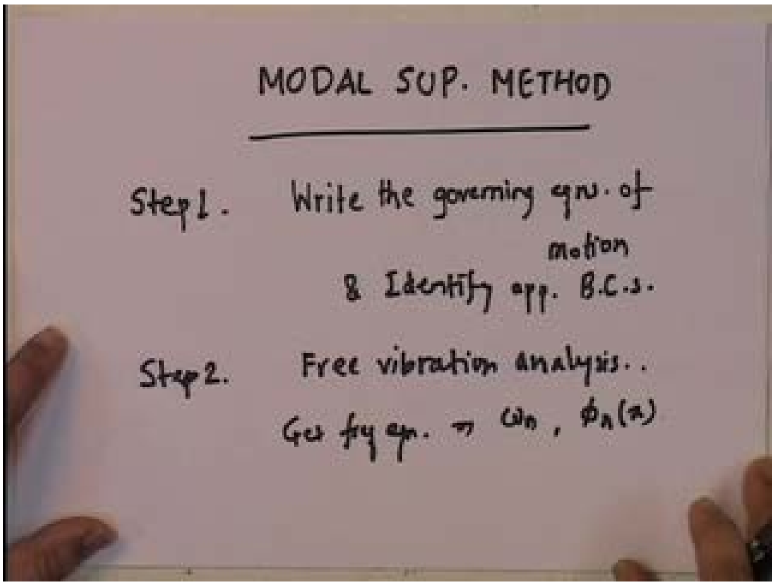
Structural Dynamics
Prof. P. Banerji
Department of Civil Engineering
Indian Institute of Technology, Bombay

Lecture - 37
Dynamic Response for Continuous Systems

Hello there, last time we started discussing, how to obtain the dynamic response of a continuous system. And we are going to be continuing today to be talking at dynamic response for continuous systems.

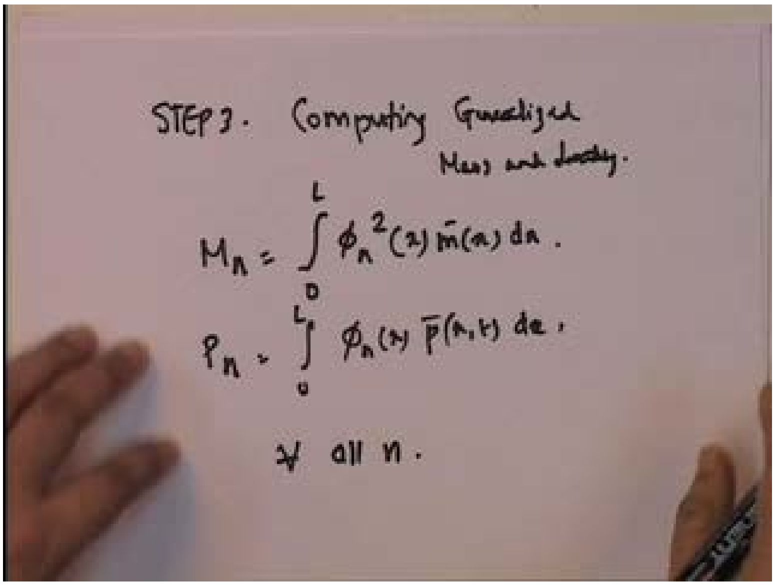
(Refer Slide Time: 00:49)



And just to kind of lay down the specific steps, this is the steps in the modal super position method, the steps. What is step 1? Well, write the governing equations of motion and identify appropriate boundary conditions, this is the first step. Today, in the later part after I have written this, we are going to see how you can develop the governing equations, what we have done is we have developed the governing equations in two situations. One is a beam in a simple beam in actual deformations to develop that equation of motion, and we have also developed the equation of motion for simple flexure.

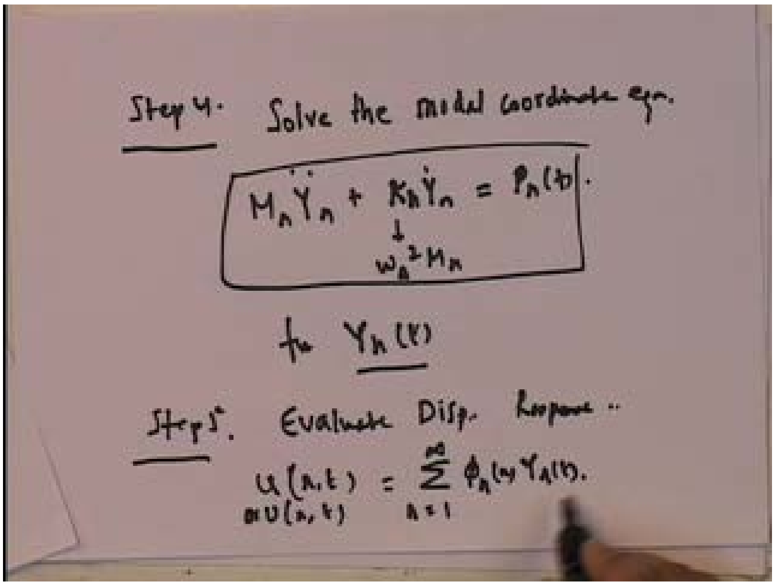
The later half of today, we are going to be developing for another type of beam. Now, in the equations of motion and also seeing what should be their corresponding boundary conditions for it, so therefore that is the first step. The next step is really, so in other words you know get the frequency equation, then derive omega n and phi n x, in particular case we are talking about we are talking about beam and it is always phi n x, so that is step 2.

