Fluid Mechanics

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Lecture 12
Dynamics of Fluid Flow

Welcome back to the video course on fluid mechanics. So in the last lecture, we were discussing about the dynamics of fluid flow, we were discussing about the energy equation and we were discussing about the corresponding Bernoulli's equations and its applications.

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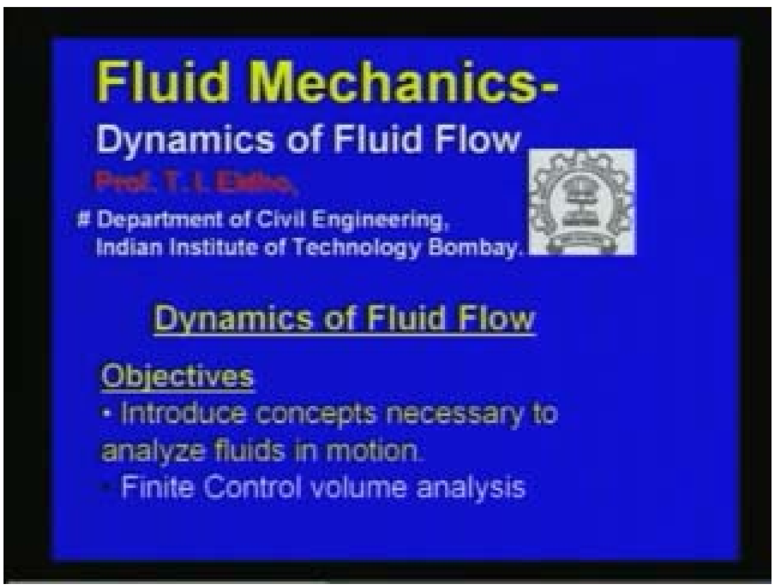

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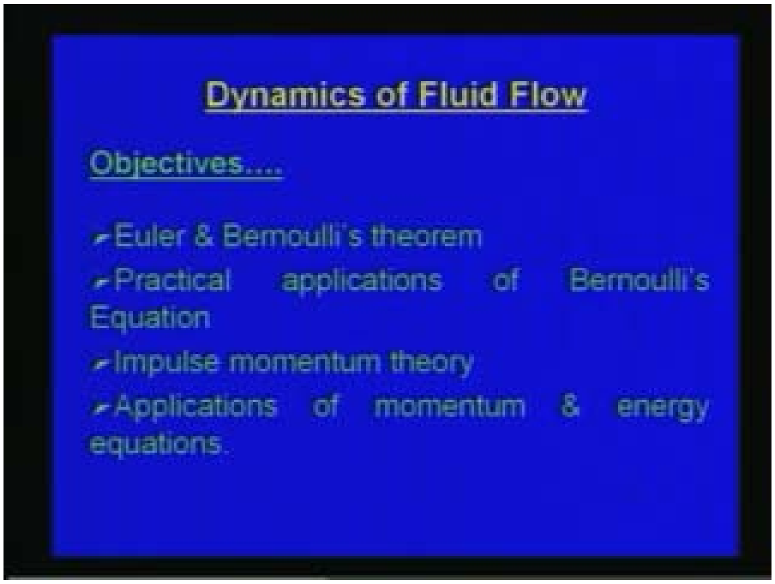

And we have also seen the various aspects of the dynamics of fluid flow which we will be discussing further like analysis of fluids in motion, though we have already seen in fluid kinematics without considering the various forces.

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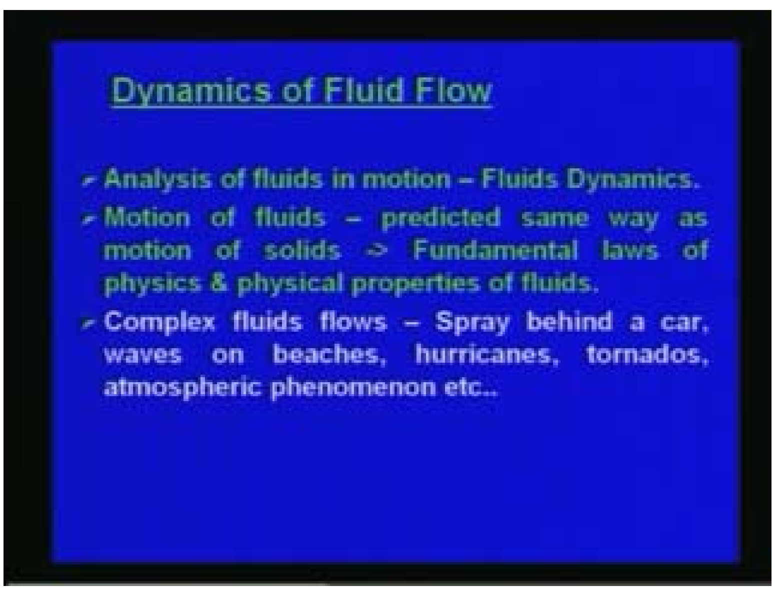

But in dynamics of fluid flow as we have seen, we will be discussing the various fluid motions including whenever forces are applied. Some introductory aspects we have seen various cases which we will be discussing and then we have also seen various forces influencing the fluid motions like gravity force, fluid pressure force due to molecular viscosity, surface tension, compressibility Reynolds stresses etc.

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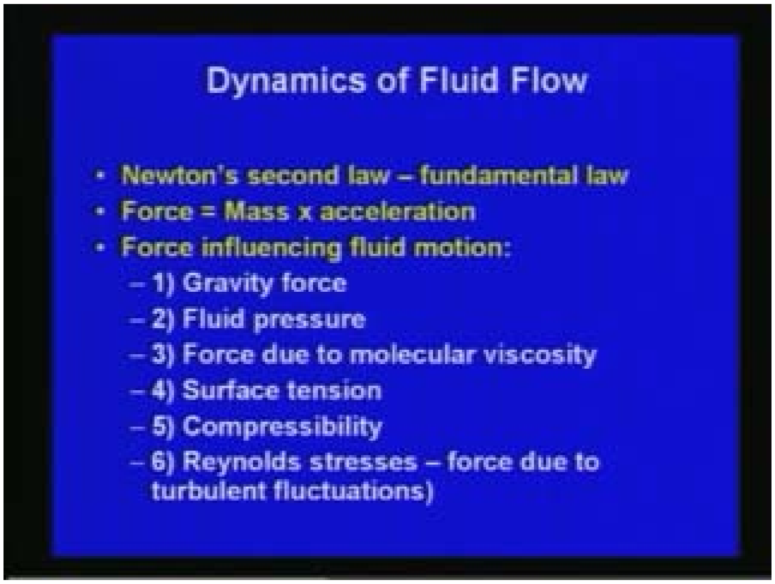

Also as we have seen that fluid motion can be either 1 D_1 dimensions two dimensions, three dimensions or steady state and transient or the uniform flow or non uniform flow with respect to dynamics of fluid flow also.

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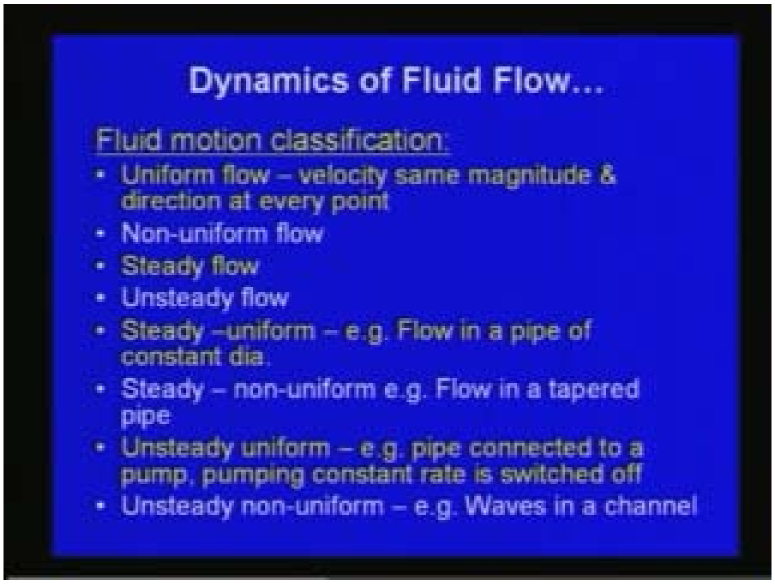

And then we have also seen the various aspects.

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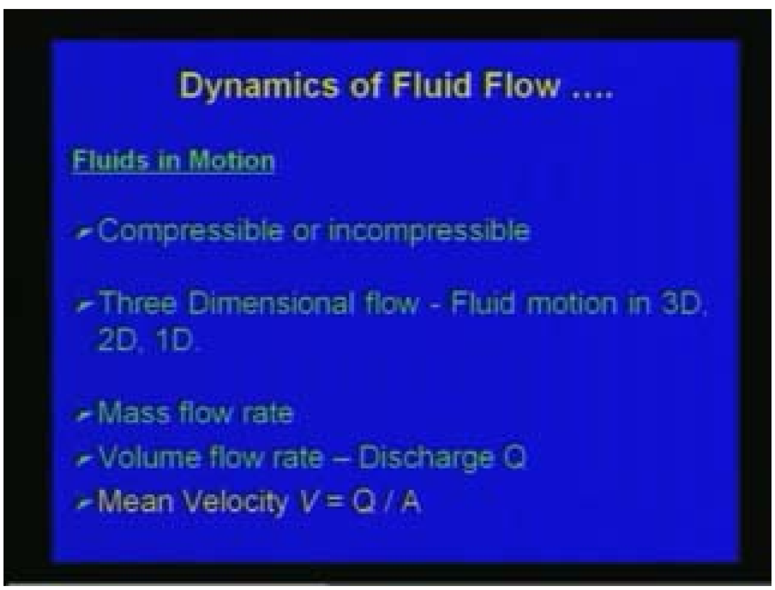

Like how to analyze when we are analyzing the fluid flow.

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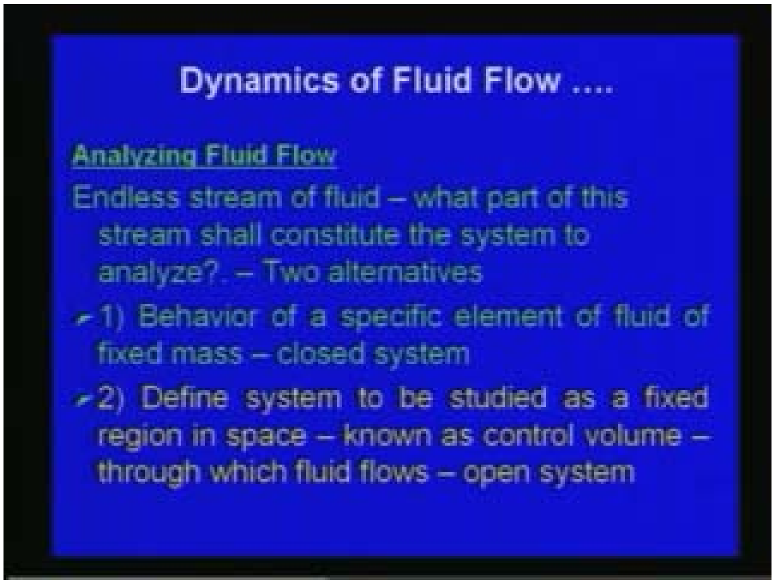

We will be either using endless stream of fluid, so that what part of this stream shall constitute the system which we will be analyzing. So we have seen two alternatives, one is behavior of a specific element of fluid of fixed mass so in a crossed system.

