Fluid Mechanics

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Lecture - 14
Kinematics of fluid flow

Welcome back to the video course on fluid mechanics. In the last lecture we were discussing about the dynamics of fluid flow we were discussing about the energy equation and we were discussing about the corresponding Bernoulli's equations and its applications.

Today we will before further proceeding to other topics we will be discussing more about the limitations of the Bernoulli's equations which we have already seen earlier, as we have seen here large number of applications of Bernoulli's equations the general energy equation with respect to the Bernoulli's equations already we have discussed here it is p_1 by gamma plus alpha v_1 square by 2 g plus z_1 plus q w plus h e is equal to p_2 by gamma alpha v_2 square by 2 g plus z_2 plus e_2 minus e_1.

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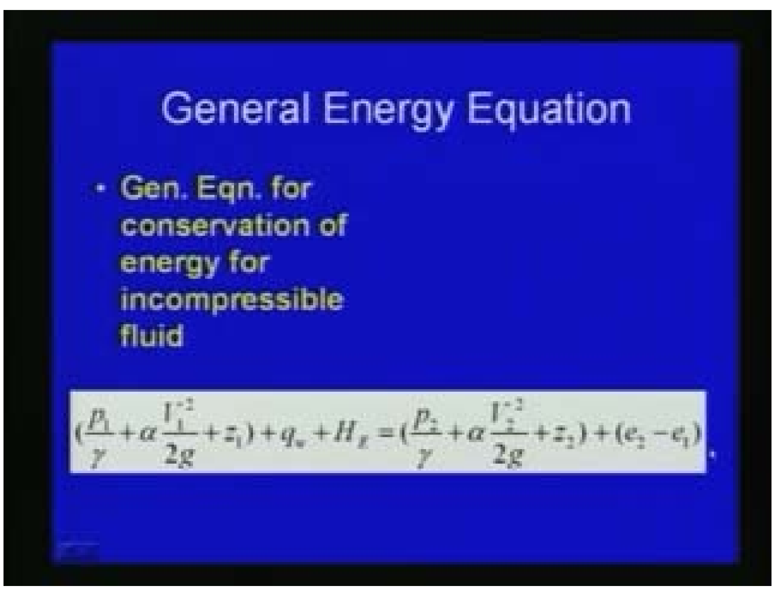

We have already seen considering with respect to two sections of flow between for example if you consider a pipe flow like this, we have already seen here between sections one and two here the total energy equation or general energy equation is written as p_1 by gamma plus alpha v_1 square by 2 g plus z_1 plus q w plus h e is equal to p_2 by gamma plus alpha v_2 square by 2 g plus z_2 plus e_2 minus e_1.

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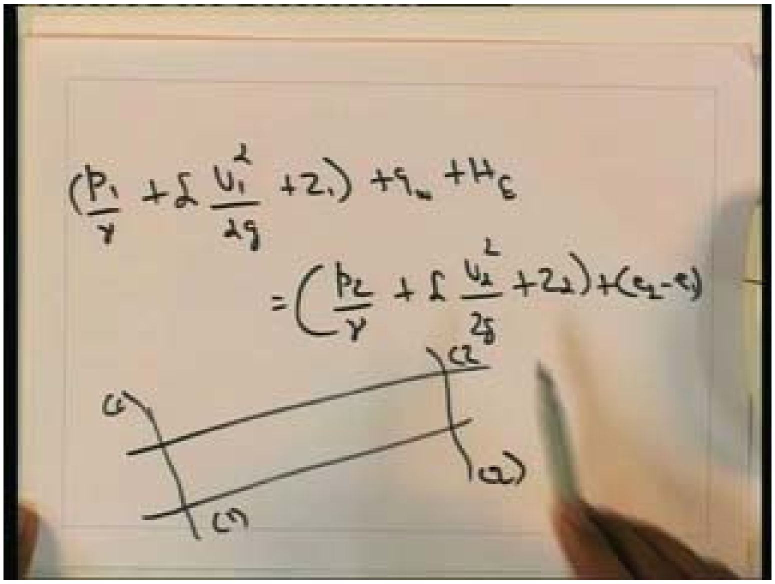

Various terms like q w is the heat added for with respect to the system which we discuss q w is the heat added per unit weight of fluid e_1 e_2 are the internal energy per unit weight of the fluid at respective states and h e is the external work done.

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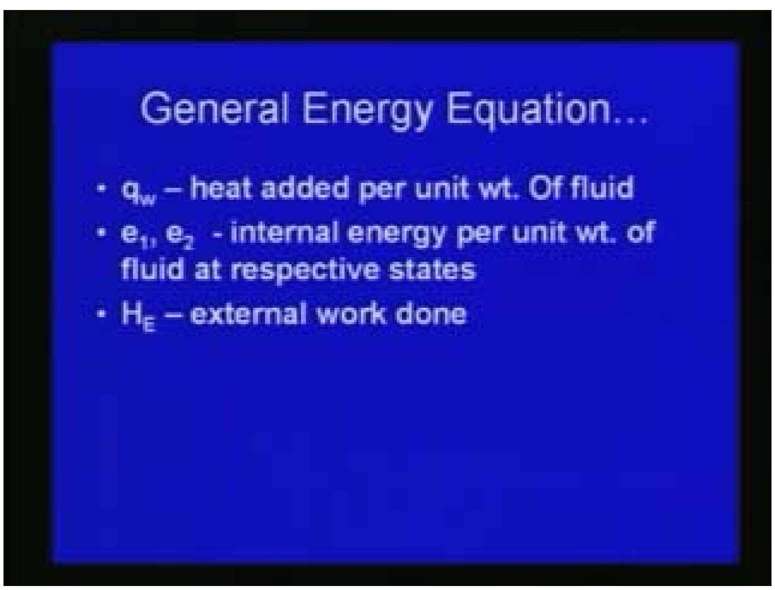

We have already seen this e_2 minus e_1 minus q w is equal to reversible and irreversible, irreversible head is the head loss h l is the energy loss per unit weight of fluid due to the friction and other causes, for incompressible fluid total head at 1 plus heat added due to machine or head loss is equal to total machine minus head loss is equal to total head.

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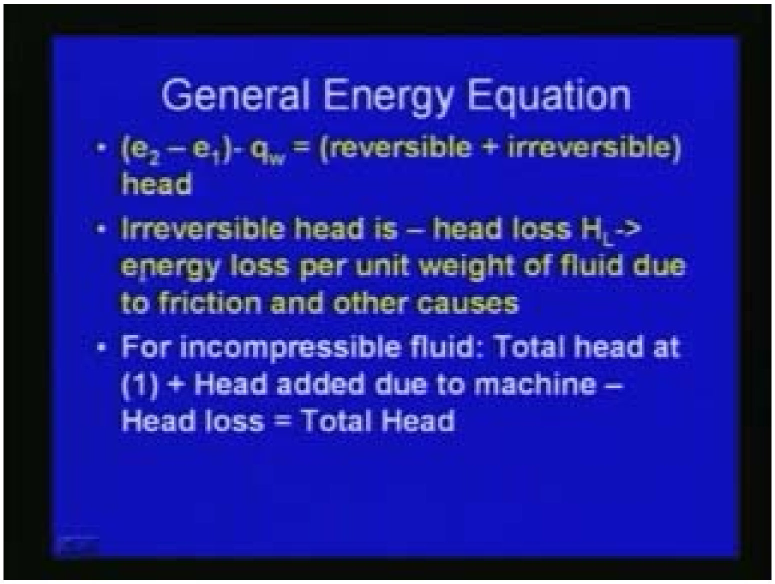

Finally, the general energy equation we can write as h_1 plus h e minus h_l is equal to h_2 and the work done over a fluid is equal to power input into power input flow that is equal to p is equal to gamma q h m in watts and where gamma is the unit weight of fluid q is the discharge in meter cube per second and h m is the head added to the flow in meter.

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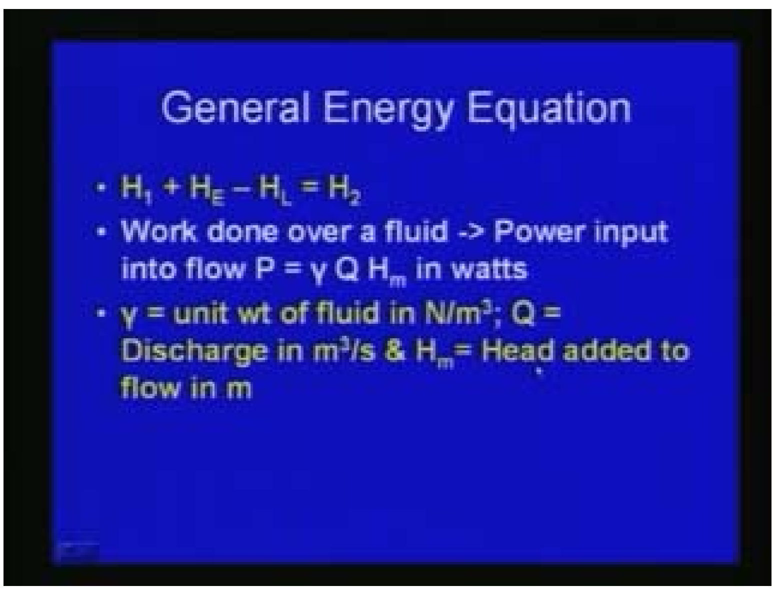

We have already seen a large number of applications of the Bernoulli's equation and now we have already discussed the general energy equations, the general energy equation is the Bernoulli's equation which we have derived is, for in viscid flow the Bernoulli's equation can be generalized with respect to the equations which we have seen as general energy equation. Now the Bernoulli's equation which we have discussed, have number of limitations since we have put certain assumptions with respect to the derivations of the Bernoulli's equations.

The limitations important limitations are first one is the compressibility effects of the fluid., the Bernoulli's equation which we have derived or we have discussed far, the compressibility effect is not taken into account. We have assumed that the fluid is incompressible if the compressible fluid fluids are considered then, we have to slightly modify the Bernoulli's equation. The Bernoulli's equation can be modified for the compressibility effects and al the Bernoulli's equation which we have discussed for steady state whenever the problem, we have to deal with the unsteady conditions then the equation is to be modified for the unsteady flow.

In the next slides here with the limitations of the Bernoulli's equation which we have already seen earlier. Here you can see the Bernoulli's equation for the unsteady case. Here we can see the ro into del v by del t. The steady state equation is modified like this rho into del v by del t d s plus d p plus half ro d v square plus gamma d z is equal to 0 along a streamline and similarly we can rewrite as p_1 plus half rho v square plus gamma z_1 is equal to rho integral s_1 to s_2 del v by del t d s plus p_2 plus half rho v_2 square plus gamma z_2 along a streamline.

