Fluid Mechanics

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Lecture - 18
Laminar and Turbulent flows

Welcome back to the video course on fluid mechanics. In the last lecture, we were discussing about the laminar and turbulent flow. We have seen how laminar and turbulent flow is creating nature and we were discussing about its various kinds of behavior and how we can demonstrate or how we can classify between laminar and turbulent flow by Reynolds experiment. We have seen that with respect to the dimensionless number, Reynolds number. We can classify the flow either in open channel or in pipes as laminar or turbulent; that classification also we have seen. Then, we were discussing in detail about various kinds of laminar flow. We have seen in one of the first demonstrations that the flow between two fixed parallel plates and then we have seen the plane poiseuille flow.

In today's lecture, we will be discussing further on the flow between parallel plates but whenever, say, a plate is moving. The first case which we have seen last time is: whenever two plates are fixed like this here you can see if these are two plates then it is fixed then what happens a flow is in between. So, that is the plane poiseuille flow that we have seen. Then we have derived the expression for velocity and shear stress and discharge. Various flow parameters we have derived for plane poiseuille flow that means the flow between two parallel plates. The plates are of infinite line then all the properties we have seen. In today's case, we will be discussing the laminar flow between parallel plates but one plate is moving.

Here, (Refer Slide Time: 03:05) if we assume that there are two infinite plates like this and then in the flow is between the plate and one plate is moving with the velocity, say, velocity u. Then how we can derive various flow parameters like velocity, discharge and then the shear stress?

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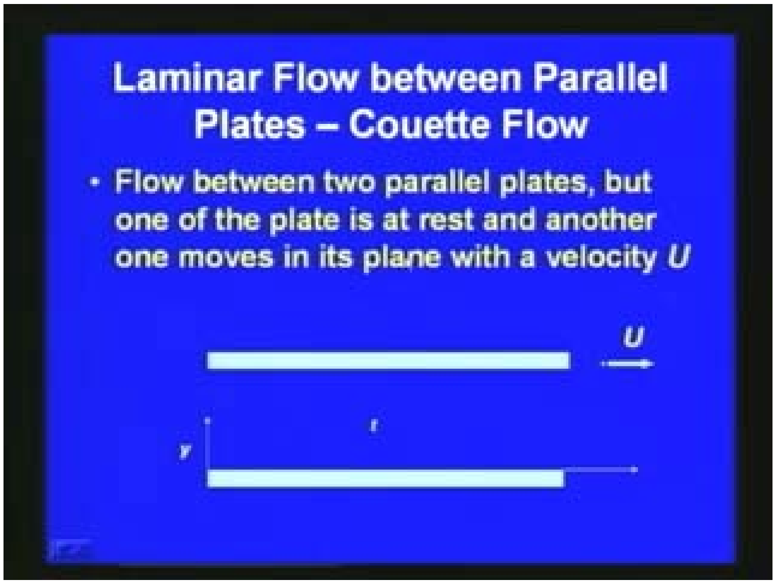

You can see in this slide the flow between two parallel plates but one of the plates is at rest and another one moves in its plane with a velocity u.

Here, the figure shows here is the x axis if you draw and y axis is here the distance between the plates is t and then u is the velocity of the upper plate. So this is the case typical flow has got number of application like whenever belt is moving in a fluid stream and then carrying some chemicals. These different applications are there with respect to this coquette flow or the laminar flow between parallel plates. Here the case is demonstrated. Now, we want to derive the velocity shear stress and the discharge.

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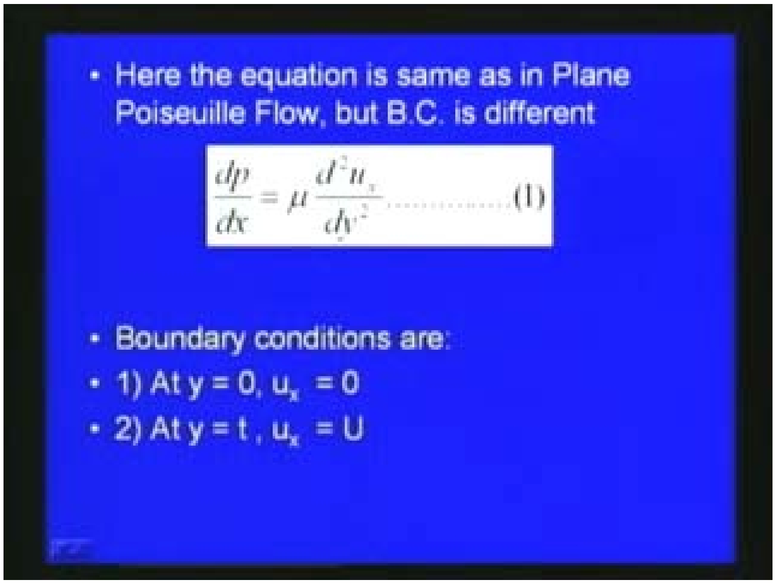

We have seen that earlier plane poiseuille flow we have seen the case that the forces acting are the shear force and then the pressure force. If we consider just a fluid element of thickness dx and then various forces acting on the pressure force on both sides of the element and then the shear force.

The earlier case for the plane poiseiulle flow we have seen that the expression for this kinds of flow when we write the for example if we consider the case here now the one plate is here and now the second plate is moving with a velocity u so this is u and t is the distance.

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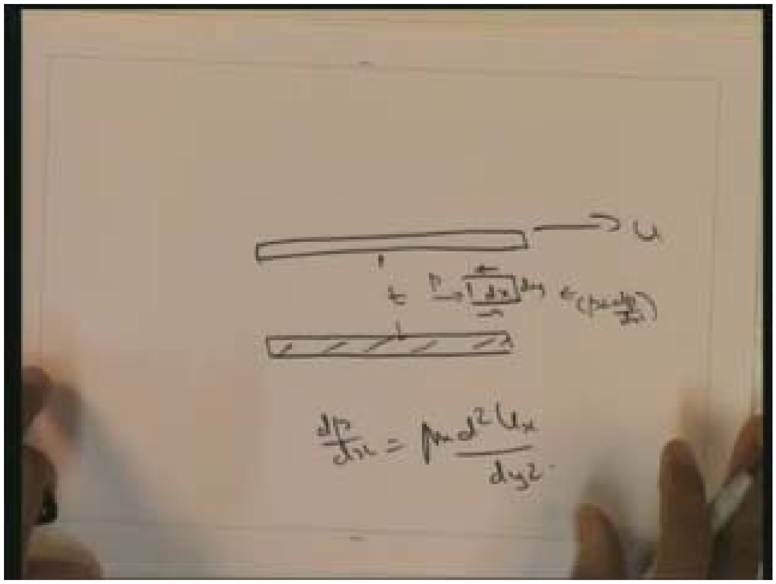

Now we will be considering here to derive an expression for the velocity for flow between the plates. We will be just considering a fluid element of size dx by dy and then here the pressure force p and the other side p plus dp by dx we will be considering and then the shear force on both sides. Correspondingly, with respect to the plane poiseiulle flow we have seen the equation we have derived dp by dx is mu d square u_x by dy square.

Correspondingly, with respect to the plane poiseiulle flow with respect to the flow between two fixed parallel plates only the difference is that the upper plate is moving.

So, the basic equation is same, the basic equation is written here: dp by dx is equal to mu d square u_x by dy square. This is the equation with respect to the coquette flow and then the boundary conditions are at y is equal to 0 that means on this plane at y is equal to 0. Due to the no slip boundary conditions, the velocity u is equal to 0 and at y is equal to t then we can see that the velocity u is equal to U. Those are the boundary conditions. So the boundary conditions at y is equal to 0, u_x is equal to 0; at y is equal to t, u_x is equal to U. As we discussed in the case of plane poiseiulle flow between two fixed parallel plates to derive an expression for the velocity we can integrate this expression twice so that we get the expression for velocity.

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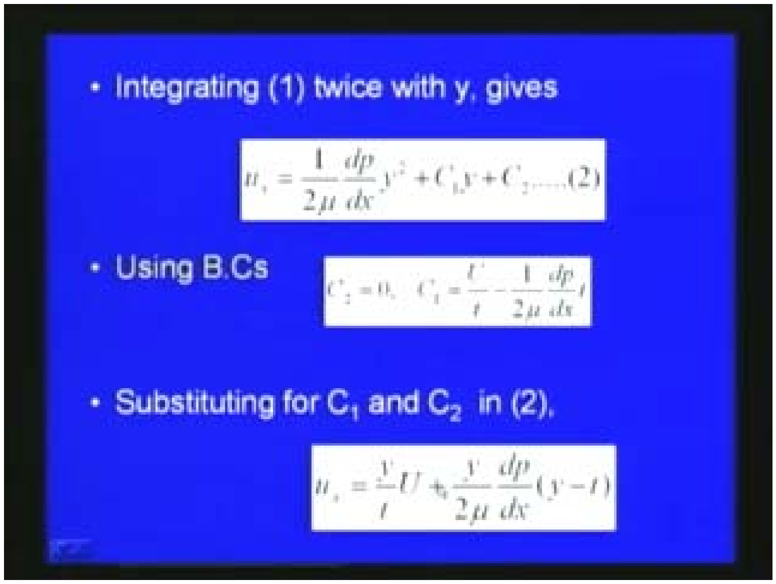

If we integrate twice with respect to y gives u_x is equal to 1 by 2 mu dp by dx y square plus C_2 y plus C_1 where C_2 and C_1 are constants of integration then we can use this both boundary conditions as we have seen here the boundary condition here u is equal to 0 at t is equal y is equal to 0 that is one boundary condition.