(Refer Slide Time: 03:05)



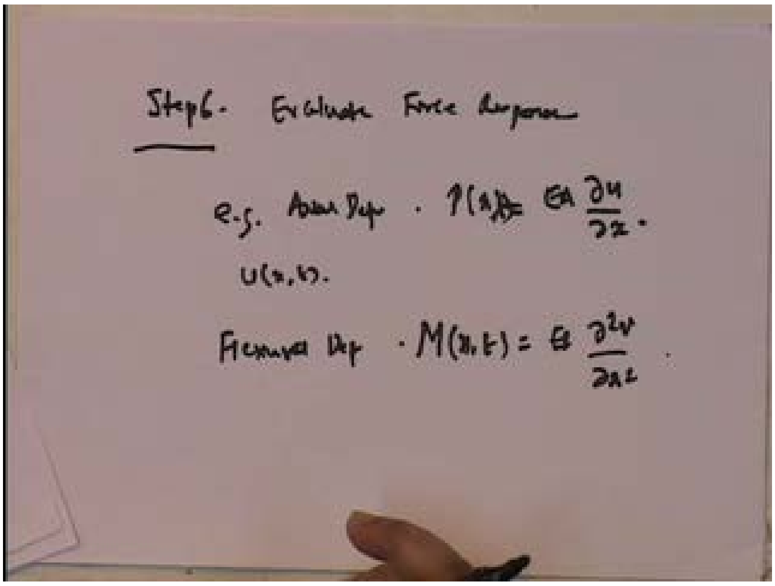
Then step 3 is writing down the computing generalized mass, and loading how do you do that, well M n is equal to integral 0 to L phi n square x into m bar x d x that is M n and p n is equal to 0 to L phi n x p bar x up to t d x; and once you have done once you have found out for all n, once you have done.

(Refer Slide Time: 04:12)



The fourth step in the process is to solve the modal coordinate equation, which is what M n Y n double dot plus k n Y n dot is equal to p n, where k n is actually omega n square M n, solve this is a single degree of freedom, solve this for Y n of t, then step 4. In step 5 is nothing but evaluate displacement response for example, either u x of t or v x of t is equal to n equal to 1 to infinity phi n x Y n of t. So, you have got phi n you know phi n x and you can get that, so that gives you your evaluate displacement response. What is the next step?

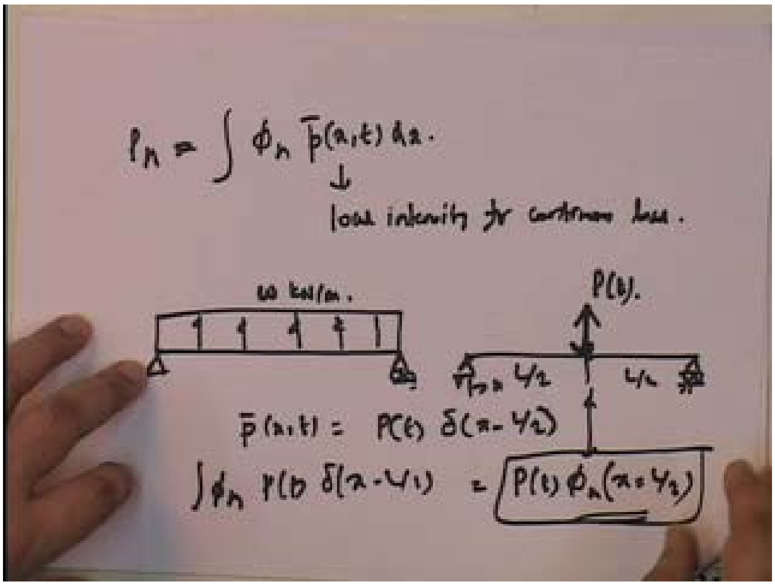
(Refer Slide Time: 06:00)



Well, you had evaluated displacement response the next step is to evaluate, so step 6 is to evaluate force response. For example, actual deformation p of x is equal to EA del u by del x, so now, since you have got u x of t del u by del x into e will give you p x of t. And if you have flexural deformation then movement M x of t would be equal to EI into del square v by del x square, so you can evaluate the force response and that is it.

So, these are your dynamic analysis for a continuous system, first step write down the particular equations of motion and the boundary conditions, second step you write you find out the solve the few vibration problem and obtain omega n and phi n of x. The third step well evaluate the generalized mass and loading which is M n and p n and M n is equal to integral phi n square into m bar x d x and p n is equal to integral over the whole length phi n into p bar x of t d x, and that is what your system looks like.

(Refer Slide Time: 08:15)

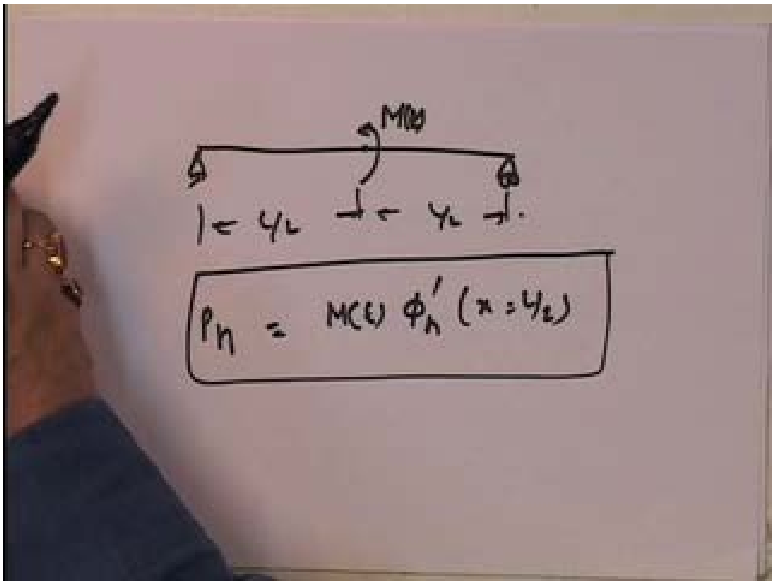


So, now, let me you know I will solve some example, for both flexural response and this thing, but I just like to say one particular thing, and that is p n into phi n this is by definition this is a load intensity for continuous loads. Now, suppose you have a situation where let us say severe like this, let us say if it was this say u do then this would become a constant, and that this p bar would be w, and that is no problem you can do that.

