Fluid Mechanics

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Lecture - 23
Dimensional analysis

Welcome back to the video course on fluid mechanics. We will start the chapter dimensional analysis. Dimensional analysis is one of the important aspects in fluid mechanics. Since fluid mechanics to do various experiments for the various analysis or investigation purpose or theoretical development, we will be doing number of experiments especially. In this experiment, we have to generate large set of data and out of this data, we have to choose particular sets or we have to do lot of manipulation of the data to derive the specific results or specific tasks. Dimensional analysis is one of the most important aspects in fluid mechanics. As you can see here are the some of the important objectives of the dimensional analysis here.

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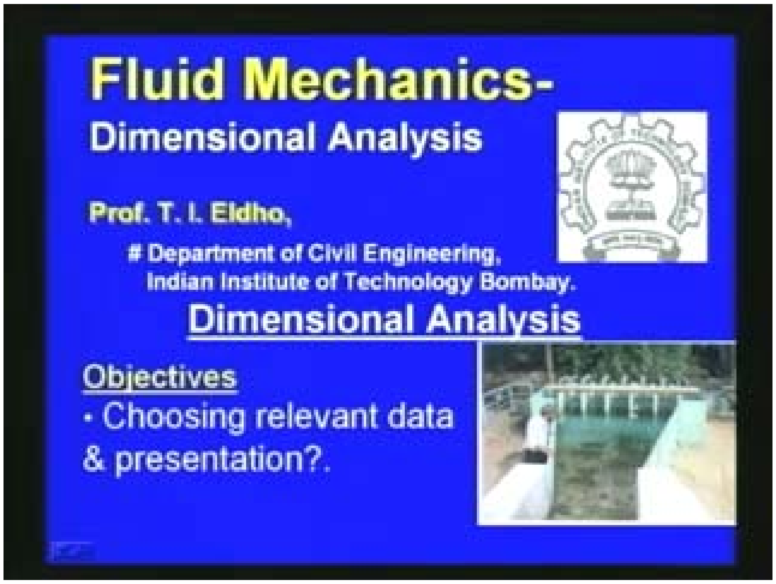

I have noted first one is: Choosing relevant data and presentation. As I mentioned while conducting experiments in fluid mechanics, we will making models for particular prototype of particular fields problem; that particular model should be made to certain scales, while desiring this scales while doing the analysis, we have to see the important parameters to be considered in this process, the parameters that should be given as input and parameters that should be measured. In this aspect, one of the main objectives the dimensional analysis is to choose the relevant data, do the modeling and present it in appropriate format. This is possible through dimensional analysis. Next objectives are: data relationship and analysis. As I mentioned, once we get the data, we have to see that which way we can relate between the data, we can analyze with respect to this data to derive meaning full relationship. The third objective with respect to dimensional analysis is model scales. As I mentioned while conducting experiments in the laboratory or while representing the field problem with respect to the model, we have to choose appropriate scales. The model may be very same very small or very medium or slightly large depending upon the real field problem. Real field problem generally we can represent very large real field problem with respect to scale down or with respect to small scales with respect to the laboratory experiments. Models scaling generally we will be getting through this dimensional analysis process.

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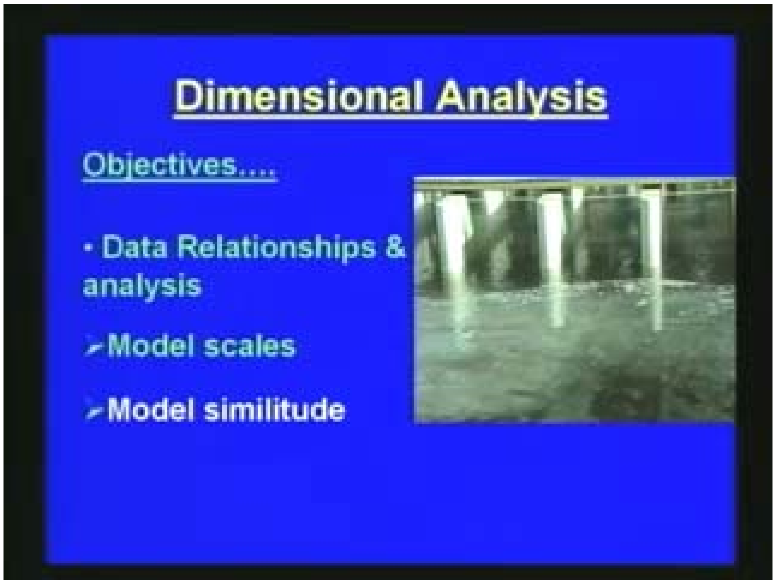

The other objective here is model similitude. With respect to the field problem and with respect to the model which we develop in the laboratory, we have to see the similarities of which we are keeping with respect to the field problem and the model which we develop. Finally, the theory and the applications with respect to the dimensional analysis these are the objectives for this particular topic on dimensional analysis. As you can see here (Refer Slide Time: 05:11) this is a typical model which we developed in hydraulic lab to see the flow variations with respect to some pumps. Here, you can see a channel flow is coming and open channel and here number of pumps are put; this is the study we have done for a particular power project where, large quantity of water is taken. The purpose of these kinds of model is to see that, what should be the carrying, what are the important parameters we have to consider and conduct the experiments. We have to derive and we have to obtain the results finally to get certain meaningful and relationship so that, we can represent the problems. For this particular model which some of the photograph shown here, we first did a dimensional analysis, we derived particular model scales for this particular problem 1:8 scale is used with respect to the problem. We will be approximately checking and putting various parameters; we will be conducting the experiments to derive various relationships and to obtain various results. These are the important objectives with respect to this topic on dimensional analysis.

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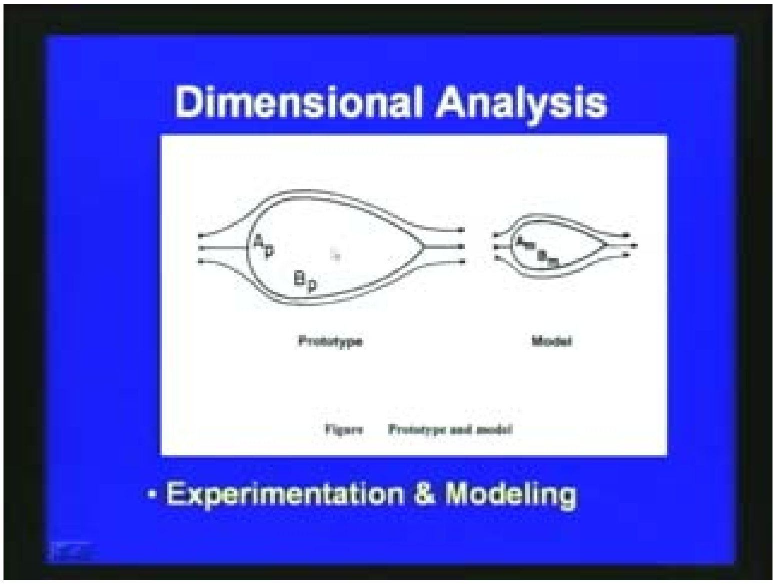

Now, what is dimensional analysis, as I mentioned generally in fluid mechanics, we conduct large number of experiments with respect to see that how the real actual field problem is behaving. We can see that here in this slide we have a prototype of larger the size; we will make a small model with respect to a scale of this prototype. This is the prototype and a model. For any kind of experiments in fluid mechanics this kind of model we will be developing as I mentioned this dimensional analysis is one of the most important aspects which we will be using for fluid mechanics experiments and modeling.