Second boundary condition is u is equal U at y is equal to t. So here to find out these two unknown constants C_2 and C_1 we can use the boundary condition. If you use the boundary condition with respect to the first boundary condition y is equal to 0. If we apply then we will get this and so we will get C_1 is equal to 0 and if we apply the second boundary condition at y is equal to t, u is equal to U then we get C_2 is equal to U by t minus 1 by 2 mu dp by dx into T.

We got the constants C_2 and C_1 and then we can substitute back the constant C_2 and C_1 into this equation number 2 so that we will get an expression for the velocity. Finally, we get velocity in the x direction u_x is equal to y by t into U, which is the velocity of the moving plate plus y by 2 mu dp by dx into y minus t. This gives an expression for the velocity between the flows between two parallel plates; upper plate is moving with the velocity U.

This we can say as I mentioned earlier there are number of applications for this coquette flow so that we can utilize this analytical derived expression to find out the velocity in a various practical cases.

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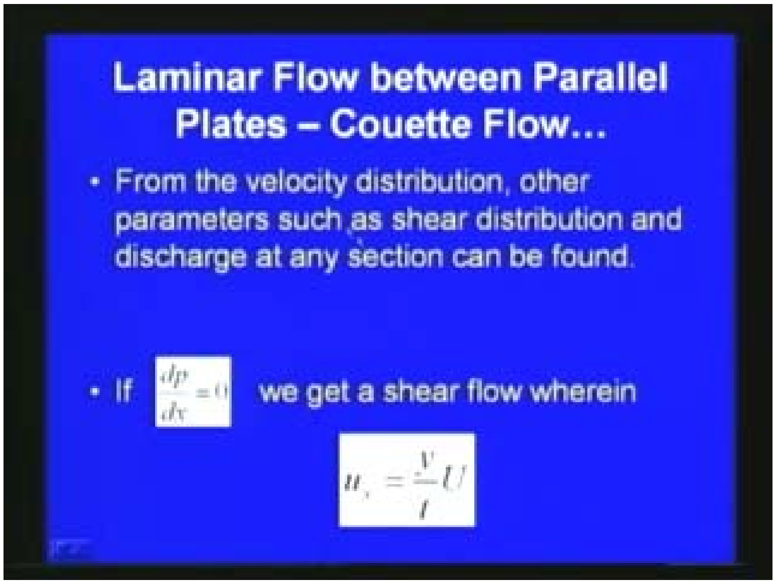

From the velocity distribution, other parameter such as shear distribution and discharge at any section can be found. So as we have seen earlier in the case of plane poiseuillie flow first we get an expression for the velocity and then from that we can derive the discharge passing or we can derive the other parameters like a shear stress. For example, this particular case in dp by dx is equal to 0 then we say that here in the earlier expression if dp by dx is equal to 0 means this second term goes and then we get u_x is equal to y by t into u; so, this kind of flow we say that shear flow.

Wherever dp by dx pressure gradient in the x direction dp by dx is equal to 0 that kind of flow is called the shear flow. In that case, we can get the velocity is equal to a simple expression y by t into u. This is called a shear flow.

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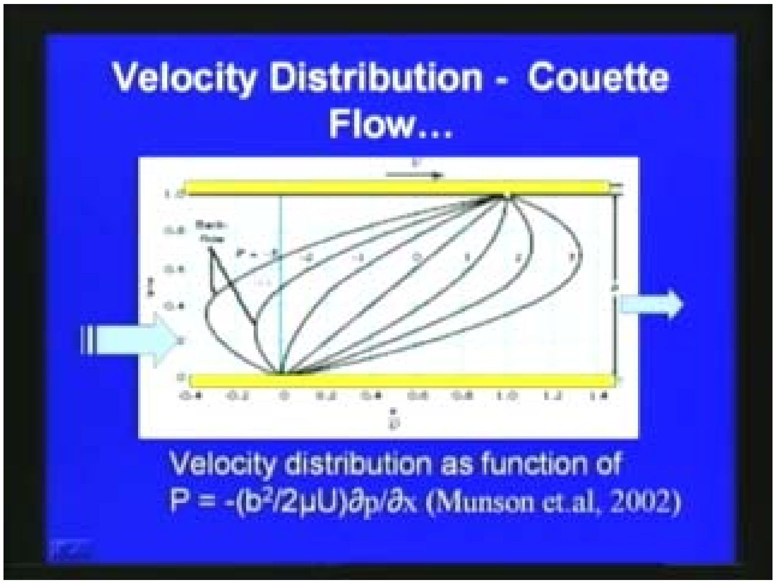

Then, the velocity is concerned; the velocity distribution for coquette flow if we say here you can see that this plate is fixed; lower plate is fixed and upper plate is moving with a velocity U and the flow is in this direction S. If you drop the velocity distribution if the flow between the two plates for various parameter called p, which we can define as minus p square by 2 mu dp by dx, where d is the distance between this two plates. For various p we can drop the velocity like this. You can see that when p is equal to 0 then it is linear, when p is equal to 1 it is going like this and p is equal to 2 it is in this direction.

Like that for various parameters we can drop the velocity distribution. So here this p is equal to t, which means the distance between the upper and lower plate and this figure is taken from the fluid mechanics book by Munson et al, 2002.

Like this now we can got the velocity and then as I mentioned once we get velocity we can get the expression for the shear stress. So, shear stress- tow is equal to mu into du by dy. From which we can just take a derivative of the velocity and then we can multiply by mu, the dynamic coefficient of viscosity that we can get the shear stress. If we want to find out the flow between these two parallel plates then we can integrate between 0 to t of this velocity going. So, the velocity into area that will give the discharge passing so that we can integrate between these two to get an expression for the discharge passing through between these two plates. So like that various parameters can be found out.

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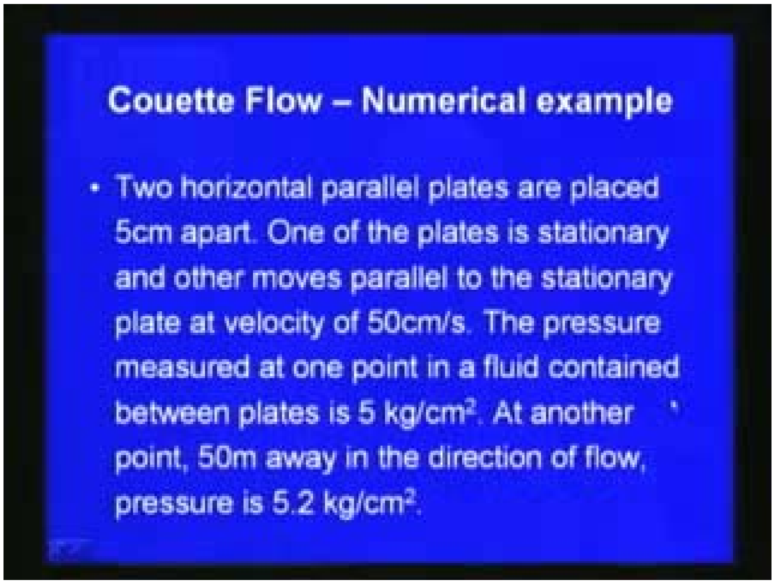

This is called coquette flow which has got number of practical applications in various engineering problems. Before further discussing one of the applications we were discussing the one of the numerical example with respect to this coquette flow. Here, we consider two horizontal parallel plates which are placed 5 centimeter apart. One of the plate is stationary and other one moves parallel to the station plate at velocity of 50 centimeter per second; the pressure measured at one point in a fluid contained between plate is 5 kilogram per centimeter square and at another point, 50 meter away in the direction of flow pressure is measured as 5.2 kilogram per centimeter square.

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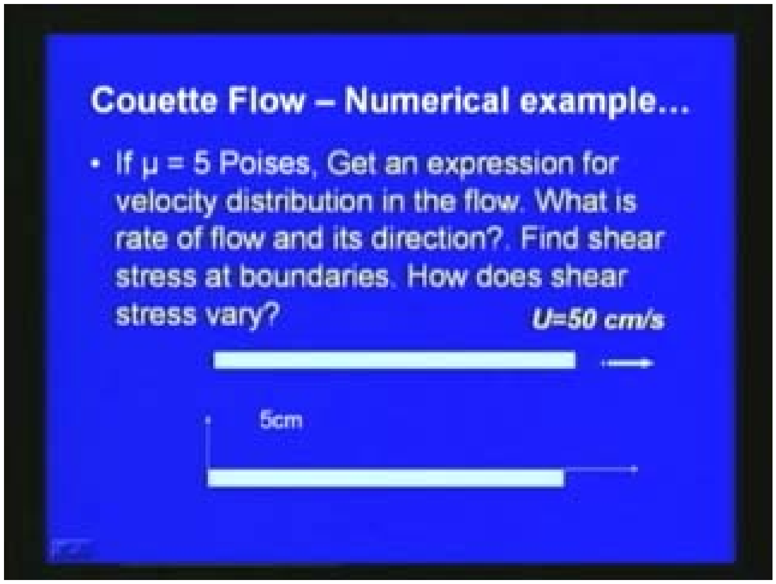

We have to find if the coefficient dynamic viscosity is given as mu is equal to 5 poises we have to get an expression for the velocity distribution in the flow and what is the rate of flow that means how much the discharge passing and its direction? Find shear stress at boundaries. How does shear stress vary?