Now, suppose on the other hand you have let us say a load like this, a concentrated load of p how would you solve this problem. Well, there are two ways of solving this problem writing this as a continuous, and then the p bar x t essentially becomes p of t direct x minus L over 2, where x starts from this is like let us say L over 2 L over 2. Then direct function is nothing but a value that is 0 for every value which is not x not equal to L by 2 and it is an infinite value at, so that when you integrate it over the whole length you all you get is when you integrate.

Like let us say phi n into p of t direct into x minus L upon 2, what you get is p over t times phi n x equal to L over 2 this is what you get. Now, note this, what is this as long as you understand that the basic concept in p n, note is what in a way it is the work done by a load as long as you remember that you see, what is this p n into the phi n x of t?

(Refer Slide Time: 11:03)

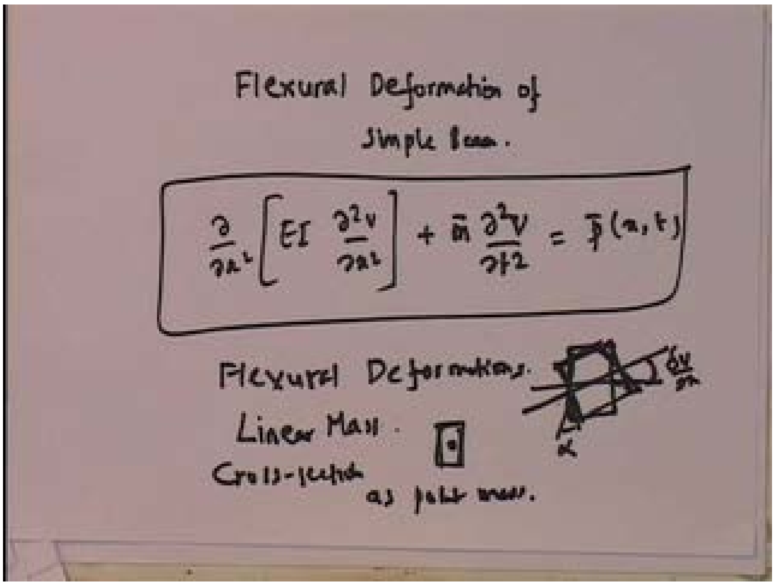


Suppose for example, I had a situation where I was to say this that look, I had a applied movement at L by 2, then what would be p n in this particular case. Well obviously, in the same procedure this would be equal to m of t into phi n prime at x equal to L by 2. So, you understand the point, the point that I am trying to make is that in this p bar phi n is essentially for load intensity for continuous load.

So, this defines a load intensity and of course, if you have continuous load you can always define it in this fashion, and the other way to do it is p bar if it is m of t it will be d prime x minus L over 2 which is that the slope at particular point. So, these are issues that you know we can always handle as long as you realise that, even for continuous systems the load p n really is in terms of being a particular work done in a particular sense, so as long as you understand that there are no issues associated with this particular problem.

Now, what would I like to do this, so that gives you an overview of dynamic response in the next lecture, I am going to actually solve some specific problems on you know maybe towards the end of this lecture, I will define some of the problems. That I will talk, but I will take it up in the next lecture, I have actually solving some problems both for actual deformations and flexural deformations, just to give you an idea of the procedure. The procedure the steps that I have given you this we will need to put it down, so but before I go into those let me look at a particular problem that I had talked about and that is remember the case, where we only looked at flexural deformation of simple beam.

(Refer Slide Time: 13:24)

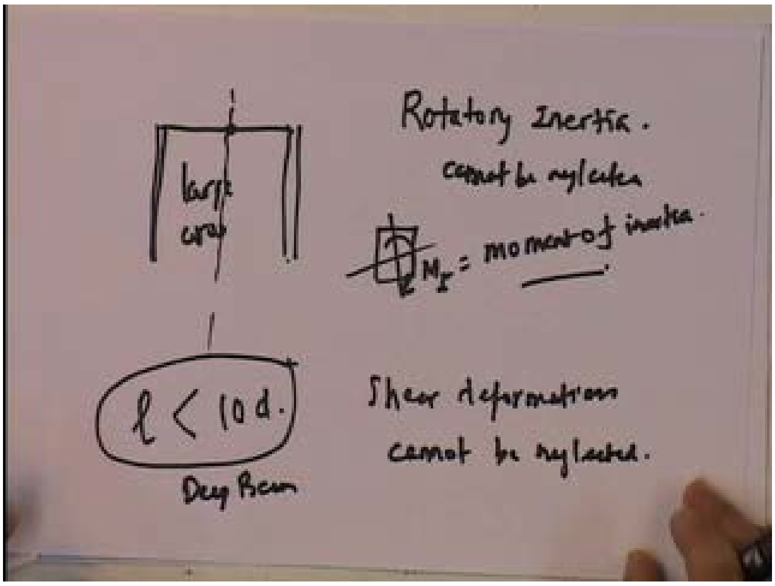


And we got the thing as d upon d x square EI del square v by del x square plus m bar d square v by d t square is equal to p bar x of t this was the equation that we wrote down for a flexural beam. Now, in this particular case this was where we considered only flexural deformations; in other words if you looked at this and the relative motion of this was the let us say that the slope alpha was the same as the slope of the neutral access.

