Fluid Mechanics

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Lecture - 21
Laminar and Turbulent Flows

Welcome back to the video course on fluid mechanics. In the last lecture we were discussing about the turbulence models. We have derived the basic equations for turbulence; we have seen the Reynold's equations; then we have seen that the complexities for solving the turbulence problems. Since, we have got four equations in three dimensions, three momentum equations and one continuity equations. But we have generally ten unknowns, so it is very difficult to get a mathematic solution for these turbulence problems. So, we have seen by considering this various methodologies for solution for a turbulent flow problem.

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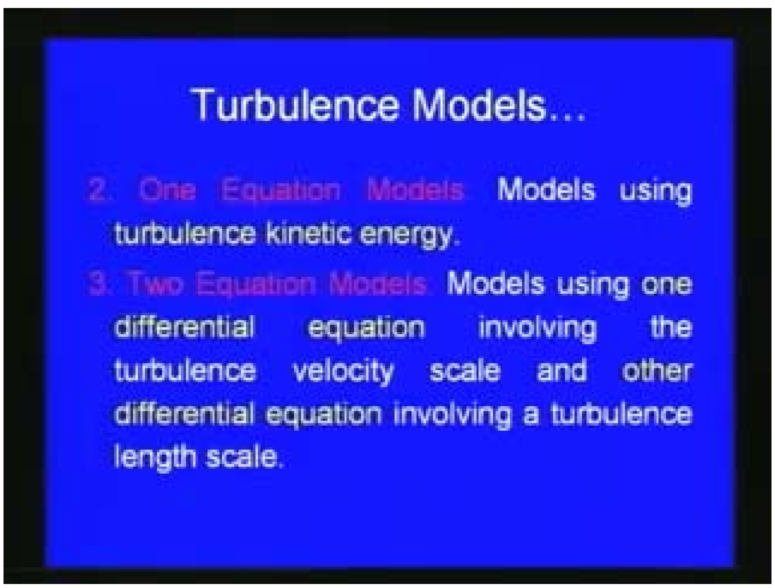

So we have discussed about five models for the turbulent flow simulations. First one is zero equation models, which we have seen in last time. Second one is one equation model and third one is two equation models.

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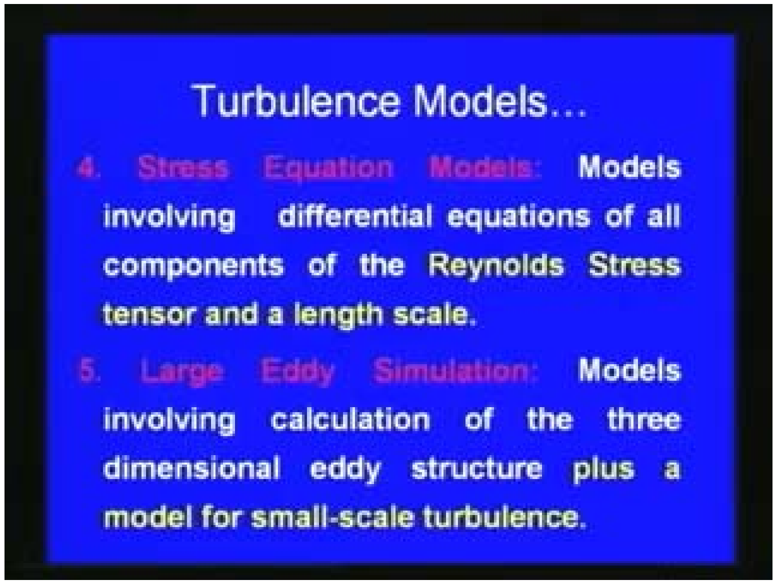

Fourth one is stress equation models and fifth one is large eddy simulation models. Now out of these five models which are generally used, which is available in literature, as the turbulence problems are very complex always better to go for a complete solution or either larger simulation or completing equation models, but even the solution of this is very complex large computer capacities required and then now in recent times we have got number of computer fluid dynamic packages which we take care of these kinds of models.

In this course, to understand the fundamental theories or the principles of turbulence we will be discussing one of the models, the first model zero equation models, since these zero equation models are derived based upon some of the experimental observations and then some of the empirical theories. It is a mix of some of the experimental observations and some of the empirical equations.

So in this field, the zero equation models, two names are very important who have developed this theory: one is Prandtl's, the other one is Karman. The zero equation model briefly will be discussed here for the turbulence flow simulation and that will give rate of picture of how difficult are these problems and then how we can approach with a simplified theory. So out of this, first one which we are discussing here is the Prandtl's mixing length theory.

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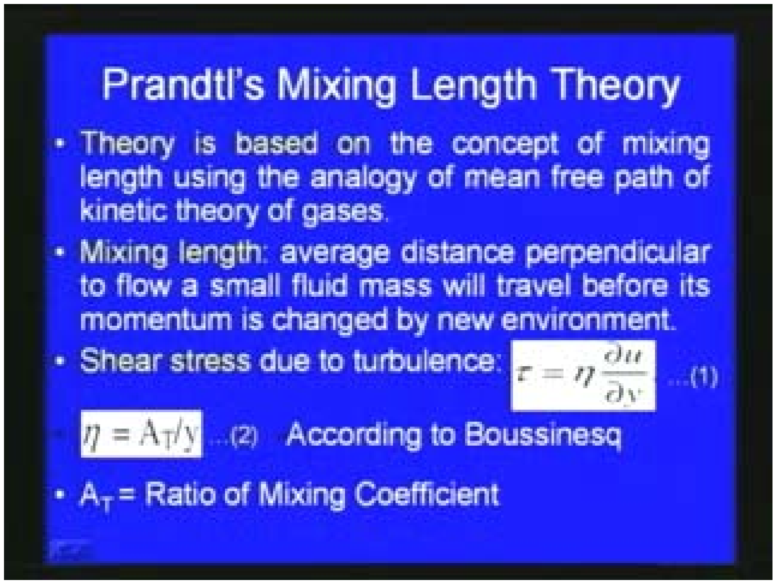

So this Prandtl's mixing length theory is based on the concept of mixing length using the analogy of mean free path of kinetic theory of gases. So kinetic theory of gases has been well developed in the beginning of 20th century. Then, Prandtl's used this kinetic theory of gases, to analyze the turbulent flow for simplified cases, like a flow over a parallel plate and then flow through pipes, so that some aspect of the turbulence or the theories behind turbulence can be understood, so he started with the kinetic theory of gases. What he did is, he defined a length called mixing length which is the average distance perpendicular to flow a small fluid mass will travel before its momentum is changed by new environment.

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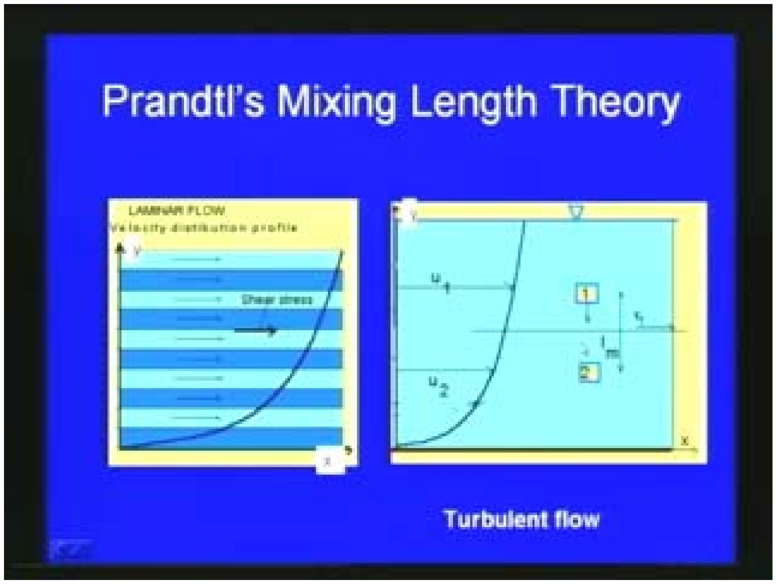

Here this is explained in this figure. First one is the case of laminar flow velocity distribution profile is given and second case is for turbulent flow. As for PPrandtl by using the kinetic theory of gases, what he found is he defined a length called mixing length and then this length is actually the average distance perpendicular to flow, a small fluid mass, say if we consider lump of fluid or a small fluid mass it would travel, the average distance which travel perpendicular to flow before its momentum is changed from one environment to another environment or a new environment. This is clear from this figure, so if we consider a small mass here, this 1 and then it is jumping to other location 2 so that the momentum is changed in this process. This length Prandtl defined as mixing length and then he proposed the mixing length hypothesis which is known as the one of the fundamental development in the area of turbulence.

So then he defined the shear stress due to turbulence as tow is equal to eta into delta u by delta y where u is the velocity in x direction and then where eta is equal to A_T by A subscript T by y where A_T is the ratio of mixing coefficient.

He also used this Boussinesq approximation which is given by eta is equal to A_T by y where y is the vertical distance. At the ratio of mixing coefficient, so by using this further as shown in this with respect to this figure y is in this direction and x is this direction, velocity for the turbulence is plotted and then he derive the shear stress as the tow is equal to minus rho u_x dash u_y dash bar.

So this is the definition of the shear stress and by using the Boussinesq's hypothesis considering a parallel flow Prandtl's showed that this u_x bar can be written as a function of the u_x y and he considered u_z is equal to 0 and u_z bar is equal to 0 and u_y bar is equal to 0.

