

Structural Dynamics
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Lecture – 8
Transmissibility and Base Isolation

Good morning, we have been looking at the response of a single degree of freedom system to harmonic loading. We have seen you know how to look at the equilibrium? We have looked at what are the steady state forces then I started looking at transmissibility and it is find that there are two ways of looking at transmissibility. One is where a force from a machine is transferred to the floor and what we want to do is, keep the floor forces as low as possible relative to the floor amplitude floor force amplitude to a minimum as relative to the amplitude of the excitation force.

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Today what we are going to look at is, we are going to look at the same problem which we call as... And we will look at the specific problem of the base isolation. So now, the key point here to note is that you know when you look at force transmissibility, what kind of values did we get?

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The image shows handwritten notes on a whiteboard. At the top, the transmissibility ratio is given as:

$$TR = \frac{f_{T0}}{f_0} = \left[\frac{1 + (2\zeta\beta)^2}{(1 - \beta^2)^2 + (2\zeta\beta)^2} \right]^{1/2}$$

Below the equation, there is a diagram of a mass-spring-damper system. A box labeled \ddot{u}^t is shown above a horizontal line representing the ground. Below the ground line, the ground acceleration is given as $\ddot{u}_g = \ddot{u}_{g0} \sin \omega t$. To the left of the diagram, there is a box containing the ratio $\frac{\ddot{u}^t}{\ddot{u}_{g0}}$.

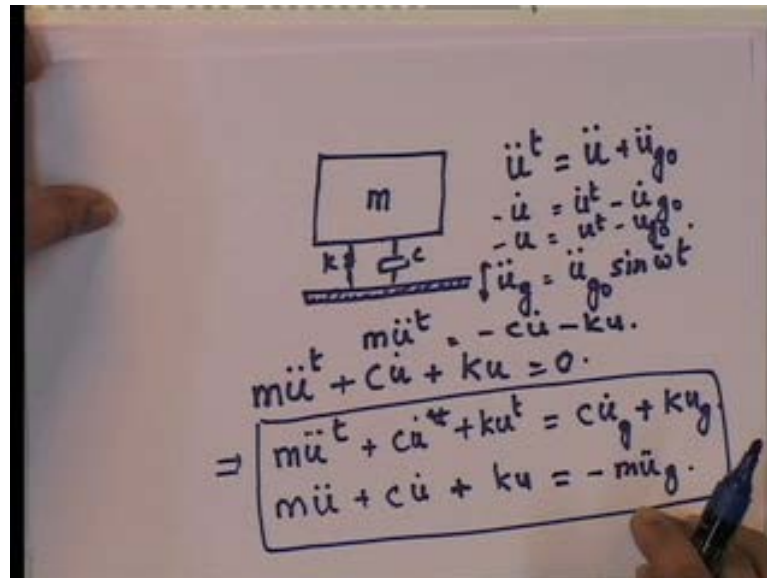
So let us look at it. We got the transmissibility ratio which was given as f_t the force transmitted to the floor upon the amplitude of the force and this was equal to that was what we derived for the transmission of the force and today what we going to looking at is, we are going to be looking at the base isolation problem. What was the base isolation problem? The base isolation problem was the following where you have sensitive equipment connected through its foundation to a floor which is and automatically because of this, this starts vibrating and what we want to do is, the total acceleration so the total acceleration, the amplitude of the total acceleration.

So I will call that as \ddot{u}_g divided by \ddot{u}_{g0} . This value is also another kind of transmissibility. So, this value we want to keep it to a minimum. So let us see how to solve that particular problem, how to write down the equations of motion and for this, I will start off by looking at the acceleration transmission by looking at this particular.

So we have a rigid mass. It is being subjected to base excitation and we will call this excitation as a sinusoidal function. It is a harmonic function, I mean later on we will see that. It does not really matter whether it is a harmonic function or not. We can still look at it, but right now let us look at it as this thing. So, if you look at this, now this is going to be subjected to an acceleration and this acceleration is going to be equal to given by $m \ddot{u} + c \dot{u} + k u = m \ddot{u}_g$. So $c \dot{u}$ which is the c mass into the

acceleration of this mass into the c of the relative velocity between the mass and the base.

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So essentially what we have is, the fact that the total acceleration is equal to the relative acceleration plus the ground acceleration. If you look at it, if the ground was steady and we looked at this that would be the relative acceleration of this system. But since this is moving also, the mass is being subjected to not only it is relative acceleration it is also being subjected to the acceleration of the base.

So, the mass does not know that the base is shifting. The mass only sees itself moving. What happens to the spring and the dashpot is that they only see the relative movement. So in other words, \dot{u} is actually velocity minus this and u is equal to u total minus u_g . So in other words, these that the spring and the dashpot see the relative motion. The mass; however, sees the total motion and therefore, if you look at it, this is equal to this plus this and there is no excitation on the mass. So, this is equal to 0.

