Fluid Mechanics

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Lecture – 10
Kinematics of Fluid Flow

Welcome back to the video course on fluid mechanics. In the last lecture of kinematics of fluid flow, we were discussing about the basic potential flows. We have seen the various aspects of uniform flow, source, sink and vortex.

In a vortex, flow in which the streamlines are concentric circles, it is either rotational or irrotational vortex. As mentioned, rotational vortex is forced vortex. For example, the motion of a liquid contained in a tank is rotated about its axis, with angular velocity omega. In the case of irrotational vortex, it is called free vortex. For example, the swirling motion of the water as it drains from a bathtub.

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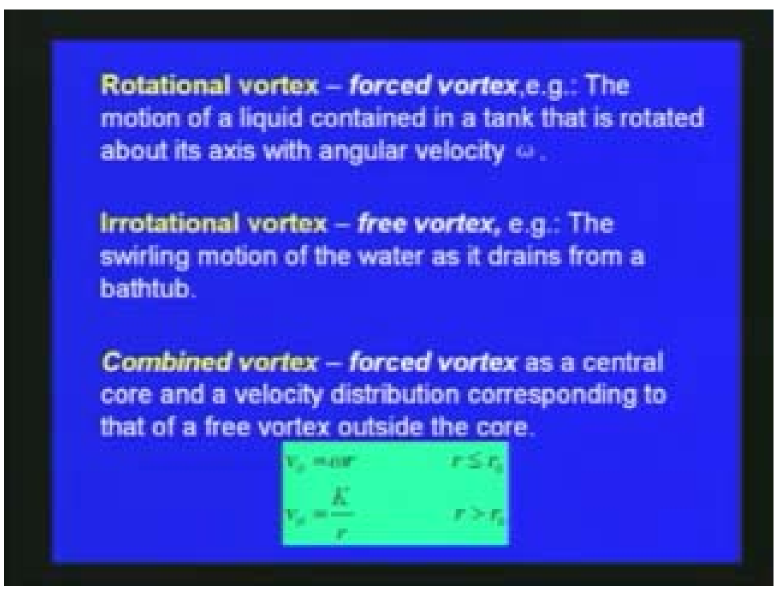

Combined vortex is forced vortex as a central core and a velocity distribution corresponding to that of a free vortex outside the core. Out of this, mainly in potential flow, we will be dealing with irrotational vortex. Irrotational vortex is one of the basic potential flow. In vortex, another important parameter is circulation, which we have already discussed earlier.

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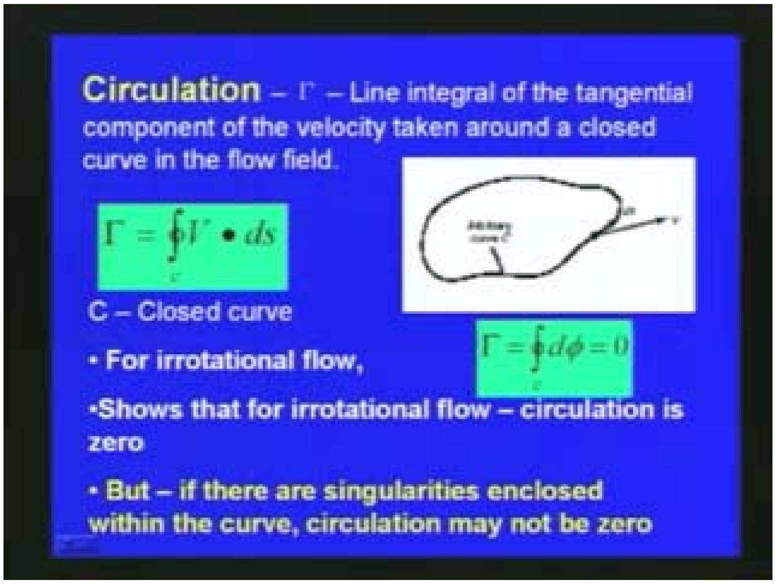

In this slide, you can see that circulation gamma; say capital gamma is a line integral of the tangential component of the velocity, taken around a closed curve in the flow field. You can see that, there is a close curve. The circulation is defined as line integral of the tangential component of the velocity and it is taken around a closed curve. Circulation is described as; gamma is equal to integral V into ds, where c is the close curve, which we have described here. For irrotational flow, gamma is equal to integral d phi is equal to 0, where c is the close curve. It shows that for irrotational flow circulation is 0. We have seen that the vortex can be forced vortex or irrotational vortex.

If there are singularities enclosed within the curve, circulation may not be 0. So, within the curve, if there is a singularity, then we can see that circulation may not be 0. In irrotational vortex also, if there is a singularity within the curve, then there is a possibility that circulation may not be 0. But, generally for irrotational vortex or free vortex, the circulation is equal to 0.

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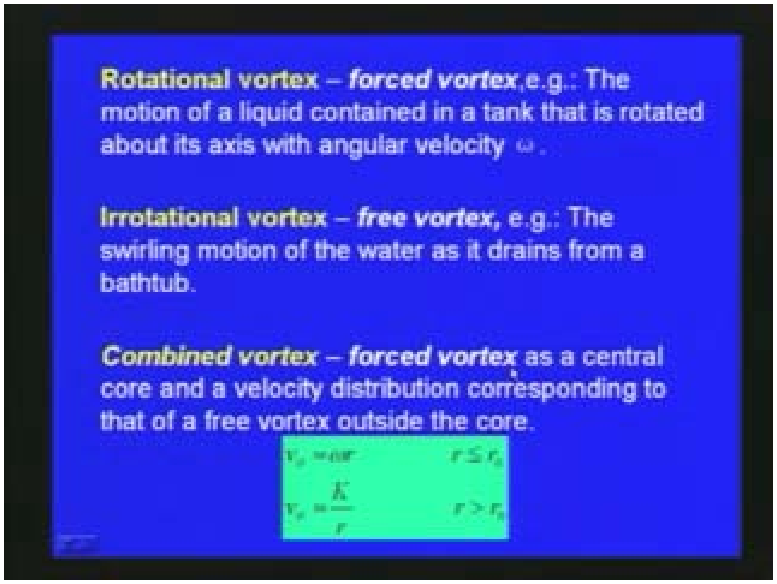

So the circulation is an important parameter which we use as for as rotational vortex or irrotational vortex is concerned. We have also defined the tangential velocity for the vortex as v subscript theta is equal to omega r or v subscript theta is equal to K by r, where r is greater than r_0, as described earlier.

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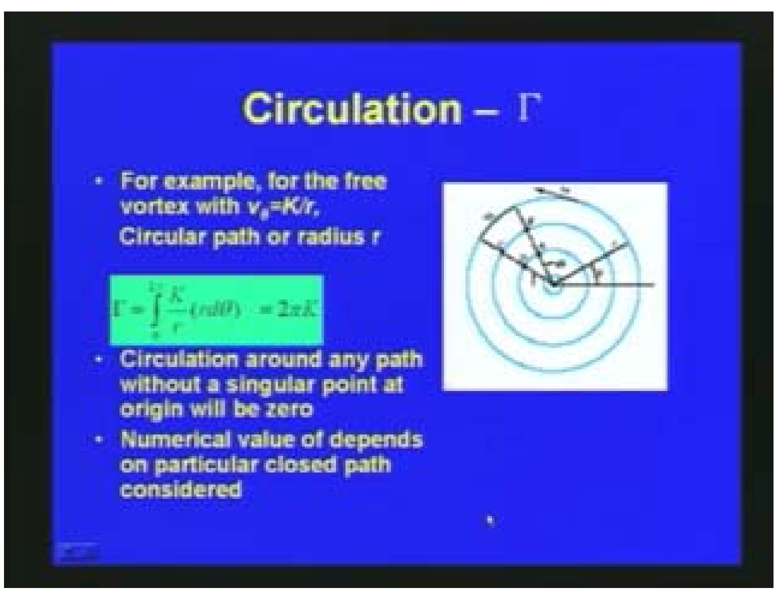

If you consider the circulations here, as you can see this figure, with respect to the velocity v subscript theta and with respect to the radius r, here theta is mentioned. For example, for the free vortex with v subscript theta is equal to K by r, circular path of radius r can be described as, gamma is equal to, integral 0 to 2 phi K by r into rd theta is equal to 2 phi K.

For free vortex, we can see that circulation within this, we can consider singularity inside with respect to the vortex and gamma is equal to 2 phi K. So, circulation around any path without a singular point at origin will be 0. If there is a singular point at origin, then gamma is equal to circulation is equal to 2 phi K. So, the numerical value of the circulation depends up on a particular closed path considered.

Therefore, whether there is a value for the circulation or circulation is 0, depends upon if there is singularity or not, within the closed curve. As shown in this figure, gamma is equal to 2 phi K, otherwise circulation around any path without a singular point at origin will be 0. Circulation is also an important factor which we should consider as for as the vortex flow in potential flow theory. We have to see whether the circulation is equal to 0 or circulation has some value, depending upon whether there is a singular point within the circulation.

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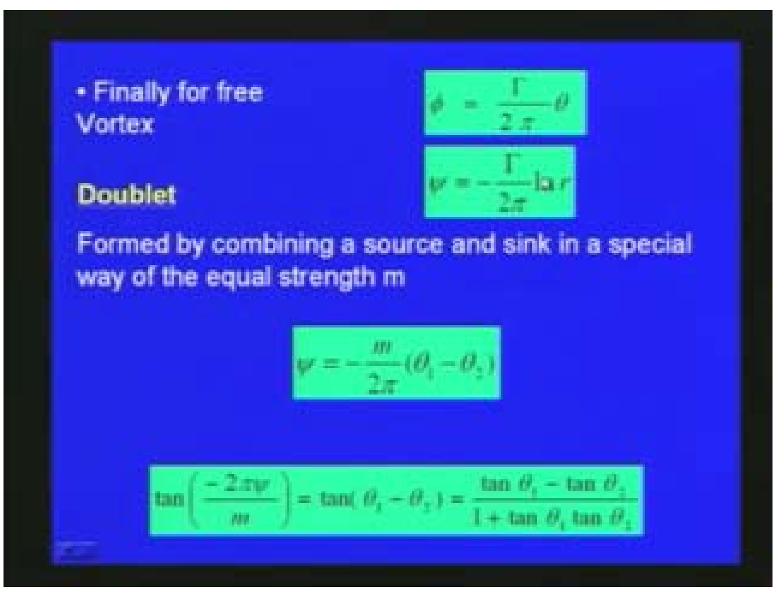

For a free vortex flow with respect to circulation gamma, the potential velocity phi is equal to, gamma divided by 2 phi theta, and psi the stream function, is equal to minus gamma by 2 phi into natural log r; where gamma is the circulation and r is defined and theta defined. We have already seen earlier the general definition of phi and psi, where phi is equal to K theta and psi is equal to minus K natural log with respect to the circulation gamma, we can define the free vortex phi and psi.

