Structural Dynamics

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Lecture -7
Response to Harmonic Loading (Continue..)

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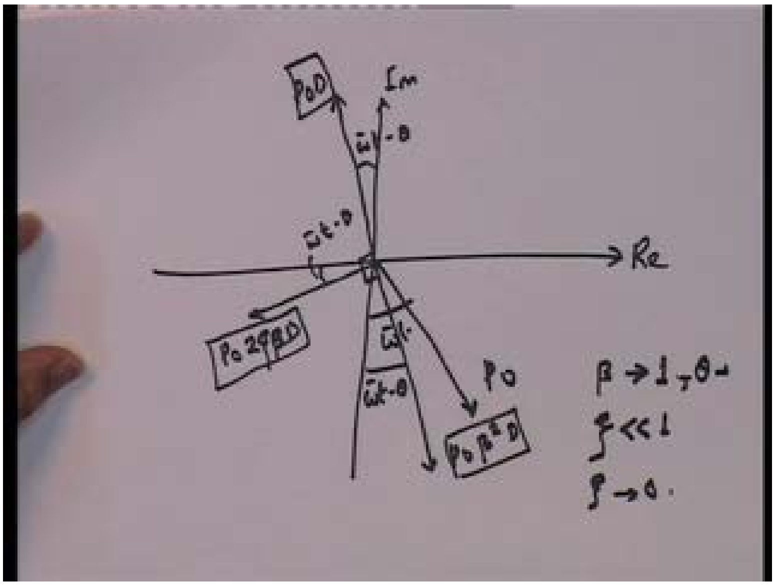

Hello, we are continuing with our discussion on Response to Harmonic Loading. So and again to go back to I did end my last lecture rather hurriedly, and so I want to go back a little bit and look at the argand diagram that I have drawn. I redraw that argand diagram again for you, I have and I am not going to spend any time in explaining, because we have already done all of this, p naught omega bar t p 0 beta square d, then this is omega t minus theta. Then opposite to this is p naught d, this is omega bar t minus theta, and then at 90 degrees to that is p naught into 2 psi beta, this these values that I am writing all of them are the magnitudes of the vectors and the direction of the vectors are given by this.

And another explained these are 90 degrees to each other, and these three balance out p naught, so this was the diagram, and then we went back to for me to show you that these three indeed give you balance out p naught and vectorially. Now, vectorially it is very difficult, so what I did was we looked at just the real the real part and therefore what we saw was that this sin omega t plus this cosine omega t is equal to p naught sin omega bar t plus p naught, so writing that properly it becomes in this fashion.

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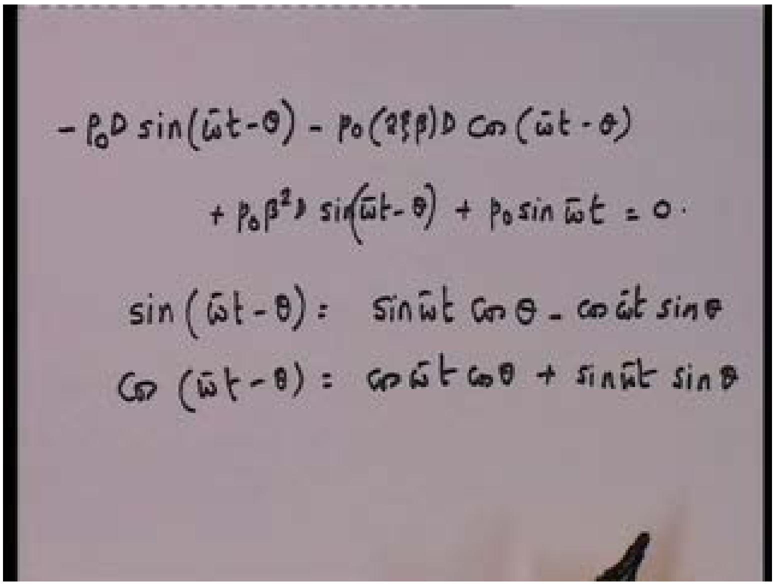

It becomes now p naught D, so that is minus p naught D sin omega bar t minus theta minus p naught into 2 psi beta into D cosine of omega bar t minus theta, then we have plus p naught beta squared D sin omega bar t minus theta plus p naught sin omega bar t is equal 0. This is if you look at this these are all the projections of each vector on the real axis, that is what I written down over here, so this part is what we are going to see that indeed that this is the truth. So, we are now what I will do is I will instead of rewriting this I will just write down, what sin omega bar t minus theta is equal to sin omega bar t cosine theta minus cosine omega bar t sin theta. So, that is the solution that you have and cosine omega bar t minus theta is equal to cosine omega bar t cosine theta plus sin omega bar t sin theta.

So, these are the expansions of these terms that we have here, for the more we see that we know that tan inverse of sorry tan theta is equal to 2 psi beta upon 1 minus beta squared. So, this do last time was this implies that if this is theta this is 2 psi beta and this is 1 minus beta squared, so obviously, this is equal to by Pythagoras theorem, because this is 90 degree is going to equal to 1 minus beta squared plus 2 psi beta squared square root, this is the Pythagoras theorem.

