Structural Dynamics
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Lecture - 33
Dynamic Analysis of Buildings

Hello, there last time we started off by talking about dynamic analysis of buildings, and so let me just continue and we showed that if we have you know in buildings you have a situation where you could do 2 d also 3 d analysis, and we showed that you know 2 d analysis can be done, when the centre of mass and the center of stiffness of the building coincide with each other, and that there are no you know should we say what are known as eccentricities in the structure.

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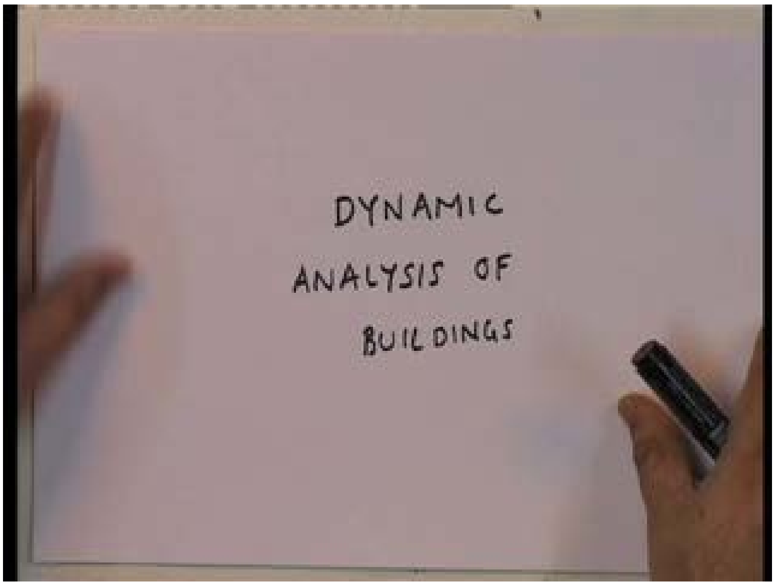

These kind of buildings are actually known also as symmetric plan symmetric buildings, because the centre of mass and the centre of stiffness are you know coincide. So, this called as plan symmetric buildings, and plan symmetric buildings we saw that you did not have to take consider the building as a whole, you can actually do what is known as 2 d analyses where you can consider you know each frame or all frames together, that is not a problem because in one direction all the frames act together and thus you can do a 2 d analysis for the structure.

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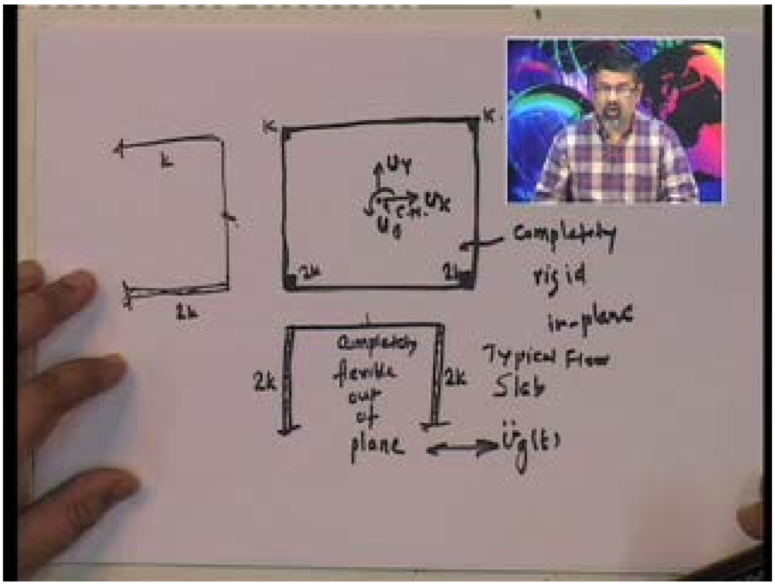

Today, what we are going to be looking at is what happens when you have you do not have plan symmetry in other words you have a situation, where you have to do 3 dimensional analysis of the building. Now typically in such a situation what advantage we take is we take advantage of the fact that typically in buildings the slab at every floor is considered to be rigid in plane, and you know in plane it is completely rigid and a slab is completely flexible out of plane. So, in other words once you have rigidity in plane you can actually define the degrees of freedom in this fashion at any point and typically what we do is we consider U x U y and U theta defined at the centre of mass. So, when you define at the centre of mass what happens is that you can define these three because you know in its plan every point is really defined in terms of these three degrees of freedom.