And that is the reason why we could say that the curve which is directly will search only the flexural deformations, and also we only took the fact that the beam was not deep enough for there to be, so the deep the beam was like a point mass, so that it only had cross section as point mass. These are the assumptions that we made in deriving this now by enlarge this is for normal beams, but suppose we are tackling the flexural deformations of a chimney.

A chimney would typically be exceeding large in dimension and therefore, you know it could not if it is a very large dimensional cross-section then it is very difficult to consider it as a point mass at this thing, there is also something called rotator inertia. So, when you have a large dimension cross-section and this is your neutral axis, it is very difficult to say that look this entire mass is concentrated at this point.

(Refer Slide Time: 16:57)

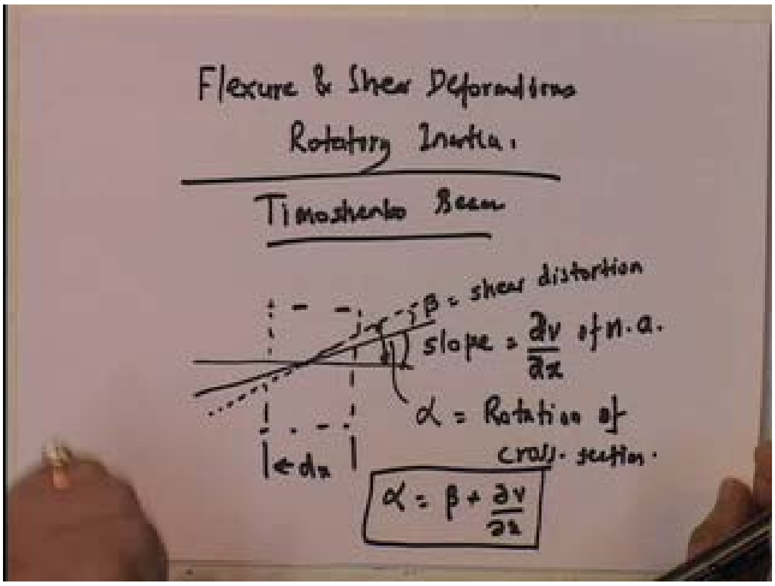


So, in such a situation you have rotatory inertia is one in which if a cross-section rotates, then the there is you know if it rotates in this direction there is a mi setup a moment of inner, you know moment of inertia it is called the moment of inertia. So, moment of inertia comes up and this is something that we have to take care of, and similarly if you have a you know if you have a cross-section which is deep the beam is deep a deep beam is one where l is typically less than 10 d.

If l is less than 10 d the beam is called you know it is not a called as slender it is called as deep beam, and if you have this then shear deformations cannot so large cross-section rotatory inertia cannot be neglected. And here if L is less than d and you have a what is known as a deep beam shear deformation cannot be neglected, so typically what happens is that in this particular case it only had the equations that we looked at the flexural deformations.

In fact, the you know the flexural motion of a simple beam only had a flexural deformations and linear inertia term, now you know if you have a deep beam you not only have to consider of shear deformation, but you also have to consider rotatory inertia. So, the question then becomes is how do I generate the equation of motion, where I consider both flexure and shear deformation and rotatory inertia.

(Refer Slide Time: 19:36)

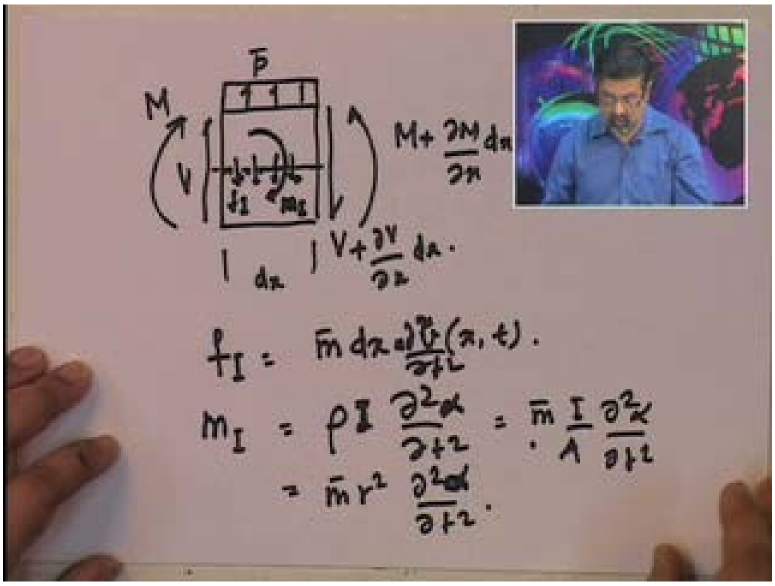


So, this is and typically this beam in regular Palace is called as a Timoshenko beam, so how do we solve this particular problem. So, now let us look at this situation what we have is that is let us look at the neutral axis and you know the cross-section, now under flexural deformations what we have is look the neutral axis itself rotates. So, and we know that this, which is the relative it is a relative motion of the you know, if you have length d x let us just take a length.

So, this length is d x the rotation of this end related to this end slope is equal to d v by d x I will call it del v by del x, because after all v is the function of x and t and so it is del v upon del x. Now, in addition to this what happens is, so you have a situation where this goes like this, and you know this is like this in addition to this if you take shear deformation. Then what you have is that the this part rotates you know this part rotates relative and you know to this and so this is known as the beta which is the shear distortion. This is the distortion of the thing and note that after all the if you look at this motion relative to this motion, these two together is alpha which is the rotation of cross-section. So, what you have in this particular case is the following, is that in this particular case if you look at this is the slope neutral axis.