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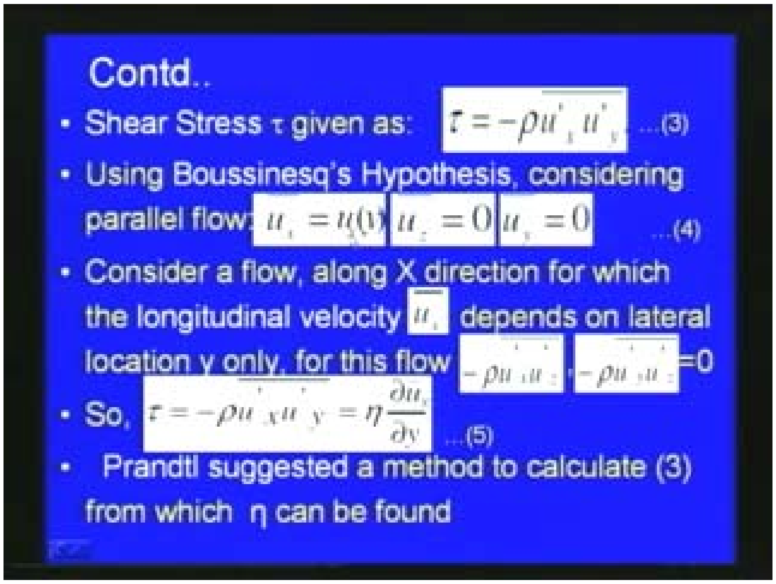

What he considered here is we can see it is a parallel flow, so in the parallel flow case for example, if we consider plate like this and then he considered parallel flow, such that for this particular case the velocity variation, if this is x direction and here this is y direction so that x variation is with respect to u_x and y only and other components like the z component and the y component are 0.

So this is how he defined a problem, if this is a parallel plate and over a flat plate and he considered the parallel flow.

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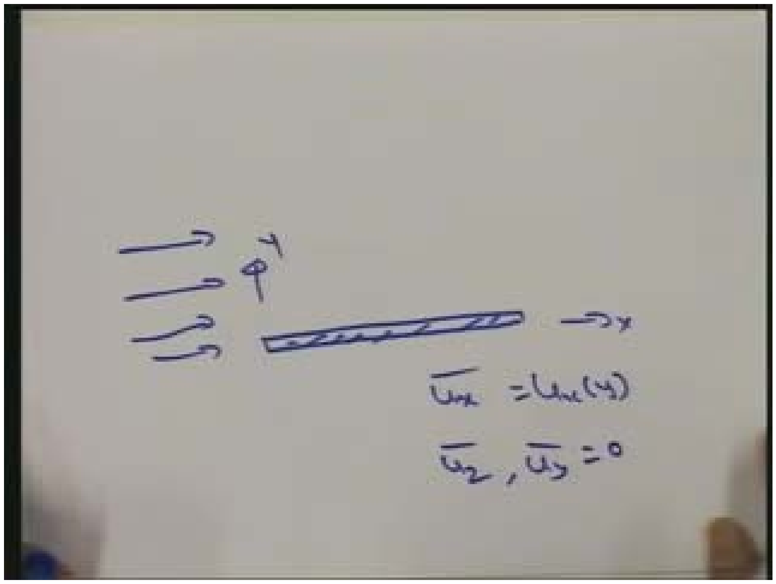

Such that u_x is the velocity in x direction, is the function of y and u_z the velocity z direction and y direction are zero. So he considered such a problem and then he tried to explain the turbulence happening with respect to this particular problem and so that further theory can be extended for other problem.

So for this parallel flow along x direction for which the longitudinal velocity is u_x this depends on lateral location of y only. As we have already seen here this u_x depends on only on y, so that the flow is other components are 0. So this is the parallel flow which Prandtl's considered to explain the turbulence.

So that, this rho u dash x u dash z mean or it rho u dash u_x dash u_z dash bar and minus rho u_y dash u_z dash bar is equal to 0, for this particular parallel flow case. So that he derived tow is equal to the shear stress is equal to minus rho u_x dash u_y dash bar that it is equal to eta delta u_x bar by delta y by using the Boussinesq's hypothesis. So finally Prandtl's got the shear stress is equal to minus rho u_x dash u_y dash bar is equal to eta del t u_x bar by delta y as given in equation number 5 and then Prandtl suggested a method to calculate this tow from which we can find out this eta.

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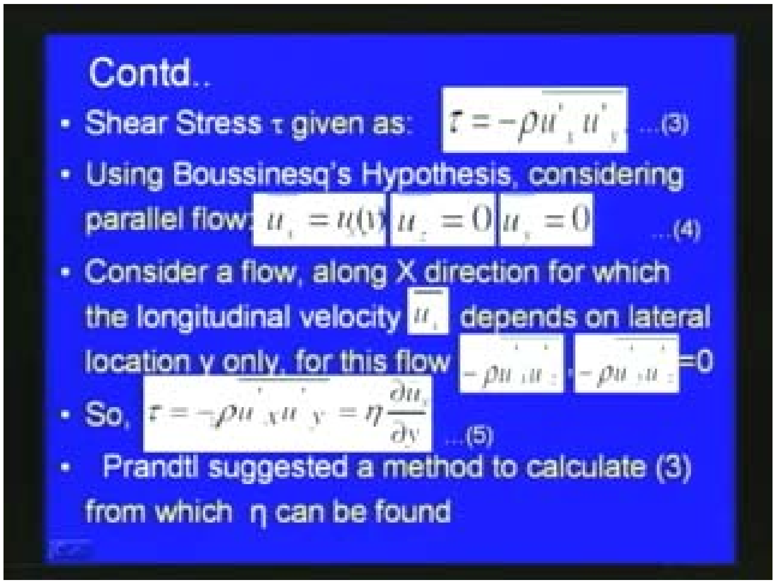

So that is the way he approached the problem and finally Prandtl introduced the concept of mixing length which is called mixing length hypothesis for the fluid which travels laterally before losing its momentum and acquiring a new momentum in the new layer.

So by Prandtl's hypothesis the mixture or mixing length the absolute value of the fluctuating component of velocity along x axis given by u_x dash bar is equal to l into the mixing length multiplied by the derivative of u_x with respect to y mixing length multiplied by u_x bar by dy.

So this l is equal to lm that is that transverse distance where the lump of fluid mass is jumping or traveling, that distance is called a the mixing length or the mixture length. In all these problems, our final aim is to find out an expression for the velocity.

So he used this theory with the Boussinesq's approximation and then finally, the fluctuating component is shown in this figure here if it is u_y dash bar is he showed that u_y dash bar is proportional to u_x dash.

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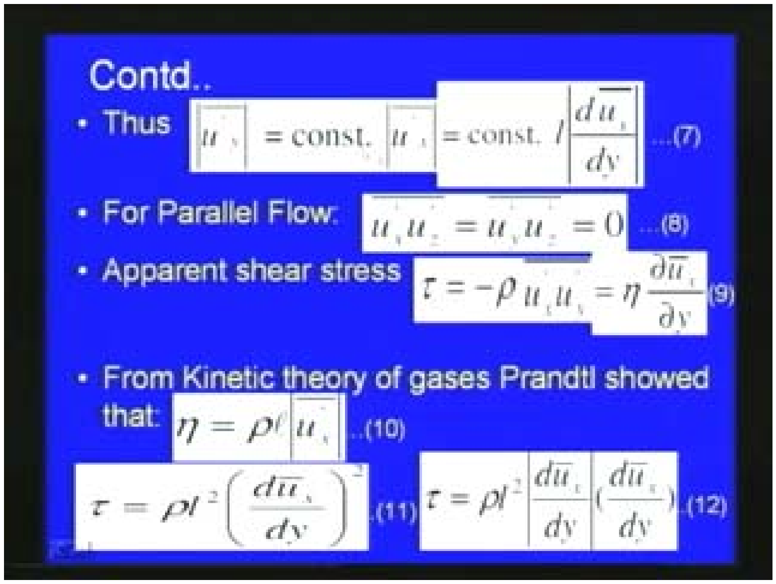

So that we can write u_y dash bar is equal to a constant multiplied by u_x dash. This constant, this u_x dash, as we have already seen, u_x dash bar can be written as mixing length multiplied by d u_x bar by dy.

So finally, we can write this u_y dash bar is equal to a constant multiplied by l into d u_x bar by dy as given in equation number 7 and for the particular case of the parallel flow as we have explained here for this particular case, Prandtl finally showed that this u_x dash u_z dash bar is equal to u_y dash u_z dash bar is equal to 0.Such that the apparent shear stress tow is equal to minus rho u_x dash u_y dash bar is equal to eta into delta u_x bar by delta y and from the kinetic theory of gases from which he has started this, he has derived this theory, eta is equal to rho l u_x dash bar and finally, we can write by using this here, we can write by using 10 in 9, we can write the shear stress tow is equal to rho l squared into dx bar by dy whole squared, so where l is mixing length.

This finally we can write, this is equal to taking the sign into consideration we can write tow is equal to rho l squared d u_x bar the modulus of d u_x bar by dy into d u_x bar dy. So the sign is taken care as shown in equation number 12.

This is finally what Prandtl did, he uses the Boussinesq's approximation and then he used the kinetic theory of gases such that, the shear stress is say approximated or shear stress is described in terms of the mixing length square multiplied by the density and then squared of the gradient of the velocity in y direction of d u_x bar by dy whole squared. That is finally he got this expression. Now using this equation number 12, this equation is the result of the Prandtl's mixing length theory.

As we have already discussed equation 12 is the resultant of Prandtl's mixing length theory. Now this mixing length theory, as you can see in equation number 12 here you can see, here this includes the fact that the apparent shear stress due to turbulence will change sign with the velocity gradient.