So, let us take a situation where you have well let us assume that there are no beams and there is just columns this is of course, an approximation, but for now you know I do not want to make it too complicated and we will see, that you have a situation where I will say that this is the slab. So, in this direction and this is completely flexible out of plane, and this is completely rigid in plane it is completely rigid and this is typically what you assume typical floor slab assumption, where it considered to be completely rigid in plane and completely flexible out of plane. So let us assume that in this particular direction this is given by 2 k in other words which ever direction it moves in its 2 k this is also 2 k. So, this is 2 k for movement in any direction and these are square columns. So, in both directions they have the same and this 1 is k and k.

So, if you look at it in this direction this is and this is a thicker. So, we look at this in this direction in this direction both of them have the same in this direction this is 2 k and this is k and this again is completely out. So, you understand it is a one store front and these are our degrees of freedom and let me assume, for now that there is a what should we say we will say that there is ground motion, let us assume ground motion given by V g (t) and this is V x V y and V theta. So, these are so first and foremost given this problem what we need to do is we need to write down the equation of motion how do we write down the equation of motion.

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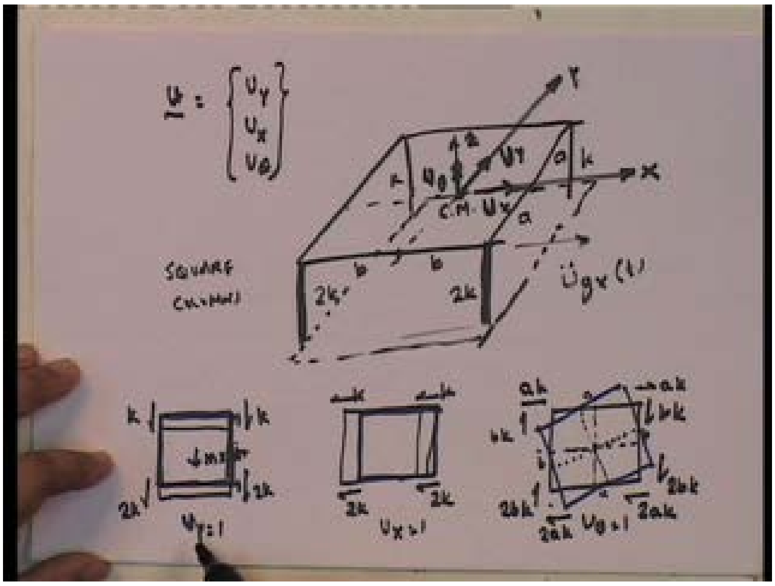

Our degrees of freedom are given as follows v is equal to V y V x V theta and the ground motion is only along the x direction. So, what you have is you have V g x. So, that only x direction this thing. So, if I were to draw this it would be like this let me just draw this and all of these are resting on the same raft. So, this all resting on the same raft this is x direction this is the y direction this is z direction. So, you have U x U y and you have I am going to show this in a vectorial notation sorry V x V y and V theta. So, this is this is in other words this is the rotational rotation is shown in right hand coordinate system in this direction, and this is 2 k in any direction these are all square 2 k and this is k and k.

So, now, and what is happening is that this entire thing is being subjected to V g x of t and this is the center of mass.

So, now, the thing that happens is how do we write the equations of motion well the way to write the equations of motion is to give each displacement quantity and find out what are the forces executed by this and. So, for that I am going to what I am going to do is I am going to show everything in plane. So, and let us also say that this is b b and this is from the centre a a and the centre is at right at the centre. So, it is right at the centre of mass. So, this is b and b and this is a. So, if I look at it in plan and I give u v y equals 1 what happens this goes this way 1. So, what happens is because of this displacement this force is set up as 2 k this force as 2 k this force is set up as k, and this force is set up as k and what we have here is also m into the acceleration.

