Fluid Mechanics

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Lecture No - 20

Laminar and Turbulent flows

Welcome back to the video course on fluid mechanics. We were discussing about the turbulent flow; we have seen various characteristics of turbulent flow; we have seen various reasons for turbulent flow. We were discussing about the various factors affecting transition flow from laminar to turbulent flow like free stream turbulence, pressure gradient, roughness of boundary, then curvature of boundary, suction of boundary etc.

Also we have seen all this turbulence is coursed into abrupt discontinuity in velocity distribution generally occurs due to various reasons and then of course, due to the shear flows. Due to the shear flow there is a mean velocity variation in space and then that causing the turbulence. Further, we have seen that generally if the turbulence flow is very difficult to solve the problems or to find out the velocity distribution is very difficult. So, generally, for practice what we do is we use a mean value components for the velocity and then what happens to the fluctuations. That is what we were discussing in the last lecture on turbulence.

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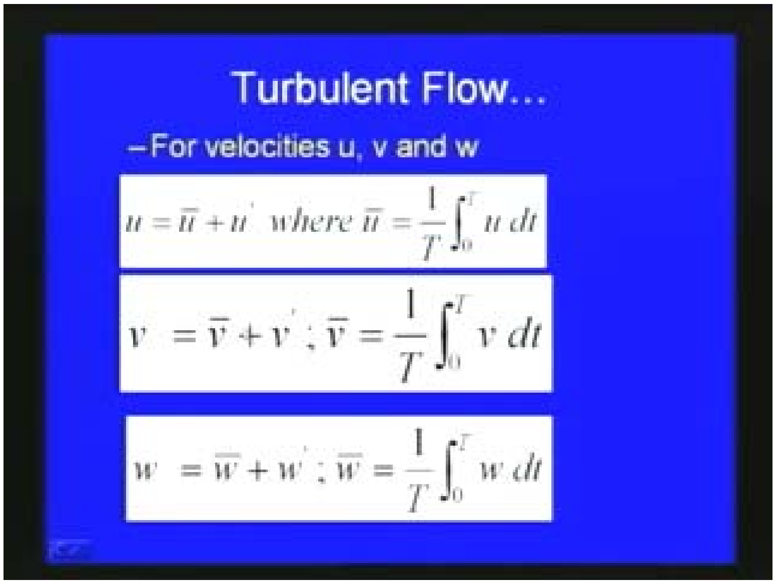

As I mentioned the velocities in three dimensions u v w can be expressed as u is equal to a mean value u bar plus u dash, where u dash is the fluctuating component; this u bar with the mean velocity component we can write that is equal to1 by T integral 0 to T u dt. where T is the time; then v, the velocity in y direction we can write v bar plus v dash, where v bar is equal to1 by T integral 0 to T v dt; and w is equal to w bar plus w dash w bar is equal to1 by T integral 0 to T w dt. So this we have seen how we are expressing the various velocity component u v and w. Similar way we can express the pressure and other components with respect to a mean component and then the fluctuating component or instantaneous component..

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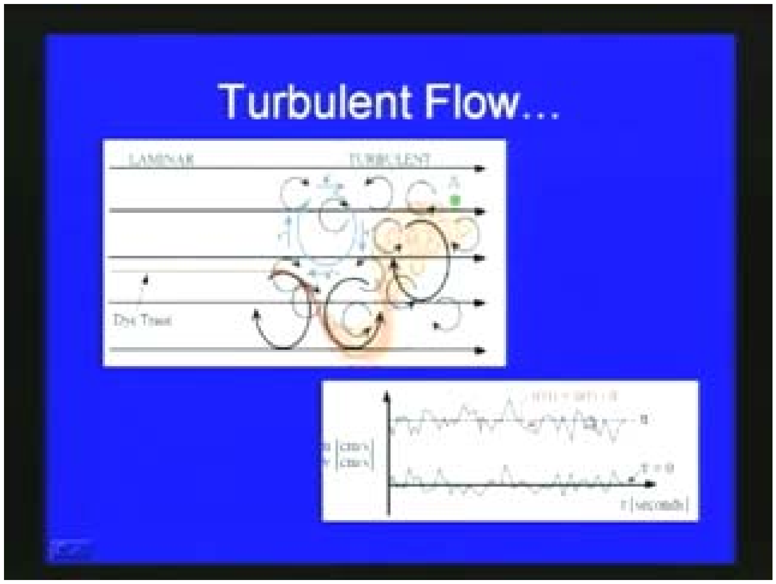

It is clear from this figure here so you can see that the velocity in x and y direction u and v is represented here; there is a mean value of mu which is called u bar and then the mean value of v in y direction v bar. Then you can see that we are expressing the turbulence is with respect to this mean velocity. So in the fluctuation we can see that it is going upon due to the various disturbances taking place in the flow. So these are called fluctuating components for u and v and then you will be expressing with respect to the mean component plus the fluctuating component.

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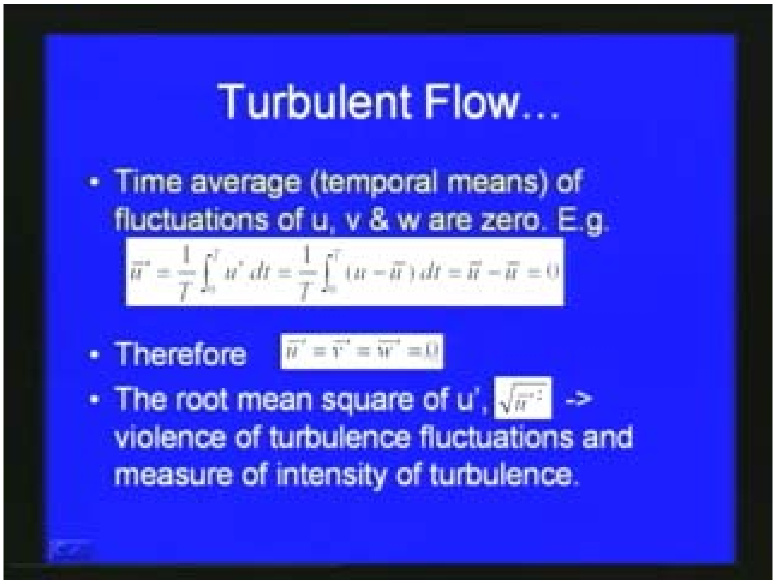

We have seen the time average of this fluctuations u v w are 0 since we can show as we have seen earlier the mean component of u dash is equal to1 by T integral 0 to T u dash dt that can be equated to 0, similarly v and w also. Generally, we express this root mean square u dash bar whole squared that gives to describe the violence of turbulence fluctuations and it is generally a measure of intensity of turbulence.

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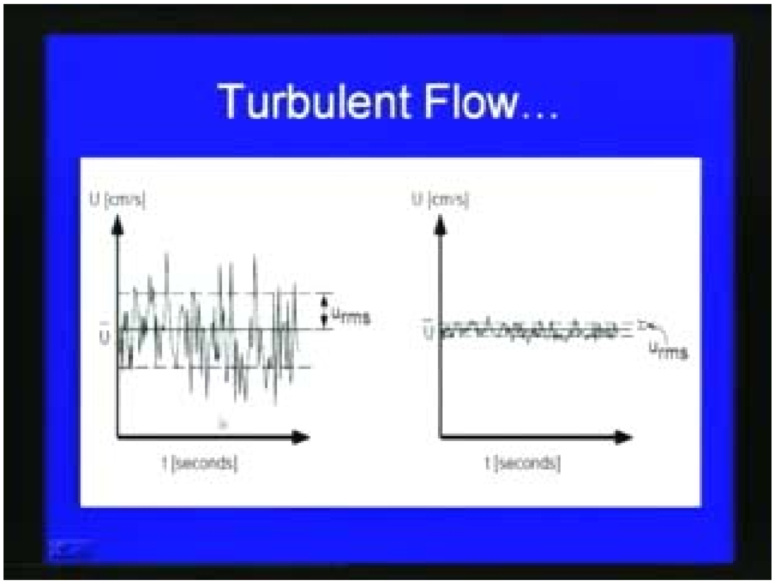

As we have seen due to various reasons it is very difficult to express the real velocity fluctuations. So generally, what we do is we express as a root mean square component. We can see here in this figure how we express this root mean square component; this is the velocity distribution versus time on x axis and velocity on y axis. We have seen how the variations takes place with respect to the mean component and then we can see that here we express in terms of U rms that means the root mean square of the velocity fluctuations. So that is also expressed in another scale we are use U rams as shown here; here root mean square of the velocities generally used to express the turbulent components in various fluctuations as far as turbulent flow is concerned.