There are two alternatives the way of analyzing fluid flow, first one is behavior of a specific element of fluid of fixed mass in a closed system, and then we can also do the analysis by defining a system as a fixed region in space known as control volume through which fluid flows so that it is an open system. So this we have seen and now in our analysis we will be using this finite control volume analysis, as we have seen and discussed in the last lecture.

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So this finite control volume analysis used in many of the theoretical development in fluid mechanics including this dynamics of fluid flow. The advantages are it is very easy to interpret physically and use and the formulas derived from basic laws, we can apply to the system and as we have seen the description, how dealing the fluid flow is concerned.

It can be either say Lagrangian description or Eurasian description but in this finite control volume analysis, we will be using the Eurasian description and then the basic development is based upon the Reynolds transport theorem, which we discussed earlier in the fundamental aspects of fluid mechanics in the earlier lectures.

So now in today's lecture we will derive some of the fundamental equations starting from the conservation of linear momentum.

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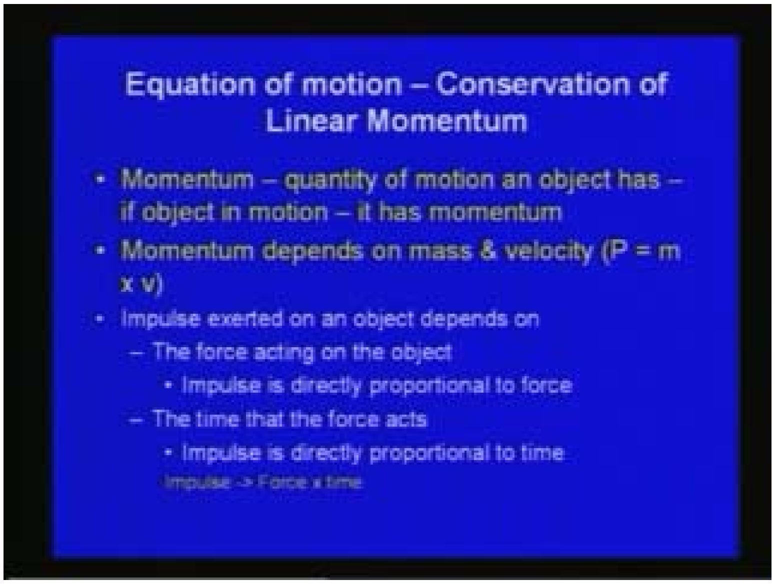

First we will derive the Euler’s equation for in viscid flow or non viscous flow and then or shall we discuss the Bernoulli’s equation and its applications. So as far as the three fundamental principles, the principles based upon the most of the theories derived we have already seen the conservation of mass based upon which the continuity equation is derived that we have already seen in the previous lectures, how to derive the continuity equation based upon the conservation of mass.

So this is one of the other most important theory based upon which most of the fundamental laws are derived, conservation of linear momentum. Today we would discuss, we would derive the Euler’s equation based upon this conservation of linear momentum and then further we will derive the Bernoulli’s equation based upon the Euler’s equation.

So say us? The word itself indicates the conservation of linear momentum means the title itself indicates the linear momentum is conserved. What is momentum? So we all know that momentum is the quantity of motion an object has, if the object is in motion then say it has momentum. So a momentum can be defined as it is a quantity of motion, an object has when the object is in motion, then we say that it has momentum. The momentum of course most of the bodies the moving bodies have got mass and then it has got velocities the momentum basically depends upon the mass and velocity so that we can express the momentum as mass multiplied by the velocity.

So here you can see that the momentum depends upon mass and velocity and then we can express the momentum is equal to mass into velocity, and then say a body under motion say a fluid in motion, then we can say that an impulse is exerted on a object which is in motion, when the force acting on the object is a force acted upon the object so that we can say that the impulse which gets to the object is proportional to the force. So impulse is also very important as far as the momentum of the system is concerned, linear momentum of the system is concerned.

So the impulse depends upon the time that the force acts upon the body or upon fluid which we are considering so that impulse is directly proportional to time. So that here as written in the slide, impulse is equal to proportional to force is equal to force into time so that impulse is directly proportional to time. So the momentum is depends upon the impulse, that when we discuss the conservation of linear momentum we have to consider an impulse that means a force applied with respect to time and then with respect to motion already it has the body or the fluid has a momentum, so both this will be playing as far as total momentum is concerned. So the impulse exerted on an object is equal to the objects change in momentum.

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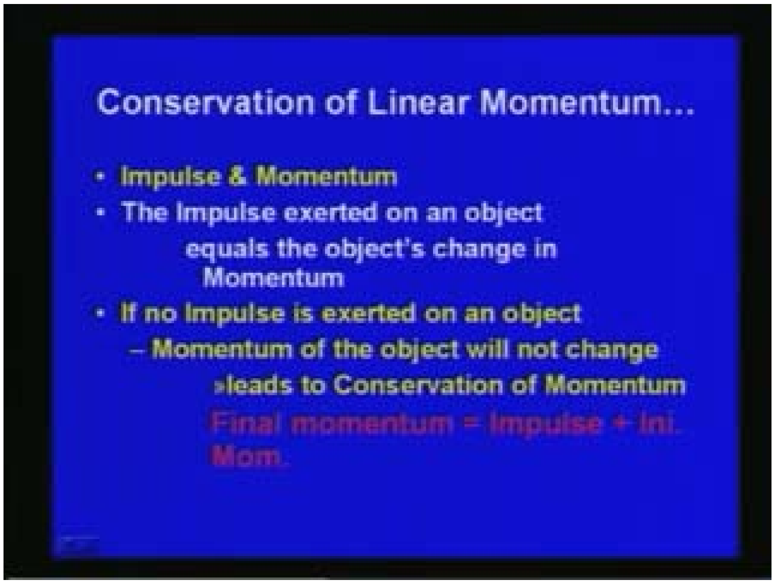

So finally we can define the impulse as the impulse is what exerted impulse is exerted on an object equals to the object's change in momentum. Whenever a fluid is moving so that we can say that with respect to the motion say there is already an initial momentum and then we are applying an extra force on the fluid or on the particle which you are considering so that there is a change in momentum. So this is so called impulse with respect to the force which is acting upon the fluid or the body which you are considering. So if no impulse is exerted on an object then momentum of the object will not change. These are some fundamentals of this mechanics, so based upon which also the fluid mechanics theories are derived. We can say that whenever there is no impulse then we can say that the momentum is not changing, momentum is constant so that means this leads to the conservation of momentum.

So according to the conservation of momentum we can say that final momentum is equal to the impulse which is acted upon the body or the fluid which we are considering then plus the initial momentum.

So the bodies or the fluid which is under motion it has got an initial momentum, so then the final momentum will be if you exert an extra impulse so that, there is a change in momentum. The final momentum will be impulse plus the initial momentum so while we are considering the conservation of linear momentum we have to see that what is the initial momentum for the fluid for the body which we are considering or what is the change? What is the impulse acted upon the body? The total or the final momentum is equal to impulse plus the initial momentum. So now as far as any fluid system which we discuss is concerned if there is no external force acted on the system then we can say that the total momentum of the system will not change.

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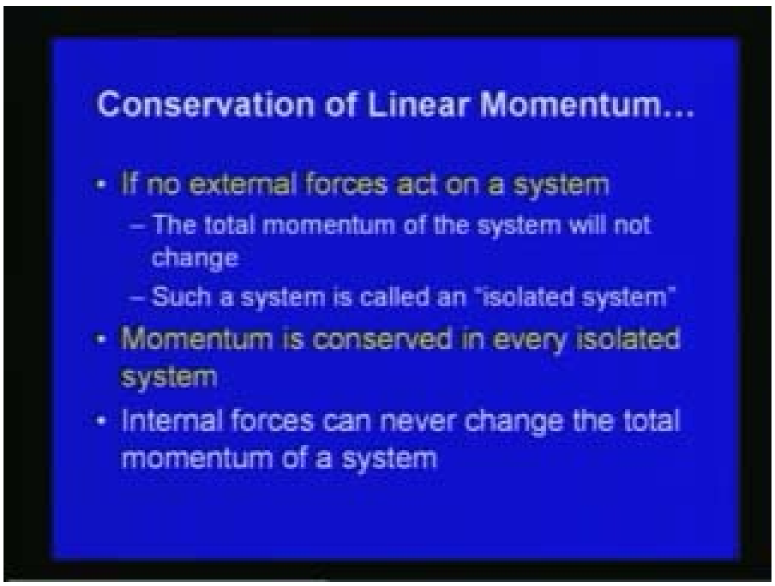

So we have already seen that the total momentum is equal to initial momentum plus the momentum change due to the impulse, we can say that if there is no external force act on a system then the total momentum of the system will not change. So such a system is called an isolated system.

So the system can be of two types wherever that is the total momentum of the system is not changing, there is no external force whatever the fluid is moving or the body is moving its all initial moment, we can say that the total momentum is not changing so such a system is called a isolated system and the other type of system is whenever we are exerting a force, there is a impulse and then there is a change in momentum so that the normal whenever we consider the dynamics of fluid flow most of the time we will be discussing due to external force is acting with respect to system but sometimes, also depending on the problem isolated system will also be considered.

Finally we can say that the Euler's equation which we are going to derive is based upon conservation of linear momentum so the momentum is we can say that, when we consider a particular control volume, the momentum is conserved in a very isolated system. If you isolate a system and there is no external force then we can say that the momentum is conserved in that particular system but as far as the internal forces are concerned it can never change.

The total momentum of the system even though there can be internal forces within the fluid system which we are considering but it cannot change the total momentum of the system. So these are some of the fundamentals when we consider the linear momentum and the impulse or the effect of force upon a particular system. So system can be either isolated system or system can be wherever an impulse is acted then there is a change in momentum. We have seen that internal forces are concerned it cannot change the total moment of a system. So now based upon these say the linear momentum equation we can write as shown in this slide here.