These are the case. The problem is that here we have got to fix one fixed plate and another plate is moving with respect with a velocity 50 centimeter per second and the distance between the plates is 5 centimeter. Then the fluid is passing between these plates and its dynamic viscosity is 5 poises. We want to derive an expression for the velocity and then rate of flow and shear stress. This is the problem. As we have seen in the previous derivation we have derived an expression for the velocity; that expression we can directly utilize here.

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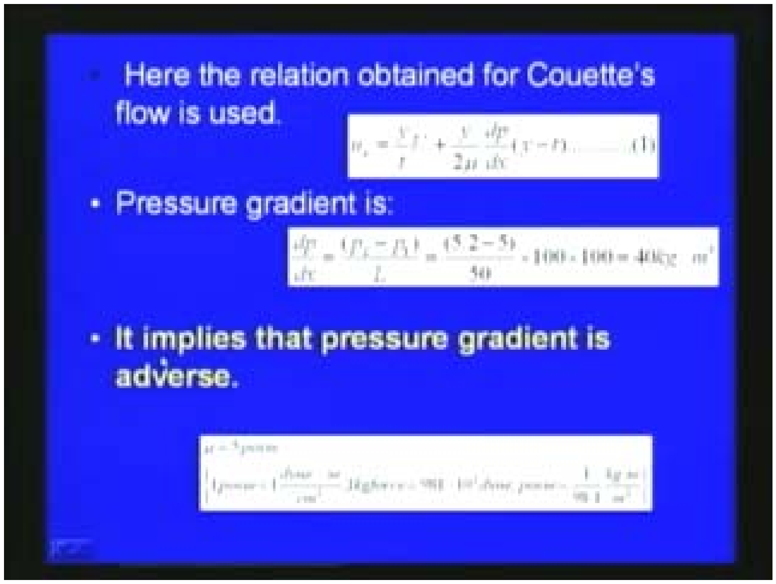

Here, the relation obtained for coquette flow is directly used. So, this expression u_x is equal to y by t into u plus y by 2 mu dp by dx into y by t that we have already derived earlier and then for this particular problem if the pressure gradient is given between two points, between these plates two points; the pressures are given. With respect to this we can find out dp by dx. We can take the difference between the pressure and the distance is also given so that we get dp by dx, the pressure gradient dp by dx is equal to p_2 minus p_1 by L. So, here 5.2 minus 5 by this distance is equal to 50; the unit is converted to meter. So that we get dp by dx is equal to 40 kilogram per meter cube with respect to this dp by dx. Now, it implies that the pressure gradient is adverse so the pressure is positive, dp by dx is positive, so pressure gradient is adverse. Now, the coefficient of viscosity mu is given as 5poise that we can write; that is equal to 1 by 98.1 kilogram per second kilogram second per meter square. So, we are converting all into the system and then finally we can substitute back to the equation.

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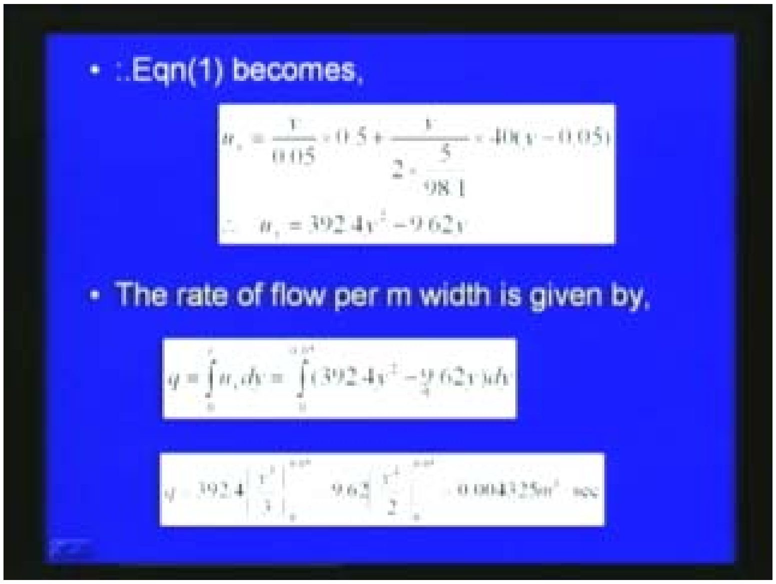

All the parameters are known here so we can get an expression for the velocity; so u_x is equal to in the previous expression this t is 5 centimeter, so 0.05 and mu is 0.5 meter; dp by dx is obtained as 40 and then t is also 5 centimeter and mu is 5 by 98.1. Now, we can substitute back so the velocity u_x is equal to y by 0.05 into 0.5 plus y by 2 into mu is 5 by 98.1 into 40 into y minus 0.05. From this, we get an expression for the velocity as u_x is equal to 392.4 y square minus 9.62 y. Finally, we got an expression for the velocity by substituting the various values. Once, we get an expression for velocity as we discussed the discharge passing between the plates, the fluid flow between the plates and then also the shear stress can be calculated. The discharge, the rate of flow per meter width is given. If we can integrate between 0 to t, u_x dy if we integrate, t is 5 centimeter, integral 0 to 0.05 and the velocity 392.4 y square minus 9.62y dy; this we can integrate between the limits of 0 to 0.05 that gives the discharge rate of flow per meter width; this is per meter width as 0.004325 q_{max} or meter cube per second. So like this once the velocity is known we can find out the rate of flow per meter width.

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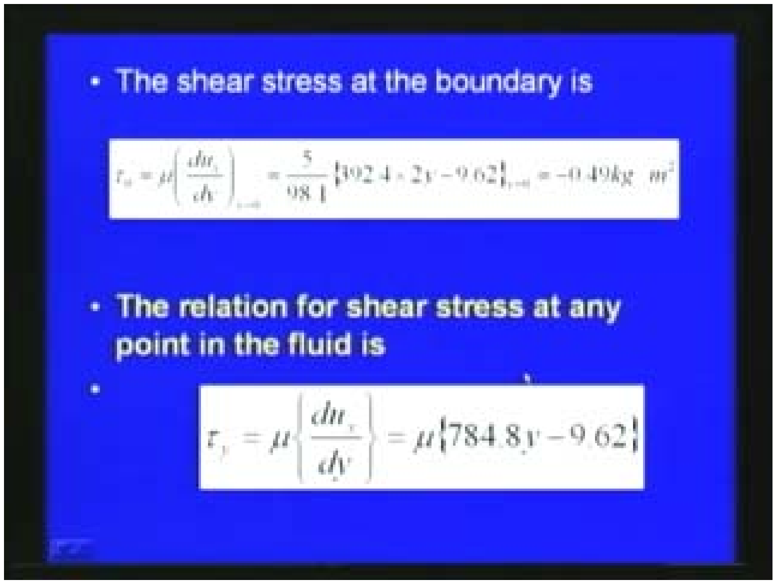

The shear stress at the boundary if you want to find out then we can write tow$_0$ is equal to mu into du$_x$ by dy at y is equal to 0. We can substitute back all various values phi by 98.1 into 392.4 into 2y. Here, already u$_x$ is known so this we can just take a derivative delt u$_x$ by delt y you can take find out and then we can put here as shown to find out the shear stress. So, shear stress tow $_0$ at y is equal to 0 mu into du$_x$ by dy that is equal to 5 by 98.1 into 392.4 into 2y minus 9.62. So, at y is equal to 0, the boundary shear stress can be found, that is equal to minus 0.49 kilogram per meter square. This gives the shear stress and then to get a relationship for the shear stress at any point in the fluid we can just take tow is equal to mu into du$_x$ by dy that gives the shear stress. We can get an expression by taking the derivative of the velocity expression that gives us mu into 784.8 into y minus 9.62. This gives the relation for the shear stress. Like this you can find out various parameters.

