

**Fluid Mechanics**  
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**Lecture - 19**  
**Laminar and Turbulent flows**

Welcome back to the video course on fluid mechanics. We were discussing about the laminar and turbulent flow chapter. In the laminar flow we were discussing about how to develop various fundamental equations; then we were discussing in the over flow between two parallel plates; then we were discussing about the flow when one of the parallel plate is moving, that means, coquette flow; then we were discussing about the pipe flow. So with respect to the pipe flow we have seen the basic equations Hagen-Poiseuille flow equations.

Now, what we are doing here is we are trying to find out with respect to the basic principles say Newton's second law. We are trying to get the basic equations for the velocity distribution; then pressure distribution; then the discharge through the pipe; then shear distribution and various other parameters. We have seen that here the basic pipe flow is concerned we can just describe the flow with respect to the laminar flow in pipes.

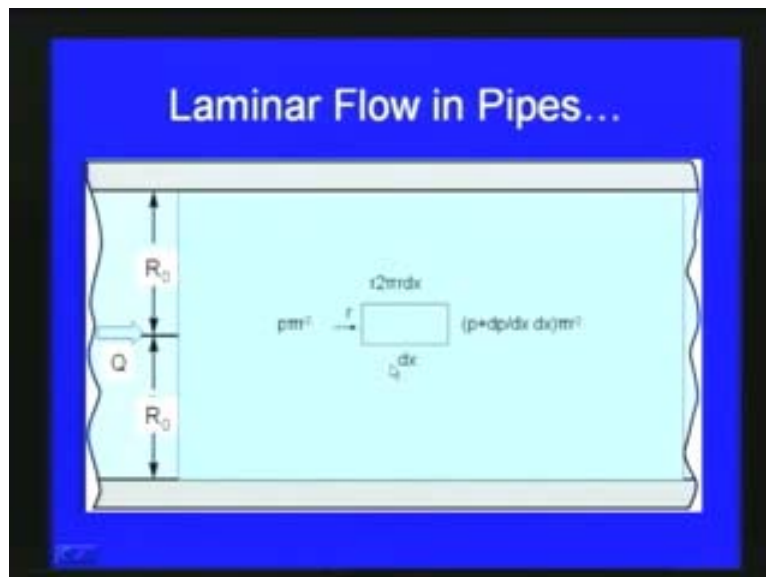
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### Laminar Flow in Pipes...

- Integrating:  $u = \frac{1}{4\mu} \frac{dp}{dx} r^2 + C_1$
- At  $r = R_0$ ,  $u = 0$ ;
- Therefore,
- Then, velocity:  $u = -\frac{1}{4\mu} \left( \frac{dp}{dx} (R_0^2 - r^2) \right)$
- For max. velocity  $\frac{du}{dr} = 0$        $\frac{1}{4\mu} \frac{dp}{dx} (-2r) = 0$
- Giving  $r = 0$ ,

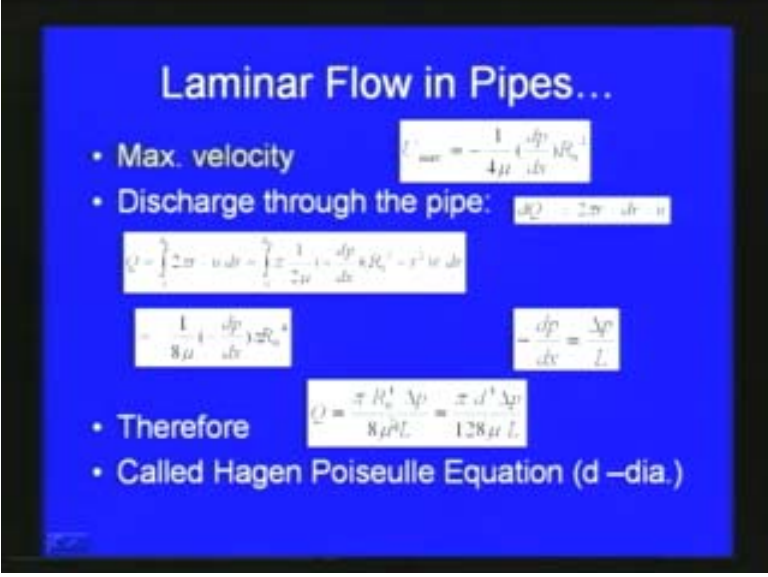
We are now considering steady laminar flow through circular tube and we have derived the basic equations for the pipe flow from the fundamental principles; then also we have seen the expression for the maximum velocity and the discharge. Before going to the further topics on pipe flow, we will continue with what we were discussing. In the last lectures, we have derived the equations for the velocity variations as  $u$  is equal to minus 1 by 4 mu dp by dx into  $R_0$  square minus  $r$  square as described in this slide.

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Here, you can see the radius of the pipe is  $R_0$  and  $Q$  is the discharge and then we consider a fluid element like this. Using this concept only we have derived the basic equation for the velocity in a pipe flow from the fundamental principles so  $u$  is equal to minus 1 by 4  $\mu$  into  $dp$  by  $dx$  into  $R_0$  square minus  $r$  square, where  $R_0$  is the radius of the pipe,  $p$  is the pressure,  $\mu$  is the coefficient of dynamic viscosity.

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**Laminar Flow in Pipes...**

- Max. velocity  $u_{max} = -\frac{1}{4\mu} \left( \frac{dp}{dx} \right) R_0^2$
- Discharge through the pipe:  $dQ = 2\pi r u dr$

$$Q = \int_0^{R_0} 2\pi r u dr = \int_0^{R_0} 2\pi r \left( -\frac{1}{4\mu} \left( \frac{dp}{dx} \right) (R_0^2 - r^2) \right) dr$$

$$= -\frac{1}{8\mu} \left( \frac{dp}{dx} \right) \pi R_0^4$$

$$-\frac{dp}{dx} = \frac{\Delta p}{L}$$

- Therefore  $Q = \frac{\pi R_0^4 \Delta p}{8\mu L} = \frac{\pi d^4 \Delta p}{128\mu L}$
- Called Hagen Poiseuille Equation (d – dia.)

For maximum velocity, we have seen that the equation is  $u_{max}$  is equal to minus 1 by 4  $\mu$   $dp$  by  $dx$  into  $R_0$  square and then we have derived the equation. For discharge we have integrated from 0 to  $R_0$ , the radius and then  $2\pi r \mu dr$  that gives the discharge as minus  $u$  is equal to 1 by 8  $\mu$  into minus  $dp$  by  $dx$  into  $\pi R_0$  to the power 4 and if you express minus  $dp$  by  $dx$  is  $\Delta p$  by  $L$  then we got the discharges  $Q$  is equal to  $\pi d$  to the power four  $\Delta p$  by 128  $\mu$  into  $L$ , where  $d$  is the diameter of the pipe,  $\Delta p$  is the pressure difference,  $L$  is the length of the pipe and  $\mu$  is the coefficient of dynamic viscosity. This equation is called Hagen-poiseuille equation for pipe flow.

If we want to find out the average velocity, then the discharge is we can just find the area flow section of the pipe and then discharge divided by area flow section gives the average velocity through the pipe; then we already got a  $V$  is equal to  $\Delta p$  into  $R_0$  square by 8  $\mu L$  by taking  $Q$  divided by  $\pi R_0$  square; then if we want to find out to the pressure at

various locations of the pipe then  $\Delta p$  is equal to  $8 \mu V L$  by  $R_0$  square that is equal to  $32 \mu V L$  by  $d$  square.

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### Laminar Flow in Pipes...

- Shear stress at the pipe wall  $\tau_w = \left( \frac{\Delta p}{L} \right) \frac{R_0}{2}$
- Avg. velocity through pipe: 
$$V = \frac{Q}{\pi R_0^2} = \frac{\Delta p \times R_0^2}{8 \mu L}$$
 
$$\Delta p = \frac{8 \mu L V}{R_0^2} = \frac{32 \mu L V}{d^2}$$
- Energy loss per unit weight of fluid in laminar flow through a circular tube 
$$\frac{\Delta p}{\gamma} = \frac{32 \mu L V}{\rho g d^2}$$

Here, the energy loss per unit weight of fluid in laminar flow through a circular tube or pipe we can write as  $\Delta p$  by  $\gamma$ , that is equal to  $32 \mu V L$  by  $\rho g d$  square. So we are generally using Darcy-Weisbach equation in head loss due to friction and other factors in the pipe flow. If we compare with the component, use Darcy-Weisbach equation which we will be discussing when we discuss about the pipe flow, so when we compare this equation which we derived here,  $\Delta p$  by  $\gamma$  and the previous slide  $\Delta p$  by  $\gamma$  is equal to  $32 \mu V L$  by  $\rho g d$  square. If we compare this with the Darcy-Weisbach equation then we can write this  $\Delta p$  by  $\gamma$  is equal to  $64 \rho v d$  by  $\mu$  into  $L$  by  $d$  into  $V$  square by  $2g$ .

