

Fluid Mechanics
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Lecture 15
Kinematics of Fluid Flow

Welcome back to the video course on fluid mechanics. In the last lecture in the topic of dynamics fluid dynamics we were discussing about dynamics of fluid flow we were discussing about the moment principle and its applications.

We have seen that in the case of steady state system with respect to the linear moment principle from the Newton's second law, we have derived the basic equation as $\sum F_x$ is equal to $\rho Q \beta_2 V_{x2}$ minus $\beta_1 V_x$.

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Apply Momentum Principle to Flow Through The Generalized System

- $\sum F = \text{Momentum out} - \text{Momentum in}$
- $\sum F_x = (\rho_2 A_2 V_2) V_{x2} - (\rho_1 A_1 V_1) V_{x1}$
- **Momentum coefficient β introduced**
- $\sum F_x = \rho Q (\beta_2 V_{x2} - \beta_1 V_{x1})$

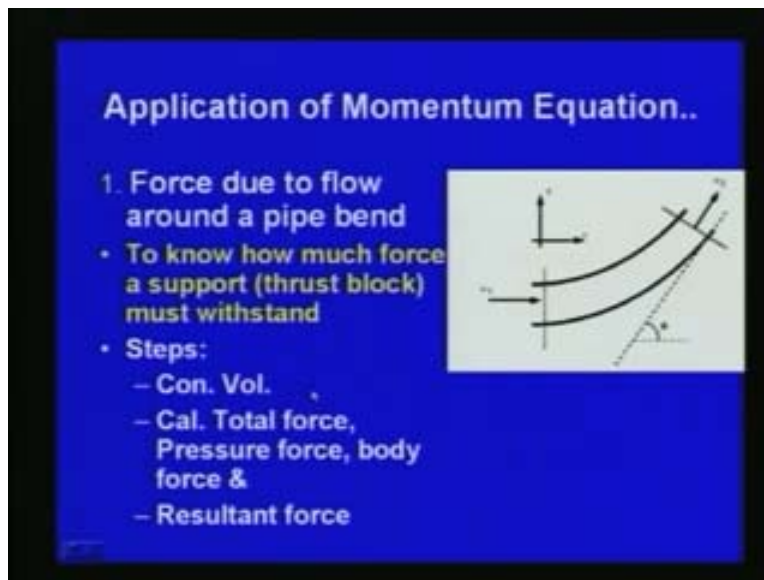
$$\beta = \frac{1}{A} \int_A \left(\frac{v}{V} \right)^2 dA$$

Where this $\sum F_x$ is the total force acting upon the control volume external force on the control volume that is equal to the linear moment change with respect to system this we have seen where β is the moment coefficients correct or moment correction factor as we have seen.

We were discussing about the applications of moment equations we have seen that the linear moment principle. We can utilize for various problems like wherever to find out the force due to flow of fluid around a pipe bend enlargement and contraction or we can use the principle to find out the force on the nozzle on the outlet of the pipe or the impact of a jet and force due to flow around a curved vane etcetera this different types of problem you can use this linear.

Moment principle or linear moment equations, that we were discussing in the last lecture and we have seen me of the applications first case what we have discussed earlier was the force due to flow around a pipe bend we have seen.

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Application of Momentum Equation..

1. Force due to flow around a pipe bend
 - To know how much force a support (thrust block) must withstand
 - Steps:
 - Con. Vol.
 - Cal. Total force, Pressure force, body force &
 - Resultant force

The diagram shows a pipe bend with an inlet velocity v_1 and an outlet velocity v_2 . A control volume is defined by a dashed line. The angle of the bend is labeled θ .

We will be considering as basic steps the basic steps involved or we will be considering control volume, we will be calculating the total force on the system and including the pressure force body force we will be finding the resultant force. We will be equating with respect to the moment change that is what we have seen and we have derived the equation for one of the example force due to flow around a pipe bend in the last lecture.

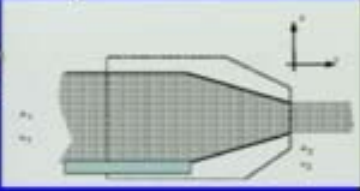
The second case we will see here the force on a pipe nozzle generally we use a nozzle is used to create a jet we want to find out with respect to the nozzle what is the force coming upon the pipe where the nozzle is connected, as we have seen earlier here all we will be considering a control volume for example here we are considering this particular

problem here you can see that a pipe is coming and nozzle is connected and the control volume considered is here.

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2. Force on a Pipe Nozzle

- Fluid contracted at nozzle – forces induced
- A) Total force



$$F_T = F_{Tx} = \rho Q(u_2 - u_1)$$

$Q = A_1 u_1 = A_2 u_2$, so

$$F_{Tx} = \rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$$

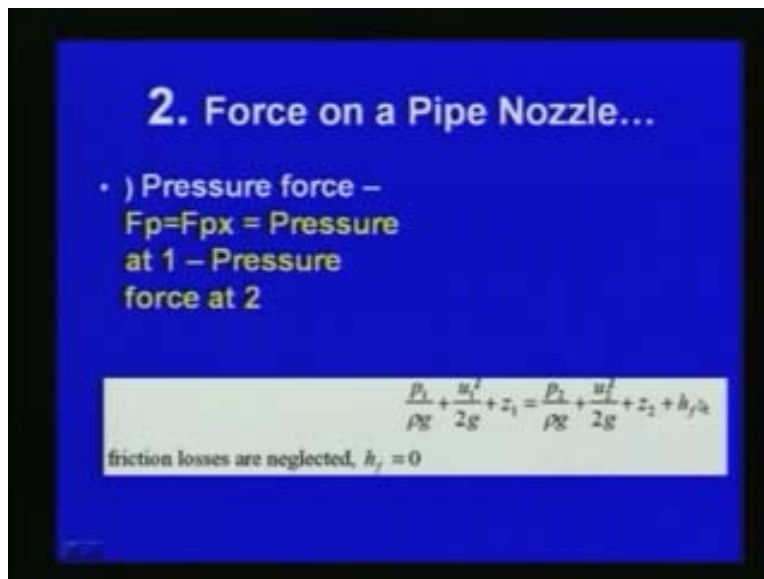
Here the velocity of entry is u_1 and velocity for exit is u_2 and the area of cross section entry at the entrance is A_1 and area of cross section at the entrance is A_2 , now the control volume considered is shown in this slide and the, with respect to the nozzle the fluid is contracted at the nozzle and the forces are induced upon the pipe where the nozzle is connected. First we will consider the total force, the total force F_{Tx} in the x direction we are considering F_T is equal to F_{Tx} is equal to the mass of fluid or rho into Q where rho is the mass density Q is the discharge coming from the pipe into the velocity difference u_2 minus u_1 where u_2 is the exit velocity and u_1 is the entrance velocity.

The total force with respect to this the nozzle which is acting upon the pipe is F_T is equal to in the x direction F_T is equal to F_{Tx} is equal to rho Q into u_2 minus u_1 , here if you use the continuity equation Q is equal to $A_1 u_1$ is equal to $A_2 u_2$, from the continuity equation. We can get the discharge Q is equal to $A_1 u_1$ is equal to $A_2 u_2$ hence we can write the total force in the x direction F_{Tx} is equal to rho Q square into $\frac{1}{A_2}$ minus $\frac{1}{A_1}$ as shown in this slide, like this we can find out the total force as per as the pressure force is concerned here we have we have already found the total force with respect to the x direction then next step is you will be finding the pressure force.

The pressure force F_p is equal to in the x direction F_p is equal to F_{px} that is the pressure at 1 minus pressure at force at section two this is section one here section two.

The pressure force the effective pressure force is equal to pressure at section 1 minus pressure at pressure force at section two using the energy equation or the Bernoulli's equation, we can write with respect to the section one and section two we can write p_1 by ρg plus u_1 square by $2g$ plus z_1 is equal to p_2 by ρg plus u_2 square by $2g$ plus z_2 plus h_f .