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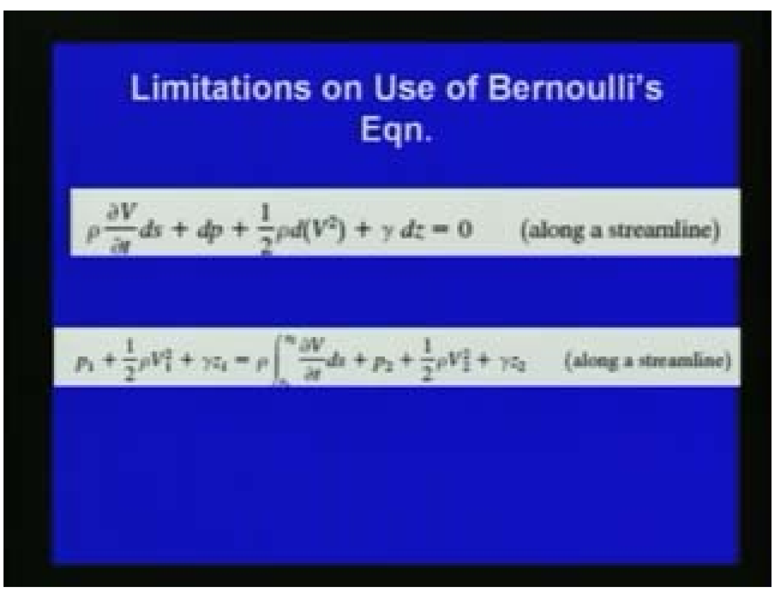

Bernoulli's equation which we have seen earlier discussed is steady state as shown here we can modify whenever we consider the transient or the unsteady conditions. These have some of the limitations of the Bernoulli's equation and further limitations are like the rotational effect. Whenever the fluids which we consider, has got the rotational effects the rotational effects care should be taken while applying for flow across the streamline.

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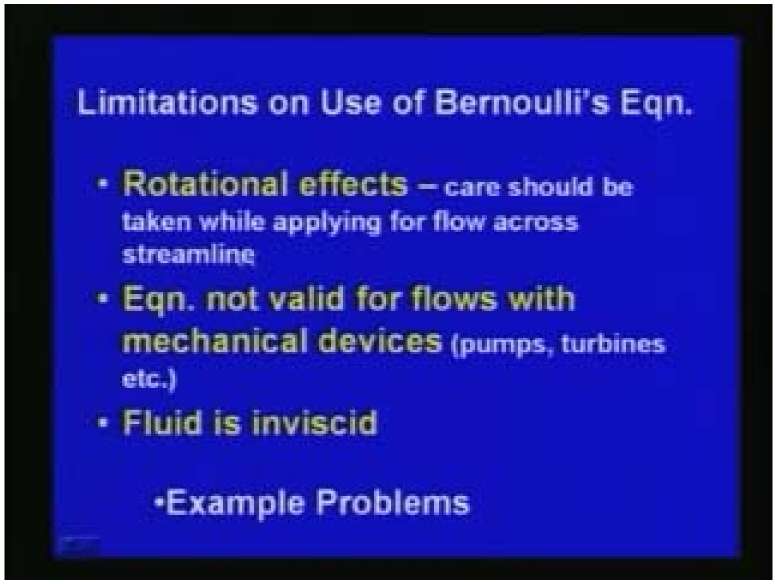

We have derived the Bernoulli's equation by considering a streamline. Whenever the rotational effects are there for the problem which we consider we have to take care the while applying for flow across a streamline. Also other limitations include the equation the Bernoulli's equation is not valid for flows with mechanical devices for example whenever a pipe line system is when we consider a problem with pumps turbines etc specific care should be taken that the Bernoulli's equation can be applied. As we have already seen the one of the important assumptions in the derivation of the Bernoulli's equation is fluid is inside whenever viscous fluids are considered still Bernoulli's equation can be applied, but there will be it is one of the limitations it is not 100 percent correct that for viscous fluid these equations the assumption the Bernoulli's equation is derived based upon the assumption is that fluid is inside these are me of the important limitations on the use of Bernoulli's equations.

Now we will see few of the example problems with respect to the Bernoulli's equations we have discussed far before going to other topics. The Bernoulli's equations the example number two earlier, we have already seen one of the example the next example is here we consider a problem, water is flowing through a smooth pipe of uniform diameter here there is a pipe the diameter is 20 centimeter at section one at elevation 5 meter. Here we consider two section one at 5 meter from datum this is the datum the pressure is 30 kilo Pascal here the pressure is 30 kilo Pascal. Section two at elevation 7 meter here at this section pressure is 15 kilo Pascal and velocity is 1 meter per second. We want to find the head loss between the sections between section 1 and two and the flow direction.

This is one of the simple one simple applications of the Bernoulli's equation. Here a pipe flow is there water is going through the pipe and the velocity is given at section two and the pressures are given at section one and two. We have to find out the head loss between section one and two and the flow direction. This is one of the simplest applications of the Bernoulli's equation.

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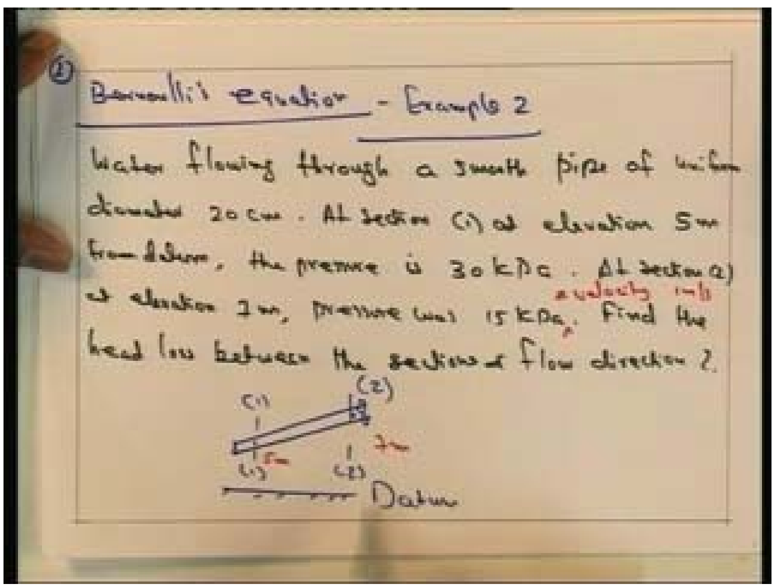

To solve this problem first we will use the continuity equation if you use the continuity equations we are considering two sections continuity equation can be applied with respect to the conservation of mass. We apply the continuity equation q is equal to a_1 v_1 is equal to a_2 v_2 here only the diameter is known the diameter is 20 centimeter q is equal to pi by 4 into 0.2 square velocity v_1 is given as 1 meter per second pi by 4 into 0.2 square into 1 that gives 0.0314 meter cube per second, here since the pipe is assumed the uniform diameter v_1 is equal to v_2 and a_1 is equal to a_2 and al this specific weight of the liquid is 998. This can be converted to 998 into 9.81 by the d rho is given as 998 kilo gram per meter cube.

That can be converted as gamma is equal to gamma is rho g 998 into 9.81 by 1000, 9.79 kilo Newton per meter cube. We are considering atmospheric pressure and hence we are taking it as 0 at section one if you apply the Bernoulli's equation if you consider section one here.

If you consider the Bernoulli's equation here we can see as per the Bernoulli's equation p_1 by gamma plus v_1 square by 2 g plus z_1 p_1 by gamma already it is given that the pressure is thirty Pascal p_1 by gamma is 30 by 9.79 then v_1 is given as 1 per second p_1 by gamma plus v_1 square by 2 g plus z_1 becomes 30 by 9.79 plus 1 square by 2 into 9.81 plus 5 that will be obtained as 8.15 meter.

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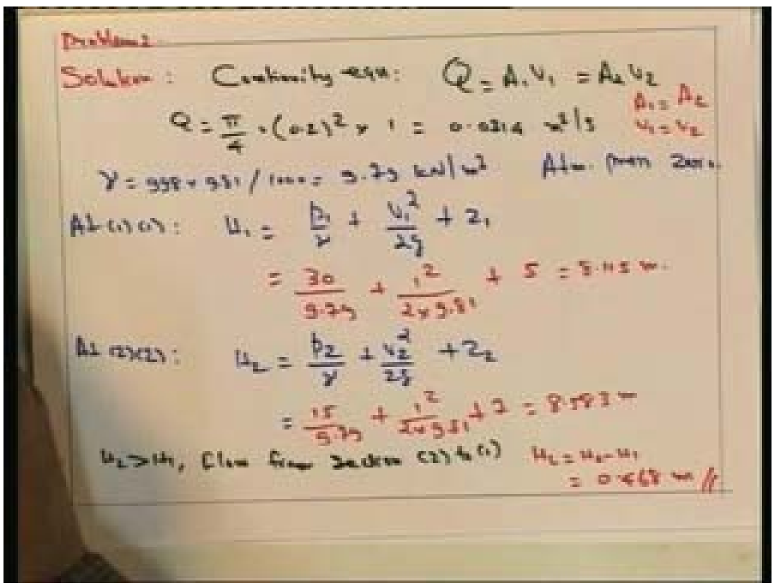

If you consider similar way the section two which we consider here for section total head h_2 is equal to p_2 by gamma plus v_2 square by 2 g plus z_2 p_2 is given as the pressure is section two is given as 15 kilo Pascal p_2 by gamma is 15 by 9.79 plus v_2 is 1, 1 square by 2 into g 9. 81 plus z_2 is 7. This we get as 8.583 meter here you can see this h_2 is greater than h_1 since 8.58 there is greater than 8.115 the flow occurs from section two to one.

We can see that the pressure head here at section two is much higher than section one. The flow takes place from section two to one and head loss is h_2 minus h_1 is 0.468 meter.

This is one of the simple application of the Bernoulli's equation we can apply the Bernoulli's equation for large number of problems. We will also discuss few more examples here example number three which we discuss is here a tank is filled with oil of relative density 0.9 and siphon of diameter 10 centimeter is used to empty the oil as shown in figure here this is the figure here.

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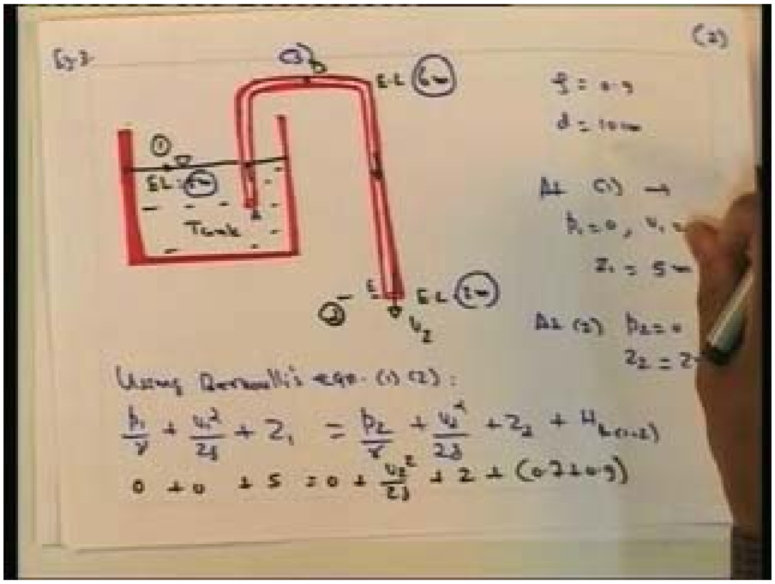

Here we have got a tank and a siphon is used to empty the oil from this tank like this using a siphon and question is the oil surface in the tank is at an elevation of 5 meter and siphon discharges to atmosphere at an elevation of 2 meter. Center line of siphon at its highest d is at an elevation of 6 meter the losses in pipe to higher is 0.7 meter and 0.9 meter from higher point to outlet.

