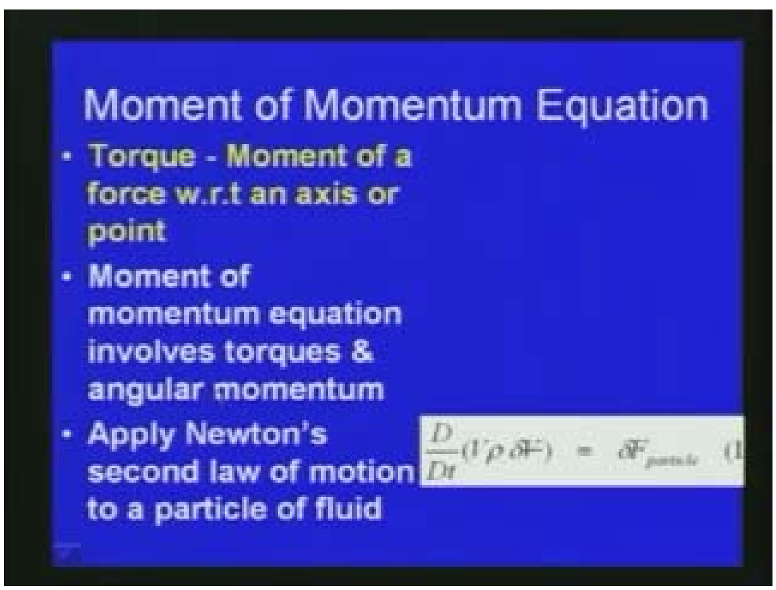
Fluid Mechanics

Prof. T. I. Eldho
Department of Civil Engineering
Indian Institute of Technology, Bombay

Lecture - 16
Dynamics of Fluid Flow

Welcome back to the video course on fluid mechanics. In the last lecture we were discussing about the linear momentum equations and its applications. We have seen the linear momentum equations and the various kinds of applications like, when I judge striking on a plate how the momentum equation can be utilized for the purpose of calculating various parameters and the force applied etc.

(Refer Slide Time: 01:20)

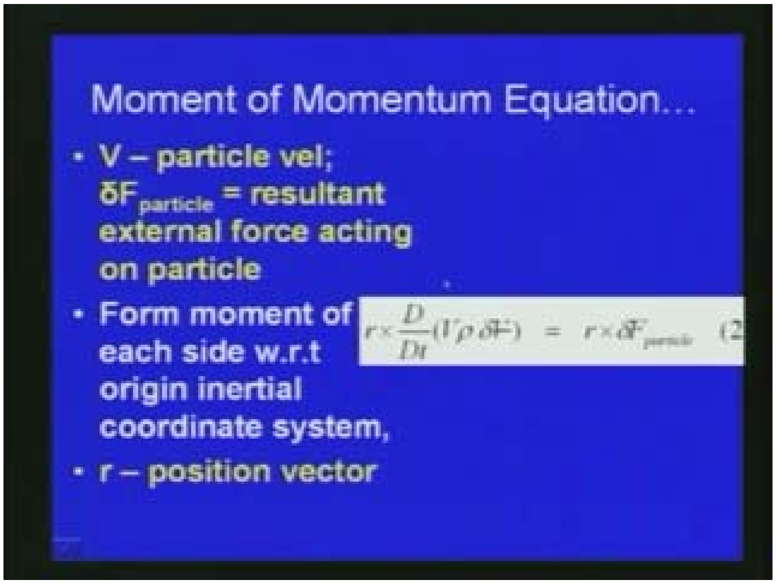


Now in this lecture, we will be discussing the moment of momentum equation. What we have seen in the linear momentum, the direct moment can be considered but in this fluid mechanics problem we will be discussing the moment of momentum that means, the angular momentum. The moment of momentum generally, what will be coming it involves torques and angular momentum; torque means the moment of a force with respect to an axis or a point.

When we consider the moment of a force with respect to a fluid or a control volume where we consider the force, the torque means, the torque of that fluid mass, which is the moment of the force with respect to an axis or a particular point. The moment of momentum what we are considering is what kinds of torques are developing with respect to fluid systems and then how the angular momentum is developed. That is what we are going to discuss in today's lecture.

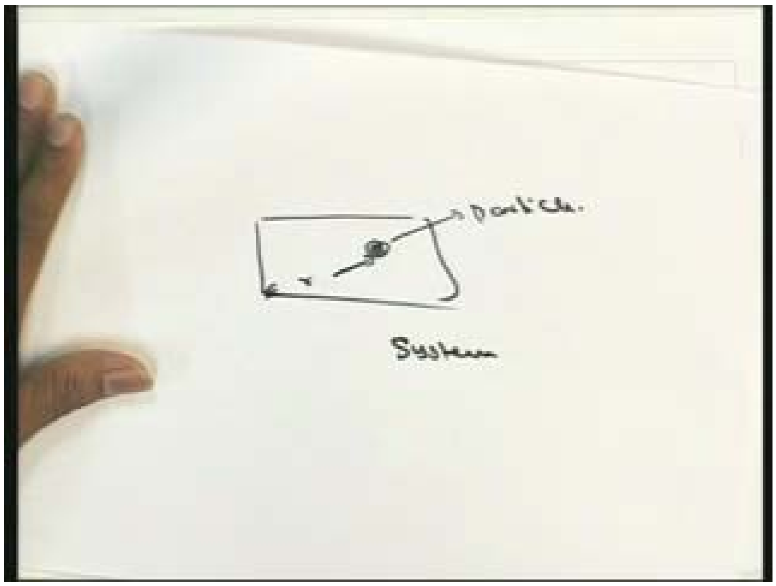
Now, to develop this moment of momentum equations we will be again using Newton's second law of motion to a particle of a fluid. Let us consider a control volume which we considered earlier. Now if we consider the total derivative d by dt of V rho delta V where delta V is the corresponding volume that should be equal to the effective force on that particle. With respect to Newton's second law we can write the rate of change of momentum that means D by Dt rho V delta delta v that should be equal to force on that particle, so that gives in this equation number one.

(Refer Slide Time: 02:58)



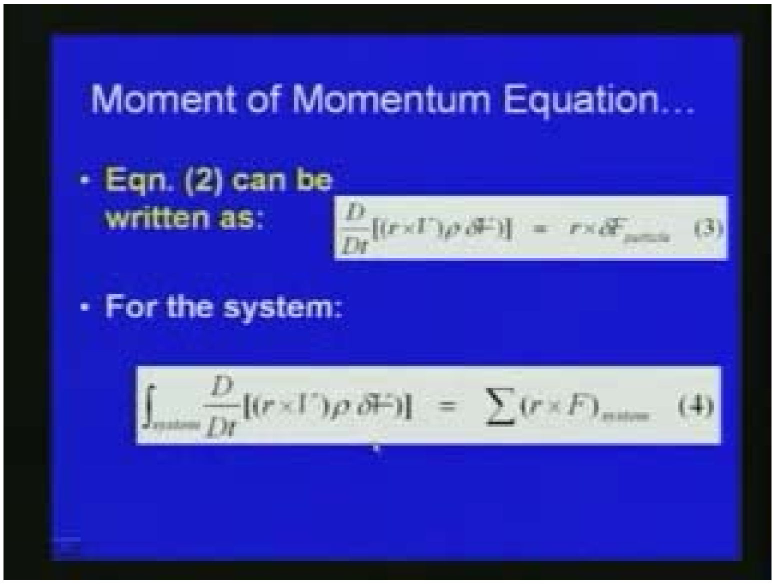
Now where, V is the particle velocity, delta F the resultant external force acting on that particle. We will form moment of each side with respect to the origin with respect to the inertial coordinate system. If we consider with respect to the earlier equation D by Dt V rho delta V is equal to delta $F_{particle}$ with force on that particle, so now if we form moment of each side with respect to the original inertial coordinate system, we can write r cross D by Dt V rho delta V that is equal to r cross delta $F_{particle}$ that means, force on the particles, as in equation number 2 where, r is the position vector which we considered.

(Refer Slide Time: 03:56)



Now, with respect to what we are discussing is we can see here, if we consider a fluid control volume like this and then if we consider particular particles see the position vector from which we are considering, so this is r and then we are taking the angular momentum. First, we are considering we have the rho V into delta V that gives the momentum and then we are considering (Refer Slide Time: 04:16) weight of the change of momentum that gives the rate of change of the momentum so that we are equating to r cross delta D by Dt V rho delta V is equal to r cross delta F particles where r is the position vector as shown in this figure.