Now the question then becomes is that, how do you want to solve this problem? Now there are various ways of solving this problem. Later on, we will see that we are really interested in the relative motion purely because we are going to be interested in the force in the spring. Right now what are we interested in?

We are interested in reducing the total acceleration that the mass sees. So therefore, it is much easier to write the equation in terms of the total you know response rather than the relative response. So if we write it in terms of relative, I mean and putting this in that u of t is equal to this into this. What we get is substituting this and this in here. What we get is $m \ddot{u} + c \dot{u} + k u = c \dot{u}_g + k u_g$. So this is the solution that we get and if you look at it in other words, we can also say the following. We can look at it in this form, this problem can also be written as and note that \ddot{u} is $\ddot{u} + \ddot{u}_g$. So this becomes $\ddot{u} + \ddot{u}_g$. So if I take it on the other side, this becomes both these equations represent the motion of this.

The only thing is that, this is in terms of total displacements and these are in terms of relative motion, total motion relative motion. Now the question then becomes is which is important since this it sits in this particular case, we are looking at specifically this problem of looking at \ddot{u} . Therefore, we are going to look at this problem. Now; however, what I am going to do is I am going to look at the solution of this equation and then say that look $m \ddot{u}$ total is equal to minus $c \dot{u}$ minus $k u$.

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The image shows a whiteboard with the following handwritten equations:

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g \sin \bar{\omega}t$$

$$u(t) = \frac{-m\ddot{u}_g D}{k} \sin(\bar{\omega}t - \theta)$$

$$\dot{u}(t) = \frac{-m\ddot{u}_g \bar{\omega} D}{k} \cos(\bar{\omega}t - \theta)$$

$$m\ddot{u} = \frac{m\ddot{u}_g c \bar{\omega} D}{k} \cos(\bar{\omega}t - \theta) + m\ddot{u}_g D \sin(\bar{\omega}t - \theta)$$

That is the way I am going to solve. So I am going to not going to solve this equation. I am going to say that look I am going to solve this equation. Note that this become something like this is the forcing function. So if I have this as the forcing function, what

is u of t ? Well, it is very simple. This is a harmonic response. So therefore, harmonic response we can always find it out and look note that this is a harmonic load, so what is the response? Let us look at it.

The equation becomes $m \ddot{u} + c \dot{u} + k u$ is equal to $-\sin \omega t$. So now, interestingly what is this problem looks like? This problem looks like, this is the amplitude of the force and this is the single degree of freedom subjected to a harmonic motion. What is the response to it? We have already know of course, only the steady state response. We are always looking at steady state response.

Understand that I am done with boundary conditions and the transient way back. Right now, we always look at a situation, where the load or the vibration has lasted for a long time and we are looking at the response at this particular instant of time. This is how it normally is in real life. Because you know when you look at sinusoidal motion, sinusoidal motion essentially comes from machinery and machinery when they start, it is a very different kind of a situation. It is when they reach the full motion. That is you know that is when we are interested in and that is like a steady state because when it started, what is the initial condition? That does not come into the picture at all in this particular case. So, when I say u of t , I am always really looking at the steady state without explicitly stating its steady state.

So, what is the response then? Well, it is f_0 . This is f_0 upon k into d into $\sin \omega t - \theta$ where we know what θ is? We know what d is? I do not want to delay by the point. So this is u of t . So therefore, let me look at \dot{u} of t . \dot{u} of t is $-\sin \omega t - \theta$ upon k into d cosine $\omega t - \theta$. So now, now let us look at this. So in other words, we have got \dot{u} and u and all we have to find out is this. So therefore, $m \ddot{u}$ becomes equal to $-c \dot{u}$.

So this is $-\sin \omega t - \theta$ so it becomes essentially $m \ddot{u} + c \dot{u} + k u = -\sin \omega t - \theta$. Then $-\sin \omega t - \theta$ so this will be $-\sin \omega t - \theta$ so this becomes $-\sin \omega t - \theta$ again plus $m k$ and k cancel each other out. So this becomes $-\sin \omega t - \theta$ into so now, we are going to divide throughout by m . If we divide throughout by m , what happens? We get the time history of our acceleration, total acceleration.

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The image shows a whiteboard with handwritten mathematical equations. The first equation is $\ddot{u}^t = \ddot{u}_{g0} \left[\frac{c\omega}{k} \right] D \cos(\omega t - \theta) + \ddot{u}_{g0} D \sin(\omega t - \theta)$, with a note 'RFB' above the $\frac{c\omega}{k}$ term. The second equation is $\ddot{u}_0^t = \ddot{u}_{g0} D [1 + (2\xi\beta)^2]^{1/2}$. The third equation is $TR = \frac{\ddot{u}_0^t}{\ddot{u}_{g0}} = \left[\frac{1 + (2\xi A)^2}{(1 - \beta^2)^2 + (2\xi A)^2} \right]^{1/2} = \frac{p}{f_0}$.