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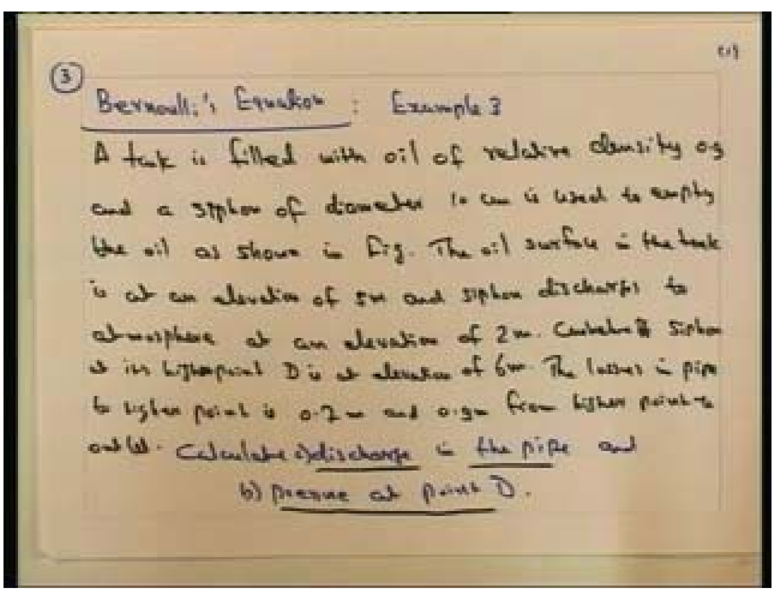

we have to find out the discharge in the pipe and the pressure at point d. problem is here we have water tank we are using a siphon to take the oil out from the tank with respect to this here you can see we have to find out the discharge in the pipe throughout the siphon we have to find out the pressure at point d. This is the problem; here it is given that the density of oil rho is 0.9 d is the diameter of the pipe is 10 centimeter.

If you consider now consider using the Bernoulli's equation here you consider three points, one is the liquid level or the oil level in the tank this is point one, we consider point two as the outlet of the siphon here this is point two, we will also consider the highest point of the siphon that is point three, we will consider these three points. We will use the Bernoulli's equation to solve this problem we want to find out the discharge passing through this pipe also the pressure at this point this is the point in the problem.

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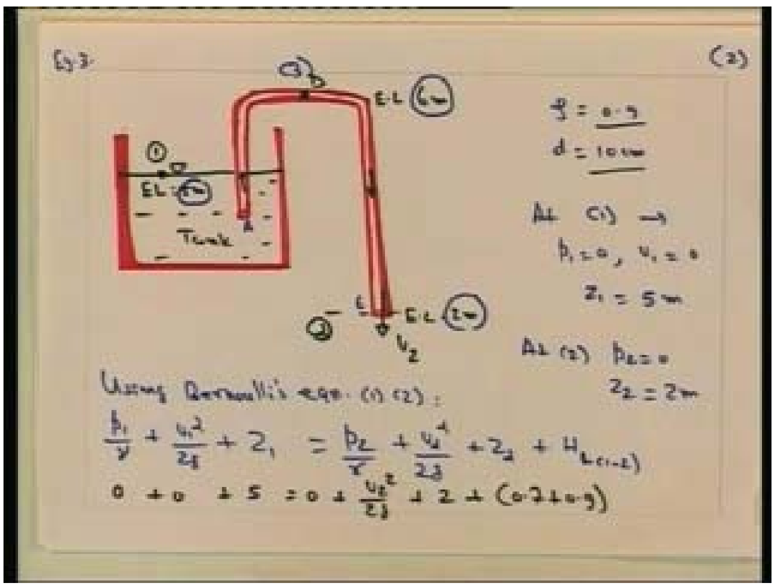

If you consider point one, we can see that here this tank is open to atmosphere that we can say that the pressure p_1 is equal to 0 here the there is no velocity of fluid v_1 is equal to 1 the elevation head is given as z_1 as 5 meter for this problem. At section two where the at the outlet of the siphon, we can see that the pressure is given as p_2 is equal to 0 that it is the flow into the atmosphere p_2 is equal to 0 we can see here the elevation head is given as 2 meter.

We will apply the Bernoulli's equations between section one and two if you consider the Bernoulli's equation between this 0.12, here or section two and section one using the Bernoulli's equation we can write p_1 by gamma plus v_1 square by 2 g plus z_1 is equal to p_2 by gamma plus v_2 square by 2 g plus z_2 plus head loss between point one to point two.

This is the Bernoulli's equation here you can see that we have already seen p_1 is equal to the pressure at point 1 is atmosphere we consider it 0 p 1 by gamma is 0 velocity at this location is al 0 v_1 square by 2 g is 0 and z_1 is already given as 5 meter 0 plus 0 plus 5 is equal to at section 2 p_2 by gamma p_2 is already 0 this al this term al 0 and plus v_2 square by 2 g here the velocity at this section to be v_2 v_2 square. By 2 g and the elevation head is given as 2.

Plus 2 plus we have got the head loss. the head loss it is already in the problem it is given here the head losses the losses in pipe to higher point is 0.7 and 0.9 from higher point to the outlet. These are the head loss from section one to two 0.7 plus 0. 9 this al added we use the Bernoulli's equation between section one and section two.

If you use this here now we get write v 2 square by 2 g is equal to, we can get an expression for the velocity v 2 square by 2 g is equal to 5 minus 3.6 that is equal to 1.4 meter. Finally we get the velocity at the outlet v 2 here we get v 2 we get the value of v 2 as v 2 is equal to 5.241 meter per second and finally the first part of the question is we want to find out the discharge. Discharge is equal to area of cross section into velocity area of cross section is the diameter of the pipe is 10 centimeter pi by 4 into 0.1 square into the velocity v_2 5.241 that is gives the discharge.

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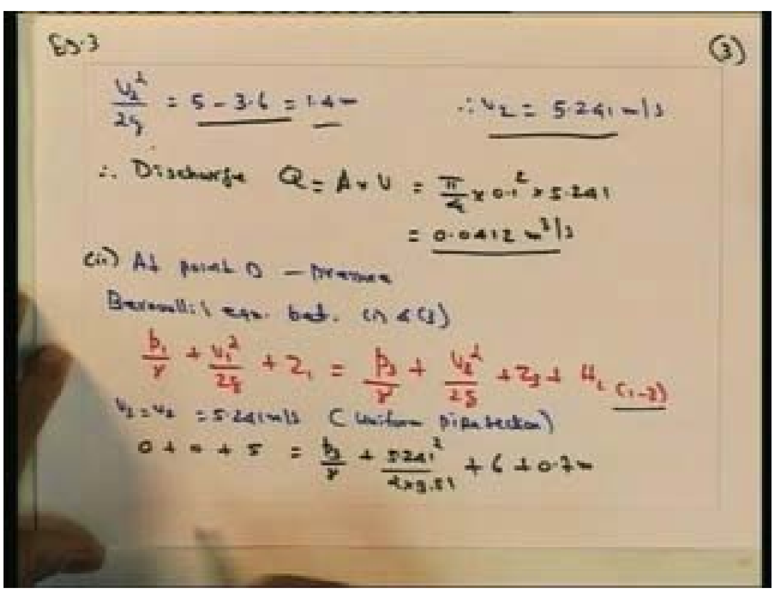

Finally, we get the discharge as 0.0412 meter cube per second. Then, the second part of the question here is we are asked to find the pressure at highest point d to find out the pressure at point d. We will again consider here the figure, we will consider the

Bernoulli's equation between this point one section one and section three between this point and this point, we will consider between one and three.

If you apply the Bernoulli's equation again p_1 by gamma v_1 square by 2 g plus z_1 is equal to p_3 by gamma plus v_3 by 2 g plus z_3 plus h l that means the head loss between section one and two one and three, here you can see that since the pipe siphon is of same size area of cross section is same the velocity also you can see that v_3 is equal to v_2 we have already found out v 2 as 5.241 v_3 is equal to v_2 that is equal to 5.241 meter per second.

Finally, if you write the Bernoulli's equation between sections one and three, we can write 0 plus p_1 by gamma is 0 v_1 is 0, 0 plus 0 plus 5 that is equal to p_3 by gamma plus v_3 square by 2 g v_3 is already found 5.241 5. 241 square by 2 into 9.81 plus 6 is the z_3 here we can see that this section is 6 meter elevation is given as 6 meter in the datum plus 6 plus.

The head loss between points one to three is given as 0.7 meter, plus 0.7 meter this is the Bernoulli's equation between section one and three. Finally we can get v_3 by gamma as v 3 by gamma is equal to 5 minus 6 minus 0.7 minus v square 5.241 square by 2 into 9.81 this is equal to minus 3.1 meter pressure at d finally we get this will be 3 by gamma gives the d that is equal to minus 27.315 kilo Pascal.

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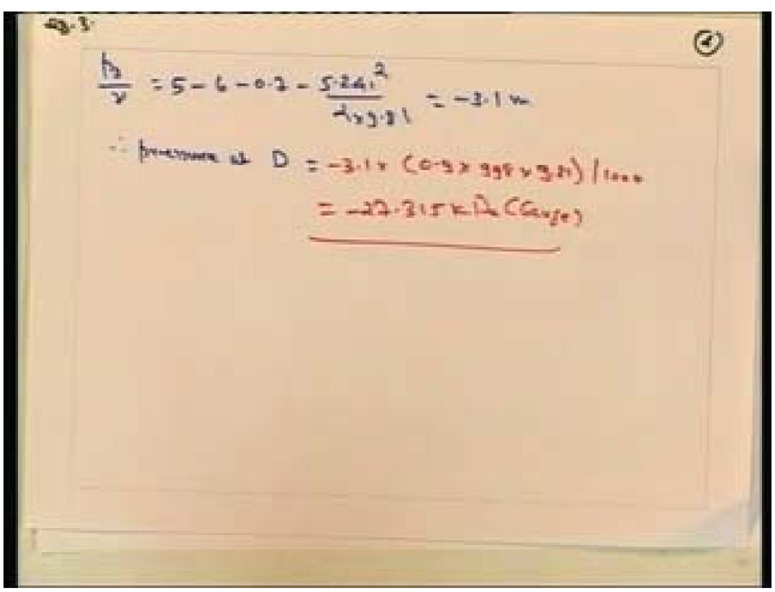

This is the pressure at d like this we can use the Bernoulli's equations for a given problem to find the discharge or the pressure or whatever the variable depending upon the problem.

Before closing this we will see one more example related to the Bernoulli’s equation. the 4th example related to the Bernoulli’s equation is here we have got a water tank, we use a pump here to pump water from this tank here water is pumped from a tank at the rate of 0.5 cubic meter per second, here the discharge the pumping rate is 0.5 cubic meter per second and the pump supplied an energy of 20 kilo watt here the energy supplied by the pump is 20 kilo watt we are asked to find the pressure at intensity at a.