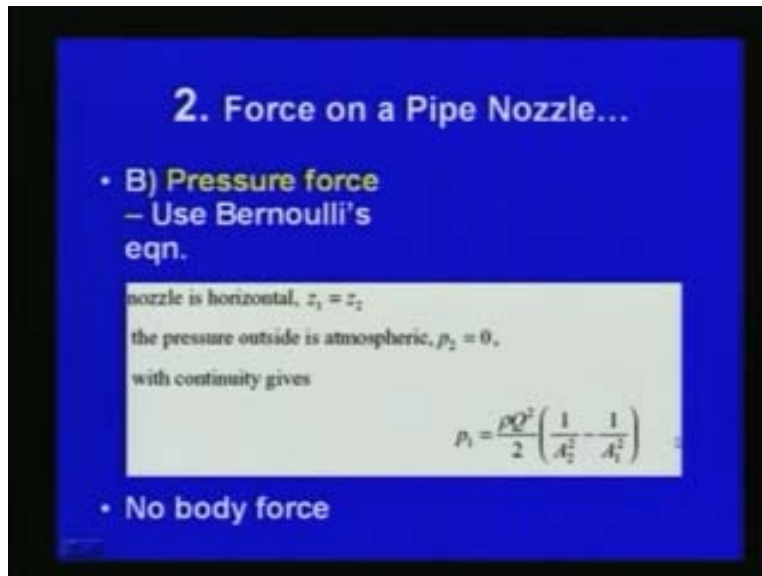
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Where h_f is the losses due to friction here if we neglect the friction losses here then h_f is equal to 0 that we can write the pressure force from the Bernoulli's equation if we consider this particular case z the data head is z_1 is equal to z_2 .

The pressure outside is al here when after the jet is here the from the it is coming out of the jet here the pressure is atmosphere as atmospheric that p_2 is equal to 0, that from the continuity equation we can write here Q is equal to $A_1 u_1$ is equal to $A_2 u_2$. Finally, we can write p_1 is equal to ρQ square by 2 into 1 minus A_2 square minus 1 minus A_1 square.

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2. Force on a Pipe Nozzle...

- **B) Pressure force**
– Use Bernoulli's eqn.

nozzle is horizontal, $z_1 = z_2$
the pressure outside is atmospheric, $p_2 = 0$,
with continuity gives

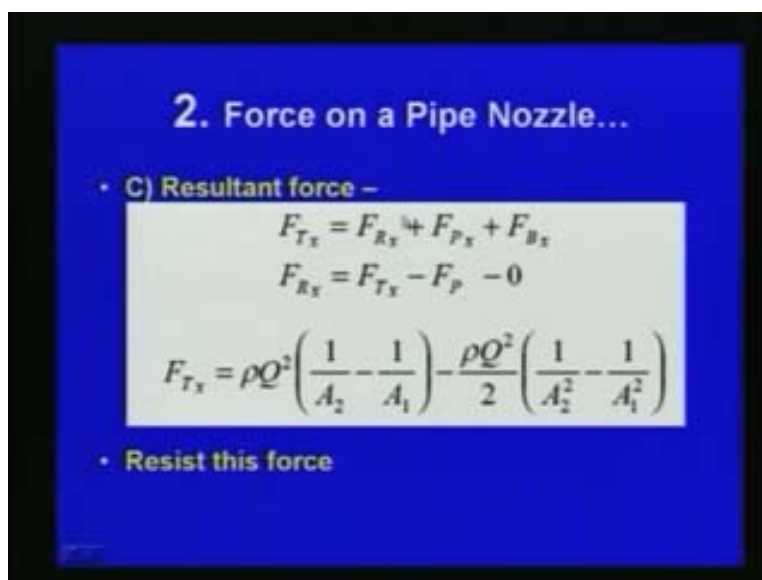
$$p_1 = \frac{\rho Q^2}{2} \left(\frac{1}{A_2^2} - \frac{1}{A_1^2} \right)$$

- **No body force**

This gives the pressure force that means the pressure here p_1 at the section p_1 we can get as p_1 is equal to ρQ^2 by 2 minus A_1 square minus A_2 square.

This gives the pressure force now this particular case is concerned there is no body force since the body force here we neglect finally, resultant force we can write as total resultant force is equal to F_{Rx} plus F_{Px} plus F_{Bx} body force is 0, P is the pressure force and F is the resultant force.

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2. Force on a Pipe Nozzle...

- **C) Resultant force –**

$$F_{Tx} = F_{Rx} + F_{Px} + F_{Bx}$$
$$F_{Rx} = F_{Tx} - F_P - 0$$
$$F_{Tx} = \rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right) - \frac{\rho Q^2}{2} \left(\frac{1}{A_2^2} - \frac{1}{A_1^2} \right)$$

- **Resist this force**

We have already found here., finally, we get the resultant force for the system F_{Tx} is equal to F_{Rx} plus F_{Px} plus $F_B x$ and finally, F_{Rx} is already can be written as F_{Tx} minus F_{Px} minus 0 this is with respect to the F_{Rx} is obtained like this now finally, F_{Rx} is equal to $\rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$ here which is obtained from the total force which we have already found F_{Tx} .

The pressure force is considered we have already calculated $Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$ square minus $\rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$ this is gives the resultant force for the system resultant force is equal to $\rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$ minus $\rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$ divided by A_2 square minus $\rho Q^2 \left(\frac{1}{A_2} - \frac{1}{A_1} \right)$ divided by A_1 square.

Where A_2 is the cross section at this location where the nozzle in this location and A_1 is the cross section at this location at section one, this gives the resultant force, this resultant force is the force which is acting upon with respect to the effect of this nozzle. The force coming on this pipe is the force on the pipe is given by this resultant force that force should be that resist the with respect to the nozzle this is the force which resist the nozzle is the liquid is passing through the nozzle.

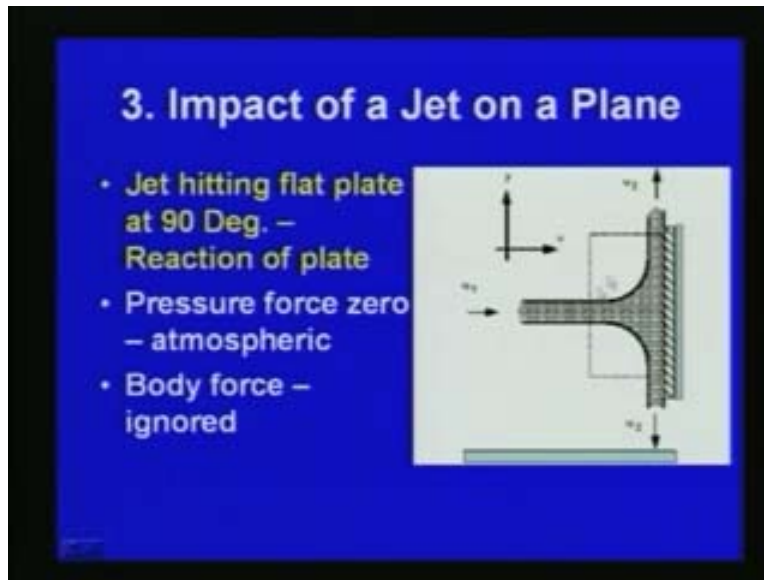
This is the case for a second case which we here we discussed is the force on a pipe nozzle as shown in this figure this is the second case, now we will be discussing another case which is the impact of a jet on a plane. As we have seen there are a number of applications as far as the linear moment equation is concerned third case is the impact of a jet on a plane.

Here you can see that a jet is coming impinging on a on a plate let us assume this case three third case jet is acting at 90 degrees here you can see the jet is coming at 90 degree impinging on this plate on the vertical plate, we want to find out the force on the plate and the reaction of the plate this is what we want to find out with respect to this case which is the third case the impact of a jet on a plane.

Here you can see that the pressure force is 0 since you can see here everything is open to the atmosphere, the jet is coming through the atmosphere it is impinging on a it is hitting on a on a flat pipe vertically placed, here for example if this is the plate the jet is coming like this vertically with respect to the this plate now the pressure force here the pressure

force is 0 or atmospheric here this is the control volume which we consider here. The control volume is with respect to this plate and where the jet is impinging on the plate.