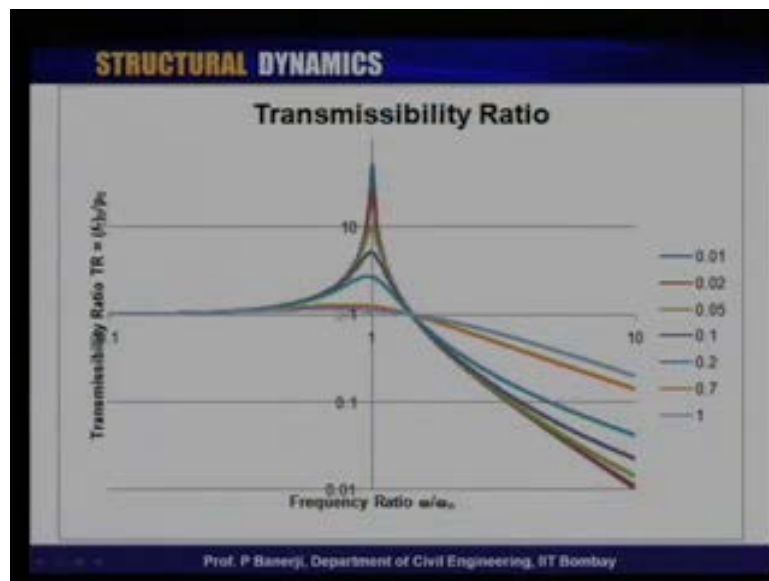
So the total time is t turns out to be equal to u double dot total is equal to $u_{g0} c \omega$ upon k into $D \cos(\omega t - \theta)$ plus u double dot $g_0 D \sin(\omega t - \theta)$. Now understand that, we are not interested in the time domain of the response. We are interested in the amplitude. So what is the amplitude? Note that this is a sin function. This is a cosine function. So therefore, the amplitude becomes the following and note what is this equal to? we know what this is equal to? this is equal to $2 \xi \beta$. So I am going to take $u_{g0} D$ outside. So, I got this into D and inside I have. So therefore, 1 squared plus $2 \xi \beta$ squared square root. This squared plus this squared under root. So, since I take u_{g0} outside all I have is inside is 1 squared plus $2 \xi \beta$ the whole squared square root.

So therefore, if you look at this, the base isolation parameter then becomes and what is D ? D is square root of one. So, I am going to rewrite this. So what is this become? 1 plus $2 \xi \beta$ the whole squared upon 1 minus β squared plus that is the so under square. So you see I have multiplied this with D . D is one upon square root of 1 minus β squared the whole square plus $2 \xi \beta$ square. So I just put it inside the square root. This is what I get and note look at this. What is this? This is the same as the transmissibility ratio which was f_{t0} upon f_0 . Two totally dissimilar problems, absolutely dissimilar problems. One is a force transmissibility problem, the other is transmission of base acceleration to the equipment acceleration. So this is a base isolation problem. We are trying to isolate the base vibration transferring on to the equipment or whatever any other

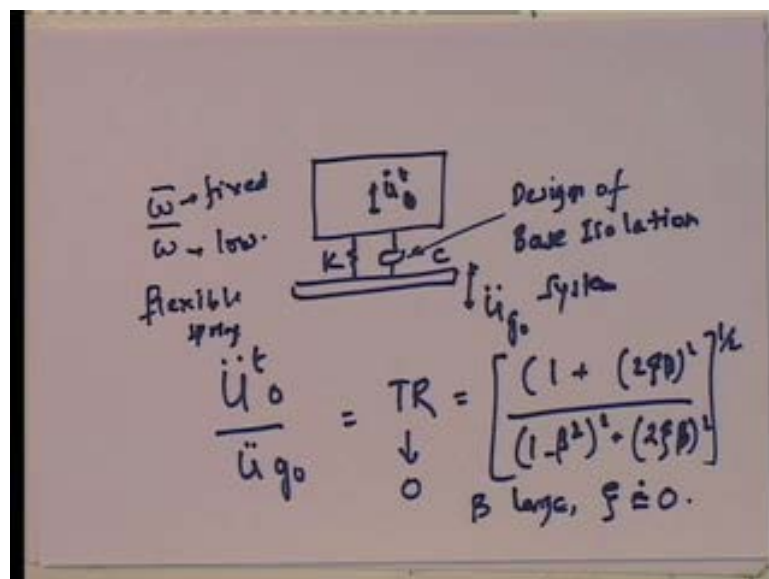
I am calling it equipment. If I want anything else and the other one is essentially transmission of a force from a machinery to the base.

So one is force transmissibility, other is acceleration transmissibility. But note that mathematically both give us the same transmissibility ratio which is what is given in here which I had looked at yesterday in terms of force transmissibility and today the same equation again is valid. So, again if I want to look at this situation where I want to isolate?

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I have sensitive equipment and it is connected through a foundation to the floor and now the whole problem is designing design of the base isolation system so that, this u_g 0 amplitude and this u total amplitude in other words, this acceleration that this base is being subjected to the base is isolated the equipment base is isolated from the floor.