So, this is the slope of the neutral axis, this is beta and what we have is rotation is alpha is equal to beta plus del v by del x, this is the kinematics of the cross-section. Now, if I look at the equilibrium, so this is the kinematics understand that what I have done over here is the kinematics of the cross-section, under both flexure and a shear deformation. Now, let us look at the equilibrium, we looked at the kinematics, let us look at the equilibrium the equilibrium says the following remember that everywhere, you have to satisfy kinematics you have two satisfy equilibrium and you have to satisfy the force deformation relationships.

(Refer Slide Time: 24:03)

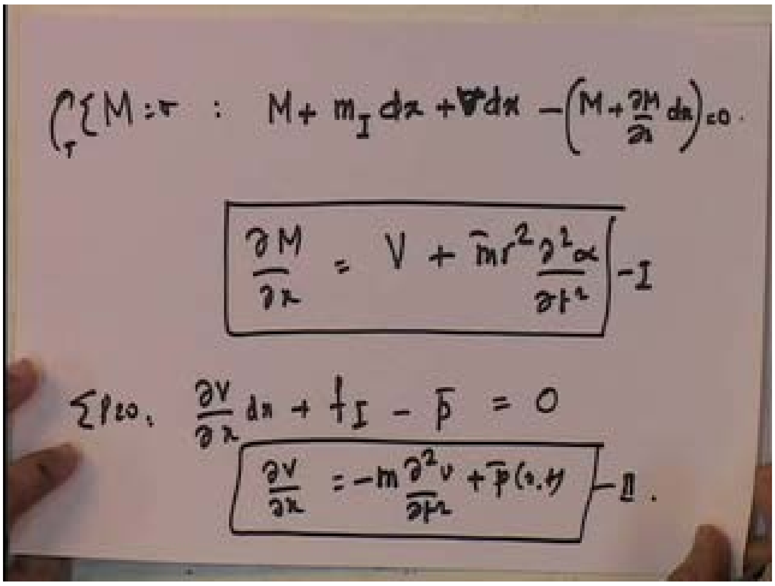


So, let us look at equilibrium this is what happens, these are the forces acting on the body m, this is m plus delta m by delta x into delta x this is length d x. So, I am looking at the equilibrium now so this is there then I have v plus del v by del x into d x, I have the load which is p bar, in addition to that I look at this as neutral axis. I have because this is the motion, then I have what is known as f I in addition this is a rotation, that is going to be a m I, so these are all the forces acting on the body.

Now, what is f I, f I is equal to m bar d x into v double dot x of t, this is what you have as f I, and what is m I, the moment is given by rho which is this thing into I which is the mass moment of inertia rho into I. So, rho is the density I is the moment of inertia of the system into, so this one is actually del square by del t square, so this is the linear and this one is the rotation.

See if your entire cross-section is going by alpha, then this is d square alpha by d, so if you look at this, this is equal to m bar I upon A del square alpha. And I upon A is what radius of direction square, we have already seen that, so this is equal to m bar r square del square alpha by del t square, where alpha is the rotation of the cross-section. So, therefore, this is the total rotation, because our table is the mass only sees the rotation, it does not see how much the, you know the neutral axis knows by etcetera, etcetera. So, these are your f I into m I, let me write down first my equilibrium equation for moment, so if I want to write down the equation equilibrium equation for the moment.

(Refer Slide Time: 27:36)

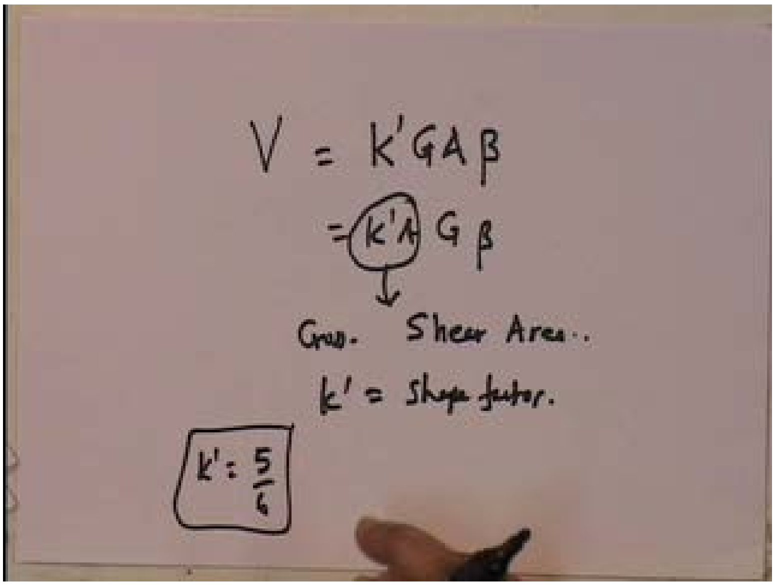


So, you know that is summation moment is equal to 0 what do I get, I get m plus m I, now this m I into d x because of course, this is only at a particular point. Then what else do we have, we have the remaining point which is d v sorry, v into d x I am taking moments about this particular point v d x. And then I have minus, so this one I have taken this as positive minus m into del m by del x into d x the entire thing is equal to 0, this is my equation.

And so if I rewrite this, what I get is dm by d x is equal to v plus m bar r square into del square alpha by this is the angular, so this is my first equation. So, this is going to be this is going to be equal to del v del x plus f I this is summation moment of force is equal to 0 is equal to del v plus f I minus p bar is equal to 0. So, now, this can be rewritten as del v by del x f I is equal to m bar d x into del square, so this is equal to minus m into del square now this one is going to be equal to del square v upon del t square plus p bar x of t, this is my second equation.

