

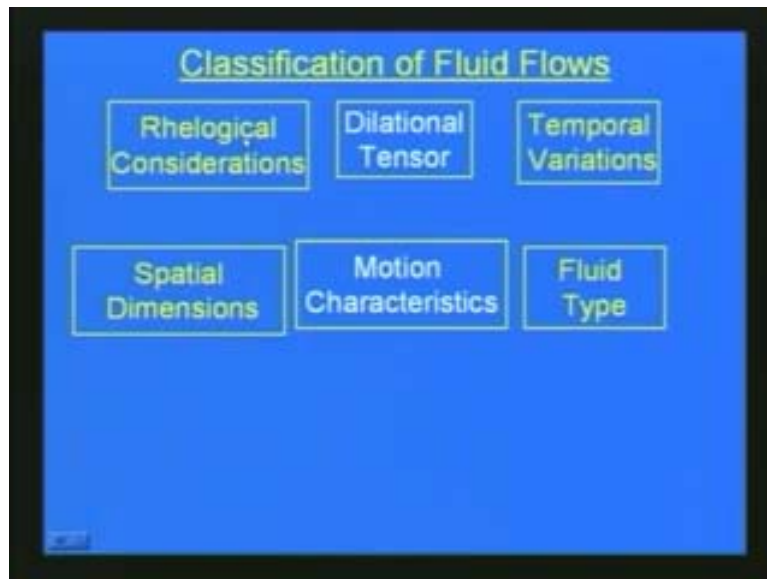
Fluid Mechanics
Prof. T.I. Eldho
Department of Civil Engineering
Indian Institute of Technology, Bombay

Lecture - 2

Fundamental Concepts of Fluid Flow and Fluid Statics

Welcome back to the video lecture on fluid mechanics. In the first lecture, we have discussed the introductory aspects of fluid mechanics. We have discussed about the various fluid properties, various theories used in fluid mechanics and how it will be used in our this video course; we discussed the various fundamental theories which applicable in the case of fluid mechanics; we discussed the various flow visualization techniques, used in fluid mechanics and the flow lines like stream lines, path lines and streak lines. Today, in this lecture, first we will discuss about the classification of fluids.

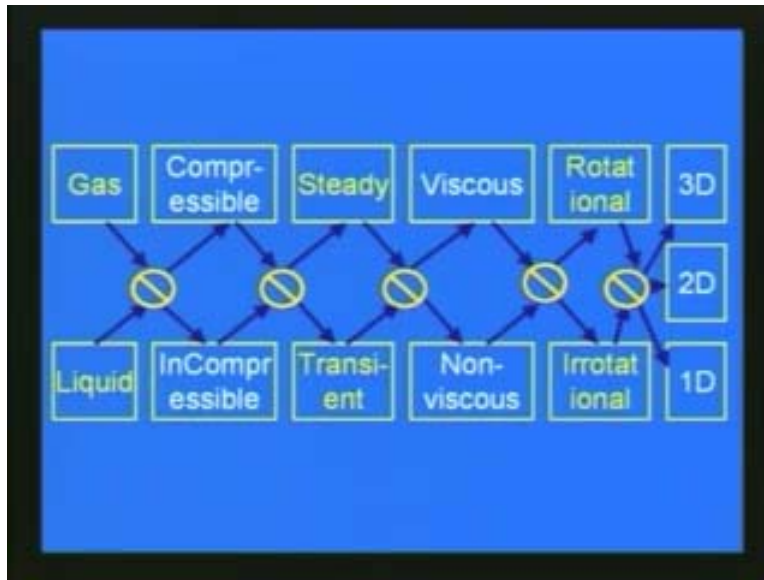
(Refer Slide Time: 22:18)



As I mentioned earlier, the fluids can be classified according to various fluid properties, fluid behavior or the dimensions on which they will be dealing with the fluids or the fluid problem which we are dealing. So, mainly the fluids fluid flow can be classified according to the rheological consideration, spatial dimensions, dilational tensor, then

motion characteristics, the temporal variations and fluid types. Here, we will discuss in detail about the various types of fluid flow according to various aspects.

(Refer Slide Time: 2:53)



As I mentioned earlier, the fluids can be either gases or liquids. The first classification of fluid flow is whether it is gas gaseous flow or liquid flow. The first classification, as you can see in this slide say, the fluid flow can be either gaseous flow or say liquid type fluids. Secondly, according to whether the fluids can be compressed or whether it cannot be compressed say, accordingly we can classify the fluids in to compressible fluids and incompressible fluids; mainly, the gases which will be generally dealing with will be compressible in nature and some of the liquids are may be some compressible depending up on the type of the fluids and some other liquids we can consider as incompressible. Even though sometimes we can compress to every small level but still it can be considered as incompressible. For example, water is considered as incompressible even though, may be to certain level it can be compressed by applying the pressure.

As I mentioned, say, the fluid flow or fluid can be again classified. Fluid flow can be classified as steady state fluid flow or transient fluid flow. The fluid, since it is deforming with respect to space and time, if it is with respect to time there is no variation at all, then the fluid flow is said to be steady. That means with respect to time, there is no variation

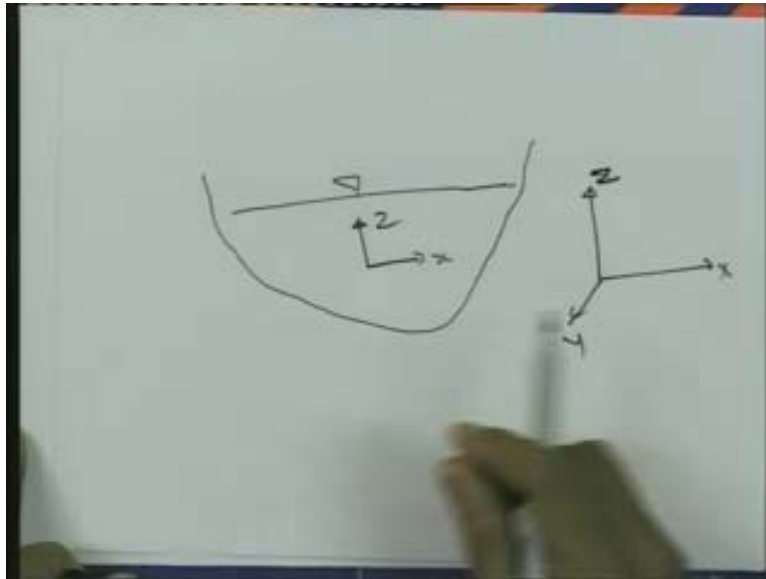
of the fluid properties but the fluid properties like velocity or depth a flow all this fluid properties are constants with respect to time and there is no variation with respect to time.

In the case of UN steady flow or transient flow the fluid property like velocity, pressure and depth, all these parameters are varying with respect to time. So, these types of fluids are called unsteady fluid flow or transient flow. As we discussed earlier the viscosity is an important fluid property. So, accordingly, whether the fluid flow which we are dealing with has got viscosity or the viscosity is negligible or there is no viscosity, then accordingly we can classify the fluids into viscous fluids or viscous flow and non-viscous flow or inviscid flow. So according to the variation or according to the fluid has viscosity fluid is called viscous fluid flow and then if viscosity is not considered then that kind of fluid is called non-viscous or inviscid fluid flow.

If there is any circulation or any rotation aspect as far as the fluid flow is concerned we can classify the fluid into rotational fluid or irrotational fluid. In the case of rotational fluid, there can be circulation there can be vorticity and all other parameters but as far as irrotational fluid is concerned circulation is not there and vorticity is 0. So, those fluids are called irrotational fluid or irrotational fluid flow.