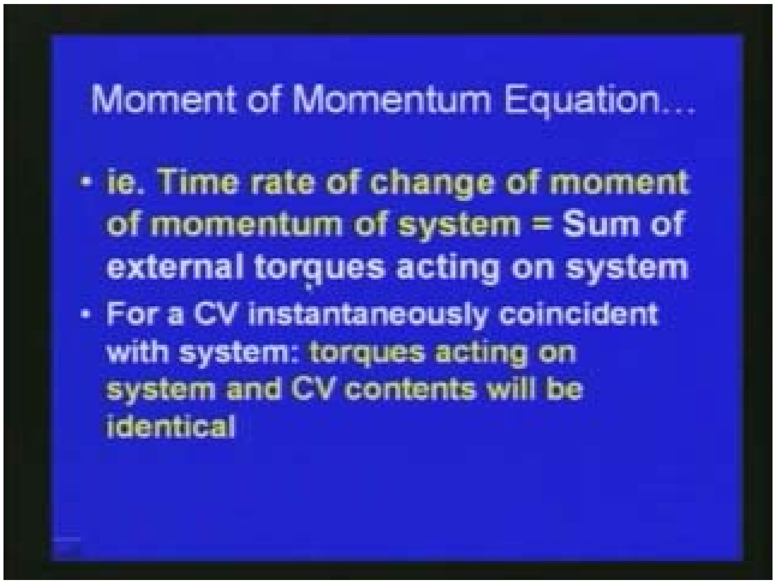
(Refer Slide Time: 04:39)



This equation number 2 with respect to the earlier equation we can write like this D by Dt the total derivative D by Dt r cross V rho delta V where rho is the mass density, V is the velocity of the particle, r is the position vector. That should be equal to r cross delta $F_{particle}$ where delta F is the force acting on the particular particle. This is as far as the particle which we considered here in a system, so if this is the system we are considering for the derivation of the equation.

Now this is the particular particle we are considering then what is its torque or its moment of momentum? So, now let us consider the total system; for the total system is concerned what we can do is we can integrate for the system D by Dt r cross V rho delta v that should be equal to integral r cross F for the system. We are considering the total system, earlier we considered for a particular particle, what are the force acting now for the total system we can integrate with respect to the left hand side of the equation number 3 and then we can sum up all the forces throughout the particle, so that sigma r cross F for the system. So, this is the moment of momentum equation for a system.

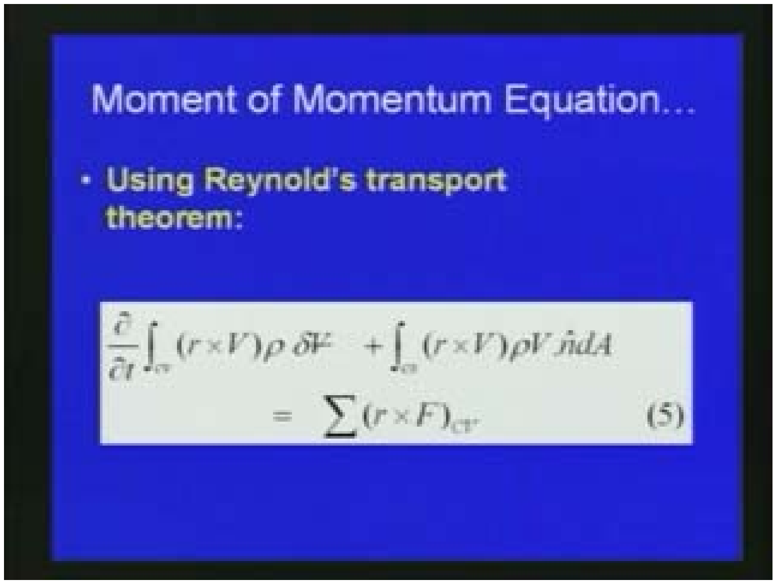
(Refer Slide Time: 06:09)



We can see that the time rate of change from the earlier equation what was written here in equation number 4 we can see, the time rate of change of moment of momentum of system is equal to sum of the external torques acting on the system. So what give this equation number 4 is you can see it gives the time rate of change of moment of momentum of the system that we are equating to the sum of the external torques acting on the system.

For a control volume instantaneously coincident with the system we can see that the torques acting on the system and control volume contents will be identical. The control volume which will consider here (Refer Slide Time: 06:50) that will be the control volume for the particular system which we are considering the torques acting on the system will be identical with control volume contents what we have considered the earlier system of the equations.

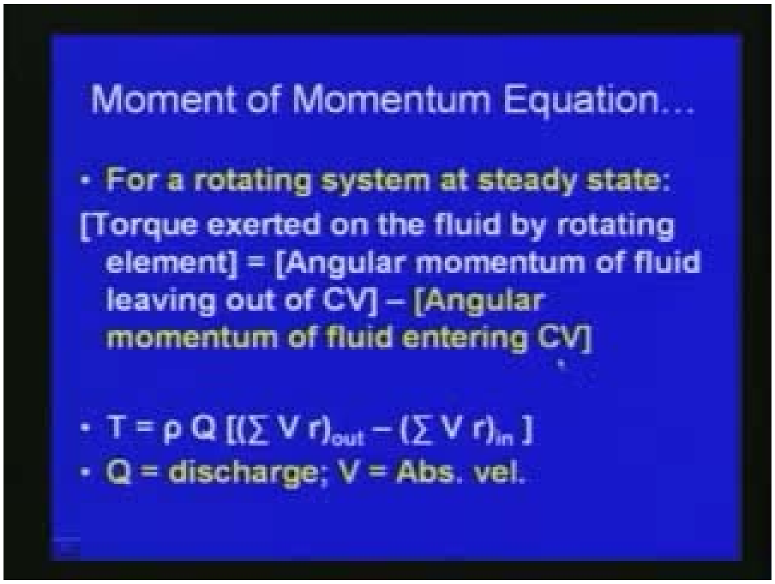
(Refer Slide Time: 07:05)



Earlier, we have seen the Reynold's transport theorem. We have already discussed the applications of Reynold's transport theorem, in many problems it is used to derive the basic equations. If we use the Reynold's transport equation to derive the final equation for the moment of momentum equation, we can write with respect to the previous equation number 4.

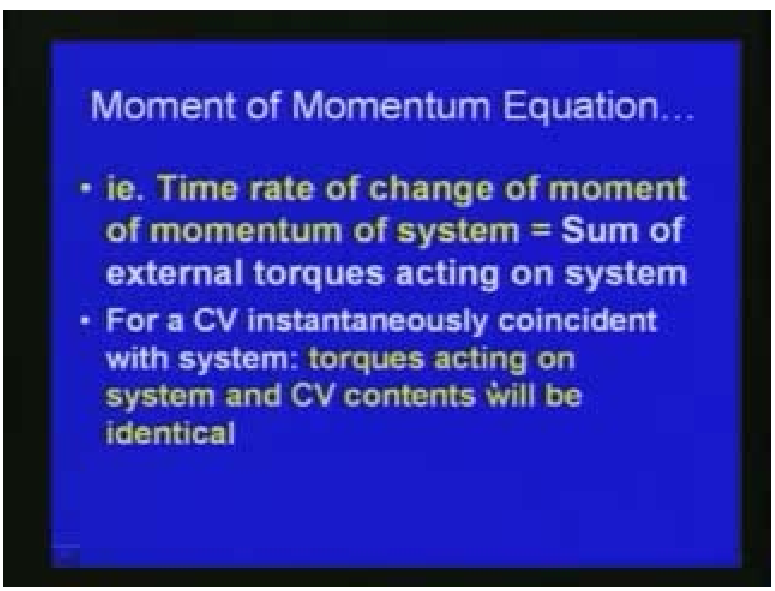
So now if we use the Reynold's transport theorem we can write delta by delta t for the control volume r cross V rho delta V so, this indicates the volume plus integral for the control surface r cross V rho V dot ndA that is equal to sigma r cross F for the control volume. The earlier equation number 4 we have used the Reynold's transport theorem to derive for the system, as we can see here, what happen if we consider the control volume inside or then on the surfaces what happens, so that is what we are doing on a surfaces control surfaces what happens and then what happens control volume so that is the basic principle we have utilized in most of the earlier derivations with respect to the Reynold's transport theorem. Finally, we get this equation number 5.