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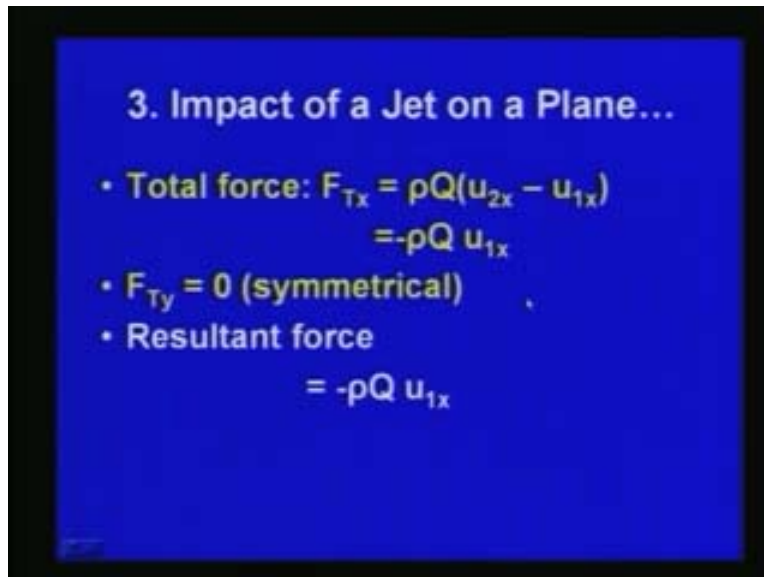
This is the control volume and the body force al in this particular place it is ignored the if you assume that velocity here of the jet is u_1 , this is the velocity in this upward direction is u_2 and the velocity in the downward direction is also u_2 , due to the symmetrical case. But you can see here with respect to here the body force is ignored total force here you can get the velocity of approach of the jet is u_1 and after hitting the flat plate the velocity is u_2 upward direction downward direction.

The total force F_{Tx} is equal to the rho Q multiplied by $u_2 \times$ minus $u_1 \times$, the we are finding the force in the x direction of this direction this is equal to total force is equal to F_{Tx} equal to rho Q into $u_2 \times$ minus $u_1 \times$ the velocity in x direction after the jet is hit on the on the on the plate and $u_1 \times$ is the velocity of approach.

This is equal to rho Q here total force is equal to minus rho Q into $u_1 \times$ here you can see that x direction is concerned x direction is considered here this plate is placed vertically. You can see here u_2 is the x direction $u_2 \times$ is here it is 0. since x direction only the y direction velocity is there u_2 is in the y direction about upward and downward you can see that x direction velocity $u_2 \times$ is equal to 0 since the plate is placed vertically.

Total force F_{Tx} is equal to $\rho Q u_2 x$ minus $\rho Q u_1 x$ since $u_2 x = 0$, this is equal to F_{Tx} is equal to minus $\rho Q u_1 x$ since due to the symmetry you can see that here the force on the on the on the y direction is equal to F_{Ty} is equal to 0 since due to the symmetry.

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We can write that is equal to minus $\rho Q u_1 x$, this gives the resultant force finally, for this particular case where jet is hitting a at 90 degree on a flat plane here we do not consider the friction on the plates we have passed that the plate is smooth.

Finally, what we get is the as far as the x direction the resultant force is concerned it is given as minus $\rho Q u_1 x$ where ρ is the density of the fluid considered, Q is the discharge of the of the jet discharge of the discharge coming and hitting the plate and $u_1 x$ is the velocity of approach in the x direction.

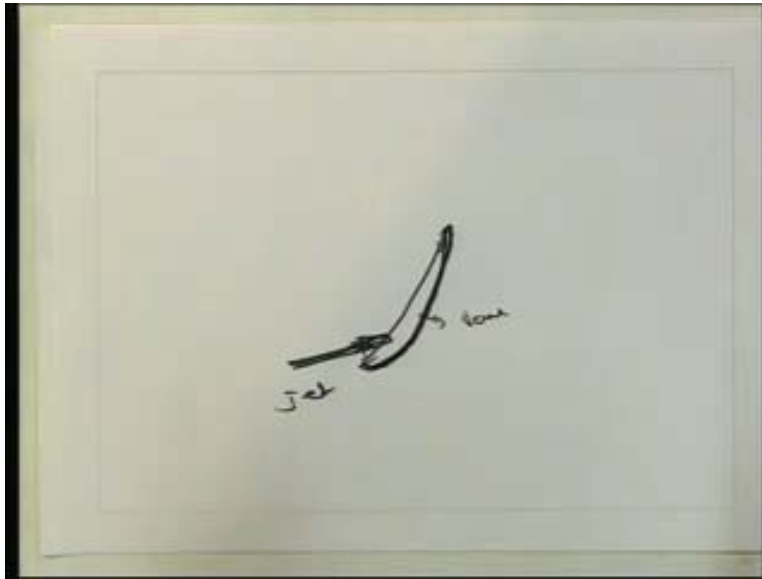
This resultant force is obtained by equal to minus $\rho Q u_1 x$ this gives if a jet is hitting on a flat plate the x direction that is the resultant force here that is equal to minus $\rho Q u_1 x$. This is the third case. We have analyzed the impact of a jet on a flat plate or on a plane like this.

This is the third case now we will be discussing the fourth case fourth case is here the application of the linear moment equation is considered. Fourth case is the force on a curved vane you can see that many problems like for example we have number of

applications like many cases jets will be coming and hitting on curved vanes like this., jet will be coming jet action will be coming like this it will be hitting on a curved vane like this.

There are many practical cases where these kinds of problems occur. Now for this particular case we want to find out the force now the jet is coming in this direction and we want to.

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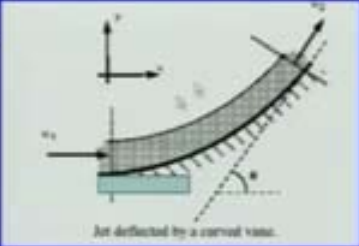
This is the vane here and this is the jet we want to find out how much is the force acted upon acted by this jet on this vane or the curved vane we consider, here for this particular case similar to the pipe pressures here similar to the pipe here the pressures are equal.

You can see that here everything is atmospheric, we do not consider the pressure force here pressure force is atmospheric we can consider 0, which is very similar to the pipe cases which we have seen pressures are equal and it is atmospheric the body force also. In particular case we do not consider the body force here F_{Bx} is equal to 0, now let us consider a control volume.

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4. Force on a Curved Vane

- Similar to pipe – pressures are equal
- Pressure force zero – atmospheric
- Body force $-F_{Bx}=0$



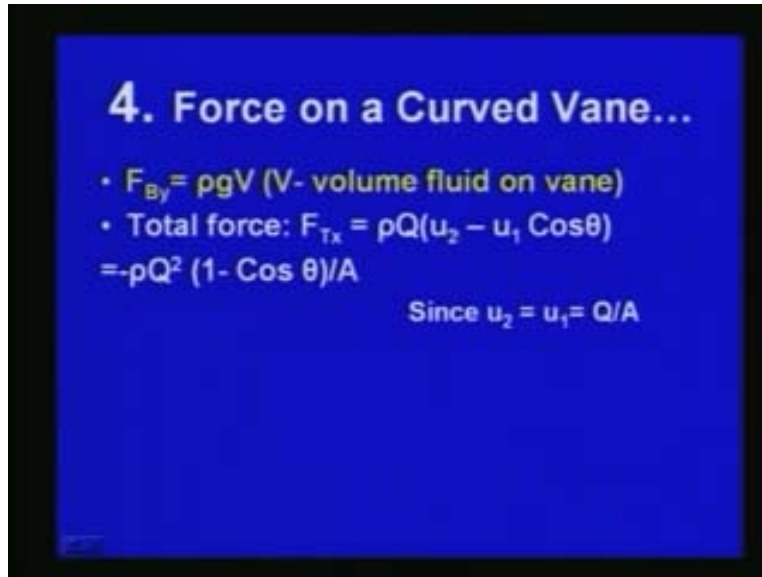
Jet deflected by a curved vane.

In this slide you can see a jet is coming it is deflected by a curved vane like this, this is the vane here and jet is coming like this here the velocity of the incoming jet is u_1 is the velocity. After deflected by the curved vane it is u_2 and x direction is here y direction is here and the angle with respect to the where the velocity u_2 this angle is theta with respect to this slide here.

If you consider for this force on a curved vane F_{By} is equal to that means the force here you can see that body force F_{Bx} is equal to 0. In the x direction the body force is equal to 0 since x direction is here the total force. We can find out the total force here with respect to this particular figure here is total force is obtained in the x direction is equal to ρQ into minus $u_1 \cos \theta$.

