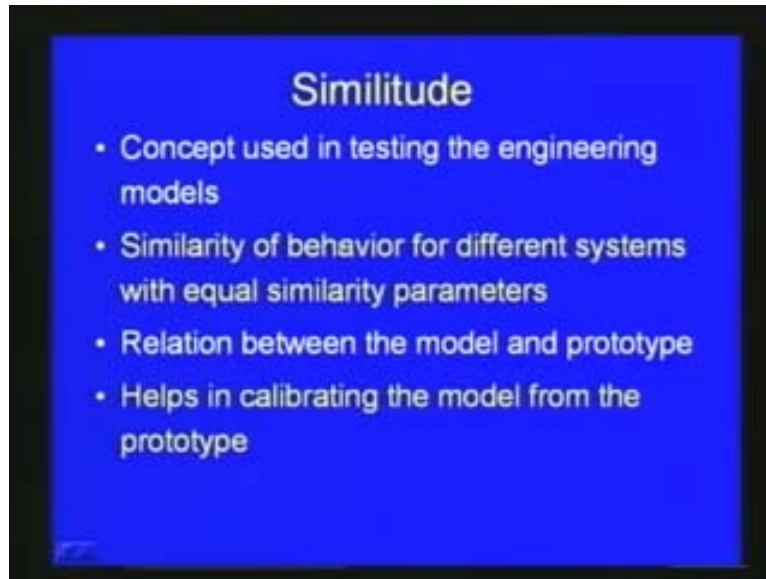
Here you can see in this slide this is the model of the excursion problem wherever a river is joining the sea, here various aspects you can see that the scale is concerned the length scale the width scale depth scale we may not be able to follow the same way.

We have to scale appropriately and also we have to see that the velocity of fluid flow coming from the river to the sea how it is behaving we have to consider various important forces, the essence of modeling here is to create a flow similar to that in prototype that is very important.

Our aim is we are investing lot of money time and doing research to develop a particular model corresponding to a prototype or corresponding to field problem.

When we do this our aim is if you can how much possible we always trying to create very similar kind of with respect to prototype up to the possible level we will be trying. The similitude of the similarity is very important in modeling especially physical modeling in fluid mechanics we will be discuss various aspects of this similitude here.

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The concept used in testing the engineering model as I mentioned when we develop the model this similitude or similarity principles used very much and the similarity of behavior for different systems with equal similarity parameters, we have to consider wide modeling. Then the relation between model and prototype we are representing the prototype with respect model this which is the similarities possible or similitude is possible.

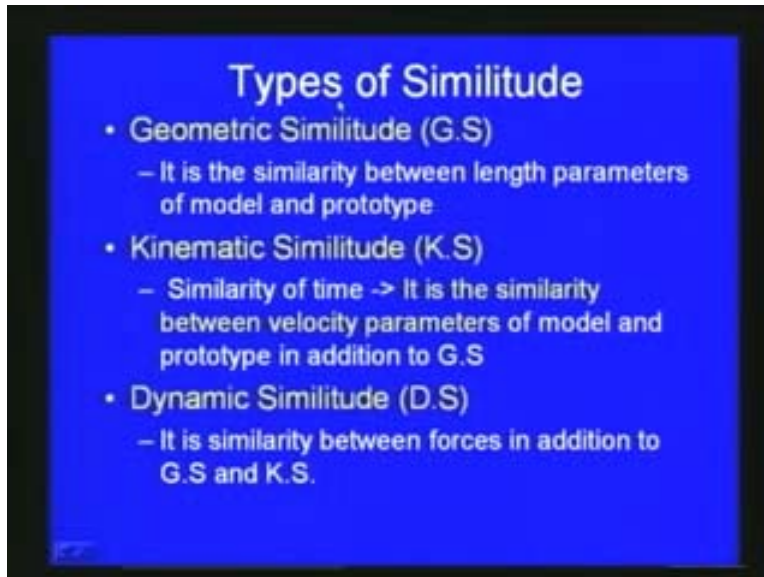
With respect to that only we will be getting various relationships. Then this similitude or the similarity principles helps in calibrating in the model from the prototype with respect to most of the time. We have to calibrate once we develop a model actually in the field problem there will be certain absurd values.