So what we have now is having put it this way then what let us do let us put x direction. So, if I have x direction displacement I have this way. So, I have again since its gone in this direction I will get 2 k here and I will get 2 k here I will get k here and I will get k here and finally, what we need to do is look at this is v x equal to 1 and the final thing is v theta equal to 1, and if I look at this again please note that this is small displacement. So, if I give theta here what happens if I look at this part if this theta is one then this will move by b this will move b this way, and what about this one this one will also go this way 1. So, this will move by a and this will move by a. So, if that is what happens if you really look at it this is what happens.

So, this point goes this way this point goes this way. So, if you look at this point this point is gone on this way, and this way this point has this point has gone this way and this way. So, essentially if I draw it in blue this is the kind of displacement pattern I will draw all the displacements patterns in blue. So, here see. So, this point goes here let us look at this point this point goes here and up. So, it is here and this point goes down and to the right. So, it comes here. So, if you look at it the displacement pattern looks like this now this may look huge, but note that this is small displacement. So, in other words this point has gone this way by a, and this way by b. So, that is the fundamental point that we have. So, therefore, if I were to draw it since this has gone this way by b.

So at this point I have 2 b k here upwards, since this is gone this way it will also have a force here and that is equal to a into 2 k. So, 2 a k similarly this 1 since it gone up it will be coming down and this is equal to 2 b k, and similarly since it gone this way it will come this way and this is 2 a k similarly this 1, since it gone this way it will go down this way and that is equal to a k this is gone this way. So, this will gone this way and it will be b k similarly since this is gone this way it is gone this way. So, it will have a force in this direction equal to a k and since it is gone up it will have a b k.

So, these are all the forces that we have and for us to find these out all, we need to do is just find out the forces at this point corresponding to this only problem is when you have crossed. So, what we need to do is these are the forces under going these displacements that is how I find out k 1 2. So, let us let us find out first k 11 k 11 is what it is it is in the y direction. So, what you have is all the forces under going this.

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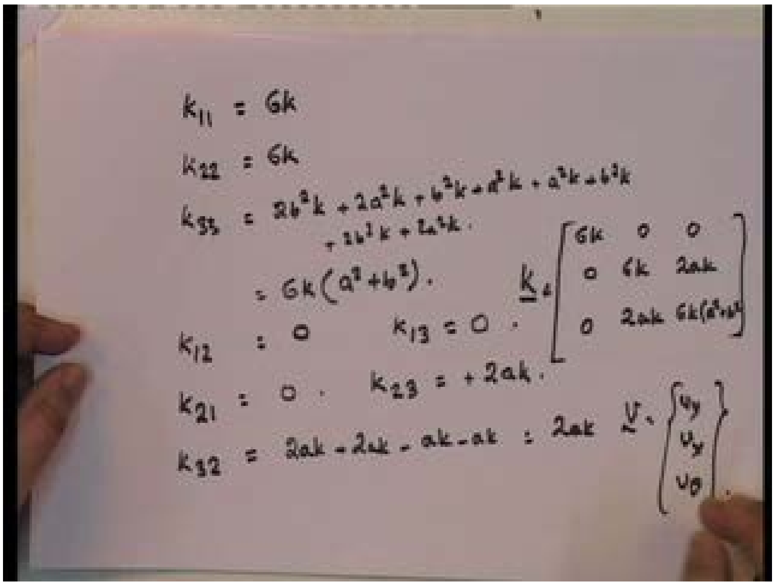

So, you have nothing, but k 11 is essentially 2 plus 2 k plus k 6 k let us look at k 22 22 is what k 22 is going to be give this displacement this is the virtual displacement also this. So, it will become equal to 2 plus 26 k k 33 give it a rotational you know virtual rotation and. So, and find out how much work is undergone. So this is going to be equal to 2 b squared. So, it is going to be 2 b squared k that is for this then for this it is going to be equal to 2 a squared k it will be 2 a squared k, then you are going to have b into this thing. So, this is going to be b squared k plus a squared k plus a squared k plus b squared k plus 2 b squared k plus 2 a squared k.