The location of the pump we want to find out the pressure intensity at a. here even though there is a pump attached with the problem pump is there but still we can use the Bernoulli’s equation here first the pump energy is given as 20 kilo watt pump energy is equal to, we have already seen the equation for pump energy as gamma q into delta h here pump energy is 20 kilo watt gamma q delta h is 20 kilo watt. From which we can write the with respect to pumping what will be the head delta h is equal to 20,000 by gamma for water.

If you take 9810 and here the discharge is given as 0.5 20 thousand divided by 9810 into 0.5 from which we can get delta h that means with respect to pumping we get is 4.07 meter of water.

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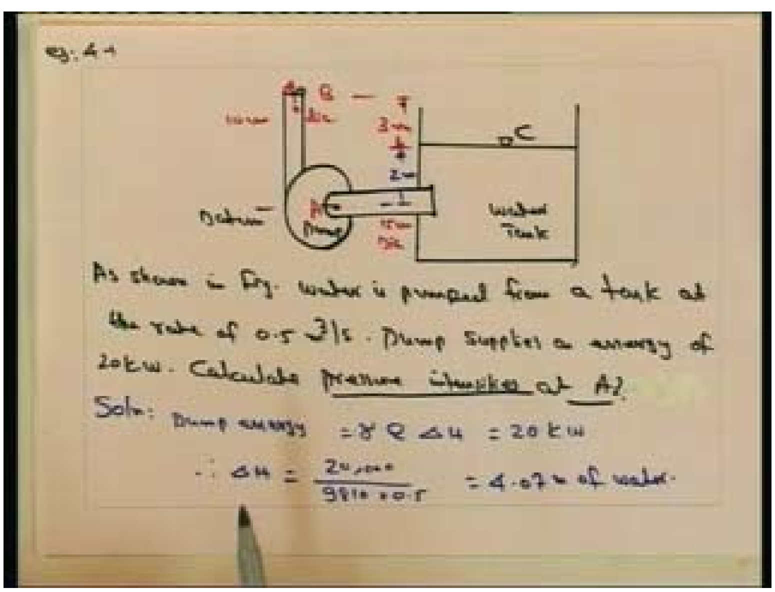

From that we get the head which can take care by the pump that is equal to 4.07 meter of water from that we get the head which can take care by the pump that is equal to 4.07

meter, we have to find the velocity at the location a here already the we have already calculated the head here from that we can try to find out the velocity already the discharge is given mew here velocity at a is equal to discharge by cross sectional area.

Here this pipe diameter is 15 centimeter here from which velocity at a, is equal to v a, is equal to q by a, we get 28.29 meter per second. If you consider here at the exit of this pipe with respect after the pumping is located here water is going through this exit at d.

Here the diameter is given as 10 centimeter the velocity at d we can find v_b is equal to 15 by ten square into v a since to conserve the continuity equation we can see that v b into a is equal to v_b into a is equal to a 1 is equal to or a 2 is equal to v a into a_1 if you use this we can see that.

We can write v b is equal to 15 by ten whole square into v is equal to 63.65 meter per second. Finally, we will apply the Bernoulli's equation between point a and c here the tank top of the water level here you can see that the pressure will be pressure is atmosphere we can take it as 0.

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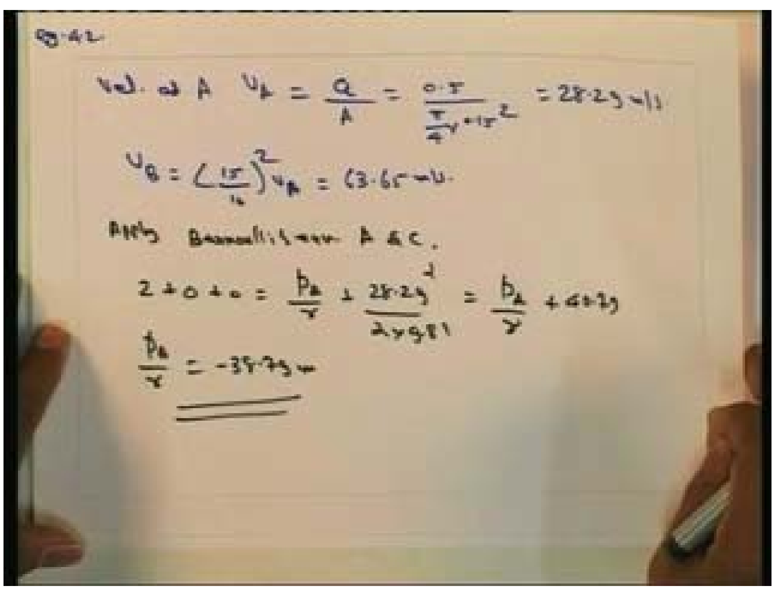

You can see here there is no flow takes place the velocity can be considered as 0 if you consider the Bernoulli's equation between points c and a that, we can find the write the equation as here the z_1 the Bernoulli's equation is applied that will give here the velocity all the values are known this is 2 plus you can see that between this level, the location a and location c the level difference is 2 meter that, we can that 2 plus it is z the location with respect to datum 2 plus 0 plus 0 this term the pressure is 0.

This is 0 the velocity is 0 again 0 2 plus 0 plus 0 is equal to p a by gamma plus the velocity here v 2 square by 2 g that is 28.29 which we already got or v a square 28.29 square by 2 into 9.81 that gives p a by gamma p a by gamma. Finally we get as minus 38.79.

We have seen few applications how we can solve various problems by using the Bernoulli's equation. Finally, we have already seen the Bernoulli's equation we have derived by certain assumptions like fluid is in viscid and flow is incompressible, we have already seen the limitations and al we have seen the various applications of the Bernoulli's equations. We have tried to solve few of the numerical examples various typical examples of this kinds of where the Bernoulli's equation can be applied, we have seen with respect to the energy equations and the Bernoulli's equations and now we will go to the next topic which is the linear momentum equations

Here we have already seen the energy equation the Bernoulli's equation now we will discuss the momentum equation linear momentum equations. In all these we have already seen that the Newton's laws we have directly applying this momentum most of the time we will be dealing with now we are discussing the dynamics of fluid flow. We are dealing with moving fluids all the time the moving fluids exert the force either one way or another way. The moving fluids the force which is exerted by the moving fluids have got lot of practical importance where we can use it for various purposes the momentum or the force created by the moving fluids;.

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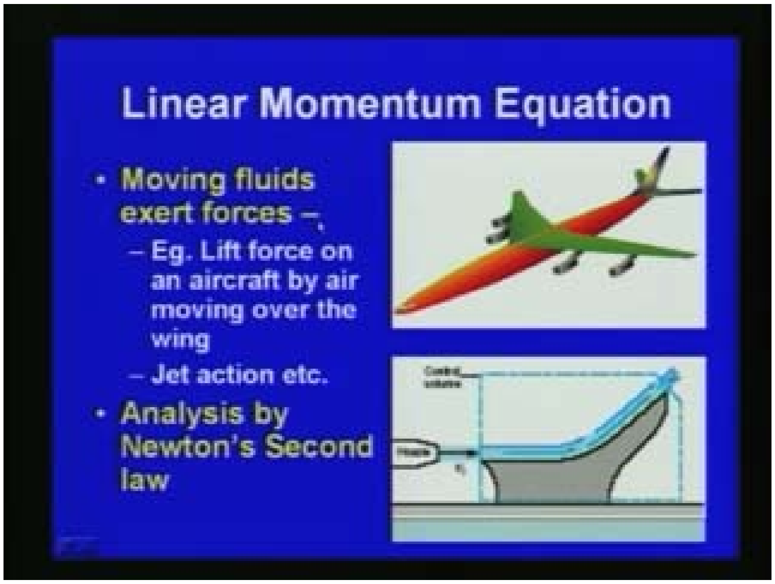

For example, if you consider the lift force on a air craft by air moving over the wing al we can easily see many times jet actions many times we use jet action. For example in turbines and me of the other kinds of problems where in engineering, we use jet action jet what is jet action fluid is coming through a pipe, we are making it just like a nozzle we are reducing is diameter the liquid will be coming at a force to a plate here you can see that here there is a plate the liquid is coming at a force you can see this is the jet action.

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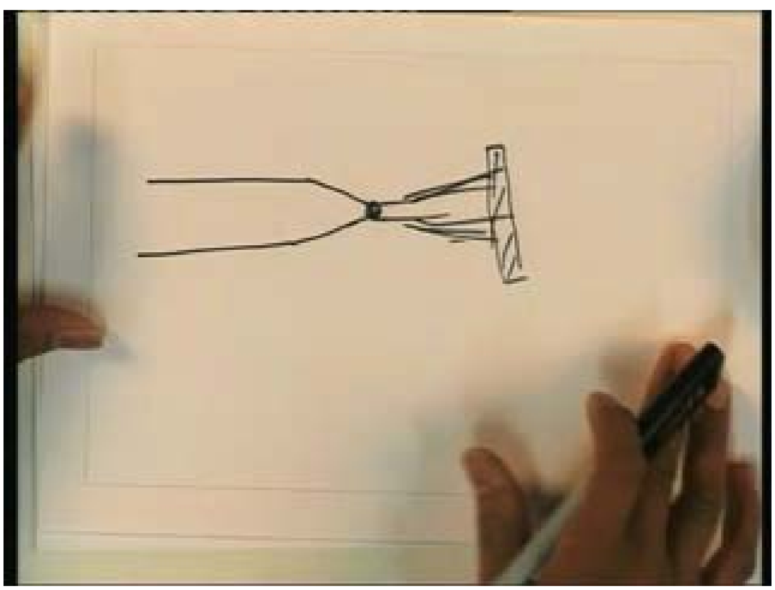

This jet action al the force is exerted by the moving fluid the moving fluids exert forces here we can see here there is a nozzle is there and jet is put on a plate like this, you can see that there is a force and this force we can utilize for various engineering purposes this moving fluids exert forces with respect to this we can use the momentum equation to find out how much is the force coming.

Here, we use the Newton's second law to analyze this moving fluid we derived the linear momentum equation with respect to the Newton's second law linear momentum equation is it is this momentum equation statement of the Newton's second law, as I mentioned moving fluids exert forces force is equal to mass into acceleration from this the Newton's second law, we can derive the momentum equation it is actually a statement of the Newton's second law

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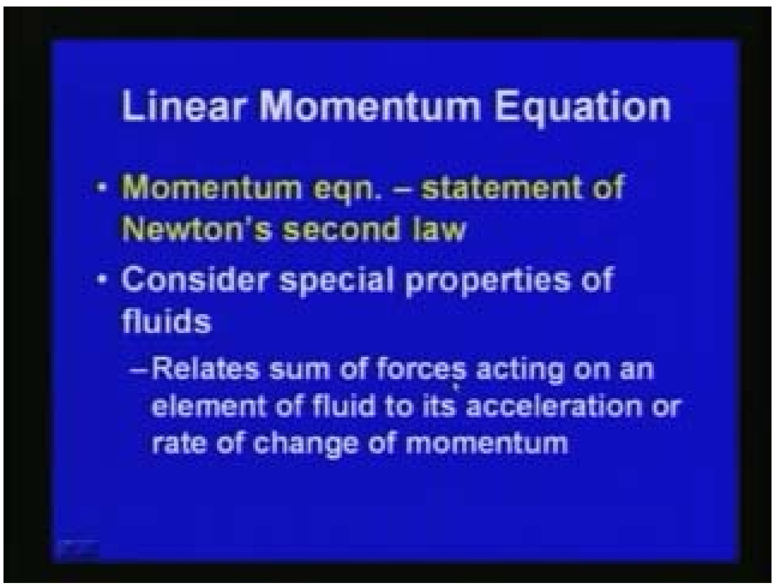

if you consider special properties of fluids like sum of forces we can with respect to movement of fluids we can relates sum of forces acting on an element of fluid to its acceleration or rate of change of momentum. As per linear momentum equation by using the Newton's second law we can write the sum of forces acting on a element of fluid to its acceleration or rate of to its acceleration or rate of change of momentum force is equal to mass into acceleration, we can equate it to the rate of change of momentum taking place between if you consider two section or a control volume.

