#
#

Fluid Mechanics

Prof. T.I. Eldho

Department of Civil Engineering

Indian Institute of Technology, Bombay

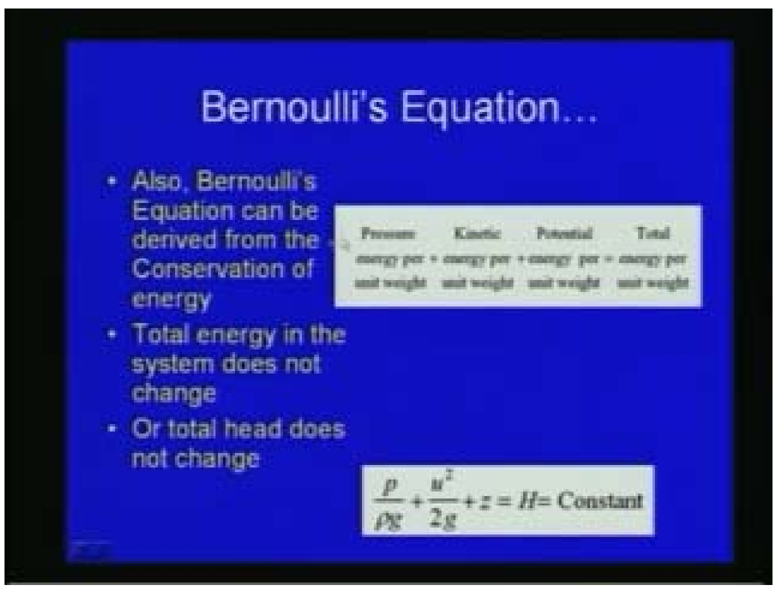
Lecture - 13

Dynamics of Fluid Flow

Welcome back to the video course on fluid mechanics. We were discussing about the dynamics of fluid flow; we have seen the derivation of the Euler's equation and how to derive the Bernoulli's equation that also we have seen in the last lecture.

Today, we will further see the applications of Bernoulli's equation and then many other practical cases where this equation can be utilized effectively that we will discuss today.

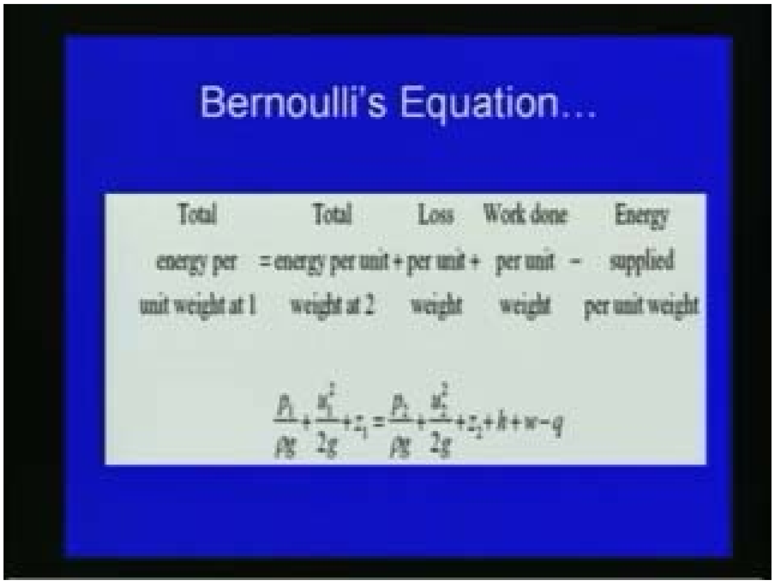
(Refer Slide Time: 01:51)



For Bernoulli's equation as you can see in this slide, the Bernoulli's equation can also be derived from conservation of energy. We have seen the energy per unit weight plus the pressure energy plus unit weight plus kinetic energy per unit weight plus potential energy per unit weight is equal to total energy per unit weight of the system. We have already seen that the total energy in the system does not change as per the conservation of energy;

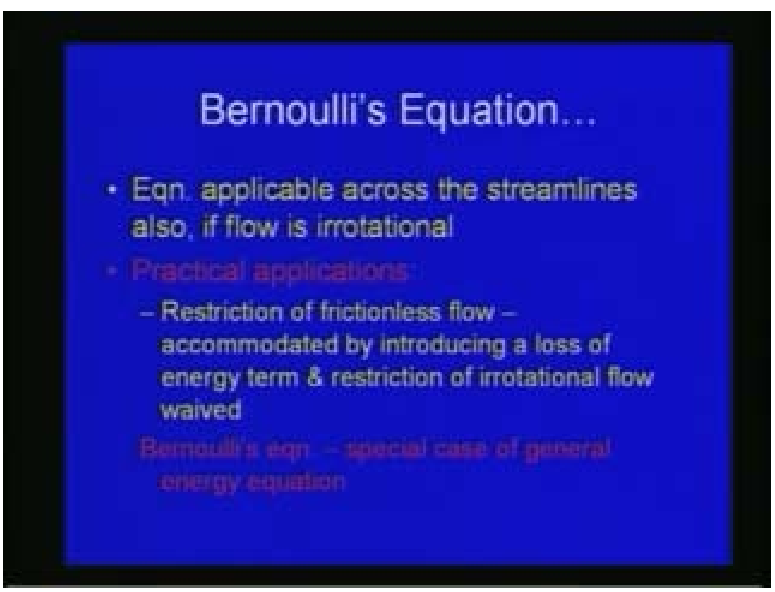
#
#
we can say that total head does not change so that this way also we can derive the Bernoulli's equation. Also, we have seen this between two points if you take total head at 1 is equal to total head at 2 if you consider in a pipe flow.

(Refer Slide Time: 02:32)



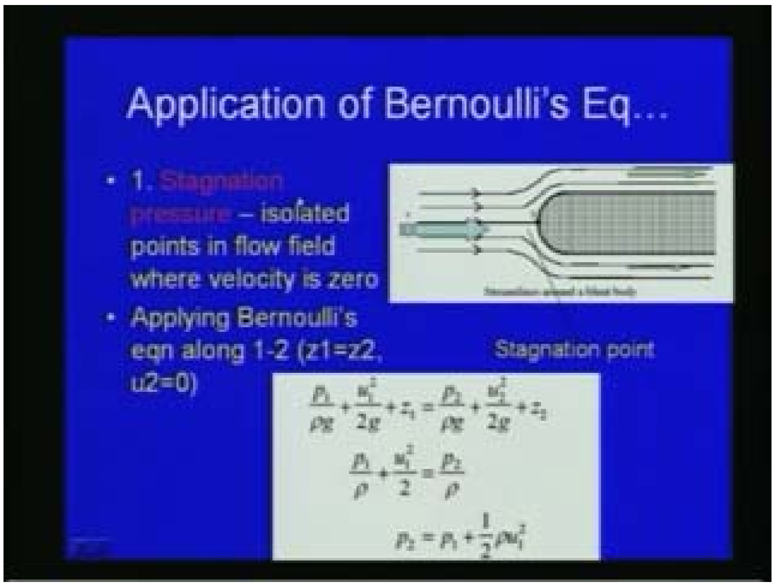
We can utilize this principle in many forms including, considering the total energy of the system including the work done per unit weight or the energy supplied per unit weight as we can see in this slide.

(Refer Slide Time: 02:55)



#
#
So, this Bernoulli's equation is applicable to many problems, but if the flow is irrotational we can stay along a streamline, but if the flow is irrotational we can also use this across the streamline; some of the restrictions we will discuss later. First, we will see some of the important applications of the Bernoulli's equation. Let us consider a streamline around a blend body in this slide.

(Refer Slide Time: 03:16)

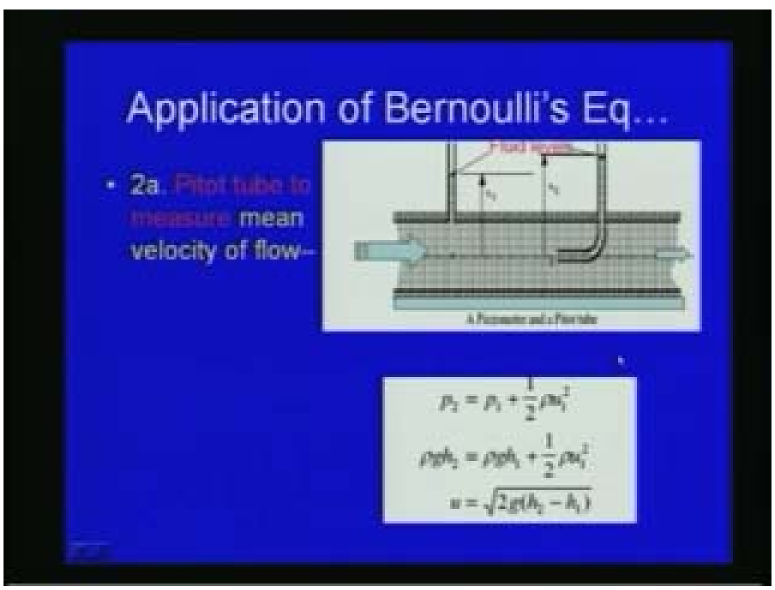


Here, you can see that the flow is coming in this direction and with the blend body effect; there will be a stagnation point like this. We can find out this in case 1. The application of Bernoulli's equation: case 1 - stagnation pressure. We can isolate a point in the field where velocity is 0 that is so called stagnation point. Here, in this particular case, in this figure, this point 2 will be the stagnation point and by considering the flow here at this location, at section 1 and section 2. Between these two sections if you apply the Bernoulli's equation along a streamline from 1 to 2, if you consider the datum like this horizontal so that flow is horizontal, so z_1 is equal to z_2. So applying the Bernoulli's equation along the streamline between section 1 and 2, we can write P_1 by rho g plus u_1 square by 2g plus z_1 is equal to P_2 by rho g plus u_2 square by 2g plus z_2, here P_1 is the pressure at section 1 , P_2 is the pressure at section 2, u_1 is the velocity at section 1 , u_2 is the velocity at section 2, rho is the density of the liquid and g is the acceleration due to gravity. We can see that we are considering the stagnation point; due to this stagnation

#
#
point the velocity at that point will be 0 so that this equation we can write P_1 by rho, if you multiply both sides by g then P_1 by rho plus u_1 square by 2 is equal to P_2 by rho. Since z_1 is equal to z_2 this is canceled and u_2 since we are considering stagnation point we can write u_2 is equal to 0. So that P_1 by rho plus u_1 square by 2 is equal to P_2 by rho.