So, if we add all of those if we look at it 22 plus 13 plus 14 plus 26. So, this becomes equal to 6 let us look at a 2 plus 13 plus 4 plus 26. So, I will put 6 k outside and inside I get a squared plus b squared. So, this is my k 112. Let us now find out k 12 k 12 is what the force at one due to displacement at 2. So, for 12 the displacement is at 2. So, this is the real 1 and this under goes this displacement. So, let us find out what is the work done all the force are in this direction displacement is in this direction work done is 0, similarly let us look at k13 13 is what force you know displacement is 3. So, this is displacement 3 and force at 2 corresponding to this. So, let us find out all the forces they are acting in this direction 2 b k b k and b k and 2 b k. So, this is 0.

so that is the work done because if I make this undergo in this direction these will do one direction work this will do opposite direction work. So, it is equal to 0 now let us look at. So, I have done k 13. So, let us now do k 21 k 21 is; obviously, it has to be 0, but let us see this is this 1 undergoing this displacement. So, the work done is automatically 0 k 22. So, is done k 23 let us look at k 23 23 is what these forces under going this displacement. So, let us look at it so; that means, this 1 is being these are the forces which have been subjected to displacement in this direction. So, the work done will be if I put. So, the work done will be the this thing is in this direction. So, the work done will be 2 k plus 2 k 4 a k minus 2 a k.

So, it is going to be minus sorry its 4 a k minus 2 a k because this is doing positive work this is doing negative work. So, in k become negative. So, this becomes plus 2 a k now let us look at k 23 23 is what these forces under going this displacement let us look at this all of them are in this direction. So, let us just find out what is the work done all of them are a. So, it is a in this direction. So, this 1 does a negative work right this one does negative work this one does positive work this one does positive work so; that means, this is negative and this is positive and then minus of that.

So, this becomes then what 2 a k plus 2 a k. So, it becomes 2 a k plus 2 a k minus a k minus a k which is equal to 2 a k well symmetric; obviously, has to be symmetric. So, we have got our stiffness matrix and the stiffness matrix is what stiffness matrix looks like this 6 k 0 zero; obviously, 0 zero here 6 k 2 a k 2 a k and this one is 6 k a squared plus b squared that is my stiffness matrix for the degrees of freedom a you know U U y V these are corresponding to V x V y V x V theta. Now let us look at the mass matrix the mass matrix since the degrees of freedom are defined at the centre of mass the mass matrix is very simple.

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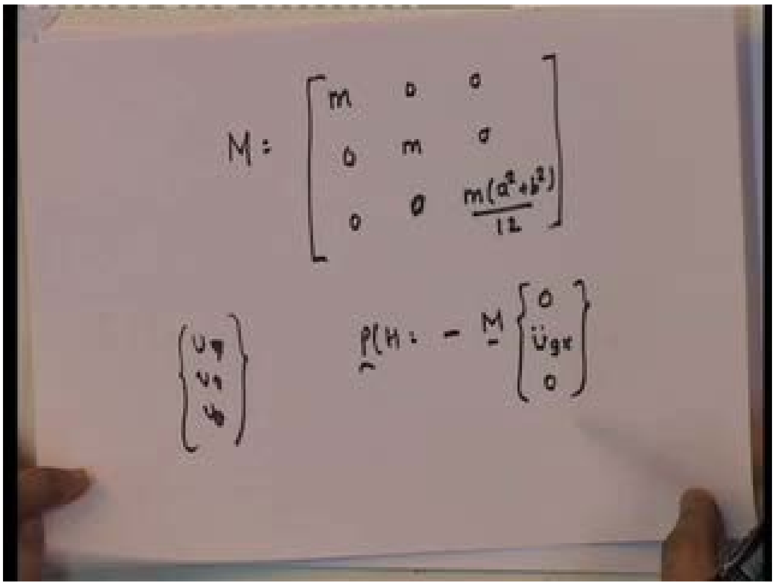

The mass matrix is equal to the linear mass 0 0 0 m 0 0 0 and the mass moment of inertia the mass moment of inertia I am not going to derive this I have already derived it for the linear term it is equal to this remember I had done m l squared by 12. So, this one also turns out to be that. So, I have my this thing, now the question becomes what is my this. So, now, on the opposite side I have U V this U. So, now, this is again corresponding to V y V x and V theta, now if you are looking at the loading vector p this is going to be equal to minus m. Now we need to see the m matrix is this one multiplied by, now let us see what it should be what is the displacement in that this thing this is 0, and this is v double dot g x and corresponding to this is 0.