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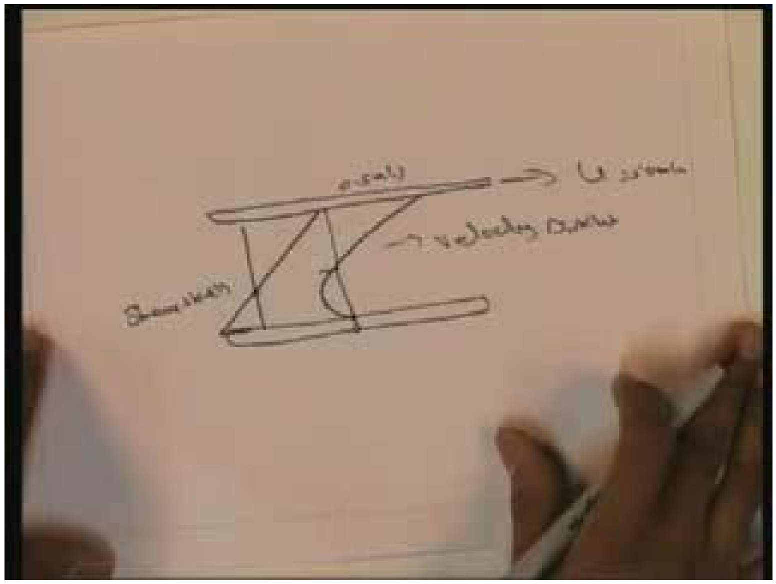

Now, for this case if you plot various velocities and then now as we have seen here we have the moving plate with 50 centimeter per second and here the fixed plate. As we have seen earlier case you can see that the velocity variation if you plot here, the bottom velocity is 0, so you will get an expression something like this. Here, it is 0.5 meter per second and here it is 0. We can find out the velocity where it will be 0 by equating this velocity expression to 0 we will get where the position where the velocity become 0.

Similar way if we plot this is the velocity distribution in this particular case and then if you brought the shear stress you can see that it will be something like; it will be varying at some point; it will be 0 and then here we have value on the upper plate also; it will be varying like this. This is the variation of shear stress. From this expression, we can find out the variation of the velocity and then the variation of the shear stress. Like this various problems with respect to the coquette flow that we have seen that means one plate is fixed and other plate is moving.

Now, we will discuss one more case here. First, we discussed two plates of infinite length that is two plates are fixed; and then we have seen that one plate is moving; and now we would derive an expression whenever both plates that means two plates are placed like this of infinite length and then first case is both are fixed and then we have derived the expression for the velocity and shear stress and the parameters; second case was first the upper plate is was moving like this with a velocity and then we found expression for the fluid velocity shear stress and then discharge and this third case we will be discussing wherever both plates one plate is moving, upper plate is moving with a velocity u in the right-hand direction and then the lower plate is moving with a velocity v in the opposite direction that means to the left-hand direction. So, two plates are there and then we want to find out an expression for the velocity and other parameters when the upper plate is moving to the right-hand side and lower plate is moving to the left-hand side with a velocity. So, here again the difference is the boundary conditions.

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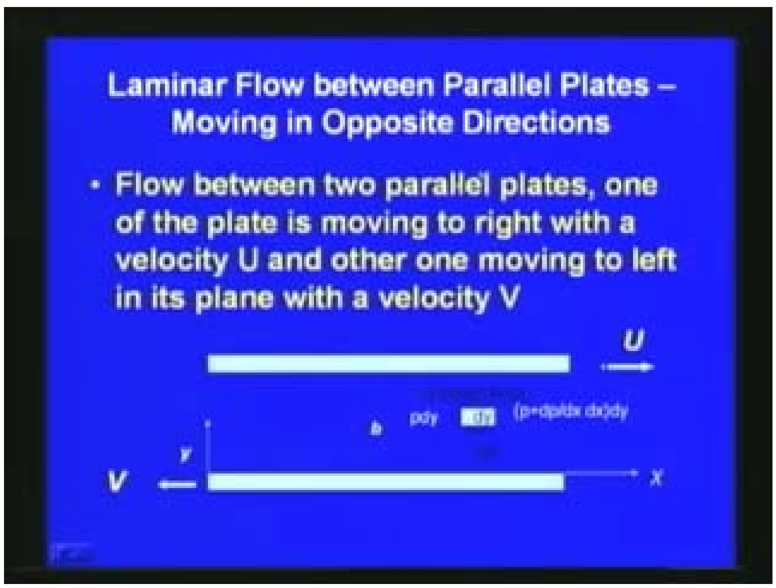

Now, the laminar flow is between parallel plates moving in opposite directions. Flow between two parallel plates, one of the plates is moving to right as here it is demonstrated in the figure. So, this upper plate is moving with a velocity u and lower plate is moving to the left-hand side with a velocity v. We want to find out expression for velocity, shear stress and other parameters. Here, the distance between the plate is b and as in the previous case we will consider fluid element of size dx dy and then we will find out the various forces on that fluid element and then we will use the Newton's second law of motion to derive an expression. Here, now all the cases what we are considering is steady state; time component is not coming and then the acceleration is also not coming to picture zone. Now, only the Newton's second law that means that the effective forces total force on the fluid element we are equating with respect to the case.

Here also, as in the previous case, the forces are the pressure force on to the left-hand side or to the both p dy on this side and other side p plus dp by dx into dx into dy and then we have the shear stress. So, here if it is tow dx on the bottom and the top of the fluid element it is tow plus d tow by dy into dy into dx. The principle is essentially same we are considering the problem in there very similar way.

In that case, if we apply the Newton's second law equation of motion for steady incompressible flow then we can write the expression as p into dy minus p plus dp by dx into dx into dy plus tow plus d tow by dy into dx minus tow dx equal to 0. The shear forces and then the normal forces pressure force gets equated to 0.

We can simplify this expression to get an expression for the velocity as we did in the previous two cases of plane poisuielle flow and then coquette flow. If you simplify and then divide by dx into dy in 1, with respect to this expression we will be finally get dp by dx is equal to dtow by dy. As we can see that finally we are getting a simple expression when we are using the first principle Newton's second law motion. Since only the forces are considered the case is steady, incompressible so obviously the equations will be coming almost the same way only with slight differences and then only changes with respect to the boundary conditions.

So here again we are getting the dp by dx is equal to d tow by dy. Now, for laminar flow as we have seen we can use Newton's law of viscosity tow is equal to mu into du by dy and then we can substitute for the shear stress expression tow mu into du by dy and then finally we can get the expression.

So if we put it back to the earlier equation here dp by dx is equal to d tow by dy we get mu into d square u by dy square that is equal to dp by dx. So, this is the final expression and then as we did in the previous case here also we can integrate twice and then we can apply the boundary conditions and then you can find out the integration constants. Finally, we can get an expression for the velocity as shown in this slide.

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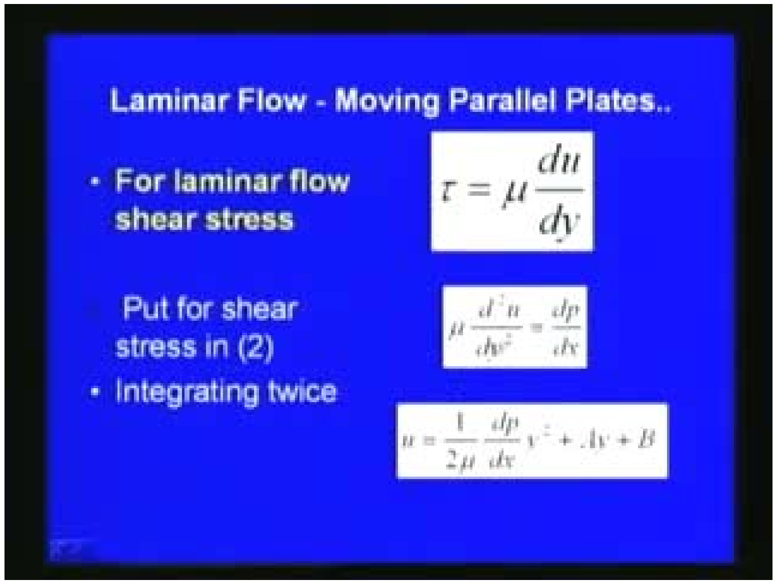

If you integrate twice we get mu is equal to 1 by 2 mu dp by dx into y square plus A y plus B, where, A and B are the constant of integration.

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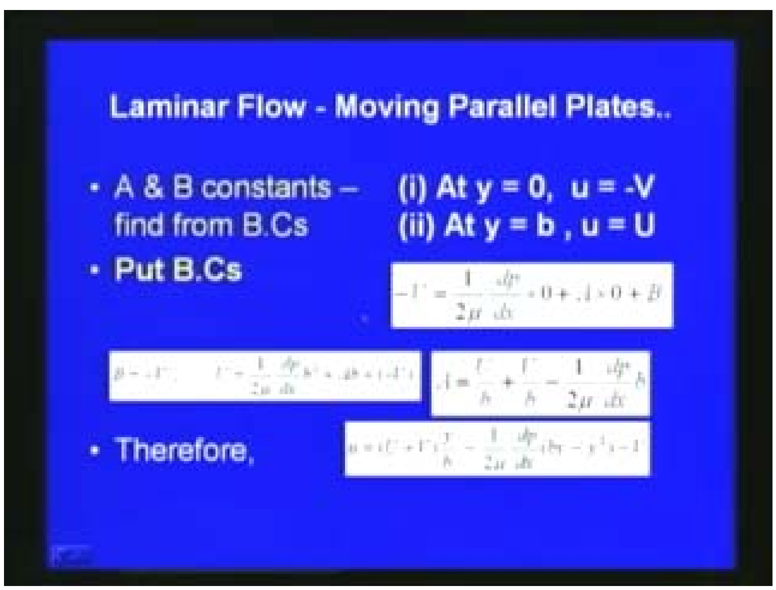

Here again we have got two boundary conditions. As I mentioned here the plates are moving with the velocity so here on the upper plate we know the velocity u is equal to U on the top of the plate and bottom of the plate v is equal to minus V in the other direction.