From this we can write P_2 is equal to P_1 plus 1/2 rho u_1 square. By considering the point 1 we can find out the pressure at the stagnation point so that is called stagnation pressure. Now, we have applied the Bernoulli's equation between two points: one is stagnation point and the other one is the particular section 1 so that we could find the stagnation pressure. This is one of the basic application of the Bernoulli's equation in fluid mechanics and there are number of other applications. Based upon this stagnation pressure let us consider a pitot tube. Pitot tube is generally used to measure the mean flow velocity especially in closed conduit such as pipes.

(Refer Slide Time: 05:42)

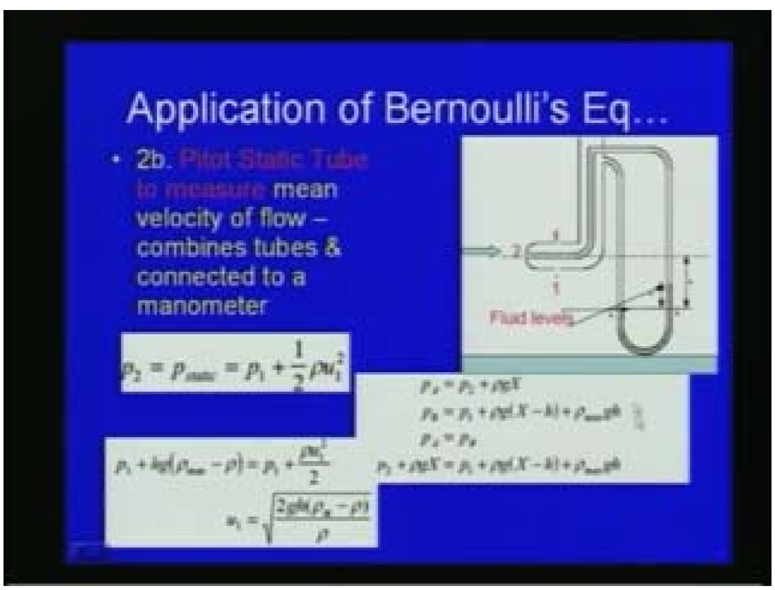


Here you can see in the slide a pipe is there; the flow direction is this and then we want to find the pressure at the central line. If you want to find the pressure at this particular central line velocity what we can do is we can immerse the pitot tube like this at this particular point 2 where we want to find the velocity. We should have another point also where we can introduce a piezometric tube like this. So this is the piezometric tube here at section 1 and at section 2 we introduce a the pitot tube so you can see the fluids level

#
#
that there will be a difference since it is mainly due to the velocity effect at the centre line. The head at the piezometric level will be h_1 from central line and on the pitot it will be h_2 from the central line. Now, if you apply the Bernoulli's equation between section this section 1 1 and section 2 2 we can write this P_2. You can see that at section 1 1 the pressure is P_1 so P_1 plus 1/2 rho u_1 square will be the total head at the section 1; at section 2 2 it will be P_2 is equal to P_1 plus 1/2 rho u_1 square so that P_2 can be written as rho g h_2 from which we can get the velocity at the centre line for the pipe loss, that is, u is equal to square root of 2 g h_2 minus h_1. This is the way while introducing a piezometer and then a pitot tube we can find the mean velocity of flow like in a pipe as shown in this figure.

We have found the central line velocity by considering two points, 1 1 and 2 2 and at point 1 we introduced a piezometer and point 2 we introduced a pitot tube and then we are trying to find central line velocity. We can get the Bernoulli's equation between section 1 and 2 as this figure and finally u is equal to square root of 2 g h_2 minus h_1, where h_2 minus h_1 is the head level between the piezometer this section 1 and the pitot tube introduced at section 2.

(Refer Slide Time: 08:32)



In earlier case what we discussed should have one piezometer and a pitot tube. But instead of this arrangement, we have another type of arrangement to find out the mean velocity of fluid, pitot static tube in this figure. You can see that a pitot static tube is

#
#
introduced with respect to the pipe where the velocity is found. In this mechanism, it combines tubes and connector to a manometer, manometer is shown here and the central line velocity where we are trying to measure velocity this manometer tube in introduced at this location. There is an opening with respect to this tube at section 1 1 and you can see that when manometer is connected, the fluid level difference will be like this at a, b as shown in this slide.

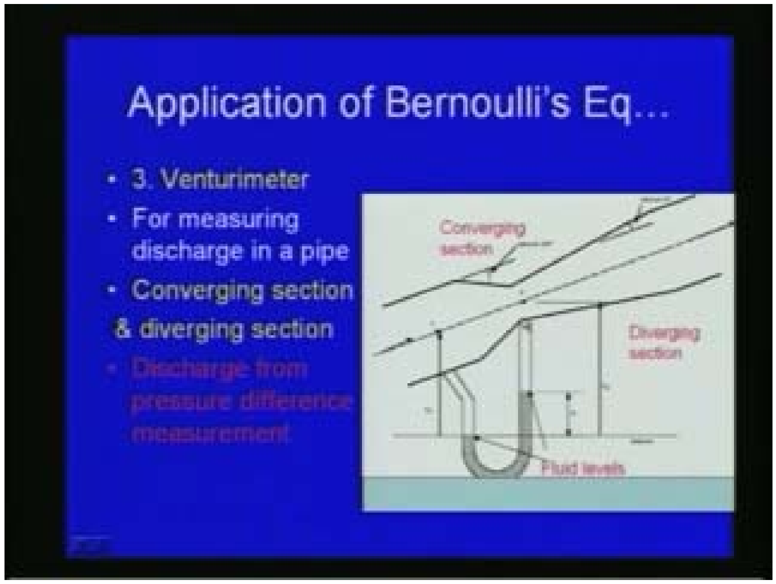
So, the pressure at the central line P_2 is static. We can write P_2 is equal to P_1 plus 1/2 rho u_1 square with respect to this section 1 1 as we have seen earlier. If you consider the manometer here this P_a is equal to the pressure, P_a is equal to P_2 plus rho g x. This is fluid level, rho g is the unit weight of the specific weight of the liquid and X is the height difference between this level as shown this line and this line is X.

P_a is equal to P_2 plus rho g and P_b is equal to the pressure at section B. At particular location b, P_b is equal to P_1 plus P_1 with respect to the fluid pressure at this section 1 P_1 plus rho g X minus h. This difference rho g into X minus h plus the density of the manometer liquid into g into, this is h, so rho g into h.

With respect to the conditions P_a is equal to P_b, we can equate this both equations. So that P_2 plus rho g X is equal to P_1 plus rho g into X minus h plus rho manometer liquid density into g h. If you use this relationship as in the previous equation you can write P_1 plus rho h g into manometer minus rho is equal to P_1 plus rho u_1 square by 2.

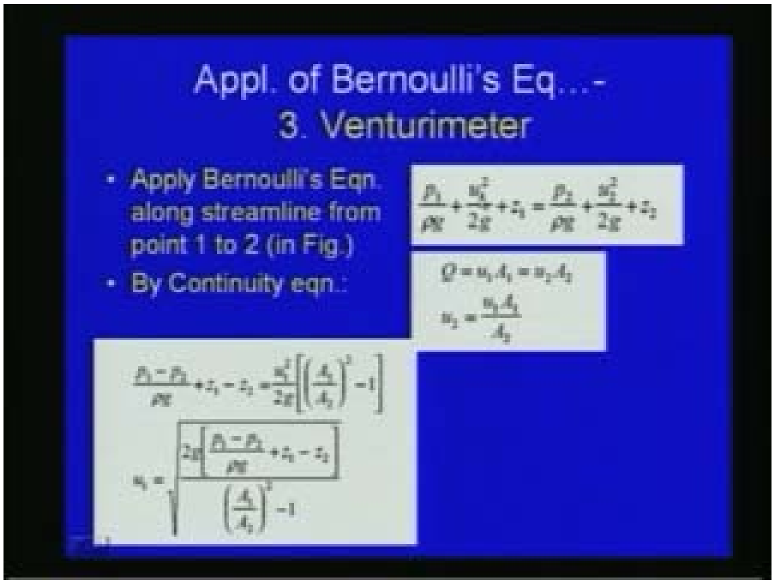
We can find P_2 with respect to this equation and then we can substitute that here so that will give u_1, the velocity. For the central line velocity square root of 2 g h rho, the density of manometer liquid minus density of the pipe divided by rho. So u_1 is equal to square root of 2 g h into rho m minus rho divided by rho. Like this a pitot static tube is used to measure the mean velocity of flow in a pipe instead of as we have seen in the previous case here we have to use piezometer as well as a pitot tube but here the mechanism is pitot static tube and we can find the velocity as shown here.