With respect to this absurd values we will be trying to calibrate by varying various parameters that here the calibration process through the calibration process the model with respect to the prototype, we will be developing various relationship this calibration is this similitude which we consider what kind of similitude or similarity or which we consider this helps in calibrating the model from the prototype.



These kinds of physical modeling generally three important similitude or similarity principles are used in fluid mechanics, first one is the geometric similitude, second one is the kinematics similitude and third one is the dynamics similitude.

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Here the geometric similitude means it is the similarity between the various length parameters of the model and prototype the real field problem which we consider with respect to the length or with respect to the breath or the depth various dimensions of the real field problems, how we model or how we do the similarity this is the geometric means length breadth and depth with respect to this parameters, how the similarities adopted called the similitude is adopted.

That is we deal with the geometric similitude. Second one is the kinematics similitude here actually most of the fluid flow phenomena vary with respect to time here this is the kinematics similitude is similarity of time, it is the similarity between the velocity parameters of model or the discharge or the acceleration various parameters which are relevant with respect to or varying with respect to time.

With respect to the model or with respect to the prototype or the real field problem how we can have the similarity or similitude, this kinematics similitude gives this is the kinematics similitude is the similarity of time.

The last one is the dynamic similitude here this dynamic similitude is the similarity between the forces in addition to the geometric similitude in kinematics similitude, the geometric similitude is the fundamental similarity which we consider when we deal with the kinematics similitude.

We have to consider the geometric similitude kinematics similitude is in addition to the geometric similitude and third one is the dynamic similitude, where we have to consider the various forces acting with respect to the prototype or with respect to the real field problem model which are the various forces acting that similitude we are considered. Then with respect to the dynamic similitude we have to also deal with the geometric similitude and the kinematics similitude which we have seen

When a particular model study or particular model with respect to the physical model; when the dynamic similitude is there then that model will be obviously similarity of geometric as well as kinematics the already met while the dynamic similitude is obtained.

While doing the physical modeling, these three important similitude geometric v kinematics similitude and dynamic similitude is considered. As far as the physical modeling is considered, now we will go in some more detail aspect of this various similitude, first one is the geometric similitude.

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**Geometric Similitude**

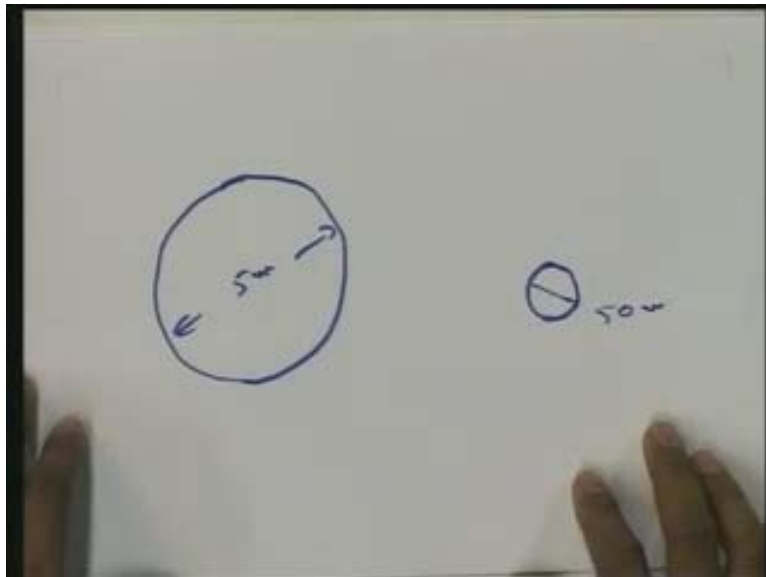
- Two geometrically similar objects must be similar in shape
- The ratio of length parameters are as below:

$$\frac{L_{\text{model}}}{L_{\text{prototype}}} \longrightarrow \frac{L_m}{L_p} \longrightarrow L_r$$
$$\frac{(\text{Area})_{\text{model}}}{(\text{Area})_{\text{prototype}}} \longrightarrow \frac{L_m^2}{L_p^2} \longrightarrow L_r^2$$
$$\frac{(\text{Volume})_{\text{model}}}{(\text{Volume})_{\text{prototype}}} \longrightarrow \frac{L_m^3}{L_p^3} \longrightarrow L_r^3$$