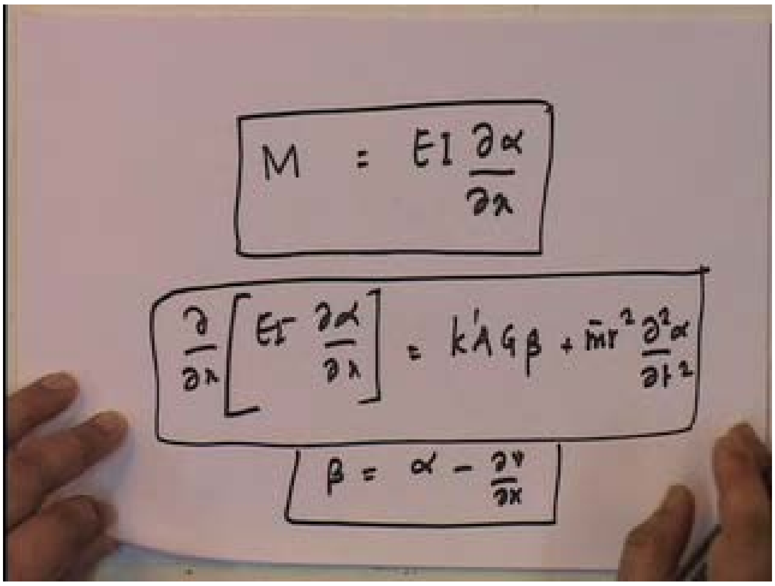
Now, let us look at certain aspects and that is these are what are known as the moment, I mean you know you have to have two relationships you have one is a relationship for v, and one is a relationship for m and these are the force deformation relationships.

(Refer Slide Time: 30:44)



So, according to definition V is equal to K prime G A into beta, where if you look at it sorry, rewrite this as K prime A into G into beta this is nothing but what is known as the cross-sectional shear area. Where, K is a shape factor, and this can be found out for anything for example, for a rectangular cross-section k bar is equal to 5 by 6, this is something that we know already from structural analysis, so I am just rewriting it. So, V equal to K bar G A beta where beta is the shear distortion, so in a way what we are writing is shear force is equal to area into G into A is like a force into the distortion angular distortion.

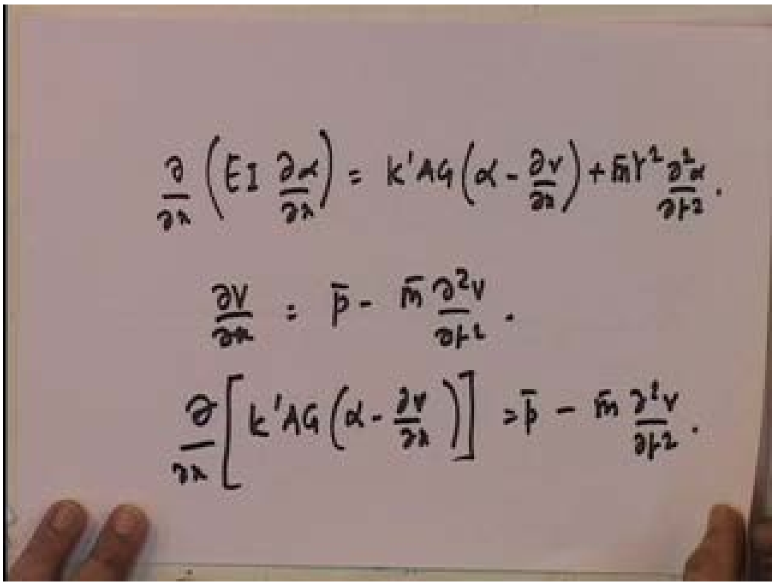
(Refer Slide Time: 32:16)



So, that is what V is and the other one is you know is essentially, what we have as M is equal to EI, and this is where it is a little bit interesting and that is EI into the rate of change of rotation, if the rate of change of rotation of the cross-section, because m is related to the rate of change of acceleration cross-section, so this is this. And of course, here the major fact that comes in is that you have a situation, so now, if we go back to our that moment equation here, this particular equation I can rewrite in this fashion.

This particular fashion del by del x into m, so m is d alpha upon d x is equal to v, v is nothing but K bar A G into beta and plus m bar r square del square alpha into del t square this is what this equation lands up being by substituting this these 2. The v and the so we are we are incorporating, the force deformation relationships into the moment the equation that we got from the moment equilibrium. Now, note very interesting that there seems to be two terms, one alpha and one beta now this well we know that the fact that actually beta can be written in terms of delta, in this fashion. Remember, alpha is equal to beta plus del v by del x, so beta is equal to alpha minus d v by del x where v is this, so this relationship exists.

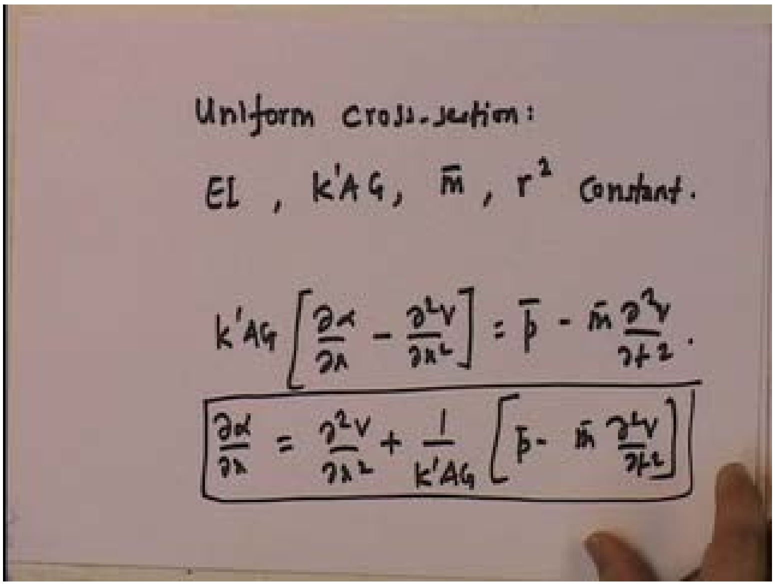
(Refer Slide Time: 34:41)



So, you can actually plug it in here and this equation then becomes nothing but this equation becomes del by del x EI del alpha by del x is equal to K prime A G into alpha minus del v by del x plus m bar r square del square alpha by del t square. Now, you know I understand this equation I would like to write it in terms of v, so I need to find out del alpha by del x in terms of view somewhere. So, how do I do that well let me look at the equation that I got from my this the which was which we had got it as del v by del x is equal to p minus m bar del square v by del x square.