#
#
(Refer Slide Time: 12:15)



The third case which we will be discussing is application of Bernoulli's equation for venturimeter. We can see that this venturimeter is used to find the discharge in a pipe and this is the pipe which we want to find the discharge and for this venturimeter arrangement there will be a converging section as you can see here; there will be a diverging section like this. At this particular point you can see a minimum cross sectional area and then we will be using here a manometer like this and it will be connected between this particular section, that is, before the divergence starts this particular section 1 and then particular section 2 as shown in this figure and then with the manometer liquid you can see the a fluid levels at a height difference of h. With respect to this convergence section and divergence section we want to find the discharge and then that is the pressure difference which we measure between the discharges obtained from the pressure difference measurement between section 1 and 2.

#
#
(Refer Slide Time: 13:28)

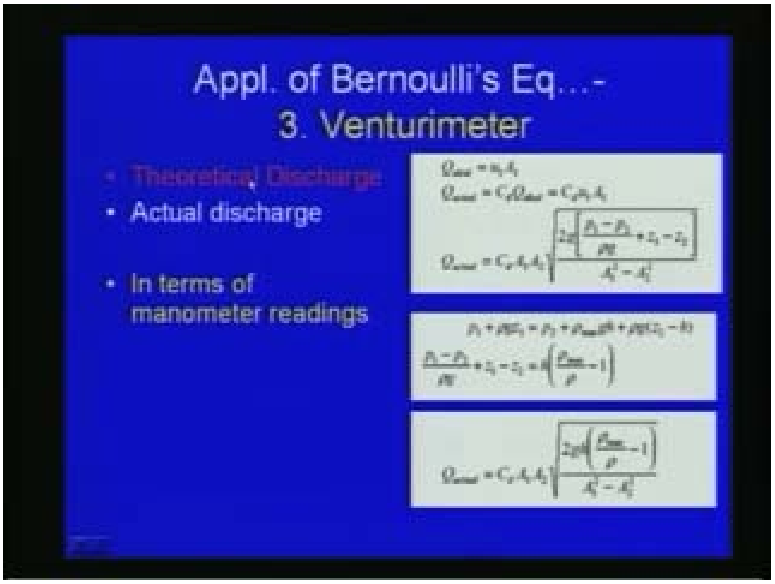


If you apply the Bernoulli's equation along a streamline from point 1 to 2 in the previous figure from point 1 to point 2 then we can write P_1 by rho g plus u_1 square by 2 g plus z_1 is equal to P_2 by rho g plus u_2 square by 2 g plus z_2, where P_1 and P_2 are the pressure at section 1 and 2 and u_1 and u_2 111110 are the velocities at section 1 and 2 and z_1 and z_2 are the high difference with respect to the datum here, this is z_1 and z_2. From the continuity equation, you can write Q is equal to u_1 A_1 is equal to A_2 u_2 with respect to the velocity and area of cross section we can write Q the discharge is same. So Q is equal to A_1 u_1 is equal to A_2 u_2 from which we can write u_2 is equal to u_1 A_1 by A_2. We substitute for u_2 in this equation so that we can write P_1 minus P_2 by rho g plus z_1 minus z_2 is equal to u_1 square by 2 g into A_1 by A_2 whole square minus 1, from which we can find the velocity, central line velocity that means at this location we can find the velocity by using the venturimeter. So once the velocity is known we can just multiply by Q is equal to A_1 u_1 that will be discharged through the pipe. This is another mechanism which we generally use to either find the velocity or you need to find the discharge depending upon the case in a pipe flow. This is another application called venturimeter.

As far as these kind of hydraulic equipments are concerned, venturimeter is concerned we can see that theoretical discharge is Q hydraulic discharge is equal to u_1 A_1 but actual

#
#
discharge you can see that there will be a reduction with respect to this measurement. So the actual discharge will not be equal to theoretical discharge.

(Refer Slide Time: 15:20)

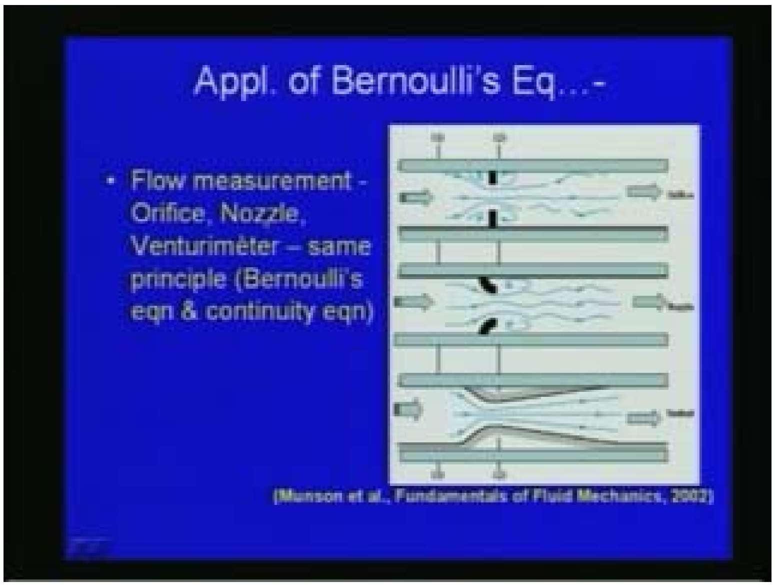


The actual discharge is obtained by actual discharge, Q_{actual}. We have to multiply by a coefficient of discharge C_d, so that Q_{actual} is equal to C_d by in C_d into Q theoretical. So that we can write Q_{actual} discharge is equal to C_d into u_1 into A_1. We can write with respect to the previous equation which we derived for u_1. The Q_{actual} is equal to the coefficient of discharge multiplied by A_1 into A_2, so A_1 is the cross sectional section at 1 and A_2 is the cross section at the converging point, A_2 multiplied by square root of 2 g into P_1 minus P_2 by rho g plus z_1 minus z_2 by A_1 square minus A_2 square. This is the actual discharge. With respect to this kind of measurement there is a difference between the actual discharge and theoretical discharge. We have to multiply by the coefficient of discharge. So, Q_{actual} is equal to $Q_{theoretical}$ into coefficient of discharge.

Now, we can say in terms of the manometer readings we can write P_1 plus rho g z_1 is equal to P_2 plus density of manometer is equal t into g h plus rho g z_2 minus h. This we can write as P_1 minus P_2 by rho g plus z_1 minus z_2 is equal to h into rho manometer by rho minus 1. Actual discharge can be written as C_d into A_1 into A_2 square root of 2 g h rho manometer by rho, the density of the fluid in the pipe minus 1 divided by A_1 square minus A_2 square.

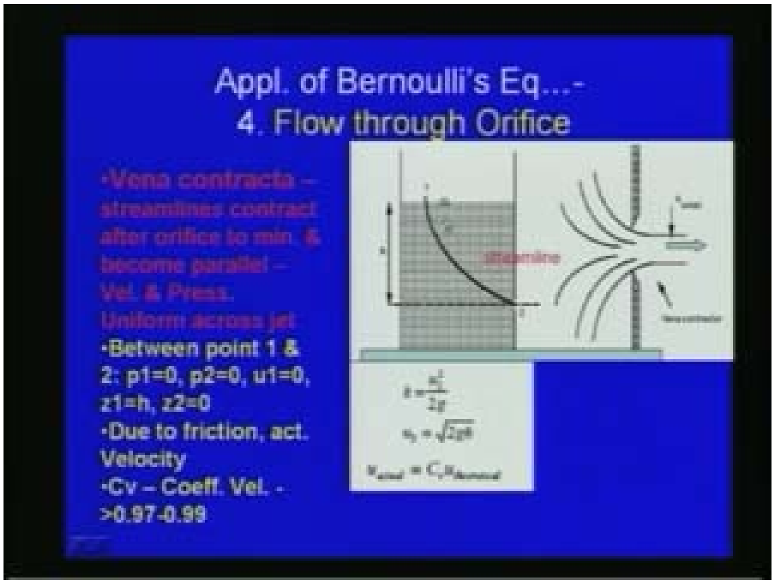
#
#
Like this by using venturimeter we can measure the velocity or the discharge in a pipe flow. Now the application as far as venturimeter is concerned we will see further applications of Bernoulli’s equation. We have seen for the venturimeter; we have also seen another mechanism for flow measurement, the discharge of velocity like the various flow measurement equipments are shown here orifice as shown in this figure.