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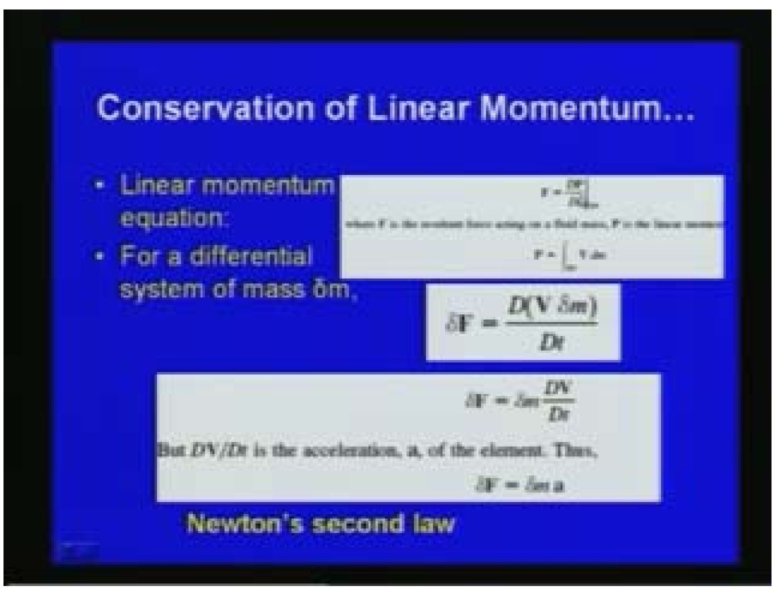

Say, if F is the resultant force acting on a fluid mass F is equal to d P by d t where P is the linear momentum. So the force is equal to the total derivative of linear momentum with respect to time. So F is equal to d P by d t for the particular system. So we can write the momentum is equal.

So the integral of this V d m, so this is the basic definition when we consider the linear momentum equation, and for differential set of mass say mass of delta m, we can write delta F is equal to V the total derivative V into delta m so that this is the velocity multiplied by the small mass delta m. So here in this equation p is equal to integral for the system V d m where V now delta F say for a differential system of mass which we will be generally considering while deriving the equations. So delta F is equal to the D by D t of V delta m as written here.

So that we can write delta F is equal to delta m is constant. So delta can be taken out so delta m D V by D t but here you can see that, here the V is the velocity and then D V by D t is the acceleration a of b element which we are considered considering here. So finally with respect to this we know that from Newton's second law this force is equal to mass into acceleration, so this delta F also we can write as delta m into a that means effectively the change in force or delta F is equal to the mass into acceleration that is what we are getting with respect to the equations here.

So now if you consider the resultant force acting on a fluid mass, here we consider as in this slag we consider a fluid element here of small area delta a here with respect to x y z axes say the force is acting can be put as delta F s is written delta F1 and delta F 2 and then delta F a with respect to normal.

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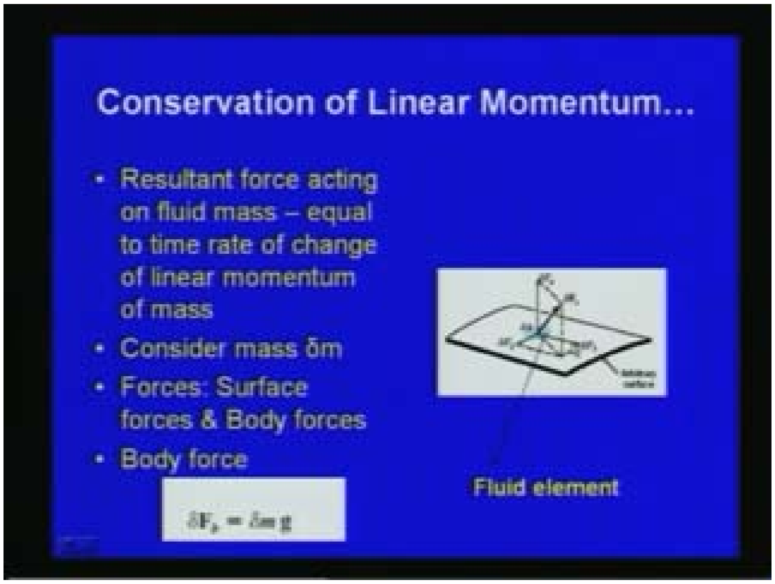

So the arbitrary surface which you are considering, the resultant force acting on fluid mass is equal to the time rate of change of linear momentum of mass. The conservation of linear momentum we can say now in the previous slide we have already seen here. So the delta F which we derived is equal to D into D by D t of V D m delta F is equal to delta m into D V by D t again that comes as the mass into acceleration Newton's second law delta m into a from this we consider here the resultant force acting on a fluid mass shown in this figure is equal to the time rate of change of linear momentum of mass.

So finally with respect to the consideration of linear momentum say now also considering the Newton's second law we can write the resultant force acting on fluid mass is equal to the time rate of change of linear momentum mass.

So now as in this figure we consider a mass of delta m and as far as this figure is concerned here various kinds of forces will be acting on this particular fluid element of mass which we are considering, So this is the fluid element which we are considering, so the forces generally are the surfaces forces and the body forces. The body forces concerned here you can see with respect to this figure, delta F d with respect to this particular fluid element which we are considering, the body forces mainly due to acceleration, due to the gravity so that we can write that delta F d is equal to delta m into g where g is the acceleration due to gravity and delta m is the small mass which we are considering.

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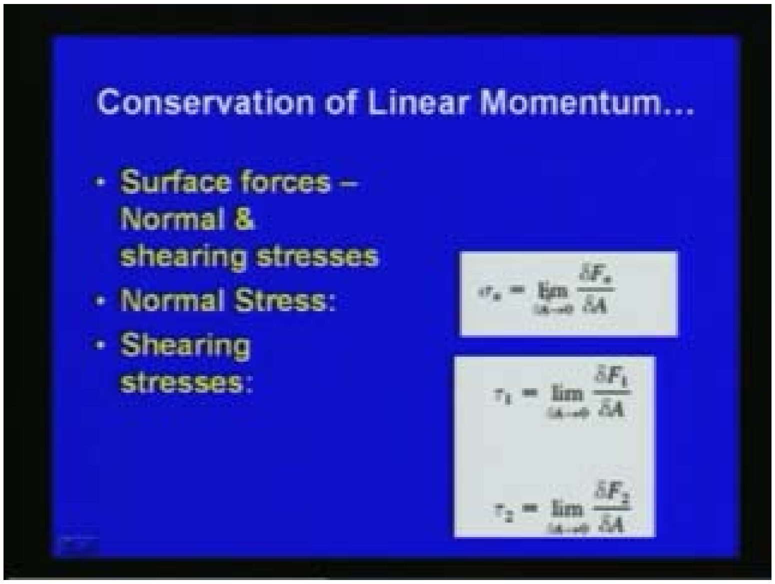

So this is the body force so for this particular fluid element so now based upon this, we are now deriving this Euler equation. Before that we are now seeing for a particular fluid element which we are considering when we derive these kinds of fundamental equations we are considering a fluid element of mass delta m and then we are now checking what are the various forces acting upon that particular fluid element, So now the surface forces they can be either normal to the surface which we have considered or it can be also with respect to the shearing stresses. Here if you consider the normal stress with respect to the previous figure here if you consider this particular fluid element the normal stresses ,we can write sigma s is equal to when the limit delta A approaches to 0, this can be written as delta F n by delta A.

So the normal stress is equal to delta F n by delta A when the limit delta A approaches to 0 with respect to this previous figure here. The other surface force is called shearing stresses. So the shearing stresses we can write as shown in this figure it can be spitted into these forces F1 in this direction and F 2 the other direction.

F 1 in this x direction and F 2 in the y direction. So if you consider that we will be having two components for the shearing stresses, these components have first one is tau_1, tau_1 is limit of delta F_1 by delta A as delta A approaches to zero and tau 2 can be written as limit delta F_2 by delta A as delta A approaches zero.

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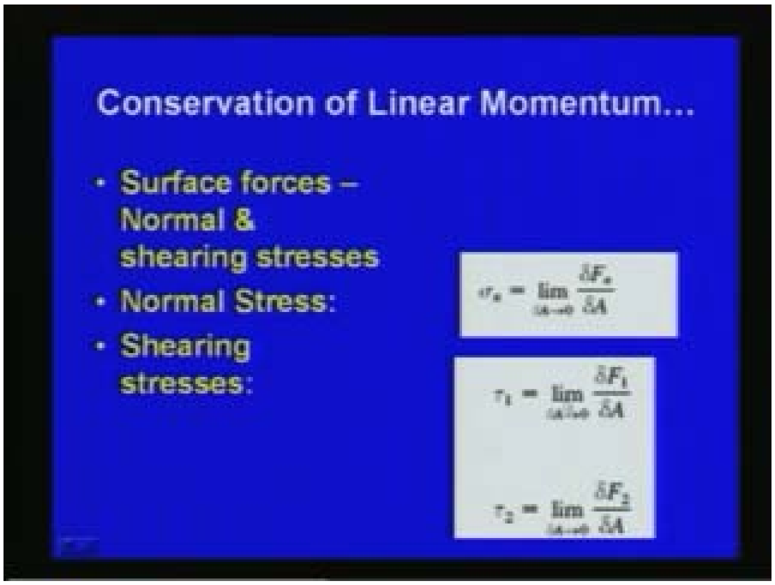

So these are the main surface forces, so we have already seen as far as a fluid element is concerned the important forces are the body forces so that body forces mainly due to acceleration, due to gravity as we have seen here the body forces delta F is equal to delta m into g and then the surfaces forces are concerned there can be normal and shearing stresses.

Normal stresses we can write with respect to normal force. So here sigma n is equal to delta F n by delta A as limit delta A approaches to 0. Similarly shearing stress is tau1 and tau 2 we have already here the expression for tau1 and tau 2. By using all this the conservation is based upon the conservation of linear momentum we are going to derive the one of the fundamental equation of say fluid mechanics so called Euler's equation.

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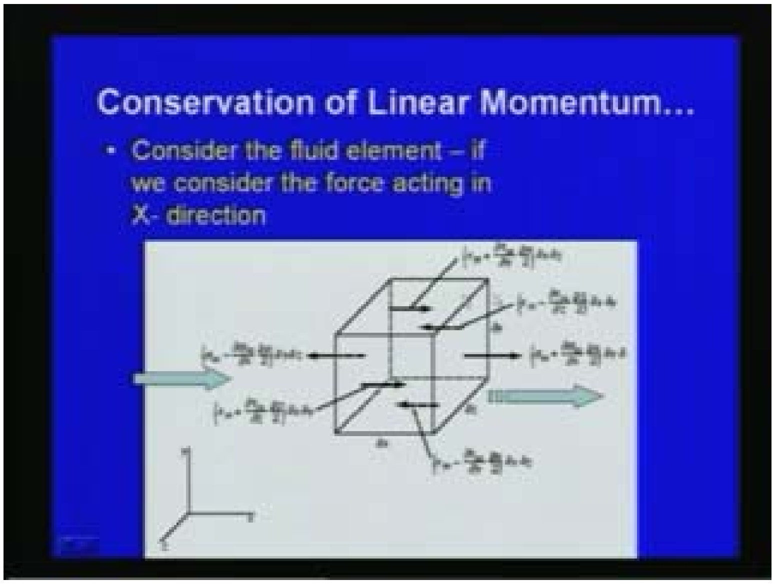

So this is mainly for in viscid flow that means the flow is considered without the viscosity so that is we derived based upon the conservation of linear moment.

Now to derive this Euler’s equation let us first consider a fluid element like this of size delta x delta y delta z so as shown in this figure the fluid element is considered.

For the fluid element the forces acting let us consider now the x direction. So, x is in this direction, y is this direction and z is the other direction. The various forces acting here you can see that as we have seen earlier the two types of forces we have seen surfaces forces and body forces. So the surface forces we have already seen it can be the normal stress with respect to normal stress or shear stress here with respect to the normal stress say for this particular fluid element.