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### Laminar Flow in Pipes...

- Comparing this eqn. with Darcy – Weisbach eqn.:
 
$$\frac{\Delta p}{L} = \frac{64}{\rho V d} \mu \frac{V}{d} = \frac{64 \mu V}{\rho d^2}$$
- Therefore,
 
$$f = \frac{64}{\rho V d} = \frac{64}{Re}$$

(Re – Reynold's number)

Darcy-Weisbach equation is given as  $f L V^2 / 2 g d$ , where  $f$  is the friction coefficient for the pipe,  $L$  is the length of the pipe,  $d$  is the diameter,  $V$  is the average velocity, and  $g$  is the acceleration due to the gravity. Therefore we can write the friction coefficient  $f$  is equal to 64 divided by  $\rho V d$  by  $\mu$  that is equal to 64 by  $Re$ , where  $Re$  is the Reynold's number.

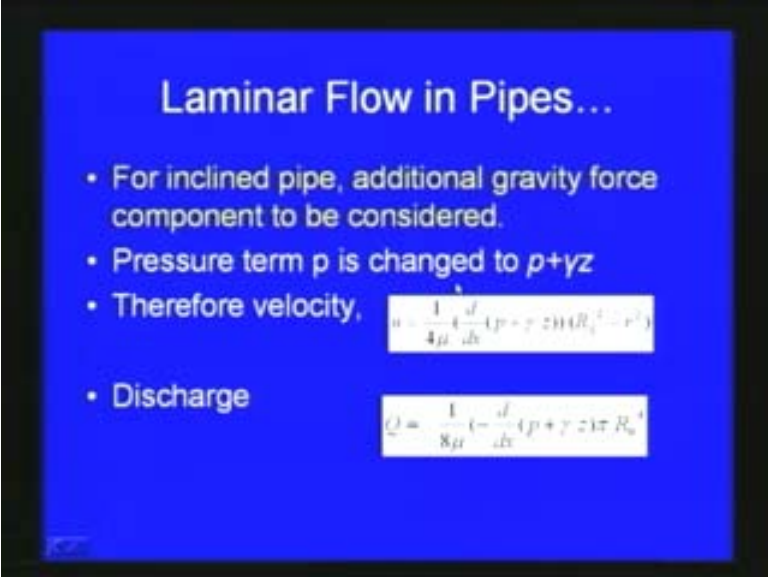
In comparison with the Darcy-Weisbach equation, the Hagen-Poiseuille equation which we derived here we can write with respect to this friction factor, it can be expressed as 64 by Reynold's number for the case of pipe flow from the case of laminar flow which we discussed. So, sometime the pipe flow **can be...** what we discussed here is when the pipe is horizontal in position; sometimes the pipe can be inclined. When we consider the inclined pipe then we have to consider this angle  $\theta$  and then we have to consider the weight of the pipe or the gravity effect on the pipe.

When the pipe is horizontal then what the equation which we derived earlier for inclined pipe, the pipe we can see here. If you consider this as a pipe then this is the horizontal position; then we do not have to consider the gravitational effect or with respect to the weight of the fluid but when the inclination comes we have to consider the gravitational

effect here with respect to the weight of the fluid. So for inclined pipe, additional gravity force component to be considered here so pressure term  $p$  is changed to  $p$  plus  $\gamma z$ .

Therefore we can write the velocity  $u$  is equal to... Now, earlier expression for  $u$  we got this  $u$  is equal to  $\frac{1}{4\mu} \frac{dp}{dx} (R_0^2 - r^2)$ , there is minus sign will be there depending upon  $\frac{dp}{dx}$  is negative or positive. In this case, when we consider the inclined pipe, we have to consider the additional gravity force component as we have seen that means the horizontal pipe and when the pipe is inclined we have to consider the additional gravity force and this we can consider by changing the pressure term  $p$  we will change  $p$  into  $p$  plus  $\gamma Z$  that means the additional gravity forces will be considered for inclined pipe,  $p$  will be equal to  $p$  plus  $\gamma z$ . Therefore the velocity can be written as  $\frac{1}{4\mu} \frac{d}{dx} (p + \gamma z) (R_0^2 - r^2)$ , where  $R_0$  is the radius of the pipe and small  $r$  is the distance from the center of the pipe.

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**Laminar Flow in Pipes...**

- For inclined pipe, additional gravity force component to be considered.
- Pressure term  $p$  is changed to  $p + \gamma z$
- Therefore velocity, 
$$u = \frac{1}{4\mu} \frac{d}{dx} (p + \gamma z) (R_0^2 - r^2)$$
- Discharge 
$$Q = \frac{1}{8\mu} \left( -\frac{d}{dx} (p + \gamma z) \right) \pi R_0^4$$

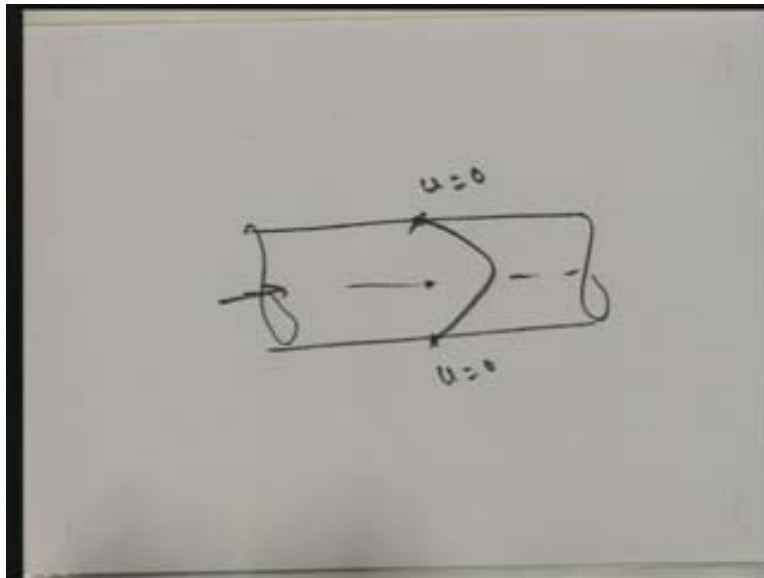
So this way for inclined pipe we can modify the earlier equation for the velocity. This equation we can find out the velocity variation at various locations of the pipe. In similar way the discharge equation and all other equation we can modify in the case of inclined pipe as we have seen the  $\frac{dp}{dx}$  term, the pressure term we will be changing into  $p$  plus

$\gamma z$ . So, the discharge term here we can just rewrite our earlier equation like this  $q$  is equal to  $\frac{1}{8\mu} \frac{dp}{dx} \left( \frac{R_0^4}{4} - \gamma z R_0^4 \right)$ .

Here, we can see our earlier equation is here. Now for  $q$ , this is our  $q$  equation; this is modified as say  $Q$  is equal to  $\frac{1}{8\mu} \frac{dp}{dx} \left( \frac{R_0^4}{4} - \gamma z R_0^4 \right)$  to the power 4, where  $R_0$  is the radius of the pipe. Here, this negative or positive for the pressure comes depending upon the direction of the flow which we consider, so that will be taken care with respect to the flow direction, like this we can calculate a various parameters for the inclined pipe.