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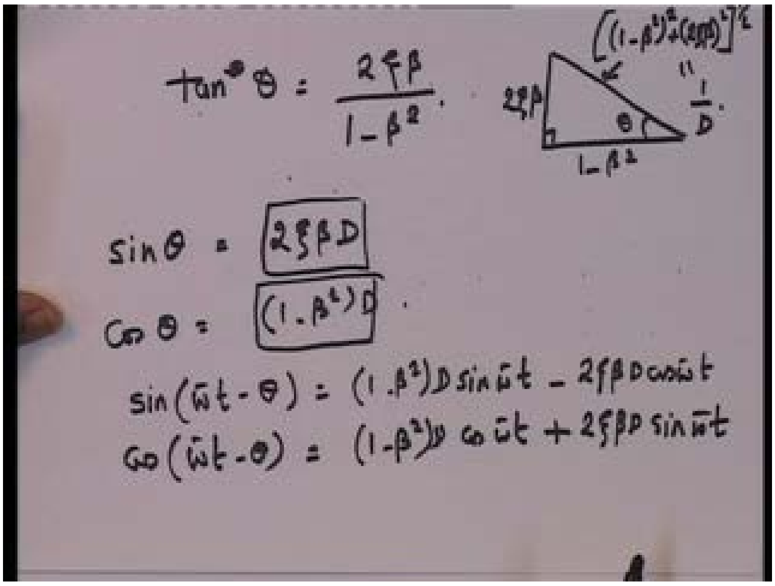

If you look at this is the Pythagoras theorem, so if you look at this, what this term, this term is nothing but it is equal to 1 upon D, so therefore, if you look at sin theta is equal to 2 psi beta upon 1 upon D upon 1 upon D basically becomes that mean 2 psi beta upon 1 over D is means sin theta is equal to 2 psi beta D. And similarly cosine theta is equal to 1 minus beta squared upon 1 upon D, so that becomes equal to D.

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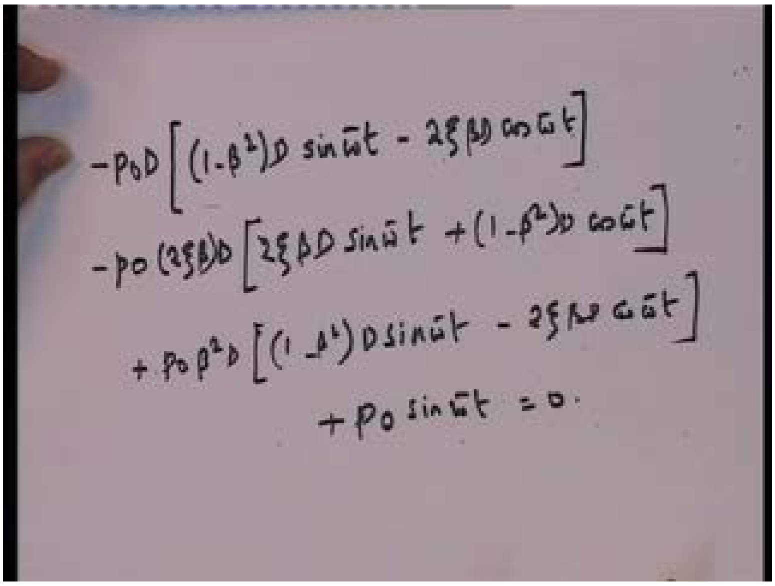

So, if we substitute this back into those equations, if you look at it what happens, what we see is the following, I am going to substitute these into sin, so this is going to be equal to cosine into cosine theta. So, that is 1 minus beta squared D into sin omega bar t minus 2 psi beta into cosine omega bar t, this is sin theta, this is cosine theta and cosine omega bar t minus theta is equal to cosine omega t into cosine theta, so that is equal to 1 minus beta squared D cosine omega t plus 2 psi into sin omega t.

So, this is what we get and substituting this into the original equation, that becomes then it becomes minus p naught D into sin omega t minus theta this is what I am substituting. So, I am substituting 1 minus beta squared D sin theta minus two into cosine omega t that is the first one, then the next one is p naught into 2 psi beta D into cosine omega t minus theta, which from here turns out to be equal to 2 psi beta D sin omega t plus 1 minus beta squared D into cosine omega t. And then finally, I have p naught beta square D into the same thing, that I have here 1 minus beta square D sin omega t minus 2 psi into cosine omega t plus p naught sin omega bar t is equal to 0. So, we have to just show that this is indeed equal to 0, so if I now look at this term this is identical.

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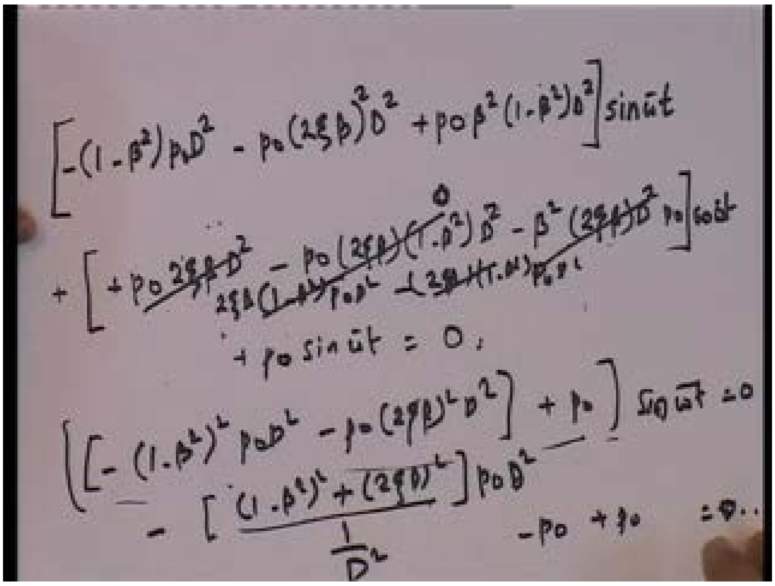

So, now let us look at putting all the sin terms and cosine terms together, so if I look at the sin term and putting together all the sin terms, what I get is for the sin terms together, it becomes 1 minus beta squared minus p naught D squared that comes from the first one. From the second one you get minus p naught 2 psi beta D into whole squared into this, so it is basically 2 terms, and then the third term becomes plus p naught beta squared into 1 minus beta squared D squared this into sin omega bar t.

