Fluid Mechanics

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Lecture - 22
Laminar and Turbulent flows

Welcome back to the video course on fluid mechanics. So we were discussing about the turbulent flow theory and then we were discussing, the solution of turbulent problems especially to determine the velocity distribution by first model zero equation models.

So, last time we have seen the turbulent flow over flat plate including smooth type and rough type flat plate, so now rough surfaces and smooth surfaces we have seen, now here in today's lecture, we will discuss mainly the turbulent flow in pipes.

Most of the theories which we have seen for the turbulent flow over flat plate or smooth surfaces or rough surfaces are very much applicable in a very similar way, we are trying to utilize for this turbulent flow through pipes also.

Here you can see now, as we are discussing now the velocity, our aim here pipe is there and flow is turbulent, so we want to determine mainly the velocity distribution for the turbulent flow through the pipe.

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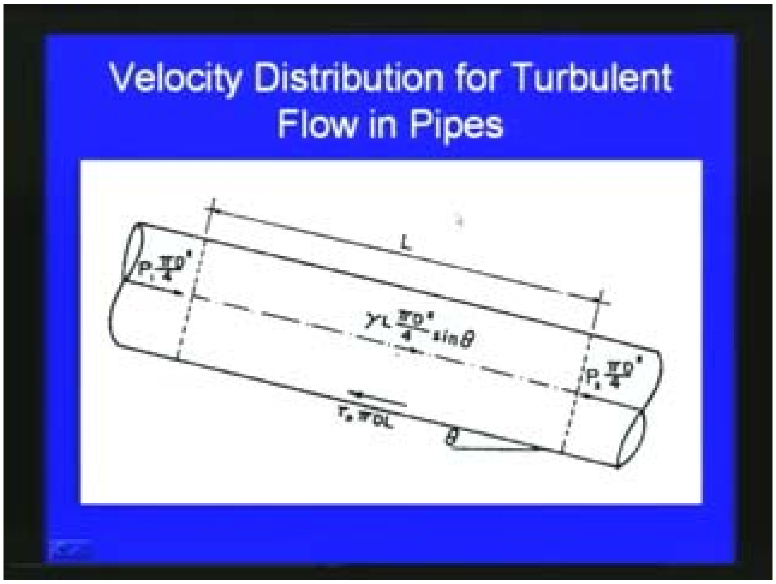

We are considering a picodiameter de and then we are considering l and then at any cross section, we want to determine the velocity distribution. As far as pipe is concerned as I mentioned earlier for the normal surface can be either smooth or rough. Similar way, here also the pipe is concerned we have to consider the pipes as smooth pipe or rough pipe. Like for example pvc pipe or that variety of pipe depending upon the roughness we will be considering the smooth and hydro dynamically will be considering smooth or we will consider like sometimes concrete pipe as rough pipe.

The theory which will be discussing here for smooth pipe and rough pipe, there are slight variation, that is why we are classifying here the pipe as smooth boundary type, and rough boundary type.

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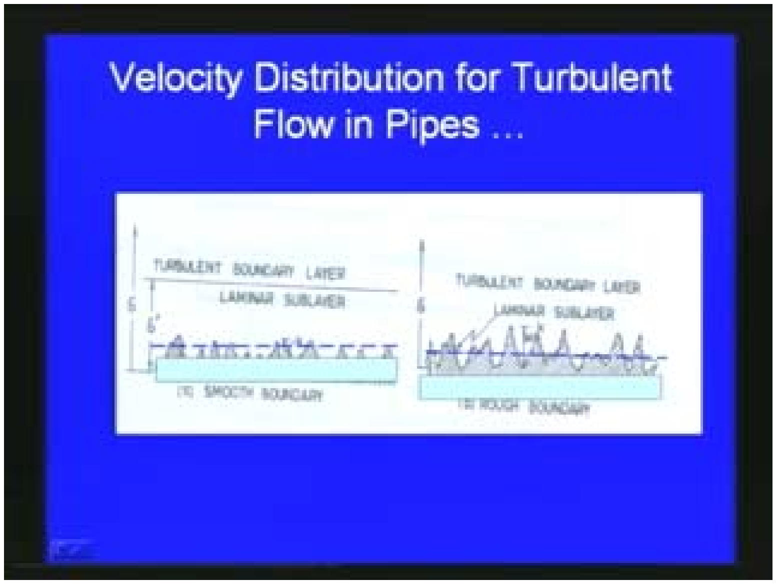

So here in this slide you can see for smooth boundary, even though it is not perfectly smooth you can see that, there are small projections like this but in comparison with the rough boundary, you can see that for this rough boundary case, the projections or the roughness is too much. So we classify this rough boundary type pipe and the first one as smooth boundary pipe. Both will be considered, we will discuss with respect to the turbulent flow theories, we will be discussing both cases.

First case is the velocity distribution for turbulent flow in smooth pipe.

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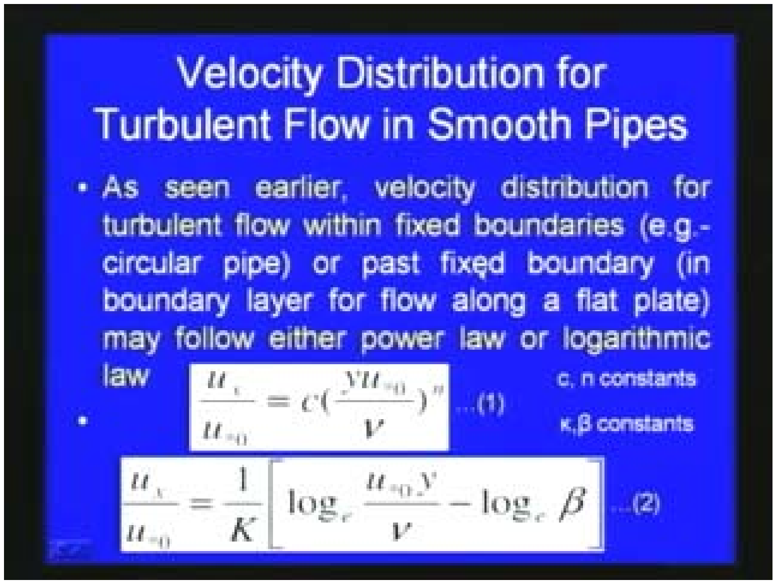

We have already seen the zero equation models or with respect to the Prandtl mixing theory which we have discussed earlier, we have seen how to develop the equation for the velocity distribution with respect to parallel flow. So in the very simple way, say here we have seen earlier the velocity distribution for turbulent flow within fixed boundaries, examples circular pipe or past fixed boundary in the case of boundary layer for flow along a flat plate, we have seen earlier may follow either power law or logarithmic law.

So we have seen in the velocity distribution is concerned either depending upon the case the velocity distribution may be following power law or logarithmic law. With respect to the power law, we have seen generally, we can use these kinds of expressions like the velocity at any location u_x by u star o is equal to C into y u star o by mu to the power n.

So this is the power law, here, C and n are constants, U star o is the shear velocity, and mu is the kinematic viscosity. So now if C and n are known then we can determine the velocity at any high, so this is from the power law and then as for as logarithmic law is concerned, we have seen this equation u_x by u star o is equal to 1 by kappa log e u star o y by mu minus log e beta.

So this equation also we have seen, which is based upon the Karman's and Prandtl approach. So one is based upon the power law other one is logarithmic law. Now what we have seen earlier with respect to flow over flat plate, now we are trying to apply very similar way to the pipe flow by using the same concepts.

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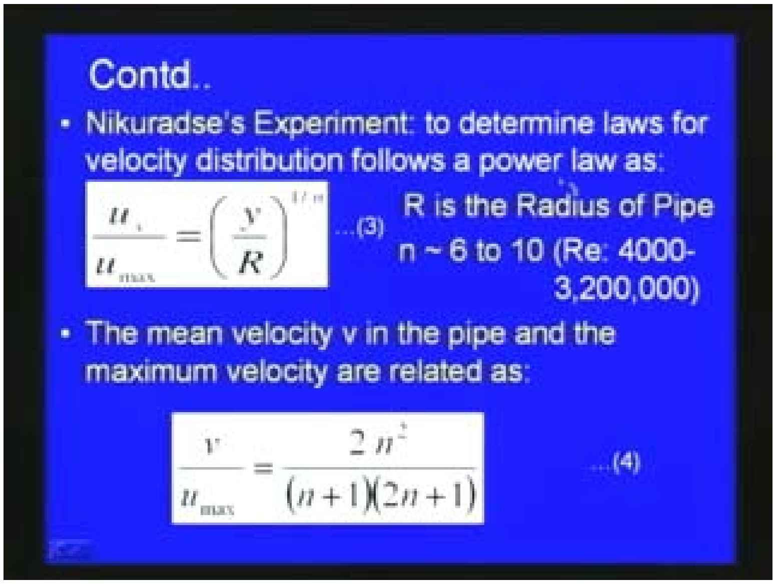

Here, by using the power law there is one equation and using the logarithmic law there is another equation and also Nikuradse's has conducted large number of experiments as far as turbulent flow in pipes are concerned, and then he has also derived some expressions for the velocity distribution in pipes. So as for Nikuradse he has through his experiments he has shown that u_x by u max that is the velocity at any location divided by maximum velocity is equal to can be expressed as y by r where r is radius of the pipe to the power 1by n.