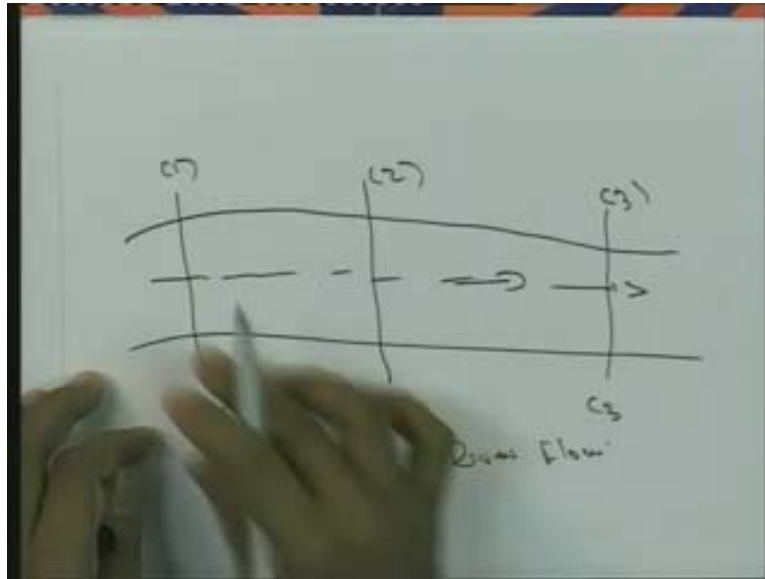
According to the space we are considering 3 dimension, x , y and z . We can classify the fluid as 3 dimension fluid flow or 2 dimension fluid flow or 1 dimension fluid flow. As all of you know fluid flow is definitely 3 dimensional in nature as you can see any type of fluid flow is 3 dimension nature.

(Refer Slide Time: 06:51)



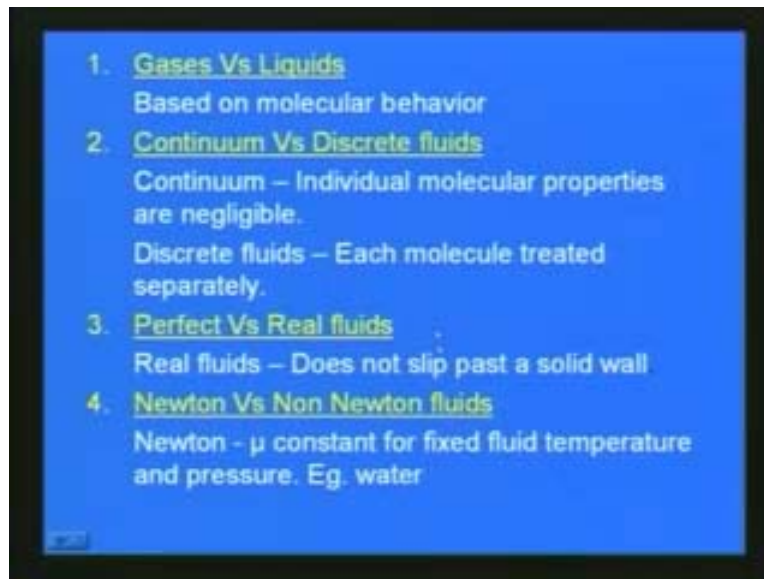
But many times, we can consider 3 dimension fluid flow as 2 dimensions, for example: here you can see say, a river. We are considering a river section like this; if you are considering here as z axis, this as x axis, here say, y axis, so this is xyz axis. Here you can see that when we consider the fluid flow with respect to this river section like this x verses z , then this other longitude dimension is not considered. So, in this case, the fluid flow can be considered as 2 dimension; we may be interested what is happening with respect to this x direction and z direction even though the fluid flow is definitely 3 dimensional in nature. We are simplifying the 3 dimension fluid flow into 2 dimension since it will be very easy to analyze and when interrupt the results also it will be very easier. So, depending upon the case, depending upon the problem and depending upon the fluid flow problem we can consider a 3 dimensional flow as 2 dimensional. So that the accuracy of the results are not much affected.

(Refer Slide Time: 8:17)



We can consider the river flow in some times as 1 dimensional flow say for example, if you are considering the river is flowing like this. In this case, we are dealing from one section to another section, say, here 1 1, 2 2, 3 3; we are dealing from what is happening from section one to section two and section three; in this case, obviously, the fluid flow is 3 dimensional in nature. We can consider the flow as what is happening with respect to this longitudinal direction as far as this river flow is concerned. We can consider in this case, the fluid flow as 1 dimension. All the fluid flow which we are dealing with is 3 dimensional in nature but depending upon the problem to simplify the problem we can consider the fluid flow as 2 dimension or whether we can consider sometimes as 1 dimension. So this simplification makes the problem analyze much simpler and the interpretation of results also much easier without losing much across. Most of the problems even though 3 dimensional in nature, we may be solving as 2 dimensional fluid flow problem or 1 dimensional fluid flow problem.

(Refer Slide Time: 9:36)

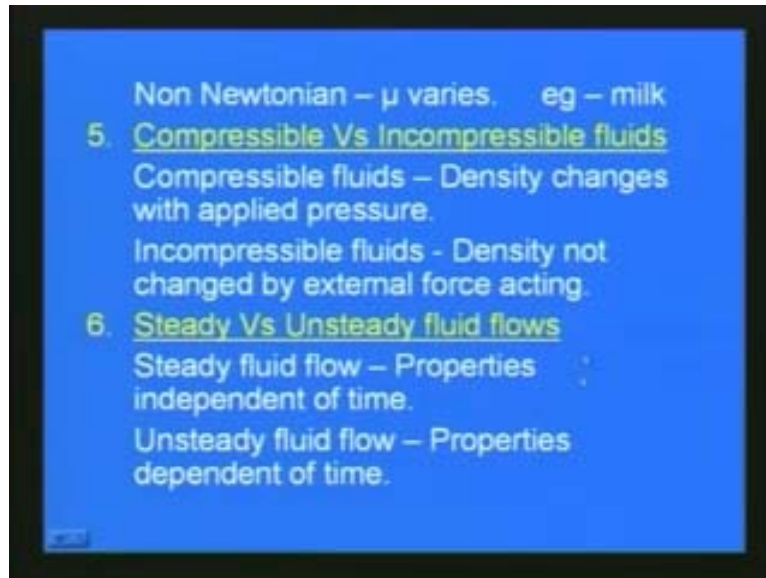


As we have seen the previous slide, the fluids can be either gases or liquids based up on the molecular behavior as you can see in this slide. Then the second case: the fluids when we are considering continuum or discrete fluids. A continuum means individual molecular properties are negligible so that we are considering as a continuum. In most of the fluid flow problems you will be considering as continuum and in other aspects, we can also consider discrete fluid; in the case of discrete fluids, each molecule is treated separately and then what is happening with respect to that molecule is studied. So, two aspects are there in fluid flow analysis: the continuum analysis or discrete fluid flow analysis. Most of the time we will be dealing with the continuum fluid flow analysis and in very rare cases we deal with the discrete fluid flow analysis.

In third case, we can again say the fluids can be either perfect your ideal fluids or the fluids can be real fluid. Real fluids, as I mentioned earlier does not slip past a solid wall and most of the fluids which we are dealing with is real fluids. So, the third case is perfect verses real fluids. Te fourth case, as I mentioned is Newtonian verses non Newtonian fluid. In the Newtonian fluids we consider the qualification of viscosity than on coefficient of viscosity μ constant for a fixed fluid temperature and pressure. For example, water is considered as Newtonian fluids and then we have already seen earlier non Newtonian fluids, when the shear stress verses shear strain various is not linear those

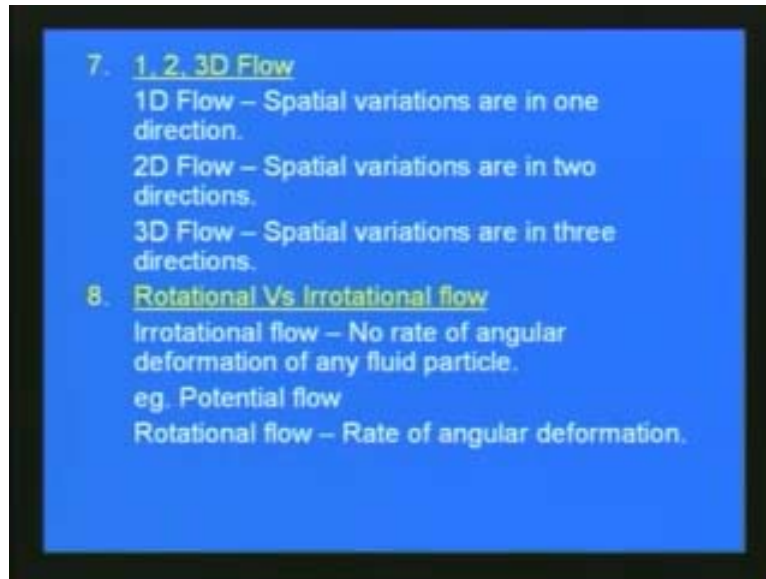
fluids are non Newtonian fluids and then the non Newtonian fluids as I mentioned in the μ varies, say, for example, milk is a non Newtonian fluid

(Refer Slide Time: 11:29)



In the fifth case the fluid can be classified as compressible or incompressible fluid. As we have seen earlier in the compressible fluid the density changes with applied pressure; if you will apply some pressure to the fluid the density changes., for example we consider just some air then we can see in a container if you put some pressure in this container the density inside changes, so this is what is called compressible fluid. In incompressible fluid, density will not change by external force, so this kind of fluid is called incompressible fluid, say, for example, water. When we apply some pressure the density is not changing accordingly, so that type of fluid is called incompressible fluid. The sixth classification as I mentioned in the fluids can be classified as that the steady state or the unsteady. In the steady fluid flow the properties like velocity, the pressure or independent of time or in unsteady fluid flow the property is depending on the time.