And then the cosine terms, if I put them together, what I get is minus minus plus, plus p naught into 2 psi beta D squared then from the other one I get minus p naught, so p naught into 2 psi beta into 1 minus beta squared D, that is what I get cosine. And then from the other one i get minus beta squared sorry, this is D squared and then minus beta squared into 2 psi beta D squared p naught into cosine omega bar t.

This is plus p naught sin omega bar t is equal to 0, I have not put the p naught inside, but I will put it in I do not worry about it, I will put that in. So, if I look at it what I get here is the following, if you really look at it I get if you look at this particular term it is one into 2 psi beta minus beta squared into 2 psi beta. So, it becomes basically these two if I put them together I get one term which is 2 psi beta into 1 minus beta squared p naught D squared and the other term is 2 psi beta into 1 minus beta squared p naught D squared; so that means, these cancel out; that means, if you look at this becomes 0.

Now, this is obvious 0 into cosine omega bar t, because there is no term, so this term should automatically become 0 and that is what it, becomes let us look at what happens here. If you look at this particular thing, this becomes one it becomes minus, if you look at this, this is 1 and this is beta squared, so this becomes actually minus 1 plus beta squared which I can write it as minus of 1 minus beta squared.

So, essentially what I am getting is this term top term becomes minus 1 minus beta squared the whole squared p naught D squared minus p naught into 2 psi beta squared D squared I put the two of them together plus p naught sin omega bar t equal to 0. Now, for this; that means, this term this term together have to be equal to 0 well let us look at it this term if you look at basically, becomes minus of 1 minus beta squared plus 2 psi beta squared the whole into p naught D squared. Now, if you look at it what is D squared D squared is nothing but 1 upon beta squared, the whole squared plus 2 psi beta the whole squared square root 1 upon D. If you remember is equal to nothing but square root of 1 minus beta squared, if you look at this particular term what is this term if you look at this term this is nothing but 1 upon D squared.

So, if I take minus 1 upon D squared into p naught D squared what do I get minus p naught and this other one is plus p naught, so indeed this is 0. So, in other words there is equilibrium, as you see it from this particular point, so after all that huge amount of calculation we showed that indeed these equilibrate each other. If you look at this diagram these three equilibrate, this now let us look at some interesting things, you know this argand diagram will show certainly very interesting thing.

Let us take a situation where beta tends to 0, and we will assume that psi is very much less than 1, in other words psi tends to 0 without being 0, understand that it tends to 0 without being 0. Why? Because, you know you do not have steady state if you have psi equal to 0, but let say that it tends to 0, then what happens, let us look at this. If you look at this, if beta tends to 1, what you have you have, p naught D remember that if beta tends to 1 what happens to theta do you remember, let us look at that.

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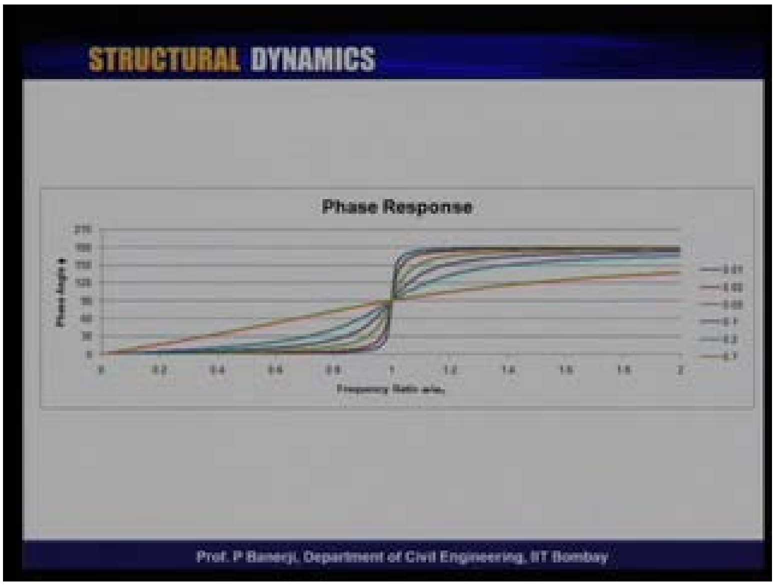

As beta tends to 0, what happens to the phase angle phase, angle theta tends to 0, in other words the phase lag if you look at if we come back to this, the lag this theta is for all practical purpose is 0. So, then if I redraw that what does is equation look like, I mean what is the diagram look like.

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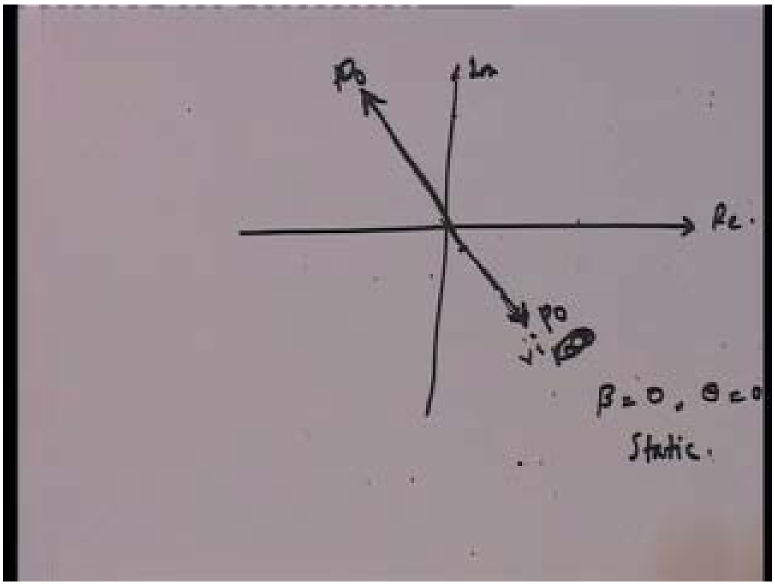

The diagram looks like this imaginary, real this is p naught, this is as beta tends to 0 what happens here, this becomes p naught beta squared d. So, in other words the inertial force, becomes just and it is along this for all practical purposes, it is it is along this and it is equal to p naught D. So, this is what are we saying and by the way, when beta tends to 0, where is D tend to D tends to 1; so that means, this one says that this is equal to no sorry, sorry I am making a mistake here, you know I made the mistake beta tends to 0.