So this n is a constant which can vary depending upon Reynolds number from 6to10 and Nikuradse's shown that this is valid between the Reynolds number four thousand to this here you can see about three point two million to this level four thousand to this three point two million Reynolds number it is valid.

So that is his experimental observation. If we consider the mean velocity with respect to the Nikuradse's experiment here shown that with respect to the mean velocity v by u max, v is the mean velocity v in the pipe and the u max is the maximum velocity of the pipe. He also derived an expression 2 n squared by n plus 1 into 2 n plus 1 as shown in this equation number 4, so this is done by Nikuradse.

These are the experimental observations. Now based upon the power law and logarithmic law which we discussed earlier, we are trying to derive some expressions first for the smooth boundaries and then for the rough type pipes. Here, the velocity distribution slide we can see for smooth pipe we can see that the variation, as we have seen the case of smooth pipe, the boundary is smooth compared to the rough type, so the velocity variation is also very smooth like this as plotted in this figure.

This will be more clear when we derive the real expression. We have also seen earlier from the relation for velocity distribution using power law and logarithmic law that is what we are trying to derive now.

So earlier Blasius has derived an expression for the friction factor f is equal to here shown that for turbulent flow in pipes, friction factor f is equal to 0.316 divided by Reynold's number to the power 1 by 4 where f is the friction factor and we can show this relation is corresponding to the one by seventh power law.

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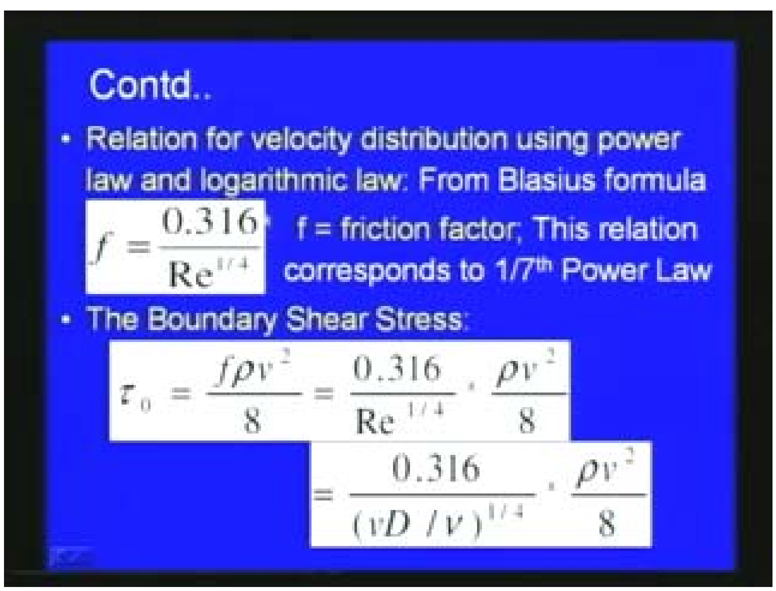

Now our aim is either by using power law or the logarithmic law we want to derive the velocity distribution expression for turbulent flow in smooth pipes. Now if we consider the boundary shear stress, sotow$_0$ is equal to f rho v squared by 8 where rho is the density of the fluid; f is the friction factor; v is the average velocity.

So now this is equal to this, so now if we substitute for f here, it is equal to 0.316 divided by Re to the power 1 by 4 into rho v square by 8. That is equal to 0.316 divided by, so this Reynolds number we can express as mu D by mu, so muD by mu to the power 1 by 4 into rho v squared by 8.

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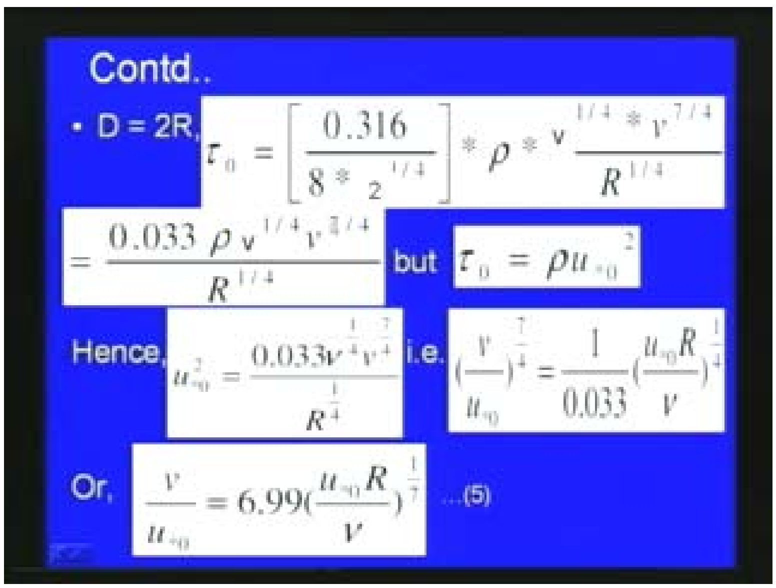

So now, if we substitute for diameter D is equal to 2R and then we will be trying to approximate simplifying this equation. So tow$_0$ is equal to 0.316 divided by 8 into 2 to the power 1 by 4 into rho into mu to the power 1 by 4 into mu to the power 7 by 4 divided by r to the power 1 by 4 as shown in this slide. So here you can see in this slide the expression for tow$_0$.

So after simplification of this we get tow$_0$ is equal to 0.03 rho into mu to the power 1 by 4 into v to the power 7 by 4 divided by R to the power 1 by 4 where R is the radius of the pipe, but now we know that the shear stress at the boundary tow$_0$ is equal to rho into u star o square, u star o is the shear velocity. If we use this relationship here, so that we can write, u star o squared is equal to 0.033 into mu to the power 1 by4 mu to the power 7 by 4 divided by R to the power 1 by 4 .

So this we can now simplify as v by u star o to the power 7 by 4 is equal to 1 by 0.033 into u star o into R by mu to the power 1 by 4. Or we can write v by u star o is equal to 6.99 into u star o into R by mu to the power 1 by 7. So this expression we are getting from the power law as given in this equation number 5.

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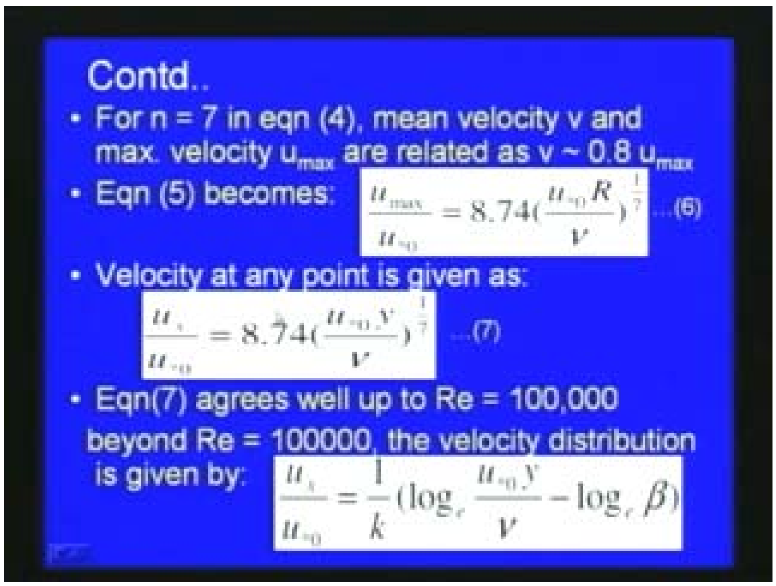

As we have seen, if we use for the power law n is the constant which is varying from 6 to 10 as we have seen, so now if we use here, for particular case say n is equal to 7 then we can see that the mean velocity v and maximum velocity generally, for pipe flow we can write approximate the average of velocity v is equal to 0.8 times the maximum velocity or 80% of the maximum velocity.

If we use this approximation n is equal to 7 and v is equal to u max as shown in this slide. You can write equation number 5 becomes u max by u star o is equal to 8.74 into u star o into by mu to the power 1 by 7, so this is equation number 6 here as shown in this slide. So now we can see that this is as I mentioned earlier, so this is coming from the 1 by 7 for the power law.