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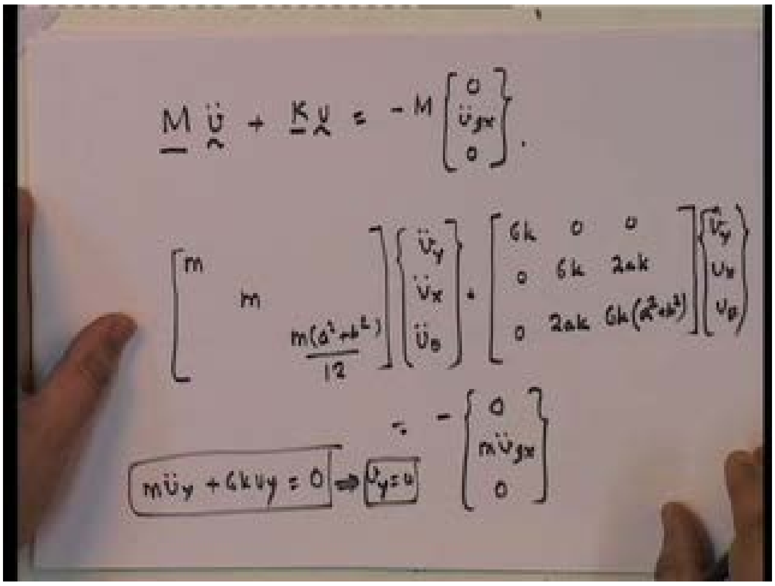

So, this is what the p looks like. So, if you look at it this becomes then equal to what well let us see this is then equal to the following m into V double dot plus k into V is equal to minus m into 0 v double dot g x 0 and this mass matrix the mass matrix is diagonal m m m a squared plus b squared upon 12, and then I will call this as a V y V x V theta plus k is 6 k 0 zero 0 zero 6 k2 a k 2 a k and 6 k a squared plus b squared. In to V x this is not this thing it is V y V x V theta is equal to minus m. So, if you look at this if you look at this into this basically becomes just 0 m v g x 0, now if I look at this first top equation what is the top equation give me m V double dot plus 6 k V y is equal to zero. This implies and this is of course, with this thing this implies that this is equal to 0, and we do not even need to consider that, so then essentially this three degree of freedom.

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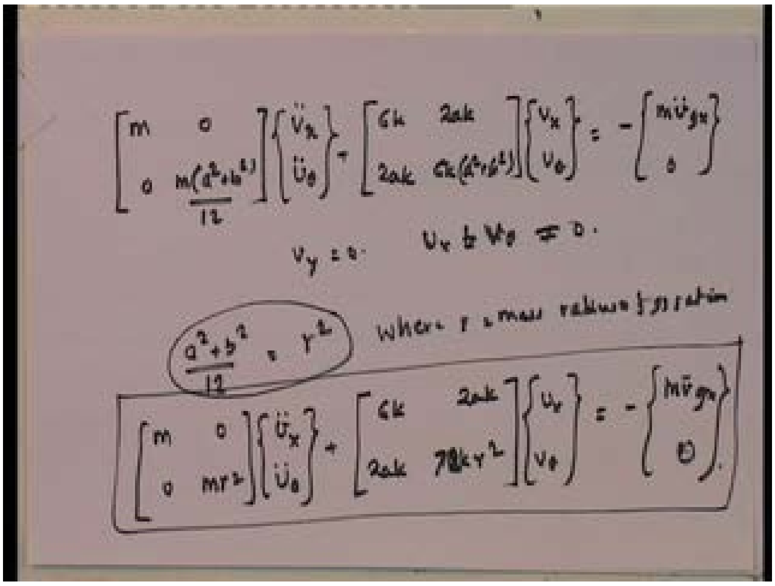