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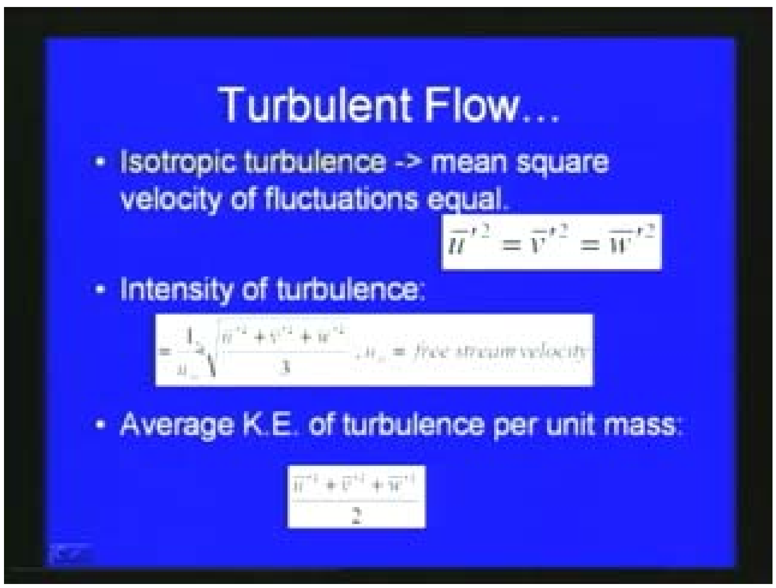

Another some of the other important definitions here is called isotropic turbulence That means if this mean square velocity fluctuations in x y and z directions are equal then we call this turbulence as isotropic turbulence, that means, u dash bar square is equal to v dash bar square is equal to w dash bar square so that case we call that turbulence as isotropic turbulence. Then the intensity of turbulence is generally expressed as here you can see that in this slide; the intensity of turbulence is equal to 1 by mu infinity, where mu infinity is the free stream velocity into square root of v u dash bar square plus v dash bar square plus w dash bar square by 3. This gives the intensity of turbulent and this is also an important definition in turbulent flow. Then if we want to find out the average kinetic energy of turbulence per unit mass we can write as u dash bar square plus v dash bar square plus w dash bar square by 2. So this is the average kinetic energy per turbulence per unit mass. These are some of the important definitions which we generally use in turbulent flow and its various parameters determination.

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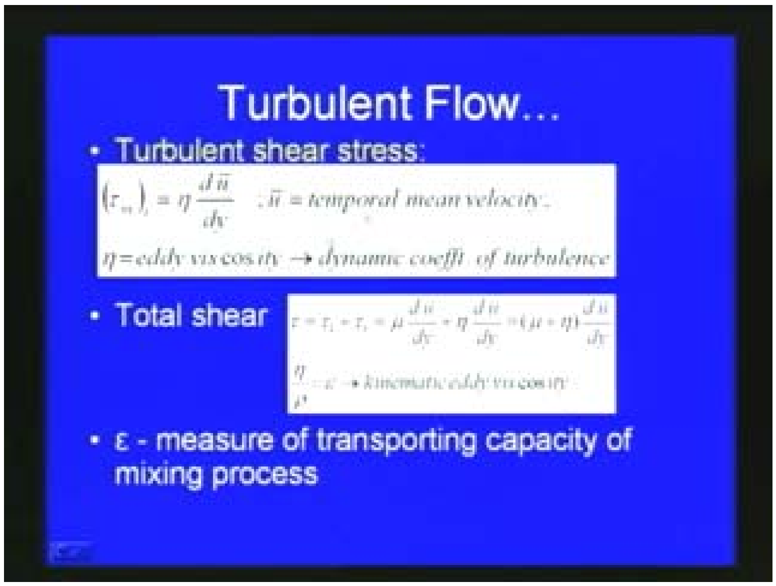

Now another important parameter is turbulent shear stress. So turbulent shear stress we can express as tau x y with respect to t that is equal to eta d u bar by dy, where u bar is the temporal mean velocity. We have already seen what temporal mean velocity is. That means with respect to the long time how the variations take place for the mean velocity. The turbulent shear stress we can express as eta into d u bar by dy, where eta is called the eddy viscosity or dynamic coefficient of turbulence.

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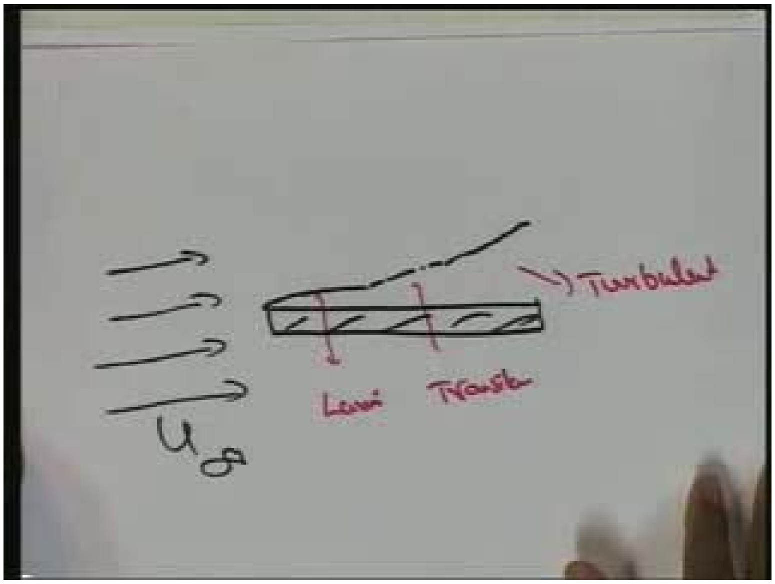

We have seen that whenever we discuss for example, when a flow approaches flat plate so you can see here, let us assume that here there is a flat plate put like this; if a free stream velocity as we described, here a free stream velocity; let us assume that u infinity is coming then we can see that there a boundary layer is generated. Initially the boundary layer will be laminar and then it becomes turbulent, so this is the general pattern whenever this is the laminar boundary layer and then here there is a transition and then here there is turbulent. So generally, when we discuss the shear stress here we can see that with respect to the flow is coming and then turbulence is generated after some time for the flow over a flat plate.

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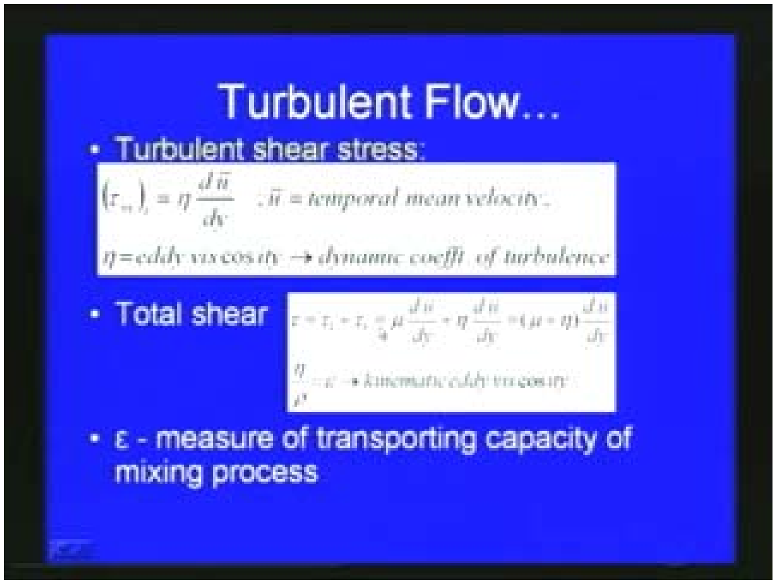

So shear stress when we consider the total shear stress with respect to this here you can see in this slide the total shear stress is equal to tau is equal to tau laminar plus tau turbulent. As we have already discussed the laminar shear stress can be especially mu into du bar by dy and turbulent shear stress is expressed as eta mu du bar by dy. So the total shear is equal to mu plus eta into du bar by dy. Wherever the turbulent flow is presence if there is a laminar boundary layer or a laminar component is there or laminar sub layer is there the total shear will be the shear stress due to the laminar flow plus this shear stress due to the turbulent flow.

Then as we have seen earlier this eta by rho eta is the dynamic coefficient of turbulent, so eta by rho is called kinematic eddy viscosity in turbulence flow; we call this term eta by rho, where rho is the mass density, eta is the eddy viscosity of dynamic coefficient of turbulence that is equal to epsilon that is called kinematic eddy viscosity. Actually this kinematic eddy viscosity epsilon is the measure of transporting capacity of the mixing process.

As we have seen in earlier cases, as in some of the slides, a lot of mixing takes place in turbulence and generally this eta by rho that means the kinematic eddy viscosity we can describe it as a measure of transporting capacity; that means, due to the turbulence and mixing process transporting transport takes place and this epsilon is same measure of the transporting capacity of the mixing process whenever we consider the mixing in the turbulence. Before deriving the basic equations of turbulent flow we will see some of the classifications of turbulence. We have already seen the various process of turbulence and then we have seen the various factors affecting how flow is changing from laminar to turbulent. So based upon this we can classify the turbulence into three categories.

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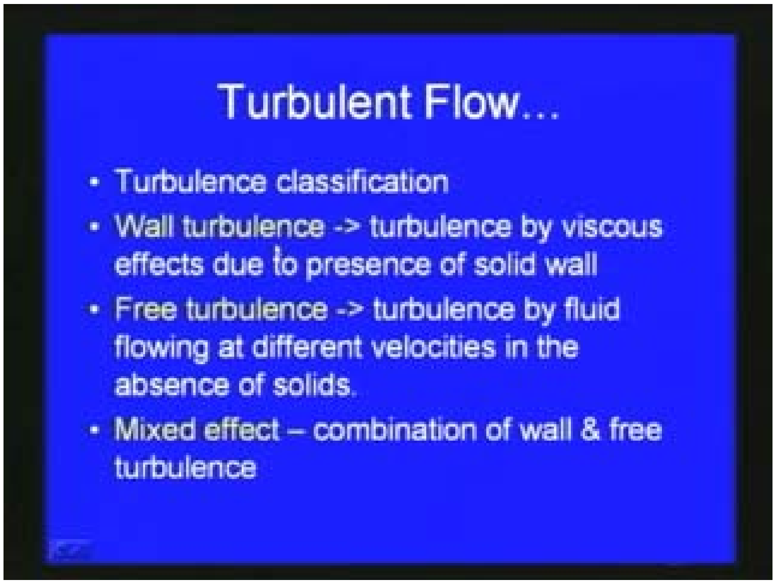

Turbulent classification we can have in three categories: first one is called wall turbulence. Wall turbulence means the turbulence by viscous effects due to presence of a solid wall, so here first category is called wall turbulence; this is due to the presence of solid wall.