So now the velocity at any point, we will derive for the maximum with respect to the central point. Now that is for R ,now if we put substitute R with respect to y at any distance from the pipe wall here, we can see that say u_x by u star o is equal to 8.74 into u star o into y by mu to the power 1 by 7. So this is the equation number 7.

This is the expression with respect to the power law, so here it is clearly shown that this expression is valid above Reynolds number hundred thousand and below hundred thousand.

So earlier we have seen with respect to Nikuradse's experiment we have shown this expression, but now when we theoretically measure very accurately then we can see that this with respect to the power law, we can get the expression generally up to Reynolds number of hundred thousand. But to that only generally the power law can be applied but beyond that we have to go for the logarithmic law, which we have seen earlier. So equation 7 here in the slide we can see equation 7 agrees well up to Reynolds number hundred thousand and beyond this hundred thousand experiment measurements shows that we have to go for the velocity distribution by the power logarithmic law.

So here as we have seen in the logarithmic law is u_x by u star o is equal to 1 by kappa log e u star o into y by mu minus log e beta. Now we have already derived an expression based upon the power law which is assume equation number 7, but various measurements in reality shows that this equation is valid generally up to the Reynolds number of hundred thousand.

So say which is observed that below hundred thousand we have to go for the logarithmic law. So for this turbulent flow through smooth pipe, now we will see to how to derive an expression based upon the logarithmic law. Now the logarithmic law as we have seen earlier u_x by u star o is equal to 1 by kappa log e u star o into y by mu minus log e beta

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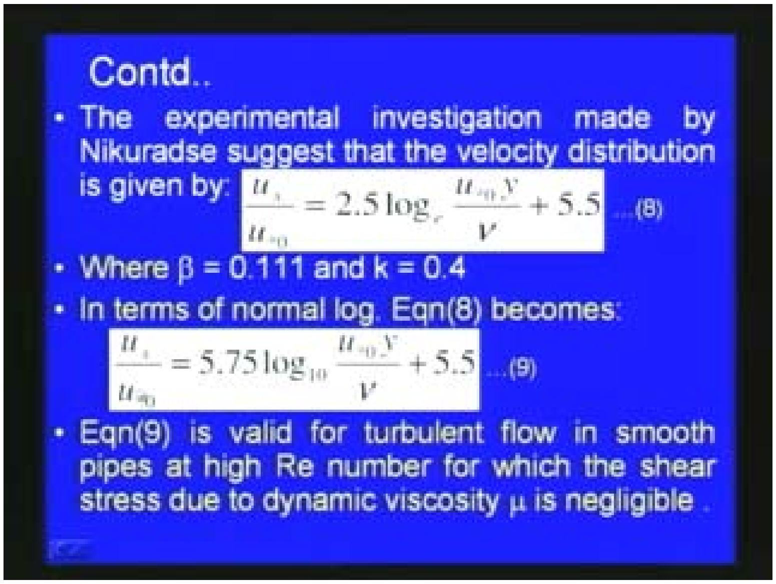

So now again the experimental investigation made by Nikuradse suggests that the velocity distribution is given by. u_x by u star o is equal to 2.5 log e u star o into y by mu plus 5.5. So here you can see with respect to this experiment and the logarithmic law, we can see with respect to the Nikuradse experiment, the constant here is 5.5 or in the previous expression kappa is equal to 0.4 and beta is equal to 0.11. Now in terms of normal logarithm this equation 8 can be expressed as u_x by u star o is equal to 5.75 log 10 u star o into y by mu plus 5.5.

So this is the expression for say turbulent flow in smooth pipes. Here you see that constants will determine based upon some of the experimental measurement by Nikuradse’s. So equation number 9 is valid for turbulent flow in smooth pipes at high Reynolds number for which the shear stress due to dynamic viscosity mu is negligible. This equation number 9 is valid for higher Reynolds number regime for turbulent flow in pipes.

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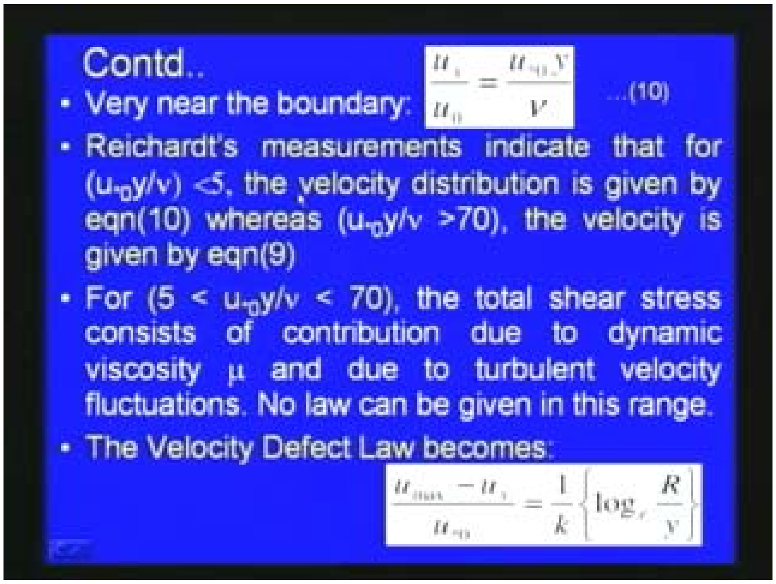

Now, you can see that as we have observed near the boundary generally, a laminar sub layer occurs and then this logarithmic law or this is not applicable. There, very near the boundary we have to use this u_x by u_0 is equal to u star o into y by mu as shown in equation number 10 and then some of the measurement by Reichardt's indicate that for u star o into y by v less than 5 the velocity distribution is given by this equation number 10 where as u star o into y by mu is greater than 70 the velocity is given by previous expression the logarithmic law in the equation number 9.

So here again as for the measurement by Reichardt's by various experiments for turbulent flow in smooth pipe, he has shown that whenever this expression u star o into y by mu is less than 5 we can use this very near the boundary this expression equation number 10 can be used but beyond that, where this value is u star o into y by mu is greater than 70, we have to get the logarithmic law and between this 5 and 70 it is actually there cannot be any general law, it is very difficult to derive, so for u star o into y by mu between 5 and 70 the total shear stress consists of contribution due to dynamic viscosity mu and due to turbulent velocity fluctuation.

So this very difficult to give an expression beyond expression for this between 5 and 70, this is as far as a very near the boundary is concerned and now if we use the velocity defect law, with respect to the earlier the expressions we can write the velocity defect as u max minus u_x divided by u star o is equal to 1 by kappa log e R by y.

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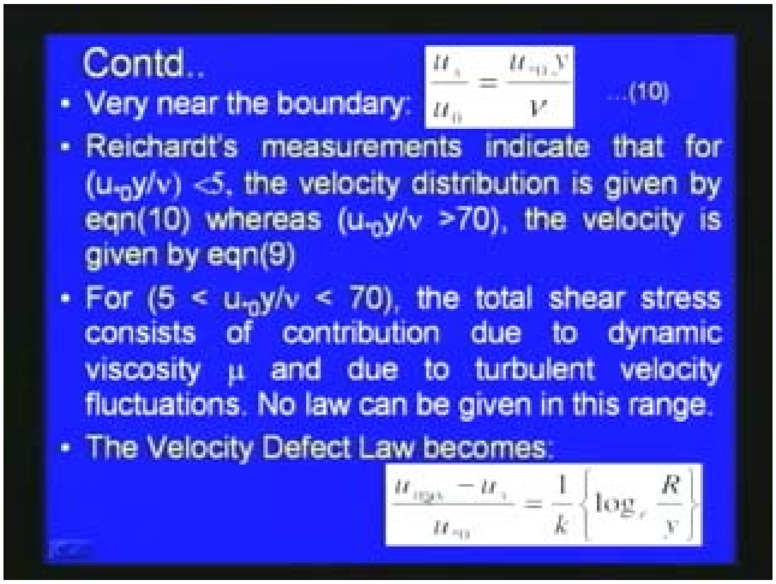

If we use this, so here, you can see that if we use kappa is equal to 0.4, this is again u max minus u_x divided by u star o is equal to 2.5 log e R by y or this can be written as u_x minus u max by u star o is equal to 2.5 log e y by R as written this is equation number 11.

So this is coming from the velocity defect, so now to find out the mean velocity distribution we can use earlier equation number 8, so if we use this equation we can write u_x is equal to u star o into 2.5 log e u star o into y by mu plus 5.5.

Discharge through the pipe we can integrate q is equal to integral 0 to R where R is the radius of the pipe u_x into 2 phi y into dy. That is equal to integral 0 to R u star o into 2.5 If this expression here u_x u star o into 2.5 log e u star o into R minus y by mu plus 5.5 into 2 pi y dy, here you see that y is measured from the center. That is why the expression is written like this, so once we integrate, we can get an expression for the discharge through the pipe pi R squared u star o into 5.75 log 10 u star o into R by mu plus 1.75, after integration.