(Refer Slide Time: 08:32)



This equation number 5 is applicable for unsteady state flow. If we consider a steady state flow system we can express this equation as for a rotating system at steady state we can write the torque exerted on the fluid by the rotating element is equal to angular momentum of fluid leaving out of the control volume minus angular moment of fluid entering the control volume.

(Refer Slide Time: 09:03)

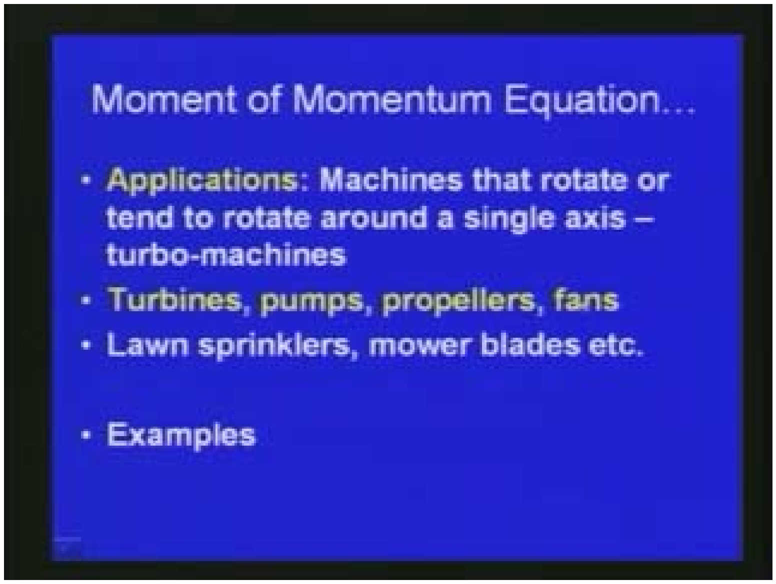


Finally, the same thing (Refer Slide Time: 09:07) what we have seen in the equation number 5 which is applicable for the for the unsteady flow conditions but if we consider the steady flow condition we can say that, for the rotating system, as we have seen here the moment of momentum equation we are deriving for system where, angular velocity or angular momentum or a rotational system like a turbines or a pumps or a that kind of system or volume lawn sprinkler, these kinds of system we are using in moment of momentum equations. If we consider for a rotating system at steady state condition we can write finally that the torque exerted on the fluid by rotating element is equal to the angular moment fluid leaving out of the control volume minus the angular moment of fluid entering the control volume.

So finally (Refer Slide Time: 10:05) this T the torque can be can be written as in this equation here T is equal to rho Q into sigma V into r out minus sigma V r in so where rho is the mass density, T is the torque, rho is the mask density, Q is the discharge, V is the absolute velocity, and r is the position vector or where the system is concerned that we are considering r as the position vector so this gives the final equation with respect to the moment of momentum equation for the steady state flow conditions.

So, the same thing (Refer Slide Time: 10:43) for unsteady state, the general equation is given in equation number 5.

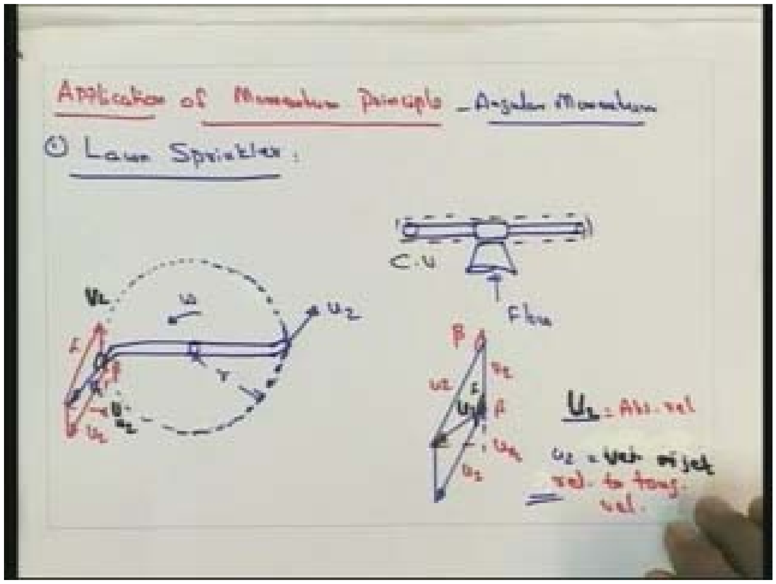
(Refer Slide Time: 10:47)



This moment of momentum equations have got large number of applications in fluid mechanics or engineering problems say wherever angular velocity of angular momentum is very important. If we consider for example, the motion of turbines and then what happens to the fluid flow taking place with respect to the turbine or while considering flowing in a pump so how the system is working or many other kinds of system where the angular momentum or the angular velocities coming to picture So where this moment of momentum equations have the large number of applications.

This we can apply for the machines that rotate or tend to rotate around a single axis just like in turbo machines, or as we have discussed turbines, pumps, propellers, fans, all these places lawn sprinklers, mower blades, etc. all these places we can utilize this moment of momentum or principle of moment of momentum equations either at steady state or unsteady state. We will be discussing few of the applications now with respect to the moment of momentum equations.

(Refer Slide Time: 12:05)

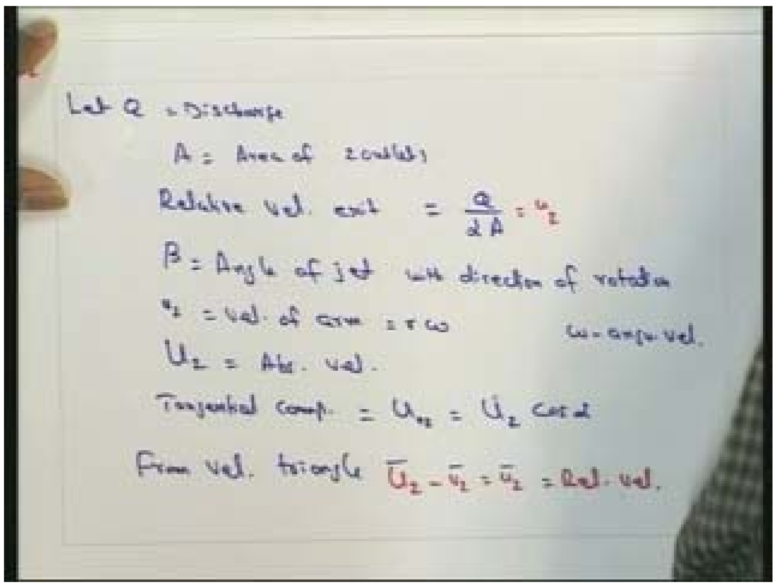


First one, what we are going to consider is application with respect to a lawn sprinklers. We will consider a single system here which is so called lawn sprinkler. All of you know, how a lawn sprinkler works so you can see that the water will be coming through a small pipe and then there will be rotating arm like this and then there will be two jets at the end if the water is coming through a pipe like this and then you can see that there would be two jets at the end and then it will be rotating like this, so this is so called the lawn sprinkler.

Here, you can see in this figure how we are just going to derive the system equation with respect to the lawn sprinkler in particular, you can see the water of the fluid is coming in this direction, flow is coming in this direction and there is a rotating arm here and at the both ends of the rotating arms there are small nozzles through which the water will be coming out. Now, we can consider a control volume like this, once the water flow is through this system the sprinkler system you can see that the water will be going this direction and the other direction through the two nozzles and then the system will be rotating.

This is called lawn sprinkler. If we consider the figure here you can see this u_2 is the absolute velocity of the water flowing through the nozzle water coming through jet action here. Similarly, on this side also, if we consider symmetric case this will be u_2 here also it will be u_2 and then the rotation of the arm if it is omega and from the central line r is the distance to the wave of the rotating arm angle, where r is shown here and then if we consider the velocity the polar vector diagram with respect to this moment of the fluid or with respect to moment of the rotating arm we can see that if this angle is alpha and here this total angle is beta and u_2 is the velocity of jet related to the tangential velocity. We can draw the velocity triangle like this.