We can see that here for this in this direction not this phase of the fluid element it can be written as since x direction is like this, so x on this phase it can be written as sigma x minus del sigma x by del x into delta x by 2 multiplied by this area of this phase of this fluid element delta by delta z and then similarly the opposite side of this other side of this fluid element we can write here the normal with respect to normal stress the force can be written as sigma x plus del sigma x by delta x into delta x by 2 into delta y delta z.

So this is as far as the normal stress is concerned. Now also see there will be surface forces with respect to the shear stress so here the shearing with respect to the shearing is concerned say shearing stress is concerned we can write on this phase as tau y x minus delta tau y x by del y into delta say delta y by 2 into delta x into delta z. Similarly the other direction to be tau y x minus del tau y x del z by del z by 2 this direction and here the other direction is tau y x plus del tau y x by del y into delta y by delta x into delta z so similarly other phase also we can.

So all the phases we can write body forces as far as this particular fluid element is we can write the surface forces for this particular fluid element is concerned one is the with respect to the surface forces with respect to the normal stress on this phase and other phase and similarly the shearing stress is concerned we can write all the components.

So now with respect to the surface forces say if you take the i j k that means the x y z direction we can write this total say surface force delta F s is equal to delta F x F s F s F x i with respect to i j k the unit vector i plus delta F s y j plus delta F z k.

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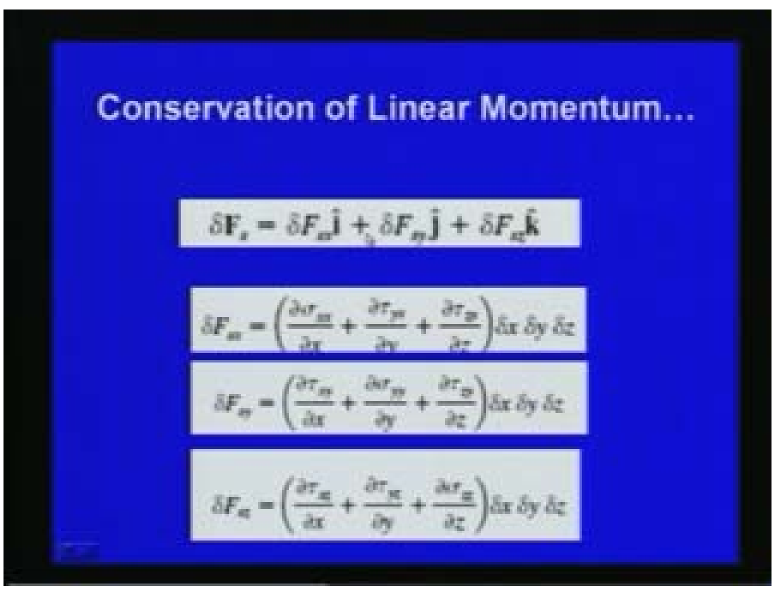

So now we have seen that this particular fluid element is considered there are three components x y z. So for the surface force we have seen so now if you consider the x direction with respect to the previous figure.

This figure we can write delta F x s x is equal to delta F s is equal to delta sigma F x by sigma delta x plus del tau y x by delta del y plus del tau z by del z into delta x del y del z so this is the body of surface forces with respect to the normal stress and shearing stress.

So similarly delta F s y can be written as del tau x y by del x plus del sigma y by del y plus del tau z y by del z into delta x delta y delta z and similarly in the z component we can write del F s z is equal to del tau x z by del x plus del tau y z by del y by plus del sigma z by del z into delta x del y del z.

So now with respect to the fluid element we have seen three components with respect to x y z direction of the surface forces. We have seen that as far as the total forces are concerned there is a surface forces and body force. So body forces as I mentioned will be with respect to the acceleration due to gravity so this also we can put in x y z direction. So here the body forces are concerned del F b x in the x direction will be mass into acceleration due to gravity in x direction so delta m into g x and in y direction delta F b y is equal to delta m into g y and delta F b z is equal to delta m into g z.

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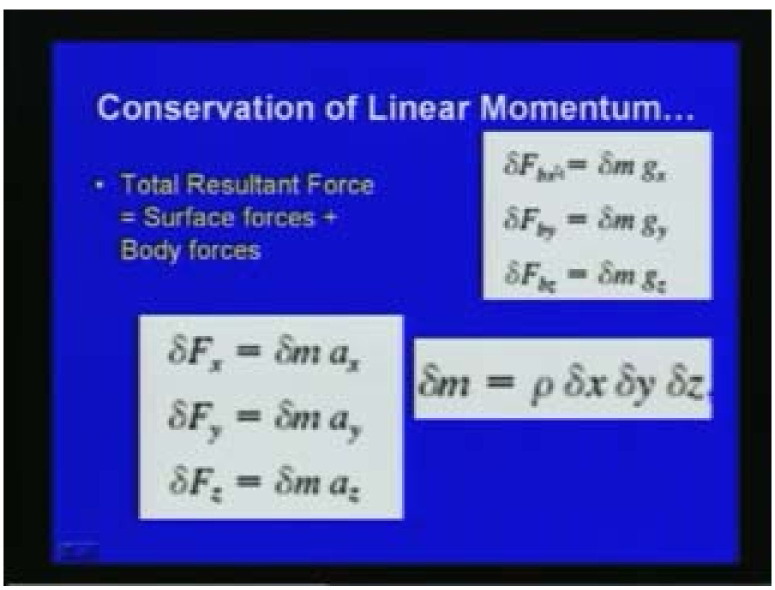

So these are the body force component in x y z direction. Now with respect to the Newton's second law also we have seen earlier with respect to the conservation of linear momentum. So finally we have come to that if you use the Newton's second law we will be getting effectively this conservation of linear momentum.

So now the various forces we have seen for the particular fluid element the surface force and body forces we have already seen various components of the surface forces and also we have seen the various components of the body forces. As per the Newton's second law in x y z direction we can write delta F s F x equal to that means the force in x direction is equal to delta m into acceleration in x direction and delta F y is equal to delta m into a y and delta F z is equal to delta m into a z, where delta m is the mass of the fluid element, so if rho is the density of the fluid element, we can write delta m is equal to rho into delta x into delta y into delta z .Now all these parameters are known.

Now if you use the Newton's second law finally, we can write the general differential equation of motion for the fluid in x y z directions.

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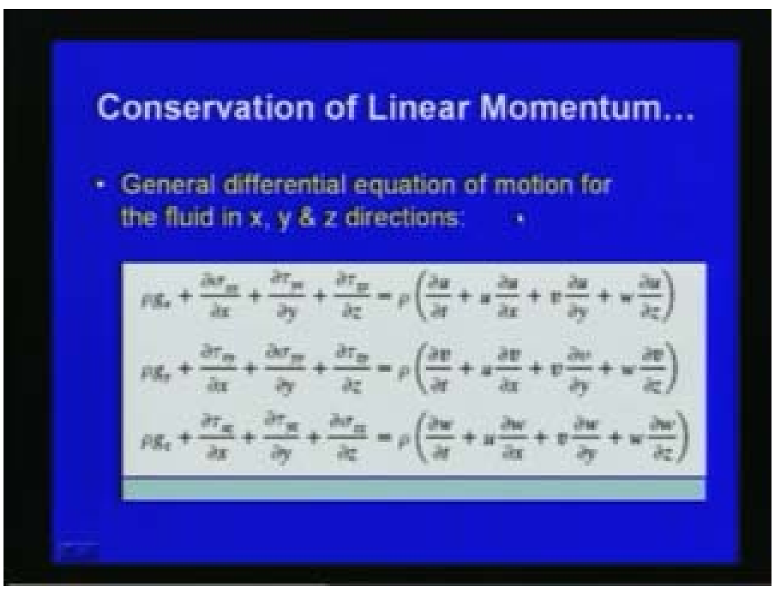

So by using Newton's second law we have equated the forces into the mass into acceleration, the forces are concerned we have seen the body forces and surface forces. By equating by using Newton's second law if you get then we can see that this is in x direction the equation will be rho g x which is the body force in x direction plus due to the stress force shearing and normal stress we can write del sigma x by delta x plus del y del tau y x by del y plus del tau z x by del z is equal to mass into acceleration.

So mass is we have already seen say rho into delta x into delta y into delta z. These deltas x delta y delta z are there in all this components which we have derived. So delta m as delta x delta y delta z and also we have seen delta F s x delta F s y delta F s z all these are delta x delta y delta z.

So while using the Newton's second law and equating force into mass into acceleration delta x delta y delta z will be cancelled on both sides. Finally, the total body forces and surface forces can be equated mass into acceleration.

Mass is effectively here now rho since delta x delta y delta z cancel and acceleration so we have already seen when we consider the concept of the Eulerian concept which we have seen here we are using Eulerian concept in the derivation of this equation.

So while considering this, Eulerian description so the acceleration can be written as, the local acceleration del u by del t plus the convective acceleration. So, del u by del t, plus u into del u by del x, plus v into del u by del y, plus w into del u by del z, this is the total acceleration in x direction so that mass into acceleration is rho into del u by del t plus u into del u by del x plus v into del u by del y plus del y plus w into del u by del z.

So this gives the general differential equation of motion in x direction. Final equation is rho g x plus del sigma x by del x plus del tau y x by del y plus del tau z x by del z equal to rho into del u by del t plus u into del u by del x plus v into del u by del y plus w into del u by del z.

So this is the general differential equation of motion for fluid in x direction. So from this only we will be deriving other equations like Euler's equation or other kinds of equation. Now this is the x equation in x direction and the general equation of motion by direction can be written as rho g y plus del tau x y by del x plus del sigma y by del y plus del tau z y by del z equal to rho into del v by del t plus u into del v by del x plus v into del v by del y plus w into del v by del z and the general differential equation of motion in z direction can be written as rho into g z plus del tau x z by del x plus del tau y z by del y plus del tau del sigma z by del z is equal to rho into del w by del t plus u into del w by del x plus v into del w plus del y plus w into del w by del z.

So where u, v and w are the velocity components in x y z direction when we consider the fluid motion in three dimensions. Now we got the general differential equation for fluid in x y z direction so now from the general differential equation of motion we will be deriving various equations. So the first case which we will be considering here is the case for in viscid flow so the equation is basic equation is Euler's equation.