Here u_2 is in this direction here which is deflected if you consider this as the normal direction. The total force with respect to x direction will be ρQ into u_2 minus $u_1 \cos \theta$ here this is u_2 and here this is the angle theta, u_2 minus $u_1 \cos \theta$ gives the total force total force in the x direction is equal to minus ρQ into u_1 minus $u_2 \cos \theta$.

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Finally, this can be written as that is equal to minus rho Q square into 1 minus cos theta divided by A which is the area of cross section since u_2 area of cross section of the jet which is coming here. Since u_2 is equal to u_1 is equal to Q by A where Q is the total discharge this is Q by A that we can write for this u_2 and u_1 we can substitute with respect to this Q by A finally, total force is obtained that is equal to minus rho Q square into 1 minus cos theta by A .

This gives the force on a curved vane total force on a curved vane finally, total force in Y direction will be equal to F_{Ty} is equal to rho Q into u_2 sine theta minus 0 here you can see that this is u_2 sine theta and here it is coming in this direction effectively. This other direction this is not concerned there is no y direction there is no velocity that is F_{Ty} is equal to rho Q into u_2 sine theta this can be written as that is equal to rho Q square by A into sine theta.

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4. Force on a Curved Vane...

- Total force in Y-Dir.
- $F_{Ty} = \rho Q(u_2 \sin\theta - 0)$
- $= \rho Q^2 / A \sin\theta$
- Resultant force:

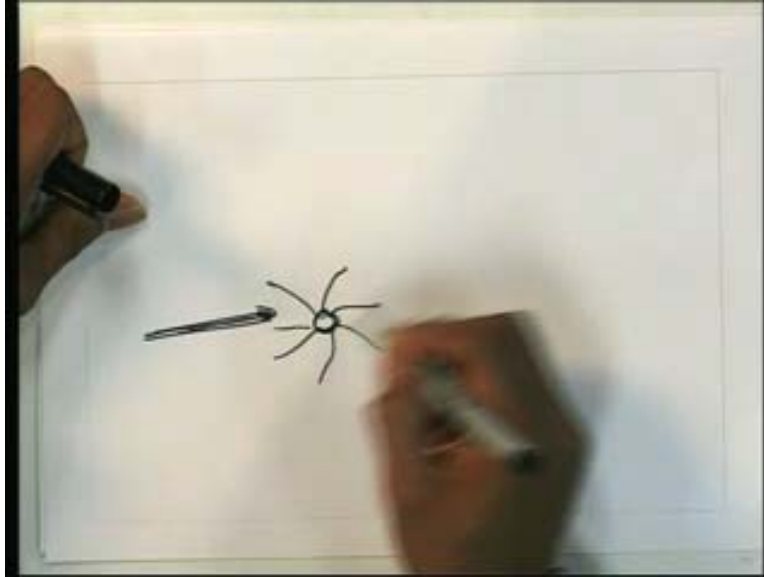
$$F_{Tx} = F_{Bx} + F_{Tx} + F_{Bx}$$
$$F_{Bx} = F_{Tx}$$
$$F_{Ty} = F_{By} + F_{Ty} + F_{By}$$
$$F_{By} = F_{Ty}$$
$$F_{\text{resultant}} = \sqrt{F_x^2 + F_y^2}$$

Finally, we get F_{Tx} in the x direction finally, we can get F_{Rx} that means the resultant force plus the pressure force any way not considered here body force is all in the x direction there is no body force. F_{Tx} the resultant force F_T F_{Rx} is equal to F_{Tx} here this with respect to the y direction F_{Ry} is equal to F_{Ty} minus F_{By} and the resultant force we can we find out the resultant is equal to square root of F_x square plus F_y square.

This gives the resultant force on the curved vane., now we have seen various cases; first one is the impact on a jet on a plane, second case third case is impact first case is the force due to flow around a pipe bend, second one is force on a pipe nozzle and third one we have seen is impact of a jet on a plane. Fourth one is force on a curved vane as here we have analyzed and now the fifth case which we will be discussing here is jets on turbine blade.

You can see that many cases we will be analyzing like peloton wheel or turbine problems. You can see that jet will be coming like this the flow with respect to jet action you can see that there will be a wheel with various vanes like this.

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You can see that due to the jet action it is it will start rotating. It is we can use this principle of the linear moment equation which we have seen here we can use the same principle to analyze the impacts of jets on turbine blades.

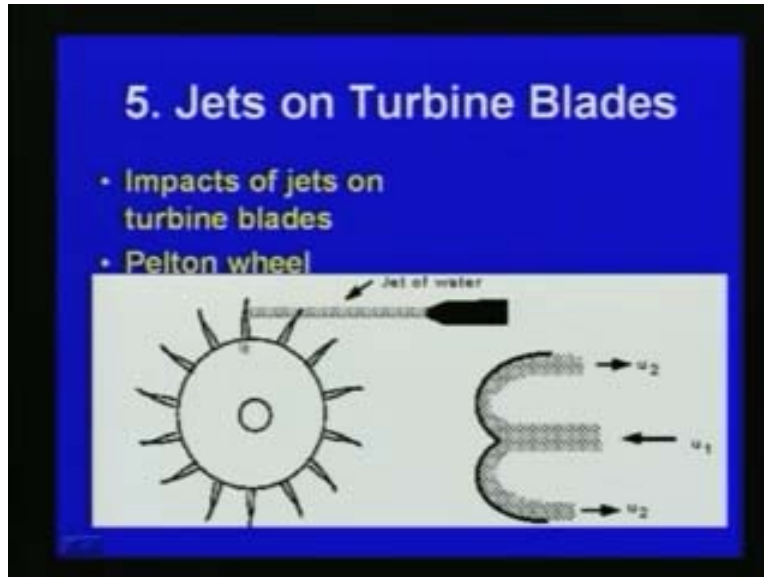
It is depicted in this slide here the jet of turbine jet of water is coming from through nozzle like this. This is the jet of water here the peloton wheel is mounted on axle and due to the jet action; we can see that a force is supplied due to that the turbine wheel is rotating.

Here we will be having either this jet can be coming directly 90 degrees perpendicular to the vane or it can also be inclined., here we will be analyzing how we can find out the force acted upon the jet on the on the peloton wheel.

If you consider this u_1 most of the time we can see that this can be either this plates which is attached with respect to the axle for the peloton wheel most of the time it will be curved one like this you can see here the curved vane type mechanism.

The jet will be coming perpendicular to this like this. After that it is hitting on the plate on the vane it is deflected like this of velocity u_2 on both direction into this way and other way the jet of velocity u_1 is hitting on the vane and it is deflected with a velocity u_2 both direction by like this.

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This is the mechanism which is generally used in a pelton wheel we want to find out the with respect to this jet action how much is the force coming on the on the on the pelton wheel.

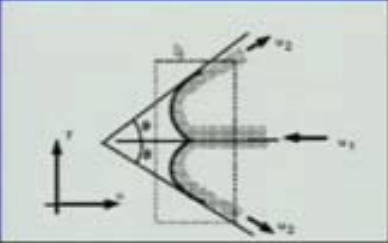
That is what we want to find out as we can see here the jet is coming perpendicular on the horizontal direction it is deflecting also in the on the horizontal direction. But it can be al inclined like this; in this slide you can see that your jet is coming horizontal direction. But due to these locations or the nature of the vanes you can see that it can be al deflected an angle theta like this. Here u_2 is deflected in this direction and other way it is with respect to angle theta.

For this kinds of problem now let us consider a control volume as in the previous cases. First we will be considering a control volume the control volume is here shown here the control volume is considered x axis is here y axis is here.

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5. Jets on Turbine Blades...

- Pressure force zero – atmospheric
- Body force – zero
- Total force in y-Dir.=0



Now again this case all here this particular all we do not consider the pressure force the pressure force is here it is atmospheric pressure or this 0. We consider the pressure force as 0 also the body force is not to be considered here this particular case due to the specific nature of the problem.

We do not consider the body force you can see that finally, the total force in the y direction will be 0 for this due to the symmetrical due to nature of the particular problem here the total force in the y direction will be 0 the total force in the x direction.