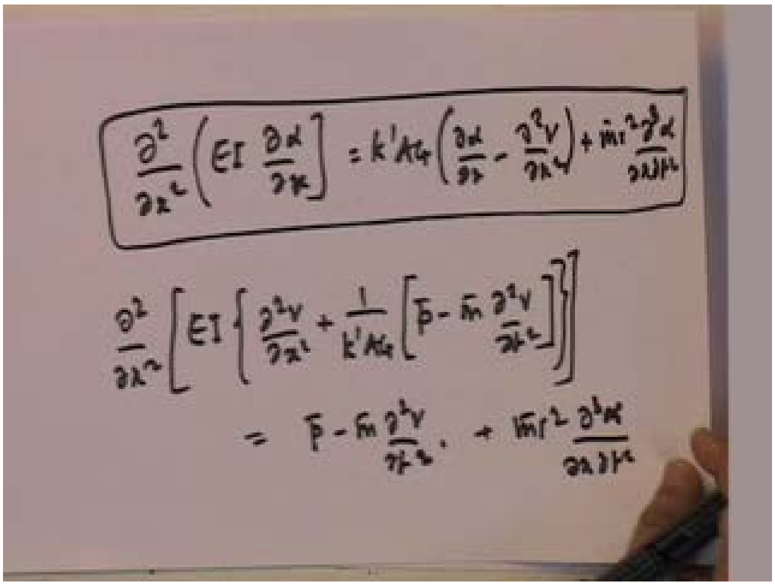
Now, what is del v by del x del v by del x is nothing but v is this so therefore, what we get is del of K prime A G alpha minus del v by del x, that is v is equal to p bar sorry, that is p bar minus m bar del square v by del t square sorry, del by del x. Now, let us assume for now that you know I mean this can this can be actually be taken forward, but let me just assume that EI, I mean this is a uniform cross-section. In otherwise I will have to put it down in some format, but the point that big happens is that if we take it to be I mean I can actually derive it in another way.

(Refer Slide Time: 36:43)



But, suppose I take it to be uniform cross-section then what happens is EI k prime A G m bar r square all are constant. If that is the case then what do we get we get this particular situation the bottom equation becomes what, it becomes K prime A G into del alpha by del x minus del square v by del x square is equal to p bar minus m del square v by del t square. So, in other words if I just put this way I get del alpha by del x is equal to del square v by del x square plus 1 upon k prime A G into p this gives me del alpha by del x in terms of these only these. So, what I need to do now, is that I have got this equation remember, in this I have 1 d alpha by d x, and the other one that I have is that I have an alpha here. So, what I need to do is I need to just differentiate throughout by the you know differentiate by d x, so if I put that in what do I get this particular equation if I put it if I differentiate it twice.

(Refer Slide Time: 39:06)

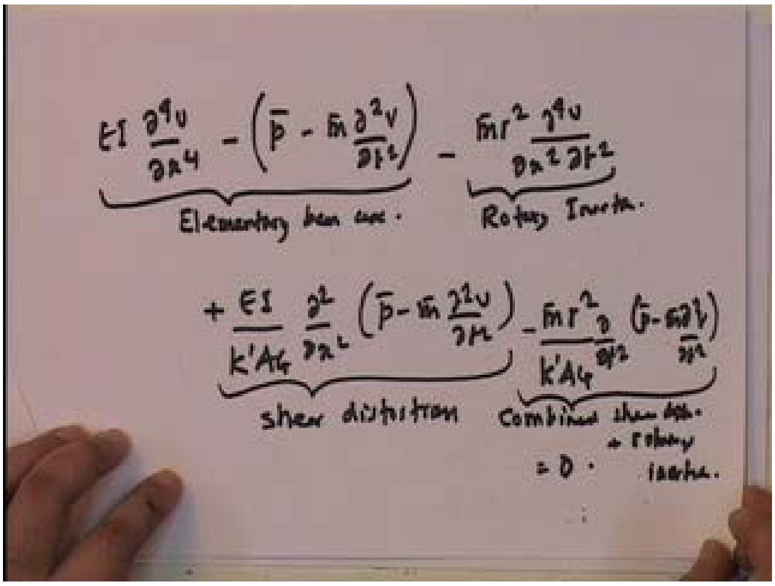


Once with respect to x what I get is the following I get d square by d x square into EI del alpha by del x is equal to K prime A G into del alpha by del x minus del square v by del x plus m bar r square del cube alpha by del x del t square, this is what we get. Now, this in this particular case you substitute the fact that the alpha by d x is given by this term, if you substitute this into this equation what you ultimately get is the following.

Now, in this particular case this becomes del square x minus x square into EI and this del alpha by del x let us just substitute it you get it equal to del square v by del x square plus 1 upon K prime A G into p bar minus m bar d square v by d t square. Then what do you have here, then you have this, then is equal to now this one, if you look at it is equal to nothing but p bar minus m del square v by del t square and this one is m bar r square del cube alpha. And now, so therefore, this one becomes this one needs to be look at a little bit more carefully d x d t square. So, now, this becomes nothing but if you look at it alpha has two parts to it. So, let us just go about it alpha has two parts to it and therefore, this one essentially if you rewrite, this part becomes here all of this comes out and ultimately the equation goes in this format.