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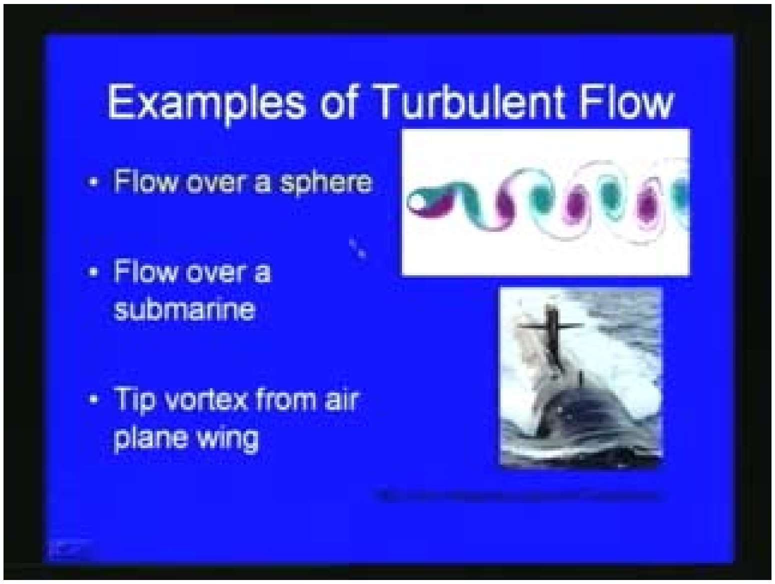

As we have seen earlier, some of the other earlier slides, you can see here whenever there is a presence of a solid wall like you can see here in this slide there is a sphere; then the flow comes and then turbulence is generated or with respect to here you can see also submarine is there. So, due to the presence of solid wall whenever the turbulence is generated that case the turbulence is called wall turbulence. This is one of the important categories of as we have seen some of the important reason is presence of solid particle fluid interaction. This is one of the important categories of the turbulence, wall turbulence.

Second one is called free turbulence. So, free turbulence means turbulence by fluid flowing at different velocities in the absence of solids. In the case of free turbulence as such there is no presence of the solid wall or solid material in the flow or there is no fluid interaction takes place but the turbulence is generated by fluid flowing at different velocities.

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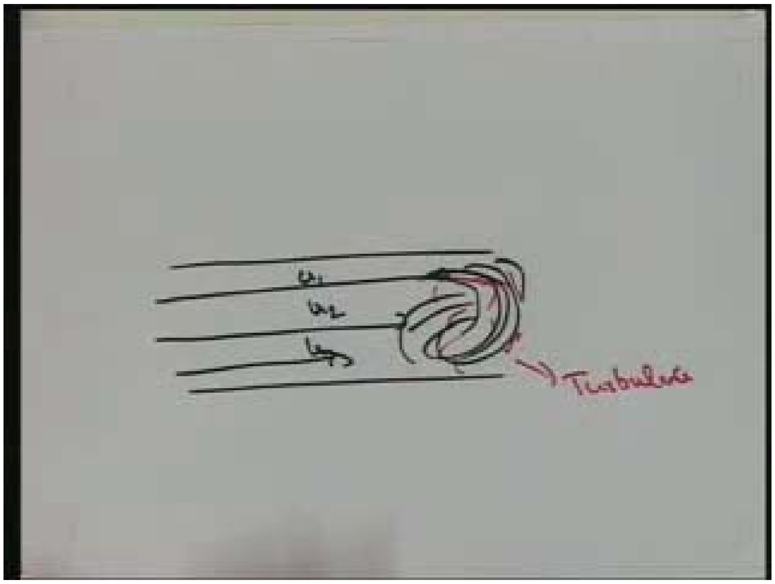

We can see that here if we plot for example the flow is in a channel like this. Then due to various reasons if the velocities here if it is u_1 u_2 u_3 and then if there is considerable changing velocity we can see that after some time a mixing starts and then the turbulence starts; the flow changes from laminar to turbulence due to this mixing. So this region is now turbulence; this turbulence which is generating is called a free turbulence. Here as you can observe there is no specific reason; the reason for turbulence is only between the different layers what happens that means the velocity changes and then turbulence is generated. So this is called a free turbulence.

Then the third category, so the turbulence can be the mixed effect. That means there can be wall turbulence as well as free turbulence. So if the mixed effect is there then third category is a combination of wall and free turbulence. These are the three important classifications of turbulence and based upon which we can sub classify and then we can try to analyse various turbulent flows. Now we have seen the classification and next we will see the theory of turbulence; then we will derive the govern equations of turbulence.

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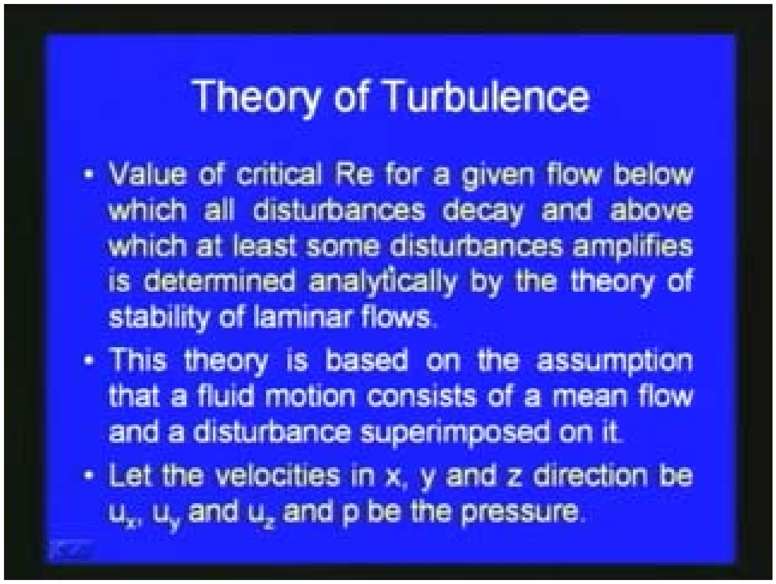

As we have seen the Reynolds experiments we have seen that once the flow reaches a critical Reynolds number the flow changes from transitional takes place and laminar flow become turbulent. The value of critical Reynolds number for a given flow below which all disturbances decay and above which at least some disturbances amplifies is determined analytically by the theory of stability of laminar flows.

Scientists try to analyse this turbulent flow for long time and then they compared with the laminar flow, why this turbulence is generating all this was critically analysed and then they used the theory of stability of laminar flow. In a flow regime, if it is laminar and if it is to be continued as laminar flow that is called a theory of stability of laminar flow so why turbulence is created? Why deviation takes place from this stability of the laminar flow? Various scientists analysed this with respect to the critical Reynolds number and they found the reasons are the various disturbances and whether the disturbance generated in the fluid system; if it is decaying then the flow become again laminar but if various reasons less as we have seen presence of a solid or the velocity variation or the boundary roughness then this disturbances are amplified; the stability of the laminar flow goes and then flow become turbulent. This theory is based on the assumption that a fluid motion consists of a mean flow and disturbance superimposed on it as we have already discussed. So this theory of stability of laminar flow is based upon assumption that fluid motion consists of a mean flow and disturbance superimposed on it. Now, let us say the velocities in x y z direction, when we consider three dimension problems then the velocities being u_x and u_y and u_z, the velocity in x y and z direction and p be the pressure component.

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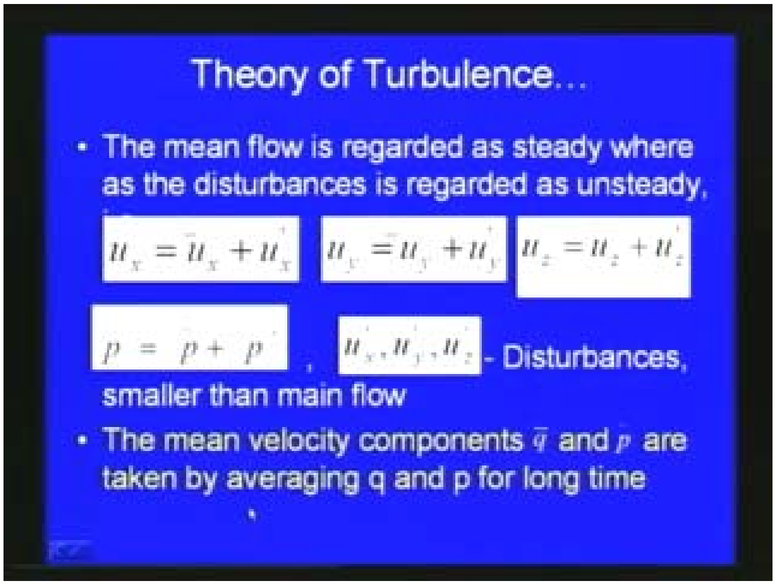

As we have seen, based upon the theory of this turbulence we can express the mean flow as ux bar plus ux dash; uy is equal to uy bar plus uy dash; and u_z is equal to u_z bar plus u_z dash. Then the pressure can be written p is equal to p bar plus p dash, where p bar is the mean pressure and p dash is the fluctuating components. You can see that all this fluctuating components are generally smaller than the main flow. The mean velocity component, if we consider the velocity components as p bar this ux dash uy dash and u_z dash and if we consider q bar and p bar is the mean pressure component we can get by averaging the velocities and pressure for long time. As we have already seen the previous slide this we can update by averaging this velocity components and pressure for long time. So the mean quantities of the fluctuation as steady as we have seen the fluctuations takes place; if this is the mean component and then up and down fluctuations takes place and then if we consider the fluctuation for long time or you can see that this fluctuations for a long time it is steady. So these fluctuating quantities are responsible producing additional stresses. As we have already seen this fluid molecules jam and then this fluctuations takes place and this fluctuations quantities are responsible in producing addition stresses called apparent stresses. This additional stresses due to the fluctuating component the additional stresses produced are called an apparent stresses or Reynolds stresses in addition to the viscous shear stresses and normal stress sigma I. So any fluid as we can see that there is already shear stress component and then normal stress component and turbulent flow other than this additionally due to these fluctuating quantities we have the apparent stresses or Reynolds stresses due to the fluctuating quantities.