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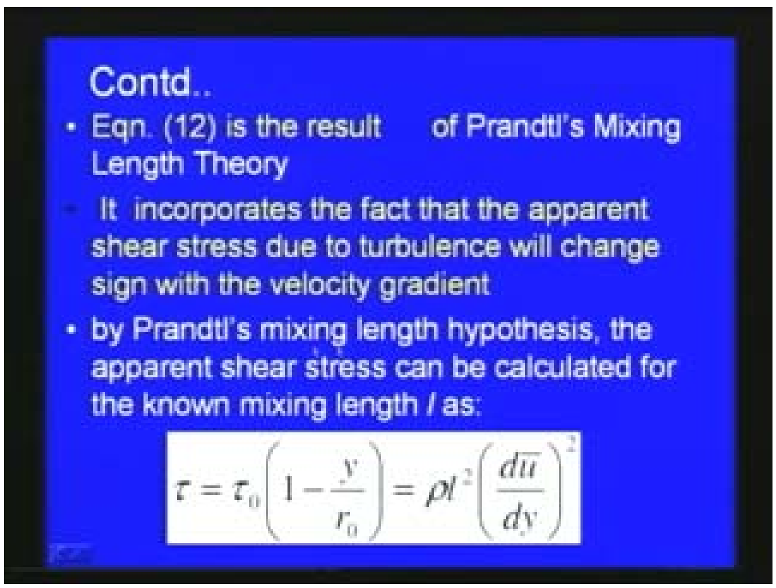

So that is obvious from equation number 12 and by Prandtl mixing length hypothesis the apparent shear stress can be calculated for the known mixing length l as tow is equal to tow_0 into 1 minus y by r_0 that is equal to rho l squared du bar by dy whole squared.

So if we consider with respect to the parallel flow, even if we consider the pipe flow as shown in this slide here, you can see that this r_0 is the radius of the pipe and then y is the we are taking from the bottom of the pipe and the velocity can be brought up like this.

So if we consider this in comparison with the pipe flow and the parallel flow which Prandtl considered, here, we can write tow is equal to tow_0 that means the shear stress of the boundary tow is equal to tow_0 into 1 minus y by r_0.So that is equal to tow is equal to tow_0 into 1 minus y by r_0, that is equal to rho l squared du bar by dy squared, so this is in comparison with the pipe flow as explained here .

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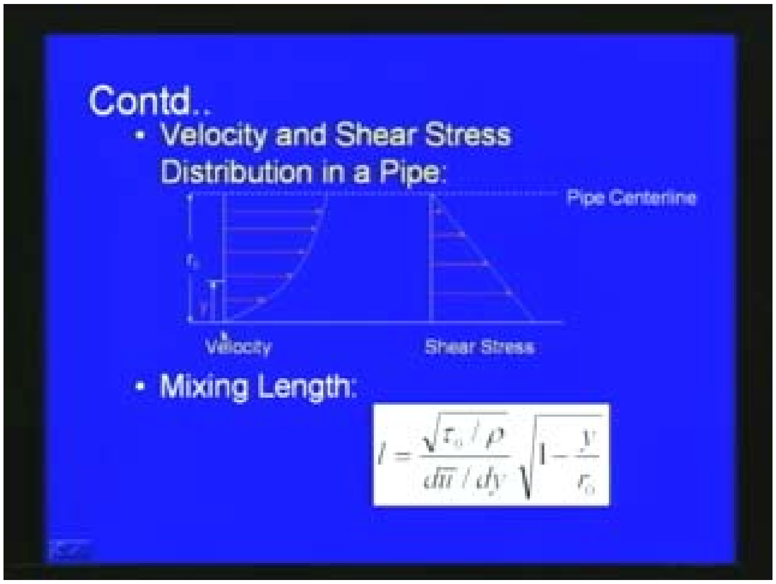

So finally by using this in comparison with the pipe flow we can write this mixing length l is equal to squared root of tow_0 by rho divided by du bar by dy into squared root of 1 minus y by r_0.

So this is obtained from this previous equation, so we can write the mixing length l is equal to tow_0 by rho by du bar by dy into 1 minus y by r_0. So this is obvious from this figure.

So now, from this once we determine the shear stress or the boundary shear stress we can get the velocity variation we can calculate and then other parameters can be calculated. This we will be explaining further, the next few slides how this mixing length hypothesis can be further used to calculate the velocity distribution and various other parameters.

So before going to the further applications of the mixing length hypothesis proposed by Prandtl's this from Karman also who was working with Prandtl's, he further modified this mixing length hypothesis and then this shear stress calculation as shown in here.

What Karman here is, he approximated this through various experiments and then through various observations Karman showed that this the apparent shear stress due to turbulence in a parallel flow can be written as tow is equal to rho into kappa squared du$_x$ bar by dy whole to the power 4 divided by d squared u$_x$ bar by dy squared whole squared. So this is the equation derived by Karman and he put forward a constant called kappa, which is for this particular parallel flow which he derived.

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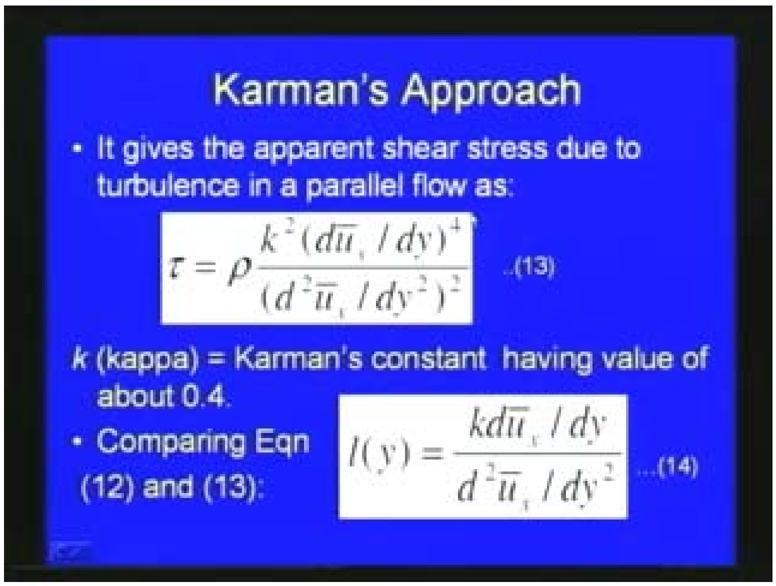

But finally, here shown this kappa is which has got value of 0.4, so he derived this kappa within terms of the shear stress in terms of this kappa and this kappa is called Karman's constant having a value of generally 0.4.

Now in comparison, by using the mixing length theory also with further experiments and observations Karman showed that the shear stress can be expressed as rho into kappa squared d u$_x$ bar dy to the power 4 divided by d squared u$_x$ bar by dy squared whole squared as shown in equation number 13 and now if we compare this equation number 13 and then if we compare equation number 12. So this is equation number 12 here this is which is derived by Prandtl that is equation number 12 and then the equation derived through experiments and observations derived by [] Karman equation number 13, here this is equation number 13.

Finally what [] Karman did is, he derived an expression for the mixture length or mixing length which is, we can write as ly which is the mixture length or mixing length is equal to kappa into d u_x bar dy divided by d squared u_x bar by dy square as shown in equation number 14 in this slide. So finally, the Prandtl's mixing length was used by [] Karman and with [] observations, finally he derived an expression for the mixing length.

In most of the problems, once we know the mixing length and then we can calculate other parameters. So that is importance of this Karman's approach or Karman's hypothesis here. Finally, he derived an expression for the mixing length or mixture length as given in equation number 14.

So this is now both Karman's theories as well as Prandtl's mixing length theory we can use together such that once the mixing length is determined we can go for the calculation of the velocity or shear stress another parameters, give some parameters already available with respect to some measurement.

So that is the application of this mixing length hypothesis and Karman's approach. Now in the next few slides, we will be discussing about the applications of this.

But before proceeding to further applications with the Karman's approach which we have seen here, we can observe here Karman assumed the turbulent flow patterns are similar in the neighborhood of any two points in the flow and then differ only in the length and time scales. So this is an important observation put forward by Karman. He assumed that turbulent flow patterns are similar in the neighborhood of any two points in the flow and they differ only in their length and time scales.

So this is obvious from this figure here.

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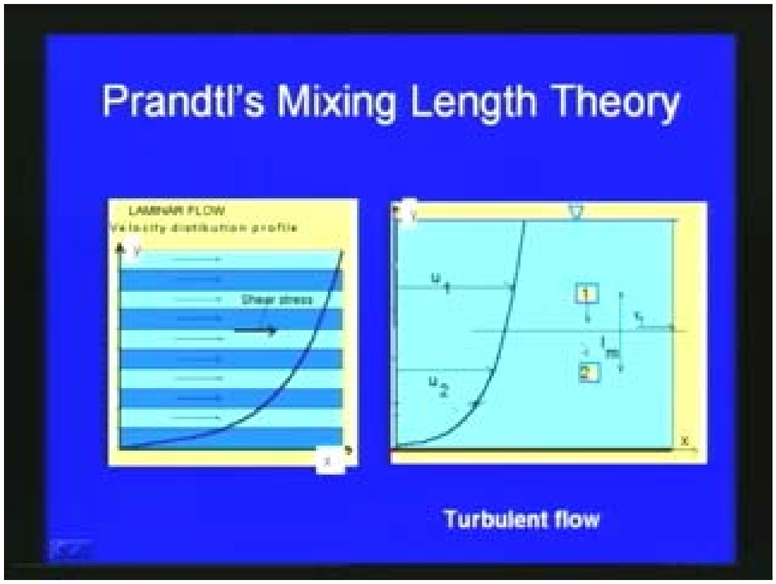

In this slide you can see as for the Karman's approach and then he also suggested that say Karman's model for the mixing length which we have already seen in the previous equation, it is a point function we can observe that it is a point function and depends only on the velocity distribution in the vicinity of a particular point. So this is one of the important observations which we can get from the Karman's approach.