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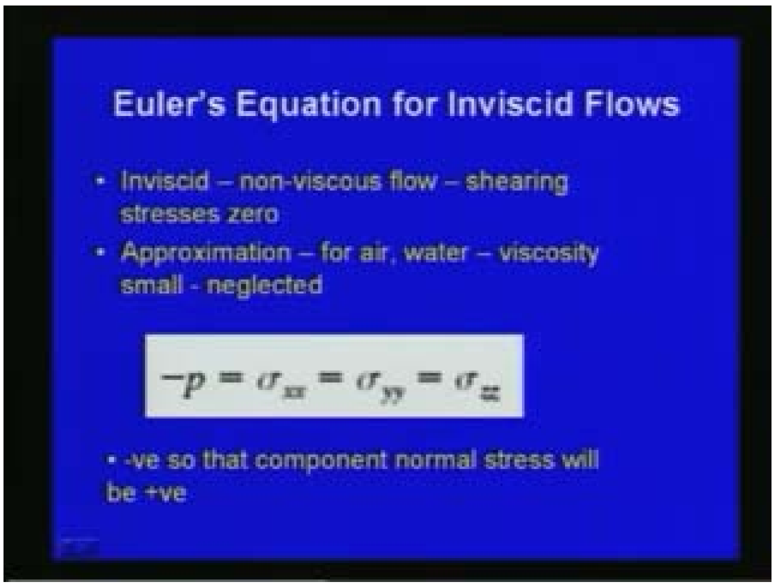

Here the Euler's equation for in viscid flow based upon the general differential equation of motion will be deriving. So Euler's equation is concerned, the equation is valid for in viscid or non viscous flow that means shearing stresses is 0. We have already seen the general equation here.

The shear shearing components are there, so when we consider the Euler's equation for in viscid flow that means there is in viscid means no viscous non viscous flow. We don't have to consider the shearing stresses, so that shearing stresses become 0. So this actually is not an exact way but we can approximate many of our fluid flow like wind flow or air flow or water, we can approximate the viscosity is small. Sometimes we can neglect the viscosity and then this Euler's equation can be utilized many of the problem but it is an approximation we are considering say the viscosity of air or water is small.

So that it can be neglected and then as far as the general equation which we are considering here generally differential equation of motion concerned now for Euler's equation or in viscid flow is concerned the shearing stresses are 0. So that the shearing terms are gone on the left hand side of this equation and then we have this normal stress component so as far as normal stress components are concerned.

So if you consider normal stress component, it will be generally the pressure which will be acting. That we can write for the fluid element is concerned with respect to x y z direction we can approximate as minus p is equal to which is the pressure force minus p is equal to sigma x is equal to sigma y is equal to sigma z. Here we use the negative; we are putting this negative so that component normal stress will be positive.

So now like this we are approximating the normal stress component the pressure term which is coming upon the fluid, so minus p is equal to sigma x is equal to sigma y is equal to sigma z. Now after this approximation based upon the general equation which we have already derived here, now the shear terms have gone and the normal stress components its derivative we are now approximating with respect to pressure components. Finally this equation can be put as general equation of motion for Euler's for in viscid flows now reduces to like this so that general equation of motion in x direction can be written as rho g x that is body force minus del p by del x.

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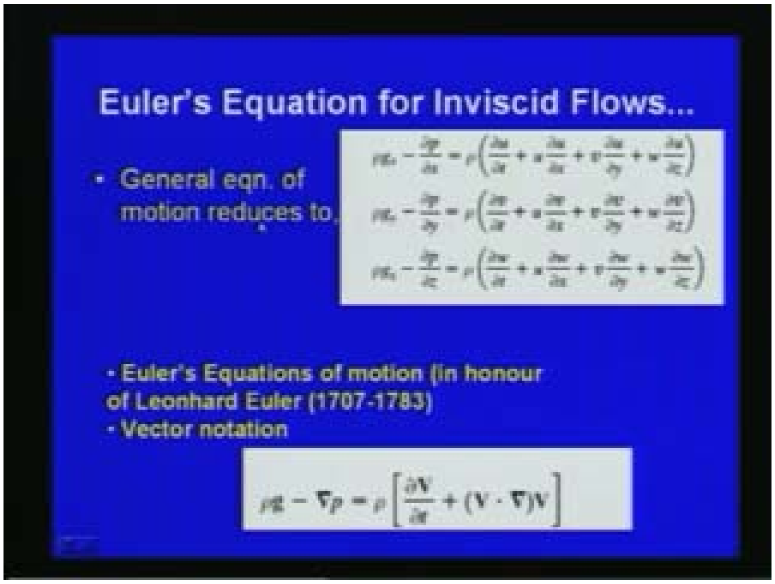

So the pressure gradient in the x direction is equal to rho into del u by del t plus u into del u by del x plus v into del u by del y plus w into del u by del z and in y direction we can write rho g y minus del p by del y is equal to rho into del v by del t plus u into del v by del x plus v into del v by del y plus w into del v by del z and in z direction rho g z minus del p by del z is equal to rho into del w by del t plus u into del w by del x plus v into del w by del y plus w into del w by del z.

So these are the Euler's equations for in viscid flow as I mentioned say this air flow or the water flow is concerned we can sometimes neglect the viscosity and then we can use this Euler's equation. So that the shearing terms are not considered so for finding the Euler's equation we can utilize but it is an approximation. So this Euler's equation, this name is given to honor Leonhard Euler, who lived from seventeen naught seven to seventeen eighty three that is the eighteenth century. In his honor only these equations are named as Euler's equation and this same equation in x y z direction which we have derived now we can express as in vector notation as rho g minus del p is equal to rho del V by del t plus V dot product del V. Here this is the vectorial vector notation form of the Euler's equation, so where V is the velocity vectors and g is also with respect to x y z acceleration the acceleration vector so the vector notation can be written like this.

So now as I mentioned this Euler's equation is 1 of the fundamental equation of fluid mechanics even though here we consider the flow as in viscid but many of the practical cases like water flow or air flow we can approximate as in viscid and then try to get a solution the advantage is that, here now we can see the equations are simple and then we can easily try to get the solution very much easier. If we do not consider the shearing terms here but it is an approximation but still this Euler's equation have have lot of applications in practical areas by wherever the fluid viscosity can be neglected or where small value of viscosity is there.

Based upon this Euler's equation, now we will be deriving another fundamental equation called Bernoulli's equation. This Bernoulli's equation is actually one of the most important or most useful equations in fluid mechanics. It is one of the fundamental equations.

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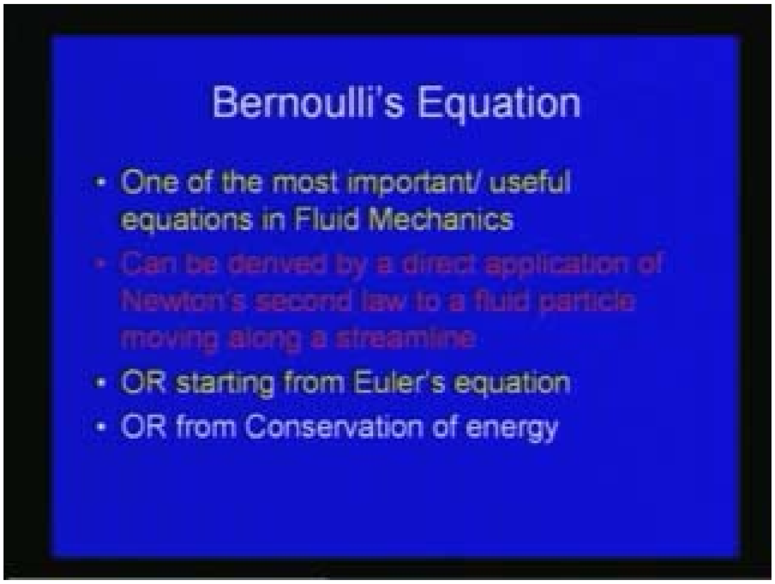

So this equation Bernoulli's equation can be derived by direct application of the Newton's second law into a fluid particle moving along a stream line. We can use the Euler's equation and then derive or we can also derive the conservation of energy. Now we will discuss we have already seen the Euler's equation for in viscid flow. Now based upon the Euler's equation here we will derive the Bernoulli's equation which is one of the most important equations.

So as I mentioned here, it can be either derived by direct application of the Newton's second law or we can use the Euler's equation or we can derive from the conservation of energy. Bernoulli's equation is concerned, it has got some restrictions. This Bernoulli's equation is derived based upon the assumption that flow is steady state, so time component is not there rho is steady state and density is constant so that fluid can be considered as incompressible.

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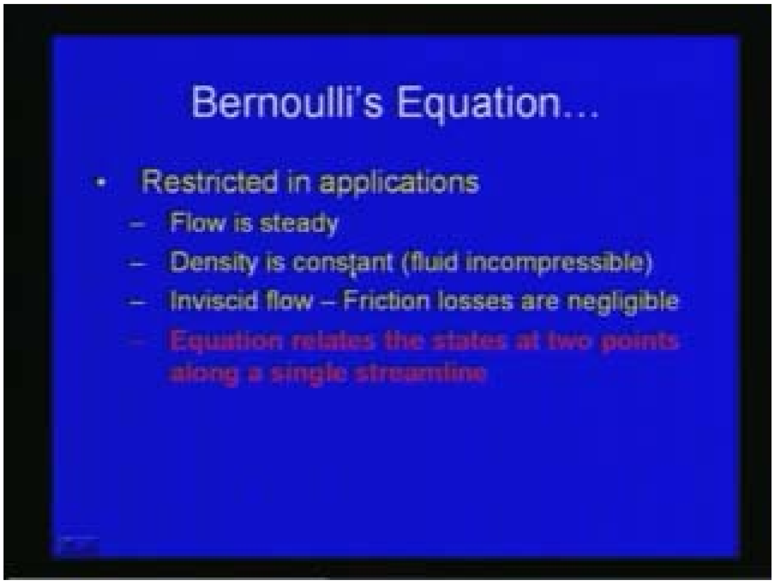

And then of course we also put the assumption that fluid is in viscid, so that friction losses are negligible so this assumption is actually many of the practical problems it will be difficult to apply but certain times, certain places we can approximate the flow is even though, say the viscosity is to be considered. Whenever water is concerned since its viscosity is small or air flow is concerned viscosity is small.

We can use this Bernoulli's equation so the assumptions are flow is steady density is constant and in viscid flow. So actually this Bernoulli's equation it relates the states at two points along a single stream line. Whenever there is a fluid flow is there, if you consider a fluid flow here in a channel like this, Here we consider two sections here, 1 and two and fluid is flowing in this direction.

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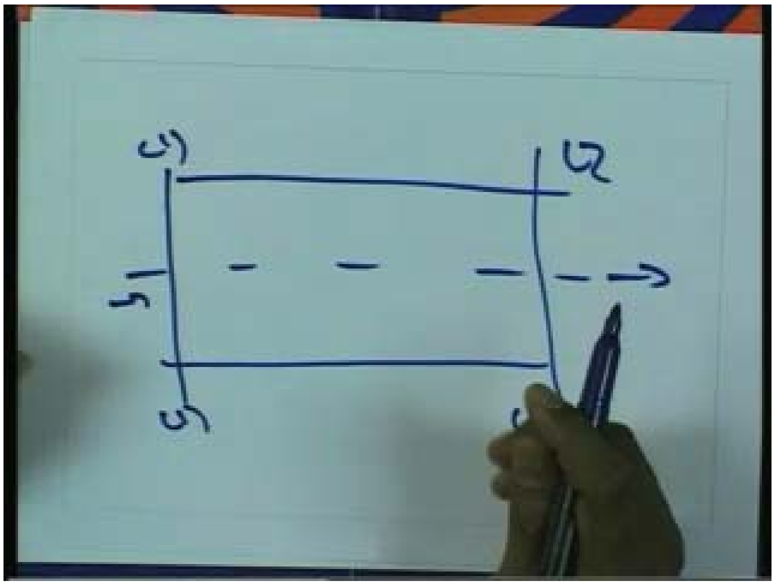

So this Bernoulli’s equation is it is relation between two points if you consider this point here and this point 1 and two, it is along a stream line if you consider a streamline like this, the Bernoulli’s equation is an expression for the states. What is the state between this position and first position and second position? So that is what the Bernoulli’s equation states.