(Refer Slide Time: 17:30)



Then nozzles and venturimeter which we have already discussed are some of the equipments used for flow measurements which we utilize the Bernoulli’s equations and continuity equation together. So that we can find the flow of velocity over the discharge at particular sections of a pipe line especially the mechanism is used for pipe line. This is one of the applications of the Bernoulli’s equation

#
#
(Refer Slide Time: 18:29)



We will go to further applications of the Bernoulli's equation. Fourth one is the flow through orifice. You can see that this orifice is a mechanism. If there is a tank like this and if there is a small hole like this and so water will be coming from the tank as in the case of a jet, so this is called an orifice. There are different forms of orifice. This is one of the simple most forms of the orifice; we can utilize to find the time to empty a tank or to particular discharge measurement can be utilized. Here for the flow through orifice, you can see that whenever the fluid is coming out of the orifice or this is small hole which is called orifice, then all the streamlines are converging to the opening at the orifice. Then you can see that there is a location where the streamlines are converging and the area of cross section of the jet is minimum. This section is so called Vena contracta, where streamlines contract after orifice to minimum and then become parallel like this. You can see that now the streamlines become parallel. At this particular location so called vena contracta there will be a velocity and pressure; the velocity and pressure will be uniform across the this particular location of the vena contracta, so this is so called orifice. As we can see that this is also the orifice, as I mentioned, you can utilize for the velocity or the discharge measurement.

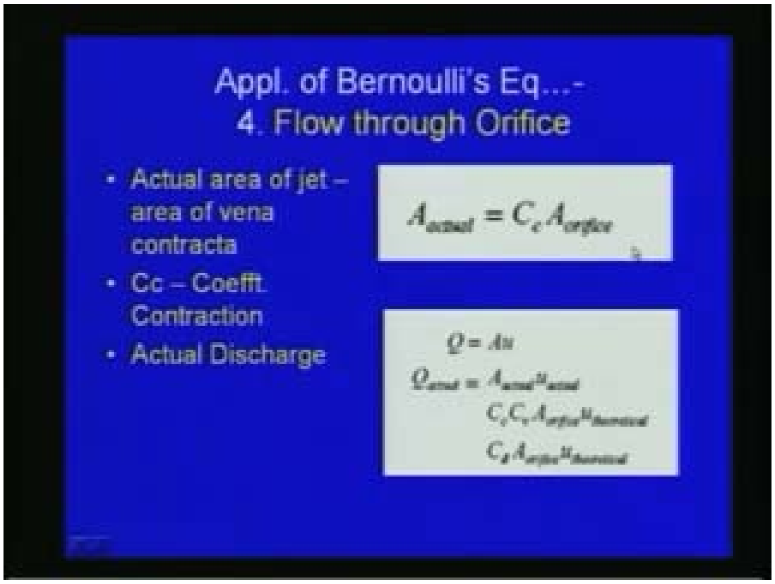
The same principle is utilized there also. Now, we are considering the vena contracta and between the tank which we are considering here, if we consider the surface of the tank at

#
#
this particular location point 1 and then if you consider the orifice location of the particular vena contracta which we are considering at section 2, we are considering two positions namely, position 1 and position 2. Between position 1 and 1we can see that jet is flowing freely to the atmosphere; the pressure at this location will be 0 and at this is an open surface. At location 1, the pressure will be 0. So, p_1 is equal to 0; p_2 is equal to 0 and this is the open surface of the tank. There the velocity u_1 is equal to 0 and then we can see that datum head z_1. If you consider this centre of the orifice as the datum then z_1 is equal to h and z_2 is equal to 0. If we can apply the Bernoulli's equation between section 1 and 2, between the section position points 1 and 2, we can see that we will get h is equal to u_2 square by 2 g or we can write u_2 is equal to square root of 2 g h. So the velocity at this location u_2 at location 2 will be u_2 that is equal to square root of 2 g h.

Now, as we have seen earlier as far as coefficient of discharge, again the actual velocity will be defined from the theoretical velocity. We have to multiply by a factor called coefficient of velocity for these kinds of problems. Due to the friction, the actual velocity will be different. So when the fluid is coming out of the orifice there will be friction with respect to the atmosphere and the then due to the friction effect the actual velocity will be slightly different. We have to multiply by a coefficient so called velocity. So, the actual velocity is equal to coefficient of velocity into the theoretical velocity.

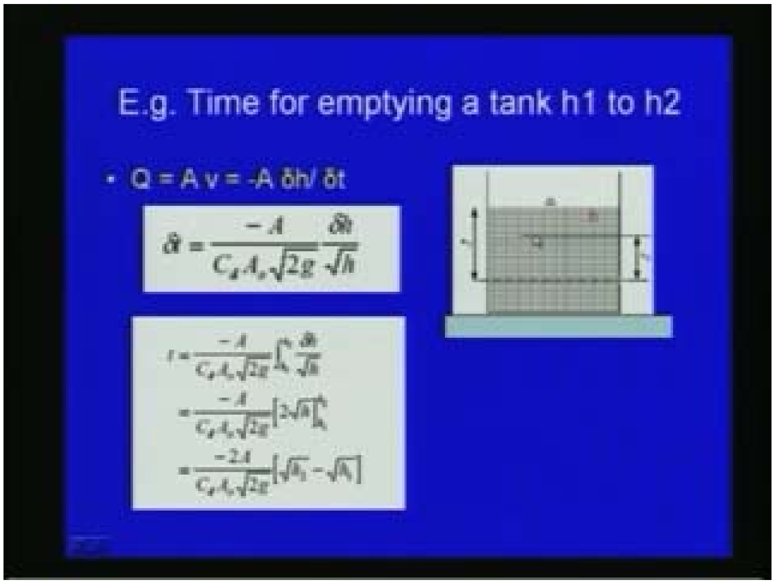
This coefficient of velocity for these kinds of problem varies from 0.7 to 0.9 depending upon the various conditions like orifice locations or orifice diameter and other conditions. So, the actual velocities obtained as the coefficient of velocity multiplied by the theoretical velocity is flow through orifice.

#
#
(Refer Slide Time: 23:07)



Finally, for the orifice you can see that if you consider the vena contracta here then with respect to the vena contracta the actual area of the jet is the area of vena contracta. So, the jet which we are considering out of the orifice the actual area of cross section of the jet is the area of the vena contracta. So A_{actual} is equal to we have to multiply by a coefficient of contraction into A into area of cross section of the orifice. So, actual area is equal to coefficient of contraction multiplied by the cross sectional area of the orifice. Finally, now if you want to find the actual discharge Q_{actual} is equal to area of cross section into u. So, Q_{actual} is equal to A_{actual} that means A_{actual} is C into $A_{orifice}$ into u theoretical. Finally, Q_{actual} is equal to coefficient of discharge into area of cross section of the orifice into the theoretical velocity. This gives the actual discharge from the orifice. This has got many practical applications as I mentioned. Even flow measurement orifice can be used so many other applications are there in fluid mechanics for this flow through orifice. We are now using the three coefficients: one is the coefficient of contraction to find the actual area; second one is the coefficient of velocity to find the actual velocity and the actual discharge; finally, we are getting C_d is equal to C into C_v that means coefficient of discharge is equal to coefficient of contraction multiplied by coefficient of velocity so that is C_d, finally, we get the actual discharge. This is another application of the Bernoulli's equation.

#
#
(Refer Slide Time: 25:00)

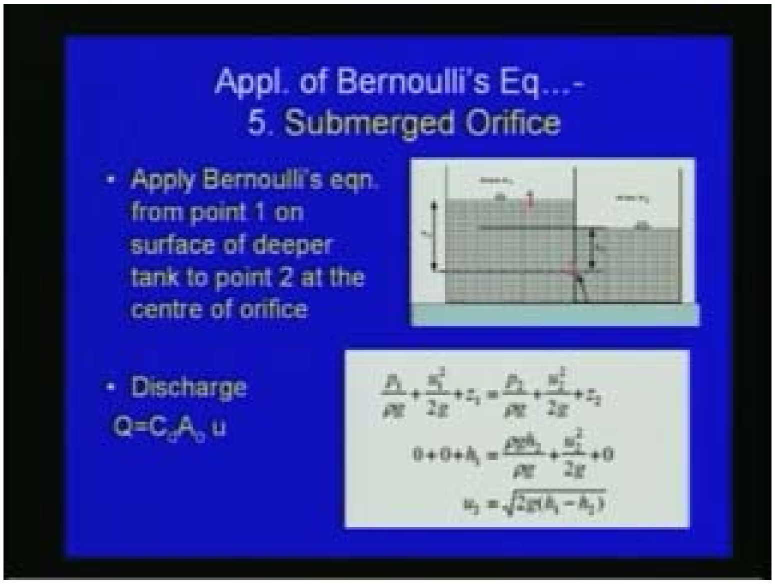


Now, just find the time for emptying a tank. As I mentioned with this Bernoulli's equation again can be utilized. If you want to find how much time is taken for a liquid in a tank to come from level h_1 to h_2. If you want to find Bernoulli's equation can be utilized here and then say if h is the level difference between h_1 minus h_2 we can write the discharge Q is equal to A into the velocity, since the level is going down we are using minus here. So this is equal to minus A into del h by del t.