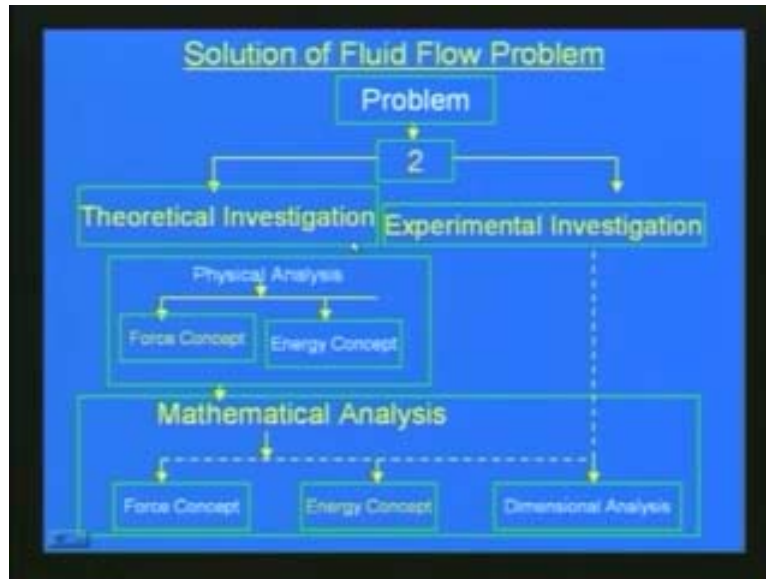
(Refer Slide Time: 12:30)



We have already discussed the 1 dimensional, 2 dimensional and 3 dimensional flows according to the spatial variations. In the eighth classification, we have already seen the rotational versus irrotational flow. Irrotational flow- there is no rate of angular deformation of any fluid particle, for example, potential flow can be considered as irrotational flow. The rotational flow-there is a rate of angular deformation with respect to the fluid flow. These are some of the important classifications of fluids and fluid flow used in fluid mechanics. Based up on these classifications the study of the fundamental principles also slightly varies. In the coming lectures, we will be discussing in detail with respect to various types of fluids; also how it will be dealing, what would be the principle, what are the changes as per the principles compared to the incompressible real fluids or compressible fluids. That we will be discussing in later lectures. Now, finally in this introductory lecture we will discuss how we can solve a fluid flow problem.

If we get a fluid flow problem how can we solve the problem? The problem may be to find out the velocities or the problem may be to find out the pressure distribution or the viscosity changes or density changes or any type of engineering fluid flow problem. If we get how we can approach the problem and then how we can solve the problem, these questions we will be asked generally.

(Refer Slide Time: 14:32)



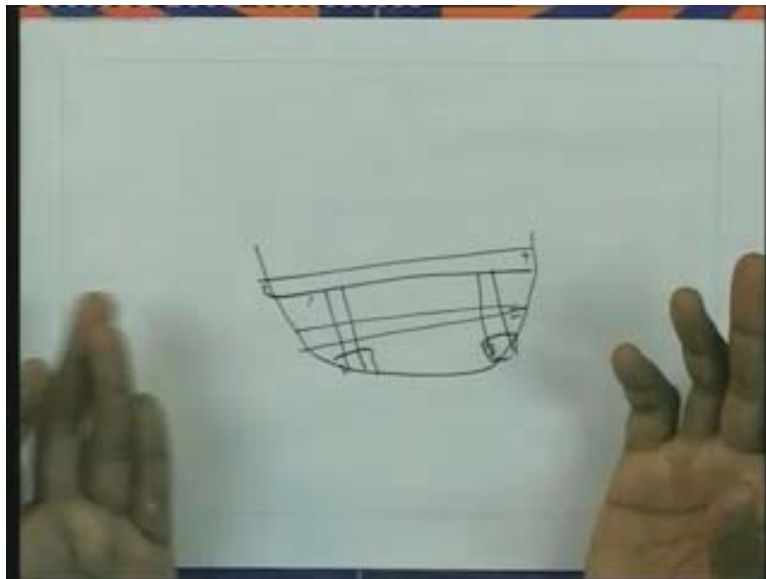
Here this flow chart briefly indicates how the fluid flow problem can be solved. In general, as far as fluid flow analysis is concerned there can be two types of investigations: first one is it can be a theoretical investigation; second one is the experimental investigation.

In theoretical investigations, what we are doing is based up on the available theory or based up on the fundamental principles or based up on the available literature published papers we are trying to approach the problem and we are trying to solve it theoretically; it may be theoretical background or some exact solutions are available or analytical solutions are available or it may be some numerical metrologies which we can directly apply for the problem. This is actually a mathematical way of approach since we are trying to solve the problem theoretically.

In case of excremental investigations, what we are doing? We are trying to replicate the problem in the laboratory to certain scale so a scaling is to be done; most of the real field problem will be very large in nature; we cannot replicate the same in the laboratories. What we can do is that we can reduce the dimensions; we can change it to a certain scale so that the problem will be small in nature and we can easily deal in the laboratory. So, experimental investigation is very much used in fluid mechanics. In the last two hundred

to three hundred years experimental fluid mechanics is very much developed in the solution of various problems. As far as the experimental investigations are concerned there are certain limitations. The limitations are: it is very expensive in nature and we have to do the scaling; through the scaling certain aspects of the problems may not be able to replicate in the laboratory. So certain level of the reality of the problem may be lost in the replications of the laboratory and then running the model in the laboratory, the accuracy may be reduced. So, experimental investigations have certain limitations but if theoretical investigations or analytical solutions are not available then it has got some limitations, the accuracy again may be reduced. Now, as far as theoretical investigations are concerned how are we approaching the problem? If the engineering fluid flow problem is given then first we can understand the problem in a physical nature, what is the problem, what are the inputs data available and what are the outputs expected what the physical principles are based up on the problem which we have to investigate. So this analysis is called the physical analysis.

(Refer Slide Time: 18:00)

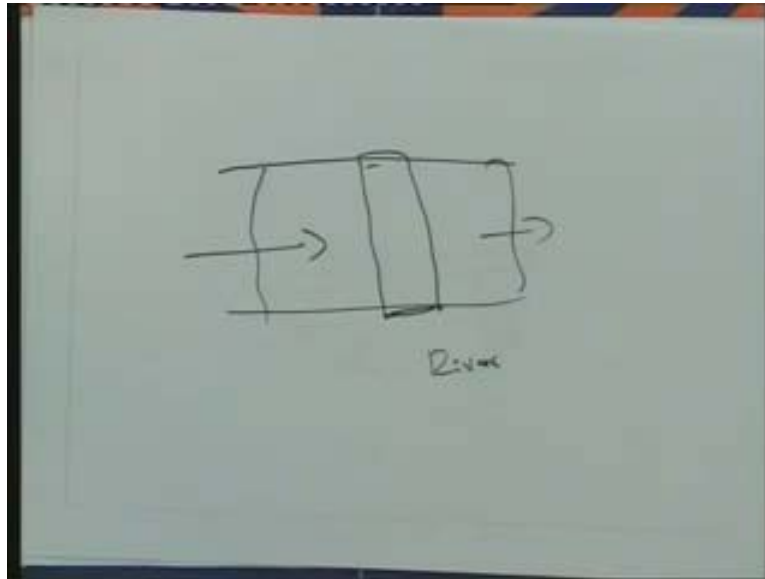


The physical analysis can be done based up on the representation of the problem by a client or so; we can go to the real problems for example, if we are constructing a bridge across a river then we have to construct pillars and then we have to understand the flow behavior across the river section so that we have to do fluid flow analysis as far as the

section is concerned where the bridge is constructed. We may go to the field and see how the river is flowing, what are the important parameters which we have to deal here. We have to analyze and we have to understand the problem physically. So we may have to go to the field or we have to analyze in our office depending up on the data available given by the client.