That control volumes what changes takes place this we can equate to the rate of change of momentum that is that way we can derive the linear momentum equations from the Newton's second law we can write the rate of change of momentum of a body is equal to the resultant force acting on the body and takes place in the direction of force. This gives the Newton's second law the Newton's second law states that the rate of change of momentum of a body is equal to the resultant force acting on the body using this Newton's second law, we can we can see that here it deals with system momentum and forces.

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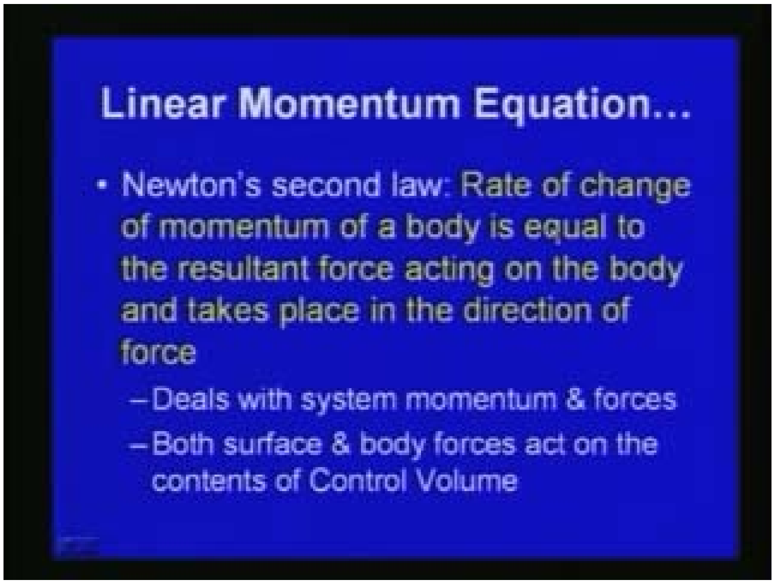

If you consider a open system or closed system the Newton's second law deals with the momentum and the momentum occurring with respect to control volume or the between the sections which we consider and the forces acting on with respect to control volume. As we have already seen the control volume concept.

We have to consider both surface forces as well as body force acting on the control volume control volume we have already seen most of the problems which we will be solving here. We are using the control volume for the control volume we have to consider what happens within the body the volume itself the surfaces what happen, we have to consider the linear momentum equation here we have to consider we are using the Newton's second law.

Force is mass into acceleration then we are equating that to the momentum changes we have to see the system the control volume of the system which we are considering its momentum and the forces as the forces or the moment are concerned what happens on the surfaces as well as the inside the body forces. Here in this slide if you consider typical control volume for flow in a tunnel shaped pipe this is pipe coming it is just opens like this fluid is flowing in this direction. If you to consider the linear momentum equation for a finite control volume like this

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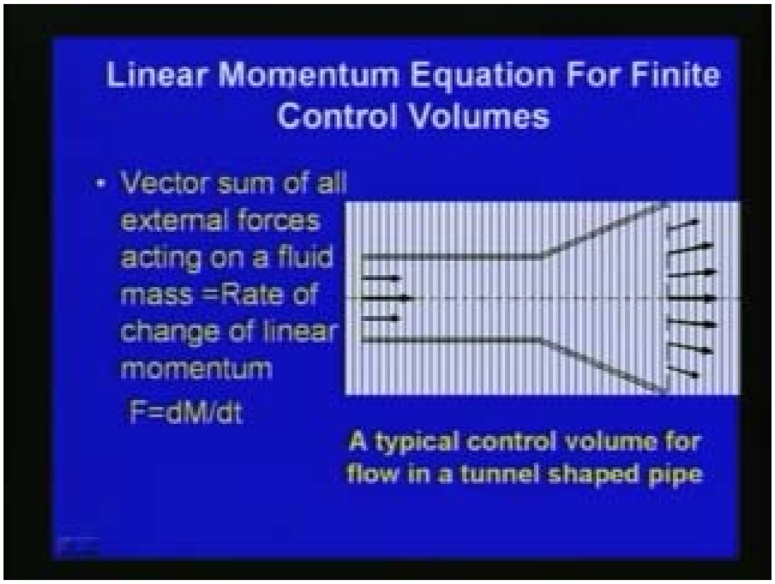

If you consider section between this section and this section we can find out the vector sum of all external forces acting on a fluid mass that will be equal to the rate of change of linear momentum.

As per Newton's second law we can find out the vector sum of all the external forces and that will be equal to the rate of change of linear momentum finally linear momentum equation is obtained, as we can find out the rate of change of linear momentum f is equal to the force is equal to the vector sum of all external forces equal to dm by dt or the rate of change of linear momentum, if you consider this control volume what are the forces acting on this control volume, we can equate the with respect to the fluid incoming fluid and outgoing fluid within this control volume. We can find out the rate of change of linear momentum equate to the vector sum of all external forces within this control volume.

If you since we are considering here the forces when we deal with the forces any body or any control volume which we deal we have to deal with the internal forces as well as external forces. Here the forces which we are considering the external forces which we are considering first one is the boundary forces what the if you consider the control volume what are the forces on the boundary that includes the boundary forces the normal forces with respect to the flow coming inside and flow going outside there will be normal forces. We have to consider this normal forces of course the internal forces of the body or field forces we have to consider.

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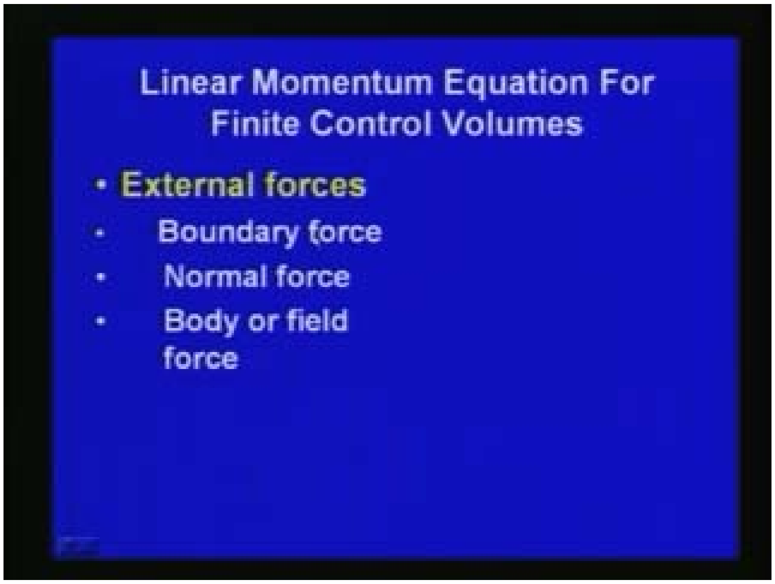

The forces which we have to consider include the boundary forces the normal forces and the body or field forces. These are the forces when we consider the linear momentum equation these are for a control volume these are the forces which we have to consider finally with respect to this we can write the equation as.

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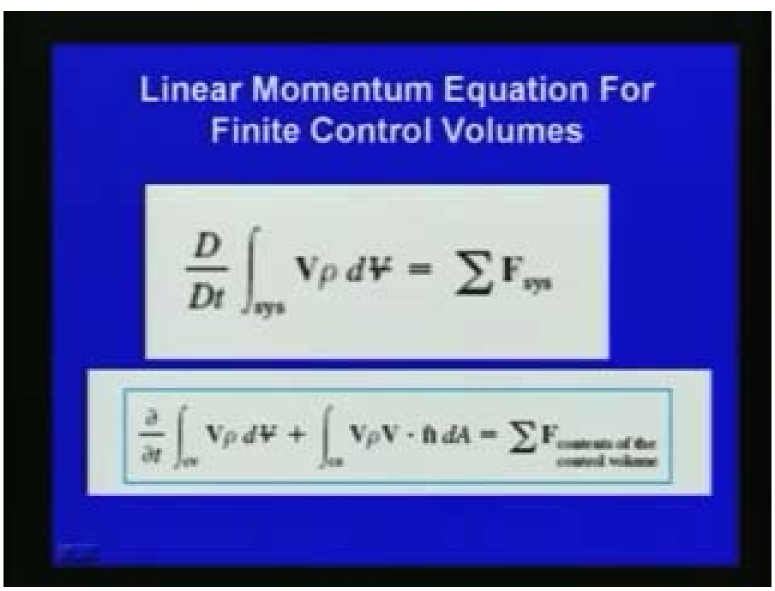

With respect to the Newton’s second law, finally, the equation already shown here it is written here d by dt, if you consider the system integral of the system v rho d v is equal to sigma f system you can see the total the adverse me of the forces is equal to the rate of change of momentum as we have seen the rate of change of momentum is equal to resultant force acting on the body we can write d by d t integral system for the system v ro d v is equal to sigma f system. Finally, this we can be written as I mentioned we have to consider the control volume inside the volume what happens on the surfaces what happen.

We can this term we can write as del by del t of the on the control volume v rho d v plus the control surface integral control surface v rho v dot n d a that is on the control surface that is equal to sigma the forces of the system that means sigma f contents of the control volume. This gives finally the equation which we are looking for with respect to the linear moment of equation for finite control volume derived based upon the Newton's second law.

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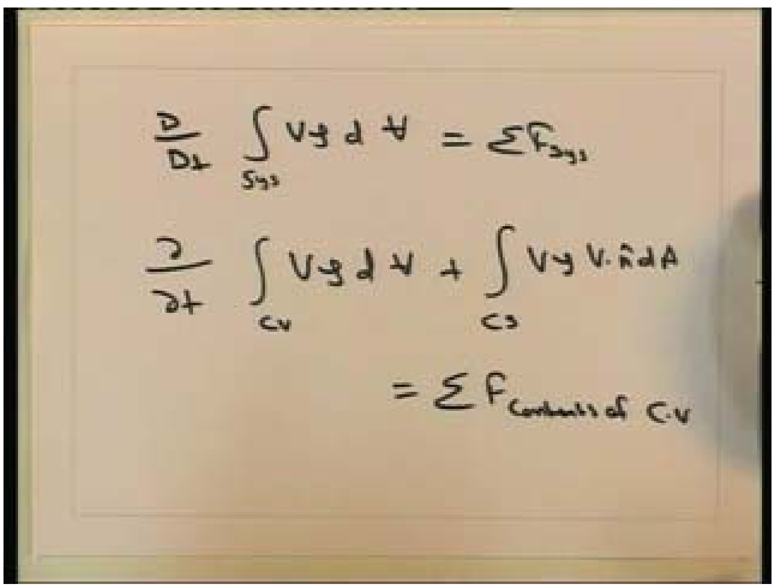

Now with respect to the equation which we derived if you consider here the linear momentum equation for finite control volume, shown here, considers a pipe flow like this.