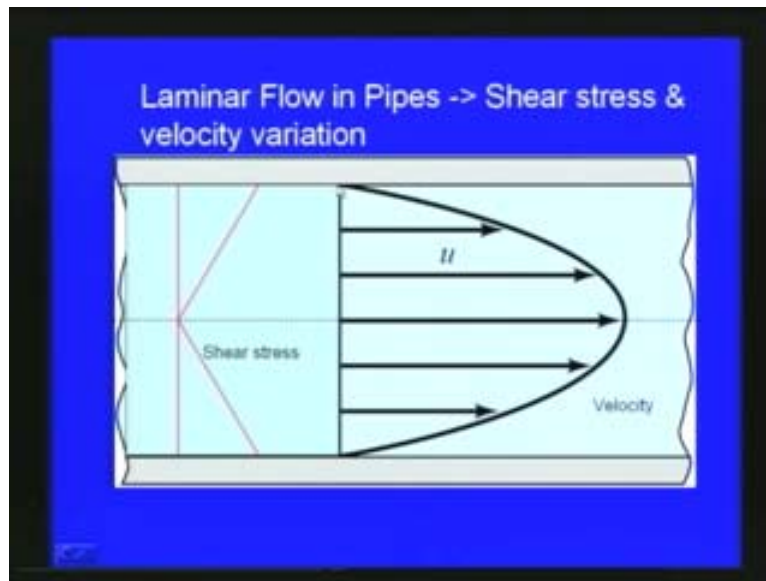
Finally, as for as pipe flow is concerned, we have seen that the velocity is varying. If we consider for pipe flow for example, if we consider the pipe like this here, then you can see at the center we always expect the maximum velocity and then the velocity is varying with respect to parabolic variation.

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So, due to the no slip condition on the pipe boundaries the velocity will be 0 on the pipe wall; then we can see that the pipe velocity will be maximum at the center; then again say other side it will be 0. So with respect to this we can now bring the velocity variation for the laminar flow in pipes we can just here in this slide we can see that the velocity is varying, the velocity variation is here.

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Here it is 0 and then it is going to the maximum at the center line and then it is coming back to 0 the other side of the pipe. So this is the velocity variation in the pipe, maximum velocity is at the center line. Similar way we have already seen earlier the expression for the shear stress at the pipe wall, shear stress equation is here. If we find out the shear stress variation as pipe flow is concerned here you can see the shear stress is here, this will be 0 at the center line where the velocity is maximum and then it will be shear stress will be maximum on the pipe wall and it will be a linear variation.

For pipe case laminar flow in pipes, we can see that the velocity will be minimum on the pipe wall at the pipe wall and maximum at the central line and shear stress is concerned you can see that shear stress will be 0 on the central line of the pipe and it will be maximum on the pipe wall.

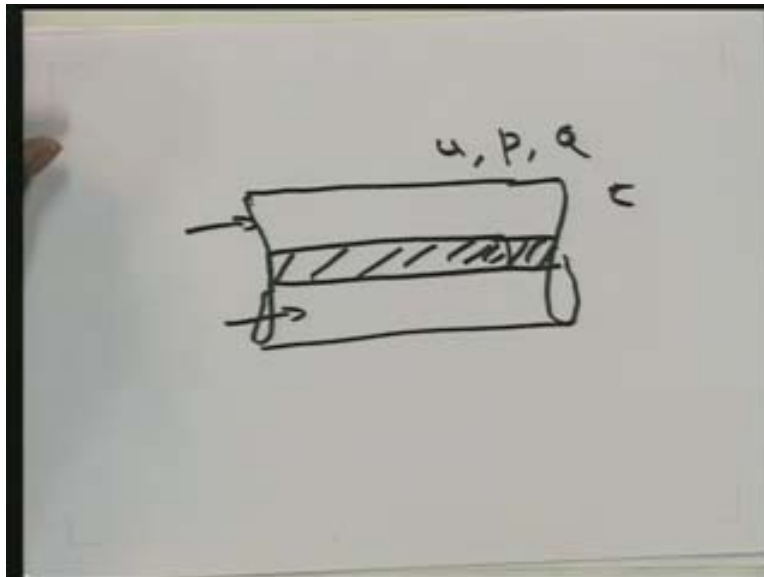
The variation we can find out from the earlier equation which we have derived in the case of velocity as well as the shear stress now for the pipe case for laminar flow we have found out the various parameters pressure variation, the shear stress variation, velocity variation and discharge variation.

Before closing this section on laminar flow, we will consider one more with special case where the case is called annulus. We can see especially in mechanical engineering and

many other engineering cases say this annulus is especially for many kinds of machines are used, so there will be one axial or the flow is between two sections like this.

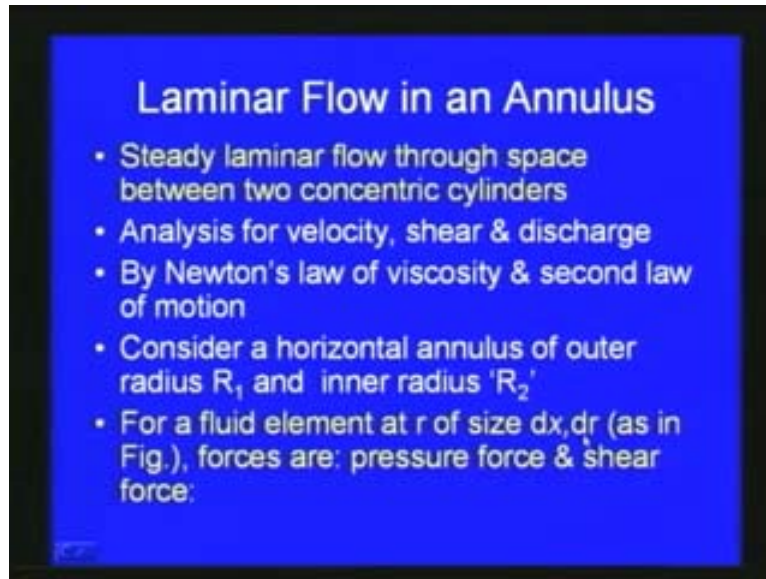
So here coming like this here you can see the flow is between say one central axial and then between the space of another pipe. We want to find out here the velocity variation, the pressure variation or the discharge or shear stress and all other parameter in the case of annulus the case is steady laminar flow through space between two concentric cylinders.

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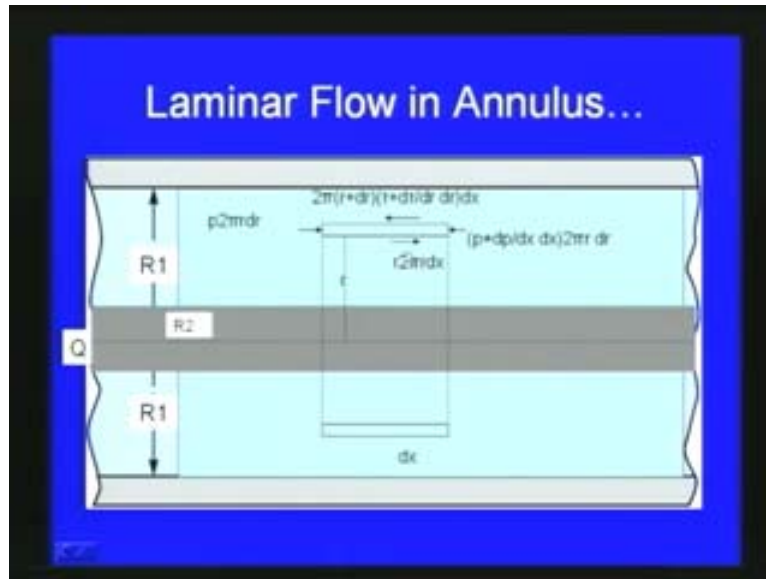
Here, we consider this is the first cylinder and the second cylinder is here so we want to find out the various parameters for this laminar flow in annulus case.

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The analysis is for velocity shear and discharge; here also we are using the fundamental principle that means the Newton's second law. Here we consider fluid element and then we consider say the forces acting on that and now since it is steady state case we consider we will be equating the forces acting on the fluid element to 0 and then we will be finding out the various parameters as we dealing the case of laminar flow in pipes which we discussed earlier. If we consider a horizontal annulus of outer radius  $R_1$  and inner radius  $R_2$  then here we will be deriving the equation for velocity and discharge and other parameters, the fluid element at  $r$  of size  $dx$  into  $dr$  here, we consider here this shows the figure.

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This is the inner cylinder and this is the outer cylinder and flow is between these two cylinder the flow is direction is like this, like this; then to derive an expression for the velocity variation and discharge the parameters here we consider fluid element like this at a distance  $r$  from the central line and the length of the fluid element is  $dx$  and then we can see that the size  $dx$  by  $dr$  is considered and the forces acting here are the pressure force and shear force.

We can see that since it is steady state condition here the forces acting, first one is the pressure force here  $p$  into  $2\pi r \cdot dr$  on this side; this side of the fluid element  $p$  plus  $dp$  by  $dx$  into  $dx$  into  $2\pi r$  into  $dr$  then shear is concerned here  $\tau$  into  $2\pi r \cdot dx$  on this side if we consider and if we here on this side it will be  $2\pi$  into  $r$  plus  $dr$  into  $\tau$  plus  $d\tau$  by  $dr$  into  $dr$  into  $dx$ .