So, boundary conditions are known. We can just substitute here to find out the constant of integration A and B. We can write at y is equal to 0, u is equal to minus V and then at y is equal to b, u is equal to U, now these are boundary conditions. These boundary conditions we can substitute back so that finally minus V is equal to 1 by 2 mu dp by dx into 0 for the first boundary condition plus A into 0 plus B. So we get B is equal to minus V and then we substitute for at B; u is equal to V so we get u is equal to 1 by 2 mu into dp by dx v square plus AB plus minus V and we get either constant A is equal to U by b plus V by v minus 1 by 2 mu dp by dx into b. Finally, the constants A and B are found. We can substitute back to the earlier equation and then finally we get the expression for the velocity for this particular case. As u is equal to U plus V into y by b minus 1 by 2 mu dp by dx into by minus y square minus V so this is the expression for the velocity.

Then, as we have discussed earlier, once we will get the expression for velocity we can find out the flow between the plates by flow, between the plates by integrating between 0 to b and also we can find out the shear stress by taking derivative of the velocity and then using Newton’s second law. So, the velocity is known and then also you can see that now the distance y at which u is equal to 0 obtained here.

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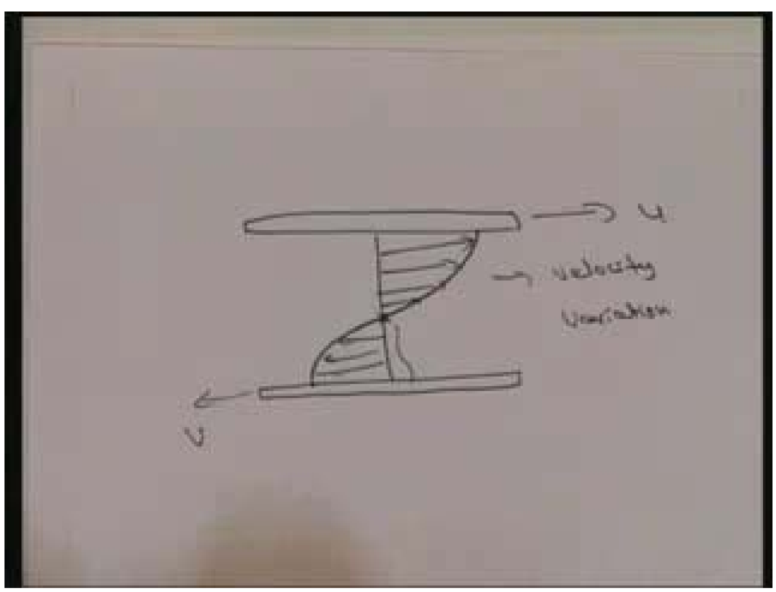

The case is that I will just draw here the variation with respect to the velocity since both plates are here: this is moving with velocity u in this direction, and here it is v. Then you can see that if we plot the velocity then the velocity variation will be something like this. Since here it is positive, this is the velocity variation; the other direction is like this. This gives the velocity variation for this particular case of whenever two plates are moving the opposite direction. So you can find out here at this particular point you can see that there is a place where velocity will be 0. You can just substitute the velocity expression which we got to 0 and then we can find out what we do in this distance where this velocity will be becoming 0. We can substitute to the expression velocity is equal to 0 and then you will get the point.

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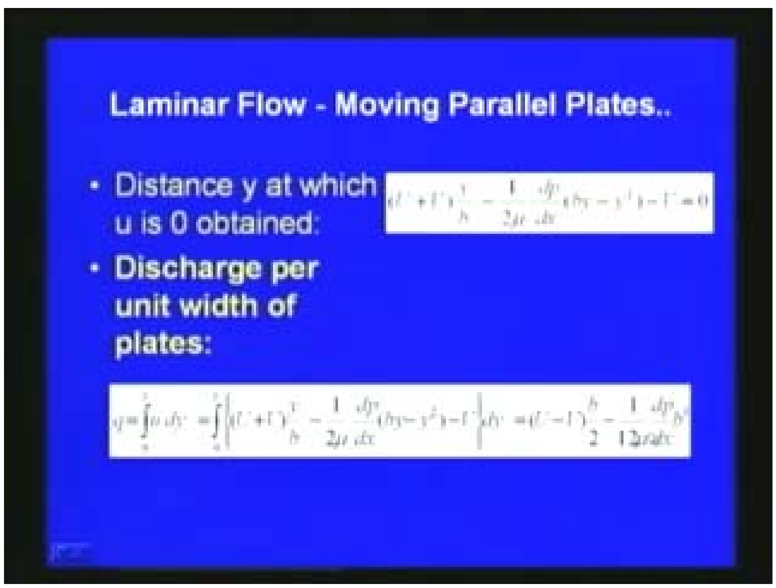

Then the discharge also as I mentioned you can just integrate between 0 to b integrate 0 to b u dy mu is known and then we can just get an expression like this. So, it is integrate 0 to b U plus V into y by b minus 1 by 2 mu dp by dx into by minus y square minus V dy. After integration we can get U minus V into b by 2 minus 1 by 12 mu dp by dx into b cube.

Like this we can find out q, the discharge per unit width of plate and then also we can find out the shear stress distribution we can just use the newtons law of viscosity tow is equal to mu into du by dy, u is known you can just take a derivative u with respect to y and then multiplied by the dynamic coefficient of viscosity we get an expression for the shear stress distribution.

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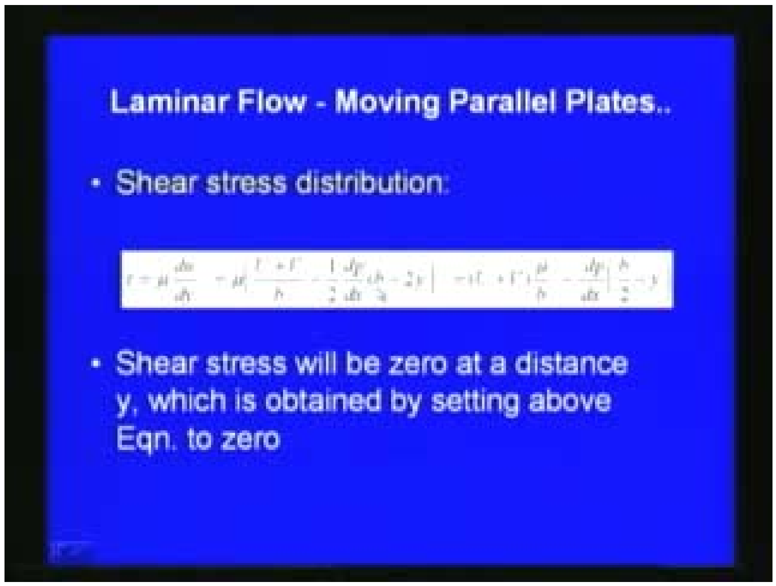

That we have shown in this slide, tow is equal to mu into du by dy plus mu into U plus V by b minus 1 by 2 into dp by dx into b minus 2 y s. This can be simplified as U plus V into mu by b so minus dp by dx into b by 2 minus y. Similar to this is a shear stress will be 0 at a distance y, which is obtained by setting above equation to 0. Very similar way we have seen in the case of velocity shear stress also between some locations it will be 0 we can equate the shear stress is 0 we have to find out where it will be 0.

So, in all this three cases what we have discussed so for is the laminar flow between two fixed parallel plates and then second case one plate is moving and third case both plates are moving in the opposite direction. Essentially, we are using the Newton’s second law of motion and then we consider a fluid element and then this are called first principle to derive the expression for velocity and other parameters from the first principle. Finally, we are getting an analytical expression for velocity and other parameters.

Before closing this section we will just discuss, as I mentioned earlier, the number of applications for these kinds of problems in fluid mechanics. We will just discuss one of the practical applications as an example what will be discussing here is coquette flow.

The coquette flow we have already seen that the lower plate is fixed and upper plate is moving and then how it can the application comes. We will be discussing it here.