With respect to time this head is changing; so Q is equal to minus A into del h by del t. This we want to find here, the time for emptying the tank from h_1 to h_2. From this equation we can write delta t is equal to minus A into, we will write the discharge equation which we have seen earlier, delta t is equal to minus A divided by C_d into A 0 into root two g into delta h by root h.

Now, we can integrate this expression with respect to the levels h_1 to h_2. So, t is equal to minus A into C_d into A_0 into root 2 g into integral h_1 to h_2 del h by square root of h. If we integrate this expression we will get finally the time for emptying a tank h_1 to h_2 will be equal to minus 2 A divided by C_d into A_0 root 2 g into square root of h_2 minus square root of h_1. Here again we have to use the Bernoulli's equation for emptying a tank from one level to another level. So like this various applications are there.

#
#
(Refer Slide Time: 26:45)

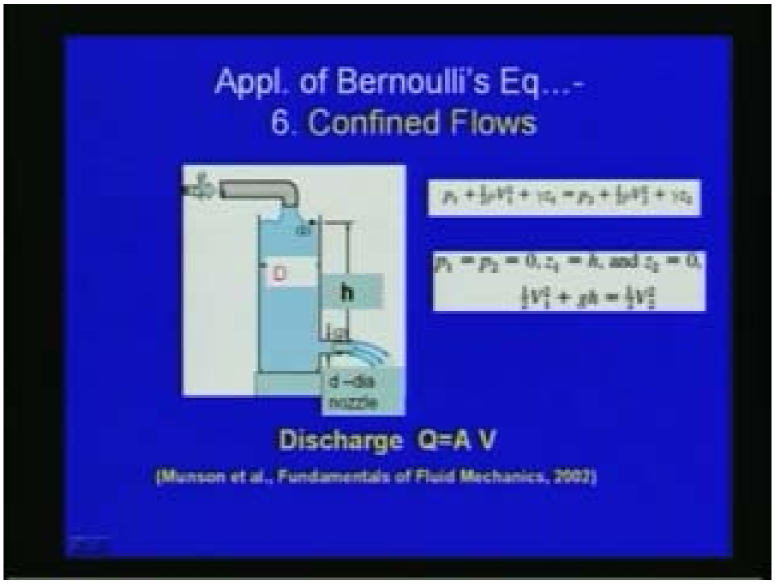


Earlier, we have seen just an orifice which is directly emptying the liquid to or water to the atmosphere. If we are considering a submerged orifice as shown in this figure you can see there is an orifice here at location 2 and then through this orifice the liquid is passing from tank 1 to tank 2, very adjacent tanks. Finally after some time this become a submerged orifice. To find the discharge passing through submerged orifice and the velocity of flow through the submerged orifice we can apply the Bernoulli's equation from this point 1 to this point 2 at the centre of the orifice. We can write from Bernoulli's equation p_1 by rho g plus u_1 square by 2 g plus z_1 is equal to p_2 by rho g plus u_2 square by 2 g plus z_2. We can see that since due to that atmosphere pressure, p_1 rho g will be equal to 0 and here the velocity will be 0. So u_1 square by 2 g also is 0 and here z_1 is equal to h_1 if you take the datum as the centre line of the orifice.

So h_1 is z_1 and then p_2 by rho g, you can write as rho g h_2 by rho g plus u_2 square, the velocity is u_2 so u_2 square by 2 g since the datum is taken z_2 is equal to 0. Finally, we can get this velocity of flow u_2 is equal to square root of 2 g into h_1 minus h_2. h_1 minus h_2 is actually the level difference between this level and this level, so this gives the h_1 minus h_2. The velocities are found and then we can find the discharge of Q is equal to, we can use the continuity equation, Q is equal to A- area of cross section area of the orifice into velocity and finally the actual discharge is equal to we have to multiply by the coefficient

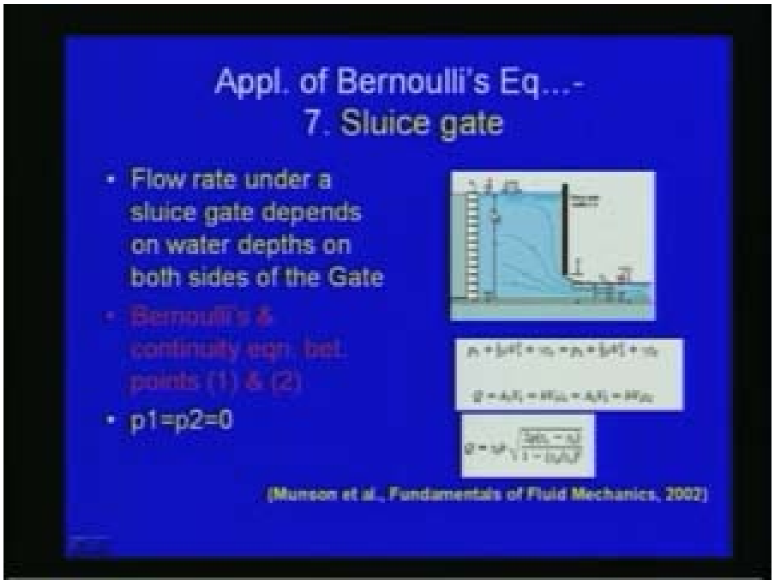
#
#
of discharge area of cross section of the orifice into the velocity which we are calculating here. So this is the case of a submerged orifice.

(Refer Slide Time: 28:52)



Now, we will see another case so called confined flows, if you consider which is a confined with respect to this tank as shown here. The flow is coming in this direction and then flow is going through the other nozzle at location 2. If you want to find for this confined flow the discharge or any other parameter or velocity of flow at section 2 again we can use the Bernoulli's equation as shown between the section 1 here and section 2. When we apply the Bernoulli's equation, we can write p_1 plus 1/2 rho V_1 square plus gamma z_1 is equal to p_2 plus 1/2 rho V_2 square plus gamma z_2. You can see this is open to atmosphere. So, p_1 is equal p_2 is equal to 0 and z_1 is equal to h and z_2 we are taking it as the datum; so z_2 is equal to 0. Finally, we get 1/2 V_1 square plus g h is equal to 1/2 V_2 square and then we can find the velocity. From that we can get the discharge. This is as far as if you consider a confined flow as shown in this figure. Now, there are many other applications for Bernoulli's equation. Some of the open air flow types again we will discuss here.

#
#
(Refer Slide Time: 30:28)



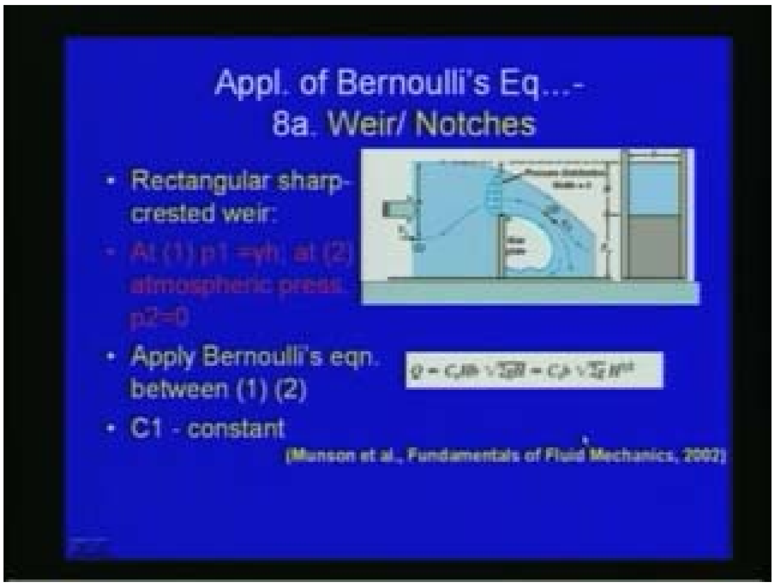
Next one is sluice gate. If you want to find the discharge passing through sluice gate so here you can see this figure here. In this figure, you know that the sluice gate is generally in a reservoir or different chemical plants or water supply. You just use this gate to pass particular amount of discharge and if you want to find how much discharge pass through this particular opening again we can use the Bernoulli's equation and continuity equation for a sluice gate.

So sluice gate is here and the liquid level is at this level and the flow is coming in this direction. After the gate again as we have seen again a vena contracta will be formed that means we know cross sectional area streamlines will be parallel and then we will be applying the Bernoulli's equation between the section 1 and section 2 of this area of cross section here between section 1 and 2. Applying the Bernoulli's equation and continuity equation, if you apply the continuity equation between section 1 and 2, we can write Q is equal to A_1V_1 is equal to, b is the width of this opening or with respect to this equation. So, b into V_1 z_1 that is equal to A_2 V_2 that is b V_2 z_2, where b is the thickness or the width which we are considering.

Now we can apply the Bernoulli's equation between point 1 and point 2. So that we can write p_1 plus 1/2 rho V_1 square plus gamma z_1 is equal to p_2 plus 1/2 rho V_2 square plus gamma z_2, here p_1 and p_2 is atmospheric pressure. So, p_1 and p_2 have to be considered.