This is the fluid element which we consider and forces acting on the pressure force on both sides of the fluid element and then the shear force So on this side  $\tau$  into  $2\pi r \cdot dr \cdot dx$  and then with respect to the increment on the other side it is  $2\pi$  into  $r$  plus  $dr$  into  $\tau$  plus  $d\tau$  by  $dr$  into  $dr$  into  $dx$ .

Now with respect to this we will be considering the Newton's second law to derive the equation. From the Newton's second law of motion in the direction of flow we can write

$v$  into  $2\pi r dr$  minus  $v$  plus  $dp$  by  $dx$  into  $dx$  into  $2\pi r dr$  minus  $2\pi$  into  $r$  plus  $dr$  into  $\tau$  plus  $d\tau$  by  $dr$  into  $dr$  into  $dx$  plus  $\tau$  into  $2\pi r$ ,  $dx$  is equal to 0; all the forces we equate to 0 and then if you divide by volume of the element here this  $2\pi r dr dx$  and if you drop the infinitesimal term and simplify.

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**Laminar Flow in Annulus...**

- From Newton's second law of motion in the direction of flow,

$$p \cdot 2\pi r \cdot dx - \left( p + \frac{dp}{dx} dr \right) 2\pi r \cdot dx - 2\pi (r + dr) \left( \tau + \frac{d\tau}{dr} dr \right) dx + \tau \cdot 2\pi r \cdot dx = 0$$

- Divide by volume of element,  $2\pi r \cdot dr \cdot dx$ , drop infinitesimal term and simplify,

$$\frac{dp}{dx} + \frac{d\tau}{dr} + \frac{\tau}{r} = 0$$

This basic equation for this laminar flow through annulus we can write  $dp$  by  $dx$  plus  $d\tau$  by  $dr$  plus  $\tau$  by  $r$  is equal to 0. So this after simplification of the application Newton's second law equating all the forces to 0 and then after simplification we get  $dp$  by  $dx$  plus  $d\tau$  by  $dr$  plus  $\tau$  by  $r$  is equal to 0. Here our aim is to get an expression for velocity  $p$  is depend on  $x$  and shear stress on  $r$  this equation the earlier equation we can write as  $dp$  by  $dx$  plus  $1$  by  $r$  into  $d\tau$   $r$  by  $dr$ . This equation we can write in this form  $dp$  by  $dx$  plus  $1$  by  $r$   $d\tau$   $r$  by  $dr$  is equal to 0 so this we can integrate with respect to  $r$  to get an expression so if we integrate this  $dp$  by  $dx$  integral  $r dr$  plus integral  $d\tau$  into  $r$  into  $dr$  is equal to  $A$ , where  $A$  is the constant. If we integrate this here we have put a constant  $A$  and now this we can write the integration we can write  $r$  square by 2 into  $dp$  by  $dx$  plus  $\tau$  into  $r$  is equal to  $A$ .

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### Laminar Flow in Annulus...

- Since  $p$  is dependent on  $x$  and shear stress on  $r$ , we can write:  $\frac{dp}{dx} + \frac{1}{r} \frac{d}{dr}(r\tau) = 0$
- Integrating w.r.t.  $r$ :  $\frac{dp}{dx} \int r dr + \int \frac{d}{dr}(r\tau) dr = \int \tau dr$  ( $\tau = \mu \frac{du}{dr}$ )

$$\frac{r^2}{2} \frac{dp}{dx} + r\tau = A$$

$$\tau = -\mu \frac{du}{dr}$$

Now, this constant A we have to find out from the various boundary conditions for this particular problem. Here, we substitute here  $\tau$  is equal to minus  $\mu$  into  $du$  by  $dr$  here this minus symbol is used since the shear stress is increasing the direction of the pipe valve that is  $\tau$  is equal to minus  $\mu$  by  $du$  by  $dr$  is used here. Therefore the laminar flow in the annulus we can finally write as  $dp$  by  $dx$  into  $r$  square by 2 minus  $\mu$  into  $du$  by  $dr$  into  $r$  is equal to A or  $dp$  by  $dx$  into  $r$  by 2 is equal to minus  $\mu$  into  $du$  by  $dr$  is equal to A by  $r$ . Now, again this expression says our aim is to find out the velocity variation, we got an expression with respect to the pressure gradient and velocity gradient, so final aim is to get an expression for velocity.

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### Laminar Flow in Annulus...

- Therefore:  $\frac{dp}{dx} \frac{r^2}{2} = \mu \frac{du}{dr} r = -rA$  or  $\frac{dp}{dx} \frac{r}{2} = \mu \frac{du}{dr} = -\frac{A}{r}$
- Integrating w.r.t.  $r$ :
- B.Cs  $\frac{dp}{dx} \frac{r^2}{4} = \mu A \log r + B$  ( $B = \text{constant}$ )
- At  $r = R_1$ ,  $u = 0$  & at  $r = R_2$ ,  $u = 0$  from which we get A & B.

We will integrate with respect to  $r$  once again that will give  $dp$  by  $dx$  into  $r$  square by 4 minus  $\mu$  into  $u$  is equal to  $A \log r$  natural log  $r$  plus  $B$ , where  $B$  is another constant. Now final equation  $dp$  by  $dx$  into  $r$  square by 4 minus  $\mu$  is equal to  $A \log$  into natural log  $r$  plus  $B$ , where  $A$  and  $B$  are constants. Using this  $A$  and  $B$  constants, we can just find from this boundary conditions here you can see that due to the no slip boundary condition here the pipe valve, here the inner cylinder valve and here also at the inner cylinder valve.

Here you can see that when the radius is  $R_2$  that means on this phase here the velocity is 0 and here again the velocity is 0. Similar way if you drop the velocity variation here at  $r$  is equal to  $R_2$ , the velocity is 0 at  $r$  is equal to  $R_1$  the velocity is again 0. The velocity variation is parabolic between the cylinders so these boundary conditions we can apply here to find out the constants  $A$  and  $B$  at  $r$  is equal to  $R_1$ ,  $u$  is equal to 0 and at  $r$  is equal to  $R_2$ ,  $u$  is equal to 0 from which we can find out the constants  $A$  and  $B$ .

If you substitute this for once you can substitute here and then you can find out  $A$  and  $B$  and then we can put back to get an expression for the velocity this mathematical steps can be done here and finally we can write the velocity expression as  $\mu$  is equal to minus 1 by 4  $\mu dp$  by  $dx$  into  $R_1$  square minus  $r$  square plus  $R_1$  square minus  $R_2$  square divided by natural log  $R_2$  by  $R_1$  into natural log  $R_1$  by  $R_2$ .

This  $r$  is the distance at which we will be finding the velocity and  $R_2$  is as shown in this figure here  $R_2$  is the cylinder inner cylinder,  $R_1$  is the outer radius and  $r$  is the distance at which we are finding out the velocity here so the final expression is  $u$  is equal to minus  $\frac{1}{4\mu} \frac{dp}{dx} \left[ \frac{R_1^2 - r^2}{2} + \frac{R_1^2 - R_2^2}{4 \ln \frac{R_1}{R_2}} \right]$ .

Once the velocity is found as we did in the earlier case for the pipe flow we can just find out the discharge passing through the annulus between the two cylinders just we can integrate  $Q$  is equal to integral  $R_2$  to  $R_1$   $2\pi r dr u$ , this integration will give the discharge flowing through the annulus  $Q$  is equal to integral  $R_2$  into  $R_1$   $2\pi r dr u$  that is equal to minus  $\frac{\pi}{8\mu} \frac{dp}{dx} \left[ \frac{R_1^4 - R_2^4}{2} + \frac{R_1^2 - R_2^2}{\ln \frac{R_1}{R_2}} \right]$ .