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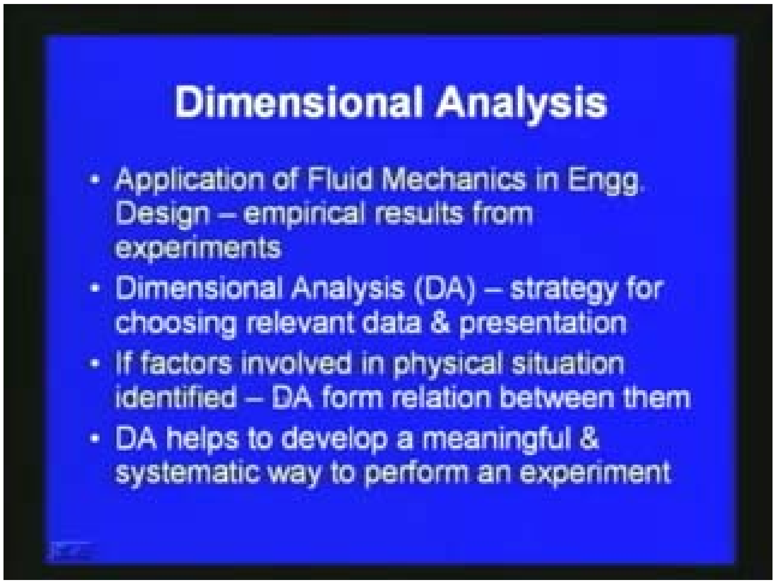

Now just first let us see what dimensional analysis is. Let us see the different methods available for dimensional analysis and various important relationships available with respect to the dimensional analysis. As I mentioned here; dimensional analysis is mainly application of fluid mechanics in engineering design to get an empirical results from experiments. The dimensional analysis is a strategy for choosing relevant data and presentation. We have to choose particular number of variables and number of parameters which will be there for particular problem we will be solving. We have to choose this relevant data for relevant parameters with respect to these parameters, we have to either build a model or we have to investigate the problem get appropriate results. Dimensional analysis; its strategy for choosing relevant data and presentation If factors involved in physical situation are identified then, we can see that the dimensional analysis from relation between them. If we identify we can find out the real factors which are dealing with the physical situation then, we can get various relationships using the dimensional analysis between these variables. Thus, we can see that the dimensional analysis helps to develop a meaningful and systematic way to perform an experiment. Finally with respect to this discussion here, we can see the aim of the dimensional analysis is to develop a meaningful and systematic way to perform an experiment and that means to perform an experiment with respect to an actual field problem, we have to make a model, we have to scale down or we have to choose particular scale, we have to see that which are the important parameters or which are the factors affecting the particular problem and then with respect to that factors we have to see that the model is made. We have to conduct the experiment so that we get a meaningful and systematic result from the models. These are some of the objectives and the aims of the dimensional analysis as I mentioned dimensional analysis.

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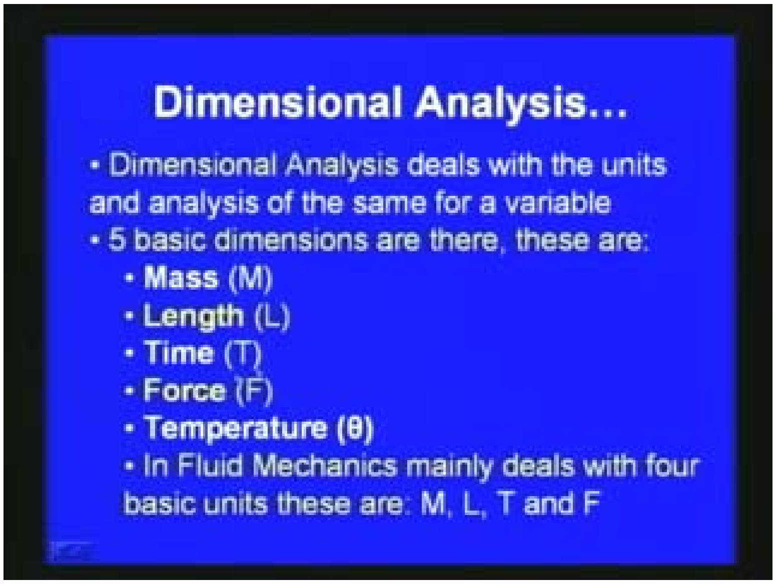

You can see that most of the physical variables or physical parameters have dimensions. In the dimensional analysis we deal with the units and analysis of the same for a variable. Number of variables like in length temperature time then velocity or different kinds of parameters will be there for each problem for which, we have to analyze the typical what is the kind of parameter variable and what are the dimensions for each variable. In a fluid mechanics generally, we can see that there are five fundamental dimensions generally used.

First one is the mass M second one is the length represent as L and third one is time represented T, instead of mass sometimes will be using in terms of force is represent as F temperature theta. These are five basic dimensions: mass, length, time, force and temperature. Out of these most of the fluid mechanics problem we will be generally dealing with mass, length, time or force, length and time, temperature depending upon the problem. Especially, for dynamic problem will be dealing with temperature; mass length time and force are the four basic units which will be generally dealing in fluid mechanics problem and between mass and force depending upon the problem sometimes you may be using mass sometimes you may be using the force. Generally again we can put into a system of three basic dimension either mass length and time or it can be force length and time depending upon the problem. These are the fundamental dimensions based upon these fundamental dimensions only we will be generally doing the dimension analysis.

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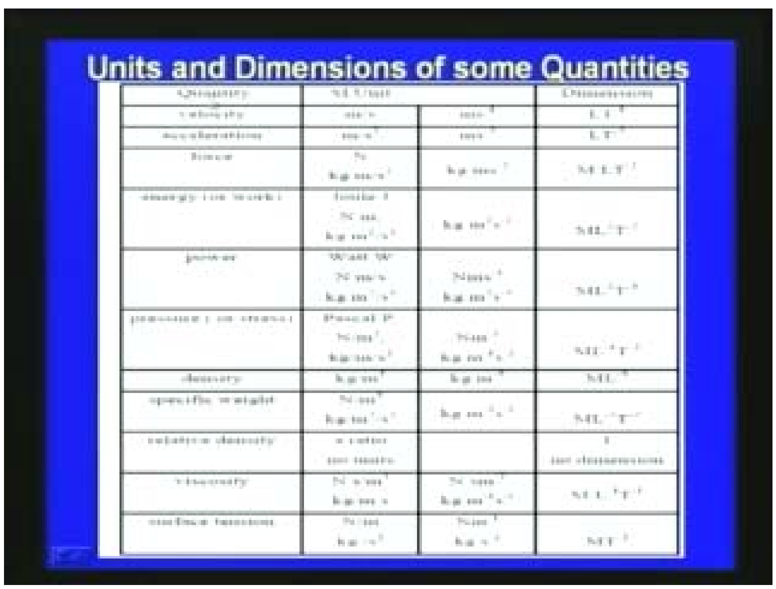

Now in this slide you can see me of the important units and some of dimensions of some of the important quantities generally which will be using fluid mechanics. Here, in this tabular form it is given first column shows the quantity, second column shows the SI unit and its dimensions. Some of the important parameters which we use in fluid mechanics include the velocity the unit is generally meter per second and dimensions L T to the power minus1, L is length and T is the time, second one is acceleration unit is meter per second square and dimension is L T to the power minus 2 and third one is force the unit is Newton or kilogram meter per second square, this is kilogram meter S to the power minus 2, the dimension is ML T to the power minus 2. Third one is energy or work generally used unit is joule or Newton meter or kilogram meter square per second square it is represented as kilogram meter m square S to the power minus 2 the dimension is ML square T to the power minus 2. Next one is power represent as watt or Newton meter per second or kilogram meter square by second cube it is NM S to the power 1 and dimension is ML square T to the power minus 3, pressure or stress the unit is Pascal or Newton per meter square, its correspond dimension is meter L to the power M L to the power minus1 T to the power minus 2. Next one is density unit is kilogram per meter cube and dimension is ML to the power minus 3 M stands for the mass L stands for the length and T stands for the time and for specific weight and the unit is Newton per meter cube or kilogram per meter square per second square the dimension is M L to the power minus 2 T to the power minus 2. Next one is relative density you can see it is a ratio there is no dimension. Viscosity it is generally the dynamic coefficient viscosity and Newton second per meter square or kilogram per meter second kg M to the power minus1 S to the power minus1 dimension is M L to the power minus1 T to the power minus1. Surface tension Newton per meter or kilogram per second square, it is M T to the power minus1.