Then the mean velocity once the discharge is known we can determine the mean velocity is equal to the discharge divided by the area of cross section. So here v is equal to u star o into 5.75 log 10 u star o into R by V plus 1.75, this we can write in this form V by u star o is equal to 5.75 log 10 u star o R by V plus 1.75 as in equation number 12.

Again in this equation number 12, you can see here, this is with respect to the mean velocity equation number 12 and here in equation number 9, this is the velocity at any expression. We can get a velocity defect law for turbulent flow in smooth pipes as u_x minus v by u star o is equal to 5.75 log 10 y by R plus 3.75 as given in equation number 13. So this is the velocity defect law for turbulent flow in smooth pipe. Now you can see that in most of the pipes the friction factor is in either component.

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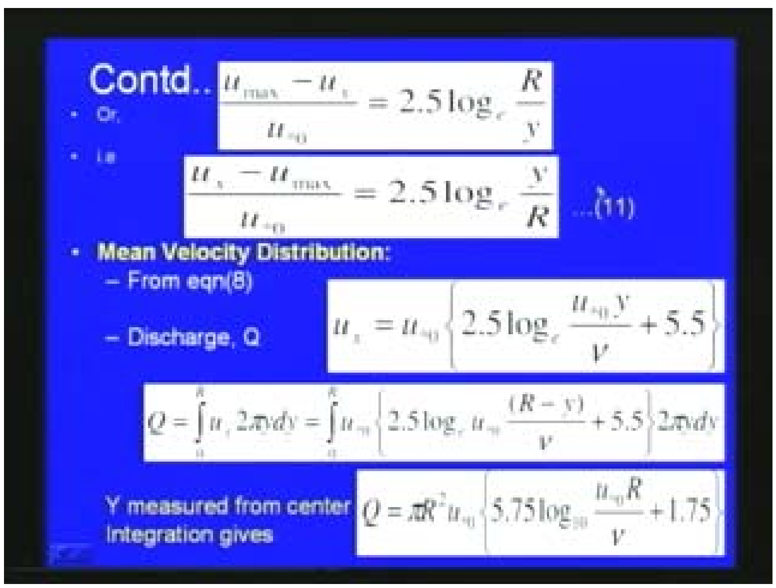

Now the friction factor f and shear stress we can relate tow $_0$ is equal to f rho v squared by 8 where v is the average velocity of flow, f is the friction factor, rho is the mass density, now by using this, also we know that tow $_0$ is equal to rho into u star o squared. Hence we can write the friction factor f is equal to 8 into u star o by V whole squared. Now this tow $_0$ is here and then we are using this tow $_0$ rho u star whole squared. Both we use f is equal to 8 into u star o by v whole squared as in equation number 15.

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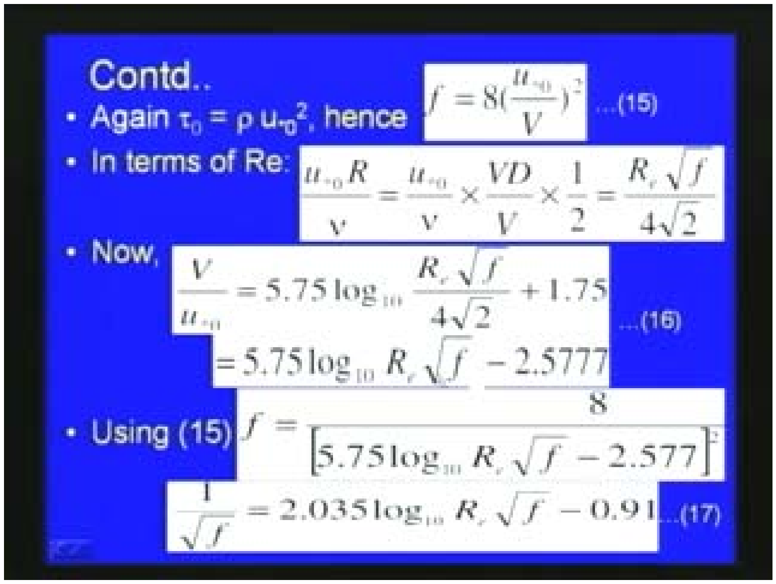

Now, in terms of Reynold's number, we can write u star o into r by V is equal to u star o by v into VD by V into half, so this is equal to Re into square root of f by 4 into square root of 2, so after substituting back we can write v by u star o is equal to 5.75 log 10 and Re root f divided by 4 root 2 plus 1.75.

So that is equal to v by u star o is equal to 5.75 log 10 Re root f minus 2.577 as in equation number 16. Or we can write f is equal to 8 divided by 5.75 log 10 Re root of minus 2.57 whole squared, from this again we can write 1 by root f is equal to 2.035 log 10 Re root f minus 0.91 as in equation number 17.

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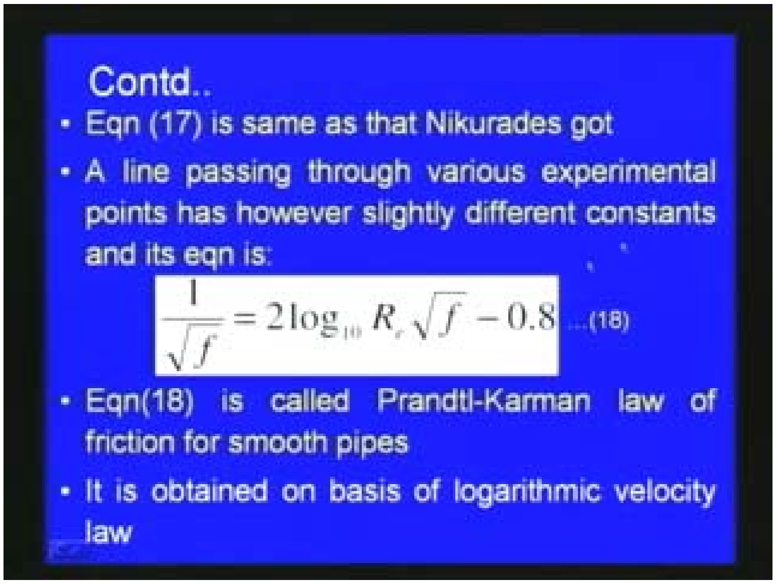

So this equation number 17 is 1 by root f this equation, now this equation 17 is almost same as that Nikurade's got. Here a line passing through the various experimental points has however slightly different constant, so Nikurade's with respect to his experiments here shown, this even we got here is 1 by root f is equal to 2.035 log 10 Re root f minus 0.91.that with respect to Nikurade's experiments, he could get 1 by root f is equal to log 10 Re root f minus 0.8 as in equation number 18, so here this equation number 18 is written here, this equation is called a Prandtl Karman law of friction for smooth pipes.

Here, we can see that the pipe friction is connected with respect to only the Reynold's number. So this equation is called the Prandtl's Karman law of friction flow of smooth pipes. This is based upon the logarithmic velocity law as described in the previous slides. Now here you can see the velocity distribution for smooth pipe if you plot, here as shown in this figure. Here on y axis it is u by u star and x axis it is u star o y by v.

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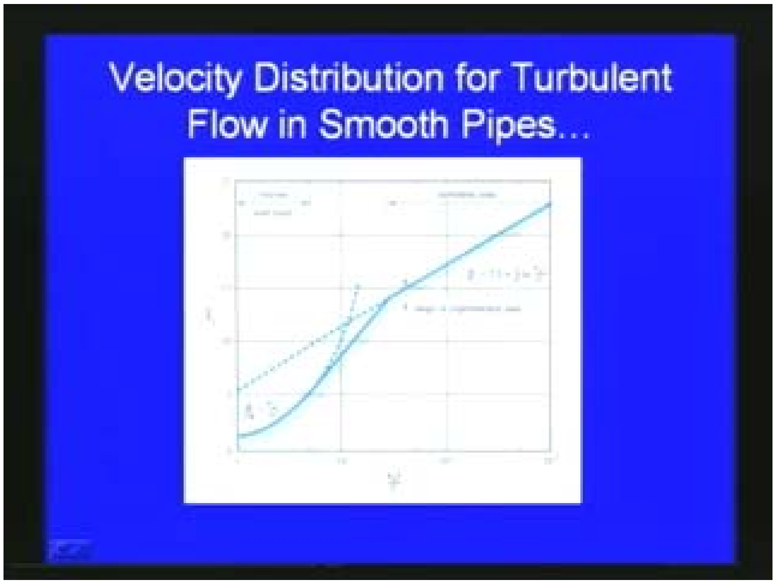

So if we plot this you can see when with respect to the various range of experimental in later, first for the viscous valve layer we can see that the variation is going like this, and then for the turbulent shown, the variation is with respect to this. Here you can see v by u star with respect to this equation which we have seen. The variation with respect to v by u star and u star o by v can be expressed like in this curve here, so this is u by u star with respect to equal to 5.5 plus 1 by kappa natural of u star o by mu. So this is with respect to logarithmic law in the turbulence shown and here with respect to the viscous valve layer you can see that u by u star o is equal to u star o by v plus that variation you can some places we can approximately like with respect to power law or linear variation can be for some range we can use and then the turbulent regime we can use the logarithmic law.