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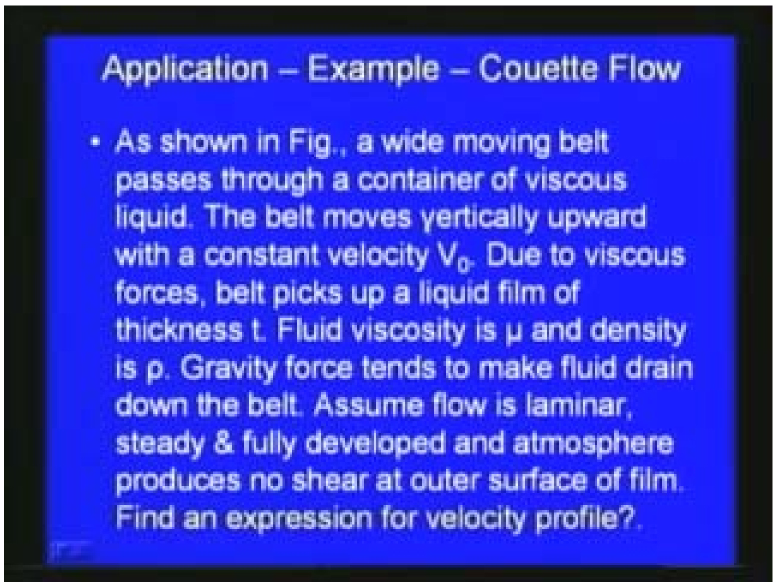

The problem here is as shown here: A wide moving belt passes through a container of viscous liquid. The belt moves vertically upward with a constant velocity V_0. Due to viscous forces, belt picks up a liquid film of thickness t. Fluid viscosity is mu and density is rho. Gravity force tends to make fluid drain down the belt. Assume flow is laminar, steady and fully developed and atmosphere produces no shear at outer surface of film. Find an expression for velocity profile?

This is a practical case, wherever a bath of chemical or tank of chemical is kept in fluid form is kept and then we just pass a belt through that and then that belt will be carrying. Since the belt is the moving plate and then the bottom of the tank is the fixed plate and then with respect to the movement of the belt it will be carrying some liquid and then that is one of the applications here. We want to find out an expression for the velocity for this particular case; once the velocity is known as we have seen earlier we can find out the shear stress distribution or we can find out how much is the carrying capacity or how much the belt will be carrying?

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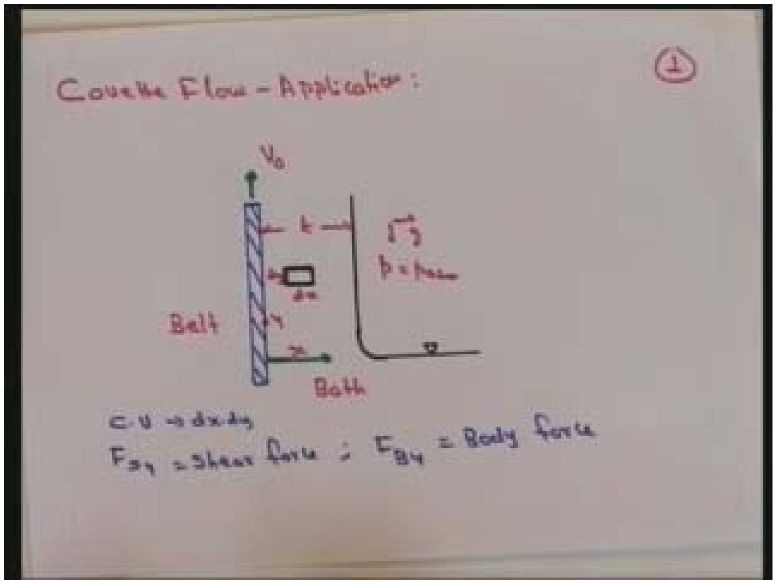

This particular case is explained here in this figure so here the application if coquette flow here. This as the numerical example discussed here. Now the belt is moving; this is the direction of the belt; here the belt is here the belt is moving with a velocity V_0 and here this is the tank where chemical in fluid form is got and then this is bath of fluid and then we can see that since the belt is moving it will carry some liquid with it. We want to find out an expression for the velocity. As described in the problem here some of the assumptions are assume flow is laminar steady and fully developed.

So these are the assumption and we are also assuming atmosphere produces no shear at outer surface of film. Here, this is the atmosphere which we consider; the belt is moving up. So we want to find out an expression for the velocity. As in the previous case here also we consider a fluid element of dx by dy; so this is x axis and this is the y axis.

To get an expression for the velocity, we consider a fluid element of size dx dy; the control volume is considered here dx dy. As we can see, here also the forces are compared to the earlier case there is body force also so shear force is there and then the body force; the forces are shear force and the body force.

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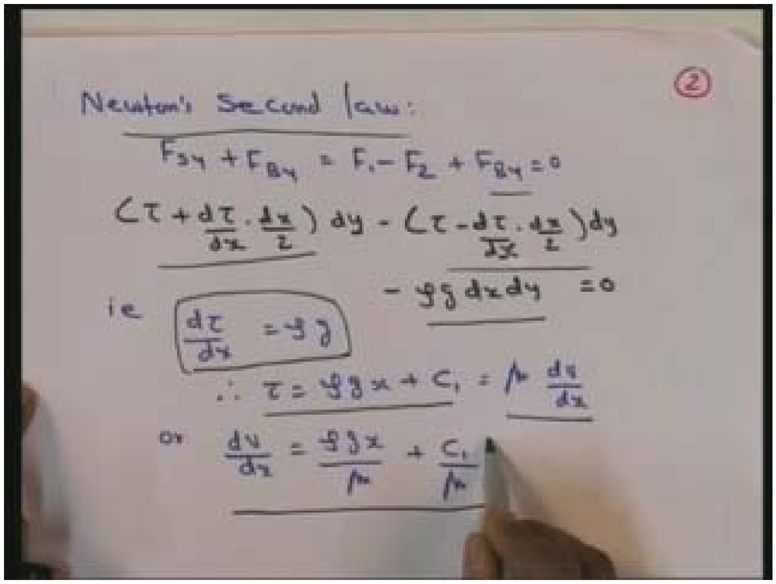

Now, with respect to this figure, if we use the Newton's second law then we can write F_{sy} plus F_{By} that means with respect to shear force and body force that will be equal to 0. With respect to the previous figure, if we consider this element, F_1 on both phases then F_1 and F_2 are the shear forces and then F_{By} is the body force. We can write the expression as tow plus d tow by dx into dx by 2 into dy minus tow minus d tow by dx into dx by 2 into dy. This is with respect to the fluid element which we consider tow is the shear stress minus rho g dx dy is equal to 0.

This is finally if we substitute the shear forces and body force with respect to the fluid element which we consider here. (Refer Slide Time: 35:58) this is the fluid element which we considered. Now, with respect to equating the body force and the shear force we get finally an expression d tow by dx is equal to rho into g, where rho is the mass, density g is the acceleration due to gravity and tow is the shear stress. So, this is the expression for this particular case which we consider here. So, d tow by dx is equal to rho into g and now once we integrate tow is equal to rho g x plus C_1, that is, after integration and that we can equate to tow is mu into dv by dx for this particular case Newton's law of viscosity. tow is equal to rho g x plus C_1 that is equal to mu into dv by dx. So that finally we can write dv by dx is equal to rho gx by mu plus C_1 by mu. Finally, we get an expression in terms of velocity. Our aim is to find out the velocity distribution; we get dv by dx is equal to rho gx by mu plus C_1 by mu.

So, this is the expression. Now, as in the previous cases, here again we can integrate; since we got an expression for dv by dx we can integrate to get an expression for the velocity. Once again if we integrate then we get v is equal to rho gx square by 2 mu plus C_1 mu into x plus C_2. This will be expression for the velocity; C_1v is equal to rho gx square by 2 mu plus C_1 by mu into x plus C_2.

Again, here we can have two boundary conditions to find out C_1 and C_2. The constant of integration C_1 and C_2, we can find out by using the boundary condition. For the particular problem is concerned we have seen that here this is one of the boundary. So, here at whenever at x is equal to 0 that means on the place of the belt you can see that x is equal to 0, and then the velocity is already known, V is equal to V_0; the belt is moving with a velocity V_0, at x is equal to 0, we can write v is equal to V_0. Then, it is given as per the assumptions we can get that at x is equal to t that means on the top of the fluid element which will be carrying there it will be shear stress, we can write tow is equal to 0; so that db by dx is equal to 0, so that first boundary condition x is equal to 0 v is equal to V_0; from which we can get C_2 is equal to V_0. The second boundary condition is at x is equal to t tow is equal to 0; so that we can write dv by dx is equal to 0 from which we can get C_2 is equal to minus rho g into t, where g is the acceleration due to gravity, rho is the mass density of the fluid. Now, C_1 and C_2 are obtained.

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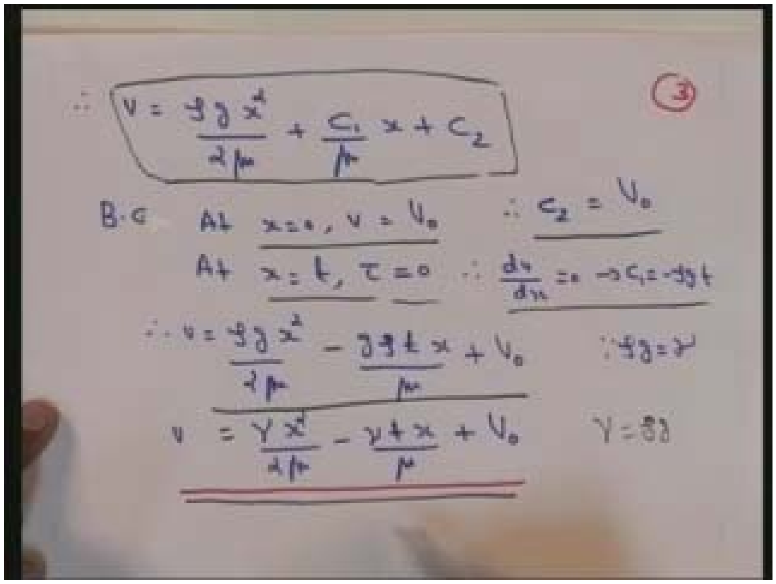

Finally, we can put back the C_2 and C_1 to the expression for the velocity. So, v is equal to rho g x square by 2 mu, C_1 is minus rho g t, so minus rho gt into x by mu plus C_1 is 0. So this is the expression for the velocity which can be simplified as v is equal to rho g is equal to gamma is equal to rho g. We can write v is equal to gamma x square by 2 mu minus gamma t into x by mu plus v_0. So this is the expression for the velocity.