#
#
From this equation, we can find out the velocity, for example, here V_2. Then, we can get an expression for the discharge passing through the gate, that is, Q is equal to z_{2d} into square root of 2 g into z_1 minus z_2 divided by 1 minus z_2 by z_1 whole square, this is z_1. The bottom of the sluice gate is considered as the datum and this is z_1 here and z_2 is depth of at this particular location section 2 and z_1 is the depth at section 1.

(Refer Slide Time: 33:20)

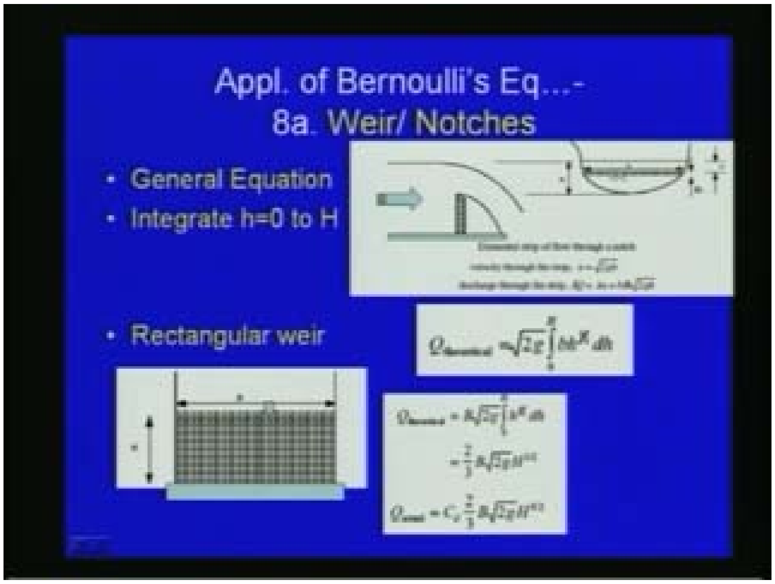


This way here also p_1 equal to p_2 equal to 0 and then by using the Bernoulli's equation and continuity equation we can find the discharge passing through this sluice gate. This is one of the applications as far as open channel flow is concerned and then another application is application for the Bernoulli's equation for weir or notch. So, here we can see that weir is here so we want to find how much is the discharge passing over the weir um or a notch.

This is rectangular sharp crested weir. If you want to find, we can say here again this particular figure you would be considering this location a streamline like this. Then, between section 1 and 2, we will be considering, this is 2 and this is 1, we will be considering the Bernoulli's equation. If you consider this particular point p_1 is equal to this height, depth of flow with respect to this point 1 p_1 is equal to gamma h and then here it is atmospheric pressure so we can neglect the pressure.

#
#
So p_2 is equal to 0. If you consider between section 1 and 2, if you apply the Bernoulli's equation we can show that Q is equal to C_1 into h b root 2 g h or this is equal to C_1 into b into square root of 2 g into h to the power 3 by 2, where C_1 is a constant as far as this weir or notch is constant. Like this we can apply this Bernoulli's equation combination with the continuity equation to large varieties of problem in practical cases.

(Refer Slide Time: 34:45)

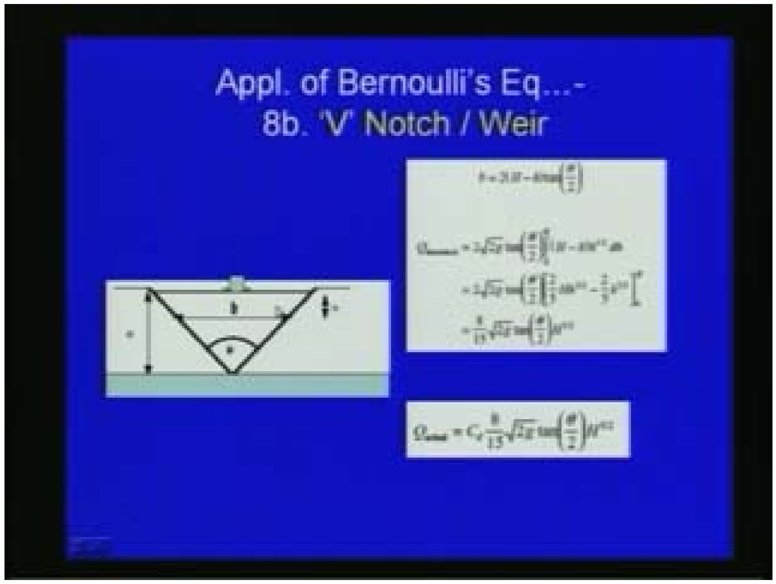


Again, here if you consider a general equation for the weir or notch the general equation you can see is the sharp crested weir and then you consider a small strip of elemental strip through a notch. You can see that this velocity u is equal to square root of 2 g h and discharge through the strip delta Q is equal to area of cross section into velocity V into delta h into square root of 2 g h with respect to this figure here.

Then, we can just integrate between 0 to H; H is the depth of flow with respect to crest of the weir as shown in this figure. So, this is the depth of flow. We can integrate from 0 to H, so that $Q_{theoretical}$ the theoretical discharge is square root of 2 g integral 0 to h b H to the power 1/2 d h and then we can integrate, the theoretical discharge is equal to b root 2 g integral 0 to 2 H square root of h d H that is equal to 2 by 3 b root 2 g H to the power 3 by 2 and the actual discharge as we have seen here again. We have to use a coefficient of discharge. So Q_{actual} is equal to C_d into 3 by 2 b root 2 g, H to the power 3 by 2. So this is

#
#
another application of the Bernoulli's equation in combination with the continuity equation.

(Refer Slide Time: 36:06)



Similarly, if you consider a V notch as shown in this figure, again, this V notch is just a triangular weir, here this b width at this particular location can be found at depth h from the surface of the water in the total depth is H; so b is equal to two times H minus H into tan theta by 2 is the angle of the V notch. Then, $Q_{theoretical}$ is equal to 2 into root 2 g tan theta by 2 integral 0 to H minus h into square root of h d H.

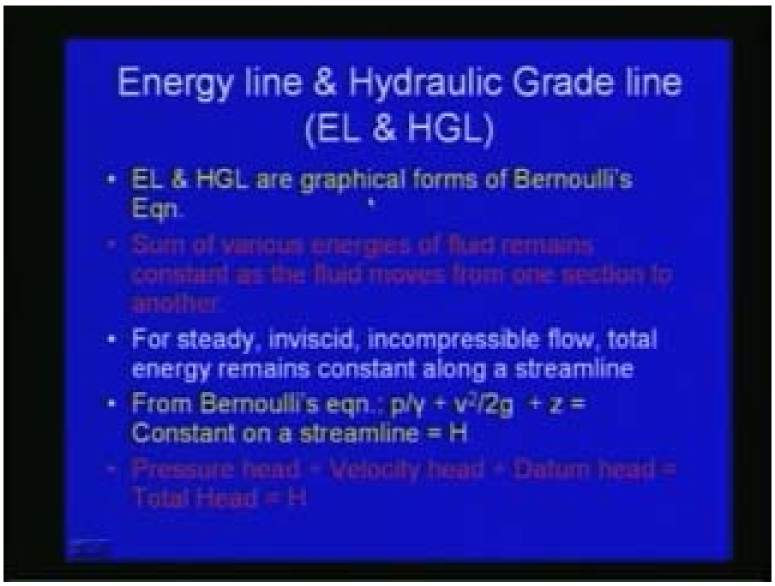
Finally, the equation is this is equal to 8 by 15 root 2 g tan theta by 2 H to the power 5 by 2. In a triangular weir or V notch the discharge will be varying with respect to H to the power the total depth of floor H to the power 5 by 2. Actual discharge is equal to the coefficient of discharge multiplied by the theoretical discharge as shown in this slide.

Like this by using the Bernoulli's equation and the continuity equation we can solve many practical problems in hydraulics or in the closed quantity flow like the pipe flow and also open channel as we have seen in the notches, weirs or the sluice gate problem. We can solve different kinds of problems using the simplified form of the Bernoulli's equation and the continuity equation you can see all this problem is very simple approach. Bernoulli's equation is a simple equation so that we can easily without many

#
#
complexities can easily approximate what kind of flow is coming and then we can apply the Bernoulli's equation between two sections between two points on a streamline. We can find either the discharge or the velocity or the pressure between the two sections.

These are the most important applications of the Bernoulli's equation. So, before going further with the derivation of other fundamentals momentum equation we will just see some aspect of the total energy with respect to hydraulic grade line and the energy line.