So when we analyse turbulent flow we have to consider the normal stresses; we have to consider the shear stresses and then also we have to consider the apparent stresses or Reynolds stresses. Due to this and more over this apparent stresses Reynolds stresses are due to the fluctuating quantities it is also fluctuating and then it is very difficult to quantify, that is the real challenge in the turbulence modeling.

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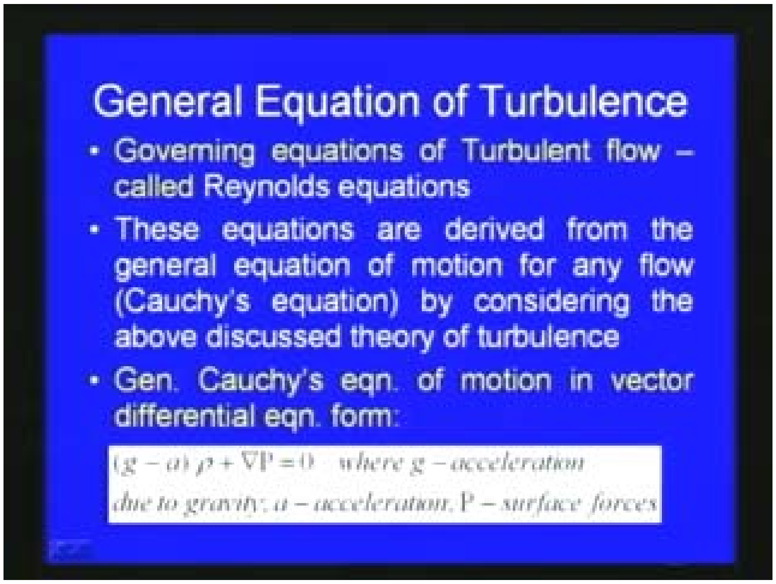

Now we will go to the general equation of turbulence. Here we will be trying to derive the general equations based upon the existing theories. The Governing equation of turbulence flow is generally called as Reynolds equation. So, these equations consist in three dimensions; it consist three momentum equations and the continuity equations. These equations are derived from the general equations of motion for any flow the

Cauchy’s equations by considering the above discussed theory of turbulence. The Reynolds equation, now we will be deriving; these equations are generally starting from the Cauchy’s equation. So we will be discussing in a later chapter on Cauchy’s and real stress equation but we will start now to derive this Reynolds equation from the basic Cauchy’s equation for fluid flow.

From the general equation of motion this is the Cauchy’s equation by considering the theory of turbulence. General Cauchy’s equation of motion in vector differential equation form we can write as g minus a into rho plus delta p is equal to 0, where g is the acceleration due to gravity, a is the other acceleration components, p is the surface forces So, P is the surface forces. We can see in standard literature of general Cauchy’s equation. Anyway we will be discussing it further when we discuss the in nevier stokes equation also. So the general Cauchy’s equation is actually we can see that with respect to the acceleration components due to gravity and then other accelerations are there and then multiplied by rho plus the surface forces the variation surface forces delta p is equal to 0. So this is the general Cauchy’s equation. Based upon this we can write the three equations of motions for the general fluid flow and then those equations are transformed into the Reynolds equations by using the theory of turbulence.

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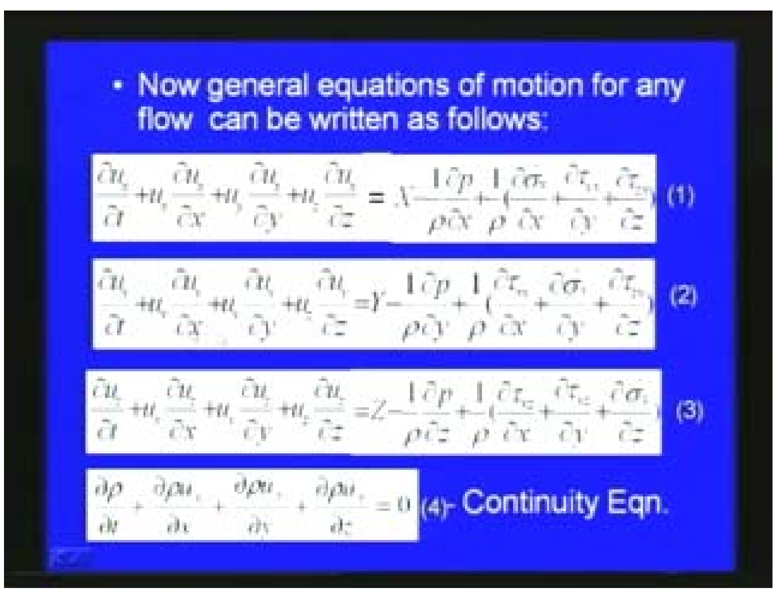

Now, the general equations of motion, so called Cauchy's equation we can write as I mentioned this equation we will be discussing later in the nevier stokes equations in chapter. This equation in x y z component we can write as del u_x by del t plus u_x into del u_x by del x plus u_y into del u_x by del y plus u_z into del u_x by del z is equal to X minus 1 by rho del p by del x plus1 by rho del $sigma_x$ dash by del x plus tau_{yx} by del y plus del tau_{zx} by del z. As we have already seen u_x u_y u_z are the velocity components in the x y z direction, t is the time, X is the there in body force, then p is the pressure, rho is the density, sigma is the normal stress and tau is the shear stress.

So similar way in y direction, we can write the general equation of motion so called Cauchy's equation as del u_y by del t plus u_x into del u_y by del y plus u_y into del u_y by del z is equal to Y minus 1 by rho del p by del y plus1 by rho del tau_{xy} by del x plus del $sigma_y$ dash by del y plus del tau_{zx} by del z, where y is the body force in the y direction

In z direction, del u_z by del t plus u_x into del u_z by del x plus u_y del z by del y plus u_z into del u_z by del z is equal to Z minus1 by rho del p by del z plus 1 by rho del p by del z plus 1 by rho del tau_z by del x plus del tau_z by del y plus del tau $sigma_{dash\ z}$ by del z. These are the three general equations of motion so called Cauchy's equation in x y z directions and then we have the continuity equation in the flow with we considering the density, the continuity equation we can write as del rho by del t plus del rho u_x by del x plus del rho u_y by del y plus del rho u_z by del z is equal to 0.

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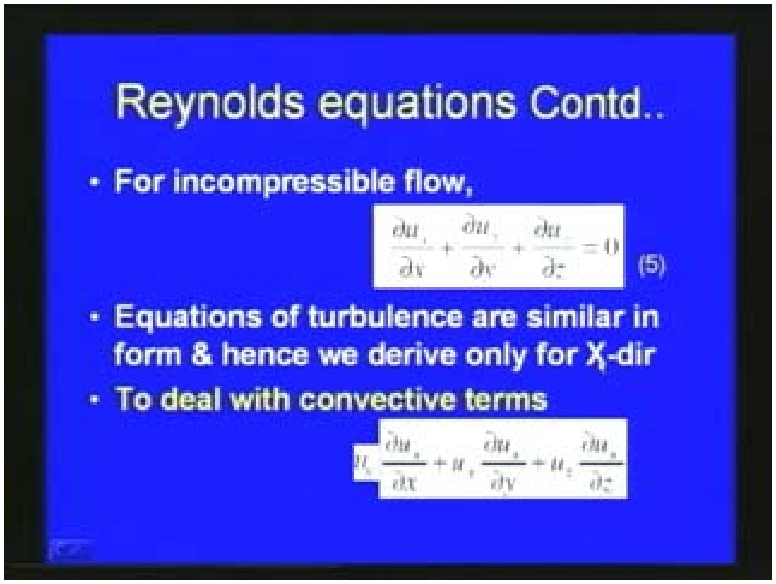

Now, we are going to derive the Reynolds equations based upon these three equations of Cauchy's equations of three general equations of motion and the continuity equations. Now most of the problems if we consider the flow are to be incompressible. rho is constant so we do not have to consider. So that the continuity equation number 4 we can write as del u_x by del x plus del u_y by del y plus del u_z by del z is equal to 0.

So this is the continuity equation for incompressible flow as shown in this slide. Now we are in the process of deriving the Reynolds equation; now the equations are turbulence as similar in form so we derive only for x direction. As I mentioned we are now trying to get the equations for Three d flow problem or 3 d turbulent flow. We will be having three equations of motion in x y z directions and continuity equations we have already seen. So in x y z directions we have three equations. There are various steps involved in deriving this equation is same either is x or y z direction.

So now we will derive the equation here in x direction and then we will write the equation in y and z direction in a very similar way. That is the procedure we adopt here. Now the earlier Cauchy's equation here we have seen that we have got this convective terms. So if we consider the equation in x direction we have this three convective terms u_x into del u_x by del x plus u_y into del u_x by del y plus u_z into del u_x by del. This we will consider to deal with these convective terms. We would be initially considering some specific steps and then we will be simplifying this term by using some mathematical approximations so the convective terms here u_x into delu_x by del x and u_y into del u_x by del y u_z into del u_x by del z. So these three terms we will be considering as convective terms so here u_x into del u_x by del x plus u_y into del u_x by del y plus u_z into del u_x by del z.