These are some of the generally used quantities in a fluid mechanics and corresponding units and the dimensions. Before going to analyze or before we discuss the different methodologies of dimension analysis, one of the important aspects which we use here in dimension analysis is called dimensional homogeneity. Dimension homogeneity means generally we will be representing in many of the either through experiment of theoretical investigation, finally the outcome will be an equation in terms of mathematical form. You can see that with respect to this equation only we will be further investigating the problem.

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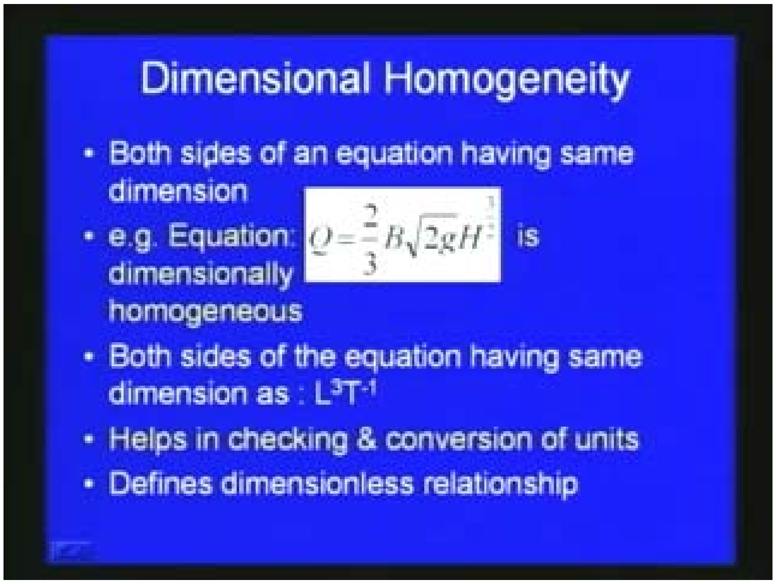

The dimensional homogeneity means as written in this slide here both sides of an equation having same dimension then that kind of equations called the equation said to be dimensionally homogeneity or dimension of homogeneity is there for the problem. When we write the equation both sides of the equation having same dimension. For example, if we consider this equation for discharge through a weer. You can see here Q is equal to2 by3 B into root2 g into H to the power3 by2. Here Q is the discharge B is the weer and g is the acceleration gravity and H is the depth of water over the rectangular wear which we consider here. We can see that this equation is dimensional homogeneous, if you analyze this equation on the left hand side it is discharge which unit is meter cube per second. We can see that it is the dimension is L cube into T to the power minus1 the unit is meter cube per second. L cube into T to the power minus1 and here2 by3 are these constants are there B is the unit is length or L H is the unit is L, L into L to the power3 by2 and g we have already seen is acceleration due to gravity meter per second.

If we use that, we can see that finally both sides the dimension will be same. You can see that L cube both sides of the equation we can see that having the same dimension as L cube into T to the power minus1. This important aspect is used generally in the dimensional analysis for various problems. This dimensional homogeneity helps in checking whether the equation which we derived is right or not. With respect to various experiment measurements or with respect to theoretical analysis in fluid mechanics, we will be deriving the equation. To see that whether one method of checking whether this equation derived is right or not is through this to see that whether dimensional homogeneity is there. You can see that left hand side of the equation the dimensional should be same as the right hand side of the equation so this is one way of checking. Also sometimes we may have to convert the units. The units may be given in weer system or SI system convert from one unit to another we can use the dimensional homogeneity for this dimensional homogeneity aspects, it also defines the dimensionless relationship. We can see that, the with respect to the equation which we derived, the dimensional homogeneity is there then we can that that relationship are properly defined. The dimensional homogeneity is an option for checking whether equation derived is right or for conversion of the units from FPS system to SI system or SI system to FPS system like that and also it defines the dimensional relationship between various variables. Dimensional homogeneity is one of the important aspects in dimensional analysis and this dimensional homogeneity is we are using or dimensional analysis based upon the dimensional homogeneity. Now let us see the importance of dimensional analysis. We have seen what dimensional analysis is and principle of dimensional homogeneity. With respect to what we have discussed let us see the importance of dimensional analysis.

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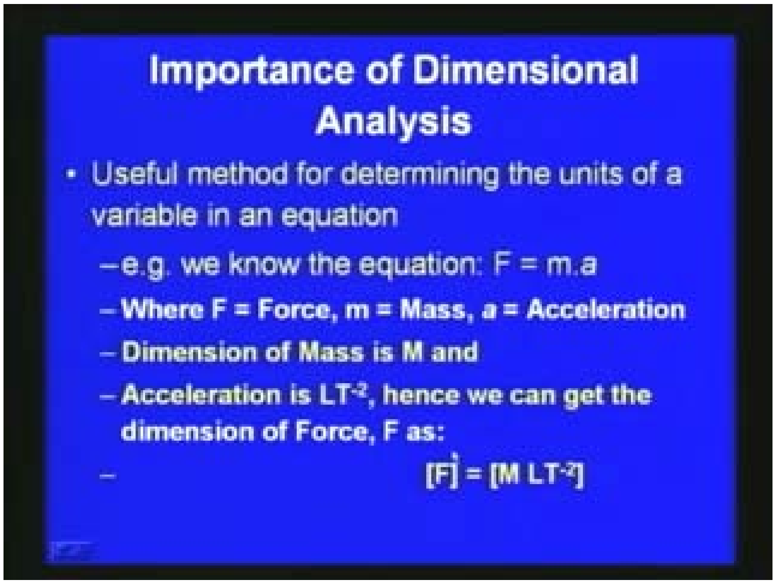

First you can see in this slide here importance of dimensional analysis is listed here first one is: it is a useful method for determine the units of variable in an equation. If there is an equation and we want to determine the units of variable, we can use the dimensional analysis. For example, if we consider this equation force is equal to mass into acceleration where, F is force m is mass a is acceleration the dimension of mass is M as we have seen acceleration is L into T to the power minus 2 hence we can get the dimension of force f as MLT to the power minus 2. Like this force is equal to mass into acceleration, like what we have done here, we can determine the units of variable by using the dimensional analysis. This is one of the important aspects of the dimension analysis.

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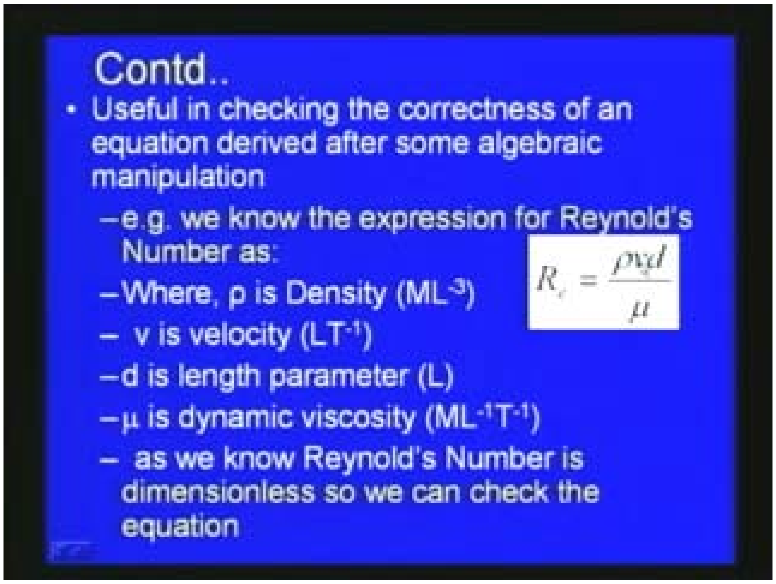

Second one is the dimensional analysis is useful in checking the correctness of an equation derived after some algebra manipulation. As we have already seen in fluid mechanics, we will be deriving the equations fundamental equation either through theoretical investigations or experiment investigations. In any of this case through dimensional analysis, we can check the correctness of the equation. For example, we know the expression for Reynolds number as in pipes rho vd by mu Reynolds number is equal to rho vd by mu where, rho is the mass density of the fluid. V is the average velocity it is the diameter of the pipe mu is the coefficient of dynamic viscosity here rho is the density the dimension is M L to the power minus 3 and v is the velocity the dimension is L T to the power minus 1 d is the length parameter of diameter, the dimensional is L and mu is the dynamic viscosity the dimension is M L to the power minus 1 T to the power minus1. As we know the Reynolds number is dimensionless, we can check the equation the equation is right or not. If we use this you can see that, both numerator as well as denominator have the same unit, we can say that, the correctness or the Reynolds's equation we have written for the Reynolds's number is correct or not by doing the dimensional analysis.