So now we will derive here the Bernoulli’s equation based upon the Euler’s equation which we have already seen earlier. From Euler’s equation at steady state the equation, we have already seen earlier here, the Euler’s equation with respect to vector notation, if you consider the flow at steady state condition. Then we can write rho g bar minus del p is equal to rho V bar dot del V. This is the Euler’s equation at steady state.

So now to get the Bernoulli’s equation we will be integrating this equation along some arbitrary streamline so before the integration. The acceleration due to gravity we can write as g bar can be with respect to the normal direction minus g del z so and then also this other term del dot V so or V dot del V bar this right hand side of the equation the steady state equation Euler’s equation can be written as can be approximated using the vector notation mathematics as half del V dot V minus V cross del cross V.

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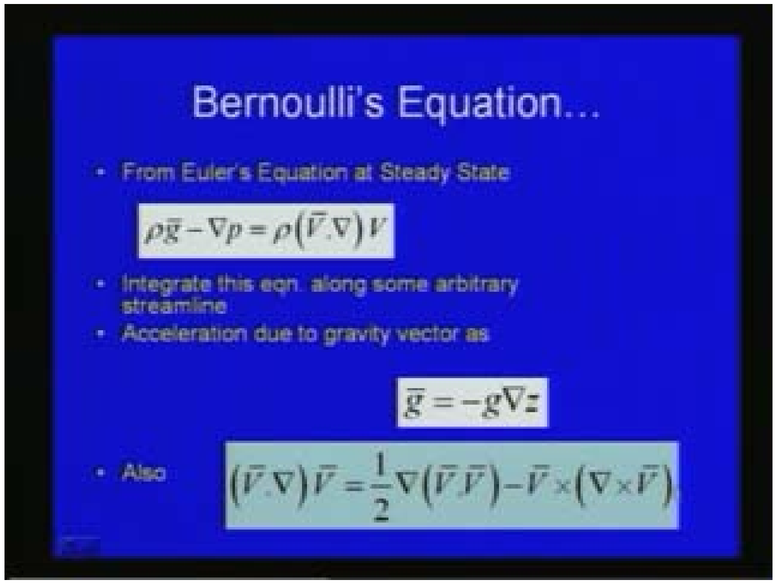

So this is from the vector algebra the mathematics, vector product, and dot product we can get this equation, and now Euler’s equation finally, becomes rho g del z minus del p is equal to rho by 2 del V dot V minus rho V cross del cross V. This we can rearrange as deep p by rho plus 1 by 2 del V square, plus g del z and that is equal to V cross del cross V.

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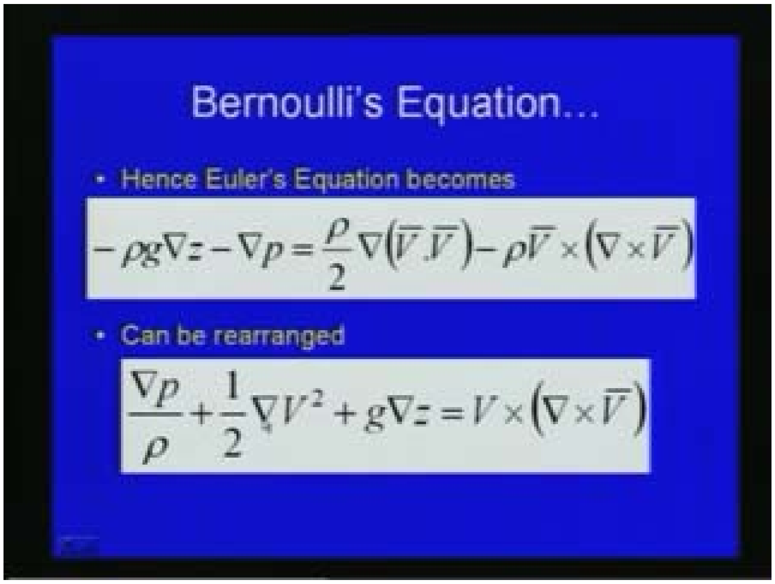

So now we are approximating the Euler's equation into derive the Bernoulli's equation. Now say let us consider a stream line like this as shown in this figure here, so if you take the dot product of each term with a differential length ds along a streamline as shown here this is streamline so that equation will become del p by rho dot d s plus half into del V square d s plus rho del z d s is equal to V cross del cross V d s.

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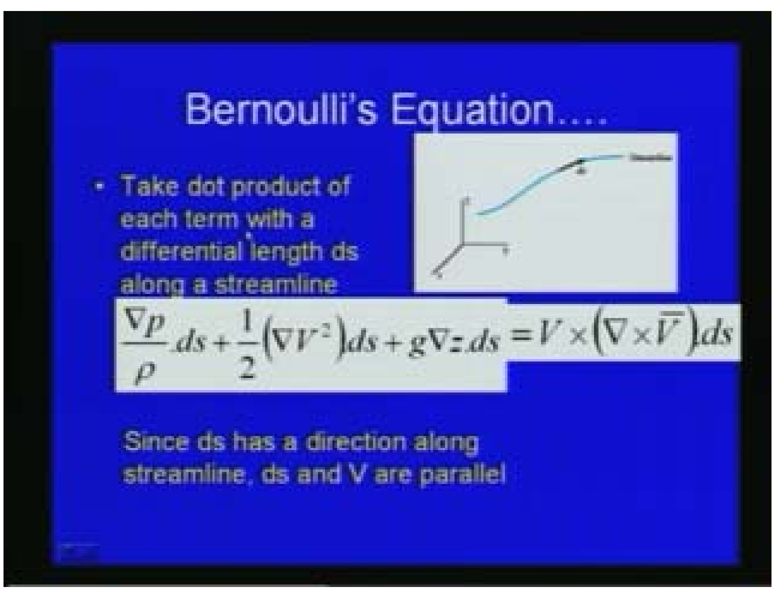

Now this d s has a direction along the streamline, so here the direction is shown here and d s and V are parallel. So since this streamline is derived like that, so that we can write also V cross del cross V is perpendicular to V bar so that we can write V cross del cross V d s is equal to zero and then this d s can be this d s in this previous figure ds this vector can be spitted into, here it can be written as, d x i plus d y j plus d z k. So that finally we can write del p dot d s is equal to and also d p del p d s can be written as del p by del x d x plus del p by del y d y plus del p by del z d z that is d p.

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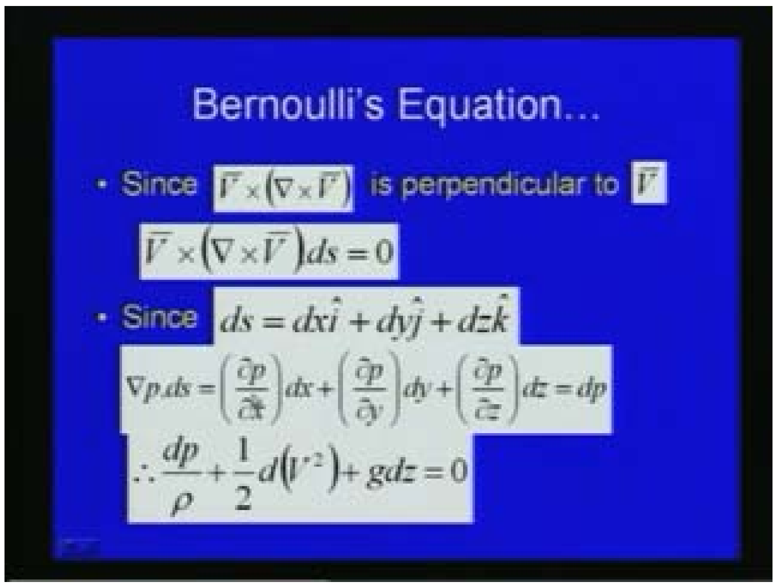

So that finally after using all these approximations the Euler's equation in steady state become d p by rho plus half d V square g d z is equal to zero. So after using all these approximations put forward here in the slides, finally we get this dp by rho plus half d V square plus g d z is equal to zero. So now along the streamline as we discussed here we integrate so that integral d p by rho plus V square by 2 plus g z is equal to constant.

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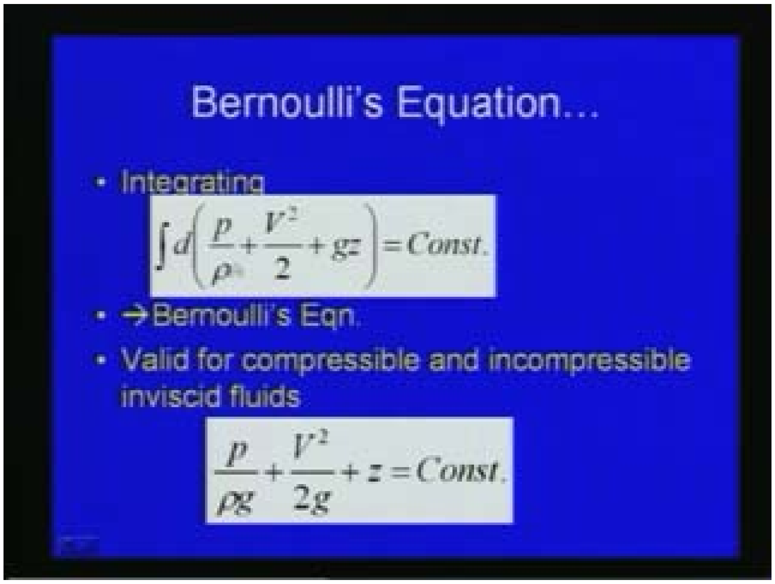

Finally, this gives the Bernoulli's equation, so Bernoulli's equation is actually here are three terms: one is with respect to pressure, other one is the velocity, with respect to velocity square, and the position head gz with respect to z. So this is the integral form this is the general Bernoulli's equation. As we have seen this is valid for incompressible and inviscid fluid.

Actually, we can write this general equation, since rho term is there, This is valid for compressible and incompressible viscous fluid so that, we can write p by rho g plus V square by 2 g plus z is equal to constant and now between two sections if you consider two sections as we have seen here in the previous here.

If you consider the section 2, section 1 and section 2 then finally, we can write p_1 by gamma plus V_1 square by 2 g is equal to z plus z_1 is equal to p_2 by gamma plus V_2 square by 2 g plus z_1 z_2.