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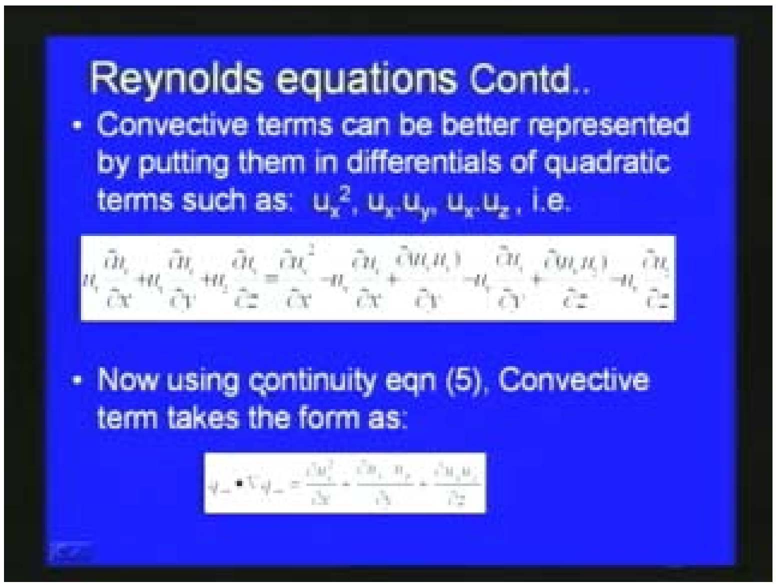

The convective terms we can be better represented by putting them in differentials of quadratic terms such as u_x square, u subscript x u_x into u_y u_x into u_z as written here, that is, u_x into del u_x by del x plus u_y into del y plus u_z into del u_x by del z. These three terms can be written as del u_x square by del x minus u_x into del u_x by del x plus del u_x into u_y by del y minus u_x into del u_y by del y plus del u_x into u_z by del z minus u_x into del u_z by del z. In this equation now the left-hand side is written into this form.

Here we can use the continuity equation which we have seen here in equation number 5. If we use that then you can see that this term u_x into del u_x by so this u_x can be taken out so we can write this term u_x into del u_x plus del x and the second terms this term, this term, three times we can combine together so this on the right-hand side second term, fourth term and sixth term.

We can combine this three together and then its continuity equation become 0 since del u_x by del x plus del u_y by del y plus del u_z by del z is equal to 0 and finally this convective terms which is represented as in this equation as in the flowing, we can write this as a convective term becomes del u_x square by del x plus del u_x u_y by del y plus del u_x into u_z by del z. So the convective terms in the Cauchy's general equation is transformed initially in this form by the mathematical simplifications.

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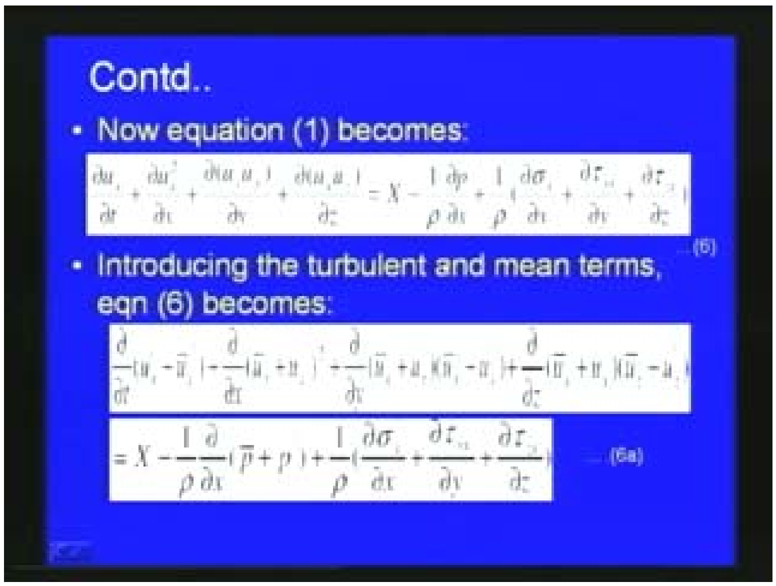

Finally, this Cauchy's equation for the equation of motion we can write in this form after changing the convective terms as in the previous slide so we can write del u_x by del t plus del u_x square by del x plus del u_x u_y by del y plus del u_x del u_z by del z is equal to x minus 1 by rho del p by del x plus 1 by rho del $\text{sigma}_{\text{dash } x}$ by del x plus del tau_{yx} by del x plus del tau_{zx} by del z. So now we will introduce the turbulent and mean terms; this is now the Cauchy's equation of motion is transformed as in equation number 6. As I mentioned, we will be changing this with respect to the theory of turbulence where theory of turbulence is there is same mean component for the velocity and pressure and plus fluctuating component for the turbulent flow. We will introduce the turbulent and mean terms so that now the equation 6 becomes here del by del t of u_x dash plus u_x bar plus del u del by del x $u_{x\text{ bar}}$ plus del u del t by del x $u_{x\text{ bar}}$ plus u_x dash square plus del by del y $u_{x\text{ bar}}$ plus u_x dash into $u_{y\text{ bar}}$ plus u_y dash plus del by del z $u_{x\text{ bar}}$ plus u_x dash $u_{z\text{ bar}}$ plus u_z dash that is equal to x minus 1 by rho del by del $x_{p\ bar}$ plus p dash plus 1 by rho del $sigma_x$ dash by del x plus delta y x by del y plus del tau_{zx} by del z. So equation 6 is changing to equation 6a by applying the theory of turbulence the velocity and pressures are expressed in terms of a mean value and the fluctuating component.

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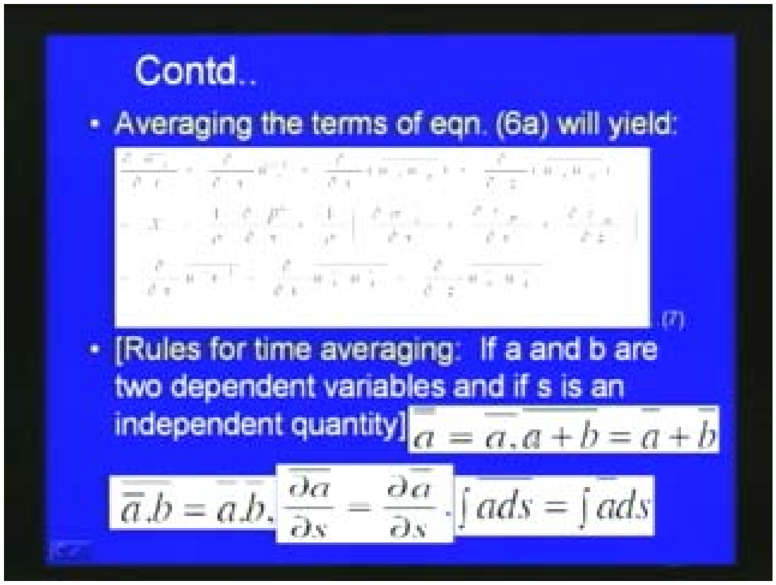

Now, in an advance mathematics there is a term called averaging the terms which is defined here. So the rules for time averaging is for example, some of the terms is expressed here if a and b are two dependent variables. Then time average has given this equation a double bar is a bar a plus b bar is like this various time averaging can be as in standard advance mathematics test given in advance mathematics test books. So this time averaging concept we will be using here for the earlier equations 6 a. If we use the time averaging concept we can write by averaging the terms we will get del u_x bar by del plus del by del x of $u_{x\ bar}$ square plus del by del y of u_y, its mean value bar plus del by del z u_x $u_{z\ bar}$ is equal to x minus 1 by rho del p bar by del x plus 1 by rho and del $sigma_x$ del by del x plus del tau_{yx} by del y plus del tau_{zx} by del z minus del by del x u dash x square bar minus del by del y u_x dash u_y dash bar minus del by del z u_x dash u_y dash bar. So this is equation number 7. Here he process is the average the terms of equation 6a to get equation 7. If you do the time averaging concept for the continuity equation which we have seen as we have already seen we are transforming the turbulent flow we are using in mean flow component and then fluctuating component.

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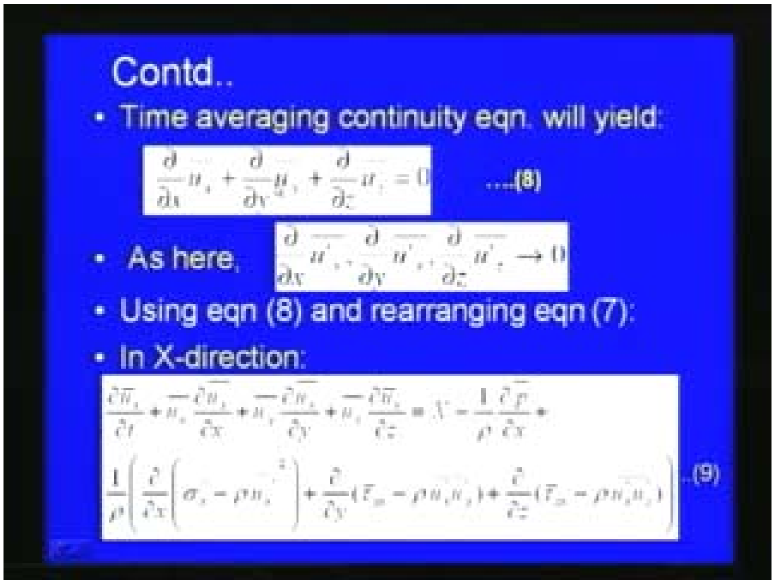

If we use the time averaging concept for the continuity equation we can write the continuity equation as del u_x bar by del x plus this equation slide here plus del by del y u_y bar plus del by del z u_z bar is equal to 0 equation number 8. Here you can see that as far as some of the other terms are concerned this we have discussed earlier del by del x of u_x dash bar and del by del y of u_y dash bar and del by del z of u_z dash bar tends to 0.