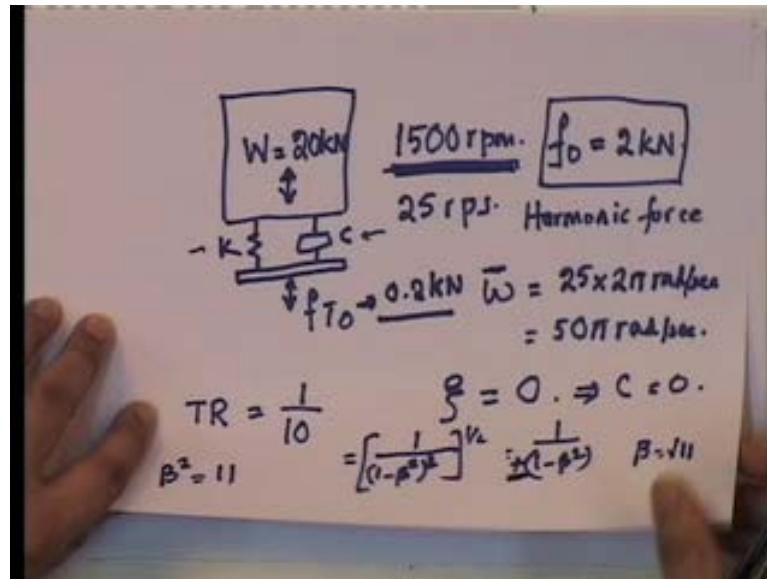
So this is the base isolation problem and essentially what we are saying is this is the same as the transmissibility ratio which is $1 + 2 \xi \beta$ upon $1 - \beta^2 + 2 \xi \beta$ the whole square under square root. So this is the design of the base isolation. So I have to design k and see such that the transmissibility ratio tends to 0. That is my design problem. Let us look at this thing. Again where do you get transmissibility ratio 0. You get it, when β is very large and ξ equal to 0, because the lower the ξ the faster it goes to 0. So therefore, again the same thing and what is β equal to 0 imply? Let us look at the same problem. ω bar is fixed. So β large means low ω . Low ω means flexible spring.

You see the force transmissibility problem and the base isolation problem are identical and therefore, both of them essentially are looking at, if you want to design an ideal force transmission to be 0 or base to be completely isolated in other words, acceleration transmitted from the ground to the equipment or force from the equipment to the ground. Both of them essentially require a flexible system. Again you know again the same thing. Here is the equipment. The base is able to move. The base is moving. If the base is moving, if you have a flexible thing, this does not feel it. Think I mean, think from your personal experience.

If you are driving a Maruti car on a very bumpy road and on the other hand, you have a Mercedes Benz. Where do you feel more comfortable? You feel more comfortable in a Mercedes why? Because it has extremely flexible shock absorber systems.

So what happens is, if you actually look at a Mercedes going on the road, you will see that the car goes like this and the wheel just keeps going up and down because the shock absorber is essentially a very flexible spring which continues. So this is exactly the problem that we have solved and we have shown that mathematically both the problems are identical and they essentially come around the concept of the transmissibility ratio. So let us look at a couple of problems to see how we can solve this particular issue. So let me look at a particular problem and let me take a problem that I have.

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I have a machinery, whose weight is note, that you know typically we say weight is 2 tones. It is wrong. Because weight is a force and in si units, force is always Kilo Newton's. So, let us say that weight is 20 Kilo Newton's and the excitation that we have is this force is transmitting a force it is operating this. This should we say the equipment is operating at 1500 revolutions per minute. So and it is transmitting a force the weight is not the force by the way because the weight is a constant weight which exists. But it is transmitting a force of amplitude equal to say 2 Kilo Newton's and this is a vertical force being transmitted and we have to design k and c such that we reduce this, the force transmitted here to as low a value as possible. This is our design problem.

So I have a rotating machinery operating with weight whose weight is 20 Kilo Newton's. It is operating at 1500 r p m and because of some eccentricities in this rotating machinery its actually subjecting you know, it is actually transmitting a force whose amplitude is given by 2 Kilo Newton's. So this is the force that is being transmitted and by the way this is a harmonic. We know that this is the harmonic force. Why? How do we know it is a harmonic force? Well, it is moving at a constant r p m. What is 1500 revolutions per minute mean? Well, let us put it. Divide by 60. What do you get? 25 revolutions per second. Revolutions per second means, every 125 of a second, the force goes round and comes back to 0.

In other words, the omega bar is equal to $25 \times 2\pi$ radians per second because every 1 circle is 2π radians. So omega bar, the excitation frequency is equal to 50π radians per second. So therefore, we know the excitation frequency, we know f_0 and so therefore, what we need to do now is, look at the possibility of transmitting as low and that let us put it this way.