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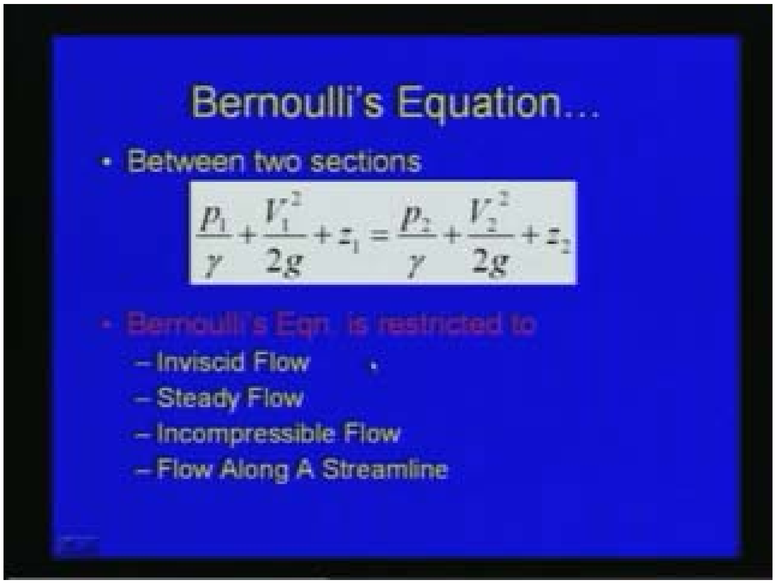

So here this if we consider in this figure here this is the datum and here z_1 the position head and this is z_2 position head. So that finally with respect to Bernoulli's equation and p_1 is the pressure here V_1 is the pressure velocity here, and p_2 is the pressure here and V_2 is the velocity on the section 2. Finally the equation become p_1 by gamma plus V_1 square by 2 g plus z_1 is equal to p_2 by gamma plus V_2 square by 2 g plus z_2 where gamma is the specific weight of the liquid. So this is the general equation when we consider between the sections.

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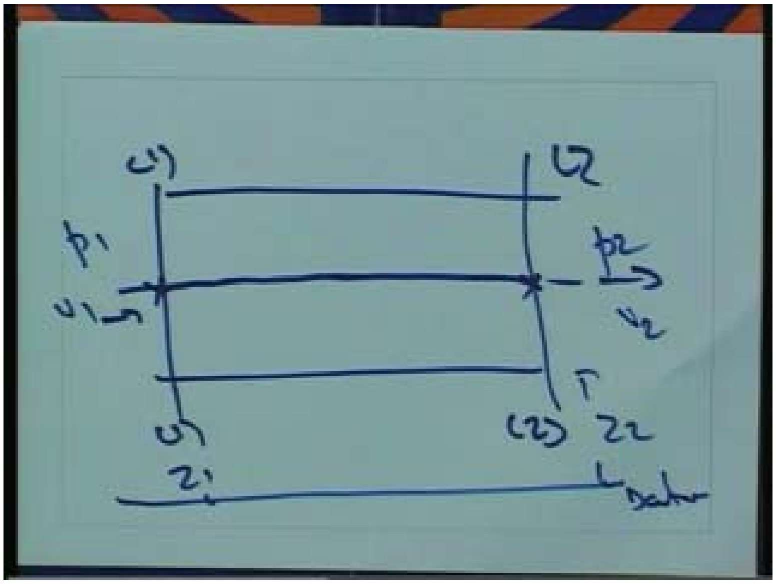

So the Bernoulli's equation, actually when we derive the equation earlier for general one but it is restricted to in viscid flow or non viscous flow, and it is in steady state and the general application is for incompressible flow and flow along a streamline. These are some of the restrictions as far the Bernoulli's equation is concerned. The equation is applicable for in viscid flow since from Euler's equation we have derived and steady state only.

We have considered so it is generally applicable to incompressible flow and the generally take a streamline and then flow along a streamline is the equation is derived. Here in the previous slide you can see here, there are three terms one is say p_1 by gamma, that is actually so called the pressure head, and then second 1 is V_1 square by2 g, which is so called velocity head, and then z_1 and z_2 which is considered here that is the datum head.

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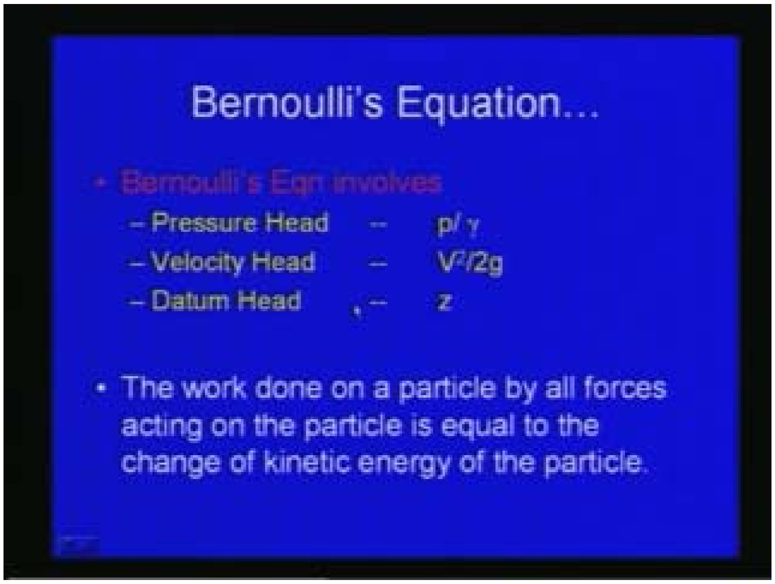

So the Bernoulli's equation holds three terms the pressure head the velocity head and datum head. Now with respect to this we can see that the work done on a particular by all forces acting on the particle is equal to the change of kinetic energy of the particle.

So if you consider particular particles which were on the streamline which we are considering, with respect to the equation, we can say that the work done on a particle by all forces acting on the particle is equal to the change of kinetic energy of the particle.

So now also the Bernoulli's equation can be derived as we have seen we have derived now the Bernoulli's equation based upon the Euler's equation for the in viscid flow but Bernoulli's equation can also be derived from the basic Newton's second law or also we can derive from the conservation of energy.

So from the conservation of energy for the system which we are considering as we have seen in this figure, here we can see that the energy like pressure energy per unit weight plus kinetic energy per unit weight plus the potential energy per unit weight is equal to total energy per unit weight.

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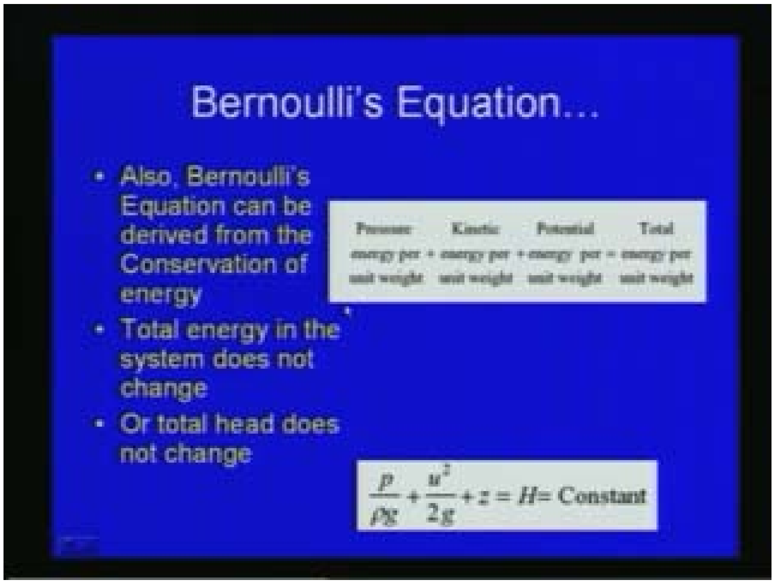

So as per the conservation of energy, this total energy is conserved so that the pressure the energy per unit weight due to pressure kinetic or potential that should be considered so this is the total energy per unit weight. Total energy in the system does not change or total head loss does not change. So since the total energy is not changing we can say that the total head also does not change. That we can write from this energy conservation we can write p by rho g plus u square by2 g plus z is equal to constant. So, this is coming from this basic energy equation. The same equation we are getting from the conservation of energy principles.

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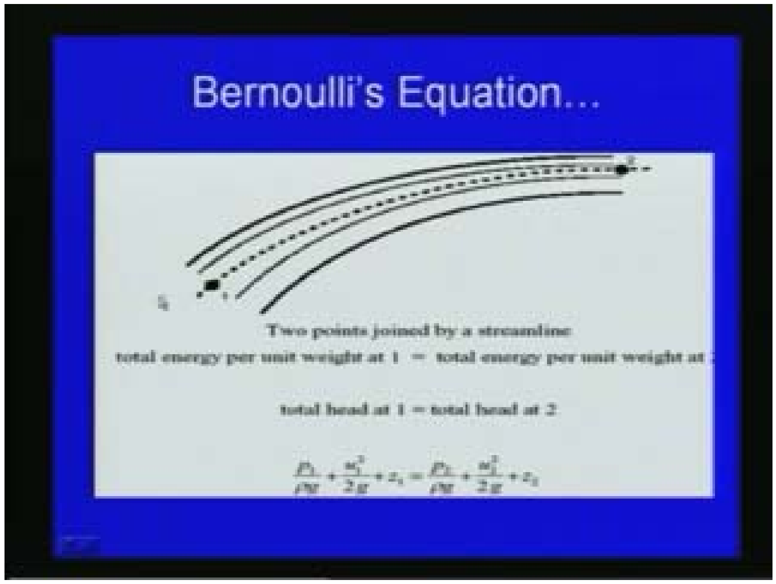

So if you consider say a flow in a channel or flow in a pipe like this, if you consider two points are shown in this slide here, two points joined by a streamline, the total energy per unit weight at 1 is equal to total energy per unit weight at 2. Total head at one is equal to total head at 2. That is why we are getting p_1 by rho g plus v_1 square by 2 g plus z_1 is equal to p_2 by rho g plus v_2 square by 2 g plus z_1 for this section 1 and section 2 which we consider here.

So finally the total energy per unit weight is equal to total energy per unit weight at 2 plus. If there is any loss in case of previous figure, if there is loss per unit weight that also we have to consider. It is considered here as h and in case between section 1 to section 2 if any work is done that also to be consider that is w.

So work done per unit weight minus energy supplied per unit weight so we consider all the aspects then the equation becomes like this p_1 so with respect to this previous figure if we consider the work inside between section 1 and 2 also any loss of a energy between section 1 and 2 and any energy supplied also considered then we can write the general equation as p_1 by rho g plus u_1 square by 2 g plus z_1 is equal to p_2 by rho g plus u_2 square by 2 g plus z_2 plus h plus w minus q.