So this is about the velocity distribution flow in smooth pipes. Now, here for the smooth pipe we have to find out the velocity variation we have basically used the power law and the logarithmic law and then we have made some comparison with respect to the Nikuradse's measured or experiments and then its corresponding expression which Nikuradse's has derived then this almost coming very nearby, so that we can rely upon these kinds of expression for velocity for turbulent flow through smooth pipes.

Now here we will discuss the turbulent flow through rough pipes.

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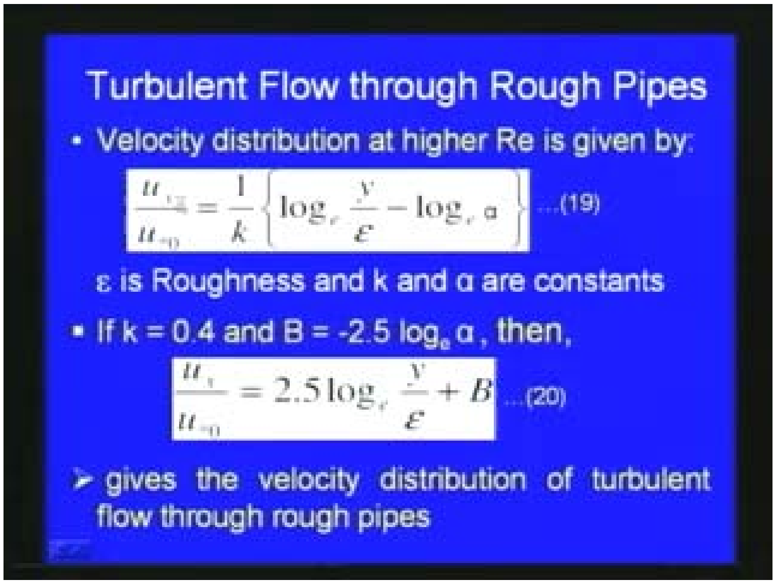

So here rough pipe is concerned the velocity distribution at high Reynolds number is again here we use the logarithmic variation logarithmic distribution. So u_x by u star o is equal to 1 by kappa log e y by epsilon minus log e alpha as in equation number 9, where epsilon is the roughness and kappa and alpha are constants. So where kappa is the Karman constant, kappa is equal to 0.4 and as we have seen this if you use this B is equal to minus 2.5 log e alpha with respect to this expression then we can write u_x by u star o is equal to 2.5 log e y by epsilon plus B as in equation number 20.

So this is the velocity distribution of turbulent flow through rough pipes, so here the major difference you can observe here is we put a term for the epsilon which is the roughness high with respect to the rough pipe.

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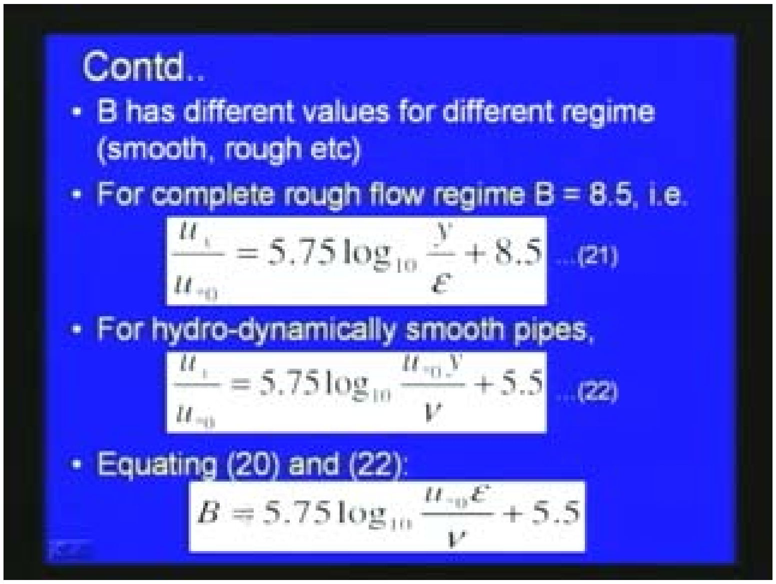

Now, this constant we have seen this constant B has different values for different regime, as we have already seen whether the pipe is rough, smooth or between which range there value of B changes. So for complete rough flow regime we can show that B is equal to 8.5, so that we can write u_x by u star o is equal to 5.75 log 10 y by epsilon plus 8.5 as in equation number 21.

And then for hydro dynamically smooth pipe as we have already derived in the previous slides we can write u_x by u star o is equal to 5.75 log 10 u star o into y by v plus 5.5 as in equation number 22. Now if we compare this expression with respect to the rough pipe as in equation number 20 and equating 21 and 22.

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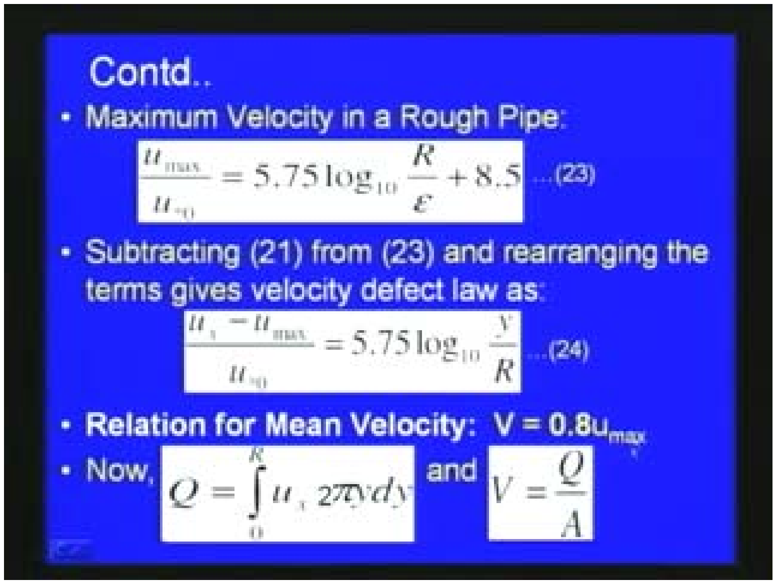

Then you can see that this, this B is equated to, B is equal to 5.75 log 10 u star o into epsilon by v plus 5.5. So for maximum velocity in a rough pipe we can write u max by u star o is equal to 5.75 log 10 R by epsilon plus 8.5 as in equation number 23 and now if we subtract 21 from 23 and rearrange the terms as, we got earlier here again, we can get a law called velocity defect law. As this velocity defect law importance is generally for pipe flow maximum velocity is known or we can easily determine.

So with respect to that we can determine the velocity variation at various locations. The velocity defect law now become u_x minus u max divided by u star o is equal to5.75 log 10 y by R as given in equation number 24.

And very similar way what we have done for mean velocity, here again if we assumed mean velocity V is equal to 0.8 into u max and then if you put Q is equal to integral 0 to R u_x two pi y into dy and V is equal to Q by A.

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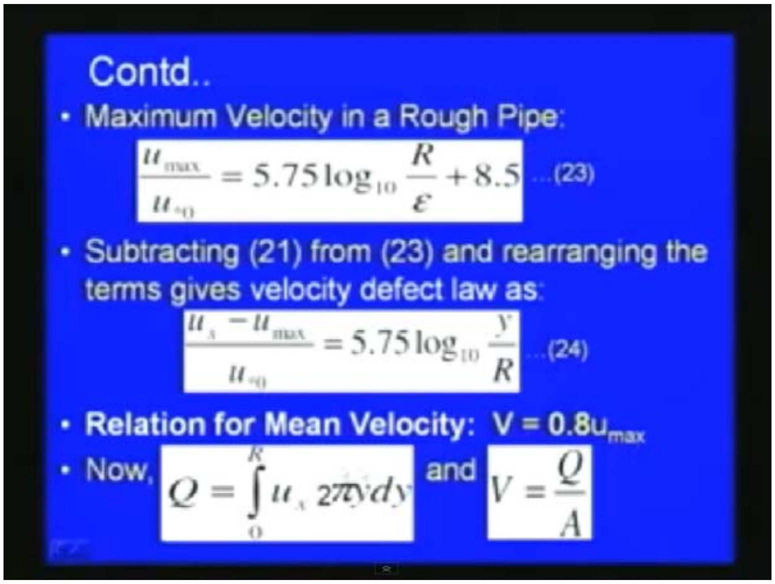

So we can change this equation for rough pipers as V by u star o is equal to 5.75 log 10 R by epsilon plus 4.75 as in equation number 25. This is a relationship with respect to the mean velocity and the shear velocity and the radius of pipe and epsilon as in equation number 25.