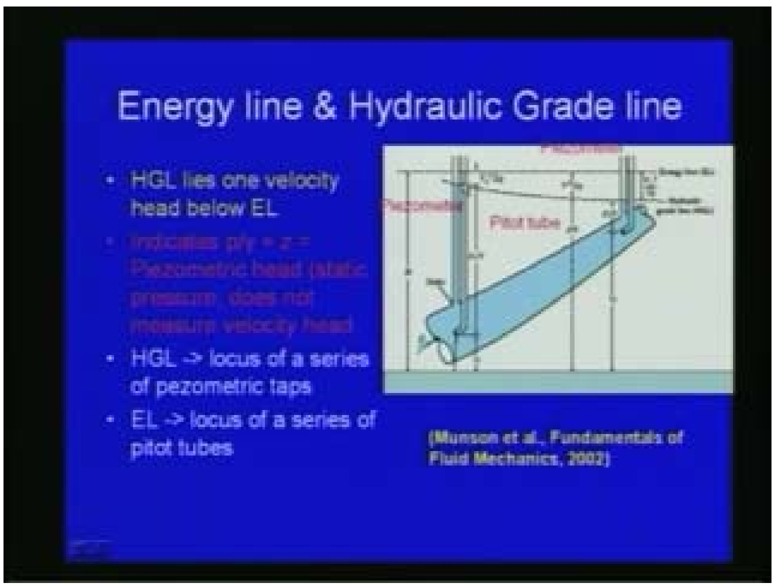
(Refer Slide Time: 38:30)



We have already seen with respect to the Bernoulli's equation total head is equal to the pressure head plus velocity head plus datum head. This total energy will be there of course, there will be losses also of the internal force, internal energy will also be there. That also we have to consider and that is the total energy. While finding this energy at various section of a pipe line or a open channel flow depending upon the case which we are doing in the problem, we can represent two lines called energy line and hydraulic grade line. So energy line and hydraulic grade line are actually the graphical forms of the Bernoulli's equation. The basic principle as we have seen is some of the various energies of fluid remains constant as the fluid moves from one section to another. The energy is conserved when we consider a pipe flow like this. If you consider this as a pipe, between one section and another section, we can see that the various energies remain constant as the fluid moves from one section to another section. Now, the equation which we derived

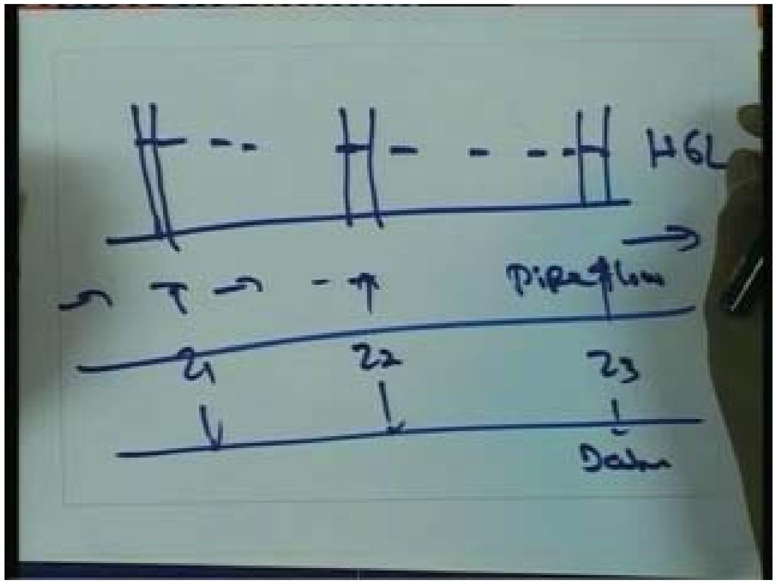
#
#
Bernoulli’s equation is derived for steady, inviscid, incompressible flow, total energy remains constant along a streamline. This is the basic equation. The energy line concept and hydraulic grade line concept is also based upon the conservation of energy that means the total energy remains constant along a streamline. If you consider the Bernoulli’s equation the pressure head p by gamma plus the velocity head v square by 2 g plus z the datum head is equal to constant on a streamline. So that we can write this is equal to H. Thus, pressure head plus velocity head plus datum head is equal to total head H as shown in this slide here.

(Refer Slide Time: 40:46)



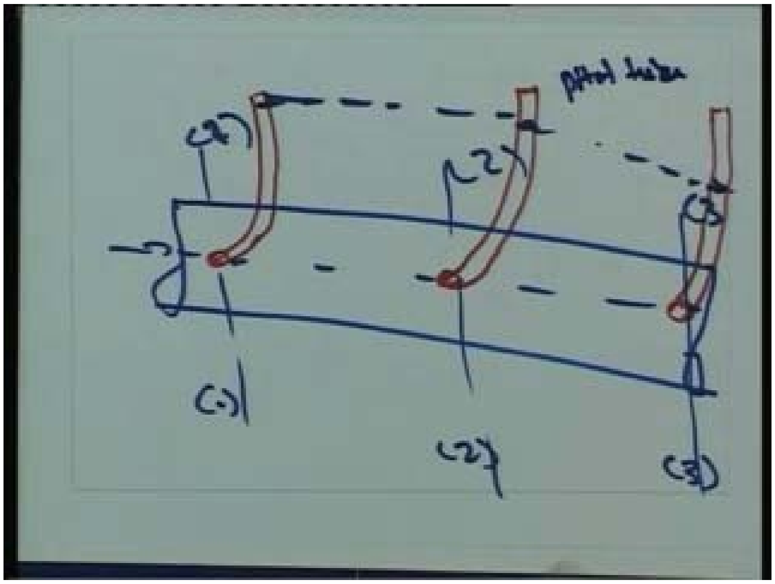
Now if you consider this figure taken from the Munson et al Fundamentals of Fluid Mechanics it is slightly modified. You can see if you consider a pipe flow like this and now if you take the piezometer level at this location and at section 1 1 and at location 2 2, you can see that piezometer indicates the pressure head and datum head. With respect to piezometer we can write p by gamma plus z, that is the piezometric head. So this as the static pressure does not measure the velocity head. This gives the piezometric head at section 1 1 and here at section 2 2, p_2 by gamma plus z_2 that is the piezometric head. So the hydraulic gradient line is actually the locus of series of piezometric taps at different locations. If you consider this as a pipe at different locations if you just find the piezometric head, that is actually the velocity head plus the datum head.

#
#
(Refer Slide Time: 42:05)



If you just draw the locus of a series of piezometric taps, you have a long pipe line as you can see here which we are considering; with respect to this is the datum. So, with reference to this pipe line we can have a series of piezometer and then we can plot the piezometric heads. You can if you just put the locus of this piezometric head that gives the hydraulic grade line. So, this is the fluid flow, this is the pipe flow, this is the datum which we are considering, this is z_3, if you can measure to the centre line this is z_2, this is z_1 to the centre line. So, when we plot the locus of the piezometric heads that is the hydraulic grade line does not include the velocity head. Here we are not considering the velocity head at various locations which we have seen. So that gives the hydraulic grade line. Hydraulic grade line is the locus of a series of piezometric taps. For example, now we consider a pipe flow like this and we have three sections: section 1 1, section 2 2 and section 3 3 as shown in this figure.

#
#
(Refer Slide Time: 43:28)



If you can introduce a piezometer at the center line as we have seen earlier. With respect to the piezometer we can see that it will be considering the velocity head also. So, for these three sections if you find the piezometric levels you can see that this is varying like this. With respect to this, if you just plot the locus where the pitot tubes levels, here this is the level, this is the level, and this is the level. So this line gives the energy lines. So now energy lines include the datum head, the pressure head and the velocity head. If you consider a pipe line at various locations and if you introduce the pitot tubes and if you just get the levels of the pitot tubes and then if you join those levels with a line, that line is called energy line. As shown here with reference to hydraulic grade line the difference is that there is extra velocity head. For hydraulic grade line we consider only the datum head and the pressure head, we are using locus of the levels with reference to piezometric tools and various equations but as the energy line is concerned we are considering the velocity head also. You can see in this figure we are introducing the pitot tubes at various locations and then we finally get a line so called energy line.

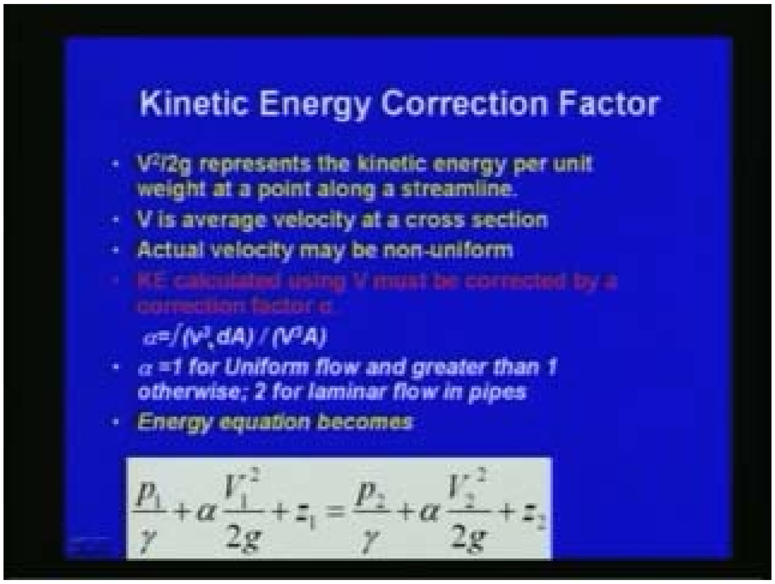
This hydraulic grade line and energy line concepts are very useful especially for pipe flow analysis and open channel flow analysis. This is coming here and again we are using the Bernoulli’s equation of total energy so that principle of conservation of energy is used.