I am going to say that look this is going to be point maximum that the ground this floor can take is 0.2 Kilo Newton's. The maximum amplitude that I have is 0.2 Kilo Newton's that is what I have given. So, this is the load coming, this is the maximum force that can be transmitted to the floor safely. So, how do I solve this problem? The problem is find k , find c . Now let us look at transmissibility. Transmissibility becomes 0.1. So, transmissibility 0.1 in other words, transmissibility cannot exceed 0.1. If it cannot exceed 0.1, what happens? If you look at it, if it cannot exceed 0.1; that means, this is the value. So, we come here; that means, we are in this zone. If we are in this zone, we have beta greater than 0.2 and if beta is greater than 0.2, we know that the lower the damping the better it is. So in other words, ξ is equal to 0.

So in other words, this implies that c is equal to 0 automatically, because if ξ is equal to 0, c is equal to 0. So, it is a un-damped system. So, if it is un-damped, transmissibility ratio is given by what? Let us see what the transmissibility ratio becomes? it becomes one upon the plus $2\xi\beta$ $\xi = 0$. So that disappears, then we have $1 - \beta^2$ plus $2\xi\beta$ that disappears. This is square root.

So, this becomes 1 upon $1 - \beta^2$ plus or minus. Remember that, that you know when you do this square and then you do the square root. A square, square root is not necessarily $1 - \beta^2$, it is plus minus $1 - \beta^2$ which ever value is needed to get as to get a realistic picture. So in other words, if I look at this, what do I get? I put this equal to so that I get. So if you look $1 - \beta^2$ is equal to -10 , it basically means, it is $\beta^2 - 1$ which is 10 . So that means, it is the minus that works and so therefore, β^2 works out to be 11 . If β^2 is 11 , β is equal to $\sqrt{11}$. So, β is root eleven. What does that mean? Let us look at it.

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The image shows a whiteboard with the following handwritten work:

$$\frac{\bar{\omega}}{\omega} = \sqrt{11} \quad \omega^2 = \frac{k}{m} = \frac{\bar{\omega}^2}{11}$$

$$= \frac{(50\pi)^2}{11} = \frac{k}{m} \quad \boxed{k = \frac{5000\pi^2 \text{ kg/m}}{11}}$$

Below this, the following values are written:

$$W = 20 \text{ kN} \quad g = 10 \text{ m/s}^2$$

$$m = 2^t = 2000 \text{ kg}$$

A final boxed equation is shown:

$$\boxed{k = \frac{(50\pi)^2}{11} \times 2000 \text{ kg N/m}}$$

Beta is equal to root 11 implies that omega bar upon omega is equal to root 11. Again I want to find out omega square because omega square is equal to k by m which is equal to if you look at it, it is equal to omega bar squared upon 11 and omega bar squared omega bar is 50 pi. So this squared and this upon 11 is equal to k upon m. Now what is m? We have given weight is equal to 20 Kilo Newton's. If g is, I normally take it as 10 meters per second squared is actually 9.81, but for all practical purposes, I can take it as 10 it just makes life simpler.

Mass then is equal to 2 tones or 2000 k g. Mass is 2000 k g. Then k becomes equal to 50 pi square upon 11 into 2000 and the units of this since this is k g and these are radians per second, this becomes Newton per meter. That is my k value. Well, you can get the value. It does not really matter what this value is.

It is you know, if you look at it, you will see that this becomes Kilo Newton's per meter and this is 2500 5000 pi square. So, k is equal to 5000 pi square upon 11 kilo Newton per meter. You know it seems, you know, we talked about the fact that this looks you know, this seems very soft. But note that you know despite the fact that to get a transmissibility ratio of 0.1. We see that it is 5000 kilo Newton per meter. So let us now look at, under the weight, what kind of displacement do I get? I mean, what kind of travel do I minimum requires?

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$$\begin{aligned}\frac{W}{K} &= \frac{20 \text{ kN} \times 11}{5000 \pi^2} \\ &= \frac{227}{50000} \quad 4.4 \times 10^{-3} \text{ m} \\ &= \boxed{4.4 \text{ mm}}\end{aligned}$$

The travel, minimum travel required is well weight upon k. So, that is equal to 20 kilo Newton's divided by so into 11 into 5000 pi square. Pi square is approximately about ten you know, I mean, I can take it as 10. You can do a much better computation. If you want to do that, I can take it as 10 for this particular purpose and so if you look at it, this goes, so this becomes 4.4 into 10 to the power of minus 3 meter which essentially means approximately 4.4 millimeter. That is not a very soft spring.

A two tone mass is only transmitting 4.4 millimeters. So, you see when we talked about, we were talking about that we require soft springs and you know, I was saying that sometimes soft springs cannot be provided because they become you see, I went to the computations to show you that even to get a transmissibility ratio of one tenth, which is like you know 0.2 kilo Newton. When you have a 2 kilo Newton force, its 0.2 kilo Newton's practically negligible.