If you consider this the jet is deflected at an angle theta like this in the x direction total force in the x direction that is the resultant force F_{Tx} that is equal to here with respect to this figure (Refer slide time on 24:45) $\rho \cdot Q \cdot u_1 \cdot \cos \theta$. Here if the discharge due to symmetry at such discharge is $Q/2$ is passing Q is coming and hitting on this and $Q/2$ is passing in this direction and other direction all $Q/2$ is passing.

F_{Tx} is equal to $\rho \cdot Q \cdot u_1 \cdot \cos \theta$ plus $Q/2 \cdot u_2 \cdot \cos \theta$ minus $Q/2 \cdot u_2 \cdot \cos \theta$ Q is the discharge coming to the hitting on the peloton wheel or the turbine blade the velocity back is due to at an angle theta let us assume that in this particular case half of the discharge is going up this direction and other half is going down this direction.

That the x direction the resultant force in the x direction can be written as F_{Tx} is equal to $\rho Q u_2 \cos \theta + \rho Q u_1$ as shown here F_{Tx} is equal to $\rho Q u_2 \cos \theta + \rho Q u_1$, where u_1 is the velocity in x direction and $u_2 \cos \theta$ is the velocity in the with respect to the x direction and ρ is the density of the liquid or here water is considered then density of water.

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5. Jets on Turbine Blades...

- Total force in X-direction
- Resultant force $= F_{Tx}$

$$F_{Tx} = \rho \left(\frac{Q}{2} u_{2x} + \frac{Q}{2} u_{2x} - Q u_{1x} \right)$$

$$u_{1x} = -u_1$$

$$u_{2x} = u_2 \cos \theta$$

$$F_{Tx} = \rho Q (u_2 \cos \theta + u_1)$$

Now the resultant forces obtained as F_{Tx} here you can see that the u_1 is the velocity we are assign for this particular problem the velocity is coming with respect to the x direction like this in this direction.

u_{1x} is equal to minus u_1 u_{2x} here you can see that u_{2x} is equal to $u_2 \cos \theta$, u_{2x} is equal to $u_2 \cos \theta$ finally, total the resultant force F_{Tx} is equal to $\rho Q u_2 \cos \theta + \rho Q u_1$.

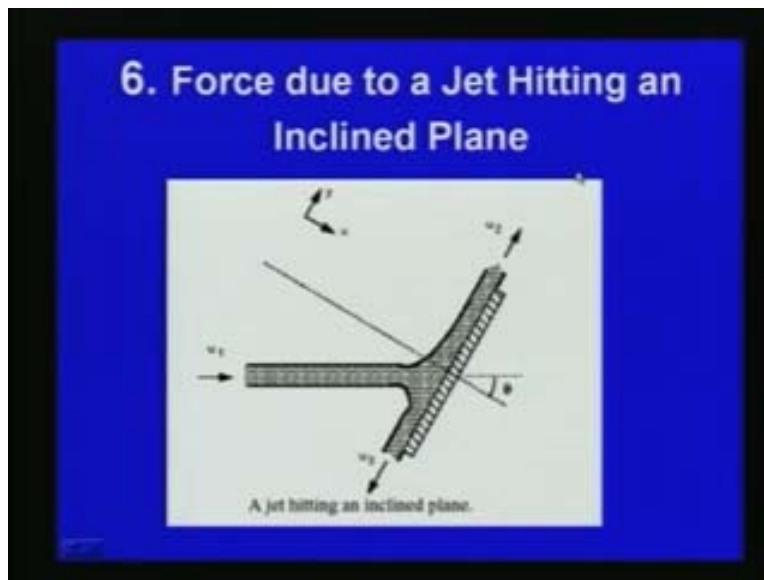
For this particular case F_T is the resultant force is equal to $\rho Q u_2 \cos \theta + \rho Q u_1$ like this we can analyze the force. Finally, we have found the total force acting upon this on this on this peloton wheel or on the turbine blade.

We consider like in a peloton wheel we have considered the turbine blade, how much is the force acted upon the turbine blade is calculated from this expression F_{Tx} is equal to

ρQ into $u_2 \cos \theta$ plus u_1 where ρ is the density Q is the discharge and jet u_1 is the velocity of the jet u_2 is the velocity after deflecting from the from the turbine blade.

We can find out the force acted upon the jets on turbine blade the force jets on turbine blades can be calculated, this is the fifth case we have analyzed the application of the linear moment equation which we have seen and now the last case we will be discussing here is force due to a jet hitting an inclined plane.

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As we have seen the jet if the plate here we have seen that whenever the plate is vertical placed then the jet is acting like this the analysis is very simple, we have already seen we have analyzed here in the previous case the impact of a jet on a plane.

Plane is it is placed perpendicular the analysis is much simpler but here in the last case here we discussed the force due to a jet hitting at an angle or at an inclined plane. If you consider a plane like this here a jet is coming and hitting on the plane like this the jet position is horizontal jet is acting coming horizontally but the plane is the plate or the plane is inclined. You can see in this slide how the mechanism is this al many problems you will be of this kind.

Here jet is coming with a velocity u_1 it is hitting on an inclined plane if you consider the normal to the plane which we consider this is at angle θ like this with respect to the horizontal.

After here the velocity of the jet is u_1 in the previous case we have seen earlier that if it is normally placed the velocity will be same to the both sides as u_2 and u_2 but here you can see that due to the inclination of the of the plane.

The velocity to this direction u_2 is the velocity in this direction, the other direction which is u_3 . It is not symmetrical or the plane is inclined or the velocity in this direction is u_2 and the downward direction it is u_3 here we consider the angle θ .

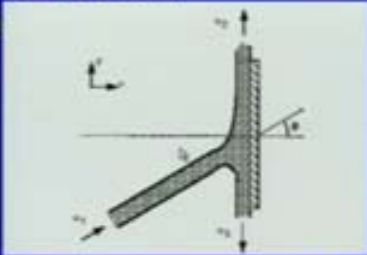
To analyze this problem, we will be just considering this problem like this here the same plane what we consider the jet is in the horizontal direction and plane is or the plate on which the jet is hitting is inclined but then the same case. We can in a very similar way we can consider the plane is vertical and jet is inclined here you can see the same problem it is very similar problem very same problem.

We are considering the plate is or the plane is vertical like this, the jet is acting inclined., we have seen the case is here the actual case is the plane is or the plate is inclined and jet is coming horizontal. We are converting this problem into we place the plate as vertical. We consider the jet is acting inclined that is the way which we analyzing this problem.

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6. Force due to a Jet Hitting an Inclined Plane...

- Diagram rotated for analysis
- By Bernoulli's equation

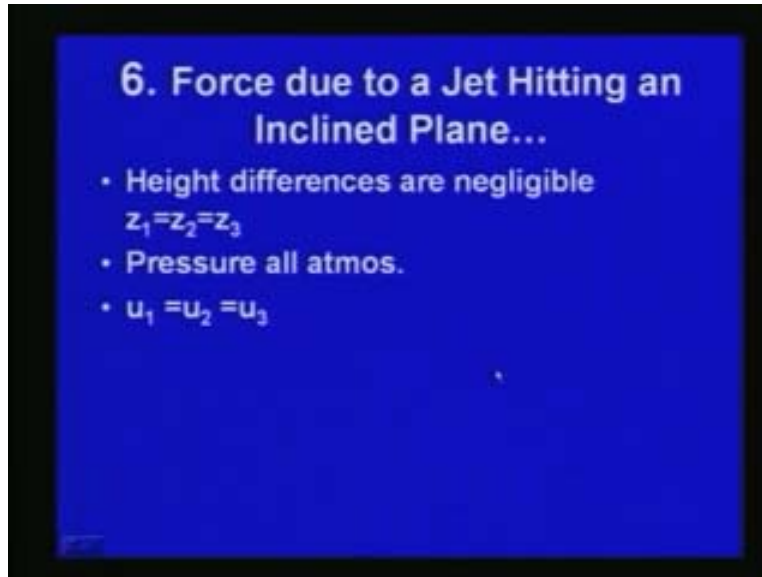

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 = \frac{p_3}{\rho g} + \frac{u_3^2}{2g} + z_3$$

Here the diagram is rotated for analysis now we can see this is angle theta and u_1 is the jet velocity is u_1 and now x is horizontal and y is vertical direction and u_3 is the velocity downward u_2 is the velocity upward.