Here you can see that what we are dealing is to geometrically similar objects must be similar in shape, here for example when we deal with the flow over a cylinder you can see that here real field problem will be the diameter will be very large and a cylinder of this glass size is considered, when we do the modeling we consider a small cylinder like this.

For example this can be five meter diameter and here if you consider a one is to ten scale it can be 50 centimeter diameter.

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Here you can see that the shape the physical modeling which we are doing with respect to this flow over a cylinder.

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### Geometric Similitude

- Two geometrically similar objects must be similar in shape
- The ratio of length parameters are as below:

$$\frac{L_{\text{model}}}{L_{\text{prototype}}} \longrightarrow \frac{L_m}{L_p} \longrightarrow L_r$$
$$\frac{(\text{Area})_{\text{model}}}{(\text{Area})_{\text{prototype}}} \longrightarrow \frac{L_m^2}{L_p^2} \longrightarrow L_r^2$$
$$\frac{(\text{Volume})_{\text{model}}}{(\text{Volume})_{\text{prototype}}} \longrightarrow \frac{L_m^3}{L_p^3} \longrightarrow L_r^3$$

Here this is the fluid flow is coming here this is the real case and this is the model and here the real problem, when we do the scaling we have to see that the two geometrically similar objects must be similar in shape that is what this geometric similitude is with respect to this. Generally, we can see that the ratio of various length parameter as like the length of model the ratio of length of model to the length of prototype  $L_m$  by  $L_p$  that is the length ratio  $L_r$  or the area of model to area prototype we can see that area is described as discover of length the model length square divided by prototype length square.

$L_r$  square similarly the volume is considered the volume of model divided by volume of prototype  $L_m$  cube divided by  $L_p$  cube we get the cube of the length ratio. As far as geometric similitude is concerned it can be either the length ratio. As far as length is considered it can be either the length geometric length or width or the depth various concern we will be having a scale ratio law square and volume is considered we will having scale ratio of with respect to  $L_r$  cube.

This is as far as geometric similitude is concerned, now the kinematic similitude as we have seen the time scale is important in the when we considered a time scale then the kinematic similitude is considered.

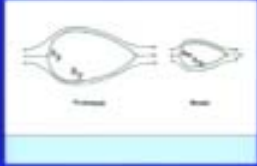
When a geometrically similar points on model and prototype various parameters which deals with the time like velocity discharge or acceleration are in a constant ratio, here we can see that the velocity of the model divided by velocity prototype  $V_m$  by  $V_p$  is the ratio is  $V_r$  and time ratio when we consider time is al  $L_r$  by  $V_r$  that we get the dimension of V is meter per second and L is meter this comes as time.

Time ratio is generally represented as a  $L_r$  by  $V_r$  the acceleration ratio is when we consider here  $V_r$  square by  $L_r$  that is equal to here  $L_r$  by T square  $T_r$  square that meter per second square is concern various parameters like time acceleration this velocity .

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### Kinematic Similitude

- Discharge ratio =  $Q_m/Q_p = Q_r = L_r^3/T_r$
- Also, paths of moving particles are geometrically similar
- Hence streamline patterns are the same



Then you can see the discharge is concern discharge ratio is  $Q_m$  by  $Q_p$  that is the with respect to the stage of the model divided by this state of the prototype that is the ratio that is equal to  $L_r$  cube by  $T_r$   $L_r$  is the length ratio and  $T_r$  is the time ratio  $L_r$  cube by  $T_r$

When we deal with the kinematic similitude you can see that the paths of moving particle are geometrically similar, you can see hence streamline patterns are the same, you can see here the prototype and its model here you can see the streamlines. The streamlines patterns should be al same sand finally the dynamic similarities is concern.