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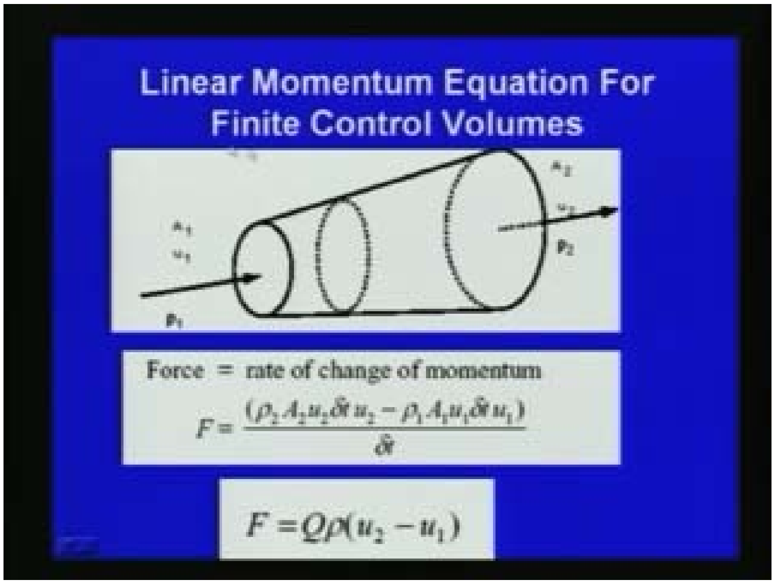

On this section we consider two sections here at this section if the area of cross section is a one and the velocity is u_1 and the density is rho 1 at section two if the area of cross section is a two the velocity is u_2 and the density is rho 2.

Force is equal to as per the equation which we force is equal to rate of change of momentum force f is equal to rate of change of moment is given as rho 2 a_2 u_2 delta t u_2 minus at section 2 minus rho 1 at section 1 rho 1 a_1 u_1 delta t u_1 divided by delta t this gives the rate of change of momentum. Here you can see this is the momentum between section one, two and the time difference delta t.

Finally, we can write f is equal to q rho u_2 u_2 minus u_1 if you consider this approximation after simplification, we can write the force is equal to q is the discharge passing through this pipe. q is constant since due to the conservation of mass between section one and 2 q is constant and if you assume the density ro is constant we can write rho 2 is equal to rho 1.

If you consider that it will be mainly the f is equal to ro q capital q is the discharge ro q into u_2 minus u_1 that gives the velocity difference between section two and one f is equal to the force is equal to q rho u_2 minus u_1.

Similarly, if you consider as a_1 dimension case and if you consider 2 dimension system we have to consider in x and y direction, we have to equate the force in x direction with respect to the change in momentum in x direction force in y direction, we have to equate to the rate of change of momentum in y direction.

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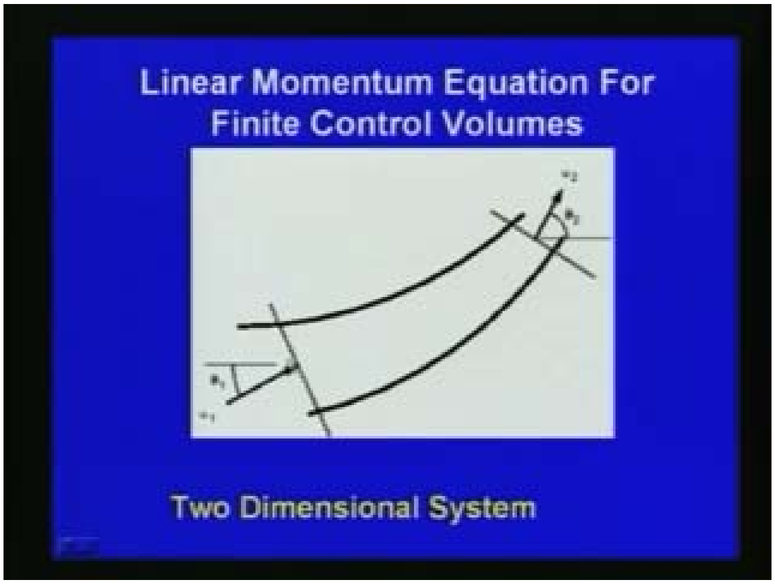

Similar way we can write for two dimension system for a two dimension system if you consider the f_x the force is in x direction is equal to rate of change of momentum is equal in x direction that is equal to rate of change of mass into change in velocity in x direction rate of change of mass if any mass change takes place. If you put a dash is the rate of change of mass change in velocity in x direction.

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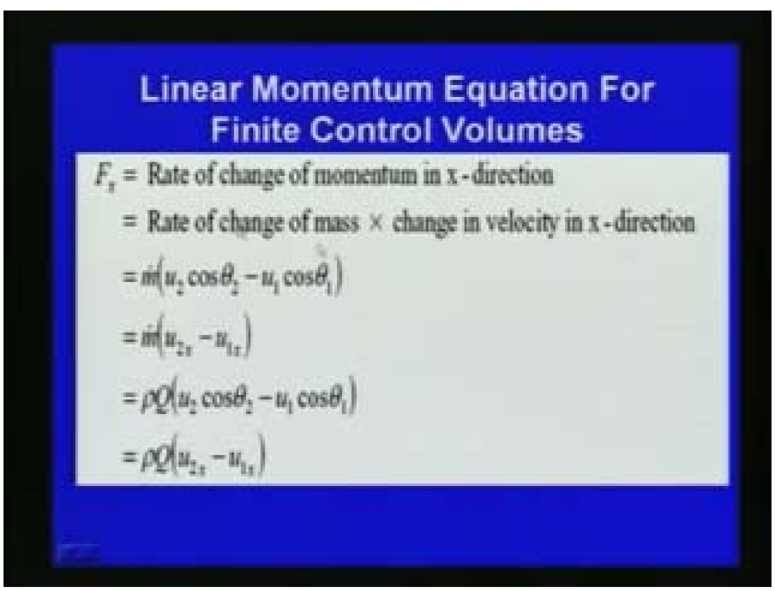

With respect to this figure (Refer Slide Time: 37:12) we can write as u_2 cos theta$_2$ minus u_1 cos theta$_1$ if theta$_1$ is the angle here and theta$_2$ is the angle here with respect to the x direction we can write as f_x is equal to rate of change of momentum in x direction that is equal to rate of change of mass into change in velocity in x direction.

Change in velocity in x direction can be written as u_2 cos $theta_2$ minus u_1 cos $theta_1$ into rate of change of mass. This is f_x, finally, we can write as u_2 cos $theta_2$ is written as u_2 x in this direction in x direction and u_1 cos $theta_1$ is written as u_1 x finally f_x is equal to m dash u_2 x minus u_1 x.

That is equal to m dash the rate of change of mass if you write the density into the discharge flows through the control volume rho q into u_2 cos $theta_2$ minus u_1 cos $theta_1$ finally this can be written as f_x is equal to rho q u_2 x minus u_1 x.

For two dimensional case can write in x direction the force in x direction is equal to the rate of change of momentum in x direction that is f_x is equal to rho q u_2 x minus u_1 x. similar way we can write for the rate of change of the rate of change of momentum in y direction f y is equal to ro q sin u_2 sin $theta_2$ minus u_1 sin $theta_1$ that is equal to rho q u_2 y minus u_1 y.

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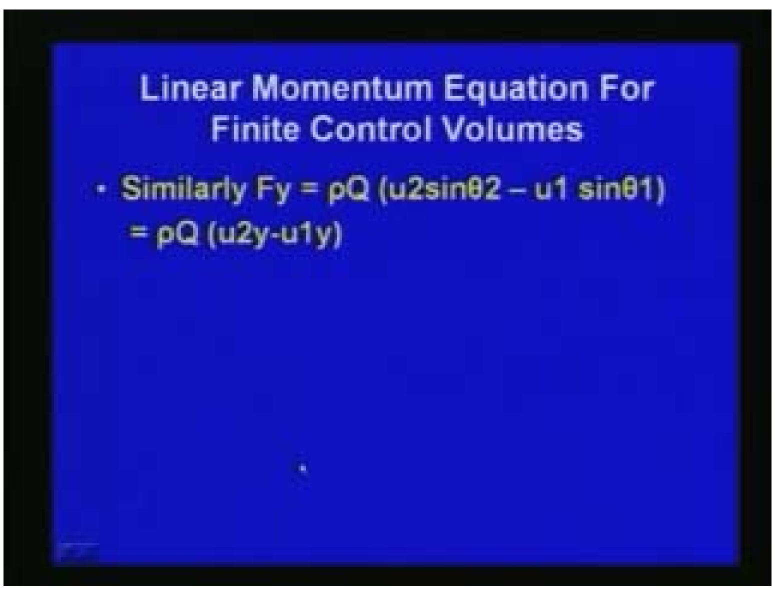

With respect to this figure u_2 y is the velocity in y direction and u_1 y is the velocity for section one and u_2 y is the velocity in section two in the direction of y. Finally for f y we can write as that means the rate of change of momentum in y direction is equal to rho q in the into velocity change in y direction u_2 y minus u_1 y this is the case for two-dimension system.

Earlier we have seen the case of one-dimension system now we have seen two-dimension system similar way we can write for three-dimensional system also now the we have already seen the when we discuss the energy equation or when we discuss the Bernoulli's equation or the total energy equation we have seen that most of the time you will be considering the average velocity.

With respect to the average velocity, when we consider the control volume or the sections which we are considering, v is the average velocity at a cross section but you can see that velocity is varying from one section to another. Here we can see that here if you consider a pipe flow here a pipe flow between section one and two the pipe flow is here you can see that velocity variation if you plot it will be like this you can see the velocity will be maximum at this center line this is velocity variation.

Velocity will be maximum at the center line of the pipe and it will be 0 on the on the side wall of the pipe but the equations which we have derived is the average velocity at a cross section. But in actual sense actually reality we can see that actual velocity will be non-uniform you can see that it will be varying from one section to another.