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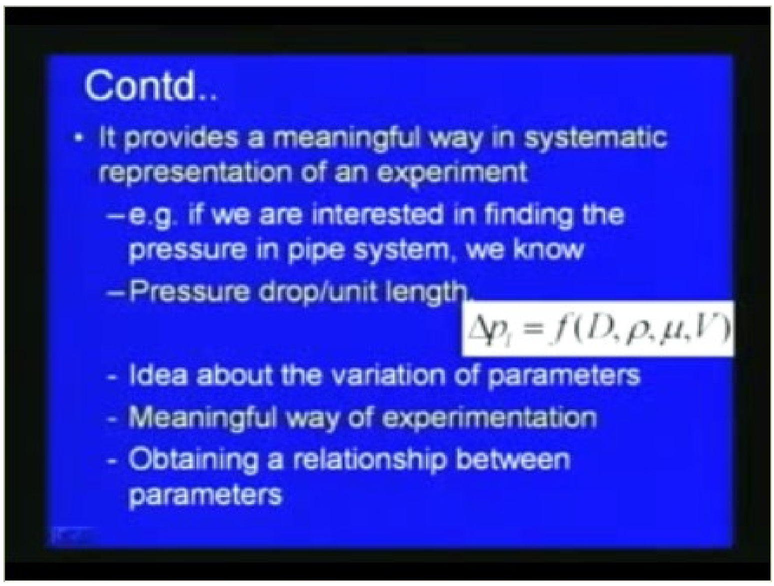

Another important aspect of dimensional analysis is to provide a meaningful way in a systematic way representation of an experiment. As I mentioned, we will be doing experiments to analyze various problems or to represent field problem in the laboratory to do various testing, while doing this kind of experiment, this dimensional analysis gives a meaningful way to systematically represent the problem. For example, if we are interested in finding the pressure in pipe system, we know that if we want to find out the pressure drop in pipe system pressure drop or unit length delta p_l for particular pipe we can see that, important parameters here are diameter, density of fluid coefficient of viscosity mu and the average of velocity V here this. Delta p is the pressure drop or unit length. From this we can see it gives an idea about the variation of parameters by representing like this it gives a meaningful way of experimentation through which we can get relationship between the parameters. This particular problem pressure drop or unit length you can we can see that through investigations we can show that this pressure drop depends upon the various parameters like d diameter rho density and viscosity mu and the average velocity. We can get a meaningful relationship.

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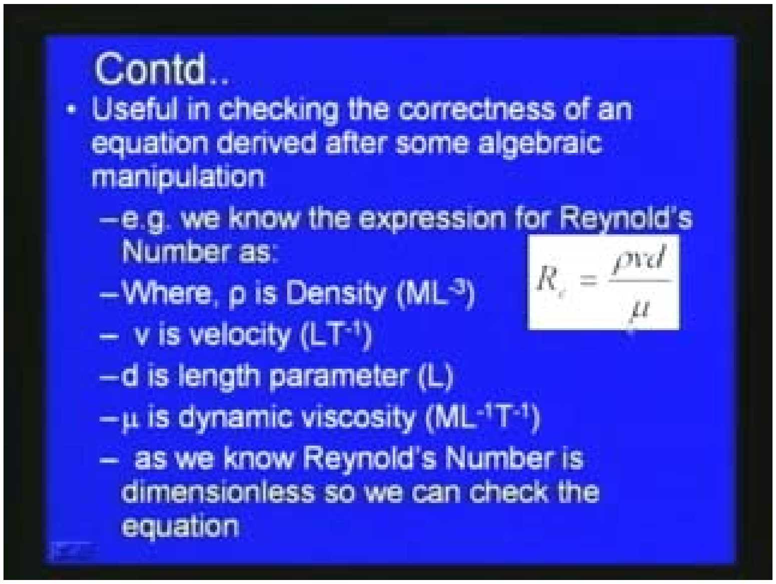

Through this once we have listed the various variables of parameters then, the dimensional analysis gives a way for use or dimensional analysis is used for interpretation representation of the results of an experiments. For example, in previous problem through dimensional analysis we can show that, the relationship is d delta p_l divided by rho v square is equal to as a function of rho VD by mu where, rho VD by mu is the Reynolds number. This pressure drop is depending upon this rho VD by mu these parameters. Here, the sides are dimensionless, we can further represent or we can interpret the results which we are getting with respect to the particular experiments. If you go for analytically showing this it will be cumbersome. But, this dimensional analysis gives number of options to interpret the results and represent the results from an experiment in an appropriate way. These are some of the importance of the dimensional analysis.

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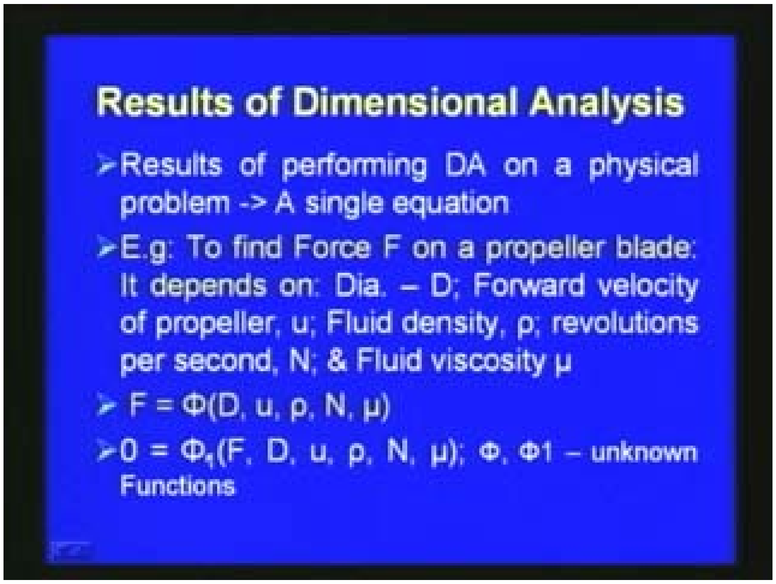

Before further going to see the different methodologies of dimensional analysis what the dimensional analysis gives or the results of dimensional analysis. Here, the results of performing the dimensional analysis on a physical problem as we have seen may be get a single equation or sometimes depending upon the problem more equations and depending upon the particular case. For example, to find a force F on a propeller blade the case of if you want to find out the force on propeller blade for a typical problem, we can see that force depends upon the diameter the forward velocity of propeller u fluid density rho revolutions per second n fluid viscosity mu. We represent this force on the propeller as f as a function of D mu rho N and mu as described here. With respect to this we can write another function phi_1 as a different variable also set for parameter phi_1 as a function of F D mu rho N mu, we can equal to 0 and you can see that phi and phi1 are the unknown functions here. Through the dimensional analysis we will be trying to find out these unknown functions for the particular case. You can see that, for this particular problem for the particular case of force on a propeller blade.