#
#
This concept of energy line and hydraulic grade line is used in many problems especially pipe line problems also sometimes in open channel flow. In all these problems what we have seen so far when we measure the velocity we are getting the mean velocity or say the velocity measurement is with respect to most of the 1 dimensional problem, V is the average of the mean velocity.

Here, we have to do a correction with respect to this. Since we are considering the mean flow if you consider a pipe then you can see that with reference to this we are considering the mean flow velocity. So, with respect to the mean flow velocity the average velocity at a cross section is taken; the actual velocity may not be uniform. The concept which we are using here is that the mean velocity is taken in such a way and then it is multiplied by all other area of cross section. Q is equal to area of cross section into velocity, V is the mean velocity. This mean velocity is not actually considered in a pipe flow like this with reference to the center line. The mean velocity you can see the velocity is varying like this; it will be maximum at the center line; then, it will be minimum 0 at both sides of the pipe valve.

The mean velocity concept will not give a correct value as far as you find the various measurements like discharge. So, it will not give a correct value, we have to use a correction factor. So as far as kinetic energy is concerned we have the actual velocity is non-uniform as you can see in this pipe flow or even the open channel flow so what we are considering will be non-uniform flow. For the non-uniform flow we have to use a correction factor called kinetic energy correction factor which is so called alpha.

#
#
(Refer Slide Time: 48:48)



We can obtain this alpha. With respect to mean velocity we can say what is the discharge or with respect to the continuity equation we can derive this alpha is equal to integral v cube dA divided by V cube A, where small v is the varying velocity. You can see that the velocity is varying. So you can consider various sections like this. Then, it is the integral of V cube, v is the velocity at any point and then capital V is the mean flow velocity. The energy correction factor alpha is equal to integral small v cube dA divided by V cube A area of cross section of the pipe or the channel section which you are considering so alpha is equal to integral v cube dA by V cube A, where V is the mean flow velocity, A is the area of cross section, small v is the velocity varying from various section which you are considering.

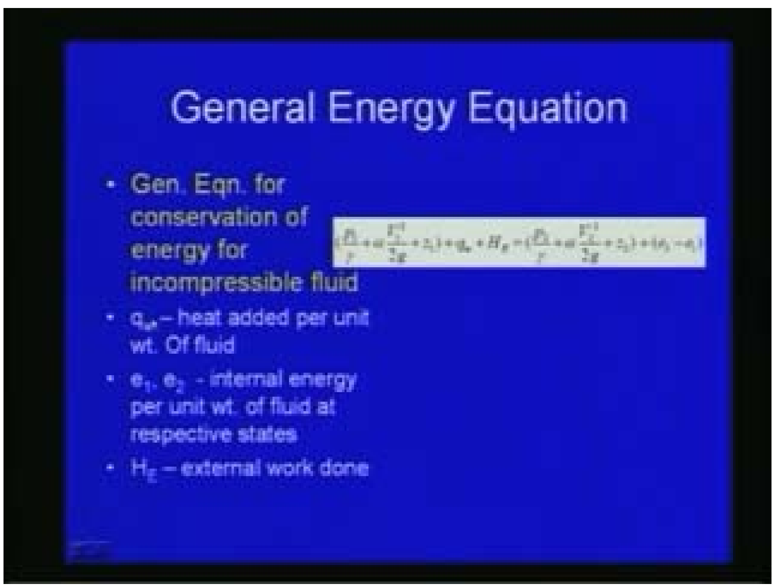
When we are consider the V square by 2 g term we have to use this. We have to multiply by the kinetic energy correction factor alpha so that we get a correct velocity head or the correct kinetic energy. If you consider the flow to be uniform you can say that alpha is equal to 1. No need of this correction and if it is greater than 1, we can say that it will be greater than 1 for all other forms. That means for non-uniform flow condition it will be greater than 1 and then for laminar flowing pipes this we can derive as equal to 2. Finally, if you use the kinetic energy correction factor the energy equation becomes p_1 by gamma

#
#
plus alpha V_1 square by 2 g plus z_1 that is equal to p_2 by gamma plus alpha V_2 square by 2 g plus z_2.

We have to utilize this correction factor and finally the equation become p_1 by gamma plus alpha V_1 square by 2 g plus z_1 is equal to p_2 by gamma plus alpha V_2 square by 2 g plus z_2. To get a correct value, this energy kinetic energy correction factor to be used, we multiply with respect to the velocity head V_1 square by 2 g or V_2 square by 2 g when we use the Bernoulli's equation or the energy equation as shown in this slide.

Finally, to conclude the Bernoulli's equation or the energy equation, as I mentioned earlier, we have to see the various losses whether any of the various energy levels are added or taken out or external work done. All these things are to be considered while we solve real practical field problem.

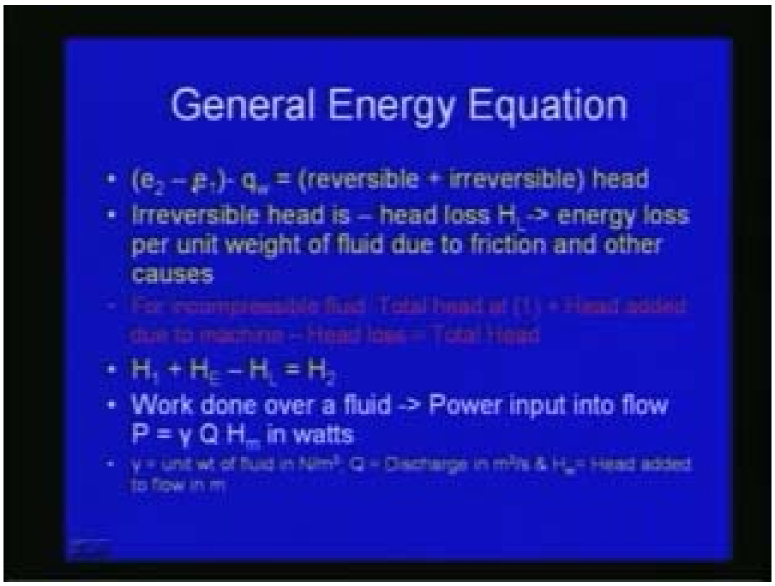
(Refer Slide Time: 51:45)



General equation for conservation of energy for incombustible fluid between two sections we can write as p1 by gamma plus alpha V1 square by 2 g plus z1 plus q_w plus H E is equal to P_2 by gamma plus alpha V_2 square by 2 g plus z_2 plus e_2 minus e_1, where this q_w is the heat added per unit weight of fluid so that effects is to be considered. e_1 and e_2 are the internal energies, it is there where the flow is considering and H E is the external work done.,

#
#
This is the final form of the energy equation even though we have seen the simple form of the Bernoulli's equation but when we solve practical problems we have to see what is the work added or energy is added or any losses of energy due to various aspects, like internal energy aspects or any heat added. All these things we have to consider and final form of the equation when we consider the conservation of energy as shown here.

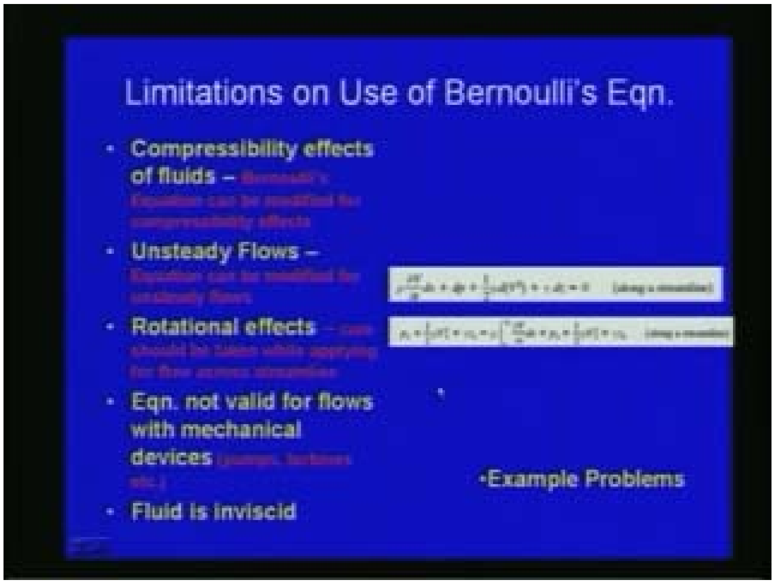
(Refer Slide Time: 52:52)



Now, e_2 minus e_1 minus q_w can be written as reversible plus irreversible head. Irreversible head is can be written as head loss and it is the energy loss per unit weight of fluid due to friction and other causes. For incompressible fluid we can write total head at 1 plus heat added due to machine like pumps or turbine and then minus head loss is equal to total head

Finally, the energy equation can be written as H_1 plus H_E minus H_L is equal to H_2 and if you want to find the work done over a fluid, power input into a fluid is equal to gamma Q H_m in watts, where gamma is the unit weight of fluid in Newton per meter cube, Q is discharge in meter cube per second and H_m is the head added to flow in meter. So this is the general form of the energy equation which is used to solve many of the practical flow problems.

#
#
(Refer Slide Time: 53:47)



We have now seen various application of the Bernoulli's equation and we have seen the general energy equation. Later, we will be discussing various limitations of the Bernoullli's equation and we will solve some of the example problems related to Bernoulli's equation.