(Refer Slide Time: 14:48)

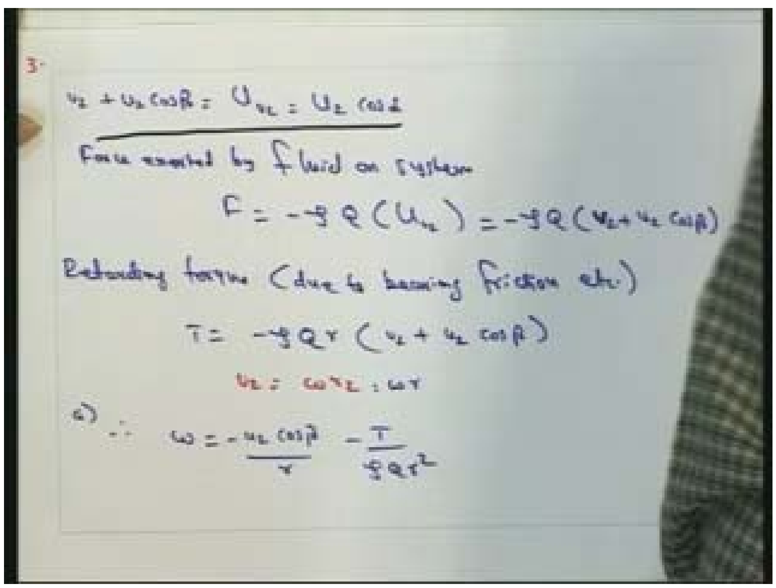


With respect to this we will be further explain how we can find out the various parameters like the torque or if one parameter is given how to determine the other parameter. Let us for this particular system (Refer slide Time: 14:59) which we consider here so let Q be the discharge coming through the pipe so that Q is divided into two nozzles. If A is the area of two outlets, so relative velocity at exit will be Q by 2A. (Refer Slide Time: 15:19) Here flow of water is coming through pipe here which is discharge is Q and that is divided into two nozzles a and b and then it is going through the two outlets. If A is the area of the two outlets then relative velocity at exit will be Q by 2A, so that is equal to $u_{2.}$

With respect to this figure this beta, (Refer Slide Time: 15:43) beta is the angle of jet with direction of rotation and then v_2 is the velocity of arm which can be (Refer Slide Time: 15:55) put as r into omega where r is the distance and omega is the rotations fluid of the arm, v_2 is equal to r into omega where omega is the angular velocity and u_2 is the absolute velocity as we have seen and then its tangential component (Refer Slide Time: 16:08) here, you can see that if this is u_2 then its tangential component you can just to draw u_2.

The tangential component u_{v2} is equal to u_2 cos alpha and from the velocity (Refer Slide Time: 1625) triangle drawn here, this is the velocity of triangle, we can draw u_2 bar minus v_2 bar is equal to u_2 bar that is the relative velocity.

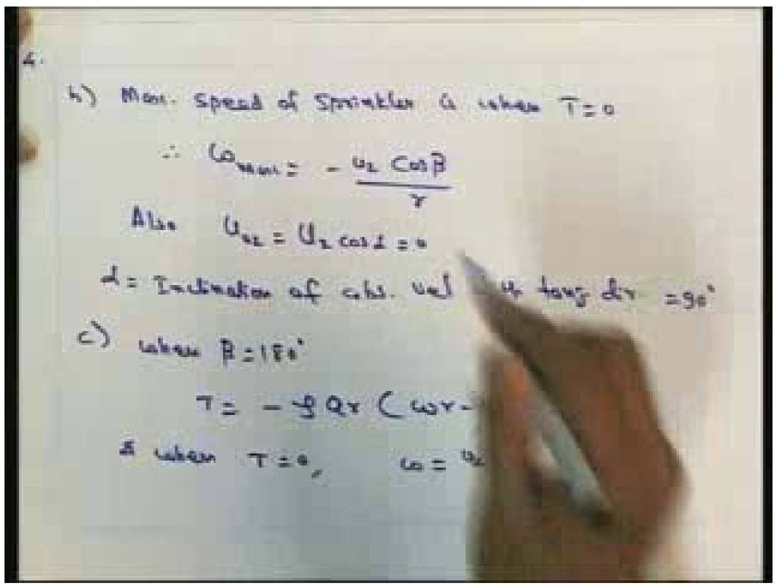
(Refer Slide Time: 16:44)



With respect to the velocity of triangle we can write v_2 plus u_2 cos beta is equal to u_{v2} that is equal to u_2 cos alpha. With respect to the system (Refer Slide Time: 16:51) here, we will find out the force exerted by fluid on the system so that can be written as F is equal to minus rho Q u_{v2} where u_{v2} is the relative velocity, so F is equal to minus rho Q u_{v2} that is equal to minus rho Q into u_2 plus v_2 cos beta where Q is the discharge, rho is the mass density. We can write the retarding torque due to the bearing friction etcetera we can write T is equal to rho Q r into v_2 plus u_2 cos beta where v_2 is equal to omega r_2 that is, equal to omega r.

If we want to find out (Refer Slide Time: 17:38) for this system, what is the angular velocity of the rotation, omega we can write as omega is equal to minus u_2 cos beta by r minus T by rho Q r squared where T is torque which we have derived. So omega is equal to we can get the omega is equal to minus u_2 cos beta by r minus T by rho Q r squared. This is the omega the angular velocity of the system.

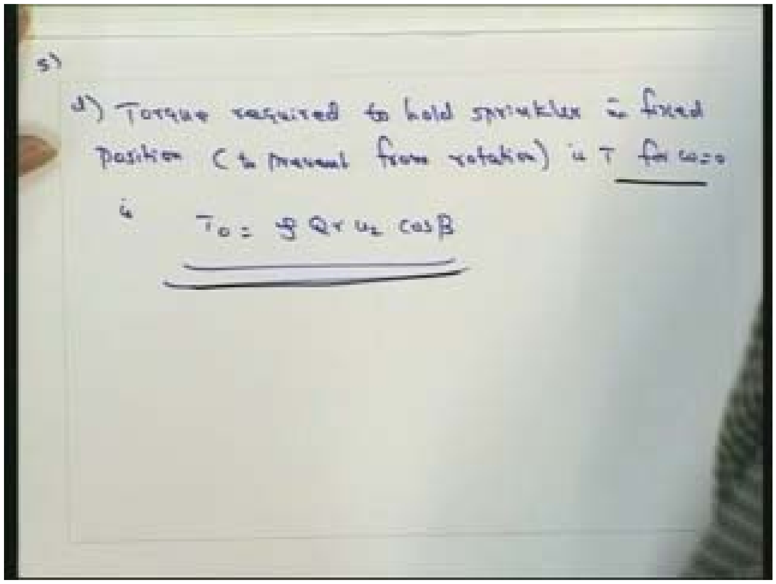
(Refer Slide Time: 18:05)



If you want to find out the maximum speed of sprinkler when torque is equal to 0 we can write omega$_{max}$ is equal to minus u_2 cos beta by r also we can write u_{v2} is equal to u_2 cos alpha is equal to 0 where, alpha is the angular of inclination of absolute velocity with respect to tangential direction which is equal to 90 degree here, and now with respect to the earlier figure (Refer Slide Time: 18:35) when this angle beta is equal to this angle beta is equal to 180 degree.

We can write T is equal to minus rho Q r into omega r minus u_2 and now when T is equal to 0 we will be getting omega is equal to v_2 by r.

(Refer Slide Time: 19:21)

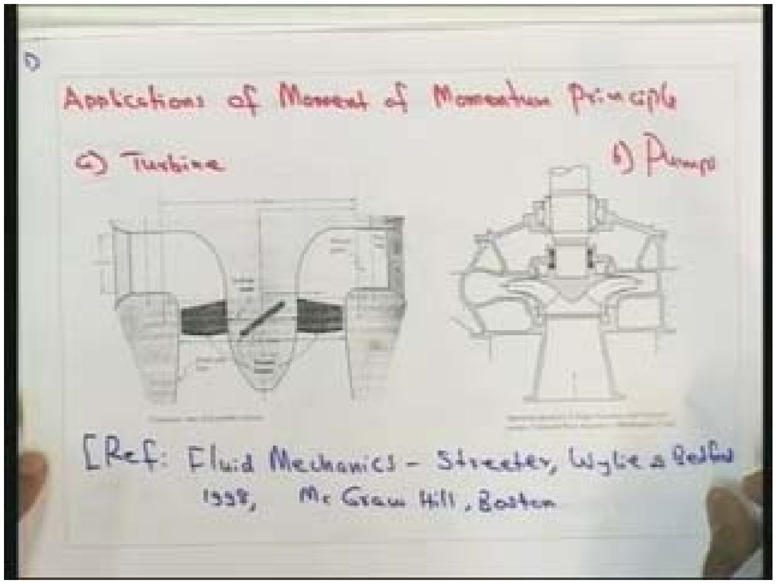


If we want to find out the torque required to hold the sprinkler in fixed position to prevent from rotation, we can write T is when omega is equal to 0, so we can write T_0 that is the torque required to hold sprinkler in fixed position T_0 is equal to rho Q r u_2 cos beta.