If we use the Bernoulli's equation here with respect to this figure you can see that p_1 by ρg plus u_1 square. As previous case we will be considering control volume p_1 by ρg plus u_1 square by $2g$ plus z_1 is equal to p_2 by ρg plus u_2 square by $2g$ plus z_2 is equal to p_3 by ρg plus u_3 square by $2g$ plus z_3 .

If you consider three position here and here three sections from the Bernoulli's equation you can write p_1 by ρg plus u_1 square by $2g$ plus z_1 is equal to p_2 by ρg plus u_2 square by $2g$ plus z_2 is equal to p_3 by ρg plus u_3 square by $2g$ plus z_3 .

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If you assume that height difference is negligible with respect to the jet action if the height difference is not much we can assume z_1 to be equal to z_2 equal to z_3 for analysis purpose the pressure the jet is acting in atmospheric pressure.

That we can assume that here all due to the atmospheric pressure here we can see that u_1 is equal to u_2 is equal to u_3 here velocity with respect to this particular problem u_1 is equal to u_2 is equal to u_3 .

With the Bernoulli's equation the possibility is that u_1 is equal to u_2 since pressure is atmospheric that is already all these terms as one z_1 is equal to z_2 is equal to z_3 finally, we get the velocity this is an approximation u_1 is equal to u_2 is equal to u_3 .

Now by from the continuity equation you can see that if we apply the continuity equation the discharge this should be what is hitting on the plate should be equal to what is going up and what is going down Q_1 is equal to Q_2 plus Q_3 .

For that we can write u_1 plus A_1 is equal to u_2 plus A_2 plus u_3 plus A_3 or we can write A_1 is equal to A_2 plus A_3 or finally, we can write Q_1 is equal to $A_1 u$ or Q_2 is equal to $A_2 u$ and Q_3 is equal to what is going down that discharge will be Q_3 is equal to A_1 minus A_2 into u Q_3 is equal to A_1 minus A_2 into U .

The pressure force since it is atmospheric that is considered as 0 and finally, we got Q_1 is equal to $A_1 u$ Q_2 is equal to $A_2 u$ and Q_3 is equal to A_1 minus A_2 into u and in this case all we neglect the body force body force is ignored.

Finally, the total force F_{Tx} in the x direction the total force is equal to ρ into Q_2 into u_{2x} plus Q_3 into u_{3x} minus Q_1 into u_{1x} .

With respect to this figure here, finally, we get the total force in the x direction F_{Tx} is equal to ρ into Q_2 into u_{2x} plus Q_3 into u_{3x} minus Q_1 into u_{1x} . With respect to this figure is rotated here you can see that in x direction u_{2x} and u_{3x} are 0 u_{2x} is 0 u_{3x} is 0 we get u_{2x} is equal to u_{3x} is equal to 0 jets are parallel to the plate.

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6. Force due to a Jet Hitting an Inclined Plane...

- Total force

$$F_{Tx} = \rho(Q_2 u_{2x} + Q_3 u_{3x}) - Q_1 u_{1x}$$
- $u_{2x} = u_{3x} = 0$ (jets are parallel to plate)
- $u_{1x} = u_1 \cos \theta$

$$F_{Tx} = -\rho Q_1 u_1 \cos \theta$$

Finally, this expression become since u_{1x} is equal to $u_1 \cos \theta$ this angle is θ u_{1x} is equal to $u_1 \cos \theta$ finally, we simplify the equation to F_{Tx} the total force in the x direction is equal to minus $\rho Q_1 u_1 \cos \theta$.

This will be the force acted upon by this inclined jet on this plane or this particular case which we consider the plane is inclined and jet is horizontal. Finally, the expression is the total force is F_{Tx} is equal to minus $\rho Q_1 u_1 \cos \theta$, like this we can analyze what will be the force due to a jet hitting at jet the plane is vertical and a jet is inclined that both case we can analyze.

Finally, the resultant force exerted on fluid is equal to body force is neglected pressure force is atmospheric, that we don't consider in the x direction F_{Rx} is equal to minus rho $Q_1 u_1 \cos \theta$ this is the resultant force exerted on the fluid in y direction F_{Ty} you can see that F_{Ty} is equal to 0 in this particular case

Finally, the resultant force is R is equal to minus F_{Rx} which is the reaction or which is the with respect to the jet what is the force acted by the plate the reaction force will be R is equal to in the opposite direction minus F_{Rx} that is equal to rho $Q_1 u_1 \cos \theta$.

This gives the reaction force in the opposite direction that is minus F_{Rx} , like this we can analyze the resultant force acting on a acted by a jet hit on an inclined plane also from what we have seen we can find out the discharge in each direction $Q_1 Q_2 Q_3$ that we have seen here.

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6. Force due to a Jet Hitting an Inclined Plane...

- Resultant force exerted on fluid
- Force on plate
- In Y-direction, $F_{Ty}=0$
- Discharge goes on each direction can be also found

$$F_{Ty} = F_{Rx} + F_{Py} + F_{By}$$

$$F_{Rx} = F_{Ty} - 0 - 0$$

$$= -\rho Q_1 u_1 \cos \theta$$

$$R = -F_{Rx}$$

$$= \rho Q_1 u_1 \cos \theta$$

That also can be calculated discharge goes on each direction can be al found in a very similar way this is the application we have discussed, what we now we have seen the from the Newton's second law we have derived the linear moment equation. We have seen various applications this far we have discussed various applications.

Now we from the applications we can see that some of the interpretations what we can with respect to the applications are here the linear moment we have to always consider

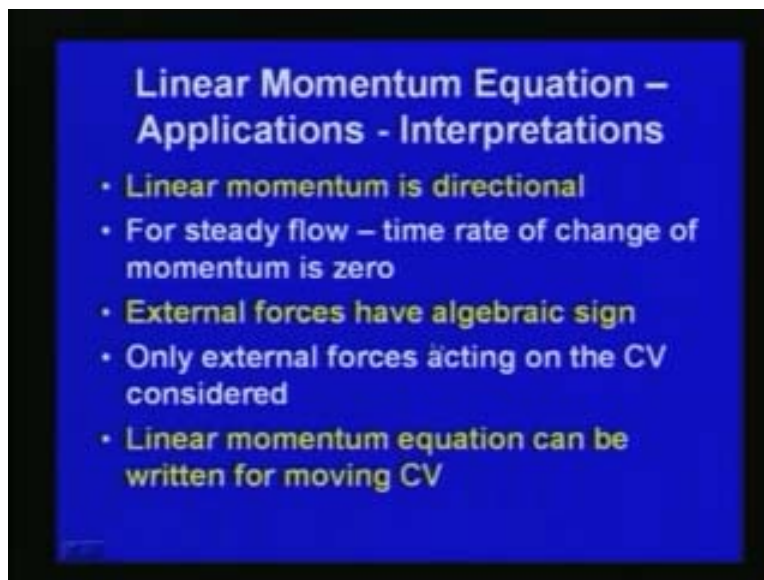
the direction, whether it is x direction or y direction that we can that the linear moment is direction that is obvious always we have to which direction it is acting the linear moment is direction.

That is the first interpretation, most of the cases we have discussed are steady state condition for steady state flow the time rate of change of moment is 0 that is not considered most of the examples we have considered is at steady state condition also we have seen we are using the Newton's second law.

We are considering the external force we have to consider the sign that means whether the force is acting which direction whether plus or minus the sign is important we have to consider the external forces. We have to consider its sign algebraic sign of the external force to be considered and also here as far as the control volume which we consider the analysis is we are starting the analysis by considering a control volume.

For the control volume considered only external forces acting on the control volume to be considered for the problems which we have seen here also in a very similar way. The cases which we have analyzed is most of the time the plate or the vane. All the cases are fixed it is not the vanes or the plates are not moving but also the linear moment equation we can write in a very similar way for moving control volume.