The problem is again very similar to what we have seen in the earlier three cases. Here one practical application we are discussed; we will finally get an expression for the velocity and once we get an expression for the velocity, if you want to find out how much will be the carrying capacity or how much fluid will be carrying with respect to movement of that the belt the velocity is known so we can find out the flow rate per unit width.

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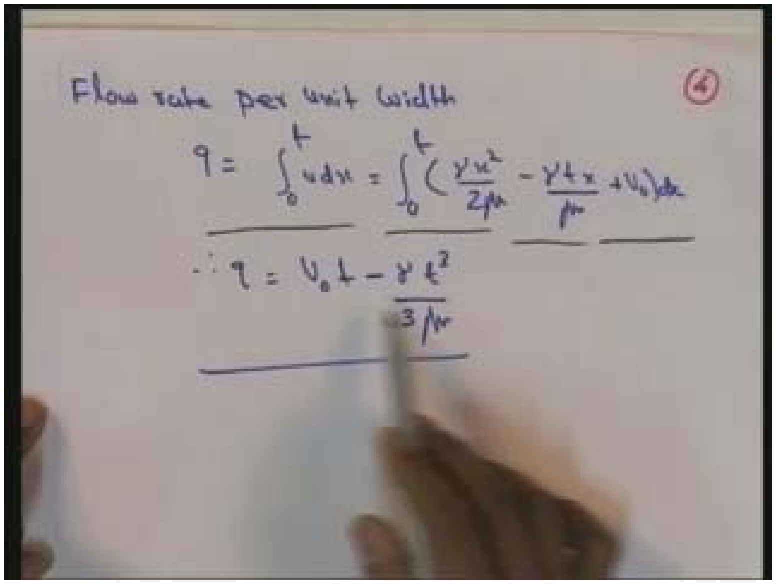

That is obtained as q is equal to integral 0 to t v dx as shown here, that is, integral 0 to t gamma x square 2 mu minus gamma t x by mu plus V_0 dx, from which we can get q is equal to V_0 into t minus gamma t cube by 3 mu. So, this gives t is this thickness. With respect to this how much fluid it can be carried, we get q is equal to V_0 into t minus gamma t cube by 3 mu. Like this we can get the expression for how much is the carrying capacity of the belt. Like this various problems we can solve. This is one of the applications of the coquette flow. There are various applications like this. Since most of the problems what we consider so far we are considering flow is steady laminar and then the incompressible fluid is considered.

With respect to these assumptions, we are using the first principle that means Newton's second law of motion and then we are using the Newton's law of viscosity to supplement it and then we are getting the expression for the velocity, shear stress and other parameters. So, this is the way which we can solve various kinds of problems. In all this cases, we are able to get exact solution such in the problem is simple compared to other cases. We are able to get finally the exact solution for velocity and other parameters.

This is about the first case: first part of this laminar and turbulent flow is laminar flow between parallel plates this we can see that here also lot of similarity can be practical cases like in an open channel; whenever a flow will be considered, flow in an open channel then we can if we consider both sides of the channel as fixed; then you can see how the flow is behaving and then with respect to as I mentioned the application of coquette flow whenever one plate is moving various applications in mechanical engineering, chemical engineering etc., as we have seen.

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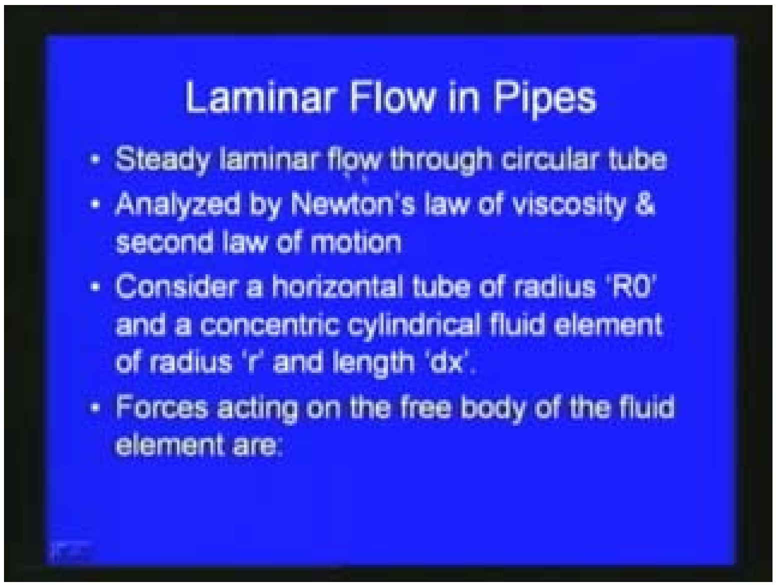

So, now the second part of this chapter: laminar and turbulent flow. We are discussing the laminar flow between pipes. Here, we are going to discuss the laminar flow in pipes so the pipe flow is considered. Now we consider here again steady laminar flow through circular tube.

First, we will be discussing the flow as I said laminar flow and then we will be discussing turbulent flow in a pipe. So, first cases laminar flow in pipe. We consider steady laminar flow through circular tube and here again we take the help of Newton's law of viscosity and second law of motion as we have seen in the previous cases.

If we consider just like here we can see that if this is a circular tube and flow is entering from here and flow is leaving from the other side; for this kind of simple pipes our aim is always to derive an expression for the velocity variation, velocity distribution of flow within the pipe and then we can derive an expression for shear stress. Also, we can find out how much is discharge passing through the pipe as we have seen in the earlier cases. If we consider a pipe this is a simple tube.

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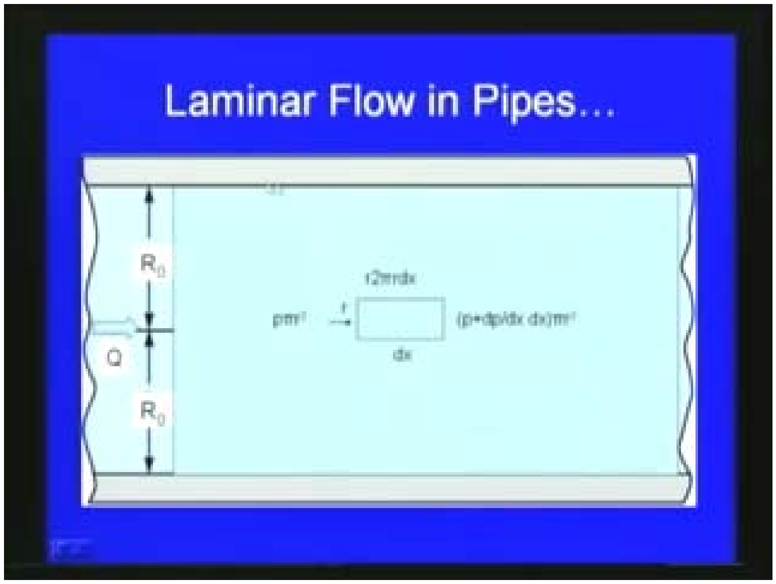

If we consider a simple tube like this and then consider a horizontal tube of radius R_0 and concentric cylindrical fluid element of radius r and length dx forces acting on the free body of the fluid element we are considering. Here, this is the case we consider a flow is this is the pipe so and the flow is coming from this direction and going like this; the pipe is of radius R_0 is the radius of the pipe. As in the previous case again here we consider a fluid element. This is the first principle as we have seen. We consider a control volume, a small control volume and then we see what happens in the control volume.

Here, in the case of pipe flow also, laminar flow in a pipe, we consider a small fluid element of size dx length and then from the central line of the pipe we consider the radius to that top of this is R. We consider to a small tube inside the pipe at R. If you for this fluid element are considered what are the forces acting and then once the forces are known here it is a steady state flow. So we can equate this two fluid by using Newton’s second law of motion, we can equate the forces to 0 since acceleration is not considered in this case also.

So, now if we consider a fluid element cylindrical, a concentric cylindrical fluid element of radius r and length dx and the radius of the pipe is R_0. This is the case here so the forces you can see for this fluid element is considered here the normal pressure force will be p into phi r square and on the other side it will be at distance dx will be p plus dp by dx into dx into phi r square; we have to consider the shear force so the shear force we can write with respect to this fluid element, we can write tow into shear stress multiplied by 2phi R dx; these are the forces.