When since we take the mean component if we use the this continuity equation 8 and then using this concept the time averaging concept we can rearrange the earlier equation number 7 into this form equation number 9. So finally next direction we can write del u_x bar by del plus u_x bar into del u_x bar by del x plus u_y bar into del u_x bar by del y plus u_z bar into del u_x bar by del z is equal to x minus x is the body force x minus 1 by rho del p bar by del x plus 1 by rho del by del x $sigma_x$ dash minus rho into u_x dash square plus del by del y tau_{yx} bar minus u rho u_x dash u_y dash bar plus del by del z tau_{zx} bar minus rho x u_x dash u dash z bar.

This is now the final equation in x direction. We have used sum of the time of averaging concept, then we use the turbulence mean flow theory and then finally we have transformed the Cauchy's equation- the general equation of motion into the turbulent flow equation. Similarly, as we have seen in the x direction in equation number 9, in similar way we can write in y direction and z direction

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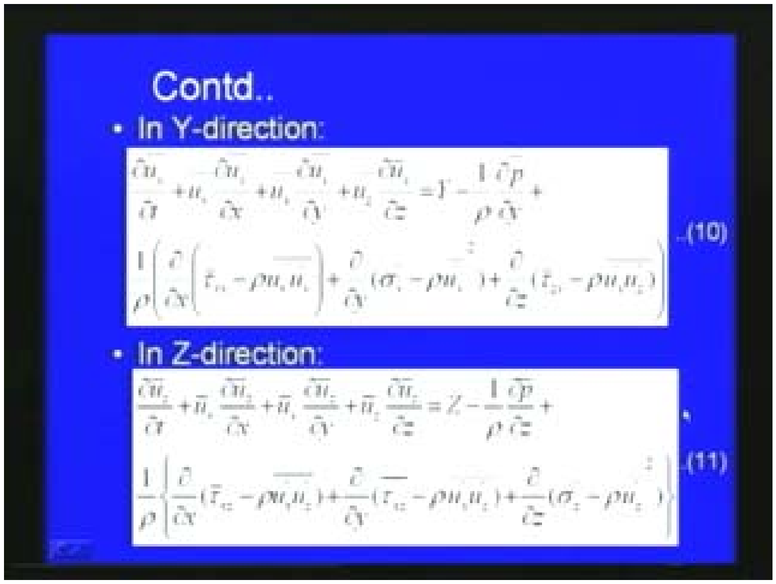

So here in y direction we can write del u bar by del plus u_x bar into del u_y bar by del x plus u_y bar into del u_y bar by del y plus u_z into del u_y bar by del z is equal to Y capital Y minus1 by rho del p bar by del dy plus1 by rho del by del x of tau x_y minus rho u_x dash u_y dash its mean plus del by del y sigma y dash minus rho u_y dash bar square plus del by del z tau_{zy} bar minus rho u_y dash u_z dash bar. This is the equation number 10 and this is the equation of motion for turbulence in y direction.

Similarly, in z direction we can write del u_z bar by del plus u_x bar into del u_z bar by del x plus u_y bar into del u_z bar by del y plus u_z bar into del u_z bar by del z is equal to Z, where Z is the body force in z direction minus 1 by rho del p bar by del z plus 1 by rho del x of tau_{xz} bar minus rho u_x dash uz dash bar plus del by del y tau_{yz} bar minus rho u_y dash u_z dash bar plus del by del z tau_z dash minus rho u dash z square.

So this is equation number 11, the equation of motion for turbulence in z direction. Now we have seen three equations in the x y z direction; these equations are called Reynolds equations of turbulence.

These are the fundamental basic equations of turbulence which we generally using to solve most of the problems but here these equations you can see it is in terms of normal stress and shear stress. Generally, we will be transforming those equations in terms of the velocity components itself. That is the process which we do generally in nevier stokes equations. So this process of converting we will be discussing in the chapter on nevier stokes equations how we are doing this transformation.

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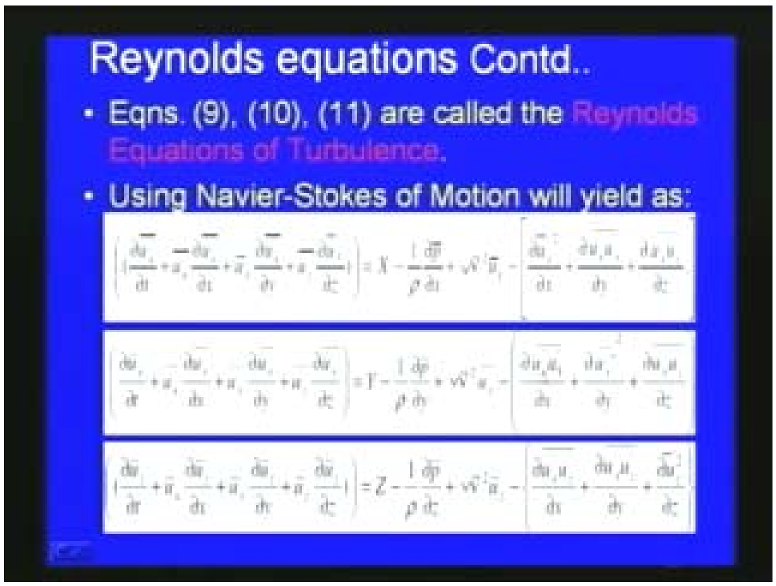

Here, as far as the turbulent flow is concerned now the basic equations here are the Reynolds equations of turbulence equations 9, 10 and 11 as we have already seen here equation 9, 10 and 11. So these equations are called the Reynolds equation turbulence which is the basic fundamental equations of turbulence.

As I mentioned these equations we can convert into nevier stokes form by certain transformation. So this transformation how we transform we will be discussing the theory how we do the transformation when we discuss the chapter on nevier stokes in another separate chapter. Here we will be just write this Reynolds equations of turbulence as in the form of nevier stokes equations of motion of turbulence as follows. Here, the three equations 9, 10 and 11 are converted and written like this.

So the form of equations are del u_x bar by del plus u_x bar del u_x bar by del x plus u_y bar del u_x bar by del y plus u_z bar del u_x bar by del z is equal to x minus 1 by rho del p bar by del x plus mu into del square u_x bar so this mu is the kinematic viscosity minus del u_x dash bar square by del x plus del u_x dash del y bar by del y plus del u_x dash u_z dash bar by del z. So this is the nevier stokes form of the turbulent motion equation in x direction.

Similar way in y direction, we can write del u_y bar by del plus u_x bar del u_y bar by del x plus u_y bar del u_y bar by del y plus u_z bar del u_y bar del z is equal to y minus1 by rho del p bar by del y plus mu into del square u_y bar minus del u_x dash u_y dash bar by del x plus del u_y dash bar square by del y plus del u_y dash u_z dash bar by del z this is the nevier stokes form of the turbulent flow equation in y direction.

In z direction, we can write del u_z bar by del plus u_x del u_z bar by del x plus u_y bar del u_z bar by del y plus u_z bar del u_z bar by del z that is equal to z minus1 by rho del p bar by del z plus del u_x dash u_z dash bar by del x plus del u_y dash u_z dash bar by del y plus del u_z dash bar square by del z. So this is the nevier stokes form of the equations of turbulence in z direction.

So either we can solve the Reynolds form of the turbulence equation or we can use the nevier stokes form of the turbulent flow equations to get a solution. Here now the challenge is we have seen that we have three equations of turbulence in x y z. When we consider three dimensional flows in x y z direction we have three equations of motion and then we have one continuity equations.

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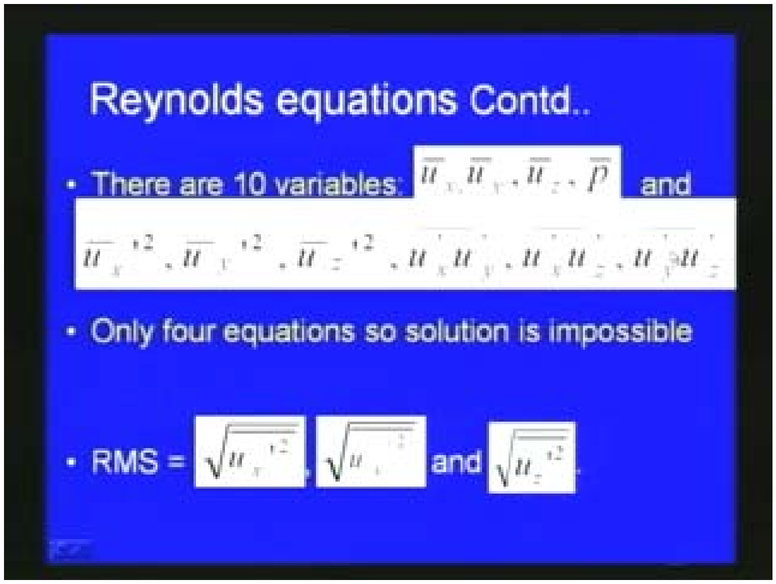

So we have four equations but you can see that the number of unknown variables even we if we consider the nevier stokes form of the turbulent equation of motion we can see there are ten variables. The variables are u_x bar, u_y bar, u_z bar, p bar and the u_x dash bar square, u_y dash bar square, u_z dash bar square, u_x dash u_y dash bar, u_x dash u_z dash bar, u_y dash u_z dash bar. There are ten unknowns and we have only four equations. So mathematically to get solution is very difficult since we have got four equations and ten unknown. We have to go for some other theories, some other empirical relations in addition to this equation to get solutions. The solutions means the velocity components at particular time step particular position if we want the velocity variations of pressure variation turbulent flow and we can use either Reynolds equations or nevier stokes equations plus we need some other relationships since the number of unknowns are ten and we have got only four equations. We have already seen this root mean square represent a general way of some of the fluctuations are turbulent quantities.