We have already seen the various forces like the real problem is concern various forces acting or the inertia forces friction forces or viscous forces or gravity forces pressure forces elastic, forces surface, tension forces, with respect to this we will do the modeling the physical modeling the dynamic similarity, we have to keep this in this case in addition to the geometric and kinematic similarity.

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**Dynamic Similarity**

- In this case in addition to Geometric and Kinematic similarity, similarity between the forces acting on the system exists, e.g.

$$\frac{(Inertia\ F)_{model}}{(Viscous\ F)_{model}} = \frac{(Inertia\ F)_{prototype}}{(Viscous\ F)_{prototype}} \Rightarrow Const.$$

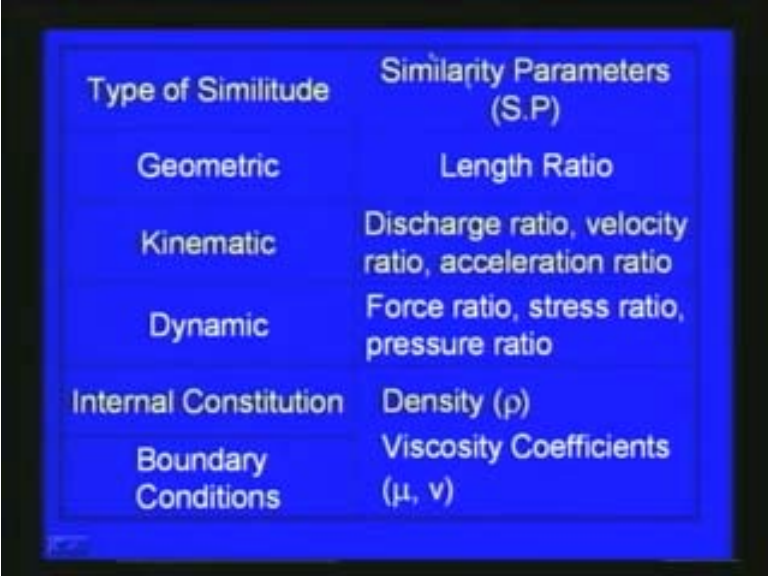
$$\frac{(Inertia\ F)_{model}}{(Gravity\ F)_{model}} = \frac{(Inertia\ F)_{prototype}}{(Gravity\ F)_{prototype}} \Rightarrow Const.$$

- F stands for Force.

Similarity between the forces acting on the system exist inertia force of the model divided by viscous force the model should equal to inertia force the prototype divided by the viscous force of the prototype.

That is a constant similarly when we consider the fluid model inertia forces the model divided by the gravity forces of the model should be equal to inertia force the prototype divided by the gravity forces of the prototype, that is constants here f stands for the force.

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Type of Similitude	Similarity Parameters (S.P)
Geometric	Length Ratio
Kinematic	Discharge ratio, velocity ratio, acceleration ratio
Dynamic	Force ratio, stress ratio, pressure ratio
Internal Constitution	Density ( $\rho$ )
Boundary Conditions	Viscosity Coefficients ( $\mu, \nu$ )

Finally, when we do the similitude we can have the geometric similitude the similarity parameters are the length ratios and kinematic similitude is concerned the similarity parameters are the discharge ratio velocity ratio and acceleration ratio.

When we deal with the dynamic similitude the parameters are force ratios or stress ratio or pressure ratio. Internal constitution is what we deal with the fluid the physical modeling and the prototype is concerned. We have to deal with the internal constitutional and boundary conditions like density and various viscous coefficients viscosity coefficients like coefficient dynamic viscosity or kinematic viscosity. Further we will be discussing about the various model laws or model scales with respect to this the similarity principles and the dimensionless numbers in the next lecture