So that means this mixing length is a point function and it depends mainly on the velocity distribution in the vicinity of the particular point which we consider in the turbulence flow regime. So that is one of the important observations which we can see from the Karman's model for mixing length.

So now further in the next few slides, we will discuss how this mixing length theory and Karman's approach can be utilized to solve various problems. First, we will see the flow over flat plate or parallel flow case further and then we will be discussing about the pipe flow.

So it is written here as the application of the mixing length theory, so, the velocity distribution in turbulent flows. As mentioned, our main purpose here is to find out the velocity distribution in the turbulence flow regime. As we have seen earlier in some of the cases for turbulence flow, total shear stress we can see that the most of the cases even the turbulence is generated especially when we consider flow over a flat plate there is even though most of the flow regime is turbulence but there can be a small layer called laminaus sub layer.

So when we consider the shear stress at any particular level then we have to consider that we can split the shear stress into the shear stress due to the turbulence and the shear stress due to the affect of the laminar nature or laminar sub layer.

So total shear stress can be written as tow is equal to tow turbulence plus tow laminar as in here in this equation number 1 further, so tow laminar is the shear stress due to dynamic viscosity of the fluid, as we can see as the flow takes place and here the tow turbulent is the additional apparent shear stress due to the turbulence.

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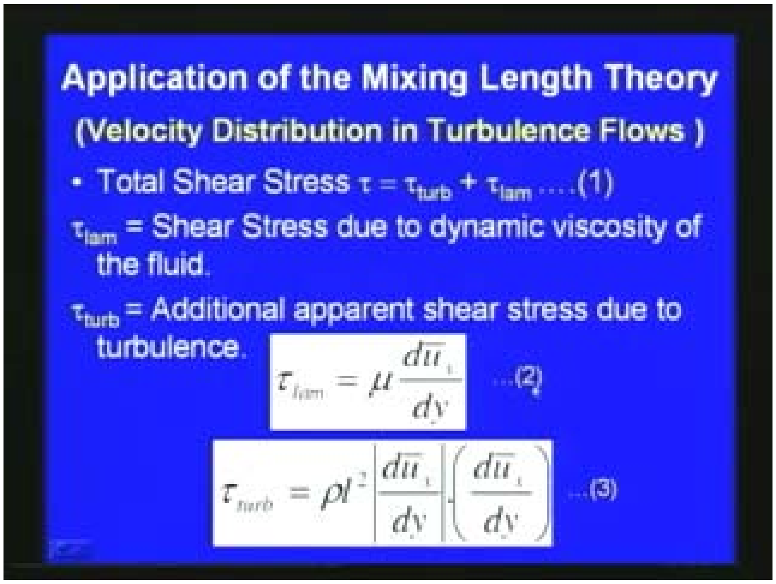

Total shear stress is the affect of the shear stress due to dynamic viscosity of the fluid and then the additional apparent shear stress and due to the turbulence. So now this tow laminar as we have seen, we can use Newton’s law of viscosity and write tow laminar is equal to mu into du_x bar by dy as written in equation number 2 and tow turbulence as we have seen the mixing length theory, we can write tow turbulence is equal to rho l squared into du_x bar by dy modulus into du_x bar by dy as shown in equation number 3.

So the total shear stress is the affect of this equation number 3 and equation number 2, tow laminar and tow turbulent. Once we know the magnitude of the total shear stress and the mixing length, as I mentioned here we are trying to use the mixing length which we have discussed already.

So by knowing the magnitude of the total shear stress and mixing length it is possible to solve this equation number 1 that means the shear stress. It is possible to solve equation number 1 for u_x that means our aim is to find out the velocity for u_x and thus to get a relation for the velocity distribution in the turbulent flow.

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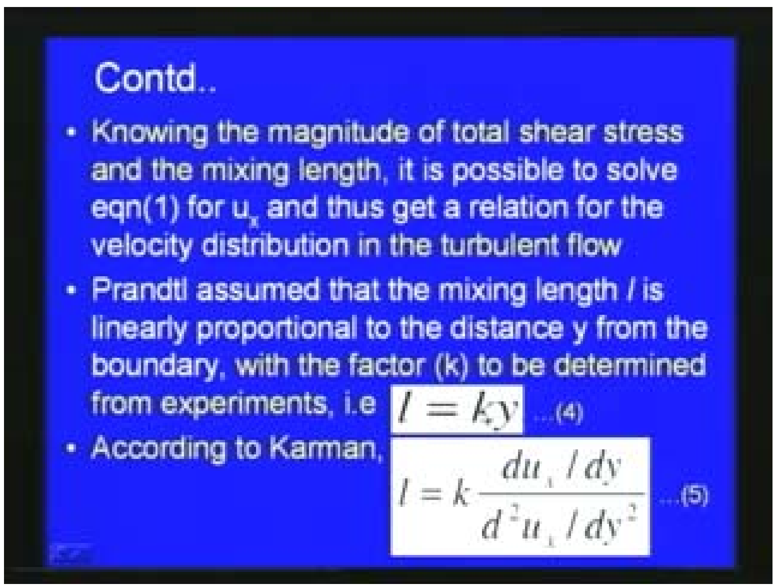

So this way we are proceeding to solve the turbulent flow. So if we know the magnitude of the total shear stress and then the mixing length, what we can do is we can store for the velocity u_x and then we can get a relation for the velocity distribution in the turbulent flow. We have already seen in the Prandtl's mixing length hypothesis. So, Prandtl's assumed that the mixing length l is linearly proportional to the distance y from the boundary with a factor called kappa to determine from the experiment.

Finally, he put forward by using this linear theory, linear proportionality by putting l is equal to kappa into y, where kappa is the Karman's constant, which we have already seen. The mixture length l is equal to kappa into y as in equation number 4. So Karman in the Karman's approach which we have already seen earlier and shown by Karman, [] Karman l is equal to mixture length and it is also equal to kappa into d u_x bar by dy divided by d squared u_x bar divided by dy squared. As shown in this equation number 5, y using this for flow along a solid boundary as we can see in this slide, when we consider the flow along a solid boundary we can see that there are three regions- one is the laminar sub layer as I mentioned initially there is a small sub layer which is called laminar sub layer; and then we will be having a transition; and then we will be having a turbulent flow.

So we can see that in the laminar sub layer the viscous shear predominates. So that we can see that the velocity is proportional to this distance y there is some what a linear variation. So this is this area where it is laminar flow say due to or the laminar sub layer and then there is a transition zone from laminar to turbulent and the turbulent flow. We can see that this transition zone is called buffer zone where viscous stress and Reynold's stress are same.

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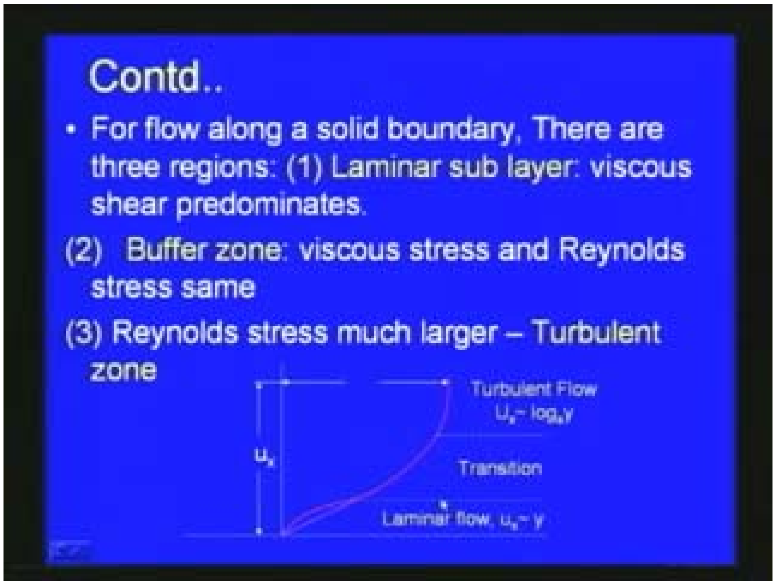

So this is the transition of buffer zone and then the turbulent zone. We can see that the Reynold's stress have larger and then here you can see that the velocity is proportional to the natural logarithmic of the distance y as shown here. These are the variations with respect to the laminar sub layer the buffer zone or the turbulent zone. So this is if this we can show experimentally also.

So now when we discuss the velocity variation within the turbulent flow we have to consider this is shown separately, the laminar sub layer or the turbulence shown separately and then for this laminar sub layer the region near the boundary, we can write by using the Newton's law of viscosity we can write tow is equal to mu into d u_x bar by dy and away from the boundary as far as shear stress is concerned we can use the Prandtl's hypothesis in combination with this Karman's approach by given by tow is equal to rho l squared d u_x bar by dy into d u_x bar by dy as shown in equation number 7 and then transition zone is concerned it is what is happening is the transition takes place from laminar to turbulent it is known as buffer zone. So two velocity distribution near boundary governed by equation 6 and equation away from the boundary governed by equation number 7.

So this is a combination, the buffer zone it is a combination between equation number 6 and equation number 7, since transition takes place from laminar sub layer to the turbulent region so the two velocity distribution should be connected suitably, so that, one gets a continuous velocity distribution from the boundary within the flow.

So due to the complexity, the problem is very complex that we cannot just clearly identify how this transition is taking place. Generally, what we do in the laminar sub layer region we will know the variation, say it is generally u is proportional to the distance y and then the turbulent zone u is proportional natural logarithmic of y.