This gives the discharge and then other parameters like shear stress then pressure variation all other parameters we can find as we have done in the previous case for the pipe flow. This finds the velocity is known we can find out the velocity gradient that will give with respect to that from the Newton's flow viscosity we get the shear stress variation and then with respect to the velocity variation also we can find out the pressure variation since that the velocity expression for velocity includes the pressure variation of from which we can find out the pressure variation for the laminar flow in annulus. So all other parameters once we find the velocity we can determine. The velocity gradient here  $\frac{du}{dr}$  is equal to minus  $\frac{1}{4\mu} \frac{dp}{dx} \left[ -2r + \frac{R_1^2 - R_2^2}{2r \ln \frac{R_1}{R_2}} \right]$  and then shear stress is obtained as I mention in Newton's law of viscosity  $\tau$  is equal to minus  $\mu \frac{du}{dr}$  that gives  $\frac{1}{4\mu} \frac{dp}{dx} \left[ 2r + \frac{R_1^2 - R_2^2}{2r \ln \frac{R_1}{R_2}} \right]$ .

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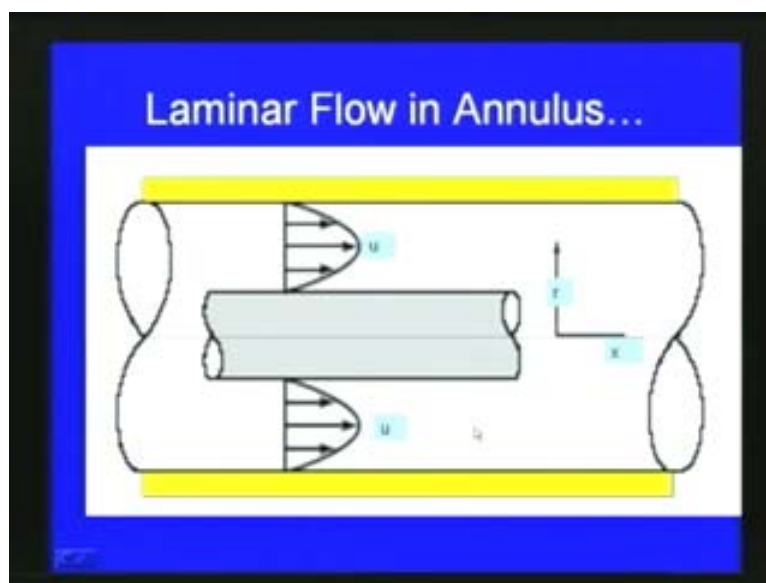
### Laminar Flow in Annulus...

- Velocity gradient  $\frac{du}{dr} = -\frac{1}{4\mu} \frac{dp}{dx} \left\{ 2r + \frac{R_o^2 - R_i^2}{\ln \frac{R_o}{R_i}} \left( -\frac{1}{r} \right) \right\}$
- Now shear stress:

$$\tau = -\mu \frac{du}{dr} = \frac{1}{4} \left( -\frac{dp}{dx} \right) \left\{ 2r + \frac{R_o^2 - R_i^2}{\ln \frac{R_o}{R_i}} \left( -\frac{1}{r} \right) \right\}$$

So this gives the shear stress. As we have plotted the velocity variation in the case of pipe flow here also we can plot the velocity variation, I mentioned due to the no slip boundary conditions on the pipe wall as well as the inner cylinder of valve, the velocity will be 0, so we can see here I have plotted the velocity variation here.

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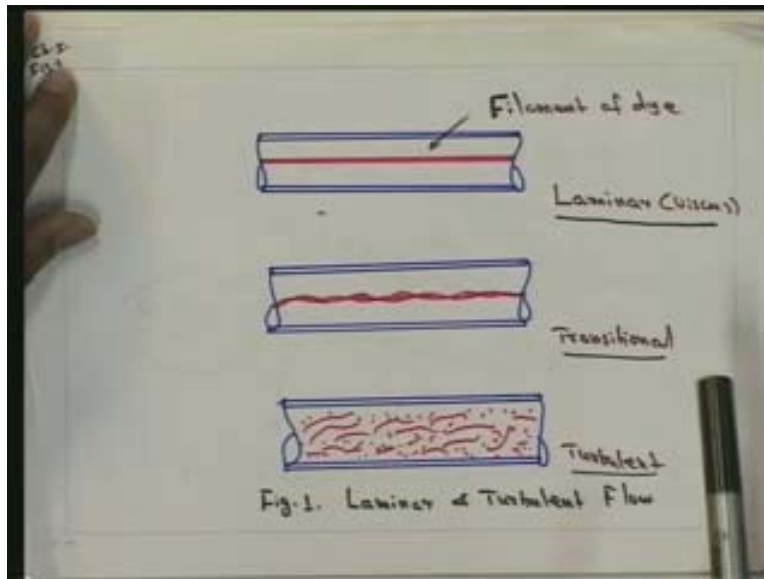
This location, this location and this location that means on the pipe valve here you can see work place the velocity is 0 and then it is going. Here, at this line it is the maximum velocity this is the velocity  $u$  and similarly here also very similar way we can plot the velocity here the minimum on both sides and here the maximum velocity.

Very similar way we can brought the shear stress variation and you can find out all other parameters so this is typical problem, so very similar way we can solve most of the laminar flow problems by considering the first principle itself, that means the Newton's second law itself we can derive various equations, various analytical expressions or exact solution for simple laminar flow steady state conditions.

So this way we can solve the laminar flow conditions, further these laminar flow problems we will be discussing when we go to the chapter on Navier-Stokes equation. Navier-stokes equation also can be used to find out the laminar flow conditions by solving the Navier-Stokes equation that will be discussing later. In this chapter since we have covered the fundamentals of the laminar flow, say first we consider the laminar flow between two parallel plates and various other problems with respect to the parallel plates, now we are consider the pipe flow.

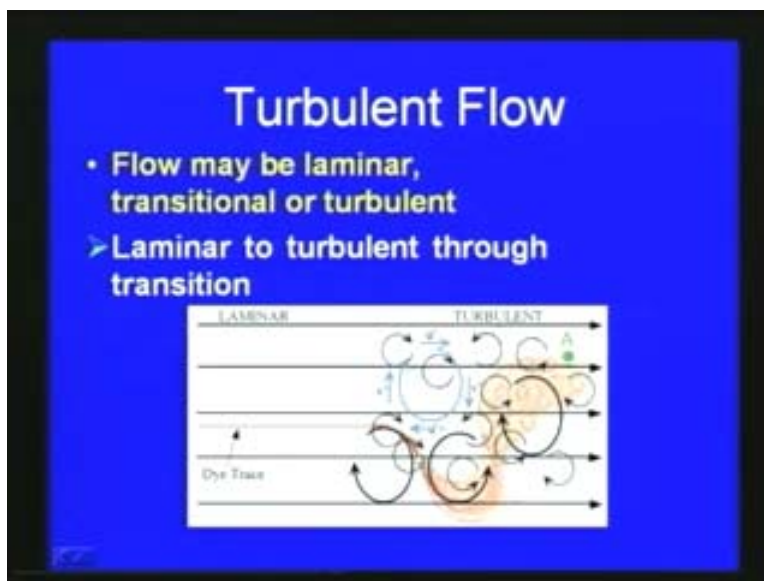
In the next section in this chapter is we are going to consider the turbulent flow. For turbulent flow as we are discussed earlier the flow in nature can be laminar or say in transitional stage or in turbulent conditions. Here, we can see that if you consider the flow through a pipe here you can see that if you put a filament of dye with respect to a pipe then say the flow is we have as we have already seen there is not disturbance to the flow and flow is in layers and that is laminar flow and then when we increase the velocity of the pipe flow you can see that there is lot of disturbance takes place and then a transitional stage comes and then finally if you again increase the velocity then you can see that the flow become turbulent.

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This is the laminar flow and turbulent flow which we have discussed in the beginning of this chapter. Now we are going to discuss more on the turbulent flow as we have already covered the laminar flow at steady state. Now we will be discussing the turbulent flow so the laminar to turbulent as we have seen it is through a transitional stage.

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Naturally the laminar flow is harm and so then you can see that it is mostly in layers. For example if you go to a river where the flow is just the slop of the river but this small and then there is not much disturbance you can see that flow is very slow and then it is can easily observed, then it is laminar in nature. It is very common and quite flow but when the slope of the river changes at some place or due to some abstraction says big stone or something is there then we can see that this laminar flow itself start to mix and then transitional takes place and then you can easily observe the flow become turbulent.