So like this for this system which we have seen here which is a lawn sprinkler which is a simple system, (Refer Slide Time: 19:30) we can find out the torque if we want to find out the angular velocity of the rotating arm, we can find out various parameter using the moment of momentum or angular momentum principle. With the moment of momentum equation we can find out the various parameters. This is one of the simple applications of this moment of momentum equation as in the case of a lawn sprinkler.

Now, other than this as we have discussed there is large number of applications we would also be discussing some of the applications like what happens in a turbine or in a pump.

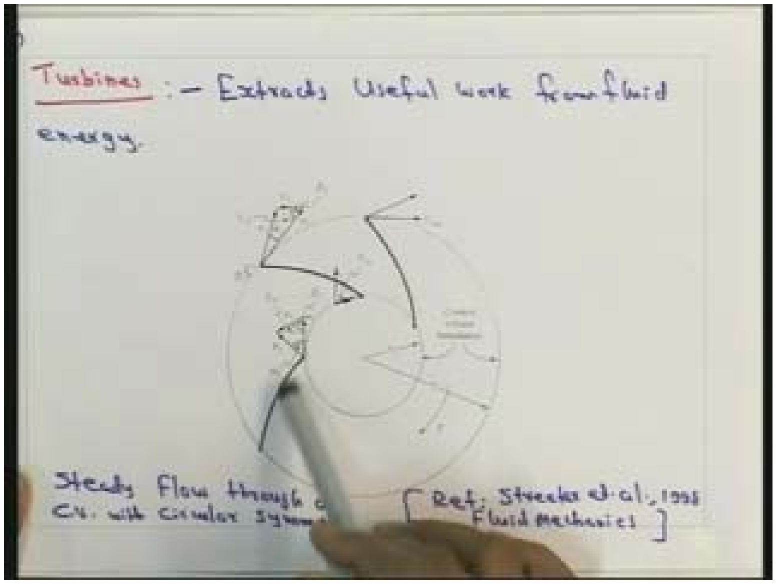
(Refer Slide Time: 20:37)



Here, we can see that the simple principle where the steady state flow condition is prevailing and then how the moment of momentum equation is applied. So we can see this figure is taken from fluid mechanics Streeter Wylie and Bedford Mcgraw Hill publication Boston. You can see how the application moment of momentum principle in a turbine case, we can see that the turbine is concerned, we can see when the water is coming and heating on the wanes of the turbine it starts to rotate and then that transplant the power to the rotation affect in the magnetic field, we get the power that is, what is happening in the turbine.

The other application is in the case of a pump, we can use pump to extract water from the well. We can see that we are using the turbine to generate power, but in the case of pumps we are using power to rotate the pump and then we are raising either water over any other fluid which we are considering. So this moment of momentum principle can be utilized to derive various relationships either in the case of a turbine or pumps or in the case of various turbo machines.

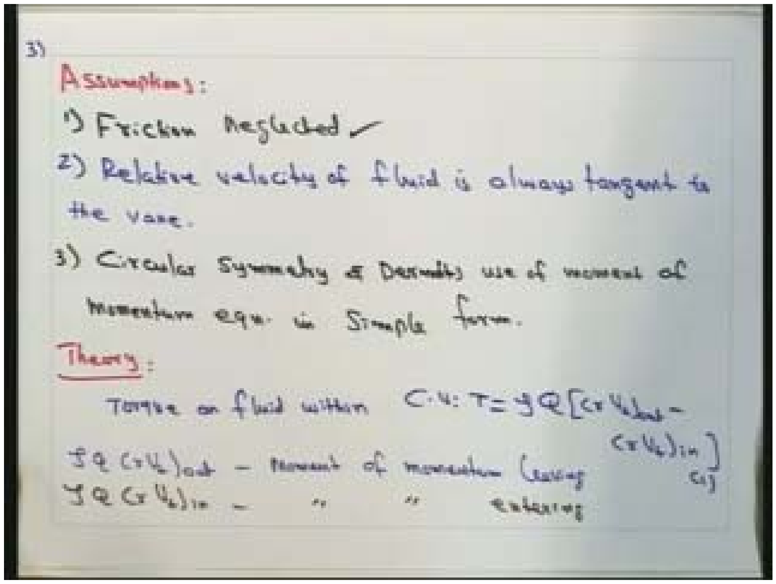
(Refer Slide Time: 21:42)



Let us consider the case of a turbine as discussed as shown in this figure. In the case of a turbine it is as we have seen in the earlier figure (Refer Slide Time: 21:56) these are all rotating mechanisms. In the case of turbine, let us see how this moment of momentum principle can be utilized. So here, the turbines actually you can see that turbine extract the useful work from fluid energy, as I mentioned in the case of turbine the large velocity jet velocities hitting on the blades of the turbine are there, it is rotating the magnetic field and then we are extracting the roes that means, in the case of turbines it extracts useful work from the fluid energy. So that is what we are doing in the case of turbine.

Let us consider this steady flow through a control volume with circular symmetry. This is where the various blades or mounted on an axle like this in the case of a turbine. This figuring is also taken from Streeter at 1998 fluid mechanics book. Various blades are located like this and now we are considering the control volume boundaries like this so this is one boundary, this is second boundary and then the from the central line of the axle this r_1 is the distance to the internal boundary r_2 is the distance to the external boundary. If we consider a particular blade where the fluid is heating whether water jetter or any other kind of fluid is heating here, alpha$_1$ is this angle and the v_1 is this velocity with respect to this blade and v_{r1} is the radial velocity and we can see that we can have tangential component and also the radial component and with respect to this figure how to derive the various how to apply this moment of momentum equations that we will be discussing here.

(Refer Slide Time: 24:00)



Before going for the how to utilize this moment of momentum principle or equation use certain assumptions here in the case of turbine or a pump, the assumptions used here are the friction force is neglected. The friction force with respect to what is happening here is neglected and relative velocity of fluid is always tangent to the vane. (Refer Slide Time: 24:29) We can see the various vanes attached to or mounted with respect to this axle so it set relative velocity of fluid, the relative velocity of fluid is always tangent to the vane and then the third assumption is circular symmetry is there and then it permits the use of moment of momentum equation in the simple form just as the steady state form which we have seen.

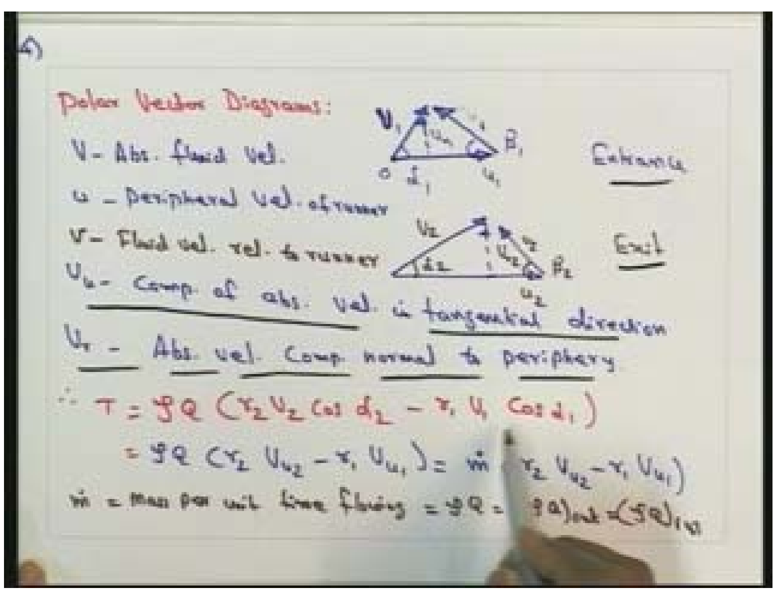
These are some of the essential assumptions used in the derivation or the application of this moment of momentum principle. Now to derive the relationship let us consider the torque on fluid within the control volume.(Refer Slide Time: 25:07) We are now considering the control volume here we can see the control volume with respect to the boundary and the blades attached, with respect to this control volume the torque of fluid within the control volume T is equal to rho Q r into V_t out minus r into v_t in where v_t is the tangential velocity with respect to out and with respect to in. (Refer Slide Time: 25:29) This is the v_t out here and then v_t in is considered here, so this is v_t the tangential velocity in and this tangential velocity out. The torque on the fluid, now we can see (Refer Slide Time: 25:44) that when the turbine is rotating, the fluid this entire boundary the control volume will be filled with fluid and then it is rotating.