Another important basic potential flow is doublet. We have seen the uniform flow, source, sink and vortex, now you will see the elementary potential flow called a doublet. The doublet is formed by combing a source and sink in a special way of equal strength m. If there is source of strength m and there is a sink of strength m, then we can get a doublet, when we combine the source and sink in a particular way. In this slide, you can see how the doublet is formed.

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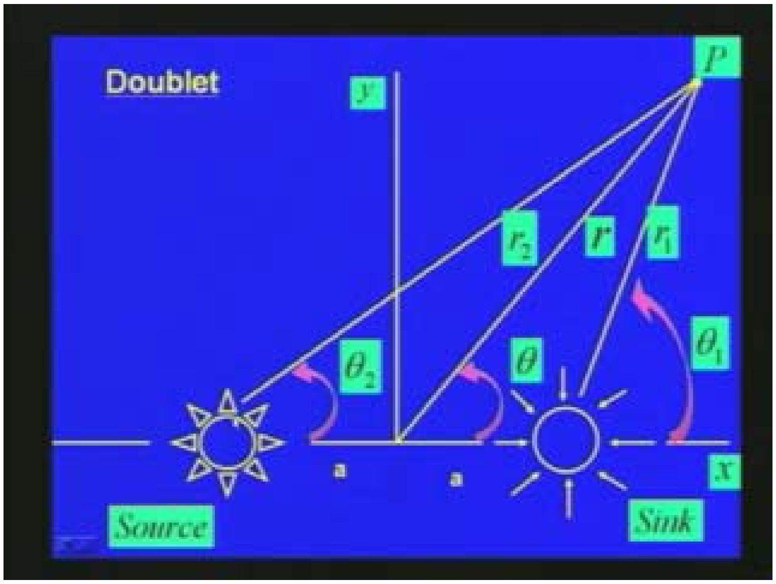

There is a source of strength m and here there is a sink of strength m, and then this is the origin. The source and sink are separated by distance, a on the left hand side and a on right hand side. The r is defined, with respect to point P, r_1 and r_2 is defined, with respect to source and sink; and angle theta one, theta two, and theta is defined. You can see the direction of x-axis and y-axis.

A doublet is formed with respect to source and sink placed at a distance and when the distance tends to 0, that means, the source and sink tends to come to the origin, same point. Further, we will see the different aspects of this doublet. The strength of the source or sink we have already see with respect to the definition, stream function psi is equal to, minus m by 2 phi theta$_1$ minus theta$_2$, theta$_1$ and theta 2 are defined in the figure. From the figure, we can define tan minus 2 phi psi by m, this is equal to tan theta$_1$ minus theta$_2$. This is equal to tan theta$_1$ minus tan theta 2 divided by 1 plus tan theta$_1$ into tan theta$_2$; theta$_1$, theta$_2$, and theta are defined in the figure.

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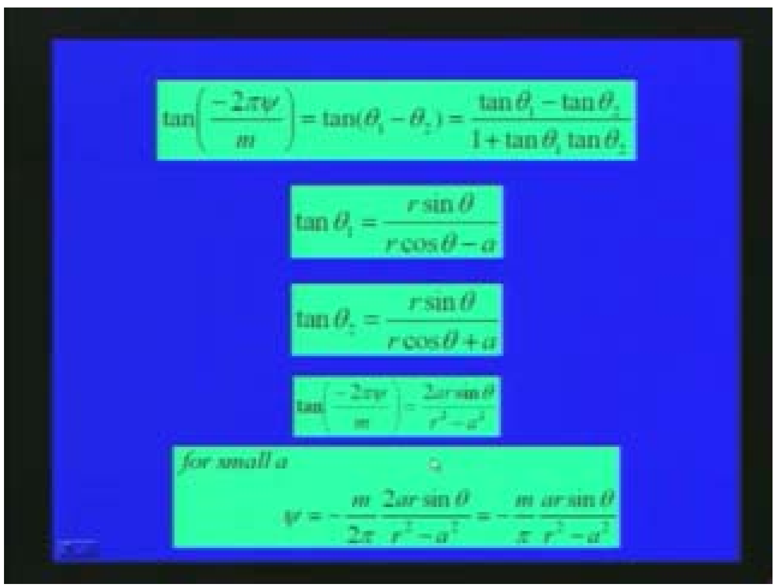

With respect to the previous figure, we can write tan theta$_1$ is equal to r into sine theta divided by r into cos theta minus a; tan theta $_2$ is equal to, r into sine theta divided by r cos theta plus a. These parameters are defined with respect to the figure. This tan minus 2 phi into psi by m, with respect to tan theta$_1$ and tan theta$_2$ defined, we can write this as 2 a r sine theta divided by r square minus a square. This is the definition of the doublet, when a tends to 0.

We can write this a as, psi is equal to minus m divided by 2 phi into 2 a r sine theta divided by r square minus a square. This is equal to minus m by phi into a r sine theta divided by r square minus a square.

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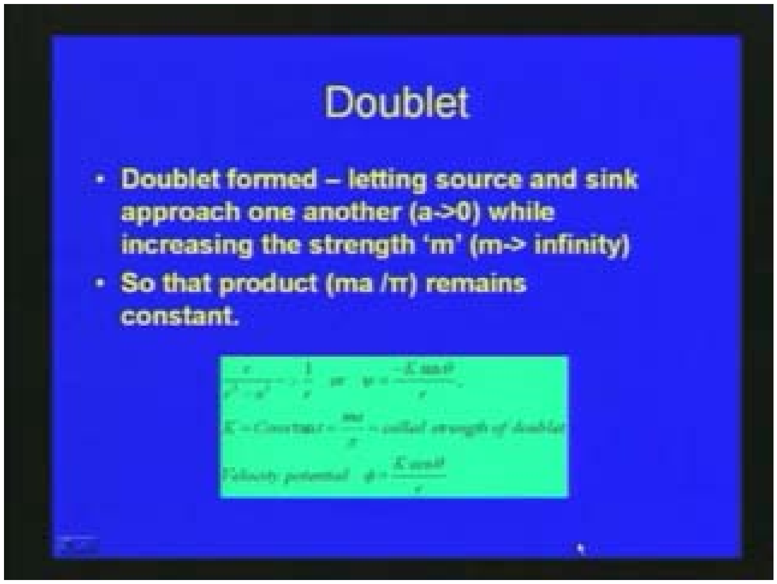

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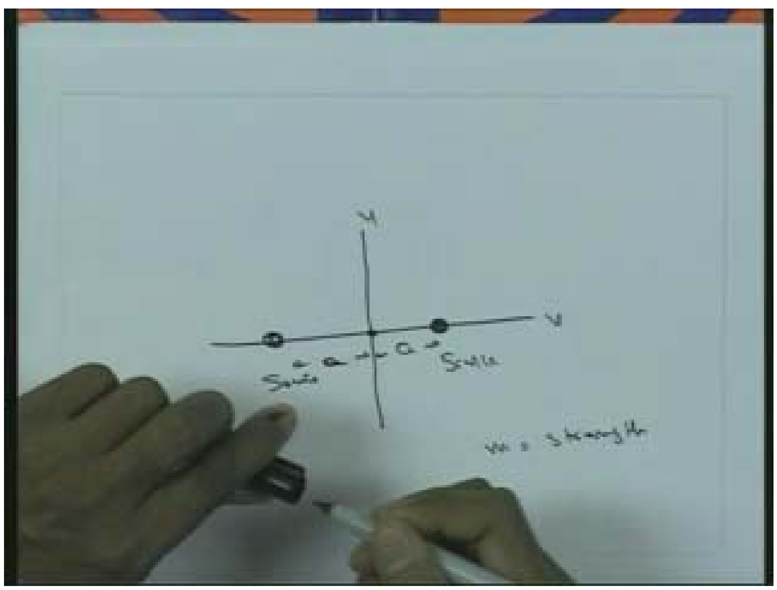

Now, we will say that a doublet is formed by letting the source and sink approach one another. As shown in the figure, if this is the origin, this is y and this is x. There is a sink and here there is a source, initially there is a distance a here on both sides. So when this a tends to 0, letting source and sink approach one another, so that a tends to 0, m is the increasing strength of source and sink, and m tends to infinity, then we say that a doublet is formed. Then we can say that the product m a by phi remains constant. This is obvious from this figure.

From previous slide, when a tends to 0, r by r square minus a square, is tending to 1 by r. Finally, stream functions psi is equal to minus K sine theta by r, where K is the constant, which is equal to m a by pi. This m a by phi is called the strength of doublet. So m, which is defined as a strength of source or sink, for the doublet the strength is the defined as this constant K, which is equal to m a by pi. Finally, velocity potential is phi, is equal to K cos theta by r.

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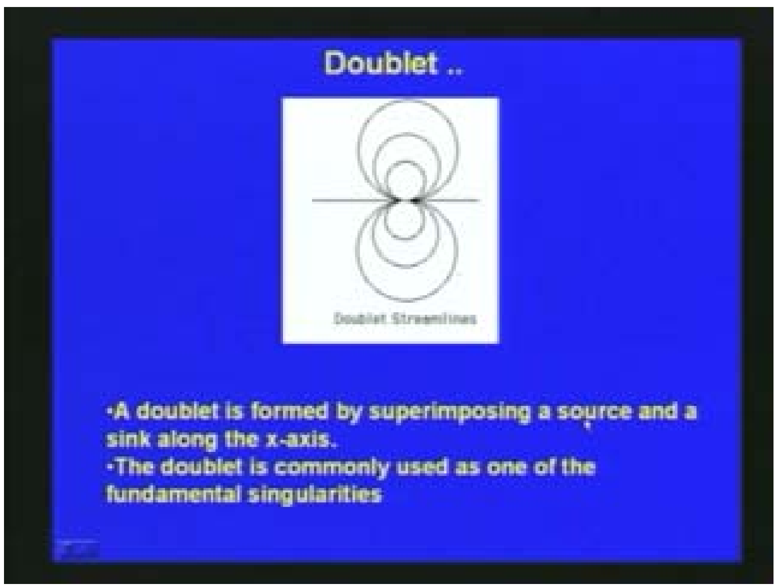

As respect to the previous slide as the source and sink approaches to the origin, you can see the doublet streamlines this are plotted like this. So finally, we can say that doublet is formed, by superimposing the source and a sink along the x-axis. The doublet is commonly used as one of the fundamental singularities. This is the definition of the doublet and the various parameters are discussed.