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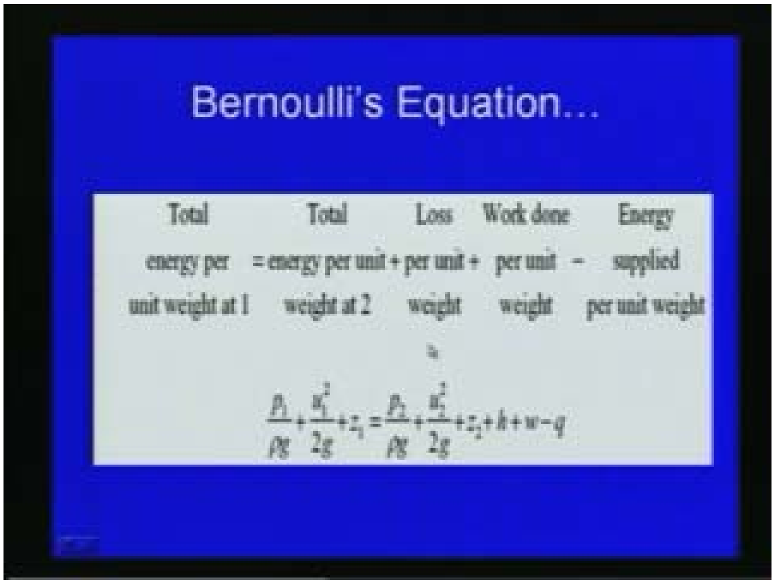

So this is the generalized equation when we consider a real system. So the earlier equation which we have considered is pressure head velocity head datum head are considered as derived here but when you consider a real system there can be loss of energy or there can be energy supplied or there can be work done. So this will be considered as shown in this general equation.

So now as we have already seen the equation is applicable across the streamline also if the flow is irrigational, so we have already seen we have derived the equation for flow along the equation is derived for streamlines along a streamline but it can be also be applied across streamline if the flow is irrigational.

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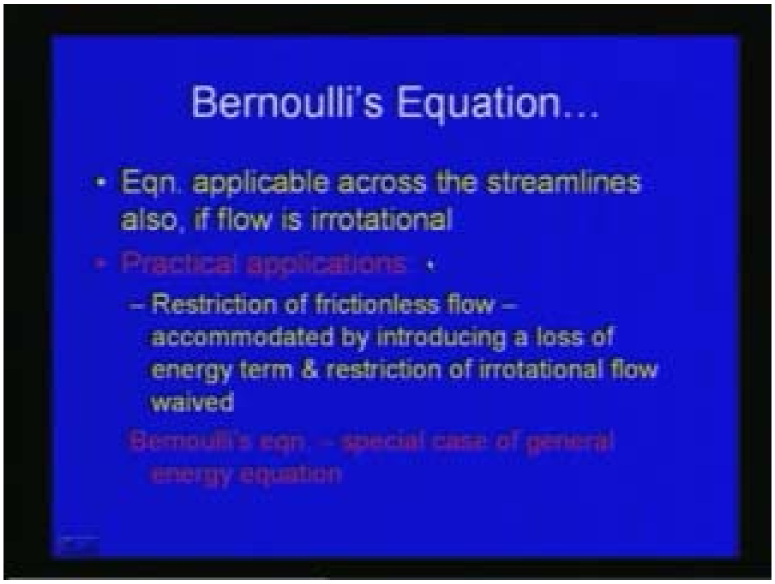

We have already seen, what is rotational flow? What is irrigational flow, in that case irrigational flow, we can say that the Bernoulli's equation is also applicable for across the streamline and some of the practical applications, the restriction frictionless can be considered then it can accommodate by introducing a loss of energy term and restriction of irrigational flow as we have seen here.

So and we can apply many of the cases which we will be discussing later. Some of the important applications of Bernoulli's equation will be discussed in later. So Bernoulli's equation as we have seen it is a special case of general energy equation, so general energy equation which we have seen here.

It is a special case of the general energy equation. So now with respect to this we will be discussing some of the applications of Bernoulli's equation so some of the things which we should always remember before using this Bernoulli's equation, are it may be applied without much error for unsteady flow also with gradually changing conditions.

We have already derived the equation for steady state condition but the variation is very gradual then still sometimes we can use this Bernoulli's equation for unsteady flow with gradual change in condition and also for flow of gases the change in pressure is small fraction of absolute pressure we can treat as incompressible and then can be considered the Bernoulli's equation.

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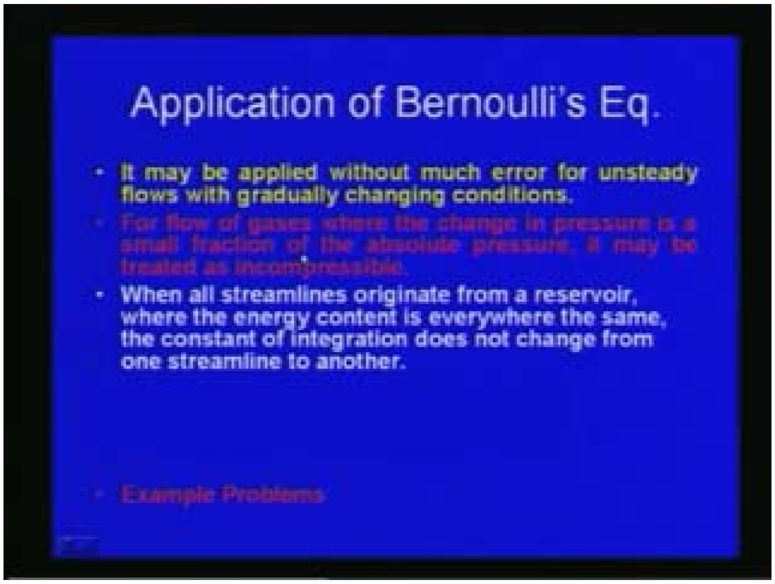

And when all streamlines originate from a reservoir where the energy constant is everywhere, the same constant of integration does not change from one streamline to another. So that this case also we can consider, so this has some of the cases which we can consider the. Bernoulli's equation further equation will be discussing later.

So before closing today's say just we will also see a simple example here. So Bernoulli's equation a small example will be discussed here.

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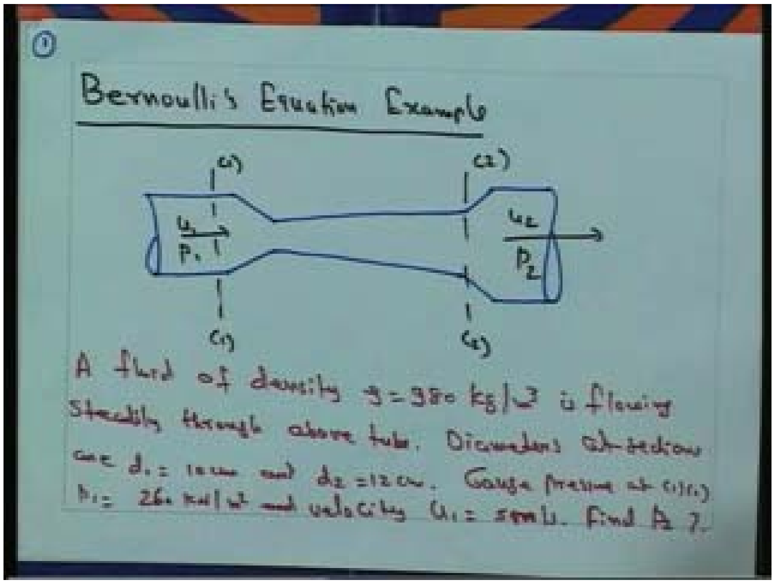

A fluid of density rho is equal to 980 kilogram per meter cubic is flowing steadily through a above tube, here you can see a tube flow a liquid is flowing through this tube and diameter at section 1 is ten centimeter and section 2 it is 12 centimeter and gauge pressure at section 1 is 260 Kilonewton per meter square and velocity here at section 1 is u_1 is equal to five meter per second we want to find P_2.

So here this simple example shows how the Bernoulli's equation can be applied. Here we can see that there is a tube and then varying the diameter is varying and the liquid is flowing through the two sections which we are considered and at one section the velocity is known, the pressure is known and other section both section diameters are known we want to find the pressure at section 2.

So that is what we want to find in particular problem and the pressure at section 1 is given as two sixty kilo Newton per meter square. So here we will be using the Bernoulli's equation and also the continuity equation. We apply the Bernoulli's equation along a streamline joining one and two. Here, as shown in this figure the tube is assumed to be horizontal, so that is z_1 is equal to z_2. The general equation we can write with respect to Bernoulli's equation P_2 plus rho by 2 u_2 square is equal to P_1 plus rho by 2 u_1 square so you can see here.

Section 1 and 2 we are equating the pressure head and the velocity head z_1 is equal to z_2 so datum head is not to be considered. So section 1 section 2 so that P_2 plus rho by 2 u_2 square is equal to p_1 plus rho by 2 u_1 square u is the velocity P is the pressure and now the velocity at section 1 is already unknown u_1 is equal to five meter per second but u_2 is not known.

But we can use the continuity equation so from the continuity equation we can write A_1 u_1 is equal to A_2 u_2 so that we can get u_2 so u_2 is equal to A_1 u_1 A_2 so the diameters d_1 is ten centimeter and diameter d_2 is twelve centimeter. So that we can write u_2 is equal to d_1 square by d_2 square into u_1 and if you substitute all the values here. We will get the velocity u_2 is equal three point four seven meter per second and then once we the velocity u_2 is known we will substitute back to this equation here the Bernoulli's equation so that P_2 is equal to P_1 plus rho by 2 u_1 square minus u_2 square.

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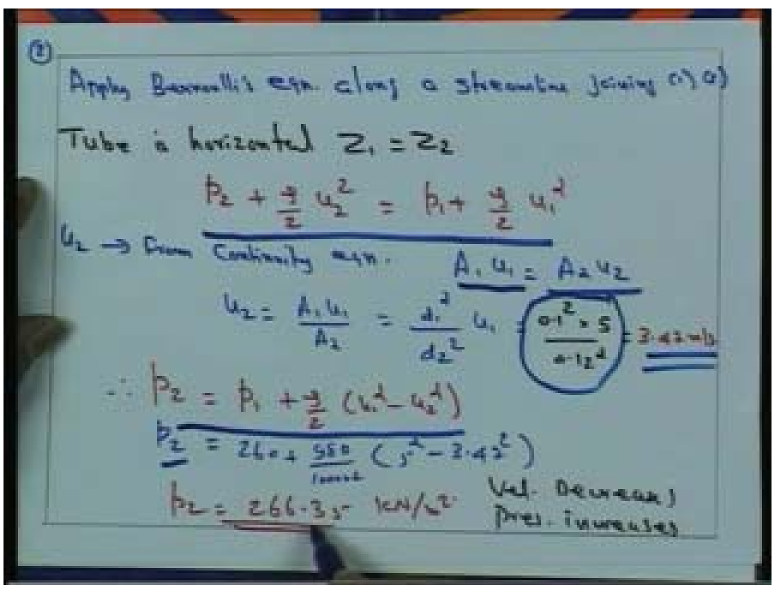

So in this equation all the values u_1 u_2 and p_1 is known. So that we can find P_2 so P_2 is equal to two hundred and sixty six point three five kilo Newton per meter square and we can see that when the velocity is decreased from five meter per second to three point four seven meter per second per pressure is increased from two, two sixty kilo Newton per meter square to two sixty six point three five kilo Newton per meter square.

So this problem shows a simple case of the application of the Bernoulli’s equations.

So very similarly way numbers of problems practical problems can be solved using this Bernoulli’s equation further applications of the Bernoulli’s equation will be discussed later.