Then theta tends to 0, psi is less than 1, so psi tends to 0, so if you have if beta tends to 0, what happens to this term disappears, the inertia disappears. What happens to this one? This is also better it tending to 0, so this term disappears and what are we left with we are left with this one, which is p naught and D is 1, so this is p naught. If you look at this, when you have beta equal to 0, theta is equal to 0, then you have a situation in which this term goes to 0, because beta is equal to 0.

This term goes to 0, because this term is equal to 0, theta tends to 0, so this tends up, so this becomes this way and what you have is what is beta equal to 0, actually means static omega is equal to 0, this is static condition. In static condition what do we have? The applied force is resisted completely by the elastic force, that is static, in static the entire load is resisted by the elastic force. So, in other words from this part, we got the situation that indeed at the static case this becomes the trivial solution, where both the equilibrating force which is elastic force is p naught, applied load is p naught, inertial force and inertial force and damping force are 0, so that is the static case understandable, very easy to understand.

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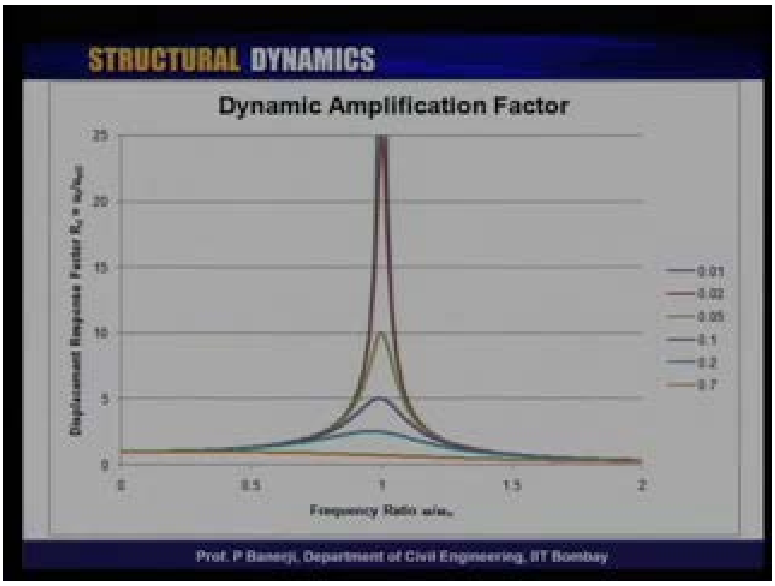

Then let us look at the situation, where you have beta tends well.

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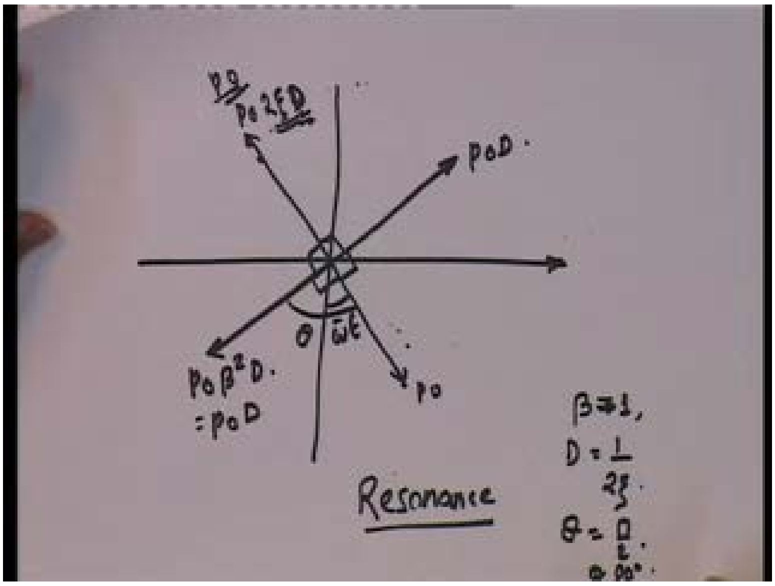

Let us put beta equal to 1, implies let see, what is delta equal to? Delta is equal to 1 upon 2 psi, we have already derived this in the last lecture and phase angle theta is equal to pi by 2 or 90 degrees out of phase. So, now, let see what does this look like, then let see I have the applied load at p naught, that is at omega theta. Now, look at this theta is equal to pi by 2, so in other words this one which is my p naught beta squared, so that is equal to p naught D, so this is 90 degrees, because theta is 90 degrees, then I have this term, where beta is equal to 1, so this becomes 2 psi D, so this is 90 degrees with that, so this is p naught 2 psi D.

And so this is 90 degrees from that, and note that this one is 90 degrees from there so; that means, this one is along this direction, and it is equal to p naught t 90 degrees. So, this is theta and these are 90 degrees, so look at the very interesting thing and look at what D, D is what? D is 2 1 upon 2 psi, so if I look at this, what is this become? This becomes p naught look at this very, very interesting thing, and that is at resonance when beta is equal to 1 resonance.

The applied load is resisted completely by the damping force, because is only resisted by the damping load that is reason why the dynamic amplification factor becomes, 1 upon 2 psi very large. Because, understand if damping is low and that has to resist this dynamic amplification has to become 1 upon 2 psi, so that this is becomes p naught, it can resist this.

And look at the inertial force and the elastic force, they are completely out of phase you know sorry, not out of phase they are 90 degrees out of phase with the applied load in the damping force, and they balance each other out. So, and note that these values would be very large, because p naught D is very large, so in other words you have this peculiar situation at resonance that since the damping force resist the applied load, therefore, the inertial force and the elastic force are very large.