As we have already seen and then in-between what we do is, the two velocity distribution can be connected suitably so that we get a continuous velocity distribution from the boundary within the flow. So this is the procedure which is generally adopted. And now the shear velocity we can define as, u star is equal to square root of tow turbulent by rho. So this we have already discussed earlier. So we can define a term called shear velocity which is equal to square root of tow turbulent divided by rho or this is a shear stress term divided by rho is square root equation number 8.

This is, the shear velocity also called as friction velocity sometimes. So this is actually the characteristics of turbulent fluctuating motion. So the shear velocity or the friction velocity it is actually the characteristics of the turbulent fluctuating motion. We can define as u star is equal to square root of tow turbulent divided by rho and then the shear velocity on the boundary we can say y is equal to 0.That we can write u star rho is equal to square root of tow_0 by rho as shown in equation number 9, where tow_0 is the shear stress at the valve or boundary. Here, we introduce a term where velocity or friction velocity is actually, it is a characteristic of the turbulent fluctuating motion.

So this way, these various parameters are now defined and then if you know either the mixing length or the shear stress or some of the parameter then we can determine the velocity.

So it is just like now this zero equation model of them Prandtl's mixing length is directly not give a complete solution but, it is we can use it in combination with some of the measured values or with some of the other observations. So that is generally this theory we have already discussed the Prandtl's mixing length hypothesis. So what are the advantages or what are the disadvantages? The next two slides we are discussing, what are the advantages. So this Prandtl's hypothesis has been put forward at the beginning of the twentieth century.

So you can see that, at that time the turbulence phenomena, what is happening in turbulent how the velocity variations and all these things were totally unknown the understanding of this turbulence flow was very little for the scientist and engineers. So at that time when Prandtl's proposed this theory, it was actually one of the important observation as far as this turbulent flow which has put forward or which has boosted the development of various fly mechanics like aero development, of aero planes and other kinds of machines. That is, one of the important theories as far as turbulent flow is concerned.

So some of the advantages of this mixing length hypothesis theory you can see the merits in this theory is simple and can be used with some degree of accuracy. We can easily explain, we can easily understand this mixing length hypothesis, it is not so complex like as we have seen the Reynold's equations or the complete turbulence theory is concerned mixing length theory is simple and even though it is not totally correct completely accurate the accuracy is less, but still we can use this mixing length hypothesis with some degree of accuracy. So that minimum of the problems can be solved or you can easily understand how the system is working to certain degree of accuracy. So if appropriate the choice of mixing length l is made.

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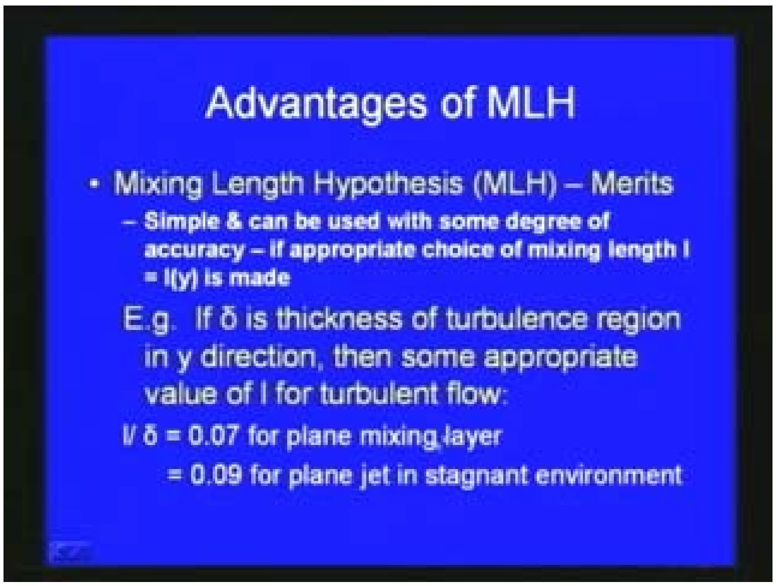

We can get a good solution, this is depends upon how effectively we can calculate the mixing length or we can find out the mixing length. As we have already seen we have derived a term for the shear stress in terms of mixing length and the velocity gradient. So if the mixing length can be correctly obtained through various means then we can calculate other parameters accurately. So this depends upon the mixing length.

So one of the important advantages is that the theory is simple and can be easily used with some degree of accuracy for various turbulent flow problems and then if delta is the thickness of turbulence region in y direction, then some appropriate value for mixing length is available in literature through various measurements. For example, if l by delta is equal to 0 point l by delta is equal to 0.07 for plane mixing layer l is equal to 0.09 for plane jet in stagnant environment like that say if delta is the thickness of the turbulence region in y direction.

Then we can say from various observations we can write the mixing length l divided by delta some values are available in literature. These are some of the important advantages of the Prandtl's mixing length hypothesis.

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Some of the disadvantages, as I mentioned is say this Prandtl's mixing length theory is based upon some of the experimental observations and some of the empirical relationship and some of the theories at that time.

Some of the disadvantages of the mixing length hypothesis do not be considered diffusion or convection of turbulence. As we have seen the turbulence in the nature of turbulence, this diffusion and convection are very important process in turbulence, actually mixing length hypothesis which we have seen here proposed by a Prandtl's, it does not consider the diffusion of convective turbulence, so this is actually one of the major disadvantage or demerit of the mixing length hypothesis and also mixing length hypothesis, takes that effective viscosity does not exist where velocity gradient is 0.

This is also one of the disadvantages as written here in this slide even though mixing length hypothesis is widely used even now. Due to the limitations of this mixing length hypothesis, some sophisticated analysis will be taken over the mixing length hypothesis.

Nowadays, with very complex CFD computers full dynamics packages are available to critically analyze most of the problems.

So generally this mixing length hypothesis is used for preliminary analysis for various problems but otherwise since we have got very good computational software or computational dynamics packages where all these various equations, momentum equations or Reynold's equations are taken help.

So we can get better solution still we use this mixing length hypothesis for preliminary analysis and understanding of the turbulence process. These are the some of the advantages and disadvantages of the mixing length hypothesis put forward by Prandtl.

So now in the next few slides and we will be trying to use this Prandtl's hypothesis as well as various other theories to derive some equations for the variation of the velocities by considering first flow or a flat plate or a parallel flow and then we will be considering the turbulent flow through pipes.

So, first case is velocity distribution for a parallel flow within smooth boundaries. As we have seen in the region away from the wall we can write by using the Prandtl's hypothesis, we can write tow is equal to tow turbulent is equal to rho l squared d u_x bar by dy whole squared equation number 10 and then by using Karman's approach mixing length is equal to kappa into y.

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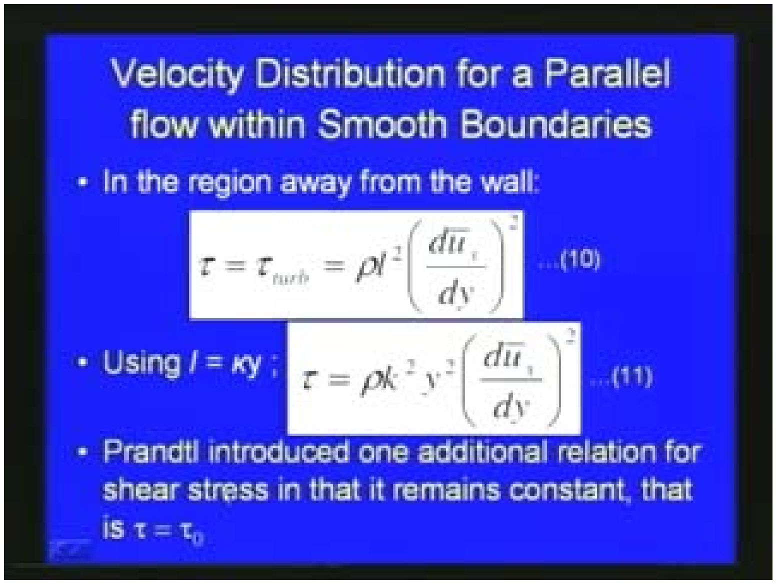

So if you put forward this here, that we can write tow is equal to rho kappa squared y squared d u_x bar dy whole squared as shown in equation number 11.Here, we are considering the parallel flow with smooth boundaries. So Prandtl's introduced one additional relation for shear stress in that it remains constant, so that is tow is equal to tow_0. So that we can write in this equation number11 here, we can write here, 11 can be written as equation number 12; tow_0 is equal to rho kappa squared y squared d u_x bar by dy whole squared as in equation number12.

So now as we have seen here, we have defined shear term called shear velocity, so which is tow_0 is equal to rho u star o squared where o is the density u star o is the shear velocity so tow_0 is equal to rho u star o squared. Now, if you substitute for this rho star o squared here in equation number 12.

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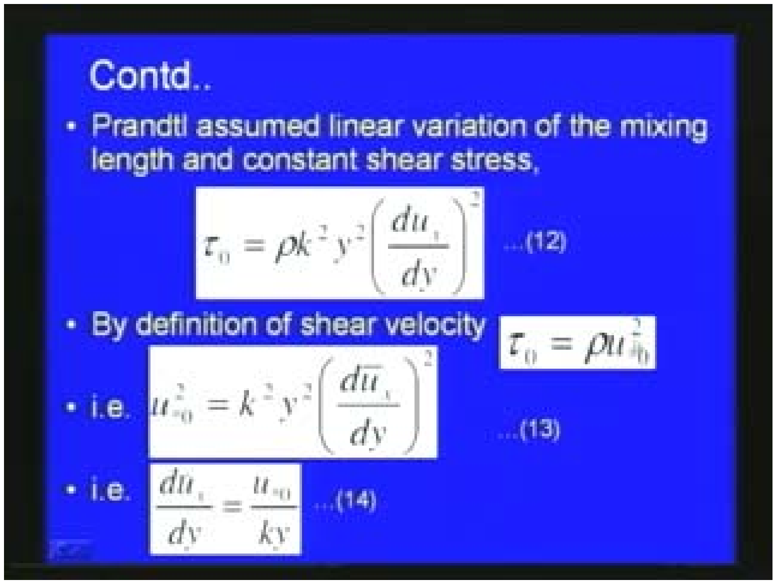

So that you can write u star o squared is equal to kappa squared into y squared into d u_x bar by dy whole squared, so that is equation number 13. Finally by using this here, we can write d u_x bar by dy is equal to u star o by kappa into y as in equation number 14.