Now a torque on the fluid within the control volume which we consider T is equal to if rho is the mass density, Q is the discharge, and then r is this either r_1 or r_2 of this r is the radius to the external or internal boundary which we consider, so T is equal to torque is equal to rho Q r into v_t out minus r into v_t in so where v_t is the tangential velocity.

Where here this rho Q r into v_t out that gives the moment of momentum leaving from the system and rho Q into r into v_t in that gives the moment of momentum entering the system which we consider. Here as we can see (Refer Slide Time: 26:40) this is a control volume which we are considering so there will be momentum entering and the momentum leaving that gives the difference is taken and then we are finding out the torque on fluid within the control volume.

So rho Q r into v_t out so that gives the moment of momentum leaving and rho Q r into v_t in gives the moment of momentum entering the control volume which we consider.

(Refer Slide Time: 27:10)



As in the previous case of the sprinkler system here also we consider the polar vector diagram. If we consider the polar vector diagram for the entrance is concerned we are as shown this figure (Refer Slide Time: 27:22) if we consider the entrance that means with respect to this system here. For this, the polar vector diagram can be drawn here where, v is the absolute fluid velocity, u is the peripheral velocity of the runner, (Refer Slide Time: 27:38) this is the runner which we consider, now this v_1 is the velocity of this direction and $beta_1$ is this angle and the u_1 corresponds for the with respect to this subscript one indicates the entrance and subscript two indicates the exit. V is the absolute fluid velocity, u is the peripheral velocity of runner and v is the fluid velocity of relative to the runner.

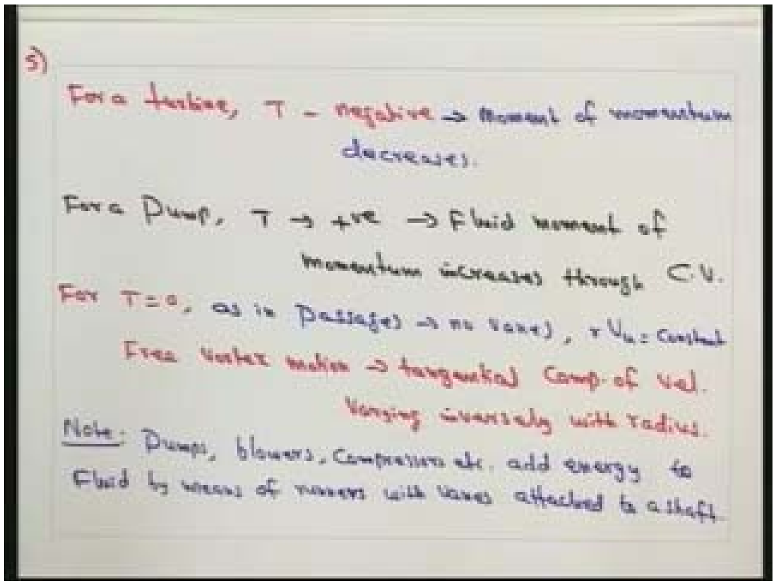
This v_1 is the absolute fluid velocity which we consider for the entrance is concerned and then u_1 is the with respect to peripheral velocity of the runner which we consider here and then v_{r1} that is the radial velocity with respect to the system which we consider, for exit conditions are concerned this gives v_2 which is absolute fluid velocity with respect to the exit. The u_2 is with respect to the peripheral velocity of the runner and $beta_2$ is corresponding angle and v_u is the component of the absolute velocity in tangential direction and v_r gives the absolute velocity component normal to the periphery.

We can see that here (Refer Slide Time: 28:57) the tangential component and then the radial component is considered. So v_r gives the absolute velocity component normal to the periphery. Finally, with respect to this system which we consider here we can write T is equal to rho Q r_2 v_2 cos $alpha_2$ and this $alpha_2$ is drawn here minus r_1 v_1 cos $alpha_1$.

That gives the torque for the system that is equal to rho Q r_2 V_{u2} with respect to the system we consider V_{u2} are defined minus r_1 $V_{u1,}$ where this if we define n dash is the mass per unit time flowing that is equal to rho Q we can write rho Q out will be equal to rho Q in, since the control volume is considering whatever the flow going in will be that will be coming out from the system.

Finally, the torque which we are considering here torque on the fluid within the control volume we can write T is equal to rho Q into r_2 v_2 cos alpha minus r_1 v_1 cos $alpha_1$ that is equal to rho Q into r_2 V_{u2} minus r_1 V_{u1}. Finally, this can be written as m dash which is the mass per unit time, flowing into r_2 V_{u2} minus r_1 V_{u1}.

(Refer Slide Time: 30:23)



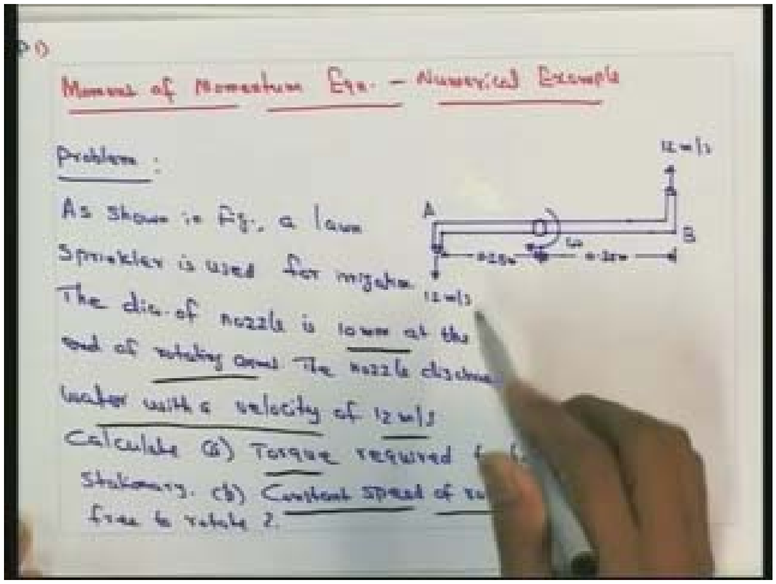
We have actually the principle what we have discussed here is applicable for the case of a turbine or a pump or a propeller whatever which we consider. In the case of a turbine we can see that the torque will be negative, the moment of momentum moment of momentum decreases. As I mentioned the fluid with high velocities entering to the turbine and then that is trying to rotate the turbine. In the case of a turbine we can see that as far as the fluid is concerned the moment of momentum decreases, so that the torque will be negative; but in the case of a pump which we have considered, we are applying electricity or the power to the pump, then pump is giving instead of the turbine which the turbine extract power from the fluid. In the case of pump, what the pump doing is it is impacting for power to the momentum to the fluid which it is pumping. For a pump T will be positive so fluid moment of momentum increases through the control volume.

In the case of a turbine we can see that moment of momentum decreases. This is the essential difference between turbines on a pump, the moment of momentum decreases, for a turbine moment of momentum increases for a pump through the control volume which we consider. If T is equal to 0 as in passages where there are no vanes we can write we can see that this rV_u will be constant. We can see that a free vertex motion which is tangential component of the velocity varying inversely with respect to radius is formed. This is called as free vertex motion where T is equal to 0 as in the case of passages where no vanes. From the discussion we can see that pumps, blowers, compressors etc. add the energy to the fluid by means of runner with vanes attached to a shaft but in the case of turbine it is extracting energy from the fluid moving. This way we can utilize the moment of momentum equation or the moment of momentum principle which we discussed here with respect to the angular velocity, angular momentum what we have discussed we can utilize in the case of turbo machines turbines, pumps, then blowers, compressors, propellers etcetera.