However, they are 90 degree out of phase with applied load, so in despite them being very large all the due is they cancel each other out. And so you have this peculiar situation where at resonance the argand diagram turns out to be in this way, and that is the reason why because of these D becomes very, very large the dynamic amplification, becomes very large for small psi.

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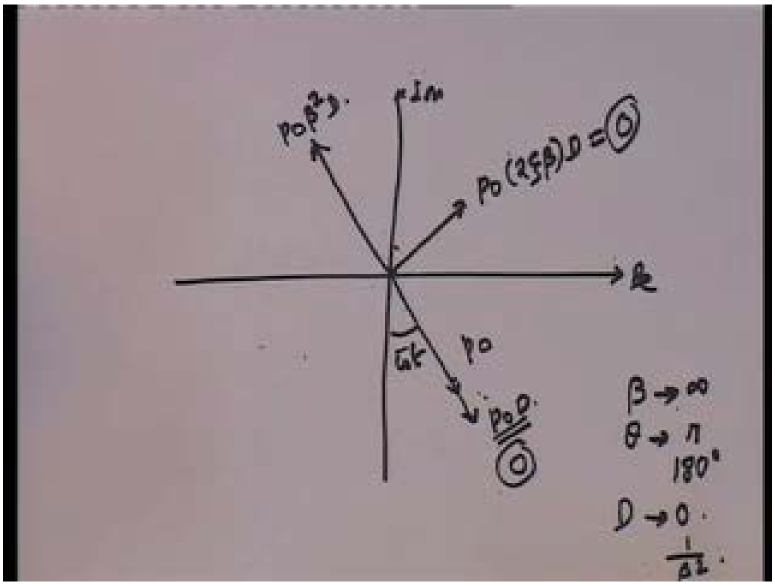

So, now let us look at the last kind of situation, which I look at which is when beta tends to infinity, as beta tends to infinity what happens, let us look at it. Phi tends to pi or 180 degrees and let see what happens, and we will see what happens to D lets draw the argand diagram, real, imaginary this is p naught. So, this is omega bar t and now phi, now this theta is 180 in other words what I have is that my p naught beta squared D is 180 degrees out of phase with p naught, because theta is exactly that term.

Then this direction becomes p naught into 2 psi beta D, and note that this direction becomes p naught what D. Let us look at this situation, what happens to D as beta tends to infinity D tends to 0; that means, this becomes 0, this becomes 0, if for all D tends to 0, so it become very small these and these become very, very small. And so let me take in the asymptote this become 0, this become 0 and what is the resistance, let us look at this what is D?

Actually, if you look at D, it goes as 1 upon beta squared, so D goes upon 1 upon beta squared, so if you look at this the elastic force is 0, the damping force is 0 and what resist the applied loading. It is the inertial force that balances the applied load and you look at, because beta squared D this goes as 1 upon beta squared remember, we done all of this last time I mean some couple of lectures ago. So, therefore, what you have is remember I say that elastic force was 0, why? Because the applied load is completely resisted by the inertial load and therefore, elastic force is 0.

So, you see the argand diagram actually tells you many, many very interesting things about the various, how what other kind of mechanisms that you have for various kinds of situations. So, I think I am very much done with response of a structure two harmonic loads, now what I would like to do is actually look at a specific case, where we are going to actually apply the response of a structure of a single degree of freedom system two harmonic loads. And this is very, very two specific situations that I am going to look at and may going to derive the kind of you know let us look at the two situations.

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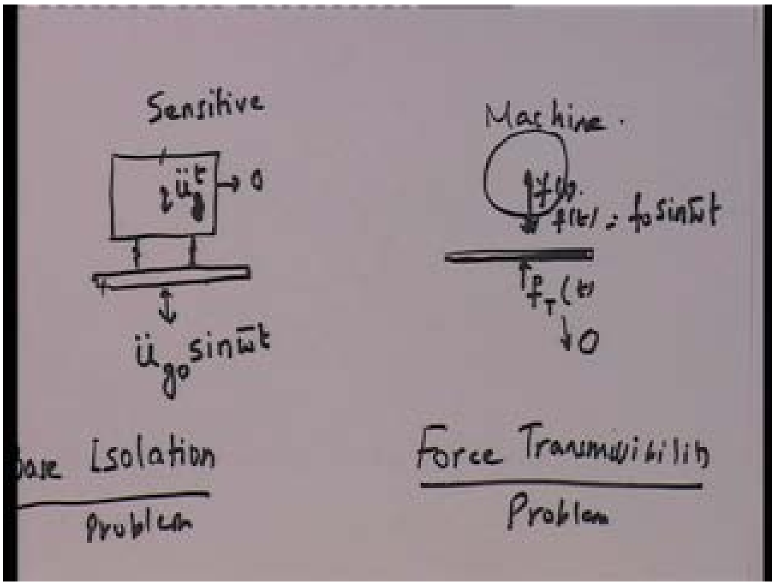

One situation is where you have a machinery, which is connected to a floor and we say that look the floor is subjected to a vibration, and the vibration is given by, so the floor vibration and this is a very, very sensitive equipment. What we do want to say is that look, what we do not want to transmit this to this sensitive equipment? In other words what I am saying is that the acceleration, that this sees the total acceleration that this sees should tend to 0, that is my problem.

The problem is that this is vibrating, the phase of the equipment is vibrating we can stop that vibration, that vibration is happening what we want to do is isolate, and this is known as isolation we want to isolate. This equipment from it is base, so this is known as something called as base isolation, this is one problem. The other problem is just the reverse of this what I have is I have a machinery, a machinery is transmitting a particular force to the floor, and I want to have to happen is that the force transmitted to the floor, I want that to be for practical 0.