I am just write the final equation and I will explain what all of those terms are, there is one EI into del fourth v by del x fourth that comes from the double different sense EI the constant. So, all we get is this is the fourth order term that comes then we have minus p bar minus m bar del square v by del t square that comes from this part which is this part just becomes this, so this part is what I put on the other side.

(Refer Slide Time: 42:09)



So, if you look at this part, this part is the elementary beam case, now in addition to that you have one term which is del fourth v by del x square del t square. This is the term that comes from this particular part, because you see del this del alpha by del x has two parts to it and that is it has it has this. So, this is nothing but if you look at it you can write it as d by d t square of d alpha upon d x this is what this is.

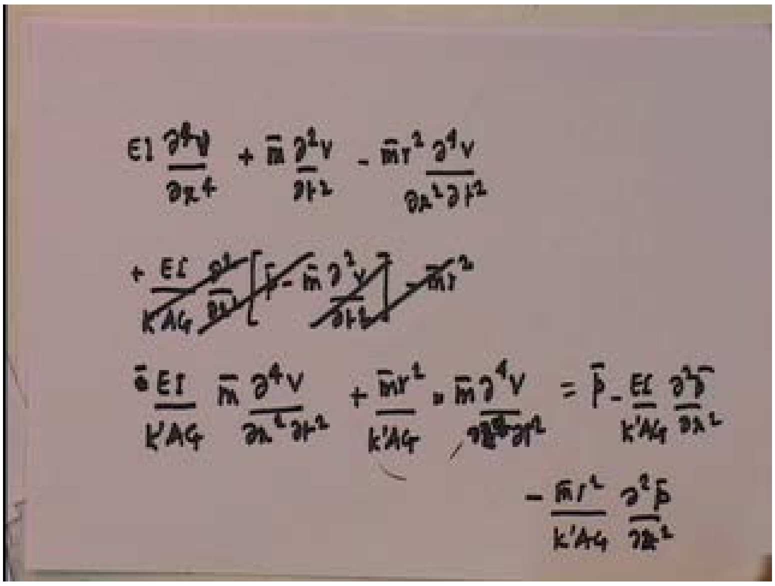
So, if you look at this m r this d cube is nothing but d square of d alpha by d x and so when have when you substitute d alpha by d x from here, you get one term which is just m r del square v by del x square and this is del t square. So, that is this particular term that you get over here this is purely due to rotatory inertia the other part that comes is EI upon K A prime G del square by del x square of p bar minus m d v.

This one comes from the other part and that is that if you look at this part del alpha by del x has this part, which is now also there in this part, so this part is been taken care of. So, that one is EI upon K I G into this is purely the shear distortion part, and now you have another term which is this term being double differentiated. So, what you have is this term double differentiated, so what you have now is the following, it comes out as minus m bar r square upon K prime A G del t square into p minus m bar del square v upon del t square, and this whole thing is equal to 0.

Now, if you look at this what is this one you look at it is purely shear distortion, because if this does not exist if this becomes infinite this disappears, remember that if shear distortion is to be neglected. You are to assume that K prime A G is infinite, this is infinite this goes to 0, so this is purely shear distortion, this term if r square which is rotatory inertia term if that goes to 0 then this term disappears, but if you look at this term.

It is a very interesting term it has both m bar r square, so it has a rotatory inertia part and it also as the shear distortion part, so this actually part is a combined shear distortion plus rotatory inertia. So, therefore, you have a very interesting situation that if I am going to rewrite this in it is proper format, and if I rewrite this in the form that you are aware of then this equation of motion becomes the following EI del fourth v upon del x fourth plus m bar del square v by del t square that is the simple motion.

(Refer Slide Time: 47:50)



Now, in addition you have m bar r fourth d fourth d v square upon now this is a combination of what the x square gives you what, because this is m bar into r square, so that there is a square of distance this part takes care of that distance. So, this in other words is a rotatory inertia purely, then we have plus EI K prime A G into I have all this is this is a minus term m bar del fourth v into del x square into del t square.

So, you see exactly the same only this part is an additional part, which come from distortion then we have minus m bar r square K prime A G this is going to be into m bar del fourth v by del x square. This is del fourth into del t fourth I am sorry, this is x square, so this is del x square del t square, this is another term is equal to now I am putting every other term on the other side and there I have p bar minus EI upon K prime A G into del square p by del x square and minus m bar r square K prime A G into del square p by del x square.

So, what we have here is the following, that we have a fourth order equation that reflects the distortion sorry, the flexural energy. Then we have one term which reflects directly the your linear term the other 3 terms that you see are really also in essence a kind of a inertia, but in a slightly different manor. This part EI upon K prime is also a double a this thing in terms of distance square and this is also distance square.

So, if you look at all of these what you have is that this particular whole thing this part basically becomes m bar into distance square, so that this part is just the distance square. So, therefore, you look at this term it is actually nothing but one part which represents like a massive rotatory inertia term, and on the left hand side also in addition to p bar. You also have terms which are d square p bar by d x square associated with it, one part with this again is a distance square, and this is also distance square, both of them are distance square.

And therefore, left hand side your right hand side is nothing but a load term only thing is that if this solution of this equation becomes that much more complicated. But, none the less this represents an equation of motion, where shear distortion and rotatory inertia are included. Of course, the boundary conditions are going to be exactly the same, the only thing is here please remember that you know if you have a simply supported beam. You have phi prime and phi sorry, phi and moment, but remember moment is EI del alpha by del x and not del v by del x. So, it does not automatically become, it becomes in a in a Timoshenko beam there are no homogenous boundary conditions, all boundary conditions are non-homogenous. Thank you very much, I will stop here as far as Timoshenko beam is concerned from next for the next class, we are going to be solving problems, from a variety of dynamic response for simple beams.

Thank you, bye, bye.