For example when we are going to construct a bridge across the river, we have to see depending up on the monsoon or depending upon the rain fall through out the year how the water level will be varying, how it is increasing or how it is decreasing and then what will be the flow velocity at the particular location. We are going to construct the bridge the bridge and then the bank of the river has any motion problems or all these things we have to analyze physically. This is called physical analysis. In the case of fluid mechanics analysis this physical analysis can be based up on the force concept or energy concept. The force concept means we will analyze what are the forces acting on the particular problem which we have dealing with and then how it is going to affect the fluid flow properties or the particular variable which we are going to understand and the second concept is called energy concept, how energy variations are like here as they would be the energy creations; we will be analyzing how it will be affecting the particular problem so the physical analysis is very important in the solution of any fluid flow problem. Physical analysis generally can be there upon the force concept or based up on the energy concept. After this physical analysis is over we can understand the problem in which way we have to approach the problem and we can try to make a mathematical formulation of the problem. After this physical analysis next step is the mathematical analysis. In the mathematical analysis with respect to the physical analysis we already know what will be the domain of the problem which we will be dealing.

(Refer Slide Time: 21: 00)

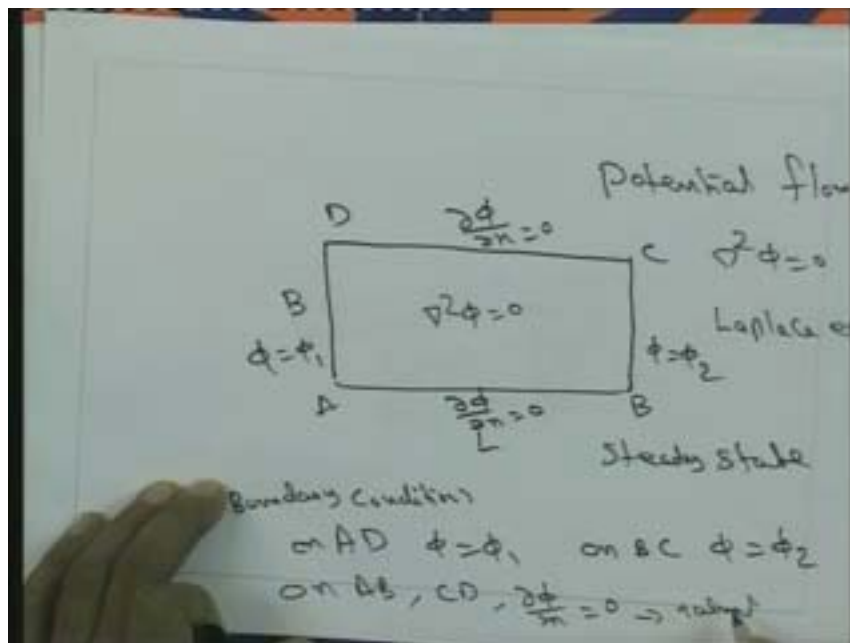


As I mentioned, we are going to construct a bridge across the river. So when the pillars will be constructed what will be the flow conditions? All these things will be analyzed. We know the domain of the problem and then also we can easily identify the data. For example, flow of the variations at various seasons with respect to throughout the days of the year how the flow depth will be varying. All these data are available; then the boundary conditions with the problem is concerned at particular location of the domain say the boundary, then how the flow will be the conditions like depth variations or velocity variation can be identified and then based upon the particular problem we can get the mathematical equations; mathematical equations can be derived based upon the theoretical analysis based upon the first concept or the energy concept.

So, once the mathematical equations are derived and then the boundary conditions are required for unsteady state problem or transient problems. But in the case of transients problems or unsteady state problems you have to also prescribe the initial conditions, time t is equal to 0 but the conditions may be the depth, velocity or pressure all these parameters will be varying. From time t equal to 0, transient analysis or unsteady state analysis of the fluid flow problem is starting. This initial condition is very much important in a fluid flow analysis. The given equations are prescribed; we have already prescribed the boundary conditions of the problem; with respect to this the mathematical

analysis is complete. Again with respect to a figure here I describe, for example, if you consider the potential flow problems, it can flow in a homogeneous isotropic axis which is given by $\nabla^2 \phi = 0$, the Laplace equations. If you consider a rectangle domain like this then we know the length and the breath. So now the dimensions of the domain are known and the domain nature is known. Now, for example we deal with the potential problem, the genuine equations is $\nabla^2 \phi = 0$, the Laplace equation. This is the steady state problem. So, the boundary conditions can be the potential ϕ is prescribed as boundary ϕ is equal to ϕ_1 here and ϕ is equal to ϕ_2 here. So the boundary condition on this way in this domain is ABCD. In the particular case, the boundary conditions on AD is ϕ equal to ϕ_1 and similarly, on BC we can prescribe ϕ is equal to ϕ_2 . Now, the given equations are known and the boundary conditions are known. On boundary AB, for example, if we know the variation or the flux, $\frac{\partial \phi}{\partial n}$ which is equal to 0 on the boundary AB and CD, here $\frac{\partial \phi}{\partial n}$ is equal to 0. That means no flow boundary condition on AB and CD. So there cannot be flux and cannot cross that means this side AB and CD are imperil in nature.

(Refer Slide Time: 25:35)



These boundary conditions are called nature boundary conditions that mean, what exists in nature. So this flow cannot cross this can imperil the boundary this side AB and on this

side CD, so these conditions are called nature conditions or [25:48] and the boundary conditions which we prescribed on AD and BC are called divisional boundary conditions or direct boundary conditions.

So as far as this potential flow problem is concerned the mathematical analysis or mathematical statement is over since we have only described the domain, we have already described the boundary condition and we have already described the given equations. Now, with respect to this, we can solve this problem either we can get analytical solution depending upon the problem or we can solve the problem numerically so depending upon the case.

This is about the mathematical analysis and as I mentioned in the experimental investigation we have to replicate the real problem in the laboratory and then with respect to certain scale we will be running the experiment what is happening in the field that will be done in the laboratory. Then we will be taking some readings sometimes it can be say either the velocity measurements are the depth variations or pressure measurements or this measurements will be done experimentally in the laboratory. Then to have some specific relationships we can give some dimensional analysis as mentioned in this slide here. (Refer Slide Time 27:10). Dimensional analysis is also very important in experimental investigation as well as mathematical investigations. So, dimensions with respect to the bucking phi theorem or various theorems, we will discuss about this dimensional analysis later.

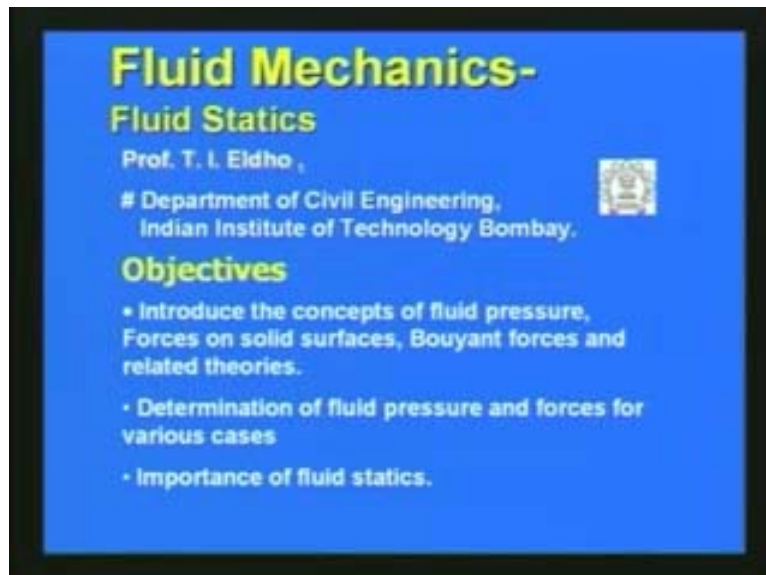
In summary, in this introductory lecture on fluid mechanics, we have discussed the various aspects of fluid flow and the various fluid properties. We have discussed the fundamental principles which will be used in the fluid flow analysis like the mass consideration of momentum, consideration of energy and then we also discussed the various fluid visualization techniques like using dyes, using smoke or other metrologies. We have discussed various metrologies of representing of a fluid flow like a stream lines, potential line, streak lines path lines will have discussed and then we have discussed based up on the various fluid properties we have classified the flows with respect to space, one D flow, two D flow and three D flow or with respect to the viscosity, that is,

viscous flow or non-viscous flow and then finally, we have discussed about how we can approach a fluid flow problem or solution to a fluid flow problem.