So, even if what I am trying to say, because you see if these are two flip size of the coin here, I have a situation by the floor is vibrating, and I want to isolate the sensitive equipment from that floor vibration. The other situation is a machine, so this is the machine vibration problem, and I want to this machine because of it is operation is going to have a force acting on it. What that force is we will come? We can discuss that later, but you know when you have rotating machinery, because of an imbalance of forces you actually land up getting forces which are you know making it vibrate in this way.

And what I want to do is that the machine vibrate, but I do not want the flow to vibrate at all, because after all on the floor some other kinds of things, where I do not want to transfer this force. So, this is actually a force transmissibility problem, so this is a base isolation problem, and this is a force transmissibility problem and in both cases let me say that this is f 0 sin omega bar t.

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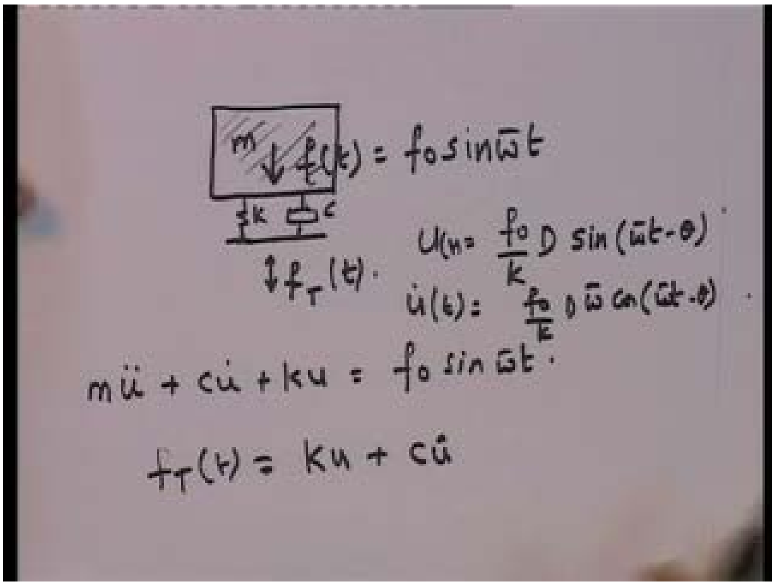

So, I will use or assume that both the excitations here the floor excitation is given as the harmonic and here the load from the structure is given as f 0 sin omega bar t, so let us first look at how we can solve these problems. So, for this problem let us start first with the base isolation problem, well let us start with the force transmissibility problem, that is a simpler problem for us understand because for the base isolation problem allow to start defining some new parameters, before I do that I will actually look at the force transmissibility problem and if I look at the force transmissibility problem it becomes the following.

I have an I mean an equipment which is transmitting a force f of t is equal to f 0 sin omega bar t, this rigid machine is connected to the floor through some spring and dash pot system. And what I want to do is I want to figure out what this floor I will call this is the transmitted force, what is this equal to well. If you look at this, this is a block machinery block of mass m, it has k and has c that spring and dash pot connected to the floor, so what is this problem, this problem if you look at it is nothing but m u double dot plus u dot plus k u is equal to f naught sin omega bar t.

This is single degree of freedom problem, now the question is what is the transmitted force, the transmitted force if you look at it is nothing,, but f of transmitted force is equal to k u plus c u dot, nothing else there is nothing else to it. So, now, the question becomes what is k u and plus c u dot equal to that is all I need to do, now note that I have already solve this problem. So, I am not going to resolve the problem, I am just going to write down these terms, and let me write down what these are what is u from this equation u of t.

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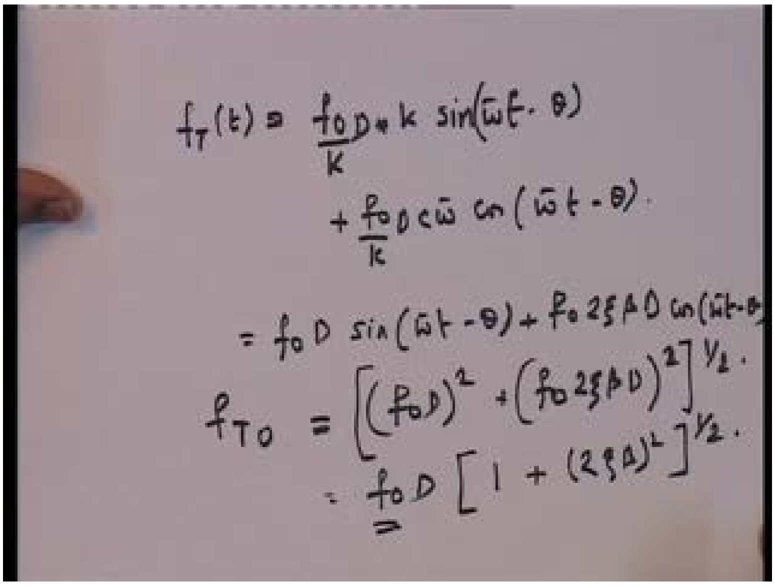

Note that I am always looking at steady state, there is understand that if there are no initial conditions, in other words this load has been there for all times there is no initial condition, then the response is only the steady state. And the steady states response becomes what, f naught upon k into D sin omega bar t minus theta, that is my u of t and what is u dot of t, u dot of t becomes f naught upon k D omega bar cosine omega bar t minus theta.

So, once I have these I plug these in and my transmitted force becomes what, my transmitted force becomes very simple, f of t is equal to k of u 0, so that is equal to f naught k D into k sin omega bar t minus theta plus c into u dot, so that is equal to f naught upon k D into c omega bar cosine omega bar t minus theta. So, if you look at this, what is this become k, k cancels out, so this becomes f naught D sin omega bar t minus theta plus f naught, what is c omega bar upon k we have already seen it 2 psi beta D cosine omega bar t minus theta.