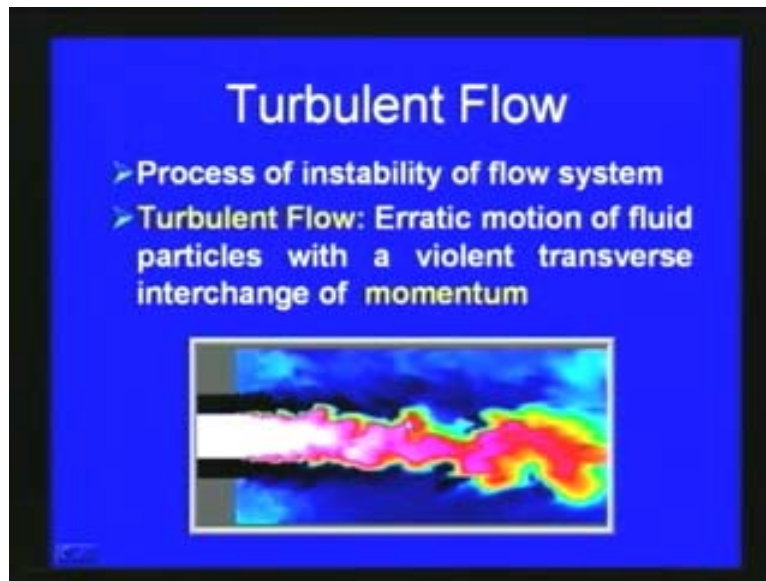
Here in this slide we can see that how this phenomenon takes place. So laminar to turbulent flow through transition, we can see that here the flow is laminar, here the flow is in layers and then if we put as we have already seen if you inject dye or tracer we can see that since the flow is also a laminar dye also moving in layers like this.

Then you can see that the turbulence conditions, if the condition is going to the turbulent then you can see that there is lot of mixing takes place and then initially this region we can say transition and then between the layers lot of inter mixing takes place and then flow disturbance takes place and then finally the flow become turbulent.

So here you can see that here this tracer or dye is here say layer are undisturbed here but since then slowly the disturbance starts the transitional stage and then you can see that lot of disturbance when the flow become fully turbulent then the dye also you can see that it is totally mixing between and then all the flow with the molecules change from one layer to another and then lot of disturbance takes place and finally the flow become turbulent.

We can easily observe this in nature as I mentioned if you go to a river and say if there is suddenly there is rain also you can see that then lot of water comes and then the flow itself is disturbed. Number of examples we can see how a laminar flow is transforming to a turbulent flow. Now, we can see here that the flow is initially coming as laminar, this is actually the gases coming out of a pipe we can see that initially it is here depending upon various conditions initially it is laminar and then lot of mixing takes place and then finally it become turbulent.

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Here you can see the process of instability of flow systems what happens in the laminar flow, the flow is stable and then for the turbulent flow lot of instability takes place to the flow system and then what happens is erratic motion of fluid particles with a violent transverse interchange of momentum. So this is what is happening as for as turbulent flow is concerned we can say that erratic motion of fluid particles with violent transverse of interchange of momentum. This is the basic definition of turbulent flow. Fluid in the laminar flow, the fluid is common quite and then it is layers and then the turbulent flow at erratic motion takes place and then fluid particles there is violent transverse interchange of momentum, this is the basic definition of turbulent flow as demonstrated in this animation and now the turbulent flow say as we have seen turbulent flow is the flow regime characterized by low momentum diffusion, high momentum convection and rapid variation of pressure and velocity in space and time.

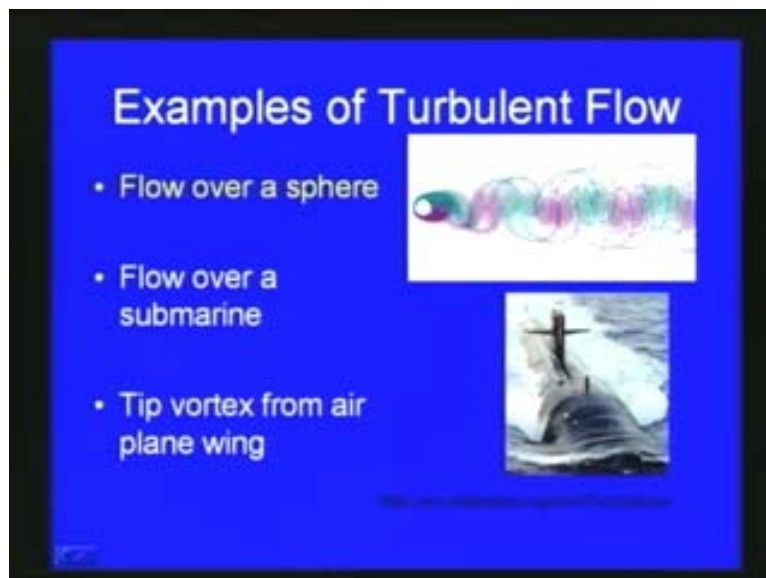
So turbulent flow characteristics we can define like this there is low momentum diffusion takes place and there is high momentum convection takes place and then rapid variation of pressure is taking place between the flow and then velocity also changes rapidly in space and turbulent flow can be characterized, that means low momentum diffusion, high momentum convection and then rapid variation of pressure and variation of velocity in space and time.

Here you can see that submarine is moving in the sea and then you can see that due to this speed of the vehicle here the submarine you can see that where the submarine started that position is laminar there is not much disturbance when its started and then we can see that lot of disturbance takes place and then high momentum convection takes place and pressure rapid variation of the pressure takes place and also you can see that the velocity is changing in space and time.

This is the phenomena taking place as far as turbulent flow is concerned. Now further we can put as I mentioned we can put a number of examples for turbulence in nature. There are many of the problems or turbulent type flow other than even normal pipe flow when it is the velocity that means when we are opening the pipe slowly then it is laminar but after some time if you open completely then you can see that the flow become turbulent.

There are number of examples we can put here in this slide, we can see that flow over a sphere we can see the sphere is here and then various flow conditions we can see that this actually an example of external flow the free stream velocity is hitting on the sphere or cylinder and then we can see lot of mixing takes place and then finally due to various conditions here the flow become turbulent.

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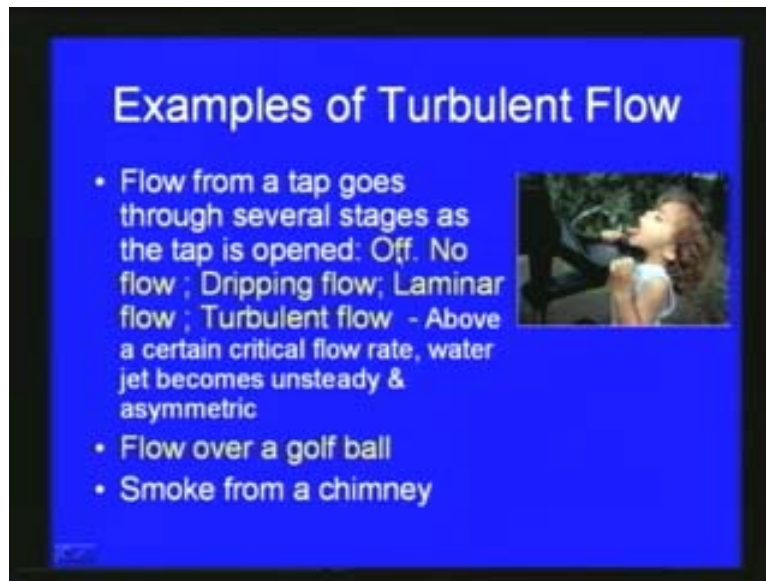
This is an example for flow over a sphere. As we have already seen flow over a submarine here you can see a submarine and then how much turbulence are created surrounding this and this figures are taking from this website as shown here and then we can see say especially when aeroplane is flying the tip vortex from aeroplane wing that also is say turbulent flow. Like this we can have a number of examples turbulent flow and this turbulent flow is very complex flow compared to laminar flow as we can observe here in various examples it is very complex flow the analysis is very difficult.

We want to find out the velocity or pressure variation or shear stress variation distribution then we have to do very complex analysis as for as turbulent flow is concerned so this is the real challenge to the engineers and fluid mechanics scientists to deal with the turbulent flow so now further to see the examples of turbulent flow say in this slide you can see when we open a tap as I mentioned say it will pass through several stages as the tap is opened.

Initially the tap is off that means no flow and then when the tap is slowly opened you can see that when the tap is slowly opened as demonstrated here then the dripping it the water is coming slowly as drips and then it is called a dripping flow and then slowly increasing then we can see that the flow become laminar then it continuous flow is there initially dropping flow then dripping flow as drips and then we can see that laminar flow is coming continuous flow and then if we increase the opening of the valve then you can see that the flow become turbulent.