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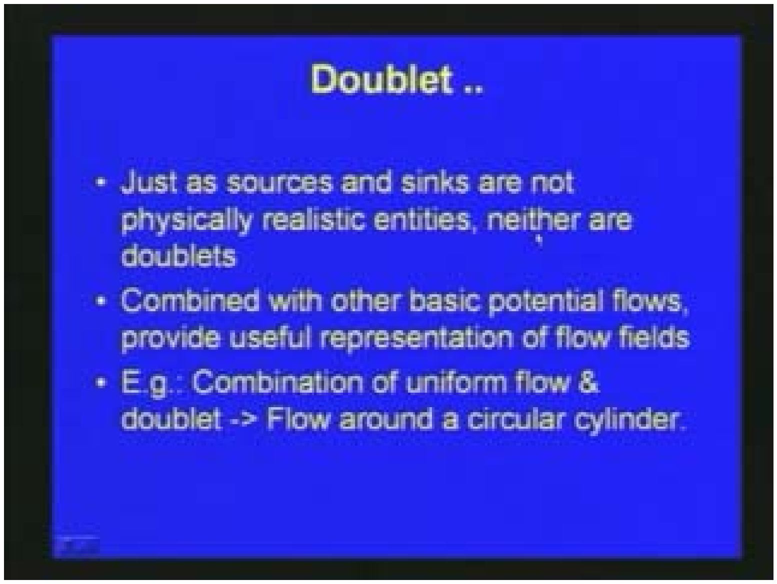

So finally, the doublet is, just as sources or sinks are not physically realistic entities, neither are the doublets. So, we are just assuming this kinds of entities, so that we can define many other complex flows. Combined with other basic potential flows, provide useful representation flow field. After this, we will be discussing about the combination of this basic flow, then you will see how this doublet is useful for further representation of the complex flows. For example, combination of uniform flow and doublet, use the flow around a circular cylinder. Flow round circular cylinder is very important and we have to determine the streamlines, potential lines and various parameters. To define various parameters, we can use this doublet. When a doublet is superimposed with uniform flow, we will get the flow around a circular cylinder.

Even though this source, sink or doublets are not physically realistic entities, we can define these assumed or unrealistic entities, in such a way that finally, this source, sink or doublet can be superimposed with the uniform flow, other flows and finally we will get various other types of complex flows, which is very useful. We can easily define various parameters, for example, flow around a circular cylinder. Finally, we will summarize the elementary flows, as shown in the slide below.

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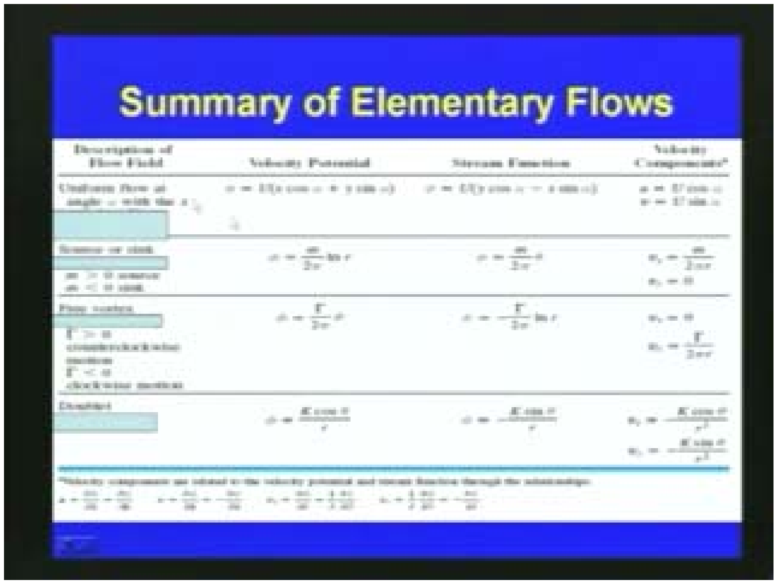

So far, we have discussed, the uniform flow, source or sink, free vortex and then the doublet. We have seen the definitions, for uniform flow, the velocity potential phi is equal to, U into x cos alpha plus y sine alpha in the Cartesian coordinate and psi is equal to U into y cos alpha minus x sine alpha. Velocity components in the x direction as, U into cos alpha and v in the y direction as, U into sine alpha.

Similarly for source or sink, we have defined the velocity potential, phi is equal to m by 2 phi natural log r, psi is equal to m by 2 phi theta, and v_r, the radial velocity is equal to m by 2 phi r and v subscript theta is equal to 0 for source or sink. It is said to be as source, when m is greater than 0 and a sink when m is less than 0.

In free vortex, the velocity potential phi is equal to gamma the circulation by 2 phi into theta, psi is equal to minus gamma 2 phi natural log r and the radial velocity, v_r is equal to 0. v_{theta} is equal to gamma by 2 phi r. For free vortex, the circulation gamma is greater than 0, it is counterclockwise motion and when it is less than 0, that means, when it is negative, we say it is clock wise motion.

Finally, for doublet, the velocity potential phi is equal to K cos theta by r, where K is the strength of the doublet and psi is equal to minus K sine theta by r. The radial velocity, v_r is equal to minus K cos theta r square and the tangential velocity, v_{theta} is equal to minus

K sine theta by r square, the other parameters, u, v in a xy-direction and v_{theta}, with respect to the basic definition, which we have already seen earlier.

We have seen four different types of elementary potential flows, uniform flow, source or sink, free vortex and doublet. Now, we are going to superpose some of these elementary flows, so that we will get some of the complex flows. We will discuss this in the next few slides. Next topic is the superposition of elementary flows.

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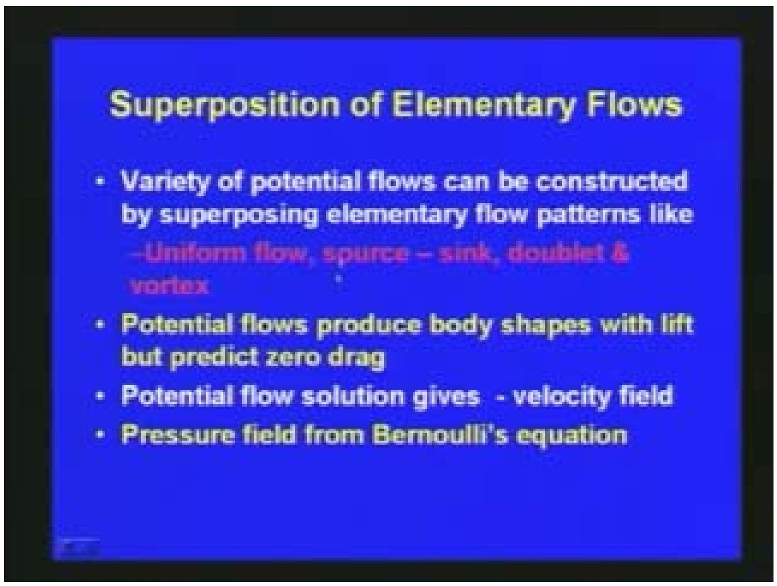

Variety of flows can be constructed by superposing the elementary flow patterns like uniform flow, source – sink, doublet and vortex. Potential flows produce body shapes with lift, but predict 0 drag. This is another disadvantage of this potential flow theory, when we compare with the real fluids. For a potential flow, we are assuming that the viscosity as negligible, so the potential flow are defined accordingly. We can see that the drag is always predicted 0, which is not realistic for real fluid. This is one of the limitations of this potential flow theory. Anyway, we will discuss this further.

Potential flow solution gives; mainly the velocity field and we can determine the pressure from the Bernoulli's equations. We can define many of the complex flows from the basic potential flows of uniform flow, the vortex, source - sink or doublet.

The limitations is because the viscosity is negligible for potential flow, we cannot predict the drag. Mainly, the potential flow solution is used to get the velocity field. After this velocity field is obtained, further we can use the Bernoulli's equation to get the pressure field, for the flow concern.

Now, we will discuss superposed elementary flows of the potential flows. We will define some of the complex flows with respect to the elementary flows. So as far as superposition of the elementary potential flows is concerned, there are two methods of comparing the elementary flows, first one is called the direct method.

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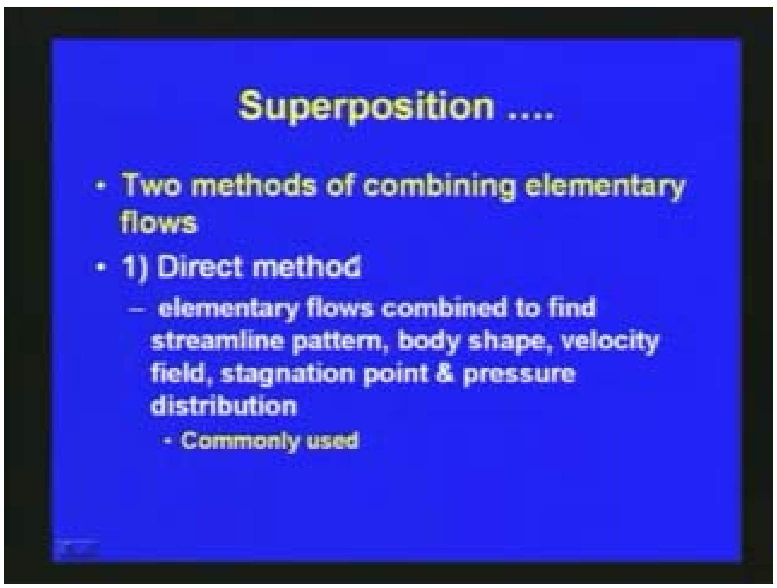

In the direct method, the elementary flows are combined to find the streamline pattern, the body shape, the velocity field, stagnation point and pressure distribution. We are directly superposing the various parameters like stream function and potential function. This direct method is most commonly used. The second method of superposition is called inverse method.