Now, this is sin into x, this is cosine into x, note that both of them have omega bar t, so this is sin x and this is cosine x. So, therefore, if I have sin x and cosine x, what is the amplitude? It is very easy, the amplitude the f t 0 which is the amplitude is equal to what, it is equal to f naught D the whole squared plus f naught 2 psi beta D the whole squared square root.

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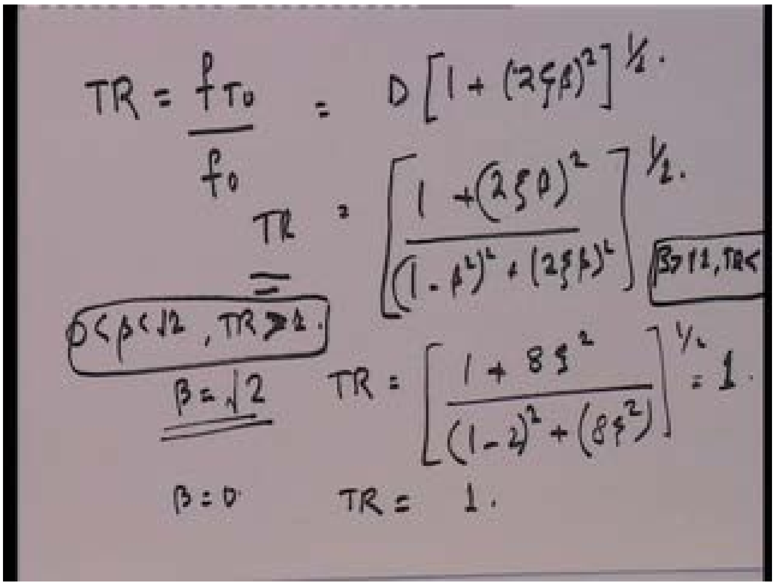

That is the amplitude, I am only interest in the amplitude, because understand that the time variation is not of importance to be I mean, I am interested in the this thing. And if you look at this if rewrite this, this becomes essentially equal to f naught and the D squared term goes out, so this becomes D inside and the inside basically becomes 1 plus 2 psi beta half, so this is the amplitude of the force transmitted to the floor and this is the amplitude of the applied force.

So, if I define a term called the transmissibility ratio and the transmissibility ratio is defined how, the transmissibility ratio is defined as, transmissibility ratio is equal to f t 0 upon f 0 which is equal to what D into 1 plus 2 psi beta whole squared square root. And in incorporating D inside the transmissibility ratio essentially, becomes 1 plus 2 psi beta the whole squared plus 1 minus beta squared plus 2 psi beta, this becomes the transmissibility ratio.

Very, very interesting let see something let us plug in beta in other words if you look at this you know for small zetas this almost looks like the dynamic amplification factor with an additional term on the top, but let us look at a particular situation beta is equal to 2. That beta is equal to 2, then what do I get transmissibility ratio to be equal to well 1 plus this is beta 2 psi beta the whole squared, so beta squared becomes if you know root 2 into 2.

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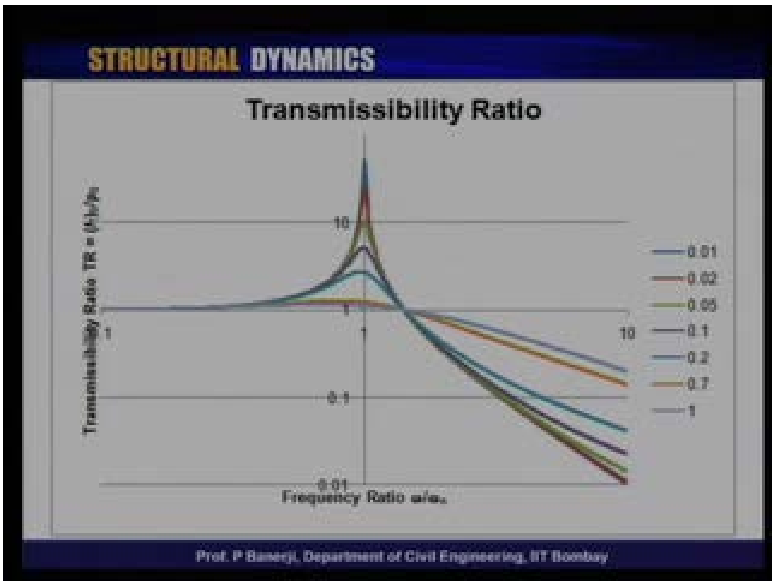

So, this becomes what, it becomes well look let us look at it this 2 psi squared, so 2 psi squared is 4 psi squared, 4 psi squared into 2 is equal to 8 psi squared. And the bottom I have 1 minus, by the way root 2, so this is 2 the whole squared plus let us look at this, this also becomes 8 psi squared square root, 1 minus 2 is minus 1 whole thing squared becomes 1 minus 1 squared is 1, so this becomes 1 plus 8 psi squared upon 1 plus 8 psi squared, so transmissibility ratio is 1. That beta equal to 0 root 2 transmissibility ratio is 1, irrespective of psi, look at beta equal to 0 transmissibility ratio is equal to beta equal to 0 this become 0 this become 0, so it is 1 upon 1 transmissibility ratio is 1.

Let us look at how transmissibility ratio looks, if you look at this, this is the transmissibility ratio T R, which is the transmitted force amplitude upon the applied force amplitude. And if you look at it at beta equal to 0, this is 0 at root 2, this is 0 and between 0 and root 2 transmissibility ratio is greater than 1. In other words for between 0 and beta root 2 transmissibility ratio is greater than 1, and for beta greater than root 2 transmissibility ratio is less than 1.

Now, the question becomes I can I want to ensure, so I want to design a support system, the machinery I want to design the support system the foundation of the machinery, such that the load transfer to the base is 0. So, I have to design a system lets look at to make it 0 what do you I need, let us look at this is very, very interesting you see how this goes, when you go you want it to become low. You want you want to take it 0 fastest possible, what do you have to do you have to ensure that beta is as largest possible, beta is large.