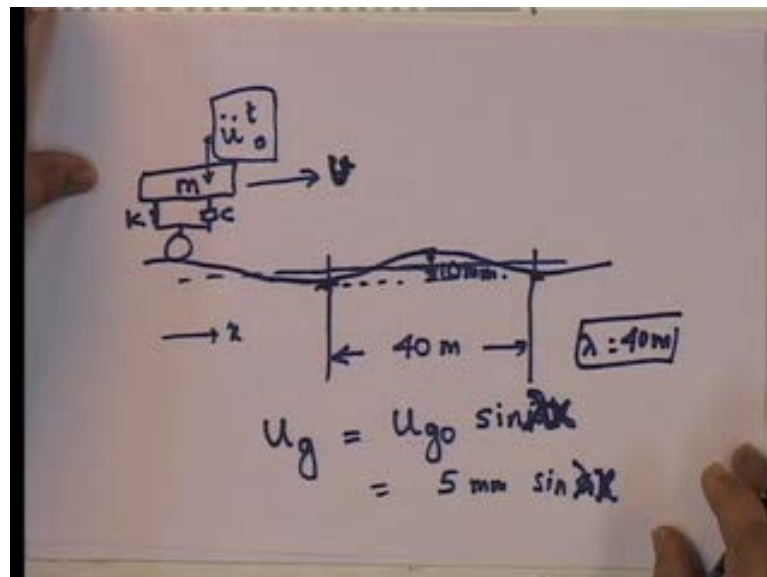
You know the transmission is very very low even then you get k which with a load of 20 kilo Newton's which is equivalent to a mass of two tones. What you get is 4.4 millimeter travel fairly stiff springs.

But those stiff springs are enough to ensure that the force from the machinery is not transmitted to the foundation and this where do we see this? This is typically in real life in designing a turbo generator foundation. A turbo generator in any power plant, a turbo generator design of the foundation this is the procedure that you go through. First you

want to ensure that there is no resonance and for that, you have to ensure that your k is soft enough and soft enough is not really soft. It is fairly stiff, but even that stiff one is good enough for us to ensure that we have this. So, that is a problem of force transmissibility. Now let me look at a problem of base isolation and for a base isolation, I will take a practical problem.

A practical problem with a slight twist in it, just to ensure that we get a harmonic. You know harmonic kind of excitation purely because you know, we have studied harmonic. Later on when we study more complex thing. You know this same concept can be applied equally well for you know more complex kind of situation. So, let us see what the problem that I have.

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You see because of foundation settlement let me look at this particular problem. What I have is a bridge, which because of the foundation settle, it become something like this. So, the surface of the road which should have been straight because of the settlement of the foundation now becomes a wavy. This is something that all of you have experienced when you have driven anywhere. So, I am actually kind of just taking a problem from real life with a little bit of you know assumptions to make it tractable right now. So, I will call this that. So, the road surface is essentially because from this to this is ten millimeter and we will say that therefore, the base which is the ground can be given by u

$g_0 \sin \omega t$, where g_0 is not 10 millimeter, 10 millimeters from the bottom to the top peak to peak.

So therefore, note that g_0 is half of that. So, that is equal to 5 millimeter $\sin \omega t$. That is my problem and what I have is, I am sitting in a car and for now let me just assume that it is one wheel just to make it simple. It's one wheel and the shock absorber system has k and c and the total mass of the vehicle and its occupants are given by m and what I want to know is I am an occupant in the car. What do I want to see? I do not want to be accelerated.

So in other words, the problem is that, I do not want to feel too much this I want to reduce this to the bare minimum that I can. So, this is a practical problem that we have you define the problem. Now the question here what is the question? the question is, well the car is moving at a particular velocity and the other thing is that, this distance is let us say 40 meters, the spacing between the two pears, what is that give me.

Can anyone tell me what that gives me? That gives me that if I look at this surface, if you look at, this is not the kind of surface that we are defining here. It is not t , it is with x . So, if I look at it, this is going to be equal to something called λx , where x is what? x is this and what is λ ? λ is what is known as the wave length. In other words, one wave this is the wave one wave what is the length from one wave to another wave? See, this is the bottom, this is the bottom again. So, from bottom to bottom or you can look at it the bottom to bottom is nothing but one cycle.

So, I am looking at the wave length is really one cycle of the wave and the distance. So, γ over here is actually sorry λ is 40 meters and this car is moving with a velocity v . So I have defined all the problems. This is moving with a velocity and the whole issue of this particular problem is finding out the shock absorber system of the car, the properties of the shock absorber system.

So, let us look at it. So how do I solve this problem? Well, let us you know I have taken this problem and written it as a wave in space. How do I transmit that wave in space into a motion of the wheel? Well, let us look at it. Let us say that this car is moving.