As I mentioned, we can approach the fluid flow problem theoretically or experimentally and then in theoretical investigation we can derive a mathematical model by describing the domain, then given equation in boundary condition. So this is about the introductory lecture on this video course on fluid mechanics

In this second chapter on fluid mechanics video course, we will be discussing mainly on fluid statics.

(Refer Slide Time: 29:19)



The main objectives in this section is to introduce the concepts of fluid pressure, forces on solid surfaces, buoyant forces and related theories. Secondly, we shall see the determination of fluid pressure and forces for various cases. Then we will emphasis in the importance of fluid statics as far as fluid mechanics is concerned.

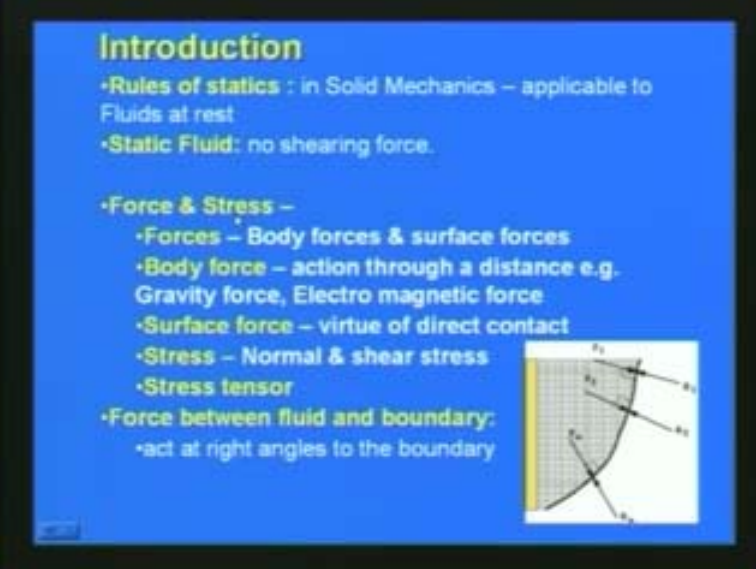
(Refer Slide Time: 30:30)



Now, as I mentioned earlier, static fluids means fluid is not moving that means fluid is in rest. So, you can see there is some water in a small basin. Here fluid is now at rest; there is no movement as far as the fluid is concerned; it is as far as the boundary is concerned the fluid is at rest so that we can say the fluid is in static conditions and the various theories on statics will be applicable as far as a static fluid is concerned.

In solid mechanics, most of the problems we will be analyzing are in that condition. So, most of the theories in solid mechanics are applicable for fluids at rest or static fluid. In static fluids you can see that since fluid is not moving it is at rest here, so that there is no shearing force. Whenever the fluid is trying to move only between the layers there will be shearing force but as far as fluid is at rest or static fluid there is no shearing force. We need not have to worry about the shearing force and now before going to more details of fluid statics we will just review some aspects of forces.

(Refer Slide Time: 31:26)

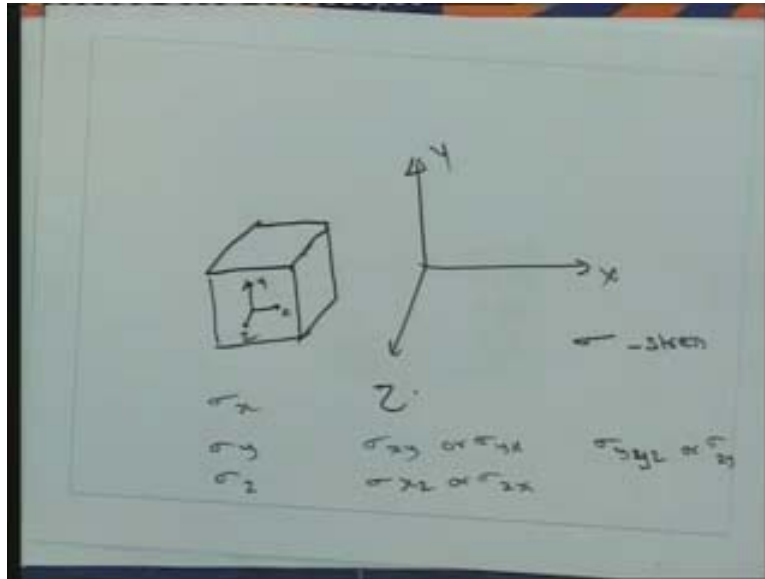


Introduction

- **Rules of statics** : in Solid Mechanics – applicable to Fluids at rest
- **Static Fluid**: no shearing force.
- **Force & Stress** –
 - **Forces** – Body forces & surface forces
 - **Body force** – action through a distance e.g. Gravity force, Electro magnetic force
 - **Surface force** – virtue of direct contact
 - **Stress** – Normal & shear stress
 - **Stress tensor**
- **Force between fluid and boundary**:
 - act at right angles to the boundary

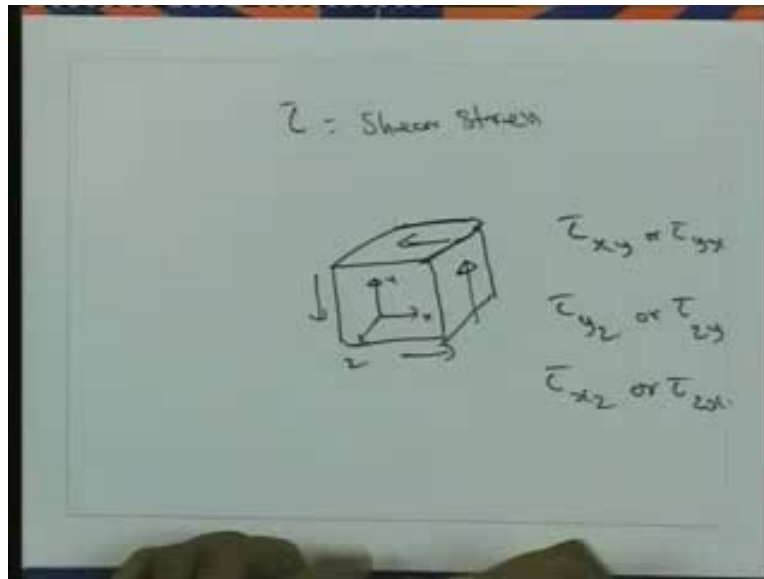
We can classify the forces into body forces and surface forces. A body force acts through a distance, for example, when there is a gravitational force the force of gravity is not directly acting, but it is acting from a distance so the action is through the distance and also if you consider the electro magnetic force most of the time it will be acting at a distance. This kind of forces like gravity force and electromagnetic force acting on fluids and that kind of study which we are dealing will be related to the body force. The second kind of force is called the surface force; it is due to the virtue of direct or in between the molecules. What is happening as far as the fluid is concerned is if you apply a force here on the fluids, then between the virtue and direct contact between the fluid molecules only there is a force. That force is called the surface force and as the stress is concerned we can also classify, as we have discussed earlier, stresses as normal stresses and shear stresses. Normal stress means when we are considering direct normal force on the body that kind of stress is called in a normal stress and if a shearing force is applied that kind of stress is called shear stress. We will be dealing with normal stresses and shear stresses in fluid mechanics and also we will be dealing with xyz directions.