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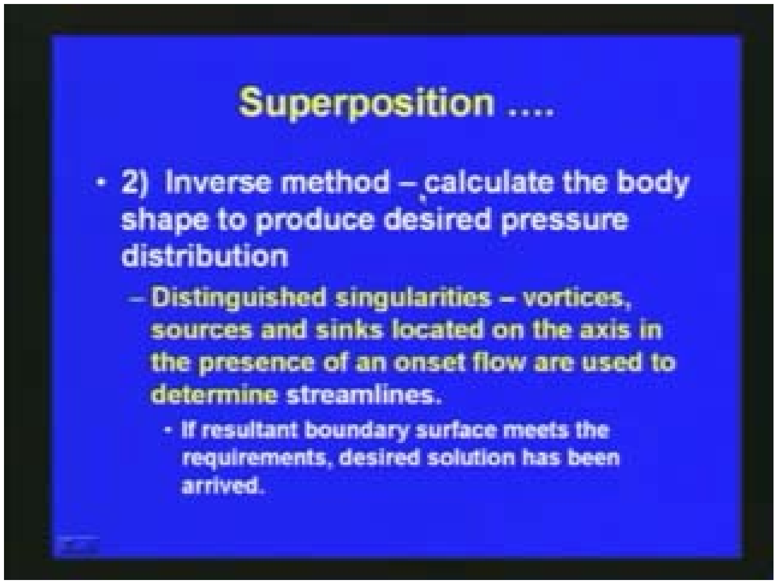

In this inverse method, we will calculate the body shape to produce the desired pressure distribution. Initially, we will try to produce the pressure distribution and then calculate the body shape, then it is an inverse process. The distinguished singularities like vortices, sources and sinks located on the axis in the presence of an onset flow are used to determine the streamlines.

First, we will determine the pressure distribution and then with respect to the singularities like vortices, sources and sink we will determine the streamlines. That is why it is called as inverse method. Generally, in both direct and inverse method, if resultant boundary surface meets the requirements, desired solution has been arrived. So, with respect to this methodologies, we say that the resultant boundary surface meets the requirement of the particular problem, then we say that the desired solution has been arrived.

As far as superposition of the elementary potential flows is concerned, there are two methodologies, one is direct method, second one is the inverse method. Mostly, we will be using the direct method for the two or three complex flows, which we will be discussing here.

In super position process, this elementary flows, which we have defined the uniform flow, source or sink, vortex or doublet, these are acting as building blocks, so that we can put one over the another and finally have the complex flow.

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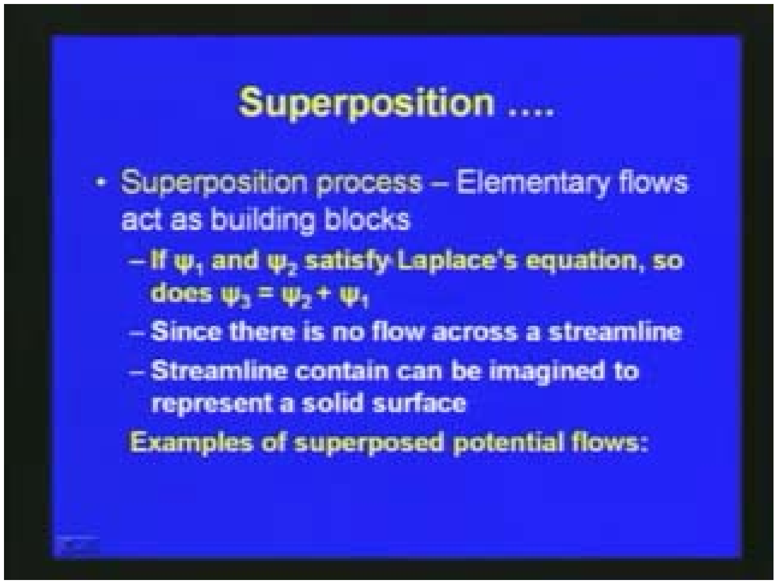

For example, in Laplace's equation, if psi_1 and psi_2, satisfy the Laplace's equation, that means, if we add this two stream function, psi_1 and psi_2, so that you will get another stream functions psi_3. This is the basic principle of this superposition. Since, there is no flow across a streamline and streamlines contain can be imagined to represent a solid surface.

If the Laplace's equation is satisfied with respect to two stream functions, then we can say that its combination, whether it is adding or detecting, also satisfy the Laplace's equations. This is the basic principle, based up on which we are doing this superposition process. Finally, we also use the streamlines flow can be imagined to represent a solid surface, so that there is no flow across a streamline.

Now, we will be discussing the superposition theories. We will be discussing three examples of superposed potential flow; first is a half body, second is Rankine ovals and third is flow surrounding a circular cylinder.

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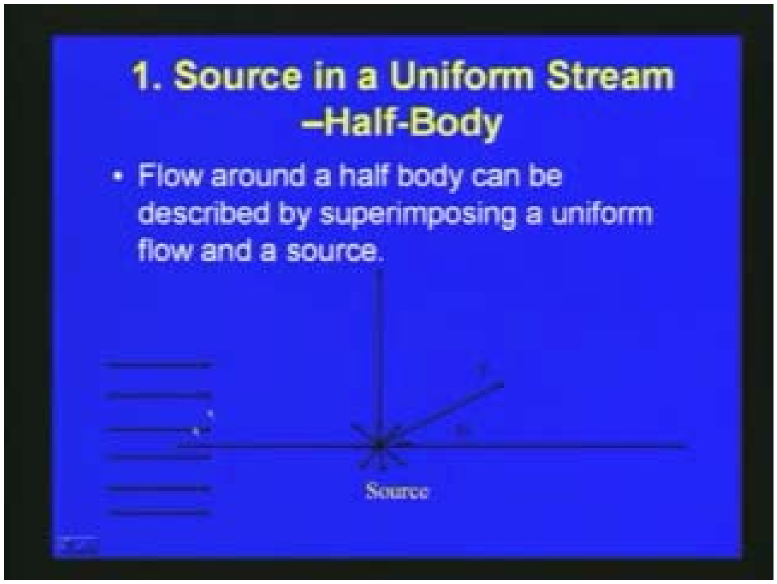

First, we will be discussing the half body. When a source in a uniform stream is superposed, we get a half body. In this figure, you can see that, we have a uniform flow of velocity U_0. The flow is in this direction. We superpose a source, there is a source here, the x-axis is in this direction and y-axis is in this direction, at the origin, we keep a source. At any point, r is defined with respect to theta. Now, we superimpose this uniform flow and source, as shown in this slide here. We can see that a half body is formed.

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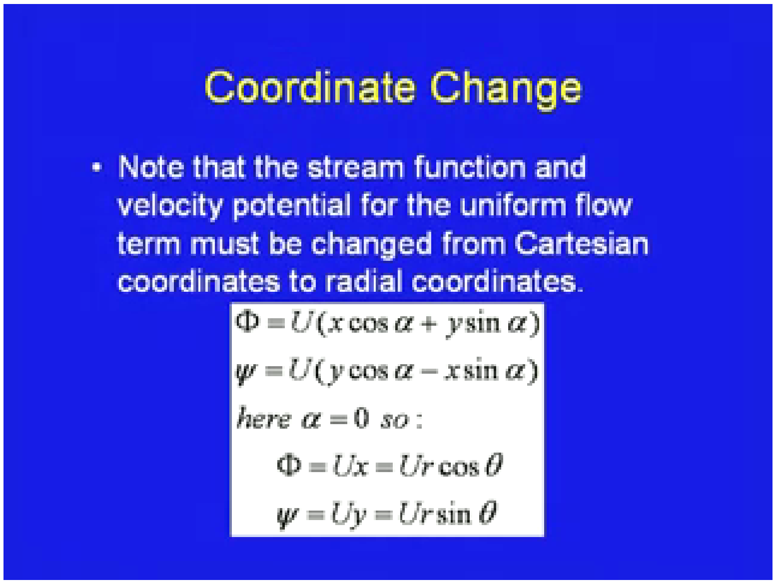

For further calculation purpose, we have to change the coordinate system for the uniform flow. The earlier coordinate was Cartesian coordinate system, now we can convert it to polar coordinate system to r and theta.

The stream function and velocity potential for the uniform flow are changed to the Cartesian coordinate. The Cartesian coordinate phi is equal to, U into x cos alpha plus y sine alpha and psi is equal to U into y cos alpha minus x sine alpha. If alpha is equal to 0, with respect to the previous figure, we can say that, phi is equal to Ux, that is equal to, Ur cos theta and psi is equal to Uy, that is equal to Ur sine theta. So, first we do this coordinate change and then the half body, we can get the potential velocity potential and stream function for the half body.

The half body is obtained by superimposing a source in a uniform flow field as described here in this figure. Since, source is superimposed over uniform flow, from the superimposition principle; we can write the potential function phi is equal to, Ur cos theta plus m by 2 phi natural log r.

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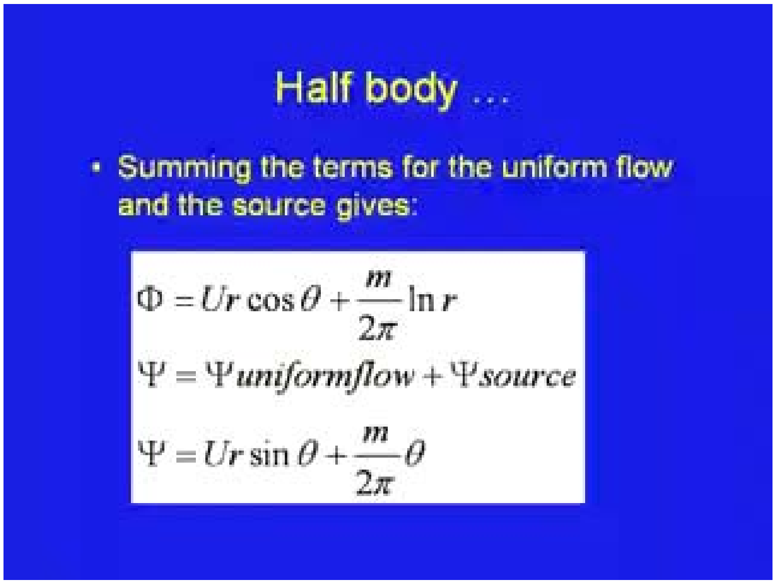

This Ur cos theta is the term from the uniform flow and m by 2 phi natural log r, is from the source. The stream function for the uniform flow, psi for the half body is psi uniform flow plus psi source. So, psi uniform flow is Ur sine theta. So, total psi is equal to Ur sine theta plus m by 2 phi theta, where m by 2 phi theta is the psi for the stream function for the source. Finally, with respect to this superimposition of the source of the uniform flow, we have defined the velocity potential and the stream function. The velocity potential is equal to Ur cos theta plus m by 2 phi natural log r and the stream function is equal to Ur sine theta plus m by 2 phi theta. When we superimpose, you can see uniform flow in the slide.