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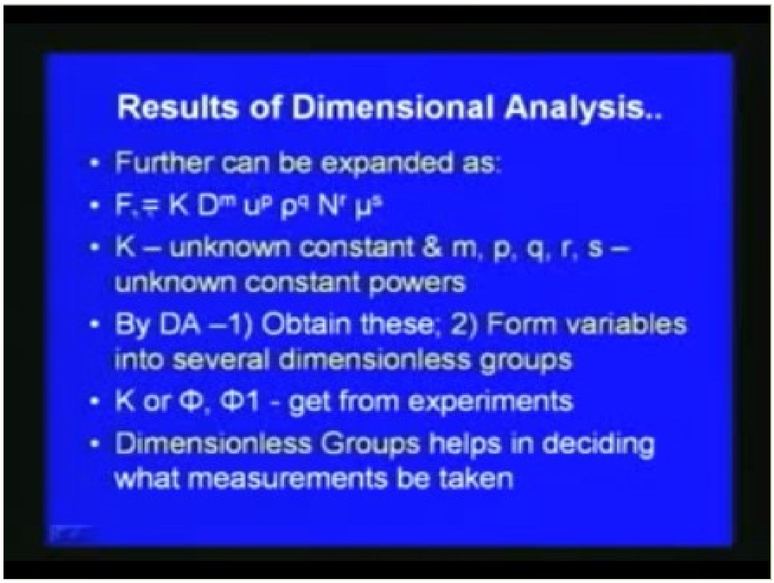

If we write F is equal to K a constant K multiplied by D to the power m, u to the power p, rho to the power q, N to the power r, mu the power S. here as I mentioned f is the force, D is the diameter, u is the velocity, rho is the density, N is the revolution per second as we have seen mu is the viscosity. Here K is an unknown constant and here these small m p q r and s are the unknown constant powers. Through the dimensional analysis we are finding out these constant values and finally we will be able to get a relationship. Through the dimensional analysis what we are doing is, number one to obtain this m this constant k m p q and r s and second object is to form variables in several dimensionless groups. For example; here, we have not put this phi1 as in terms of phi we can form different dimensionless groups here, we may get k or phi or phi1 you may get experiments. Dimensionless groups helps in deciding what measurements be taken? These are the outcomes from the dimensional analysis for a typical problem for typical problem which may be doing experiments or which may be even doing theoretical analysis. In both cases the aim is we are representing in terms of equation and we will be trying to obtain the unknown constants, finally form a relationship with respect to the dimensionless groups. These are the important objectives and the results are the outcome from the dimensional analysis.

We have seen the objective of dimensional analysis the outcomes and the importance of the dimensional analysis. Next, we will discuss the important methodologies of dimensional analysis. Generally, in literature in various standard text books, we can see two methods for dimensional analysis .First one is called is Releigh's method and second one is called Buckingham's pi theorems. Here, we will be discussing both of these methodologies with respect to examples, first one Raleigh's method.

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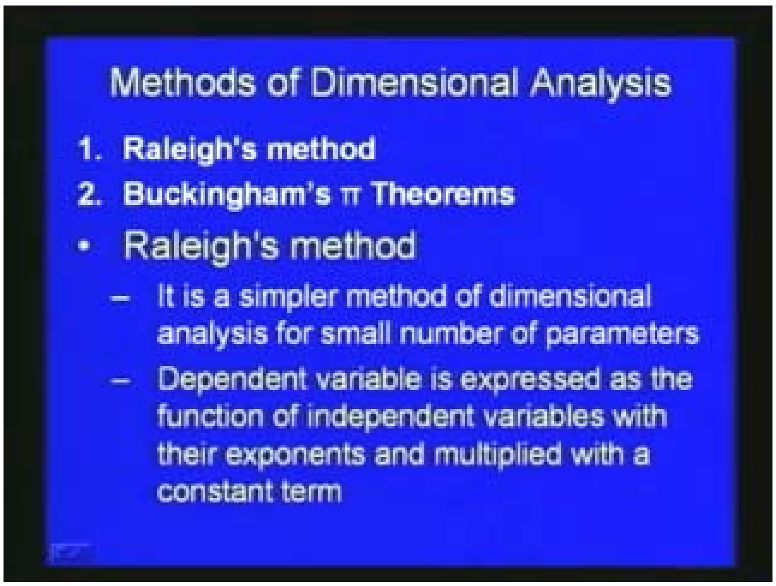

This is a simple method of dimensional analysis for small number of parameters. Some of the advance books you can see only one method as Buckingham's method. This Raleigh's method, the difficulty is that we can exceed to large number for a problem where large number of parameters are there. But, this Raleigh's method is much more suitable for a problem where small number of parameters is there. Dependent variable can be expressed as the function of independent variables with their exponents and multiplied with a constant term. The Raleigh's method it is one of the simple method. It is applicable for a particular problem where, the number of parameters less, for a particular problem for the different variable, if we can express as the functional independent variable this method can be easily applied. Further as we have seen, generally the particular problem will be trying to express in terms of the fundamental dimensions either mass length time or force length time with respect to this we will be having some of the exponents unknown exponents as we have seen in the previous slides.

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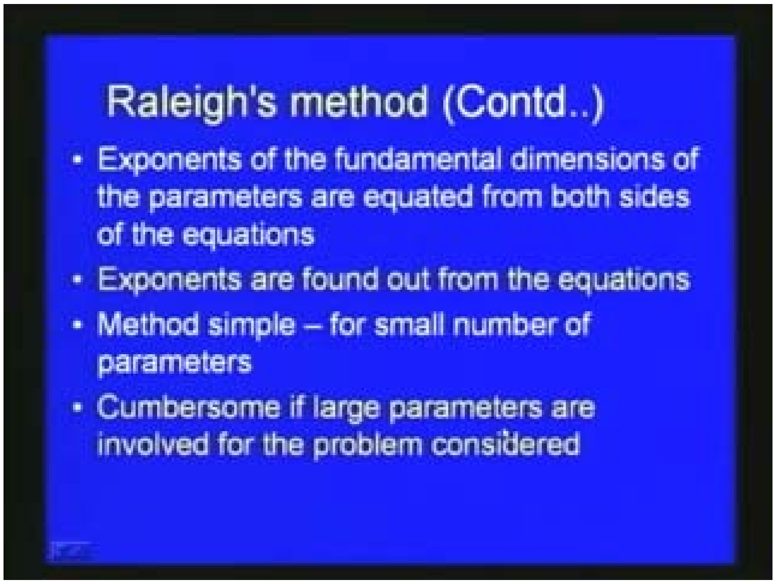

In this Raleigh's method what we do is the exponents of the fundamental dimensions of the parameters are equated from both sides of the equations. We will be discussing further with respect to an example, the method for the typical equation we will be finding out the exponents, the unknown exponents by equating both sides of the both sides of the equations to get the unknown parameters. The exponents are found out from the equations and here you can see method is much simpler and wherever for small number of parameters involve if large number of parameters are involved for the particular problem the method is much more cumbersome. Since this method cannot be large number of parameters.

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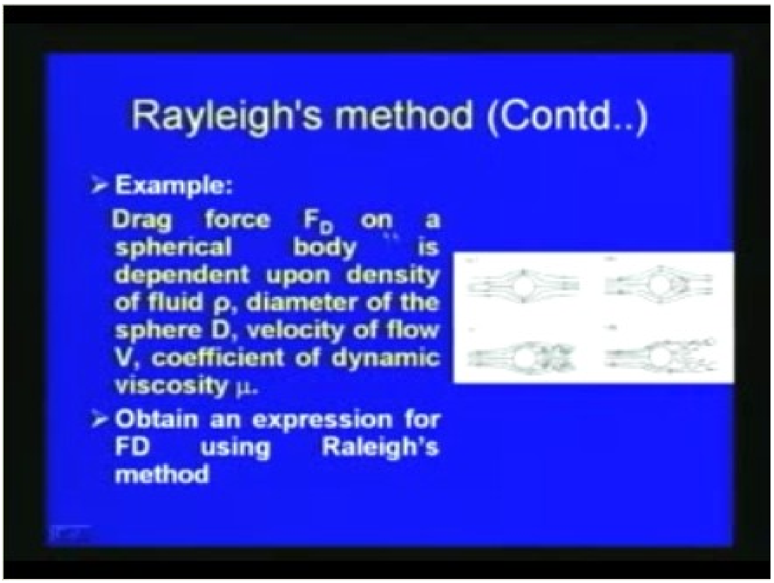

To further illustrate this Raleigh's method here we will discuss a typical problem. The problem is we will drag force to find out the drag force F_D on a spherical body, we can see it is depending upon the density of fluid diameter of the sphere, the velocity of flow and coefficient of dynamic viscosity mu. We have obtained an expression for the drag force using Raleigh's method. Here you can see that, there is this sphere which we consider the flow is coming over this sphere. We want to find out the drag force to get an expression for it with respect to various parameters involved. This is the problem.