Initially, we have seen a simple system just like in a sprinkler system and then we have seen in the case of pump or a turbine where the moment of momentum principle can be utilized. Here, we can see that we have considered a steady state situation only with various assumptions which we have discussed. Before closing this section we will be discussing one more numerical example. We have seen various applications of the moment of momentum principle or moment of momentum equation.

We have also seen the applications like sprinkler system or rotating sprinkler system or in the case of pump or a turbine. (Refer Slide Time: 33:40) Now, we will see a numerical example to further understand how we are utilizing this moment of momentum equation or the moment of momentum principle.

(Refer Slide Time: 34:10)

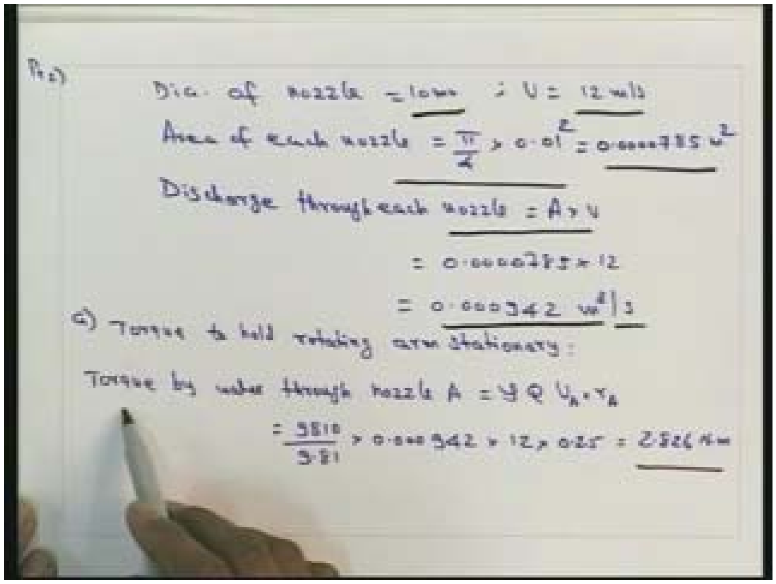


The numerical example which we are considering here is again a sprinkler system with a rotating arm with respect to two nozzles. Here, the problem is as shown in figure a lawn sprinkler is used for irrigation, so the diameter of nozzle is 10 millimeter, at the end of the rotating arms the nozzle discharges water with velocity of 12 meter per second. We have to find out the torque required to hold rotating arms stationary and then constant speed of rotation of the arm, if it is free to rotate. So this is the problem.

Here, we can seen a sprinkler system, the arm length is from the axis where the water supplied from that it is 25 centimeter to the left and 35 centimeter to the right. There are two nozzles, one nozzle at A and another at B and from the nozzle the velocity is given as 12 meter per second on this side and the other side is also 12 meter per second in the opposite directions as shown in this figure. Then the nozzle diameter is given as 10 millimeter at the end of the rotating arm and the velocity is 12 meter per second.

We have to calculate, the torque required to hold the rotating system stationary and then we have to find out the constant speed of rotation of the arm.

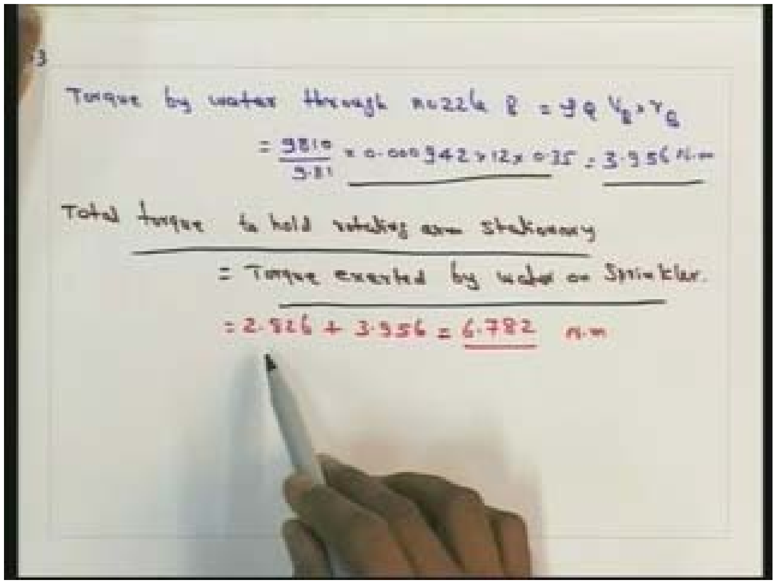
(Refer Slide Time: 35:35)



We saw this problem by using the moment of momentum equation or principle, which we have derived in the case of sprinkler system or general system which we have seen. Here, the diameter nozzle is given as 10 millimeter and velocity is given as 12 meter per second; area of each nozzle we can calculate pi by 4 into 0.01 square that will give the value of the area each nozzle as 0.0000785 meter square and then we can find out discharge through the nozzle since the velocity is given.

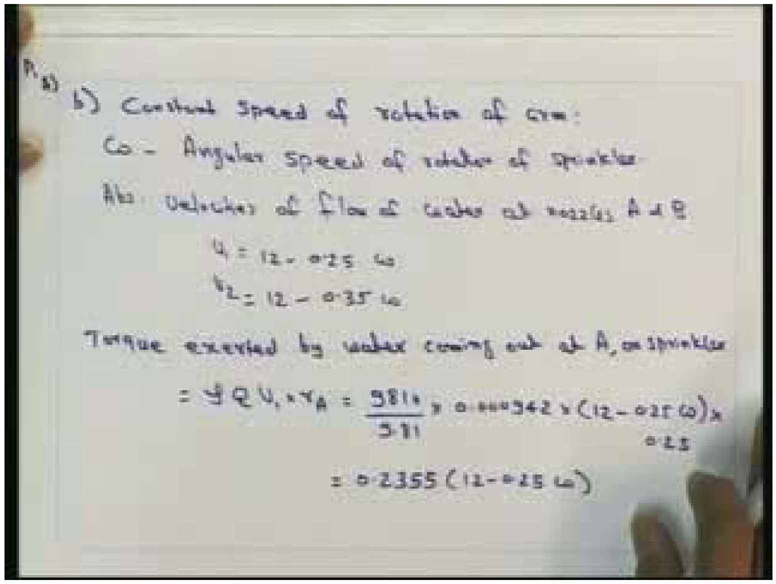
Area of section is known, so we will get discharge through the nozzle A into v so that gives the 0.000942 meter cube per second. The first part of the problem is the simple problem is to find out the torque to hold rotating arm stationary, so torque by water through the nozzle A we can write rho Q into v_A into r_A rho is the mass density, Q is the discharge coming through the system, v_A is the velocity at A and r_A is the distance from the axis to the nozzle at A. This we can write as rho Q into v_A into r_A, rho is we can write like this so, here rho is 9810 by 9.81 and Q is also found into that multiplied by the velocity v_A 12 into (Refer Slide Time: 37:13) the distance, here you can see the distance is 0.25 meter from this location from here to here, we get the torque by water through the nozzle as 2.826 Newton meter, so that gives the torque.

(Refer Slide Time: 37:28)



Now torque by water through the nozzle B so the for the second nozzle here the second nozzle is here (Refer Slide Time: 37:35) at this location so that gives we can write the same equation rho Q v_B into $r_{B,}$ so r_B is 35 centimeter here and then that gives rho is 9810 by 9.81 into Q is 0.000942 into $v_B$is again 12 into 0.35 is 3.956 Newton meter and finally, we can find the total torque to hold the rotating arm stationary, that will be equal to torque exerted by water on the sprinkler. Now you can add both cases 2.826 plus 3.956 that will give 6.782 Newton meters, that is, the torque required to hold the rotating arm stationary.