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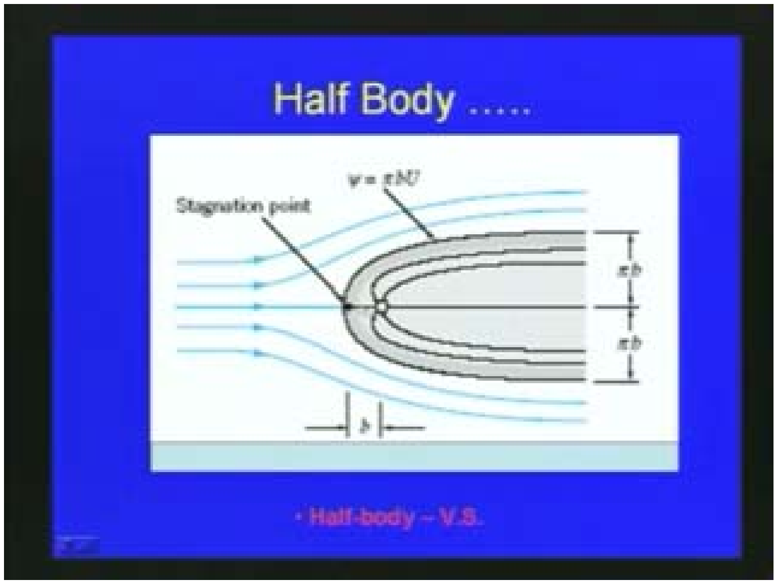

You can see the source and the uniform flow. Due to the effect of the source, the uniform flow streamlines will be deflected both sides like this. Due the source, the streamlines are coming to this point, the source. Finally, we get the half body. It is like a solid surface due to the closed streamline, that is why, it is called a half body.

The point where the velocity is 0, is called the stagnation point. On the surface of the half body, psi is equal to phi b U, where phi b is the width of this half body, in both direction, upward and downward.

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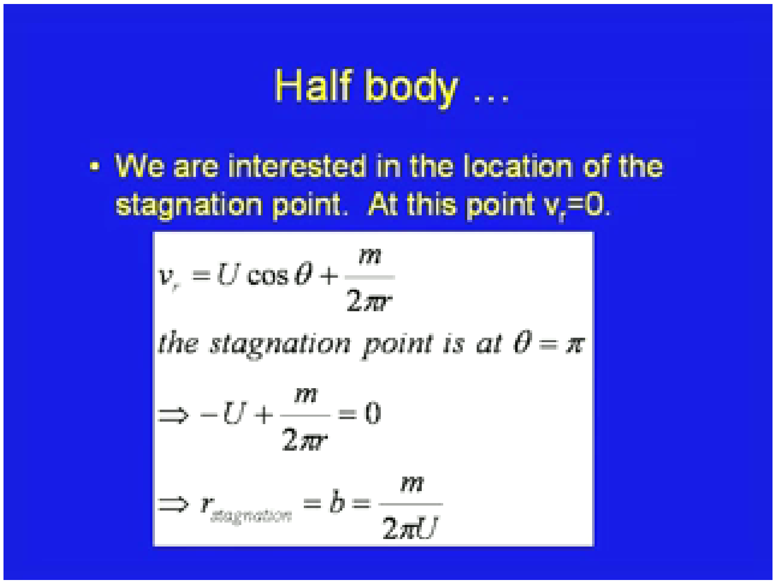

We are mainly interested in the location of the stagnation point. In the previous slide, we can see that there is a stagnation point where the velocity is 0. We already know that the velocity potential for the half body is, phi is equal to Ur cos theta plus m by 2 phi natural log r and psi is Ur sine theta plus m by 2 phi theta. Stagnation point at theta is equal to 2 pi, if the source is at the origin, then theta is equal to pi.

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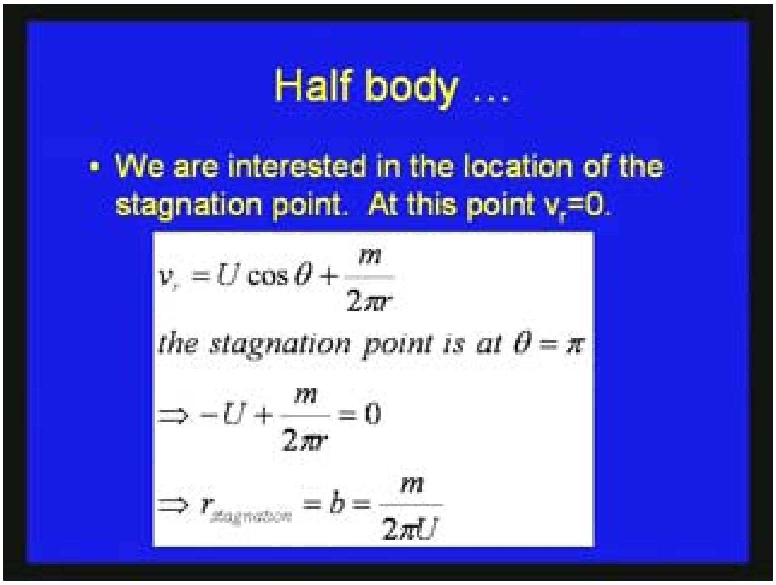

The velocity component for the half body v_r, at the stagnation point is equal to 0, that means, the radial velocity is equal to 0. v_r is defied as U cos theta plus m by 2 phi r.

If you get this to 0, then v_r is equal to U cos theta plus m by 2 phi r is equal to 0. So, theta is equal to pi, this is equal to minus U plus m by 2 phi r is equal to 0. We get, $r_{stagnation}$, which is equal b, is equal to m by 2 phi U. From the slide you can see, b is the stagnation point position. b is equal to m by 2 phi U, where m is the strength of the source, U is the velocity of the uniform flow. This is the position of the stagnation point.

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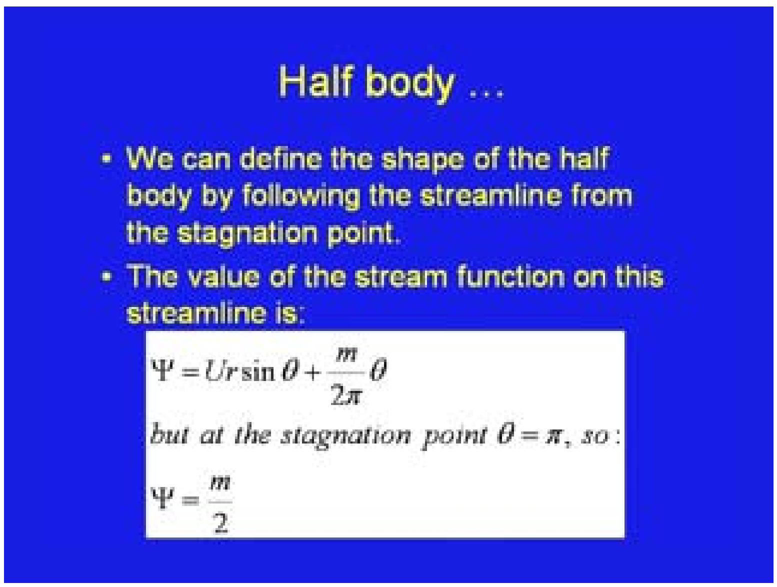

We can define the shape of the half body by following the streamline from the stagnation point. The value of the stream function on this streamline psi is equal to, Ur sine theta plus m by 2 phi theta. At the stagnation point, theta is equal to pi, so we get psi is equal to m by 2. On the surface of the streamline, psi is defined as, phi b U, finally we get, psi is equal to m by 2. Therefore, the value of the stream function on the streamline of the half body on the surface is, psi is equal to m by 2.

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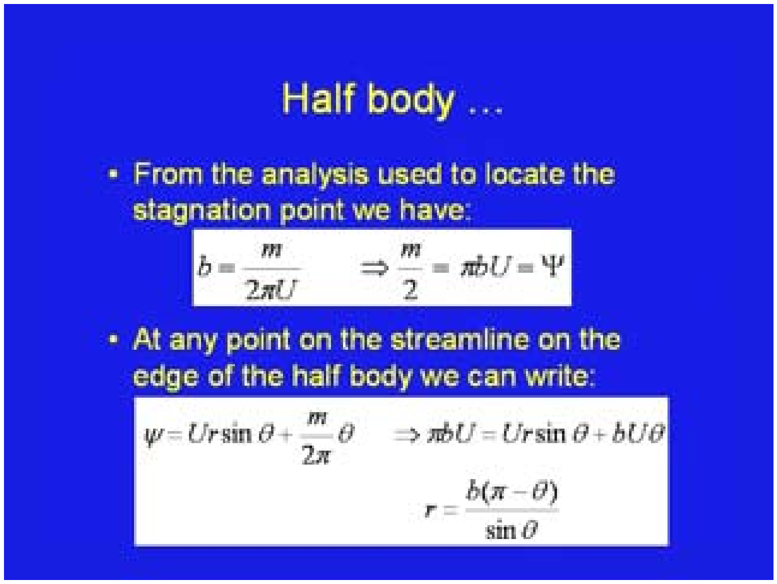

From the analysis used to locate the stagnation point, we have b is equal to m by 2 phi U or m by 2 is equal to phi b U, that is equal to the stream function. At any point on the streamline, on the edge of the half body we can write, psi is equal to Ur sine theta plus m by 2 phi theta and that is psi is defined as phi b U, so phi b U is equal to Ur sine theta plus b U theta. Finally, we can define r is equal to, b into phi minus theta by sine theta.