(Refer Slide Time: 32:21)



Here, this is x this is y this is z. So the normal stress can be in x direction, y direction or z direction or with respect to x and y, with respect to x and z or with respect to y and z. Like that the stresses will be varying, for example, if sigma is the stress in normal stress then if we consider a body like this in 3 dimension then if you consider x, y and z the normal stress we would be considering if it is acting from x direction, the normal stress can be sigma x and it is acting from the y direction it can be sigma y and from the z direction it can be sigma z. Then with respect to x y plane if it is acting, it can be sigma xy or sigma yx or if it is with respect to same x and z axis, then it can be sigma xz or sigma zx; then if it is with respect to y and z it can be sigma yz or sigma zy. Like that we can have stress tensor. So, with respect to xyz direction we can classify the stresses in x direction, y direction and z direction or xz xy or yz direction.

(Refer Slide Time: 35:22)

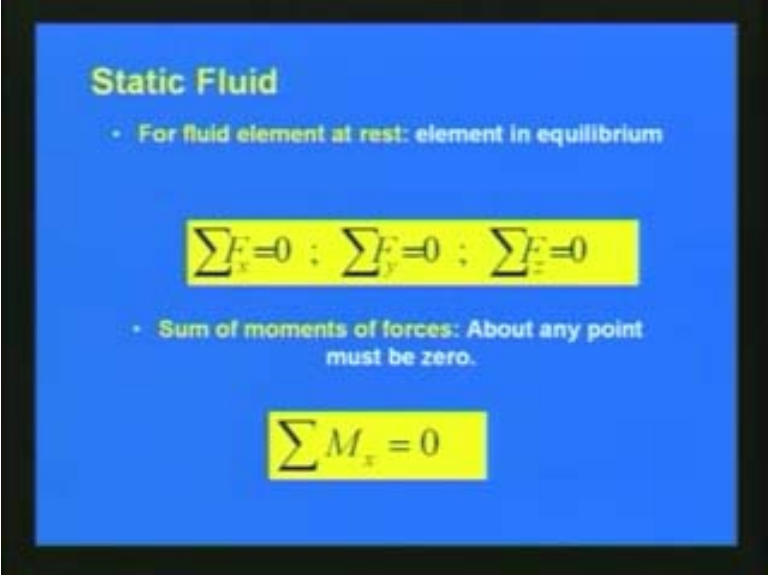


Stress tensor is very much used in many of the fluid analysis. In a very similar way, the shear stress is concerned for example tau is the shear stress; if it is depending upon the problem if you consider again a cube of three D then the shear stress also will be acting if a shear force is applied in the other direction and the shear force is applied either so this other face also. According to this we can have the shear stress in xy plane or tau yx then tau yz or tau zy or tau xz or tau zx.

According to the shear force acting it will be varying with respect to xyz. The stress tensor for normal stresses and shear stresses will be utilizing the shears stress tensor with respect to xyz direction and then as far as force is concerned, force will be especially if static fluid is concerned force will be acting upon externally. So, the force between fluid and boundaries for example if you consider it acting always at right angle boundary, so here in this slide you can see there is a boundary here and then say the force is acting. With respect to the boundary generally, the force will be acting at right angle so you can see that in the fluids that is the fluid is at rest. When we are applying a force we will be considering this acting at right angles and all the fluid static analysis will be accordingly. So as I mentioned, most of the theories which have been developed in the cases fluid mechanics are also applicable in the case of static fluid or fluid at rest.

Fluid at rest, we can consider each element for example, this basin water is at rest so when we consider the fluid statics or static fluid here. If we consider each element then the fluid element should be in equilibrium; there is no moment and the fluid is at rest. So, if there are n forces acting then finally each fluid element should be in equilibrium.

(Refer Slide Time: 38:05)



Static Fluid

- For fluid element at rest: element in equilibrium

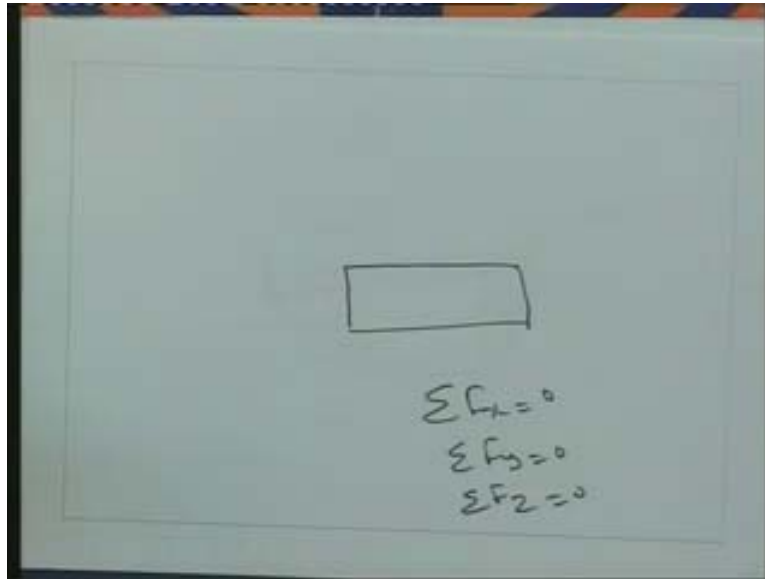
$$\sum F_x = 0 ; \sum F_y = 0 ; \sum F_z = 0$$

- Sum of moments of forces: About any point must be zero.

$$\sum M_x = 0$$

If you consider any solid with respect to solid face, if the solid is at rest and in equilibrium then say we can consider with respect to, for example, say this a solid and if it is at rest and if some forces are acting, we know that the body is in equilibrium then in solid mechanics we use the equilibrium condition. For example, if the forces in x directions $\sum F_x$ should be equal to 0; this is one of the conditions which will be generally used in solid mechanics and then $\sum F_y$ is the forces in y direction should be 0 and then $\sum F_z$ should be 0.

(Refer Slide Time: 39:07)



So, these three conditions as far as forces are concerned, considering the solid equilibrium and then we consider each element in equilibrium, so that we can say all the sum of the forces in x directions is equal to 0; $\sum F_x$ is equal to 0; $\sum F_y$ is equal to 0 and $\sum F_z$ is equal to 0.

So these three equations as far as the all the brace, some of the forces $\sum F_x$, $\sum F_y$ $\sum F_z$ are used in the solid mechanics. These equations, the solid at rest, are applicable for static fluid or fluid statics. We can use these equations $\sum F_x$ is equal to 0, $\sum F_y$ is equal to 0 and $\sum F_z$ is equal to 0. Also in solid mechanics, we know that for a solid at rest but in equilibrium condition, when there is a force acting then the sum of the moments of force at any point must be 0 as far as the particular solid is concerned.

(Refer Slide Time: 40: 28)

Static Fluid

- For fluid element at rest: element in equilibrium

$$\sum F_x = 0 ; \sum F_y = 0 ; \sum F_z = 0$$

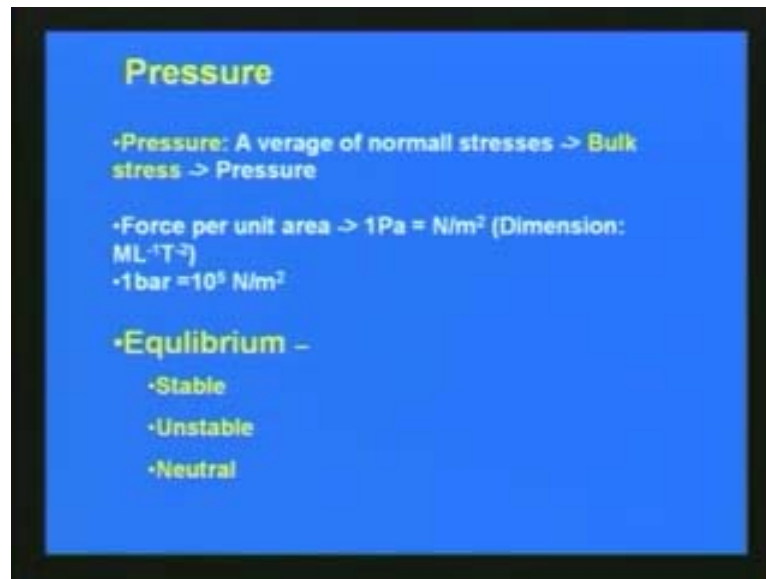
- Sum of moments of forces: About any point must be zero.

$$\sum M_x = 0$$

This equation is also very much used in a fluid mechanics for fluid at rest. If we consider that the sum of the moments, if you apply any force and then its moment with respect to that force acting upon the fluid at rest and then with respect to about any particular point $\sum M_x$ that means the sum of the moments of force should be equal to 0.