So now, here our aim is to get an expression for the velocity variation. So we are using the Prandtl as well as Karman's approach and then d u_x bar by dy is equal to, we got a relation the variation of the velocity with respect to y and d u_x bar by dy is equal to u star o by kappa y as in equation number 14.

Now, we can solve this equation number 14 to get a solution for the velocity, so as I mentioned our aim is to get an expression for the velocity. We can write u_x is equal to u star o by kappa into log e y plus C. By showing equation number 14 here, we get u_x is equal to u star o by which is the shear velocity, where kappa into log e natural log y plus C where C is the constant with respect to integration. Now, this equation number 15 indicates that the velocity varies logarithmically.

So that, we have already seen the turbulence region the velocity varies logarithmically, now the constant integration we can determined from the condition that the turbulent velocity distribution must fit in the laminar sub layer in the vicinities of the boundary.

So we have already seen earlier that there we have here you can see in this figure there is a laminar sub layer and then the turbulent. (Refer Slide Time: 42:07) So with respect to this here there is constant of integration we can determine from the condition that the turbulent velocity distribution must fit in the laminar sub layer in the vicinities of the boundary.

Now if you assume that the shear stress remains constant and it is equal to tow_0 on the boundary and assume further that d u_x bar by dy is equal to say the derivative of the velocity with respect to y direction we can approximate as u_x by y so that we can writetow_0 is equal to mu into u_x by y but we have already seen this tow 0 is equal to rho into u star o squared. We can write u_x by u star rho is equal to u star o into y by v where u is the kinematic viscosity. So this is valid for the near to the boundary so equation number 16 u_x by u star o is equal to u star o into y by v.

So hence, we can write equation number 16 gives the velocity distribution in the vicinity of the boundary, so that this is linear, so this we can say it is on the laminar sub layer.

So in equation number 15 velocity is not 0 at y is equal to 0, if not this C is equal to 0. So here equation number 15 is here, so you can see that in equation number 15 and this velocity is not 0 at y is, actually the velocity should be 0 due to no slip condition y is equal to 0. This is possible only unless C is equal to 0 you can see equation number 15 here and then we can see that velocity is not 0 at y is equal to unless C is equal to 0.

So this we explain all these we are discussing here is explained with respect to flow over flat plate. Here the free stream velocity is coming and here is the flat plate and then once the turbulence is generated you can see there is a laminar sub layer and then the turbulence is taking place. How the transition and then laminar and turbulent transition and then that laminar sub layer are explained in this figure:

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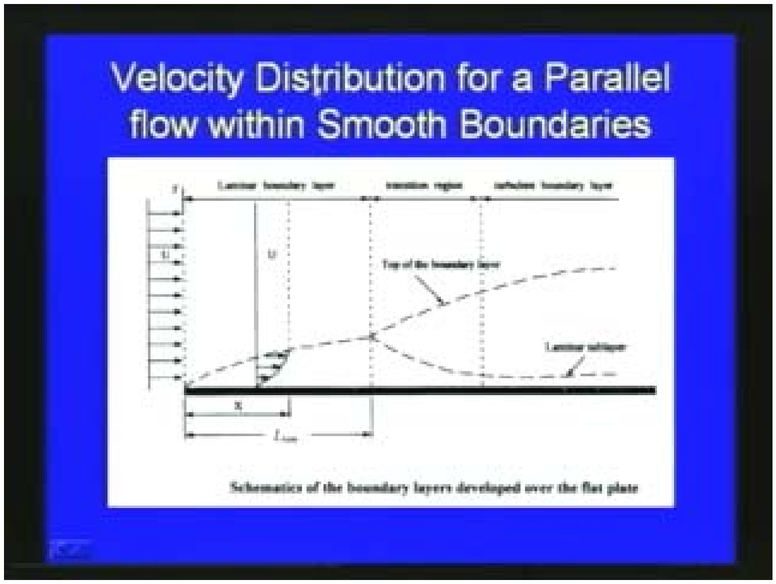

So now we want to as I mentioned our aim is to finally determine as an expression for the velocity. So the constant of integration C is determined from the condition that the turbulent velocity distribution must join the laminar velocity distribution in the immediate vicinity of boundary where laminar and turbulent shear stress is of the same order of magnitude.

So here you can see, this is the laminar sub layer and then turbulence so to determine this C the velocity distribution must join the laminar velocity distribution there is immediate vicinity of the boundary where laminar and turbulent shear stress are of the same order of magnitude. So that we can write this y is equal to we can put y is equal to y dash where u_x is say small value or u_x is tending to 0 in equation number 15 here, here is 15.

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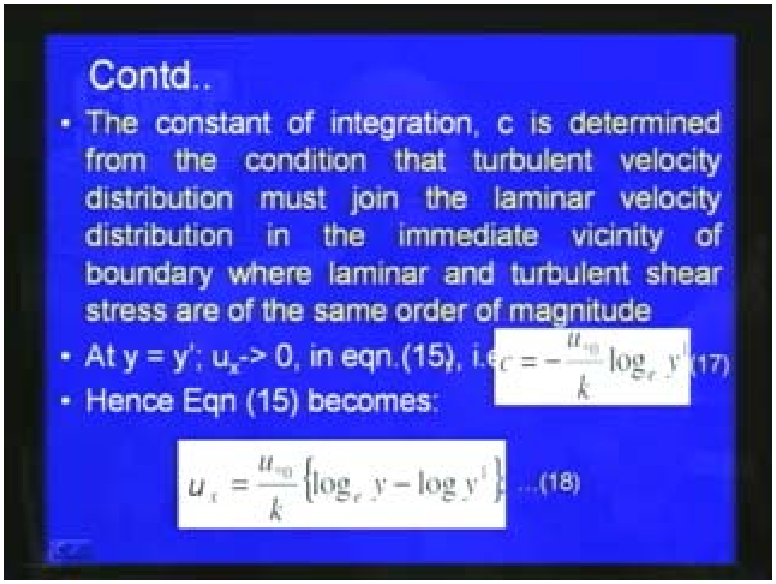

So then we can write C is equal to the constant C is equal to minus u star o by kappa log e y dash as in equation number 17. Here now equation, by using this C we can write equation number 15 as u_x is equal to u star o by kappa log e y minus log e y dash as in equation number 18.

So now from equation number 16 one can write that such a velocity distribution exists up to a distance delta dash that is proportional to v by u star o and this delta dash may be taken as representing the thickness of the laminar sub layer.

So this is the laminar sub layer you can see that a small sub layer called laminar sub layer. Actually it may be taken as representing thickness of the laminar sub layer. The distance y dash and delta dash may be expected to be independent, so that y dash is tending to delta dash so that is also approximately equal to v by u star o as in equation number 19.

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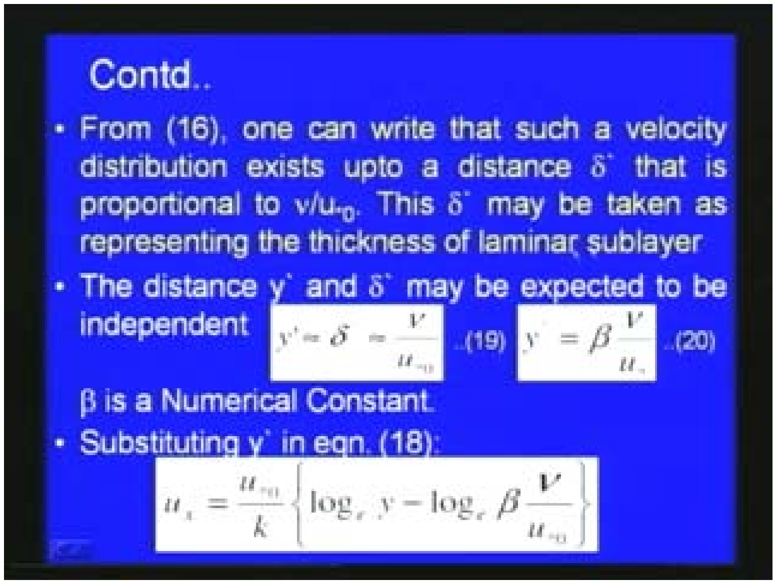

And y dash we can represent as a constant multiplied beta into v by u star so as in equation number 20, where beta is a numerical constant. So if we substitute this here in equation number 18, our previous equation number 18 we can write the velocity variation u_x is equal to u star o by kappa into log e y minus log e beta into v by u star o. So now from this, by simplifying we can write, we can get an expression u_x is equal to u star o by kappa into log e u star o into y by v minus log e beta so that is equation number 21.

So as we can see this u star o into y by v is a Reynold's number based on the friction velocity or the shear velocity u star at distance y. So this term is actually Reynold's number based on the friction velocity so for smooth boundaries at high Reynold's number, we can see that this is already in this region u_x is equal to u star rho by is equal to u star o into y by v as given in equation number 16 for laminar sub layer earlier.