So that we have already seen u_x square root of u_x dash bar square u_y dash bar square and square root of u_z dash bar square; these are the root mean square values.

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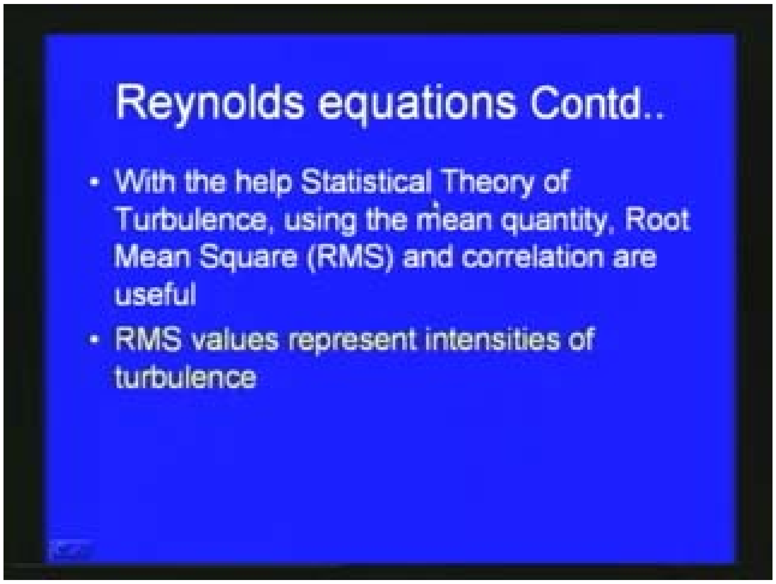

Generally, to solve this to get a mathematical solution is very difficult so in turbulence flow solution what generally scientist do is they use some of the statistical theory of turbulence here, some of the probability aspects and some of the statistical theories are used like the mean quantity root mean square and correlations. Generally, we use to get a solution in the case of turbulent flow.

The statistical theory is one of the fundamental theories which are used in the case of turbulent flow solutions. So solutions means we want to find out the velocity components and pressure components at any particular time step and at any particular point which we consider. So this root mean square values are representing intensities of turbulence as we have already seen. The statistical theory based upon the correlations or the RMS for the mean quantity we can utilize.

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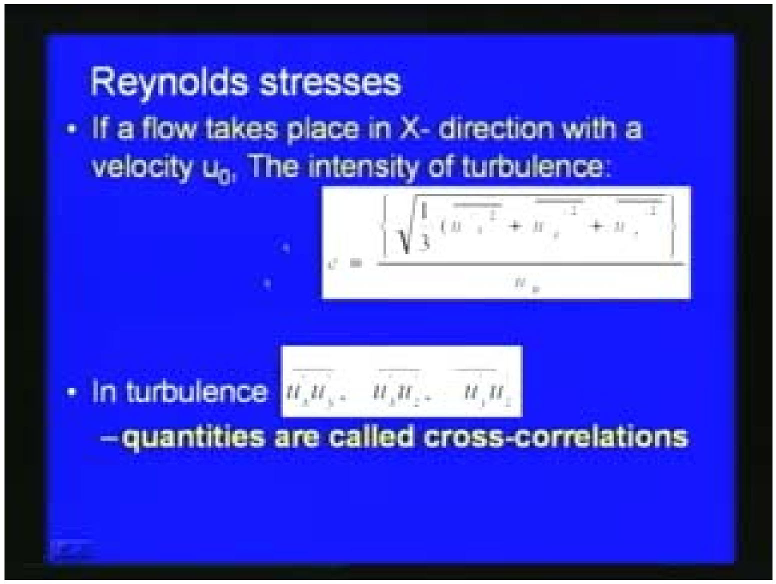

We have already seen the intensity of turbulence is expressed as e is equal to square root of1 by 3 u_x dash bar square u_y dash bar square plus u w_z dash bar square by u_0, where u 0 is the mean free mean the free stream velocity and says these quantities like u_x dash u_y dash bar u_x dash u_z dash bar u_y dash u_z dash bar. These quantities are called cross correlations.

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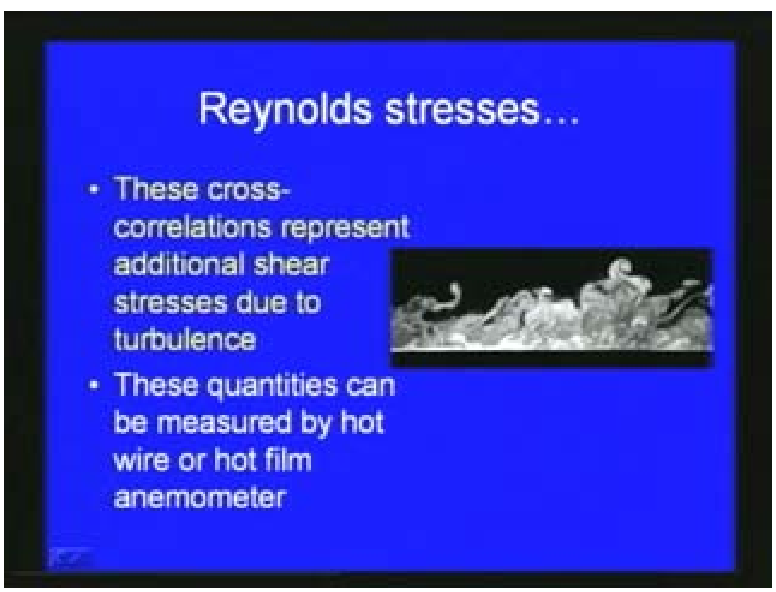

So what we do? As I mentioned turbulent flow solution is very difficult since the variations, so many unknowns and then we have got only few equations. Generally, use a theory, statistical theory so there we use this mean flow mean component, then RMS and then the cross correlations.

These cross correlations as we have seen in the previous slide here these are called cross correlations. These cross correlations represent additional shear stresses due to turbulence. Here in this slide you can see additional shear stresses will be generated due to turbulence and this cross correlations represents these additional shear stresses and these quantities can be measured by hot wire or hot film anemometer for some of the solutions.

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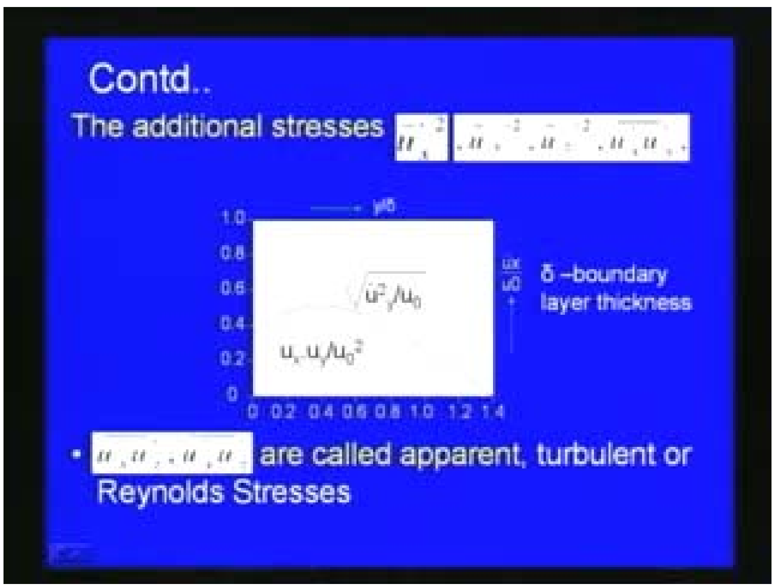

Here, this slide shows the additional stresses like u_x dash square bar square or u_y dash bar square or u_z bar square or u_x dash u_y dash or u_x u_z dash bar or u_y dash u_z dash bar. These are all the apparent or turbulent Reynolds stresses as we have seen so if some of the components we can measure using some of the equipments like the hot film anemometer or some of this and then this measured values we can utilize in some of the equations to get the solutions. So here we can see in this slide if you bring y be delta, where delta is the boundary layer thickness for a particular problem; y by delta causes u_x bar by u_0, where u_0 is the free stream velocity. Then you can see its variations for u_x dash u_y dash by u_0 squares it varies like this and then the mean square RMS value varies like this. So some of the components we can measure, some of them we can use empirical relationship and then also we can use the basic equations to get some of the solutions. As mathematically since we cannot get solutions directly that is why we go for these kinds of approach in turbulent flow theory.