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$$U = 72 \text{ kmph} = \frac{72 \times 1000}{3600} = 20 \text{ m/s.}$$
$$\bar{T} = 2 \text{ sec.}$$
$$\bar{f} = 0.5 \text{ Hz}$$
$$\bar{\omega} = 0.5 \times 2\pi = \pi \text{ rad/sec}$$
$$\ddot{u}_g = 0.05 \text{ m/s}^2$$
$$u_g = u_{g0} \sin \pi t$$
$$\ddot{u}_g = -u_{g0} \pi^2 \sin \pi t.$$

The velocity of the car is 72 kilometers per hour. Then my question becomes how long when it is travelling at 72 kilometers per hour? I can transfer 72 kilometers per hour into meters per second. How much is that? 72 into 1000 becomes meter per hour and there are 3600 seconds. So, this becomes how much? 20 meters per second so in other words, this car is travelling at 20 meters per second. So now let us look at it, how long is this car going to take going from here to here? It is 40 meters. It is traveling at 20 meters per second it is going to take 2 seconds.

So in other words, the time to go through one cycle, the time required to go through one cycle is how much? 2 seconds and this becomes, so this becomes then the excitation because in other words, the wheel every 2 seconds is going to go from here to here to here and that is a sign. So in other words, what have we done? We have transformed this problem into a problem with omega bar t and what is that equal to? Because we know that, it is travelling at 20 meters per second. It takes 2 seconds. So, if it takes 2 seconds, that means, what is the frequency? It is 0.5 hertz and what is the excitation frequency then? 0.5 into 2 pi. So, that is pi radians per second. That is my omega bar.

So in other words, I can now say that this u_g is can be written as $u_{g0} \sin \pi t$. Now the question is this is the displacement. Now remember, we were looking at acceleration. So, I have to find out the acceleration. What is the acceleration going be equal to? It is going to be equal to we gone through this many times. So, I am just going to write it

down. It is going to be $u g 0 \pi^2 \sin^2 \pi t$ minus becomes 0. You have to double differentiate it.

So, this then becomes $m \ddot{u}$ and this is 5 millimeter. So, 5 millimeter becomes 5 into 10 to the power of minus 3. This is approximately 10. So this is 0.05 meters per Second Square. So therefore, $u \ddot{g} 0$ is equal to this is 10 5 millimeter in meters is 5 into 10 to the power of 3, 50 into 10 to the power of minus 3 is 0.5 meters per second square. This is the amplitude of my acceleration. So, if this is my acceleration, then how do I solve this problem? Well, again what I need to do is, I have now, you see, I have taken that particular problem and made it into a problem that I can solve.

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The image shows handwritten mathematical notes on a whiteboard. The top part shows the relationship between acceleration and transmissibility ratio: $\frac{\ddot{u}_0}{u g_0} = TR = \frac{1}{10}$. Below this, it states $u g_0 = 0.05 \text{ m/s}^2$. A boxed equation shows $\ddot{u}_0 = 0.005 \text{ m/s}^2$ with the word "Comfortable" written next to it. The bottom part of the notes shows the derivation of the transmissibility ratio formula: $\phi_f = 0$, $\beta^2 = 11$, $\left(\frac{\bar{\omega}}{\omega}\right)^2 = 11$, $\omega^2 = \frac{\bar{\omega}^2}{11} = \frac{\pi^2}{11} = \frac{k}{m}$. The transmissibility ratio is also shown as $TR = \frac{1}{(1-\beta^2)} = \frac{1}{\beta^2-1} = \frac{1}{10}$.

So my problem then becomes this and I know that, this is equal to, so the question becomes that again I will say that, this is the transmissibility ratio and I want to say that look, I do not want, I as a car person I do not wish to feel anything more than 0.005 meters per second square. That is the maximum that is comfortable. Automatically $t r$ becomes 1 upon 10 and we go through the same steps. Note that if $t r$ is 1 upon 10, ξ has to be equal to 0.

If ξ has to be equal to 0, then what does $t r$ is equal to 1 upon $1 - \beta^2$. In other words, one upon $\beta^2 - 1$ which is equal to 1 upon 10 . So, β^2 is equal to 11 and β^2 is nothing but $\bar{\omega}^2$ upon ω^2 is equal to 11 . So again ω^2 is equal to $\bar{\omega}^2$ upon 11 . $\bar{\omega}$ is what?

Remember, omega bar was pi. So that is pi square upon 11. So, essentially this is k upon m. Again, what do you get?