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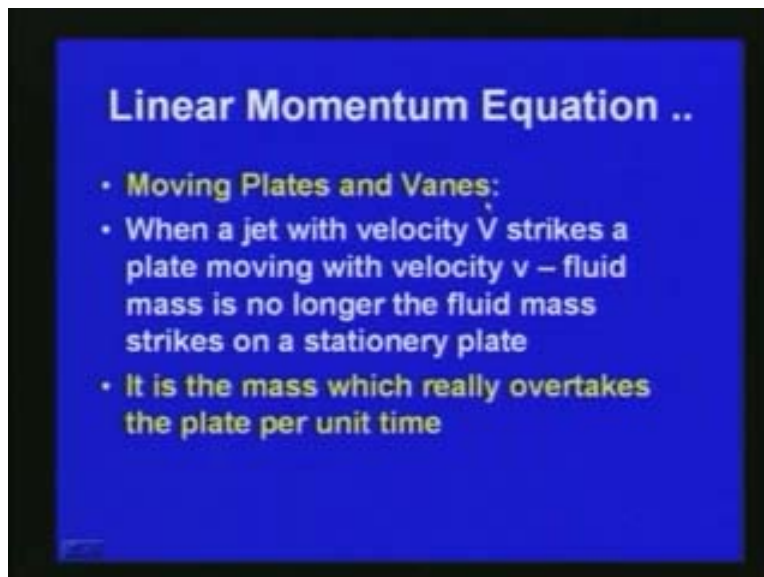
We can see that the plate which we consider with respect to the jet is now the jet is coming and the plate is fixed. We have found the force but if the plate is moving like this when the jet is acting simultaneously the plate is also moving that is the case where the moving control volume.

The cases which we have considered far is fixed control volume but some cases like you have seen the when the wheel is rotating it is moving control volume or when the plate is moving with respect to the jet action. Then we can see that the case is of moving control volume we have to consider now jet is coming in with respect to the jet movement control volume is al moving.

Linear moment equation can be transformed with respect to the moving control volume for the moving control volume like moving plates and vanes when a jet with velocity V strikes a plate moving with velocity small v fluid mass is no longer the fluid mass strikes on a stationer plate.

As I mentioned if the velocity here plate is there and the velocity of the jet is here V capital V is the velocity the plate is now moving with a velocity of small v .

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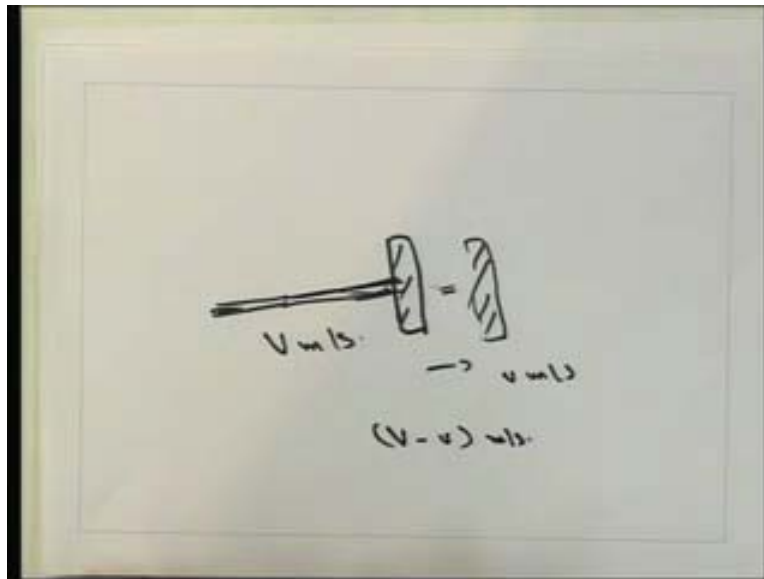


As earlier case for the fixed plate which is not the same it is not the same mass that is hitting the plate as in the case of a stationer plate since the plate is al moving, here this is the plate the initial position and now jet is coming and hitting like this.

But after some time we can see that the plate is moving the mass is changing the fluid mass is no longer the same the fluid mass strikes on the with respect to stationery plate now plate is shifted with a.

The plate is moving with velocity small v meter per second and the jet is of capital V meter per second then we will be finding out the mass which really overtakes the plate per unit time. We have to see that the relative velocity we have to consider and what mass will be overtaking the plate per unit time to be considered when the plate or the vane is moving. We have to consider the relative velocity here you can see that capital V minus small v will be the relative velocity that will be used in the calculation.

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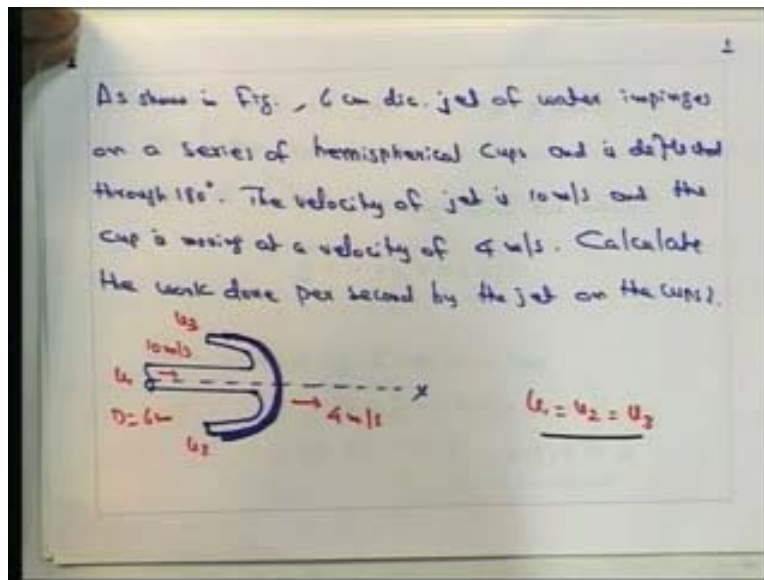


In the next slide here we can see that for the mass striking the plate is area of cross section of the jet has an area of cross section small capital A density is ρ then relative velocity we have to multiply absolute velocity V capital V minus small v . We have to find out the relative velocity, mass striking the plate is Q is equal to area of cross section into ρ into V minus small v the velocity of the jet minus the velocity of the plate or the vane we consider.

The calculation relative velocity should be used with reference to this we will be discussing few of the examples, here the first case is we consider a problem like this here there is a curved vane or a plate or cup like this a jet is coming and hitting on the on the vane or the or the cup the vane itself or the cup itself is moving with a velocity. We want to find the work done per second by the jet on the on the cup.

The problem statement is as shown in figure here figure 6 centimeter diameter jet of water impinges, here the diameter of the jet is 6 centimeter 6 centimeter diameter jet of water impinges on a series of hemispherical cups and is deflected through 180 degrees. The jet is deflected by 180 degree the velocity of the jet is 10 meter per second and the cup is moving at a velocity of 4 meter per second in the x direction calculate the work done per second by the jet on the cups.

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You can see that the jet is coming this direction and the cup is or the vane is placed curved vane is placed like this and it moving the cup or the vane is moving with a velocity of 4 meter per second. The jet velocity is 10 meter per second and if u_1 is the jet velocity then you can see that it will be after the jet reaches the cup it will be deflecting at 180 degree.

u_3 is the velocity in this direction and u_2 is the other downward direction you can see that since we consider the atmospheric pressure here, u_1 is equal to u_2 is equal to u_3 for this

particular problem. We want to find out the work done per second by the jet on the cup for this particular problem.

Since the jet is moving with velocity of 10 meter per second and the cup is moving with a velocity of 4 meter per second the relative velocity of jet is equal to 10 minus 4 that is equal to 6 meter per second.

This particular problem we do not consider the friction., the mass of the fluid striking the cup per second is the diameter of the jet is given that is 6 centimeter area of cross section of the jet is $\pi \times 4 \times 0.06^2 \times \pi \times 4 \times 10$ meter per second density is 998., finally, we get 28.217 kilogram per seconds.

This is the mass of fluid striking the cup per second now we want to find out the force exerted by fluid on the cup that is obtained with respect to the equation which we have derived. We can write that is equal to the force exerted on by fluid on the cup is equal to $2 \times Q \times \rho \times v$ where Q is the discharge ρ is the density into v that two times.

We can see it is coming both ways that is equal to we have already found the mass of fluid striking the cup is 28.217 into 2 times into the relative velocity we have calculated as 6 meter per second into 6 that gives as 3 hundred and 38.61 Newton.