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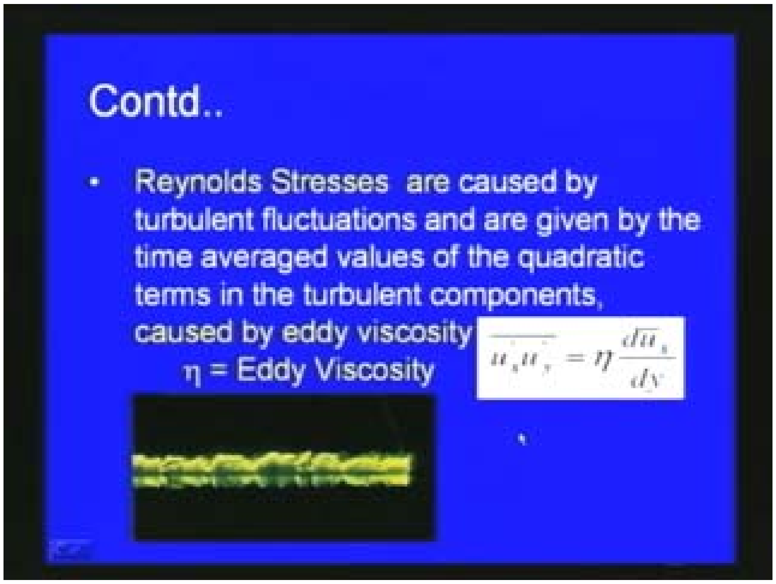

Then, we can see here Reynolds stresses are caused by turbulent fluctuations as we have already seen and are given by the time averaged values of the quadratic terms in turbulent components caused by eddy viscosity. We can represent this u_x dash u_y dash term by eta into du_x bar by dy, where eta is the eddy viscosity so this we can utilize the some of the this relationship like u_x dash by u_y dash is equal to eta dx bar by dy. This is another important relationship we use in the turbulence solutions. Now since turbulence models as I mentioned the problem is to find out the various velocity components in x y z directions and pressure components. So the difficulties as we have seen generally the govern equations only four and they we have ten unknowns and then we cannot get mathematical solution directly so generally, what we go for is a closed solution or closed soluble set is generated.

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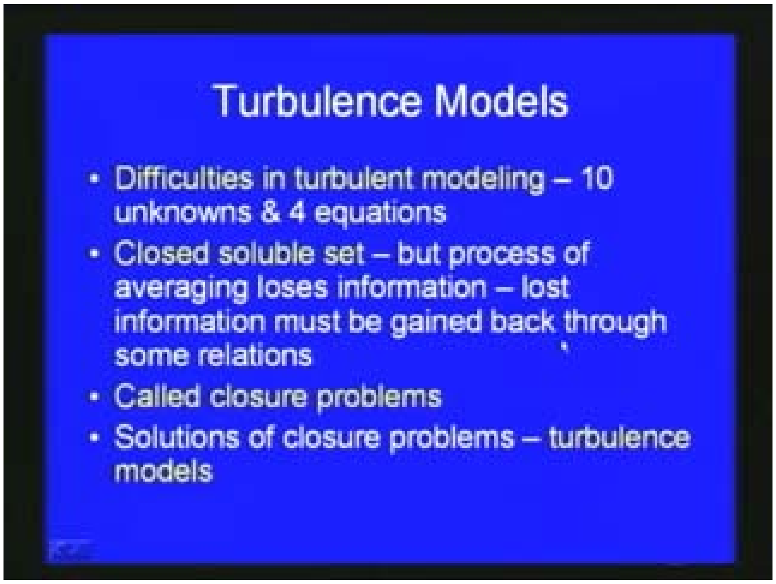

It is the process of averaging loses, in the process of solution some of the information will be lost; the lost information like the stresses as we have seen Reynolds stresses some of this will be loss and this lost information must be gained back through some other relationship. That is what we do in this turbulence models. This way of approach is called closure problems in turbulence modeling or turbulence solutions when we use some of the basic equations like nevier stokes form of equations and then some of the important information are lost. This is gained back by some other relations so this procedure is called closure problems and solutions of closure problems are through that the present the turbulence models. So the solutions of closure problems through this we get the turbulence models.

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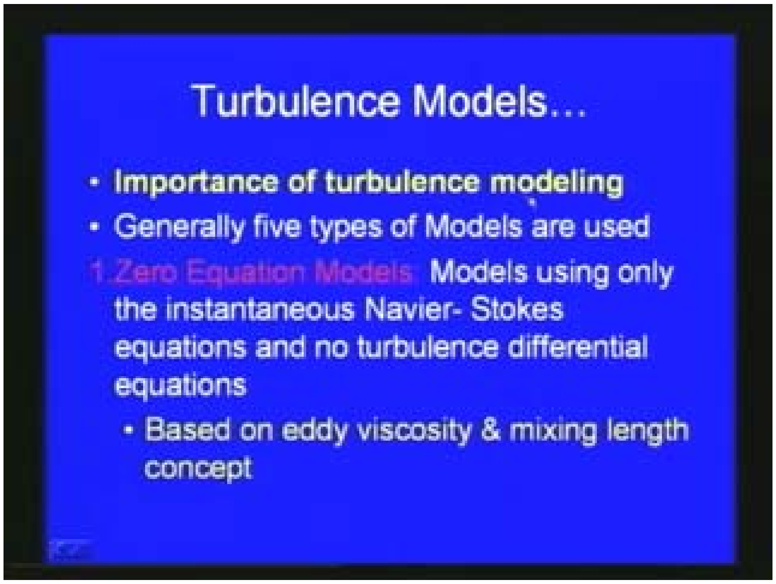

As far as turbulence models are concerned if we refer to literature we can see five types of models: first one is called zero equation models. In this zero equation models what we do is this is the process of modeling using only the instantaneous nevier stokes equations but we do not use any direct turbulence differential equations. Generally this is achieved through eddy viscosity and mixing length concept, this is the first approach.

Generally, literature has five types of turbulence models so first one this zero equation models; a zero equation model is based upon the eddy viscosity and the mixing length concept; here we do not directly use the turbulence differential equations but we use the eddy viscosity and the mixing length concept. So this is the approach called a zero equation model.

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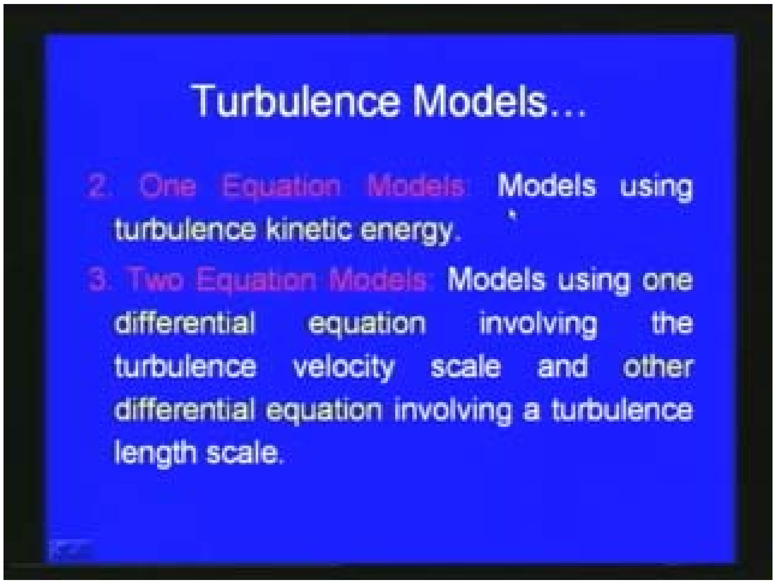

Second turbulent model is called a one equation model; here we use the turbulence kinetic energy. So instead of directly using the differential equations we use the turbulence kinetic energy to get a solution for velocity of pressure variations. And third kinds of models are called two equation models; here in two equation models what we do this gives models using one differential equation involving the turbulence velocity scale and other differential equation involving a turbulence length scale. So this is the third kinds of models which are called two equation models; here we use one differential equation and other differential equation involving turbulence length scale.

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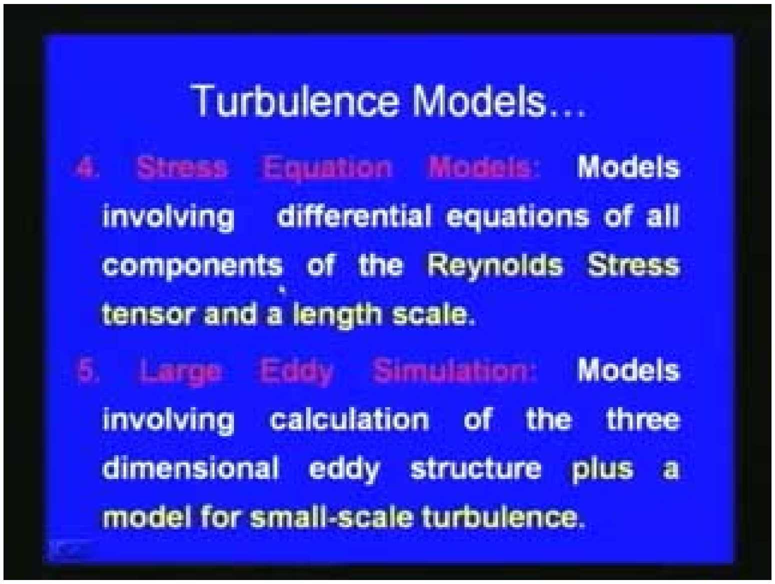

Forth kind of equation model is called a stress equation model. Here, the models involving differential equations of all components of the Reynolds stress tensor and a length scale. So this is called stress equation models, where we use all the differential equations and components of the Reynolds stress tensor and a length scale.

Finally, one of the last to declare another development in turbulence modeling is called large eddy simulation. Here, what we do is this models involving calculation of the three dimensionless eddy structure plus a model for small scale turbulence, so this is what we generally do in the fifth model it is called large eddy simulation model. In three dimensions we will be using the three dimensional eddy structure that means as we have already seen eddy will be common and in eddy structure it is used plus we use a small scale turbulence to capture the turbulence fluctuation. So that is so called large eddy simulation. In literature we can see five kinds of models: first one is zero equation models, second one is called one equation models, third one is called two equation models, forth one is called stress equation models and fifth one is called large eddy simulation models.

Due to all these complexities we will not be discussing further all this models but we will be discussing only some of the zero equation model concept like a mixing length and other things in the following lectures for the turbulence flow simulations in channels and pipes. Since other models involve a lot of mathematical and lot of complexities we will not be discussing the other four models. We will be discussing next mixing length theory and related concept.