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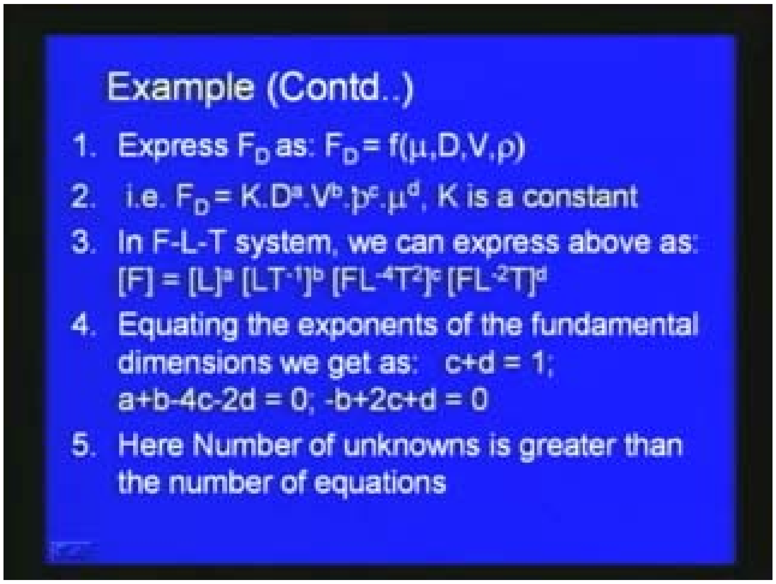

We will proceed step by step. As I mentioned earlier, first step is we have to identify for the given problem the important parameters which will be here; we have to find out the drag force for the flow over a sphere. The important parameters you can see that, the viscosity of the fluid mu of course, the diameter of the sphere is very important. The approach velocity of the flow the average velocity of flow v is important of course, the density of the fluid. The first step in this Raleigh's method is to express this F_D drag force as a function of this important parameters mu D V and rho. We write the drag force F_D is equal to as a function of mu D V and rho as in this first step you can see that, we wrote a function generally in this Raleigh's method, we represent this F_D is equal to multiplied by a constant K into D to the power a V to the power D rho to the power C mu to the power d where, K is a constant. a b c d are the unknown exponents this F_D is represent as K into D to the power a V to the power b rho to the power c and mu to the power d where, mu rho V D are the parameters for this particular problem. As I mentioned we will be using either mass length time unit or force length time unit for the dimensional analysis.

Either one of this can be used; here we are dealing with drag force. Let us try to solve this problem by using the FLT system or force length time system. This drag force you can see that drag force if you write in terms of the unit you can see the unit is force.

We can just represent as F, F is equal to, K is a constant, K has no unit, d is the length representing in terms of L, L to the power a velocity the dimension is L into T to the power minus 1, here we write L into T to the power minus 1 to the power b density as we have already seen earlier can be represent as F into L to the power minus4 T to the power two. So, rho to the power C is represent as F L to the power minus 4 T to the power two whole to the power C mu is the viscosity, that is represent as F into L to the power minus 2 into T mu is represent as this to the power d. Finally, we write in third step we write in FLT system like this both sides we have units here the unknowns are a b c d. Here to find out the value of this a b c d what we can do is we can equate the exponents of the fundamental dimensions with respect to the right hand side and the left hand side. You can see that left hand side is the unit is force; it has got F as the dimensions. F to the power 1, we have the left hand side is known but right hand side the unknowns are a b c d. With respect to this we can write here three equations. Since we have got three fundamental dimensions, here if we equate we can see that, force is concerned here two places with respect to left and right of this equation we can write c plus d is equal to 1. If you equate with respect to length L you can write a plus b minus 4 c minus 2 d here right left hand side is zero that is equal to zero, with respect to T, we can write here T to the power minus1 minus b then plus here T to the power two whole to the power c plus 2 c then, here T to the power d plus d minus b plus2 c plus d is equal to zero.

We will get three equations and unknowns are power unknowns a b c d. The number of unknowns is greater than the number of equations; we have to remodel this system of equations. Out of these four equations, we can write with respect to other parameters a b c d this can be done here.

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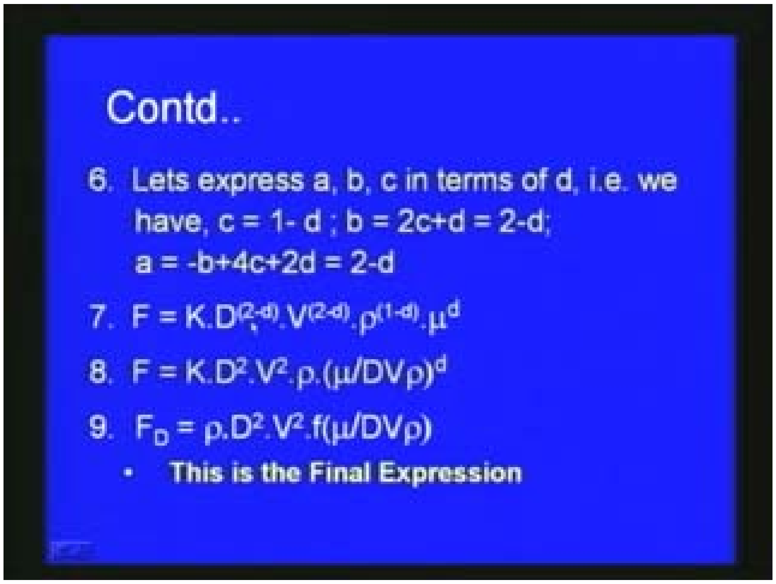

Let us express a b c in terms of d that is we have c is equal to1 minus d b is equal to 2c plus d that is equal to 2 minus d, we get a is equal to minus b plus 4c plus 2d that is equal to 2 minus d. Finally, with respect to this equation, we can write F is equal to K into D to the power 2 minus d c is represent as 1 minus d, b is represent as minus d and a is also represent as 2 minus d. In the seventh step we write F is equal to k into D to the power two minus d V to the power two minus d rho to the power 1 minus d mu to the power d.

We can see that, here if you equate with respect to this the equation F other terms, we can get the value of d. Finally here if you solve this system. We can write the system as F is equal to here you can see that this left hand side there is no F term obviously or this typical case it should be coming this way. We write F is equal to K D square V square rho into mu by dv rho VD to the power d this equation is represented this form to finally get the drag force F_D is equal to rho D square V square and as a function of mu divided by rho VD this is the final expression. This is the way which we do the dimensional analysis in the Raleigh's method. We start with first in the first, we will see that the important parameters we represent the dependant variable with respect to this parameters as a function. We put the fundamental dimensions for which either in terms of mass length time or force length time, we equate with respect to the exponents the unknown exponents are found and finally, we derive the six terms of equation. As for the drag force we obtain rho D square V square and as a function and mu D by rho VD or DV rho as shown. This is the final expression.

Here you can see that this while doing experiments for drag force this equation helps to obtain particular relationship to solve the problem in a meaningful way. Through this example, we have demonstrated the Raleigh's method. Raleigh's method, as I mentioned, it has got some number of variables are very large number of parameters and are very difficult to deal with. The system becomes quite complex. To deal these kinds of problems the further the method used is called Buckingham's pi theorems which are the most commonly used method.