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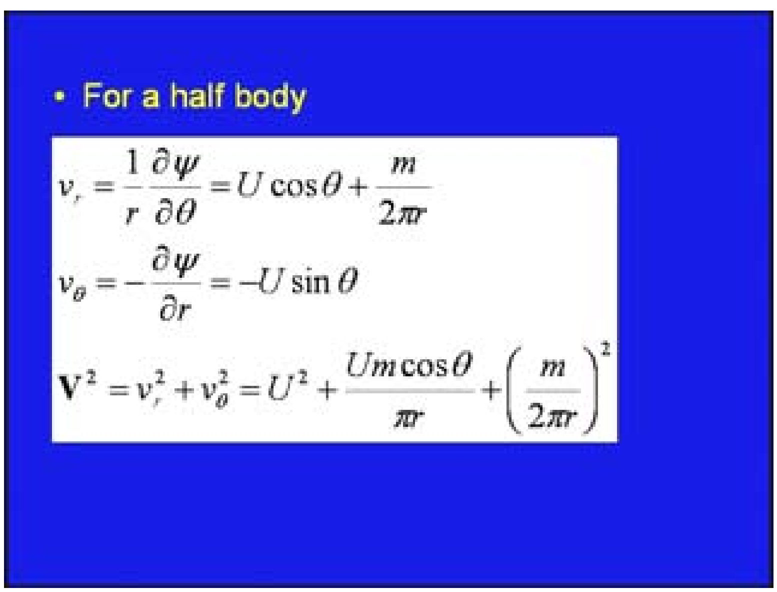

With all this definition, we have defined various parameters. The radial velocity for the half body v_r is equal to, 1 by r del psi by del theta, that is equal to U cos theta plus m by 2 phi r. v subscript theta is equal to minus del psi by del r, is equal to minus U sine theta. The resultant velocity is equal to, V square is equal to v_r square plus v subscript theta square, that is obtained as U square plus Um cos theta by phi r plus m by 2 phi r whole square. This gives all the parameters for the half body.

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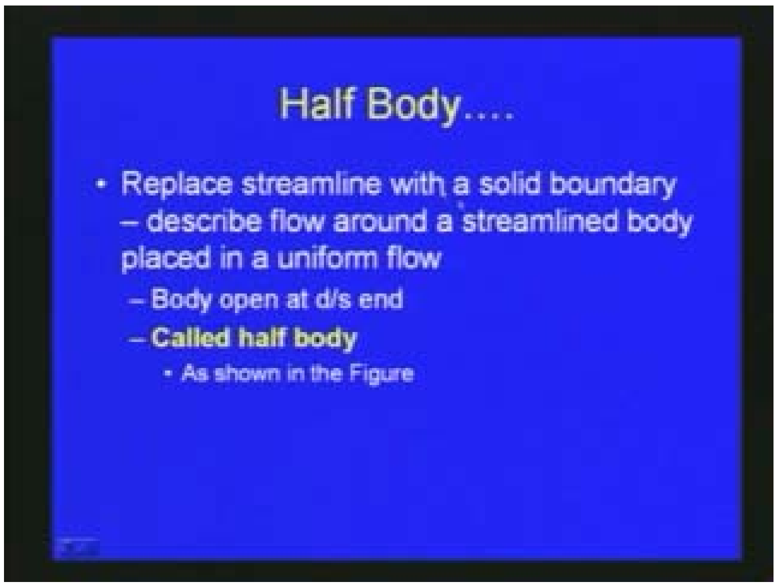

From this discussion, we can say that, if you replace the streamline with a solid boundary, this describes flow around a streamlined body, placed in a uniform flow. The body open at the downstream end is called the half body. This is shown in the figure below.

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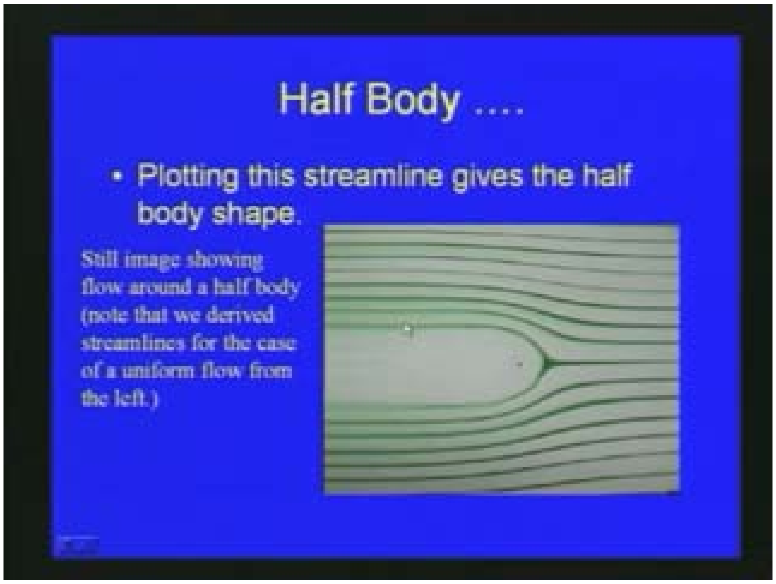

We can plot this streamline with respect to the half body shape. This is the still image showing flow around a half body. So, a half body is obtained, by superposing a source in a uniform flow. We have already defined the velocity potential, the stream function, the radial velocity and the tangential velocity. We have seen the stagnation point and also how all this parameters are derived, with respect to the discussion on this half body.

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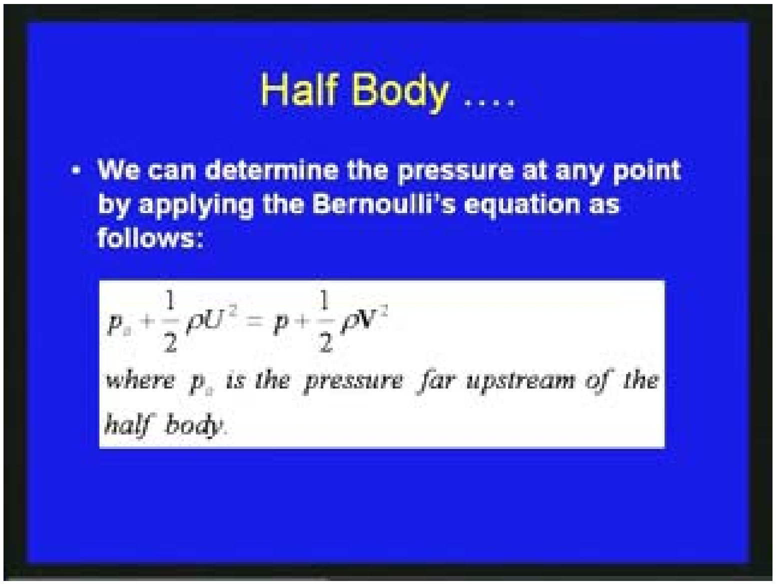

If we want to determine the pressure at any point, we can use the Bernoulli's equation. By using Bernoulli's equation, we can write, p_0 plus half rho U square is equal to, p plus half rho V square; where p_0 is the pressure far upstream of the half body; and U is the uniform flow velocity; v is the velocity at any point, which we have calculated here; rho is the density. From this equation, we can get the pressure at any point. This p gives the pressure at point; all the parameters are already defined, with the respect to the earlier discussion.

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This concept of half body is useful, as it provides useful information about flow around a front part of a streamline body, such as a bridge pier or a strut placed in a uniform stream. The limitation is, velocity tangent to the surface of body is not 0, that is, fluid slips by the boundary. So, the assumption consequence of neglecting the viscosity is that the fluid slips by the boundary and potential flow differs from real fluid. So, do not accurately represent near the boundary.

So, only wherever it is near the boundary, this theories are not valid, but beyond that, we can definitely use potential flow theory and half body concept to find this streamlines on the flow surrounding a bridge pier or a strut placed in a uniform flow.

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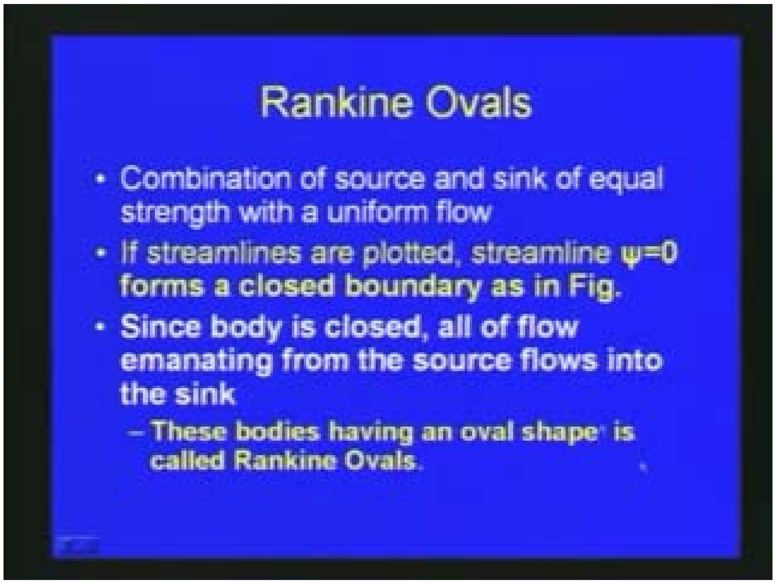

Now, we will see another concept called Rankine ovals. This is also by superposition of two types of elementary potential flows. The Rankine ovals are obtained by combination of a source and sink of equal strength, with a uniform flow. The half body, we obtained by superimposition of a source within a uniform flow, but now we will get the Rankine ovals by superimposing or by combination of a source and sink of equal strength, with a uniform flow.