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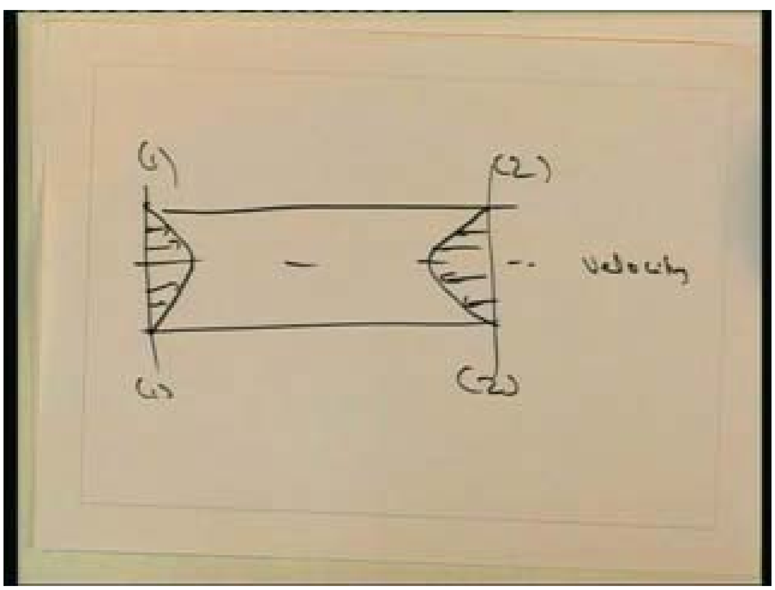

As we have already seen in the case of energy equation and the Bernoulli's equation we have applied a energy correction factor. We are discussing the momentum equation linear momentum equation for the momentum also since we are considering the average velocity at the section we have to apply the apply a correction factor called momentum correction factor momentum calculated using the average velocity must be corrected by a correction factor beta. We have to correct the momentum which we have calculated using a correction factor called beta.

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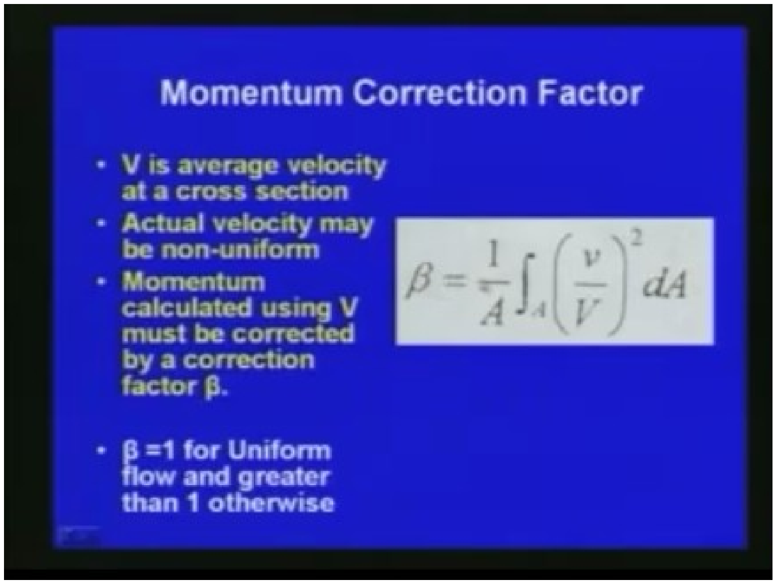

This beta as we have already seen in the case of energy correction factor we can in a very similar way we can derive this momentum correction factor beta as beta is equal to one by a integral a v by capital v whole square where capital v is the average velocity and small v is the velocity at any section which we consider as shown here this is small v and capital v is the average velocity which we consider for the case.

We have to apply for the momentum equation which we consider we have to apply a correction factor called momentum correction factor and the momentum correction factor beta is defined as beta is equal to if a is the area of cross section beta is equal to 1 by a integral a about a v by v whole square where small v is the varying velocity and v capital v is the average velocity which we consider.

If you consider the flow as uniform flow then means the velocity is same throughout the section in that case beta will be equal to one for non-uniform flow you can see that this will be beta, will always be greater than one depending upon the case whether it is depending upon the cross section, depending upon either it is pipe flow or open channel flow this beta will be varying and whether it is lamina flow or turbinal flow also the beta value will be varying which we will be discussing later.

Finally if you apply this momentum correction factor to the apply momentum principle to flow through the generalized system like here you can see the generalized system here we consider, here this is the system control volume which we are considering and here section one and section two and here if the velocity is v_1 and the discharge passing through the system is q and here at section two the velocity is v_2.

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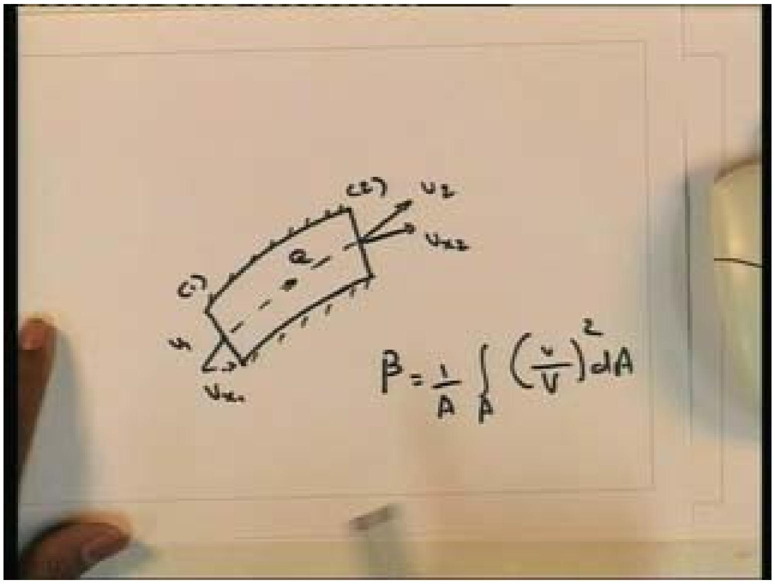

Finally we have formed beta the momentum correction factor we have to apply with respect to this we have to finally we can write the general equation the generalized equation can be written as sigma f is equal to momentum out minus momentum in sigma that is equal to the algebraic sum of the force that in the x direction can be written as sigma f_x is equal to rho 2 a_2 v_2 into the velocity in x direction v_{x2} minus rho 1 a_1 v_1 into v_{x1}. In the direction with respect to this figure sigma f_x is equal to rho 2 a_2 v_2 into v x 2 minus rho 1 a_1 v_1 into v_{x1}.

We have to apply the correction factor beta we have to use the momentum coefficient beta if you use the momentum coefficient; finally the equation should be written as like this sigma f_x is equal to rho q $beta_2$ v_{x2} minus $beta_1$ v_{x1}.

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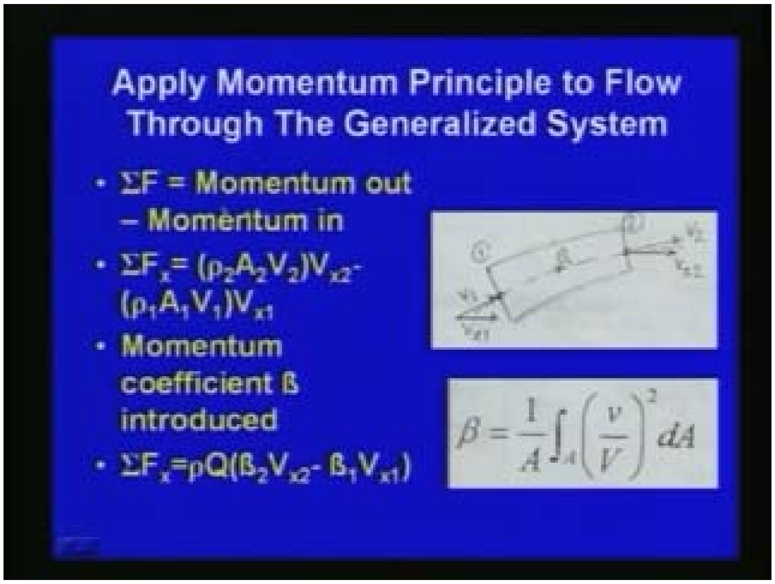

$beta_2$ is the momentum correction factor at section two and $beta_1$ is momentum correction factor at section one we can write sigma f_x is equal to rho q $beta_2$ v_{x2} minus $beta_1$ v_{x1} this in the x direction similar way we can write the equation in y direction.

We can solve the generalized momentum equation is shown here we can use the momentum principle by using the generalized equation. As we have seen the Bernoulli's equation the energy equation there are large number of applications are there for momentum equation also, here now we will discuss some of the important applications of the momentum equations.

Some of the important applications can be classified like first one is force due to flow of fluid around a pipe bend enlargement and contraction these are some of the applications. We can apply the momentum equation for pipe bend enlargement or pipe contractions al in the open channel case also bends or enlargements or expansions we can contraction we can use and second case we can use the momentum equation force on nozzle at the outlet of a pipe whenever jet action is there. We can use the momentum equations and this is the impact of jet on a plane surface or inclined surface we can use the momentum equations.

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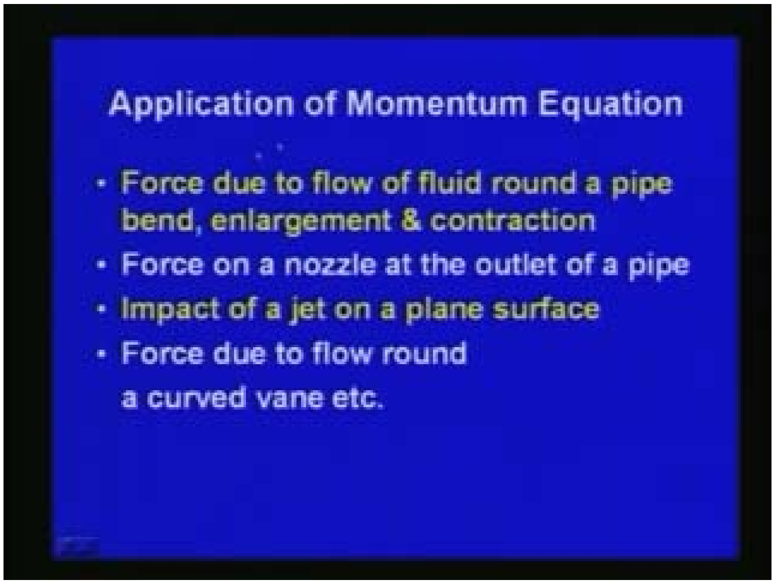

Force due to flow around a curved vane like in a turbine we can use the momentum equation momentum equation which we have derived or which we have seen far has got large number of practical engineering applications. We will be discussing some of the applications here the momentum equations. First one is the application of momentum equation force due to flow around a pipe bend. Here we will discuss a pipe bend here you can see a bended pipe like this here.

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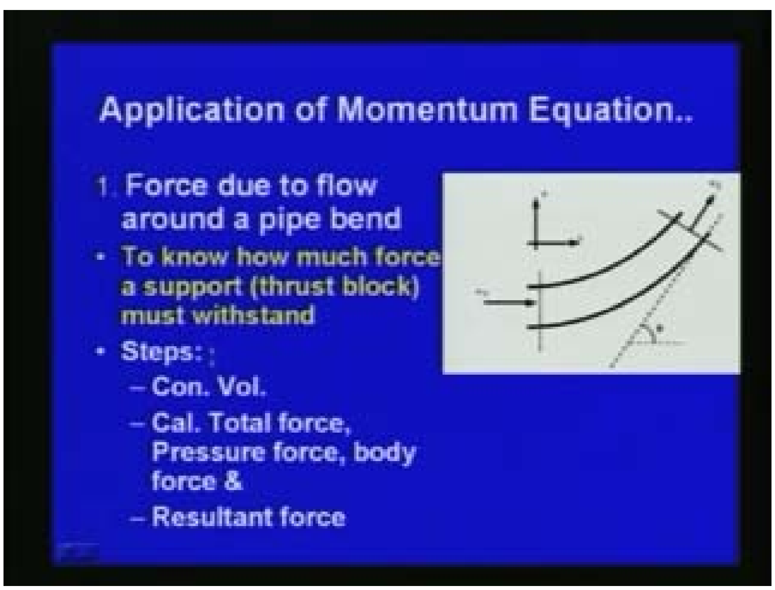

We want to know how much force is required to force should be there to support or the thrust block must withstand is there is a thrust block, here the force the flow is taking place in this bend pipe, we want to find how much is the thrust block how much thrust should be put that.