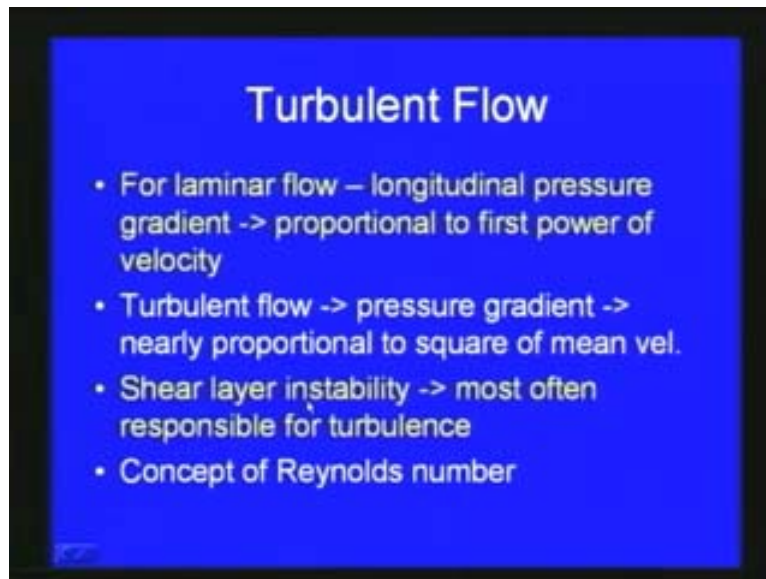
This happens above a certain critical flow rate that means say we can define in terms of Reynold's number as we have discussed in initial part of this chapter so we can define with respect to certain critical flow rate then the flow become laminar to turbulent through a transitional stage water becomes the turbulent flow we can see here the water becomes unsteady and asymmetric as the turbulent flow is concerned.

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Similar way the various way when we are playing golf we can see that when we are hitting with the golf ball then whatever if we analyze; the flow surrounding the ball or when we are throwing a cricket ball when we analyze; the flow surrounding the ball you can see that the flow is totally turbulent; then also as we have seen the smoke from a chimney is also turbulent, these are some of the important examples of turbulent flow which we have to analyze practically for various engineering problems. Before going to the fundamental equations of turbulent flow we will initially see the various fundamental characteristics of turbulent flow and then we will see the how the various parameters various in taking place; then we will go to the derivation of the basic equations of turbulent flow as we have seen you can see here in this slide for laminar flow the longitudinal pressure gradient that means it is proportional to first power of velocity if we consider the laminar flow longitudinal pressure gradient is proportional to first power of velocity as we have already seen.

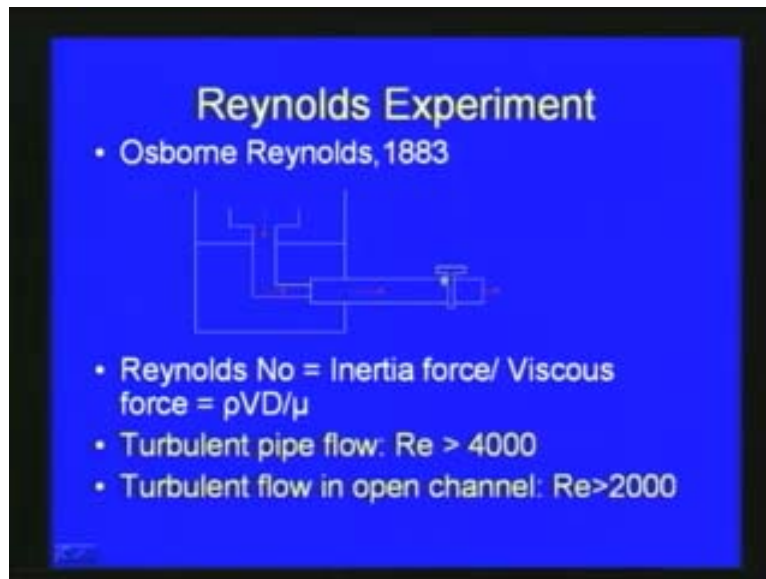
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But we can see that turbulent flow is concerned due to the intermixing; then various variations of parameters the pressure gradient is nearly we can observe that nearly proportional to the square of mean velocity. This is what is happening in the laminar flow the pressure gradient is proportional to first power of velocity. In the case of turbulent flow pressure gradient is nearly proportional to the square of the mean velocity and then other important character here is the shear layer is unstable due to shear layer instability most often this is actually the shear layer instability is the most often the responsible for turbulence.

One of the major factors for creation of turbulent, the shear layer is unstable and then this creates turbulence. Now as we have seen in the preliminary section, earlier discussion on this chapter we can define with respect to the Reynold's number we can define the turbulent flow as we have seen in the famous Osborne Reynold's experiments in 1883. So Reynold's what we did as shown in this slide, he put water in time and then a small time field the dye and then put in this form.

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And then fitted a pipe here and then with respect to time he put valve here. As we can observe when the valve is opening, initially this is here, you can see when the valve is opening a filament of dye is passing through the flow is very much laminar in nature; then we can see that after some time the flow is transitional stage when the valve opening is increased then you can see small disturbance starts and then it becomes finally total turbulent flow.

Osborne Reynolds is the first studied how this can be classified whether as flow is laminar transitional or turbulent and then he put forward the Reynold's number as we have discussed Reynold's number is the ratio of inertia force to viscous force and we can generally define as  $\rho V D$  by  $\mu$  where  $\mu$  is the coefficient dynamic viscosity  $D$  is the diameter of pipe  $V$  is the velocity  $\rho$  is the mass density and depending upon the flow condition this turbulence can be there in pipe flow open channel flow. For any flow conditions a flow can be either laminar or a transitional turbulent flow with respect to the Reynold's number. Osborne Reynolds classify the flow in pipe flow or open channel flow as we know when the flow is in open channel or pipe flow the Reynold's number variation loss will be there.

So he defined the turbulent flow as the pipe flow is concerned, the flow is set to be turbulent. When Reynold's number is larger than 4000, we can say that the Reynold's number is less than 2000 as we have seen for pipe flow it is totally laminar 2000 to 4000 it is actually in transitional stage and then the Reynold's number is exceeding 4000 we can definitely say that turbulent flow is existing the pipe flow is turbulent and similarly the channel flow is concerned as we have discussed earlier up to Reynold's number of 600 something the flow is laminar for channel flow and up to 600 to 2000 it is transitional stage and then beyond 2000 the flow is in open channel flow you can say that the flow is becoming turbulent.

So with respect to the Reynold's number we have already seen how we are classifying the flow is laminar and turbulent. Now we will see various factors affect the transitional from laminar to turbulent as we have observed in the experiments or even we can observe when we are opening a tap with valve of the pipe then flow is changing from laminar to turbulent.

What are the factors that affect the transitional we will briefly discuss here, what are the important factors that affect transition from laminar to turbulent, some of the important parameters I have listed here, and first one is free stream turbulence in incoming flow that means initial turbulence.

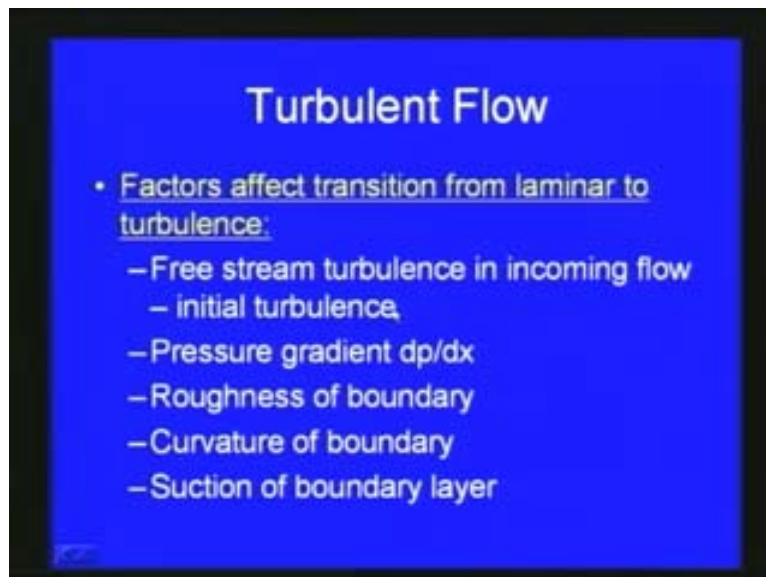
Free stream turbulence means the flow is already laminar and then you are disturbing the flow you can just say for example in a canal or a river flow is flowing laminar common quite condition and when we are putting a big stone on the river or in the canal then you can see that due to the impact there is lot of disturbance takes place and then stream turbulence takes place and this incoming flow is initial turbulence so due to this the flow become can become turbulence.