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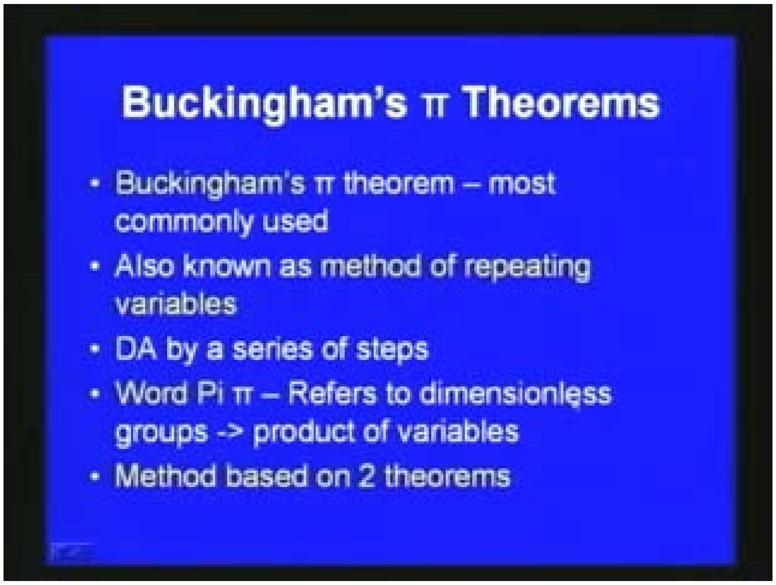

Buckingham's pi theorem is most commonly used and it is also known as method of repeating variables. You can see that, when we discussed many of the problems in a fluid mechanics there can be variables repeating, this aspect of repeating variables are taken care by Buckingham pi theorem method. That is why sometimes this method is also called as method of repeating variables. In Buckingham's pi theorem we do the dimensional analysis by a series of steps. Here in this pi the word pi refers to dimensionless groups of the product of variables. The word pi refers to dimensionless groups or products of variables.

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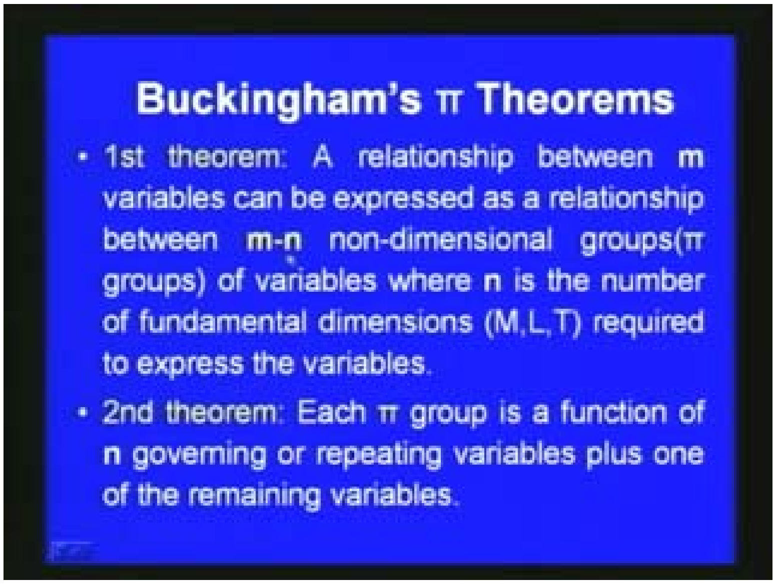

We will discuss the details of Buckingham's method. Buckingham's pi theorem is mainly based on two theorems; first one is a relationship between M variables can be expressed as a relationship between m minus n non dimensional groups or pi groups of variables where n is the number of fundamental dimensions like MLT as we have seen required to express the variables. The first theorem of Buckingham's is a relationship between with m variables there, then, a relationship between m variables can be expressed as a relationship between m minus n non-dimensional groups of variables where m is the number of fundamental dimensions.

For given problem if there are m variables and we identify n fundamental dimensions, we can have m minus n relationship this is called non-dimensional group or pi groups required to express the variables. This is the first pi theorem. Second pi theorem is each pi group is a function of the n governing or repeating variables plus one of the remaining variables. The second theorem by Buckingham is each pi group is a function of the n governing or repeating variables plus one of the remaining variables. Buckingham pi theorem or Buckingham method is of dimensional analysis based upon this two theorem, we have to identify the m variables we have to identity the fundamental dimension n fundamental dimension then, we will try to represent this with respect to m minus n non-dimensional groups or pi groups. Each pi group, we have to form as a function of governing or repeating variables plus one remaining variables. This method is based upon these two fundamental theorems. In this Buckingham's pi theorems, we use step by step method we will be starting with various steps and further proceed.

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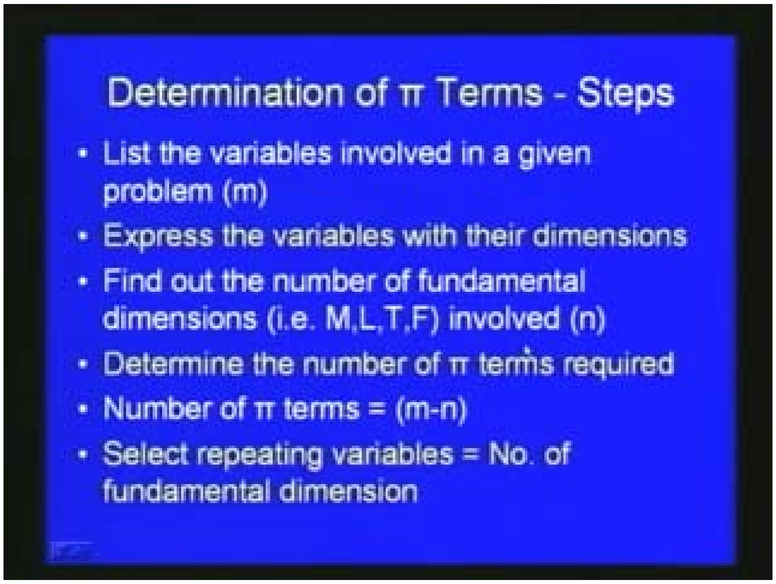

Buckingham pi theorem method the various steps are mentioned: first step in the method is list the variables involved in a given problem. As we have seen if there are m variables, we have to list the variables like velocity diameter coefficient of viscosity density or what kind of the variables, we have to list the variables. That is the first step. Let the number variables be m. The second step is to express the variables with their dimensions. As we have seen the velocity has L into T to the power minus 1 or unit is meter per second we have to express each variable with respect to the fundamental dimensions either in terms of MLT for mass length time or FLT force length time. The second step is we will be expressing the variables with their dimensions.

Third step is to find out the number of fundamental dimensions involved. Whether it is MLT or F or all this involved should be identified. Third step is to find out the number fundamental dimensions. Next step is determining the number of pi terms required. We have already seen pi term it is equal to m minus n we have to determine the number of pi terms.

Number of pi terms is equal to m minus n, based upon this we select the repeating variables is equal to number of fundamental dimensions. Further we express the pi terms as product of the repeating variables having exponents with a non-repeating variables so as to make the equation dimensionless. The next step we will be express the pi terms as the product of repeating variables for example pi_1 is equal to u to the power a v to the power b w to the power C into z. Where, u b w are the velocities which are the repeating variables and z is a non-repeating express pi_1 is equal to u to the power a v to the power b w to the power C into z.

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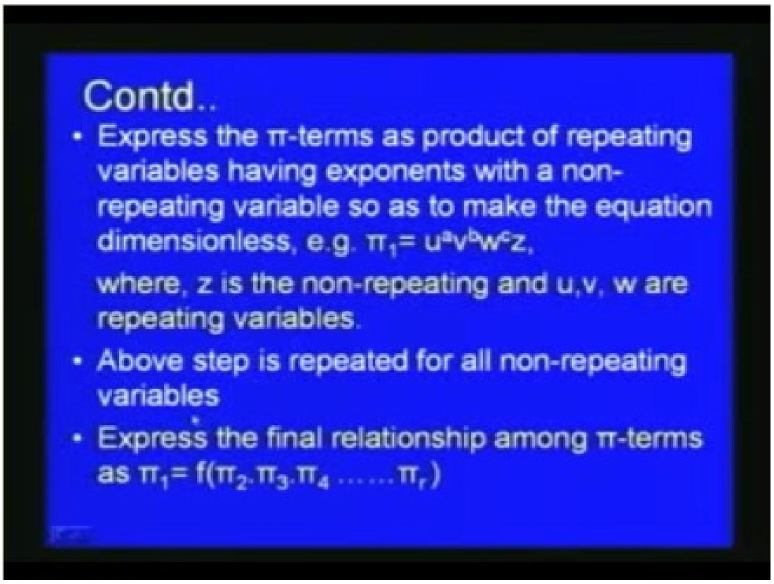

Next step is after this above step is repeated for all non repeating variables. Finally, we express the final relationship among the pi terms as pi_1 is as a function of pi_2 pi_3 pi_4 etcetera if r repeating variables are there pi_r. Finally, we express the final relationship between among the pi terms as pi_1 is equal to as a function of pi_2 pi_3 etcetera.