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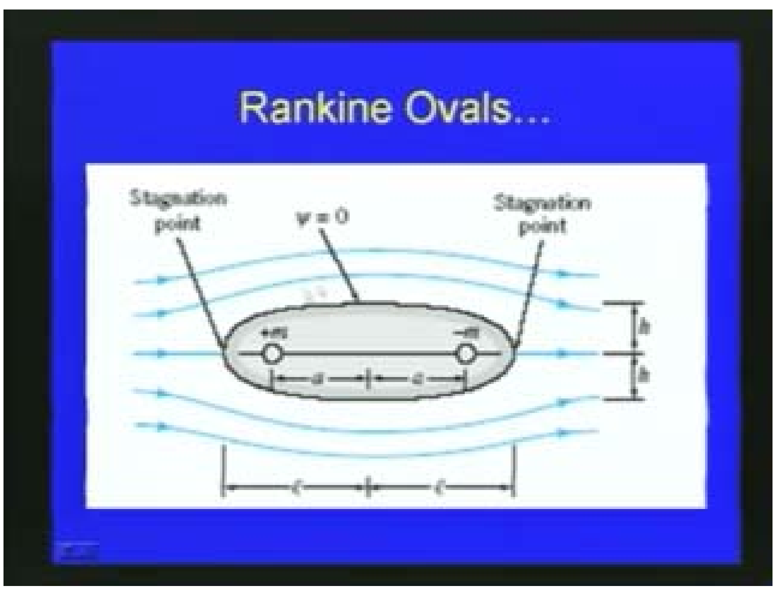

If streamlines are plotted, you can see that, psi is equal to 0; forms a closed boundary as shown in the figure. The stream function is equal to 0. Since body is closed, all of flow emanating from the source, flows into the sink. These bodies having an oval shape is called Rankine ovals.

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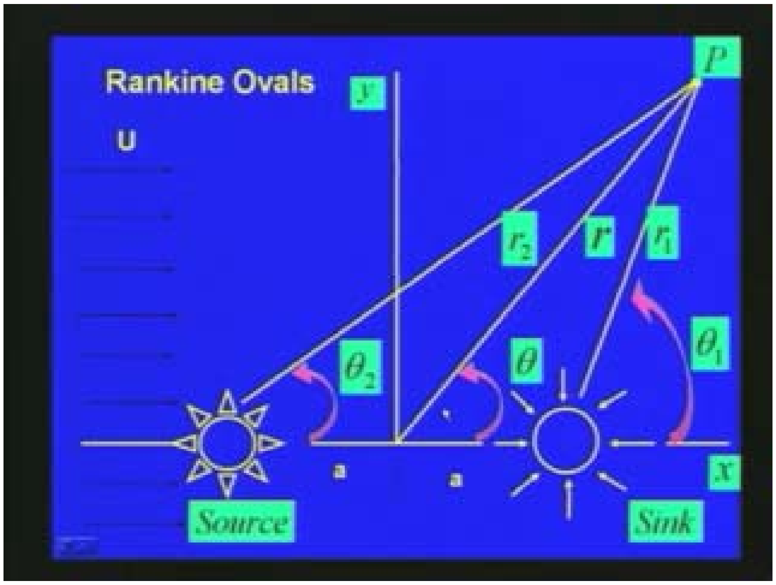

In this figure, you can see that, there is a source here and there is a sink here. There is uniform flow of velocity U. A Rankine oval is obtained by superimposition of this source and sink of the equal strength at distance a. Finally, the shape of the Rankine ovals is obtained.

As we discussed in the case of half body, here also we will discuss the various parameters like, stream function and the velocity potential function.

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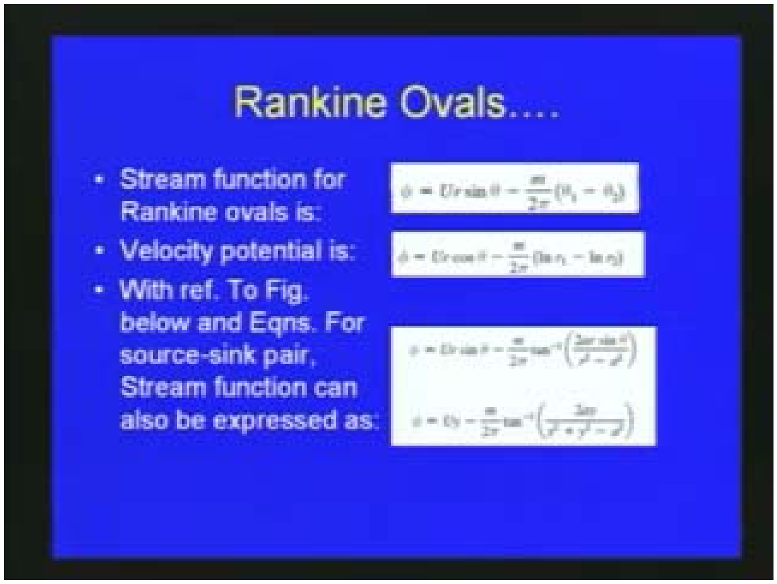

The stream function for Rankine oval can be defined; psi is equal to Ur sine theta, which corresponds to the uniform flow, minus m by 2 phi $theta_1$ minus $theta_2$. Velocity potential is defined as; psi is equal to Ur cos theta minus m by 2 phi natural log r 1 minus natural log r 2.

This Ur cos theta and Ur sine theta represents for the uniform flow and other term represents for the source or sink superimposing in the uniform flow. From the above figure, we are superimposing or we are combing the source and sink of equal strength with uniform flow. For source-sink pair, the stream function can also be expressed as; psi is equal to Ur sine theta minus m by 2 phi tan inverse 2 a r sine theta by r square minus a square. Finally, psi is equal to; if you represent r sine theta as y, Uy minus m by 2 phi tan inverse 2 ay by x square plus y square minus a square. This is with respect to the Cartesian coordinate system and above one is with respect to the polar coordinate system.

Finally, we get the velocity potential and stream function. We get the Rankine ovals as shown in this figure. There are two stagnation points, whereas in the half body there is only one stagnation point. The distance from the centerline from the origin to this stagnation point is l, in both directions. The width of the Rankine oval is defined as h, above and below the x-axis.

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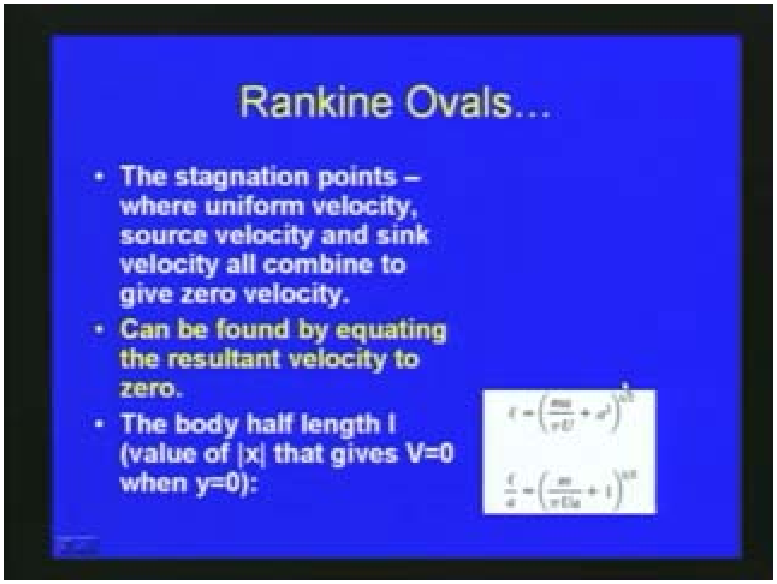

The stagnation point is where the uniform velocity, source velocity and sink velocity all combine to give 0 velocity. This can found by equating the resultant velocity to 0, so that we can find the body half-length l, value of x, that gives v is equal to 0, when y is equal to 0. We you will get, l is equal to ma by phi U plus a square, square root. The ratio of l and a, l is here and a is the distance from origin to the source and sink; is obtained as, m by phi U a plus 1, square root.

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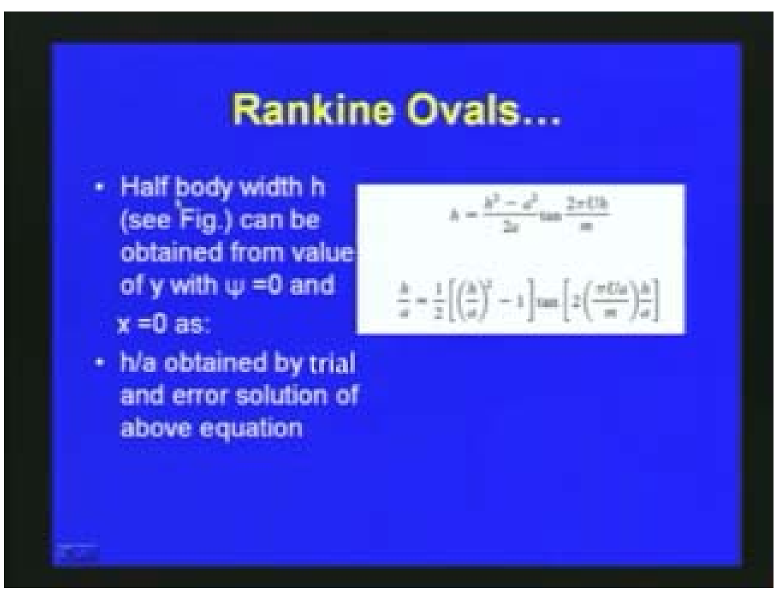

The width of half body can be obtained from the value of y with psi is equal to 0. We can get width of the half body h from the definition of the stream function, when x is equal to 0. So, h is equal to h square minus a square by 2 a tan 2 phi Uh by m, from the previous definitions. We can obtain h by a ratio as, h by a, is equal to half of h by a whole square minus 1 tan of 2 phi Ua by m h by a.

So, h by a, is obtained by trial and error, since you can see that it is a complex equation and direct solution is not possible.