This is small example it can become turbulence and then second important factor is pressure gradient  $dp$  by  $dx$ , when the pressure is drastically changing that means the pipe flow is there at one section the pressure is less and at some other section the pressure is very large so you can see that there is the pressure gradient is very high the pressure

gradient  $dp$  by  $dx$  is very high, in that case the flow can change from the laminar to turbulent and then another important reason I have listed here is roughness of boundary.

For example, in the glass flume the flow is taking place as laminar and then suddenly we will change it to a flow wherever large number of stones are put then you can see that there is roughness that means the roughness of the valve or the bottom is changed. For example, here I will just draw here the flow is coming here and then the it is laminar the flow is going as laminar, so here you can see that the bottom valve or the bottom is smooth and then if it changes to you can see that a small change here that means the surface become rough then you can see that lot of say due to the shearing affect the shear stress increases and then you can see that a small mixing starts here and then this laminar flow can become turbulent due to this reason, so third reason which I have listed here is the roughness of the boundary and then the fourth reason can be curvature of boundary so if the flow is taking place in a smooth and then normal horizontal direction and then sudden curvature is there so that you can see that in this case due to various parameters of the flow change and then the laminar flow can become turbulent.

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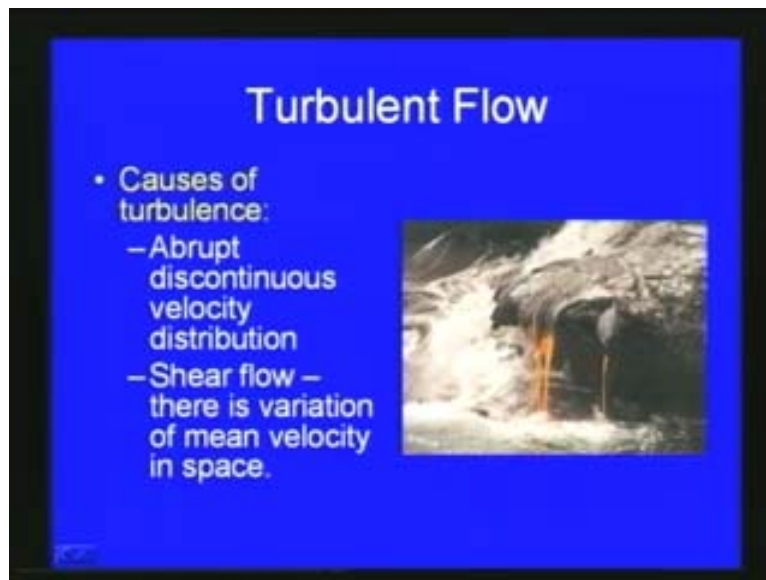


Another important factor I have listed here is suction of boundary layer, so boundary layer you can see that most of the cases boundary layer will be formed here you can see

boundary layer formation you can take place with respect to free stream flow is coming and then the suction for this boundary layer then also the flow can change from laminar to turbulent. The importance here again I will just go through the important parameters of factors affect transitional from laminar to turbulent important factors are: first one is free stream turbulence in incoming flow or initial turbulence; second one is the pressure gradient  $dp$  by  $dx$ ; third one is the roughness of boundary and fourth one is the curvature of boundary and fifth one is the suction of boundary layer.

These are some of the important parameters; there are many other parameters also where the flow can change from laminar to turbulent. Here, you can see with respect to the various parameters which we discussed in the last slide, here we can see what causes of turbulence flow are. What is finally happening is this figure shows what is happening so causes of turbulence we can just list into two major factors, first one is abrupt discontinuous velocity distribution you can see here.

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The flow is coming and then due to various parameters various factors discontinuity takes place you can see here these rock is here and then the flow is coming various obstructions are there and then due to this the velocity become discontinuous or discontinuous velocity distribution takes place, this can be one of the major cause of turbulence and

then another important factor another important reason or cause of turbulence is shear flow. Here you can see there is variation of mean velocity is space and then shear flow takes place and that can be another causes, these are two important causes of turbulence and earlier we have seen five factors of a transition and certain important causes of turbulence.

The problem as I mentioned, a turbulent flow is very complex in nature. Laminar flow it is much easier to analyze, for turbulent flow it is very difficult to analyze then how to express the turbulent flow. Generally we can see this is the flow, then say this letters consider say mean flow or the laminar flow with respect to this and then we can see that we can just superimpose say the variations like this that means say the mean we can consider a mean component of the velocity  $\bar{u}$  and then with respect to that mean component we can see what are the changes  $u'$  or the transition takes place, this is the way which we represent it.

The velocity of turbulence or turbulent flow velocity we can express  $u$  is equal to a mean flow component and then the instantaneous flow component  $u'$ , this is the way generally we express the turbulent flow. In this slide you can see in three dimensions we will be having the flow  $x$  component  $y$ , component and  $z$  component,  $u$   $v$  and  $w$  component,  $u$  is expressed as  $\bar{u} + u'$ , where  $\bar{u}$  is the mean velocity component that is equal to  $1/t$ , where  $t$  is time.

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**Turbulent Flow**

–For velocities u, v and w

$$u = \bar{u} + u' \quad \text{where } \bar{u} = \frac{1}{T} \int_0^T u \, dt$$

$$v = \bar{v} + v' \quad ; \quad \bar{v} = \frac{1}{T} \int_0^T v \, dt$$

$$w = \bar{w} + w' \quad ; \quad \bar{w} = \frac{1}{T} \int_0^T w \, dt$$

Long time is considered, so  $\frac{1}{T}$  by  $t$  integral 0 to  $T$   $u \, dt$  and  $V$  turbulent velocity direction  $v$  is equal to  $\bar{v}$  plus  $v'$  so where  $\bar{v}$  is equal to  $\frac{1}{T}$  by  $T$  integral 0 to  $T$   $v \, dt$ . Similarly  $w$  is equal to  $\bar{w}$  plus  $w'$ , where  $\bar{w}$  is equal to  $\frac{1}{T}$  by  $T$  integral 0 to  $T$   $w \, dt$ , where  $u'$   $v'$   $w'$  are the instantaneous velocity with respect to mean velocity as we have discussed.

This is the general way we describe the velocity variation in turbulent flow. There will be a mean velocity component and then there will be instantaneous velocity component and then we will be adding together that will give the velocity in  $x$   $y$  or  $z$  direction at appropriate time.

This is the way we describe the velocity component and then the time average of fluctuations you can see that this  $u'$   $v'$  and  $w'$  are the fluctuations we consider for long time and you can see that this will be the fluctuations will be 0. For example, if we consider the mean value of the fluctuations  $\bar{u'}$  is equal to  $\frac{1}{T}$  by  $T$  integral 0 to  $T$   $u' \, dt$  that can be expressed as  $\frac{1}{T}$  by  $T$  integral 0 to  $T$   $u - \bar{u} \, dt$  that is equal to  $\bar{u} - \bar{u}$  so this is equal to 0.

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### Turbulent Flow

- Time average (temporal means) of fluctuations of  $u$ ,  $v$  &  $w$  are zero. E.g.  
$$\bar{u'} = \frac{1}{T} \int_0^T u' dt = \frac{1}{T} \int_0^T (u - \bar{u}) dt = \bar{u} - \bar{u} = 0$$
- Therefore  $\bar{u'}}, \bar{v'}}, \bar{w'}}, \bar{u'v'}}, \bar{u'w'}}, \bar{v'w'}} = 0$
- The root mean square of  $u'$ ,  $\sqrt{\overline{u'^2}}$   $\rightarrow$  violence of turbulence fluctuations and measure of intensity of turbulence.

This way we can say that the time average fluctuation of the velocity component in turbulent flow is 0. Therefore  $\bar{u'}$  mean value is equal to  $\bar{v'}$  mean value is equal to  $\bar{w'}$ , mean value is equal to 0 and then the root mean square of  $u'$  we can express this will be the violence of turbulence fluctuations and measure of intensity of turbulence. The intensity of turbulence or the violence of turbulence of fluctuations we can express as the root of  $\bar{u'}^2$  whole square, this gives the violence of turbulence of fluctuations and measure of intensity of turbulence.