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Relative vel. of jet = $10 - 4 = 6 \text{ m/s}$
No friction considered
Mass of fluid striking cup per second
$$= \frac{\pi}{4} \times 0.06^2 \times 10 \times 998$$
$$= 28.212 \text{ kg/s}$$
Force exerted by fluid on cup
$$= -Q \rho [-(V-u) - (V-u)]$$
$$= 28.212 \times 2 \times 6 = 338.61 \text{ N}$$

This is the force exerted by fluid on the cup is 338.61 Newton and now finally, work can be found by here you can see that work is the force in work is equal to force into displacement. We can see that the with respect to the cup is moving at 4 meter per second that we can write the work done per second is the force exerted by fluid on cup multiplied by that is 338.61 multiplied by this 4 meter per second which is the movement of the cup into 4 divided by 1000 that will give 1.3 by 1.354 kilo watt.

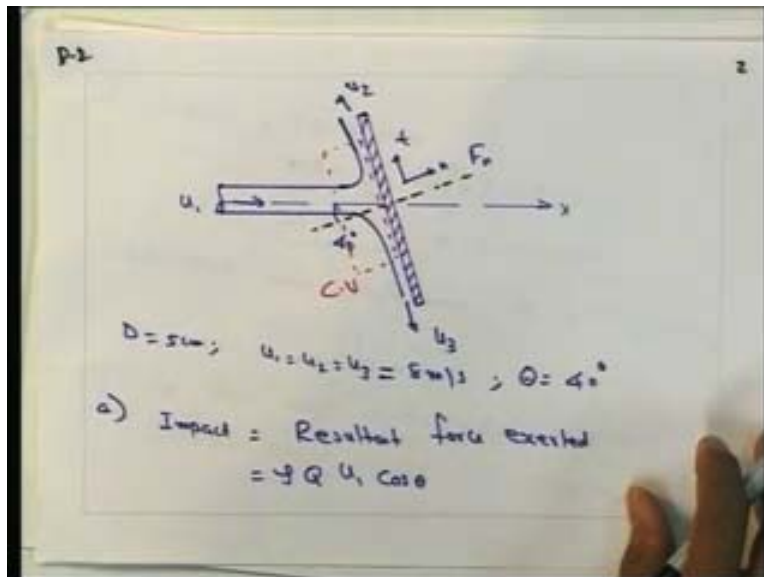
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Work done per second
$$= \frac{338.61 \times 4}{1000}$$
$$= 1.354 \text{ k.w}$$

This gives the work done per second for this problem here we have analyzed a problem with which is relative movement by where the vane or the cup is moving with respect to the jet action, here what we have to see what we have to note here is that we have to find. We have to calculate while finding out the discharge the while finding out the force which we discussing the forced exerted on the on the cup, that we have to with respect to the relative velocity, we have to consider the relative velocity is considered here in this particular case the cup is al moving in the direction of the jet.

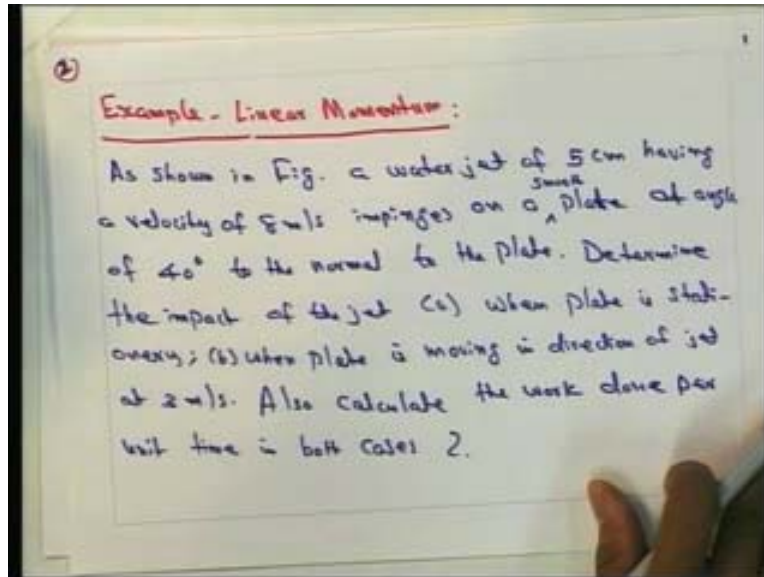
This is the first example first problem and second is with respect to an inclined plane a jet is hitting an inclined plane here the problem statement is as shown in figure here you can see the figure here.

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As shown in this figure a jet is hitting on an inclined plate like this we have to find water jet of 5 centimeter having a velocity of 8 meter per second impinges on a smooth plate at an angle of 40 degree to the normal to the plate. We have to find out the impact of the jet when the plate is stationery case a and case b when the plate is moving in direction in the direction of the jet at 3 meter per second also calculate the work done per unit time in both cases.

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The problem is we have we have a plate put at an inclination of 40 degree with respect to the horizontal that a jet is acting jet is hitting or impinging on the plane at 40 degree.

We have to find out the impact of the jet we have to find out the impact of the jet when the plate is stationary plate is not moving that is case a and case b is when the plate is al moving in a in a direction of the jet at 3 meter per second, Here it is already given that the diameter of the of the jet is 5 centimeter and also since we as we have seen in the previous discussion the all the pressure force the pressure is atmospheric we can that u_1 is equal to u_2 is equal to u_3 .

u_1 is the jet velocity u_2 is the upward direction u_3 is the downward direction u_1 is equal to u_2 is equal to 8 meter per second which is given value and this angle is given as theta is equal to forty degree.

As we discussed we will be considering a control volume as shown here this is the control volume which we consider, from the previous expression which we have derived the impact of the of the jet is obtained as resultant force exerted that is equal to ρQ into $u_1 \cos \theta$ where ρ is the density of the fluid Q is the discharge of the jet and u_1 is the velocity at angle theta the impact of the impact of the jet is equal to ρQ into $u_1 \cos \theta$.

The impact can be calculated for case a when the plate is stationary impact is equal to rho Q into u₁ cos theta., rho is 998 into Q is pi by 4 into diameter of the pipe the jet is 05 centimeter into 0.05 square into u₁ is 8 8 into eight cos 40.

This gives the impact, finally, we can see that the impact on the plate will be 96.07 Newton in the direction here in this direction this is the F m this is in this direction.

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$$P_2$$

$$a) \text{ Impact} = \rho Q u_1 \cos \theta$$

$$= 998 \times \frac{\pi}{4} \times 0.05^2 \times 8 \times 8 \cos 40$$

$$= 96.07 \text{ N} \quad \text{in } x\text{-direction}$$

Work done: Force \times Displacement = $F_x \cos 40^\circ \times$ Displacement

$$96.07 \times 0 = 0$$

$$b) \text{ When plate moves in } x\text{-direction } v = 3 \text{ m/s.}$$

Consider normal direction of relative velocity

$$u_r = 8 - 3 = 5 \text{ m/s.}$$

This gives the impact of the jet on the inclined plane that is 96.07 Newton. Work done here you can see that the plate is stationary since there is no displacement of the plate work done is force into displacement is 0. Work done here in this particular case is 0 and case two when the plate moves in the x direction, v is equal to 3 meter per second if you consider the normal direction and relative velocity you can see that the relative velocity will be the plate is moving with respect to 3 meter per second and jet velocity is 8 meter per second that is equal to 8 minus 3 is equal to 5 meter per second.

The impact of the jet will be impact is equal to rho into Q r into u₁ r cos theta rho is 998 into pi by 4 into 0.05 square that is Q r into 5 into 5 cos 40, that gives 37.53, this will be impact for whenever the plate is also moving with respect to movement of the with respect to the jet force. The relative velocity will be 8 minus 3 into 8 minus 3 that is equal to 5 meter per second with respect to when the plate is al moving the work done w is

equal to this the force into the displacement per unit time here 37.53 into cos 40 into 5 that gives 143.75 Newton meter per second.

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25) Impact = $\rho Q v \cdot u \cdot \cos \theta$

$$= \frac{938 \times \pi}{4} \times 0.5^2 \times 5 \times 5 \cos 40^\circ$$
$$= \underline{\underline{37.53 \text{ N}}}$$

Work done:

$$W = \frac{37.53 \times \cos 40^\circ \times 5}{}$$
$$= \underline{\underline{143.75 \text{ N}\cdot\text{m/s}}}$$

[Displacement per unit time]

This is the work done, like this we can solve most of the problem by considering the linear momentum equation for either a fixed control volume or for a moving control volume.