We can support the pipe the first application the force due to flow around a pipe bend then we can solve in the following steps first we will consider control volume, we have to consider a control volume with respect to flow or with respect to the case which you are considering, we will calculate the total force the including the pressure force and body force we can find out the resultant force.

As we have already seen as per the momentum equations we can equate the rate of change of momentum to the force once the force is given we can find out the momentum also we can directly apply the case.

To find out the force due to flow around a pipe bend as shown here first step is you consider the control volume here we consider the control volume that means the section between one and two we consider the control volume like this (Refer Slide Time: 47:25) here, there is an angle theta with respect to here and at section one the velocity is u_1 and section two the velocity is u_2 and x axis is in this direction y axis is in this direction the total force, which if you consider the total force with respect to section one and two. we can see here in the x direction, as we have already seen earlier before total force in x direction will be rho q is the discharge passing through the pipe and ro is the density rho q into u_2 x minus u_1 x the velocity in x direction at section two and velocity at in x direction at section one is u_1 x rho q into u_2 x minus u_1 x. If you consider u_1 x is equal to u_1 itself here and u_2 x can be with if u_2 is this direction u to x can be written as u_2 x is equal to u_2 cos theta.

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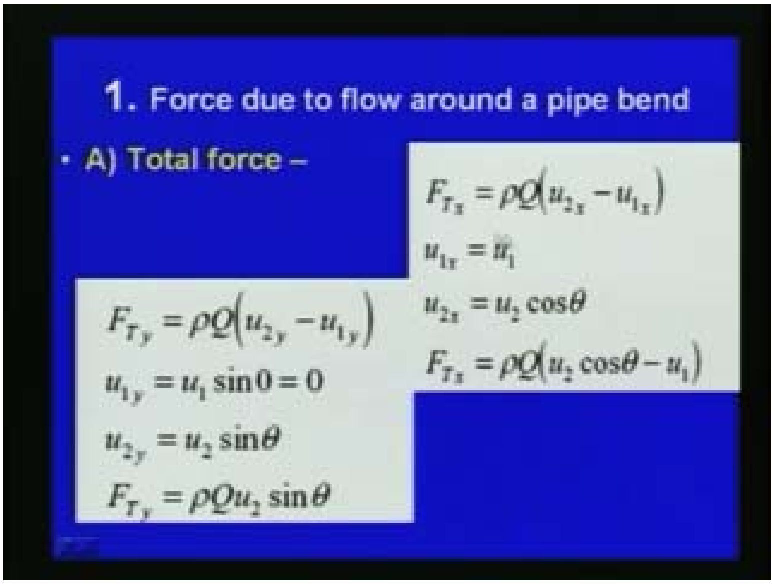

Finally, we can write f 2 x equal to f_{tx} is equal to rho q u_2 cos theta minus u_1 the force in x direction is equal to rho q u_2 cos theta minus u_1. Similarly the total force we can write f t y is equal to rho q into u_2 y minus u_1 y in y direction if you consider u_2 y u_1 y rho q into u_2 y minus u_1. Here with respect to this figure (Refer Slide Time: 47:25) if you since u_1 is in this horizontal x direction u_1 y will be u_1 sine 0 that is equal to 0 and u_2 y is u_2 sine theta .

Finally, f t y the total force in y direction will be f t f t y is equal to rho q u_2 sine theta the we have already considered a control volume and calculated the total force with respect to the fluid movement between section one and two the pressure force, we can calculate the pressure force here between section one and two the pressure force is the change in pressure force pressure force at 1 minus pressure force at 2 the pressure force p_1 is the pressure intensity at section one and p_2 is the pressure intensity at section two.

We can write the pressure force in the x direction can be written as f p x is equal to p_1 a_1 cos 0 minus p_1 p_2 a_2 cos theta here this angle is 0 here angle is theta p_1 a_1 cos 0 minus p_2 a 2 cos theta that is equal to p_1 a_1 minus p_2 a_2 cos theta.

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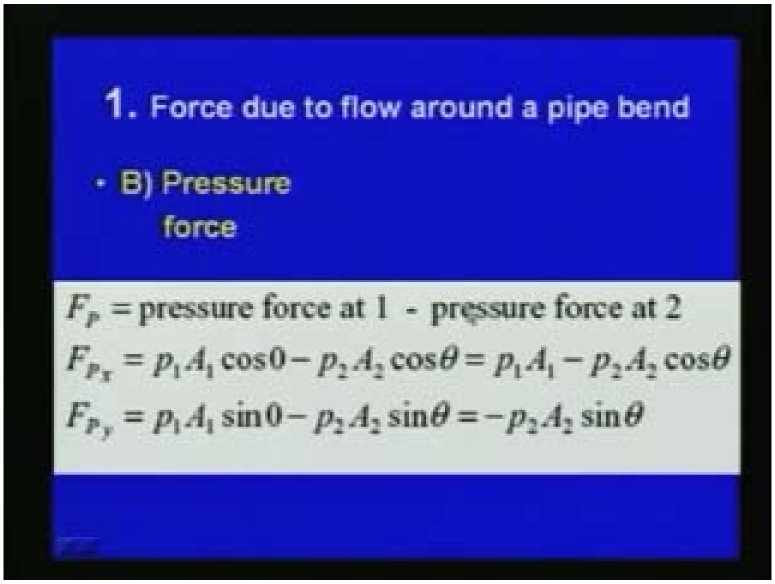

Similarly in y direction we can write the pressure force in y direction p_1 a_1 sine 0 minus p_2 a_2 sin theta sine 0 is 0 this is equal to minus p_2 a_2 sin theta if you consider the body force here mainly it is due to gravity.

Here the pipe is the bend pipe is put way such that the body force we don't have to consider here the gravity force not considered body force is 0. Finally find out the resultant force that the resultant force will be the summation of the total force. If you consider in the x direction the summation of total force in x direction that means rho q u_2 cos theta minus u_1 plus the pressure force in x direction that is p_1 a_1 minus p_2 a_2 cos theta

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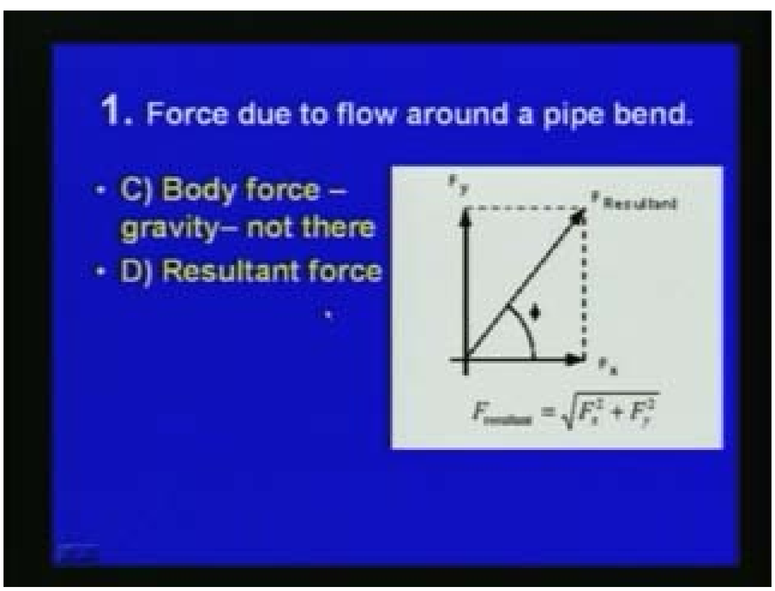

This gives the f_x force in x direction y direction will be f t y the total force in y direction is equal to rho q u_2 sin theta pressure force in y direction is minus p_2 a_2 sine theta .

That gives the resultant force in y direction and finally the f_x is plotted here f_y in this direction finally we can find out the resultant force will be square root of f_x square plus f_y square and the angle will be f_y divided by f_x tan phi is equal to f_y by f_x

We can find out the resultant force and its direction you have already seen one of the example how we can apply the momentum equation linear momentum equation which we have derived. We can apply for a particular case the force due to flow around a pipe bend

Here we have found the force the resultant force acting on the particular problem that will be equal to the rate of change of momentum. That gives the linear momentum equation using this we can find out the resultant force. For this particular problem how much thrust or how much force should be used if you want to know how much force to support or it will put as a thrust block to withstand the pipe that is obtained by the resultant force.

We can apply the linear momentum equations for various cases we will be discussing few more cases. Finally force due to flow around a pipe then it is f t x in y direction f_{rx} plus f_{px} plus f_{bx} that is already obtained as f_{tx} is already found the pressure force is found the body force is 0. Finally this expression gives the force in x direction and y direction is f_{ty} minus f_{py} minus 0. This is rho q u_2 sine theta plus p_2 a_2 sine theta that gives the force in y direction.

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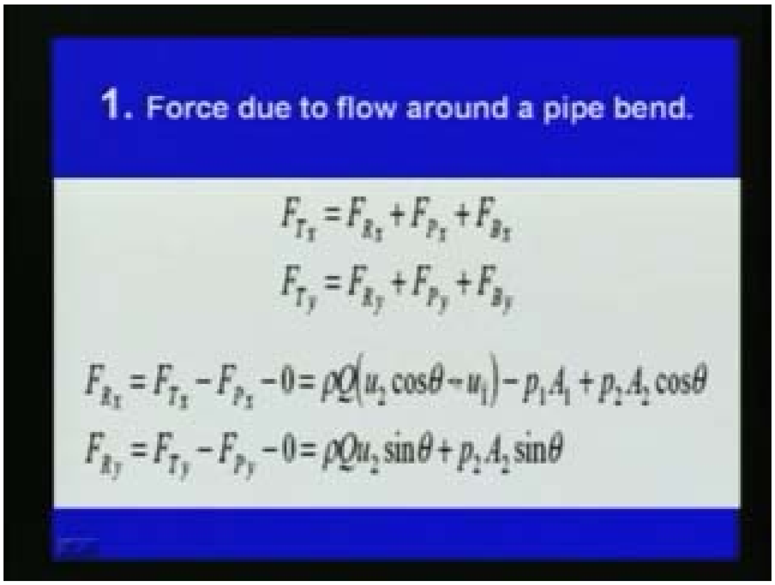

We found the force in x direction y direction we can find out the resultant force that is the thrust required to keep the pipe the that pipe the thrust block should withstand this force. This is one of the simple application of the momentum linear momentum equation. Further we will be discussing some more applications of this linear momentum equations in the next lecture.