So that finally for the turbulent region we can write u_x by u star o is equal to 1 by kappa log e natural log e u star o into y by v minus log e beta as explained in equation number 23.

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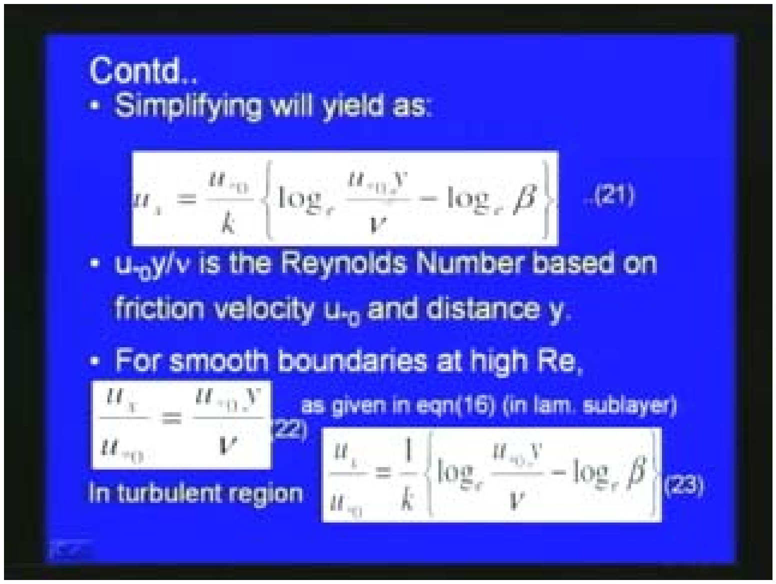

So this equation number 23 as shown here, this equation number 23 is known as dimensionless logarithmic universal velocity distribution law for smooth boundaries.

So when we consider smooth boundary, this equation is called dimensionless logarithmic universal velocity distribution law. These constants kappa and beta are empirical we can determine through experiments, already say you can see kappa is almost equal to 0.4 which is a universal constant kappa is equal to 0.4 and beta also we can determine, beta depends on the roughness at the boundaries for the particular case we consider.

So for low Reynold's number we can show that instead of this linear variation we can show that power law is valid which is expressed as u_x by u star o is equal to C into y into u star o by v to the power n where C and n are constants as shown in equation number 24.

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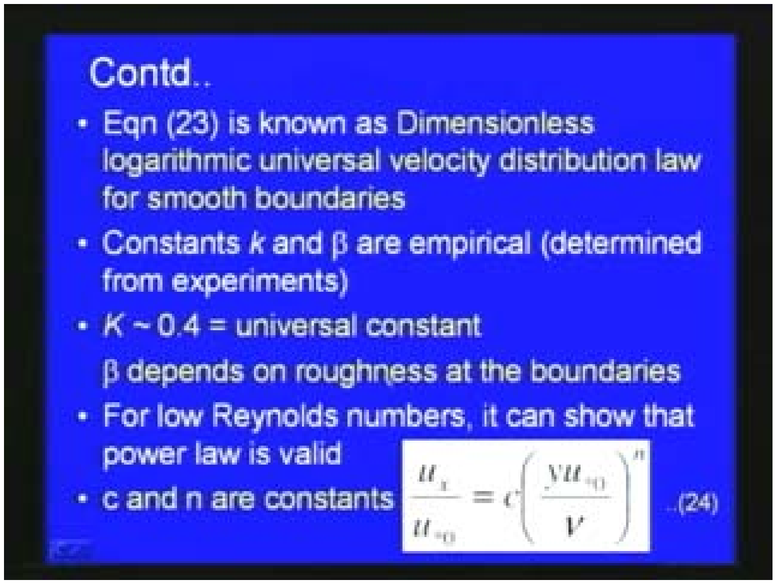

So this is for a low Reynolds number low. So this you can see this.

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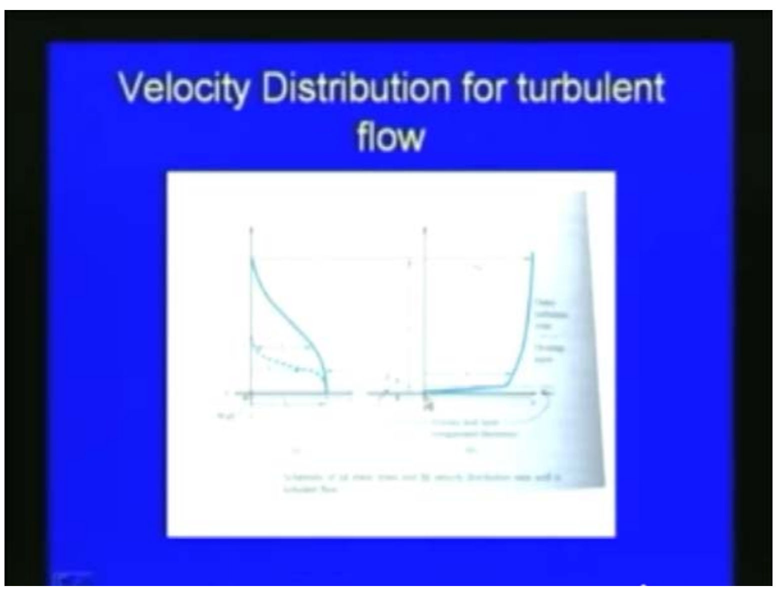

This slide shows how the velocity distribution takes place so if this is the valve and then the schematic representation turbulent shown and laminar sub layer and how the shear stress is varying is shown in this figure which is taken from the [] book.

So now for Rough boundaries we have seen how the variation taking place for smooth boundaries, if we consider rough boundary we can see that the boundary, the roughness of the boundary also attack the flow. So that this laminar sub layer thickness which we have seen earlier where rough boundaries, we can write y dash is equal to alpha into epsilon where, this epsilon is the roughness coefficient roughness factor and alpha is a constant. So that now by using our earlier equation number 18 we can write u_x by u star o is equal to 1 by kappa log e y by epsilon minus log e alpha equation number 26.

So this equation number 26 represents universal velocity distribution law for rough surfaces.

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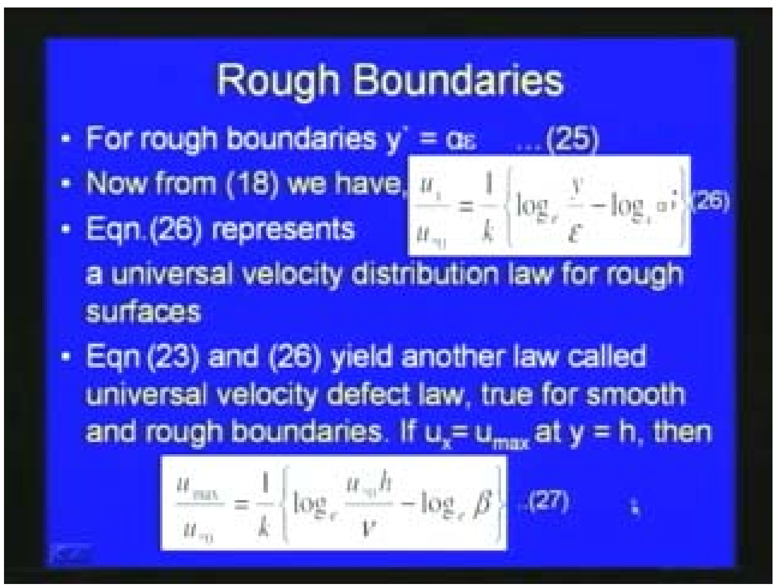

So we have already seen a universal velocity distribution for smooth surfaces, so now equation number 26 use universal velocity distribution law for rough surfaces. So this equation number 23 and 26 yield another law called universal velocity defect law which is true for smooth and rough boundaries.

So if we consider u_x is equal to u max at y is equal to h then we can write u max divided by u star o is equal to one by kappa log e u star o into h by v minus log e beta as in equation number 27. If we consider the velocity is maximum at y is equal to x so that u max by u star o is equal to 1 by kappa into log e h by epsilon minus log e alpha.

As in equation number 28, if we subtract this equation 23 and 26, from equation 27 and 28, we will get u max minus u_x divided by u star o is equal to 1 by kappa log e h by y, this is called universal velocity defect law which is valid for both smooth and rough boundaries. Finally, through this we got a general expression which can use for smooth boundaries as well as rough boundaries.

So that equation is u max minus u_x divided by u star o is equal to 1 by kappa log e x by y this equation is called universal velocity defect law which is applicable for smooth as well as rough boundaries.

So this is the general expression before going to the pipe flow, we will discuss a small example. Here the example problem is say in a meteorological station the wind velocity was measured at 2.3 meters and 6 meter above the ground, the values obtained being 2 meters per second and 2.3 meter per second respectively.

We have to compute the shear velocity u star o assuming Karman constant kappa as 0.4 and then what is the probable laminar sub layer thickness for the problem, next part of the question is, what is the velocity at 9 meter above the ground?

Assuming that, the boundary is smooth and take kinematic viscosity v is equal to 0.145 stokes so for this particular problem is concerned say this is the meteorological station, where the wind velocity is measured. We know that from the ground level the velocity is measured at 2.3 meter and 3 meter and 6 meter above the ground and we know the velocity values 2 meters per second, 2.3 meter per second respectively.