So now this equation number 25 as in this slide we can again deduct this, subtract this 25 from this equation number 21, so that we get again a velocity defect law based upon the mean velocity. So earlier velocity defect law which we have seen is with respect to maximum velocity, here we can derive the velocity defect law with respect to the mean velocity. We can write u_x minus v by u star o is equal to 5.75 log 10 y by plus 3.75 as in equation number 26. This equation number 26 is called universal velocity law or velocity defect law, since u_x minus v is considered or it is also called Karman Prandtl law for velocity distribution in a circular pipe. So this is one of the generally used equations for turbulent flow in rough pipe as expressed in with respect to equation number 26 which is the universal velocity law.

So here, this friction factor for turbulent flow in rough pipe again we can with respect to f we can write f is equal to 8 u star o by v whole squared, so u star o by v is equal to square root of f by 8.

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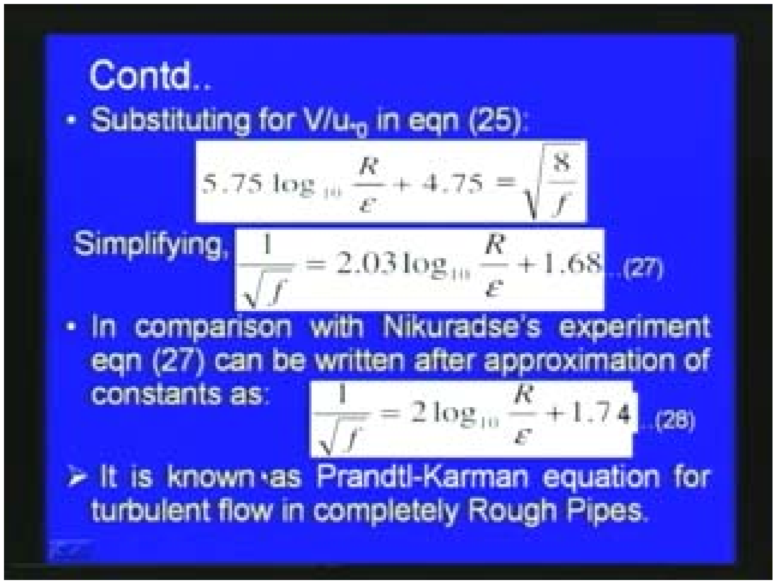

So that here for this u star o by v with respect to this expression we can substitute back so substituting for v by u star o in equation number 25 we can get 5.75 log 10 R by epsilon plus 4.75 is equal to square root of 8 by f.

So if we simplify this we can get 1 by root f is equal to 2.03 log 10 R by epsilon plus 1.68 as in equation number 27. Here again we are getting an expression with respect to the radius of the pipe and then with respect to the friction factor and the roughness high epsilon. So that in this equation number 27 and again for rough pipe also, turbulent flow in rough pipe Nikuradse's conducted large number of experiments and he has also derived an expression. Here in comparison with Nikuradse's experiment equation 27 can be written as after approximation of the constant as one by root f is equal to 2 log 10 R by epsilon plus 1.74.

So here we can see with respect to the measurement by the Nikuradse's as again small variation is there. This is the expression equation number 28 is generally used but it is almost very near to what we are getting with respect to the theoretical development here.

Here, this equation number 28 is called Prandtl's Karman's equation for turbulent flow in completely rough pipe. Now, we have seen the various expressions for velocity variation, for turbulent flow in smooth pipe and turbulent flow in rough pipe. We have to define which pipe is smooth which pipe is hydro dynamically smooth or which pipe is hydro dynamically rough .This we can use some expression here, we say that a pipe is set to be hydro dynamically smooth u star o into epsilon_S by v is less than 5 where epsilon is the equivalent roughness of pipe and u star o is shear velocity.

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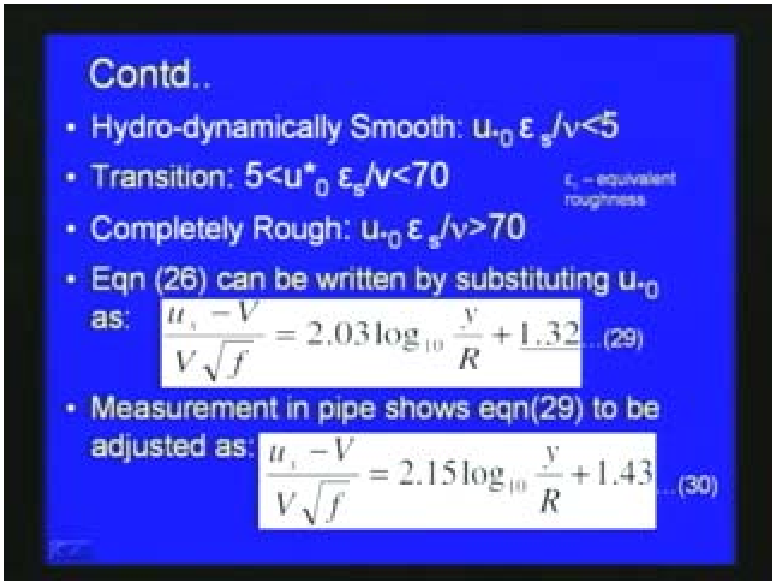

So whenever this expression is less than 5 we call the pipe as hydro dynamically smooth and then if this expression u star o into epsilon_S is by v is greater than 70 then we call the pipe has completely rough. Accordingly, we can use depending upon whether it is smooth pipe or it is rough pipe we can use the corresponding expression to find out the velocity variation for turbulent flow is smooth or rough pipe and then between 5 and 70 of this u star o into epsilon_S by v. Here when it is between 5 and 70, it is a transition between this smooth to rough.

So here it is very difficult to get an expression like what he had derived for smooth or rough. So either depending upon it we have to use either one for the equation for rough or smooth depending upon the case.

So that way, we can see whether the pipe is hydro dynamically smooth or in transitional stage or in completely rough. Now the equation 26 which we have derived here, so here equation 26 can be written by substituting in u star o so that we can write u_x minus v divided by v square root f is equal to 2.03 log 10 y by R plus 1.32 as in equation number 29.

So again the measurement in pipe shows in equation 29 to be adjusted since generally what we theoretically or with respect to various expressions, we derive may not exactly match what we really are getting the experiments. Seeing that the expression it will be written here 29. We have to slightly adjust like this, u_x minus v divided by v square root of f where v is the average velocity that is equal to 2.15 log 10 y by R plus 1.43.

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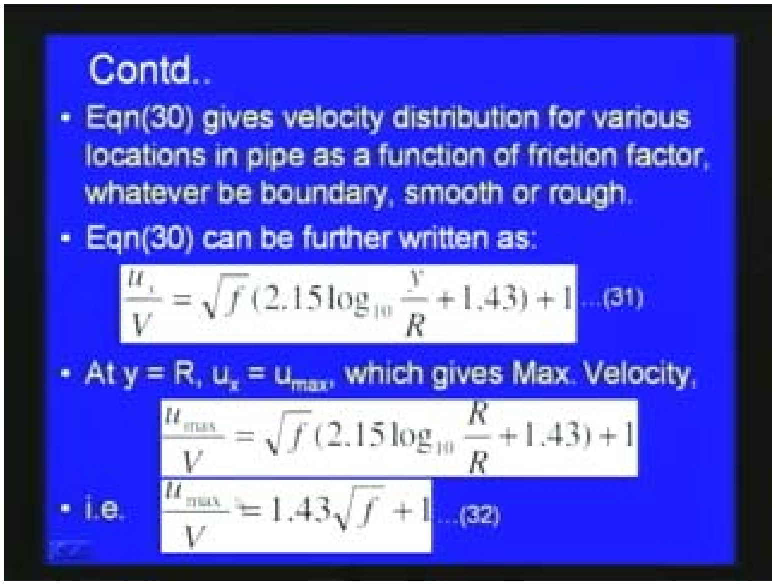

With respect to the measurement, this is based upon some of the theory which we are developing here. This is the expression equation number 30.

So equation 30 gives the velocity distribution for various locations in pipe as a function of friction factor wherever the boundary is smooth or rough. This equation number 30 which we have written here, we can use it is for either rough pipe or this is an expression between the average velocity at any location and the friction factor. So this expression we can use for smooth or rough pipe as a general tool.

So now equation 30 can be further written as u_x by v is equal to square root of f 2.15 log 10 y by R plus 1.4 plus 1, we can simplify and then at y is equal to R, say central line of the pipe, u_x is equal to u max which gives the maximum velocity. So u max by v is equal to square root of f into 2.15 log 10 R by R plus 1.43 plus 1. This we can simplify as u max by v is equal to 1.43 square root of f plus 1 as in equation number 32.