In this particular case now you can see here we consider this pipe is horizontal pipe but then if the pipe is considered in an inclined position then we have to consider the gravitational effect also that will be considering later but at present we are considering the fluid pipe is horizontal and then we are deriving an expression for the velocity. So that is our aim.

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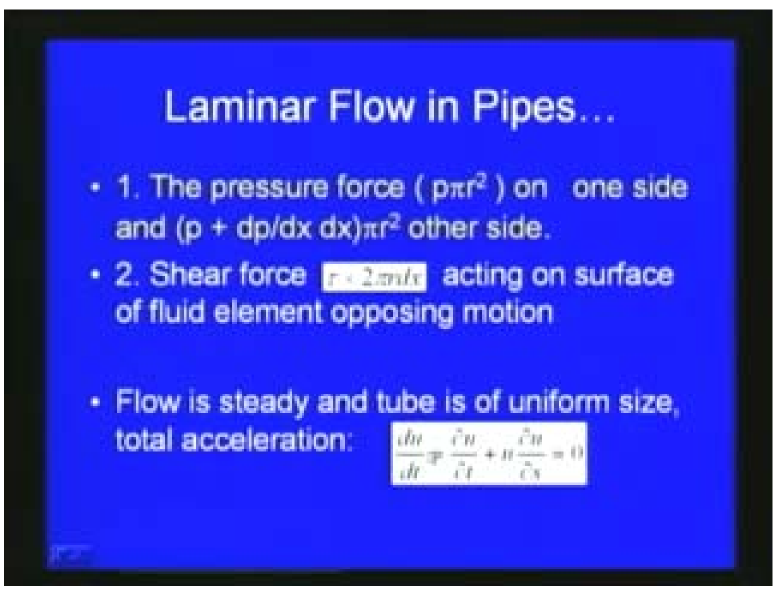

Here, the forces are the pressure force p into phi r square on one side and p plus dp by dx into dx into phi r square on other side. And then the shear force is tow into 2phi rdx acting on surface of the fluid element opposing the motion with respect to the shear force. As we have already assumed the flow is steady and tube is of uniform size; total acceleration is equal to 0 that we can write, du by dt is equal to delt u by delt t plus u into delt u by delt x is equal to 0. So the total acceleration is equal to 0. So that the Newton's second law of motion we can just equate the total forces to 0. Since Newton's second law states force is equal to mass into acceleration so as per our assumption acceleration is equal to 0.

We can equate total forces to 0. So here with respect to the fluid element p into phi r square minus p plus dp by dx into dx phi r square minus tow into 2phi r dx is equal to 0, so the pressure force and shear force. Then, we can simplify this as tow is equal to minus dp by dx into r by 2. So this expression, the force is equated to 0 so that we can equate as tow is equal to minus dp by dx into r by 2.

As we have seen in the previous case again we can utilize the Newton's law of viscosity tow is equal to…(49:16). So, here the flow is taking place in the right-hand direction so we can write tow is equal to minus mu into du by dr. So, the shear stress we can equate tow is equal to minus mu into du by dr. Once you substitute this expression tow is equal to mu into du by dr we can get du by dr is equal to 1 by mu dp by dx into r by 2. So this is the expression for tow is substituted back du by dr is equal to 1 by mu dp by dx into r by 2. Again, we get the velocity gradient du by dr with respect to r. As in the previous case again we can integrate to get an expression for the velocity and there will be the constant of integration can be obtained by utilizing the boundary conditions.

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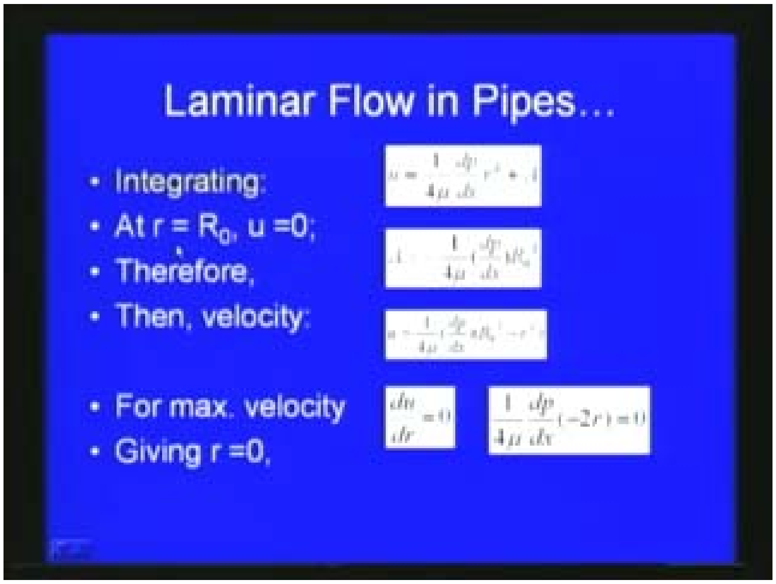

Now, on integration we get mu is equal to 1 by 4 mu dp by dx r square plus A. So, the boundary conditions here are at r; here one boundary condition is only one is known so one boundary conditions so at r is equal to R_0. That means when the flow takes place you can see that here the fluid is going like this and passing like this. Due to no slip boundary condition on the boundary of the tube or the boundary of the pipe we can assume the velocity is zero. So, at r is equal to R_0 on the boundary of the tube or the pipe we will get mu is equal to 0. Once we substitute back we get the integration here A is equal to 1 by 4 mu dp by dx into R_0 square. Then, finally, we can put it back this expression for A into here the expression for velocity so u is equal to 1 by 4 mu dp by dx into r square minus small r square. This is the expression for the velocity. From this we can find out where the maximum velocity takes place that we can get as to differentiate and equate to 0 du by dx is equal to 0. We can find out where the maximum velocity takes place so that is equal to 1 by 4 mu dp by dx into minus 2r is equal to 0 or r is equal to 0.

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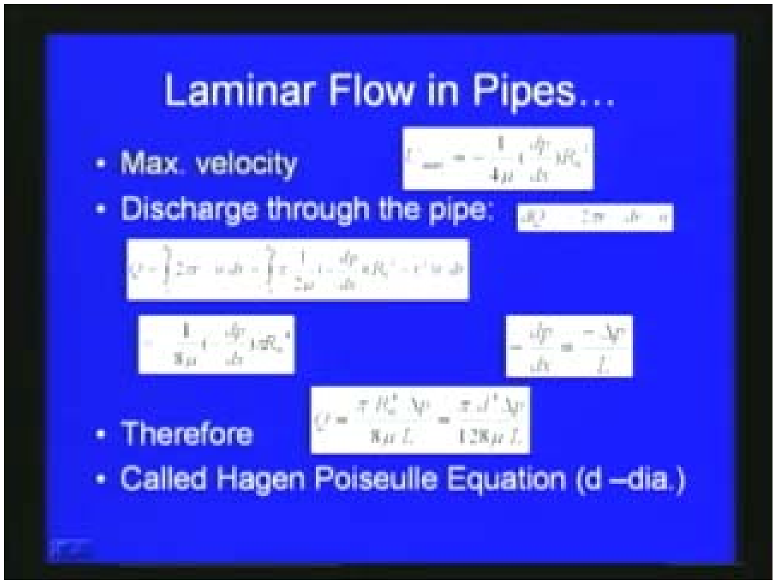

We can see at the center will be the maximum velocity so that is obvious for pipe flow and then the expression for maximum velocity will be in the previous case if you put small r is equal to 0 we get u_{max} is equal to 1 by 4 mu dp by dx into R_0 square and then very similar way discharge through the pipe we can get dq is equal to 2phi r dr into du.

So, here we can integrate between 0 to R_0 Q is equal to 2phi r into u into dr. So that is 0 to R_0 phi into 1 by 2mu minus dp by dx into R_0 square minus r square into r dr. From which we will get this is equal to 1 by 8 mu minus dp by dx into phi into R_0 to the power 4 and if we substitute minus dp by dx as minus delta p by L, the pressure drop.

Then we will get Q is equal to the discharge is equal to phi into R_0 to the power 4 into delta p by 8 mu L, that is equal to in terms of diameter phi d to the power 4 delta p by 128 mu L.

This equation is called Hagen poiseuillie equation, here d is the diameter. So the expression what we got the velocity or the discharge is called Hagen poiseuillie equation for pipe flow and then as in the earlier case we can get the shear stress tow 0 is equal to minus delta p by L into R_0 by 2 so that we will get an expression for the velocity at the pipe. Also, we can find out the average velocity through pipe. So the total discharge is known and then if you divide by the area of power section we get the average velocity. In this particular case, we will get V is equal to delta p into R_0 square by 8 mu L and from which also one of the parameter is obtained. Like velocity you can find out other parameter. So, the pressure drop also we can obtain from this expression for the average velocity as delta p is equal to 8 mu L by R_0 square that is equal to 32 mu L by d square. From this we can get the energy loss per unit weight of fluid as delta p by gamma, gamma is 32 mu VL by rho g d square. Like this various parameters we can derive and then we will be discussing for laminar flow in pipes and then further problems will be discussed in the next lecture.