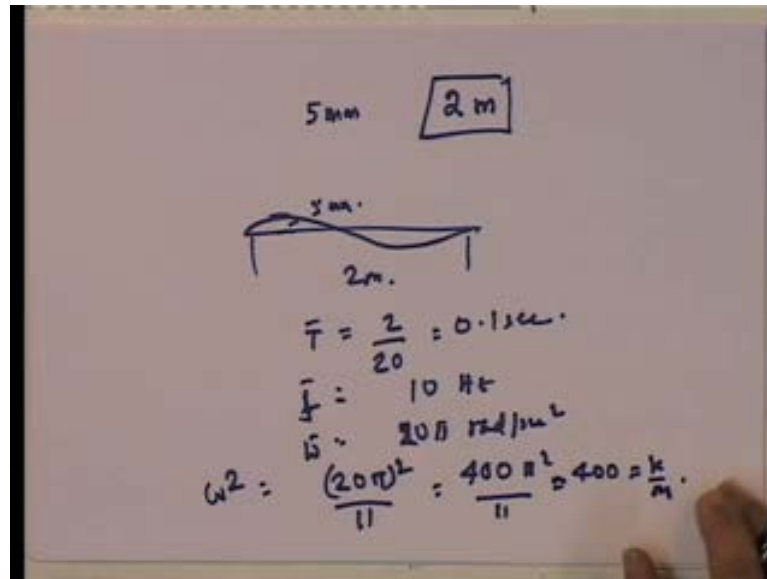
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The image shows a whiteboard with handwritten mathematical work. At the top, the equation $\omega^2 = \frac{k}{m} = \frac{\pi^2}{11} \approx 1$ is written. To the right of this, it says $m = 2000 \text{ kg}$. Below this, the spring constant is calculated as $k = 1 \times m = 2000 \text{ N/m}$, with the value 2000 N/m boxed. Further down, the weight is given as $W = 20 \text{ kN}$ and the displacement is $\delta = 10 \text{ m}$, which is circled. At the bottom, there is a large 'X' mark.

You have a situation, where omega square which is k upon m is equal to pi square upon 11. Now, let me take that as approximately one for all practical purposes. Basically that means, k is equal to 1 into m. What was the mass that we had said the mass of the car? The mass of the car we had not given. We will take the mass of the car as 2000 k gs. So therefore, k is equal to 2000 Newton per meter. Now this is a very very soft spring. Because this is going to give raise to huge displacements, which we cannot take. So in other words, designing this shock absorber, understand this that if this is k, weight is what? Weight will be 20 kilo Newton's.

So therefore, if weight is 20 kilo Newton's, this is 2 kilo Newton per meter, what is my displacement? 10 meters under the load impossible. So, this is not possible. This problem in other words, we have discovered that look under the weight, the spring will have to go 10 meters and if the spring has to go 10 meters then; obviously, we have a basic problem. The problem becomes that if I am going at 70 kilo meters per hour and this is 40 meters, we have a problem. However, let us look at the problem.

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Let us see, if this 40 meters was made into 2 meters. In other words, you are going at still the same speed, you are going in other words look at it. See understand one thing that you have a situation where your system is being subjected to very slowly varying and we see that you know the design that we have. It is practically impossible to do the design for that. Now let us look at let us let us take, it is a road surface. Let us not take a bridge. It is a road surface and road surface is very rough.

So, in other words I have 5 millimeters and the spacing is 2 meters. In other words, it is a very very rough road. Every 2 meters if I have this kind of a thing and this is my 5 millimeter think of how rough road it is, very rough road. Let us look at something very interesting. If it is this, then how long will it take to travel and if it is still traveling at 70 kilometers an hour, t prime becomes 2 upon 20. It takes 0.1 second. So, f prime becomes 10 hertz. ω bar becomes 20 pi radians per second. We still have that same problem. In other words, I still want the one tenth. So, I still have the same thing when ω bar is equal to ω bar square. Now ω bar square is what? So, ω bar square is equal to 20 pi the whole square upon 11. So, that is like 400 pi squared upon 11 which I will take it to be 400 which is k by m and if you look at this k by m then.

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$$k = 400 \text{ m} = 400 \times 2000$$
$$= \boxed{800 \text{ kN/m}}$$
$$\frac{20 \text{ kN}}{800} = \frac{1}{40} \text{ m} = \underline{\underline{25 \text{ mm}}}$$

So k is 400 times m , which is 400 into 2000. So this becomes 800 kilo Newton per meter under the load, under the load of 20 kilo Newton upon 800 kilo Newton. How might did this go? One fortieth of a meter. One fortieth of a meter is equal to 25 millimeter which is durable 2.5 centimeter. This is a viable shock absorber system. In other words, this is a very interesting problem. The interesting problem is I am traveling at 72 kilometers per hour in a car. If it is on a bridge which is 40 meters span and it is moving. It is going to feel because I cannot design it you saw that you I cannot design it. So, it automatically means the whole entire road surface is going to be felt here; however, as soon as I took the problem and made it into a very rough surface and I am going really fast. What happens?

I can design this car now the same people who are feeling the entire acceleration in a very slow wavy road. Now in a rough road, if you are traveling at 70 kilometers an hour, what happens? You do not feel it at all. You see how interesting the problem is. What is happening? What is happening is that you are moving so fast that the car does not. In other words, the mass you yourself sitting in the car does not even realize that it is being subjected to a vibration whereas, when it is slow, what is it becomes? It becomes almost like a static load. So, it feels the entire weight entire you know acceleration comes into the car.

So, this is a very interesting of course you know, it is not one is not talking about. What the tires going to? What is going to happen to the tire? I am looking purely at a personal comfort and a personal comfort is much easier in a very you know bad road on a very flexible mercedes then it is. If a mercedes is going at 72 kilometers per hour on a wavy bridge, so this is a very interesting problem and we will talk more about this as we move for.

Now, thank you very much. Bye.