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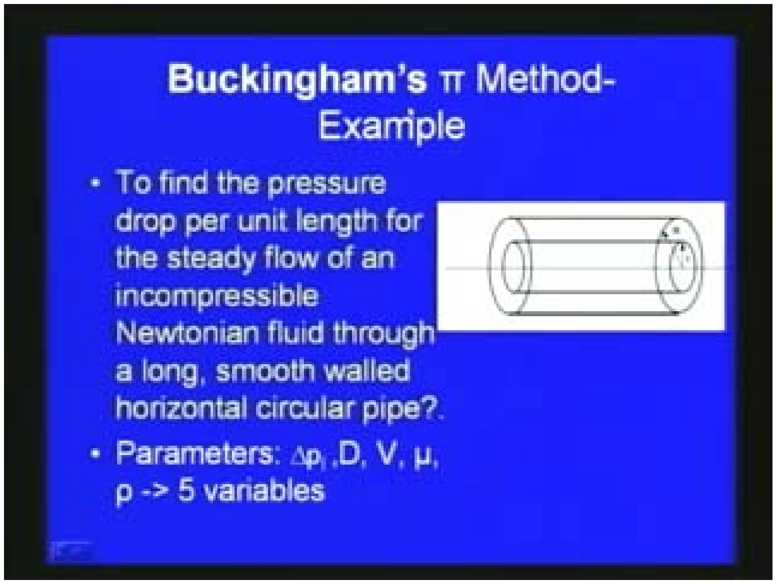

We have seen the different steps showing the Buckingham pi method, we have a series of steps, we will be identifying the variables, we will be forming the pi groups, we will be writing with respect to the repeating variables as step by step. To illustrate this Buckingham's pi method here a small problem, we discuss the problem is to find the pressure drop per unit length for the steady flow of an incompressible Newtonian fluid through a long smooth walled horizontal circular pipe. Here you can see pipe smooth wall pipe horizontal the position in horizontally. Let the diameter of the pipe is D the average velocity is V the viscosity of the fluid passing through the pipe is mu and rho VD and density we want to find out the pressure drop or unit length. By using the Buckingham's pi method, the important parameters are including this we have to find out the pressure drop that is represent as delta p delta p_l so the variables are delta p_l diameter D and the velocity V the viscosity mu and the density rho there are five variables group for this particular problem.

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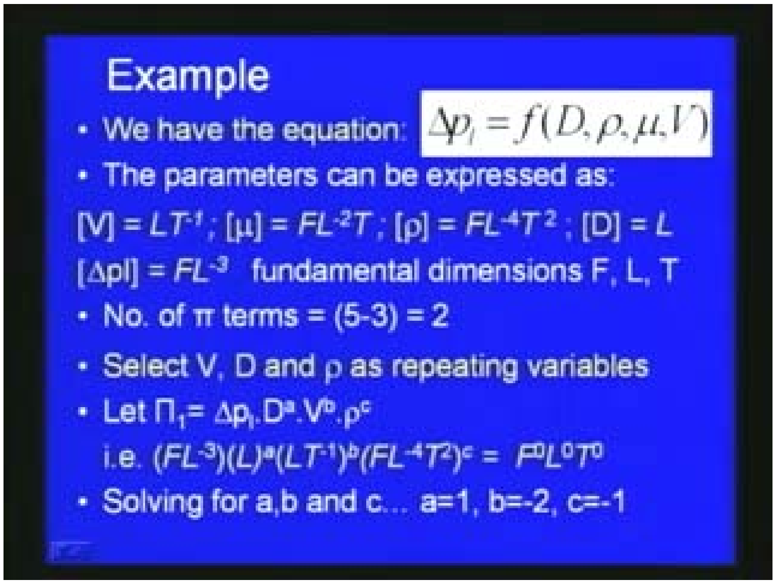

We can represent since our aim is to write get an expression for delta p. We will write delta p_l is equal to as a function of D rho mu and V. The parameters can be expressed as variable we will be representing in terms of the fundamental dimensions is V is represent as L into T to the power minus 1 mu is represent as F into L to the power minus 2 T to the power 1 and rho is represented as F to the power1 L to the power minus 4 T to the power 2 and D is the diameter represented as L and delta p_l which is the delta p subscript l which is the pressure drop that is represent as F into L to the power minus 3. The fundamental dimensions are F L and T force length and time. We can see that. we have five variables including the pressure drop we have got three fundamental dimensions, n is equal to 3 m is equal to 5, we can form two pi terms so 5 minus 3 is equal to2.

Let us choose VD and rho as the repeating variable initially for this particular problem, we can write let pi_1 is equal to delta p_l D to the power a V to the power b rho to the power C. If we choose VD rho as the repeating variable a b c are constants which we have to find, we can write with respect to this pi term if you write all the units with respect to what is given here F into L to the power minus 3 L to the power a L into T to the power minus1 whole to the power b F to the power1 L to the power minus 4 T to the power 2 whole to the power c. Right hand side is pi_1 which is a dimensional.

F to the power 0, L to the power 0 and T to the power 0, we can solve this with respect to a b c and the p_1 on the right hand side, we can solve for the unknown constant, this exponents a b and c. If you solve this we will get a is equal to1 b is equal to minus 2 and c is equal to minus 1.

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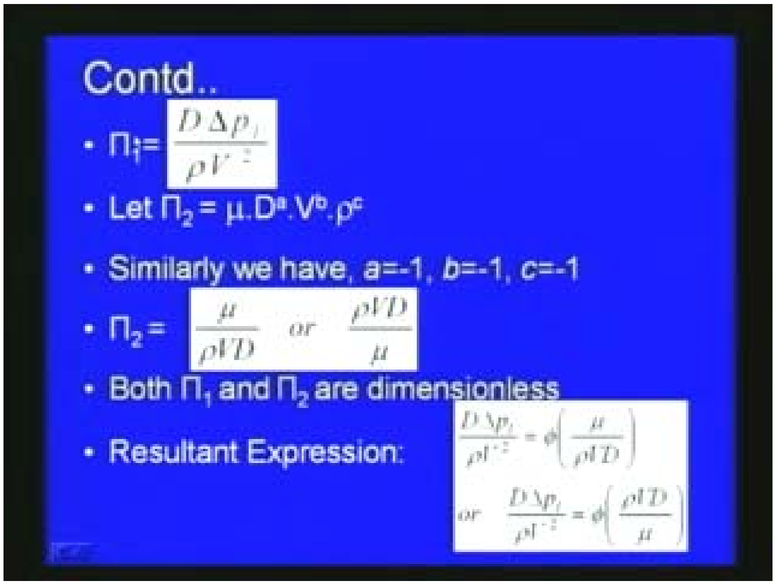

We can write pi_1 as once we get a is equal to1 b is equal to minus 2 c is equal to minus1, we can write pi_1 is equal to D into delta p_l by rho V square. This is the first pi term pi_1 and similarly, let us represents pi_2 as pi_2 is equal to mu into D to the power a V to the power b rho to the power c. As we have done earlier we can find out here a b c you can see that a is equal to minus1 b is equal to minus1 c is equal to minus1. Finally, we get pi_2 is equal to mu divided by rho VD or this mu by rho VD also expressed as rho VD by mu also, we get both pi_1 and pi_2 which are dimensionless group here. The resulting expression, we can write as: D into delta p_l by rho V square as a function of phi of mu divided by rho VD or D into delta p_l by rho V square as function of rho VD by mu. This is the resultant expression with respect to the dimensional analysis which we did here by using the Buckingham pi theorem. Further, we will be discussing the various applications of this buckingam pi theorem and importance further. This is one of the simple application how we solve a typical problem using the buckingam pi theorem.