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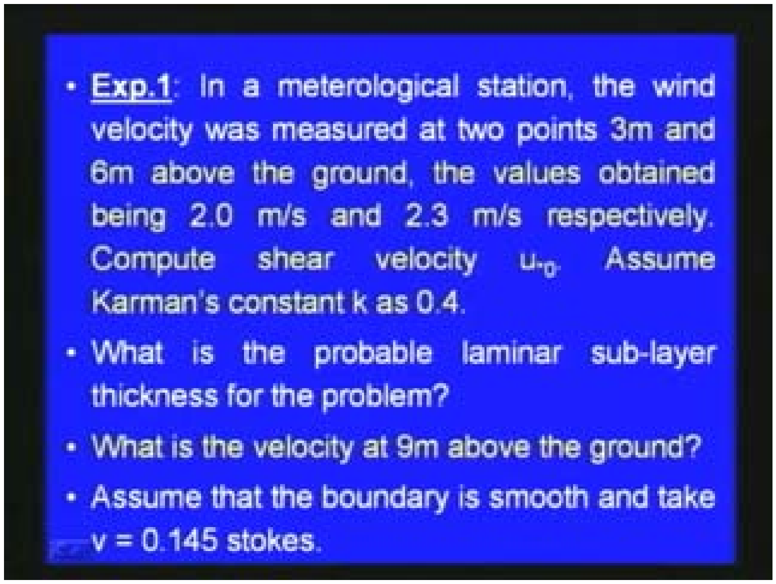

So now we know the kinematic viscosity and then we know the Karmans constant kappa and assuming the boundary is smooth, we have to determine the velocity at 9 meter above the ground and then, we have to determine the probable laminar sub layer thickness and the shear velocity. So the problem we can just explain like this in this figure.

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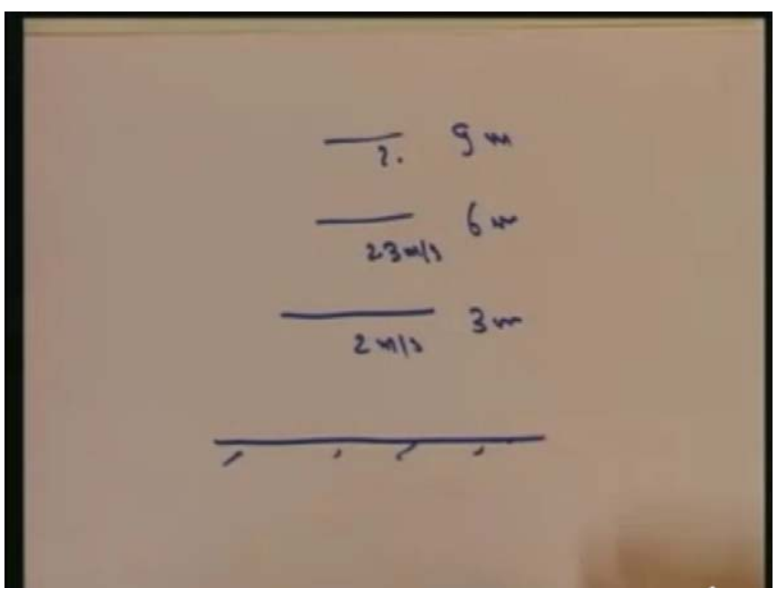

So if the meteorological station here, you can see that if this is the ground level, here we have 3 meter then 6 meter and 9 meter. Here, the velocity is given as 2 meter per second here 2.3 meter per second. So we have to determine the velocity and this 9 meter location.

So to solve this problem, we will use this equation, the velocity distribution for smooth boundary. We have given this in the equation u_x is equal to u star o by kappa into log e y minus log y dash in equation number 1 in this slide.

So now we will take the ground level as origin, so here at y is equal to 3 meter which is given u_x is equal to 2 meter per second at y is equal to 6 meter u_x is equal to 2.3 meter per second.

So we will substitute this value to this equation number 1, so 2 is equal to u star o divided by k into log e 3 minus log e y dash.

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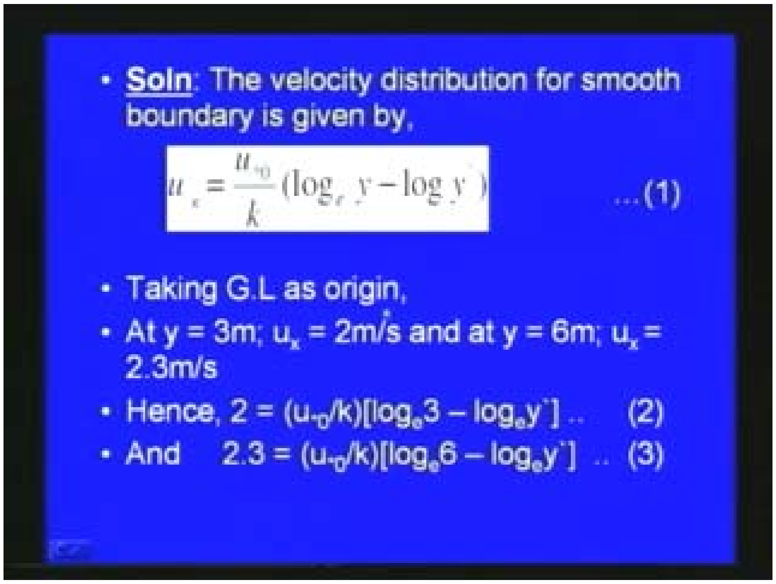

And 2.3 are equal to u star o divided by k log e 6 minus log e y dash equation 2 and 3. Now we can solve this equation 2 and 3, we get the shear velocity u star o as 0.173 and then the shear stress we can see from this slide, tow_0 is equal to rho u star rho squared the equation number 4.The laminar sub layer thickness say actually Reichardt's as measured,

for this kind of problem the laminar sub layer thickness, he found that generally it will be between this say the u star o by into y by mu is between 0 to5.m So here y is the laminar sub layer thickness. Now we will see that the Reichardt's observation, and then we will also Calculate the laminar sub layer thickness from our earlier equation, in this equation number 1.

Let us take y is equal to delta star the thickness of laminar sub layer thickness in the Reichardt's observation here, so that delta star in equation 5 mu by u star o. By solving this expression we will get the delta star as 4.19 into 10 to the power minus 4 meter. So now we can from the equation 2 here, this equation 2 since u star we have already found kappa is known.

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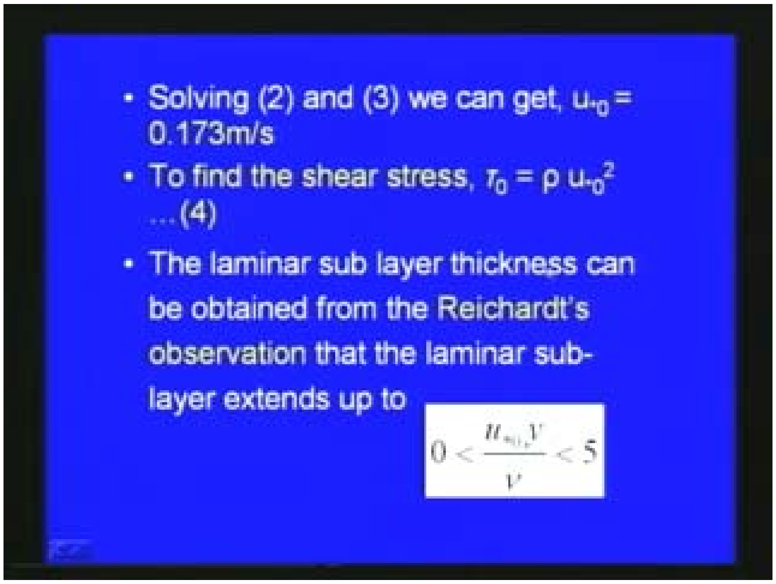

So now for this real problem we can also find out the laminar sub layer thickness. If we solve that equation we will get y dash is equal to 0.029 meter. You can see that what Reichardt's say this lot of difference is there, but here we got y dash or the laminar sub layer thickness as 0.029 meter. So now to calculate the velocity, since now the constants are unknown parameters here u star o is already determined.

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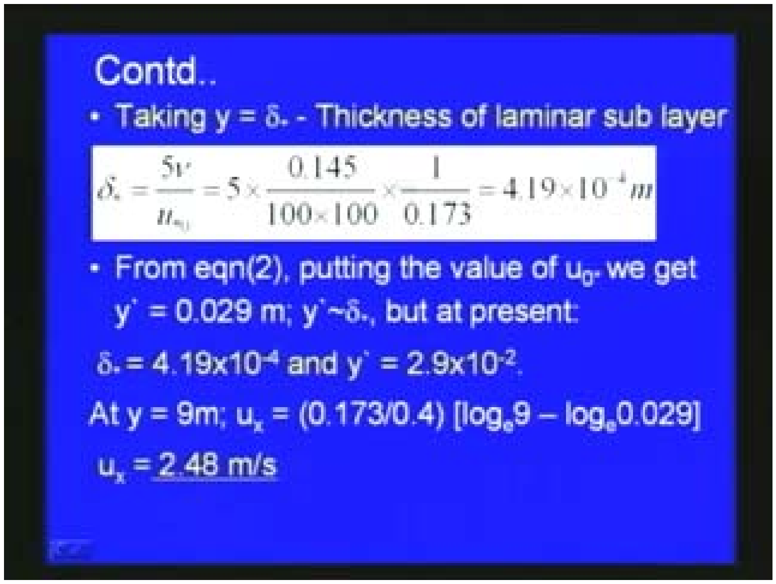

We can find out the velocity at 9 meter. We will use this equation here, the earlier equation here, and equation number 1. From that if we substitute, we will get u_x is equal to 0.173 by 0.4 into log e 9 minus log e 0.029, so that, you will get the velocity as 2.48 meter per second.

So this is a small example, so similar way different kinds of problem especially for turbulent flow are related to flat plate or similar kinds of problem can be solved. Next, we will be discussing the turbulent flow through pipes, initially, through smooth pipe and then rough pipes.