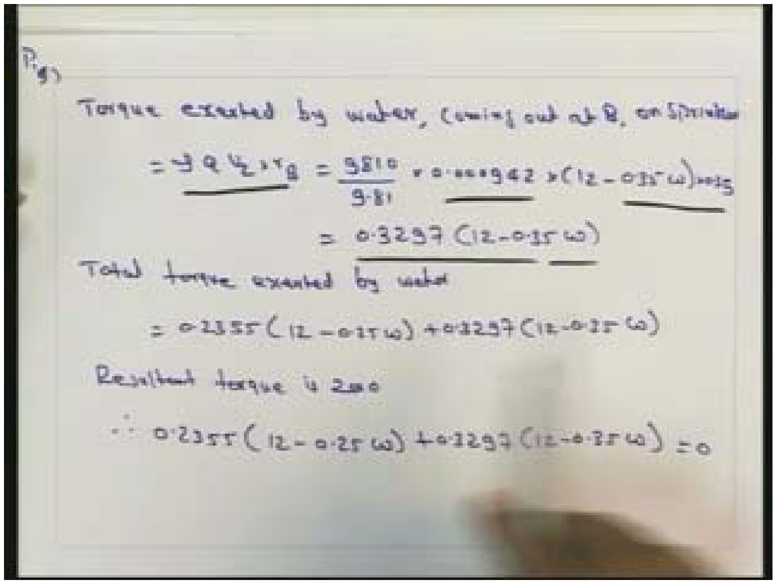
(Refer Slide Time: 38:16)



That is the first part of the simple problem, the second part of the simple problem is we have to find out the constant speed of rotation of the arm. If omega is the angular speed of rotation of the sprinkler, we can write absolute velocity of flow of water at nozzles at A and B can be written as v_1 is equal to 12 minus this 0.25 is the distance here, so v_1 is equal to 12 minus 0.25 into omega where this 0.25 is the distance from the axis to the nozzle A 0.25 omega and similarly, v_2 will be v_2 at this location will be 12 minus 0.35 omega since 35 centimeter this distance.

So now the torque exerted by the water coming out at A on sprinkler we can write rho Q v_1 r_A so that is 9810 by 9.81 into the discharge, so this v_1 we will substitute here, 12 minus 0.25 omega into 0.25. So that will give 0.2355 into 12 minus 0.25 omega.

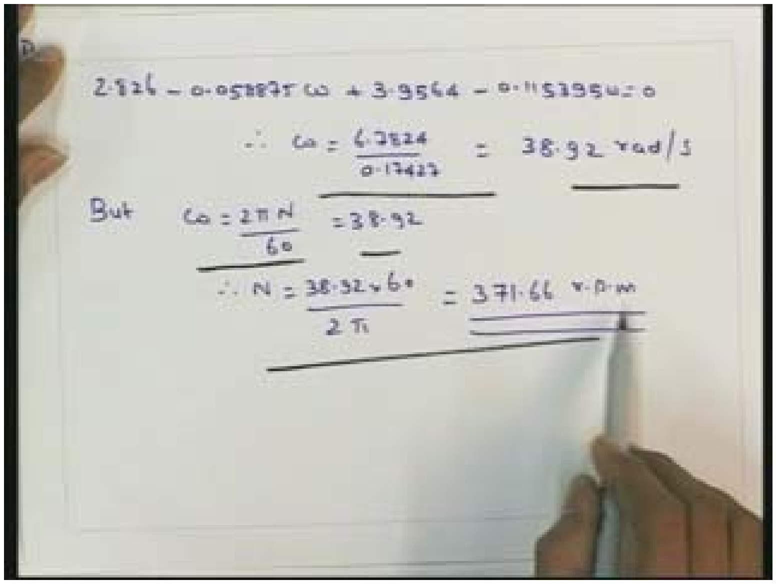
(Refer Slide Time: 39:42)



The torque exerted by water coming out of B on sprinkler, rho Q v_2 into r_B we can find out 9810 by 9.81 into the discharge Q into 12 minus 0.35 omega into 0.35; that is equal to 0.3297 into 12 minus 0.35 omega.

The total torque exerted by water we can find out by adding what happens at (Refer Slide Time: 40:09) location A and at location B, so we can add total torque exerted by water so that is equal to 0.2355 into 12 minus 0.25 omega plus 0.3297 into 12 minus 0.35 omega. (Refer Slide Time: 40:29) Now the question is we want to find out the constant speed of rotation of arm if free to rotate, we can see that resultant torque would be equal to 0 about this condition, so we can equate this to 0 from which we can find out omega, the angular velocity of rotating arm.

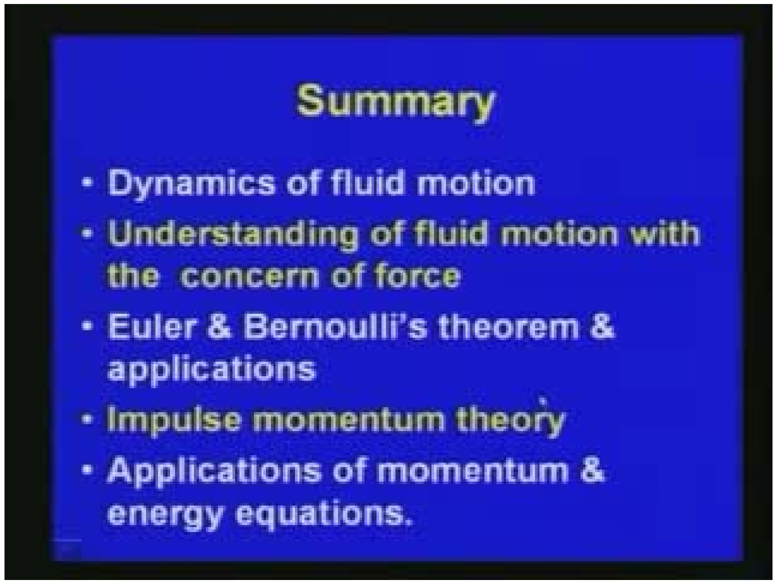
(Refer Slide Time: 40:44)



We finally, if we solve the problem we get omega as 38.92 radians per second. If we want to find out r pm (revolutions per minute) we can find out omega is equal to 2 pi N by 60, so that is equal to 38.92. That is N, we can find out N is equal to 38.92 into 60 by 2pi, from which we can find out it will be approximately 372 revolutions per minute.

So, that gives the angular velocity and the speed of rotation. This is a simple problem of the case which we considered is a sprinkler system with a rotating arm, so, we are trying to find out the torque accepted and then we were trying to find out the speed of rotation of the arm. In a similar way, we can utilize the moment of momentum equation in an affective way to solve a number of problems, either it can be the sprinkler system which is one simple most problem or the same way we can solve the problems, like in the case of turbine or in the pumps where we can approximate into steady state conditions or even in the case of unsteady flow also we can consider by using this (Refer Slide Time: 42:12) general equation as given in this equation number 5, where we can consider the general equation, this is wherever unsteady state condition is also considered. These are the some of the important say, how we are getting the moment of momentum equation and where we can apply various cases we have seen.

(Refer Slide Time: 42:48)



The last few lectures we were discussing the dynamics of fluid motion. Now we will summarize what we have seen in this particular chapter on the dynamics of fluid motions. To summarize, we have seen the various problems, various cases and how the system with respect to dynamics of fluid motion in this particular chapter. Here what we tried to do is to understand the fluid motions with the concern of force.

The dynamics of fluid motion we considered here, the force which is acting upon this control volume but in this dynamics of fluid motion, with respect to the force we are trying to understand the fluid motion. As we have seen at the beginning we have discussed about how to derive the wireless equation, what are the applications of wireless equation and also we have seen the Bernoulli's theorem. We have derived the Bernoulli's theorem with respect to flow condition but in many practical cases we can utilize the Bernoulli's equations. Further, we can extend to the general energy equations that also we have seen as in the various problems. In this dynamics of fluid flow, which we have seen in this particular chapter, various applications, we have introduced the problem with respect to various systems. Then we have seen the wireless equation Bernoulli's equations and its applications, then we discussed what is happening with respect to say, we have derived the linear momentum equations and for the linear momentum equations how the various applications we have discussed. Finally, we have seen the with respect to wherever the case where the angular momentum or the moment of momentum principle is applied. Wherever in the case of turbine turbo machines or pumps or in the simple system of a sprinkler as we have seen in the last case in the moment of momentum equation and its applications.

These are some of the important system as to summarize, what we have done in the dynamics of fluid motion. What we have seen is after the dynamics of fluid motion we will be discussing various fluid mechanics systems like a laminar flow, or turbulent flow in detail. We will be deriving various fundamental equations for the various systems and then further we will be discussing the advance topics on the fluid mechanics.

The next chapter we will be discussing is laminar flow and turbulent flow, its theory and then in the application as far as pipe flow is concerned, laminar flow in pipes turbine flowing pipes we will be discussing in the next section.