So this expression is important since this gives a relation between the maximum velocities the average velocity and the friction factor of the pipe. So for the turbulent flow in a pipe this expression equation number 32 is an important expression where relation between u max v and this friction factor f.

So now if we plot the velocity distribution for turbulent flow through rough pipes, we can see that variation can be like this. So this is the pipe which we consider here.

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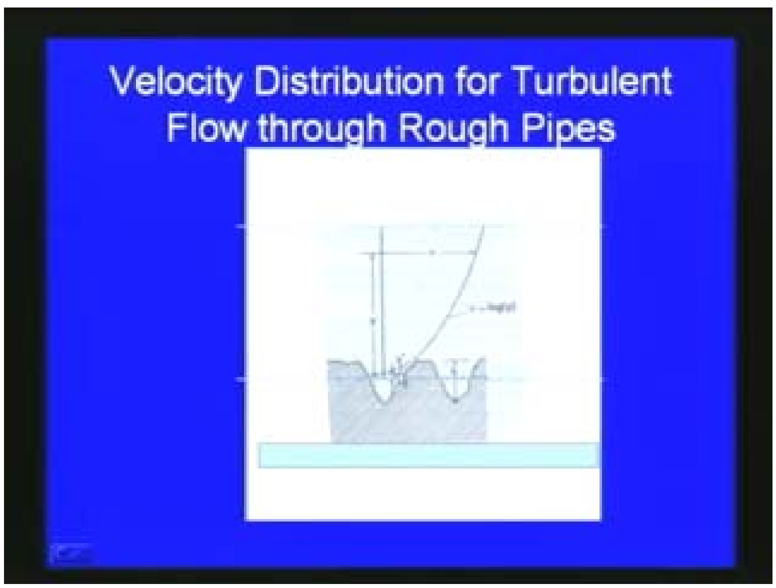

We can see these are rough type pipes. We can see here initially variation is for the sub layer which we consider the variations is like this either it is you can see linear or we can approximately using the power law and then after that laminar sub layer, we can see that we will be approximating the expression with respect to the logarithmic law or v is proportional to the average velocity v is proportional to logarithmic law of y.

So this is the general pattern which is now we have shown through experiments as derived by Nikurades or through theoretical development or the various relationships. So the laminar sub layer is either approximates either with respect to power law or with respect to the we consider say linear variation in the laminar sub layer and then beyond that, we consider the variation with respect to the logarithmic law or the variation is we are putting as the variation with respect to logarithmic variation.

So both theoretically as well as experimentally we can show this. Now we will see before closing this chapter on turbulent flow pipes, we will discuss a small example with respect to the various relationships which we have derived here.

So the example is here what we consider is a small pipe in forty centimeter diameter pipe carrying water velocities were measured and center and at quarter point where 2.425 meter per second and 2.275 meter per second. We have to calculate discharge friction factor f the shear velocity at pipe wall. Here the data is given as take the kinematic viscosity is 0.15 stokes.

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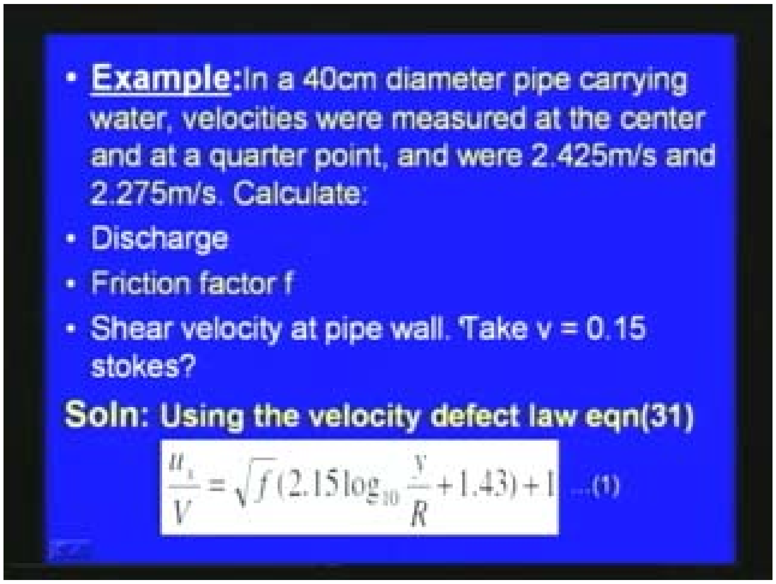

For this problem, the pipe here is for turbulent flow in pipe, this is the central line. The diameter is 40 centimeter. The velocities were measured at the center point as shown here and the velocity at the center point is 2.425 meter per second and then quarter point. So here this is R by 2, at this location velocity is measured as 2.275 meter per second.

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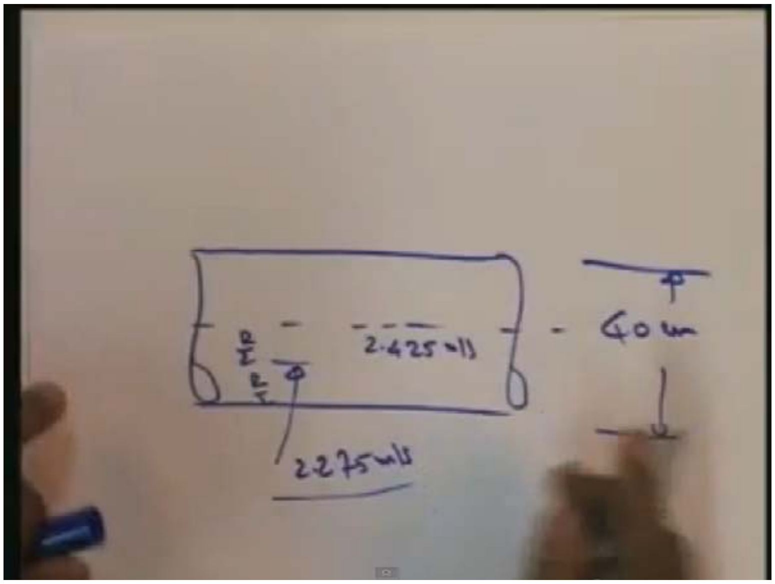

So this is the problem definition, so now for this given condition we have to get the discharge through the pipe and then we have to determine the friction factor and the shear velocity at pipe wall.

So this is the problem, so here to solve this problem we will use equation number 31. So here the 31 is given here, so u_x by v is equal to root f, this expression we will use to solve this problem. So the equation is u_x by v is equal to square root of f 2.15 log 10 y by R plus 1.43 plus 1 as given in equation number 1.

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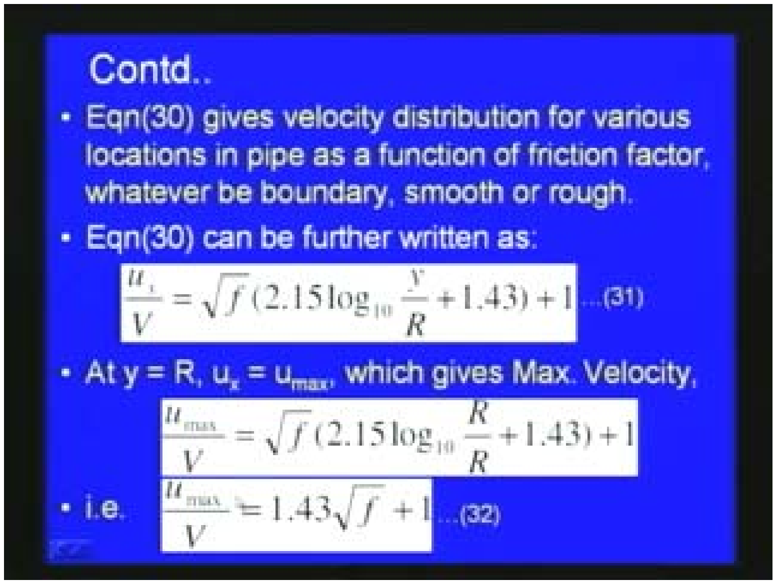

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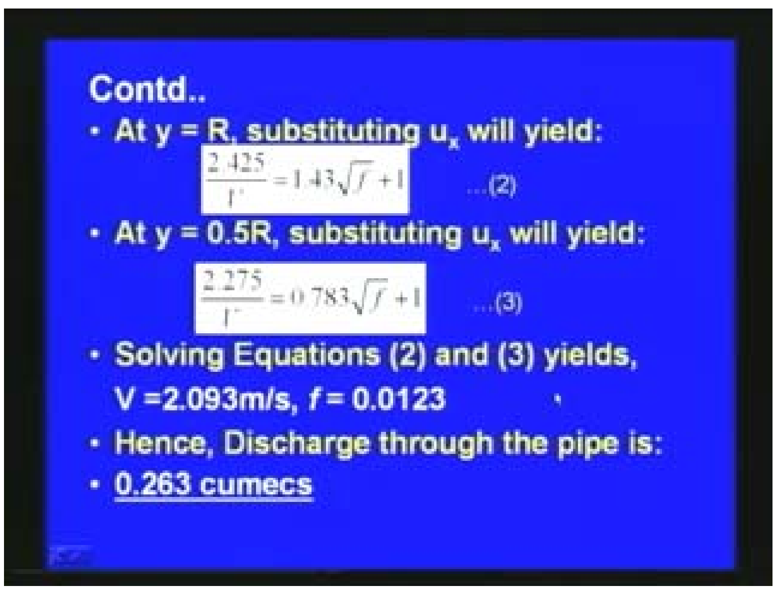

So now at y is equal to R if we substitute you can see that the velocity at the central line is given the maximum velocity is given, so if we put this expression 2.425 divided by v is equal to 1.43. So here R by R, this expression it is 1.43 root f plus 1 so that is equation number 2, similar way the velocity at y is equal to 0.5 R is given, so we can put this in this expression equation number 1. So 2.275 by v is equal to 0.783 root f plus 1 after simplification and substitution of the values. Now we got two equations with v and f, so the unknowns here are the mean velocity and the friction factor.