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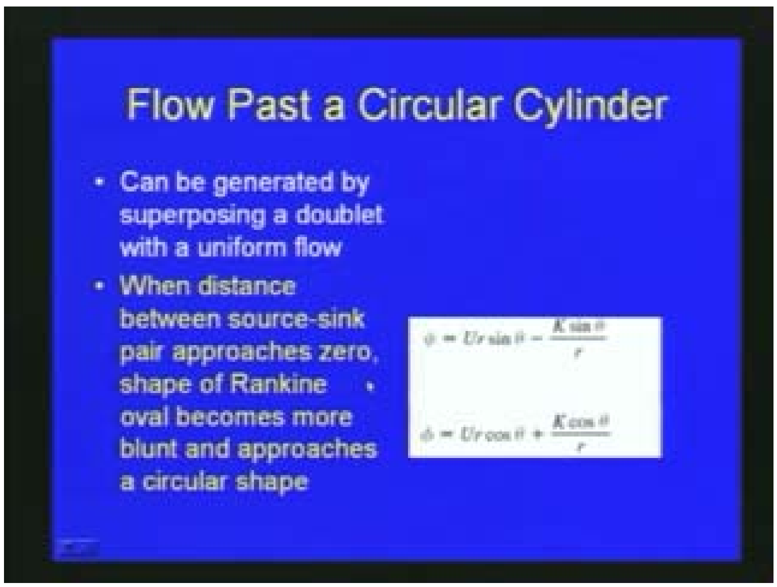

Rankine oval is obtained by, superimposition of a source and sink of equal strength, at distance a, within a uniform flow. It is another way of combination of simple elementary flows to get a complex flow system. The third case, we will discuss the flow past a circular cylinder.

The flow past circular cylinder is generated by superimposing and super proposing doublet with a uniform flow. We have already seen what a doublet is, a source or sink placed at distance a, and is tending to 0 and when the strength of source and sink is increasing.

Flow past a circular cylinder, is one of the important problem, which we will be discussing in fluid mechanics. Many of the flow parameters and fluid flow properties can be defined with this combination of doublet and uniform flow.

When distance between a source-sink pair approaches 0, the shape of Rankine oval becomes more blunt and approaches a circular shape. In Rankine oval, when a reaches to a single point, that is when source and strength approaches, this a is reduced, a blunt body is formed and approaches a circular shape.

We can define the stream function for a flow past a circular cylinder from the definition of the doublet and uniform flow, as Ur sine theta minus K sine theta by r. The velocity potential is defined as, phi is equal to Ur x cos theta plus K cos theta by r.

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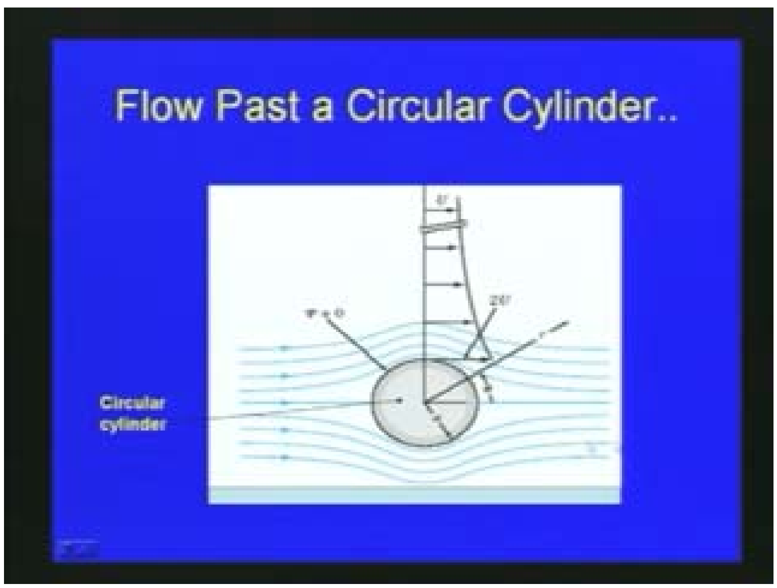

The above figure shows the flow past a circular cylinder. You can see that, this is a combination of doublet and uniform flow. Next, we will define all other properties for a flow past a circular cylinder.

So, for obtaining the stream function to represent flow around a circular cylinder, where psi is equal to constant and r is equal to a, the radius of cylinder. We can write psi is equal to, U minus K by r square into r sine theta. From the previous definition, psi is equal to, Ur sine theta minus k sine theta by r; when we put r is equal to a, we get the stream functions psi is equal to, U minus K by r square into r sine theta.

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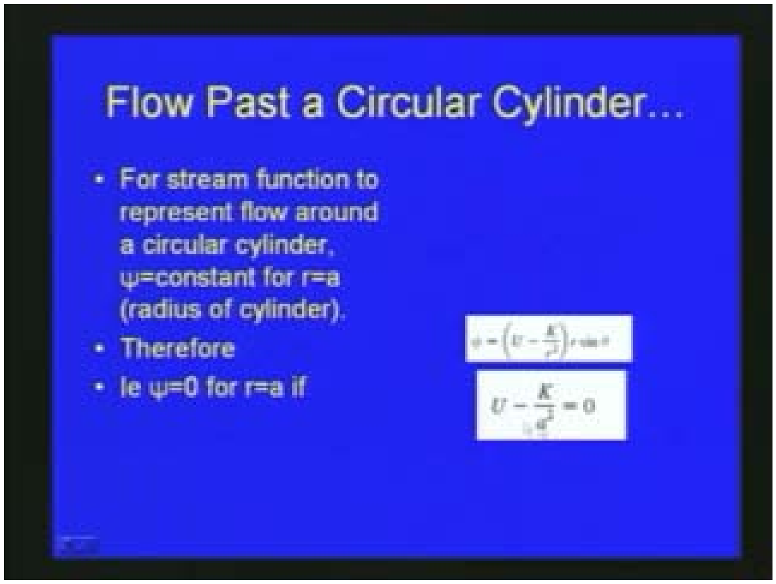

From the figure, you can see, on the surface of the circular cylinder, the stream function can be defined as psi is equal to 0, for r is equal to a; we can define U minus K by a square, is equal to 0.

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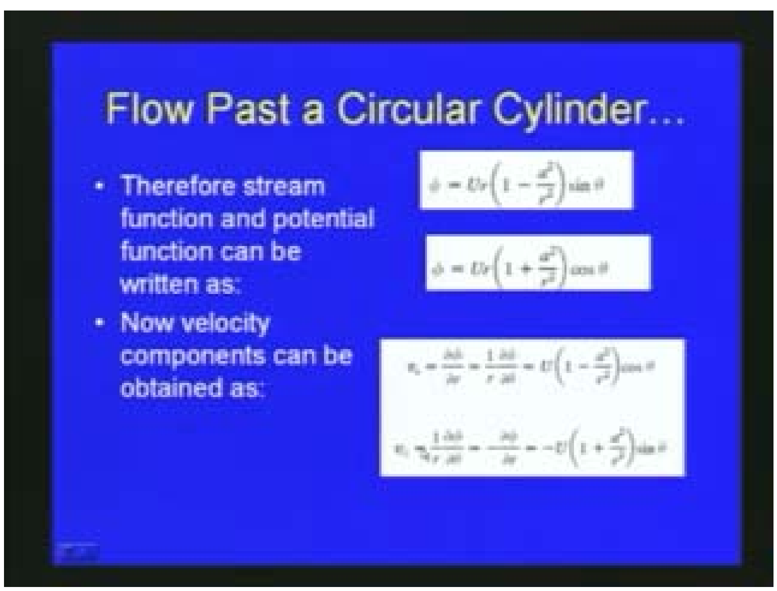

Therefore, the stream function and potential function for flow past a circular cylinder can be written as: psi is equal to Ur into l minus a square by r square sine theta and the velocity potential phi is equal to Ur into 1 plus a square by r square cos theta.

Now, from the phi and psi, we can define the velocity component; the radial velocity and tangential velocity can be defined. As per the definition, the radial velocity v_r is equal to, del phi by del r is equal to 1 by r into del psi by del theta, that is equal to, when we differentiate here, we will get U into 1 minus a square by r square into cos theta.

The tangential velocity is defined as; v subscript theta is equal to, 1 by r del phi by del theta, that is equal to, minus del psi by del r. If you differentiate phi or psi, we get v subscript theta is equal to minus U into 1 plus a square by r square sine theta.

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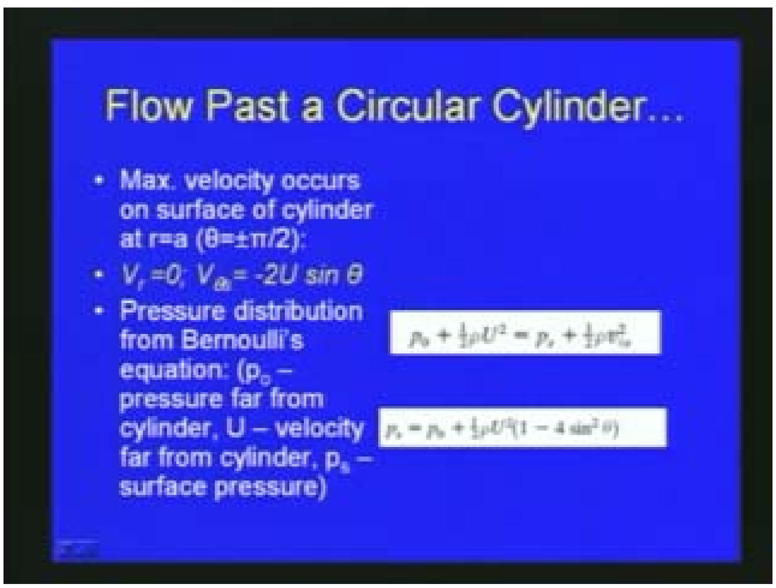

The maximum velocity occurs on a surface of cylinder, at r is equal to a; where theta is equal to plus or minus phi by 2, as shown the figure; the radial velocity v_r is equal to 0 and tangential velocity on the surface of the cylinder, v subscript theta s, is equal to, minus 2U sine theta. U is the uniform flow velocity.

We can obtain the pressure distribution function from the Bernoulli's equation for the flow past circular cylinder. So the Bernoulli's equation is defined here as: p_0 plus half rho

U square is equal to p_s plus half rho v subscript theta s square, where p_0 is the pressure far from the cylinder, U is the velocity far from cylinder, which is uniform flow the velocity and p_s is the surface pressure. The surface pressure of the cylinder is p_s, which is equal to p_0 plus half rho U square into 1 minus 4 sine square theta.

So, we got the expression for the tangential velocity, radial velocity, pressure using the Bernoulli's equation, stream function and velocity potential. We can define appropriately, the flow past a circular cylinder with all the parameters.

Further, we will discuss some more aspects about this flow past circular cylinder and then we will solve few examples for the superposition of flows, in the next lecture, before going to the fluid dynamics.