These fundamental equations or $\sum F_x$ is equal to 0 and $\sum F_y$ is equal to 0 $\sum F_z$ is equal to 0, $\sum M_x$ or the sum of the moment of forces about any point must be 0. These four equations are very much valid in the case of static fluid also. These equations generally, we use in solid mechanics and directly these equations are applicable in the case of static fluid. Now, another important aspect as far as fluid statics is concerned is the pressure.

(Refer Slide Time: 41:02)

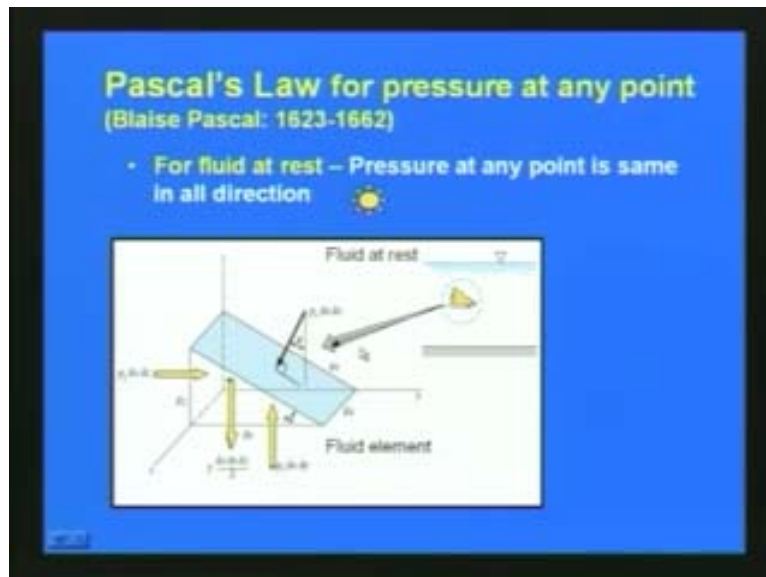


Pressure, as I mentioned earlier, pressure is the average of normal stresses. So if there is force acting, force per unit area is termed as pressure and generally we will be considering the average of normal stresses as bulk stresses and that term is defined as pressure. It is the force per unit area and the general used unit in the system international is one Pascal which is equal to Newton per meter square and its dimension is M L to the power minus 1 and T to the power minus 2 as shown in this slide and say, for example, another unit which is for bar used is 1 bar is equal to 10 to the power 5 Newton per meter square. This pressure is also very important as far as fluids are concerned. We will be discussing further about the pressure shortly for fluid in static state and another important aspect for fluid statics is equilibrium. Equilibrium condition in solid mechanics which we generally use is stable equilibrium for example, this cylinder here is said to be stable if it is place like this. (Refer Slide Time: 42:20). So that it is stable; there is no change; so it is when if we apply more force it will come back to its normal proportion with respect to small forces. That is called stable equilibrium but if you consider this cylinder if you put it in the other way like this (Refer Slide Time: 42:35) then you can see with a small force it is quit unstable, so this solid in this position it is said to be in unstable equilibrium; then third equilibrium condition used is called neutral equilibrium. So when it is lying like this on this flow it is a neutral equilibrium; whatever force is applied it comes back to the same condition. So it is said to be neutral equilibrium. Equilibrium conditions namely

stable equilibrium, unstable equilibrium and neutral equilibrium are very much solid mechanics. These equilibrium conditions are also very much valid in the case of statics fluid which will be discussing later.

Now, we will be discussing about the fluid pressure. One of the important theories is developed in seventeenth century called the Pascal law for pressure at any point. This law has been developed by balking Pascal by various experiments. Here, if you consider the fluid at rest and now we are considering this fluid element and then with respect to the fluid elements, Pascal offset with respect to various experiment that the pressure at any point is same in all directions.

(Refer Slide Time: 44:05)



We consider a fluid element like this; the pressure at any point is same in all directions. For example, if you consider the water in this basin, if you consider the pressure at any point in this; since the fluid is statics there is no moment; if you consider the pressure the pressure will be the same old directions. This is the law derived by Pascal and it is called a Pascal's law. Here you can see a fluid element in this slide. If you consider fluid element with respect to this phase, the pressure will be p_y in this direction p_y into Δx into Δz ; in this direction this is the force and the other direction it is Δx into Δy and this direction it is x direction which we are dealing is p_x into Δx into Δz .

(Refer Slide Time: 45:05)

Pascal's Law ☀

- When a certain pressure is applied at any point in a fluid at rest, the pressure is equally transmitted in all directions and to every other point in the fluid.

$$\sum F_y = p_y \delta x \delta z - p_s \delta x \delta s \sin \theta = \rho \frac{\delta x \delta y \delta z}{2} a_y$$

$$\sum F_z = p_z \delta x \delta y - p_s \delta x \delta s \cos \theta - \gamma \frac{\delta x \delta y \delta z}{2} = \rho \frac{\delta x \delta y \delta z}{2} a_z$$

$$p_y - p_s = \rho a_y \frac{\delta y}{2} \quad \delta y = \delta s \cos \theta \quad \delta z = \delta s \sin \theta$$

$$p_z - p_s = (\rho a_z + \gamma) \frac{\delta z}{2}$$

$p_y = p_z = p_s$
(at any point $\delta x, \delta y, \delta z \rightarrow 0$)

So, if you consider fluid element like this, then according to the Pascal law when a certain pressure is applied at any point in a fluid at rest the pressure is equally transmitted in all direction and to every other points in the fluid. If you consider this with respect to the fluid element which we consider $\sum f_y$ can be written as $\sum f_y$ is equal to p_y into δx into δz minus p_s into δx into $\delta s \sin \theta$, that is, equal to ρ into δx into δy into δz by 2 into a_y , where ρ is the density of the fluid and a_y is the acceleration of the fluid element which we are considering. Here, we are using the Newton's second law: the total force is equal to mass into acceleration.

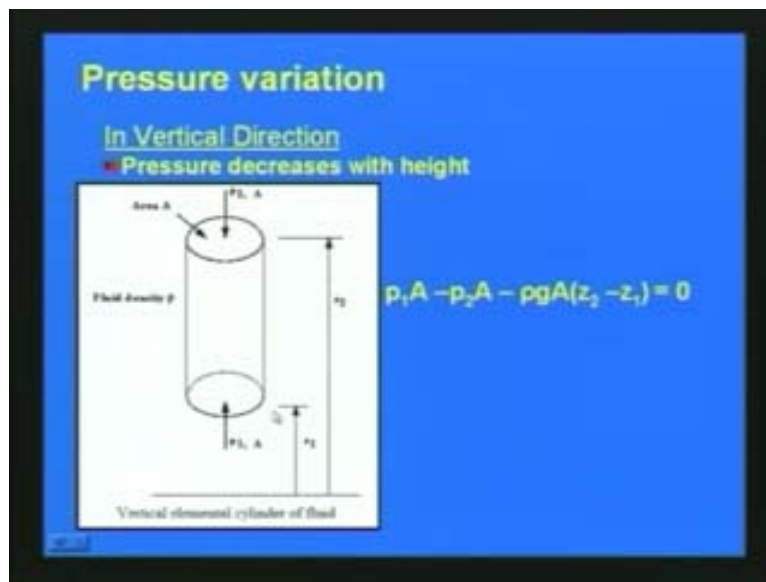
Similarly, in z direction, $\sum f_z$ is equal to p_z into δx into δy minus p_s into δx into $\delta s \cos \theta$ minus γ into δx into δy into δz by 2 since the weight of the element is considering this term which is equal to the mass into the acceleration in z direction. So, here mass is ρ into δx into δy into δz by 2 and a_z is the acceleration.