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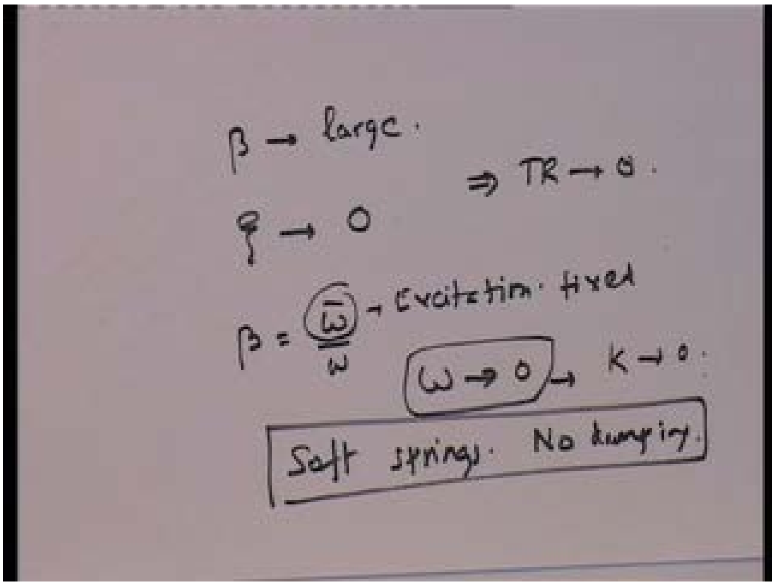

And let us look at something very, very interesting that actually damping note, that in this situation we always saw that low damping meant high transmissibility ratio and high damping reduce the transmissibility ratio. So, where as in when you go beyond root 2 damping actually harms, because the lower of the damping the faster it goes 0. So, in other words ideally speaking what kind of a system would you like to ensure that the load transferred from the structure, to the foundation is as low as possible.

It would imply that beta has to be large, what is beta is equal to large? I mean understand something, lets define it very carefully. In other words what are we saying, beta has to be large and psi should tend to 0, if beta is large psi tends to 0. This implies transmissibility ratio tends to 0, that is what we have seen from this, if you look at this, beta large psi 0, it goes to 0 fast transmissibility ratio.

So, what is beta large mean? What is beta? Lets define it omega bar upon omega is beta, now you see this excitation frequency we cannot do anything about, because that is the frequency that is coming from the machinery, I cannot change the frequency of the machinery, because that is the operating frequency of the machinery. So, omega bar is fixed, so to get beta large what do I need to do, I to make omega tend to 0 what is omega tending to 0 imply, note that the machinery mass is a fixed.

So; that means, k tends to 0; that means, soft springs no damping that is what you have to do, that mean you need to put the machinery on soft springs with no damping to ensure that very little gets transferred to the floor. Now, you know see this is where I am trying to solve what problem, the dynamic problem, but what other problem do I have note that if I make the spring too soft what happens?

Well, the mass the you know the machine has weight and if I make that so soft, what is going to happen the spring is going to bottom out purely because the weight itself takes it down soft springs means what very flexible spring. If I have a particular weight and I have a very flexible spring it is going to sit-down. So, you see there is this very, very key problem that is sometimes in trying to solve one problem, we actually create another problem, and that is a classical situation. So, please understand that the spring has to be hard enough, to be able to take the weight and not bottom out you know, because spring has a certain amount of plane.

So, it should be bottom out, because of the weight should be hard enough for that; however, it should be soft enough such that T R, I mean beta is large, so this is a, see this a design is always issue of trying to optimize, how you want to do. So, now, the question then becomes that, so therefore, ideally I would like to beta to be greater than root 2 ideally. Suppose, because of the static problem, my k value of the spring needs to be such a value, such that I cannot for the operating frequency ensured that beta is greater than root 2. What do I do then? What is the design problem? Then you would look at it come back to this, in other words I cannot be in this domain, if I mean this domain, if I can remain in this domain. I prepare as soft to spring is possible with no damping, but suppose I have a situation, where I cannot be in this zone, because of the value of omega being such that beta is less than root 2, then what do I have to do?

Then the whole problem changes, if I cannot ensure that beta is greater than root 2, I cannot ensure that there is no load transfer, I cannot because look at this situation, I can only ensure that no load, if I am in this zone. If I for operating reasons I have to be in this zone, note that the minimum value that, I can transfer is the static value of the load itself, you know other words the minimum value of transmissibility that I can keep is 1, I cannot take to greater than less than 1.

And what do I do in that situation? That situation I have to over damp in other words huge damping in huge damping into the system, such that the transmissibility ratio remains as close to 1. In other words look at it you know almost be critical damping to ensured that you see 0.7 and 1 are very close to each other, 0.7 is un critically damp 70 percent damping. So, 70 percent damping you need to introduce huge damping into the system to ensure that transmissibility ratio remains low.

So, in other words the point that I am trying to make is that a design problem is really you are trying to weight 2, ideally I would like to make this spring as soft is possible, then I do not put damping in at all and transmissibility can be as low as possible. On the other hand for operating reasons, if beta cannot be less than root two then I have to put in a design, where I have to put in huge amount of damping and keep it as soft as possible. In other words as close to root 2 or make it completely rigid, so that transmissibility ratio comes close to 1.

So, I think that is the force transmissibility problem, we will look at the base isolation problem next time, and we will see that essentially that problem also mathematically boils down to the same problem. And after that I am going to look at to see how the response of a single degree of freedom system can be used to get omega and psi, I did talk about it a little bit, but I will spend a little bit more time talking about how to get the dynamic characteristics of a structure from harmonic force response values.

Thank you very much, see you next time bye.