So you can solve either v or f so we get here after solving these two equations, we can get v is equal to 2.093 meter per second and f is equal to 0.0123. Now once we determine the velocity average velocity now v is calculated, we can easily determine the discharge.

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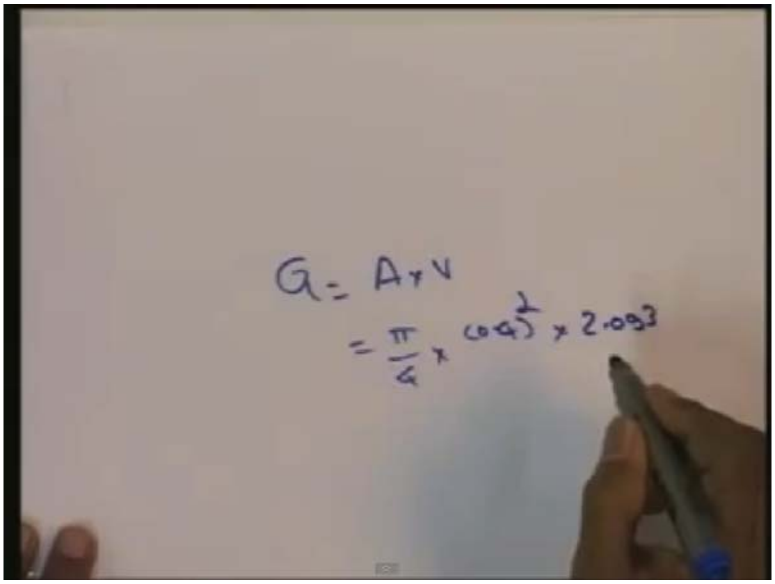

So the discharge through the pipe once the average velocity is given discharge is equal to Q is equal to area of flow section multiplied by A into Q is equal to A into v. So here A is pi by 4, v is 0.4 so pi by 0.4 squared into the velocity average velocity is 2.093 meter per second.

So from this we will get the discharge, the discharge is now 0.263 Q max, so that we can calculate. Now from equation 26 which we have already written here equation 26 is here.
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So this is equation number 26 so if you put use this equation number 26 here we can write u_x minus v by u star o is equal to 5.75 log 10 y by R plus 3.75. So here for y is equal to R we can write say u_x will be, that will be the maximum velocity, u max this expression become u max minus v divided by u star o this y by R is R by R it will be 1, this will be cancelled so that is equal to 3.75.

So u max minus v by u star o is equal to 3.75. So now here note u max u max is given as for this problem 2.425 and also we have calculated v as v is calculated as 2.093 meter per second.

So that we can find out the shear velocity u star is equal to say, from this expression u star o is equal to 0.04 meter per second. So like this we can solve various problems. What we have seen here so for is say for turbulent flow in say through pipes say, either smooth pipes or hydro dynamic smooth or hydro dynamic rough pipe, we have classified.

Then for the hydro dynamically smooth and rough pipe we have derived the expression for velocity various locations or various depths. So basically you can see that both cases either we are using the power law or the logarithmic law depending upon the Reynolds number depending upon whether the pipe is smooth or rough and then we have derived various expressions for the velocity variation. Also this what we have derived the expression we have verified with respect to the Nikuradse's experiments and Nikuradse's conducted large number of experiments for various kinds of pipes and then it is almost matching, a small variations are there or otherwise the expressions which we are deriving is almost same. That way now the turbulent flow in pipe either smooth or rough, we can utilize the expressions or the velocities equations based upon the power law and the logarithmic law.

So now to conclude this chapter, to summarize, in this chapter we were discussing about the laminar flow and turbulent flow; we have seen the various kinds of flow, types of flow then we have seen how we classify according to the Reynolds number, whether the flow is laminar or a transition or at turbulent condition by Reynolds experiment and then using the Reynolds number and then we have seen with respect to this whether the flow condition is between the laminar transition or turbulent.

So according to that we have to see the condition and then we will be generally deriving the equation and then further we have seen we have considered the initial the laminar flow between this.

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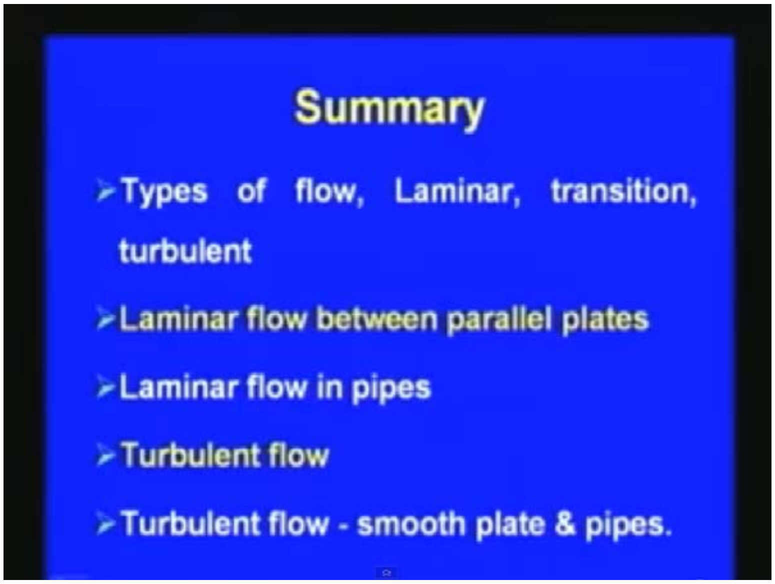

This slide is summarizing the chapter here so for laminar flow between parallel plates we have seen, we have derived various expressions for flow between two fixed plates in laminar condition and then one plate is moving, then also we have seen flow between two parallel plates both are moving in the opposite direction. We have derived the expressions generally, the laminar flow expression is based upon the Newton's second law and then various other theories and then we considered the laminar flow in pipes. There we have seen very similar way by using the Newton's law there also for laminar flow in pipes also we have derived various expressions for the velocity variation and then the average velocity, expression for discharge, expression for the pressure variation, all this we have derived for laminar flow in pipes.

And then we discussed in this chapter the turbulent flow, so turbulent flow is starting from the basic theory by considering the Reynolds equation. We have derived the basic equation for turbulent flow and then Nevier Stokes form of the turbulent flow equation.

Then we have discussed various turbulent flow models including zero equation model then les model then one equation or two equation model like that for turbulent flow we have seen. Then finally, in this chapter we have derived with respect to zero equation models since as we have seen the turbulent flow is very complex, very difficult to determine the various parameters like velocity or the pressure variation.

So as to explain further the turbulent phenomena we have used the zero equation models or Prandtl mixing length hypothesis Karman's approach, and then for turbulent flow over flat plate or the parallel flow case and then the turbulent flow through smooth as well as rough pipes, we have used this zero equation models based upon the Prandtl mixing length theory, and then we will try to derive some expressions for the velocity variations. So the importance of this equation based upon the Prandtl's mixing length theory is not completely 100% accurate, its accuracy is less compared to the Reynolds equation, if we use Reynolds Nevier Stokes equation.

But still this equation, these expressions shows how the variation takes place and it bring out the physics of the problem, how the variations with respect to the turbulence, since the turbulent phenomena is very difficult to quantified or very difficult to explain. So these expressions based upon the power law and logarithmic law for turbulent flow in pipes or flow over flat plate, this expression can be used to see how the turbulent phenomena and here we have tried to quantify based upon this expressions. So these are the topics which we have covered in this chapter. So further we will be discussing the various other chapters on the fluid mechanics.