Now, if you simplify the force, $\sum f_y$ the total occupied force in y direction which is equated to the mass into acceleration and $\sum f_z$ which is equated to mass into acceleration in dz direction. So if you simplify this you will get p_y minus p_s is equal to ρ into a_y into δy by 2. Since, δy can be written as $\delta x \cos \theta$ and δz can

be written as $\delta x \sin \theta$, the second equation can be written as p_z minus p_s is equal to $\rho \delta z$ plus $\gamma \delta z$ by 2. Now, with respect to the Pascal law if you consider the fluid at a particular point then you can see that δx , δy and δz will be tending to 0. So that we can see p_y is equal to p_z is equal to p_s as shown in this slide. That means the pressure at any particular point is same in all directions. So, here, if you consider the pressure at particular point at all directions it will be same according to the Pascal's law

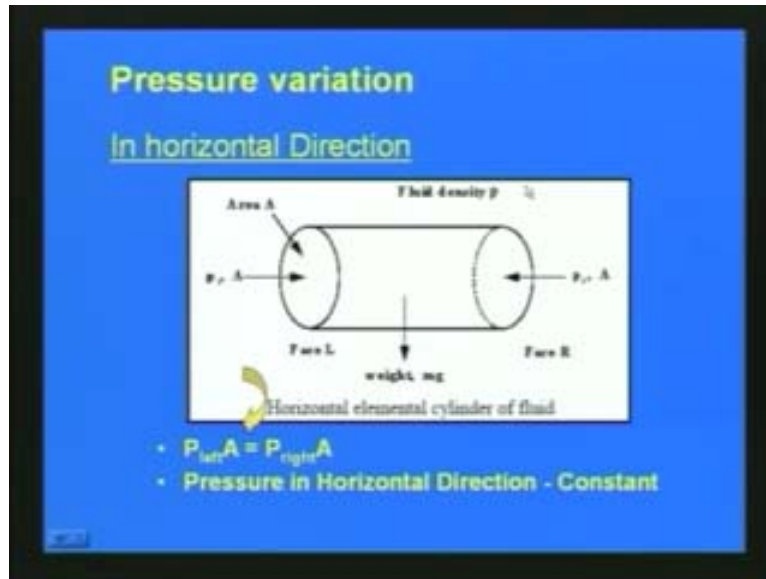
After discussing the Pascal's law, we will be discussing the pressure variations at various vertical direction are shown in a direction and then we trying to derive general equations which can be applied in all the cases.

(Refer Slide Time: 48:15)



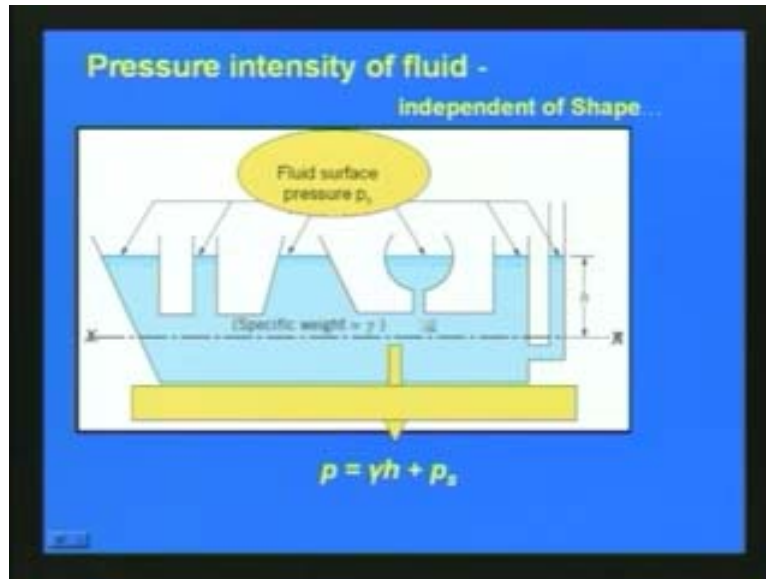
So, if we consider the vertical element cylinder of fluid like this, on this phase the pressure is even and the area is a, here the pressure is p_2 and the area of cross section is A and the fluid density is ρ and from the datum this distance is z_1 to this phase and z_2 is to the second phase; you can see that the pressure decreases with height in vertical direction. We can write the equation p_1 into A minus p_2 into A minus $\rho g A$ into z_2 minus z_1 that is equal to 0. So this gives the pressure variation in the vertical direction.

(Refer Slide Time: 48:57)



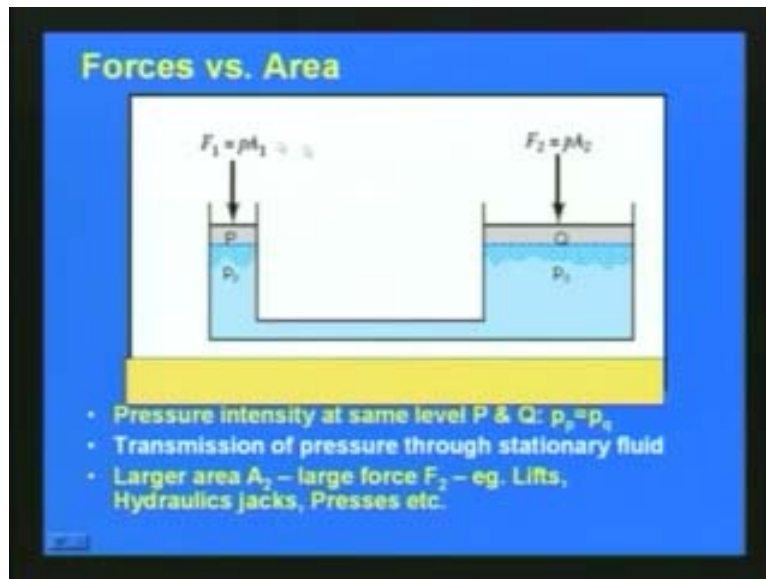
Similarly, if you consider the pressure variation in horizontal direction here a horizontal fluid element, say, cylinder of fluid is considered here and the area of cross section is same A ; the fluid density is ρ and on this left-hand side face the pressure is p_1 and the p_{left} multiplied by A is equal to $p_{\text{right}} A$. Since it will be in horizontal direction on this position in the total pressure the total force is equal to this position, so that pressure in horizontal direction is constant; accordingly, the pressure variation in horizontal direction is constant

(Refer Slide Time: 49:49)



So, with respect to this, if we consider the pressure intensity of fluid, say, for various shapes you can see that with respect to in this slide the shapes are varying from the rectangular, then the circular, then say conical differentiates. Now, if you consider an excess line this is at a height x , so here the fluid pressure will be same as the surface of the water; it is p_s and then if you consider this excess line for a fluid with specific gamma you can see that the fluid pressure density p is equal to gamma into h plus the fluid surface pressure p_s . So, this will be same at all the points in this line excess. So, this is the pressure intensity of fluid at this particular section excess.

(Refer Slide Time: 50:37)



So, these principles we can utilize in many areas. Now, if you consider a piston type mechanism here, force verses area. If we consider a small piston here and for small cylinder here and larger cylinder which is interconnected through in the mechanism like this, the pressure density is same at level p and q.

So, the pressure intensity is at same the level p and q. P_p is equal to P_q . This is the small cylinder and this is the large cylinder of area A_2 . Here, area of small cylinder area force section is A_1 and pressure intensity is same p at this point and at this point on lead of the cylinder and then you can see that since the area of force section A_1 is smaller here and here the area of force section is larger, if you get the total force f_1 will be equal to p into A_1 on this phase here and here it will be f_2 is equal to p into A_2 . Since the pressure intensity is same we can transmit large quantity force. Here f_2 is equal to p into A_2 is much larger compare to f_1 . So, the transmission of pressure through stationary fluid is possible with respect to this mechanism since the pressure intensity is same and area of force section A_2 is much larger compare to A_1 .

So, for larger area we apply larger force f_2 . This principle is very much applicable in many of the mechanisms like hydraulics utilizing these principles. We can transmit the larger pressure by applying more force here and we can obtain larger force here. This

principle is used in lifts, hydraulic jacks, pressure extra. What we are doing here is we are applying a small force here f_1 that will give correspondingly large force; input will be small and larger force output is produced here which is used in lifts hydraulics jacks pressures and many other hydraulics equipments. So this principle is very much very important in fluid statics which is based upon the theory that the transmission of pressure through stationary fluid.