This kind of loading becomes what it becomes equal to the following it becomes equal to a two degree of freedom problem which is m 0 zero m a squared plus b squared upon 12 V x double dot v theta double dot plus 6 k 2 a k 2 a k 6 k into a squared plus b squared V x V theta is equal to minus m V g x 0. Now, note you know it is because of this the 2 a k into v theta that you have coupling. So, essentially all though V y is equal to 0 both V x and U theta V x and V theta are not equal to 0, because of this coupling term if there was no coupling if this was 0 then note that you would get them as separate terms, and you get that as a completely separate problem.

So, now, if you look at it and I am going to define your a squared b squared upon 12 by r squared where r is the mass radius of gyration. So, then this become m r squared. So, the whole problem then becomes m r squared and what we are going to do is do the following yeah does not matter lets go ahead and do it. So, it become m 0 0 m r squared v x v theta plus 6 k 2 a k 2 a k 6 k no a squared. So, this becomes essentially 72 k r squared 72 k r squared into V x V theta is equal to minus m V 0 now let us say that you know my I have defined instead of defining V double. So, this becomes my equation of motion, where r squared is given in by this term. So, this becomes my equation and the advantage of this equation is that is two degrees of freedom, and you know with the x only load and one and 0 for the other. So, what actually happens is if you really look at it this problem is actually a two degree of freedom problem, and I want and I say that look in this particular case.

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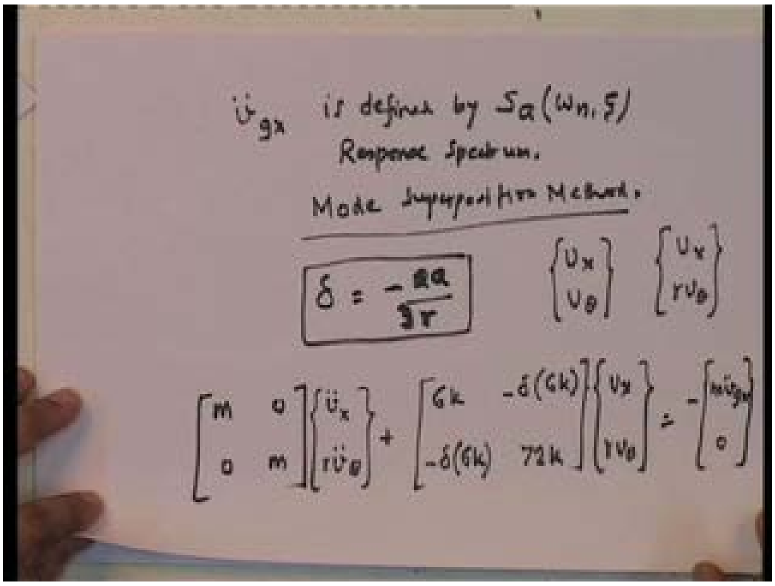

I am giving the fact that is defined by s a. So, response spectrum. So, I am going to do a response spectrum analysis and let us you know rather than put in these values of this thing etcetera let us just assume certain parameters. So, now, first and foremost if I am going to define by this thing, I have to do mode super position mode super position method and let me define certain parameters. Let me define parameter in this fashion I will define a parameter delta which I will say is equal to minus 2 a upon r. So, no it is not two a k let us see the way that defined is its 2 a k upon 6 k. So, that is a upon 3. So, this is going to be a upon 3 r let me define my this parameter, where a upon 3 r and instead of V x and V theta as my degrees of freedom I am going to consider them as V x and r V theta.

So, this becomes an equivalent you know degree of freedom. So, now, if I re write this you will see this in this format this will become m 0 zero m r v theta this will be 6 k minus delta into 6 k minus delta into 6 k so. And over here I will call this as 72 k because my r squared comes in here this becomes V x r V theta is equal to minus m V g x and 0. So, this becomes my new equation you will see that this can be put together now let us let us go ahead and look at this try to solve this 1 hmm and let us assume another parameter another parameter that I will define is.

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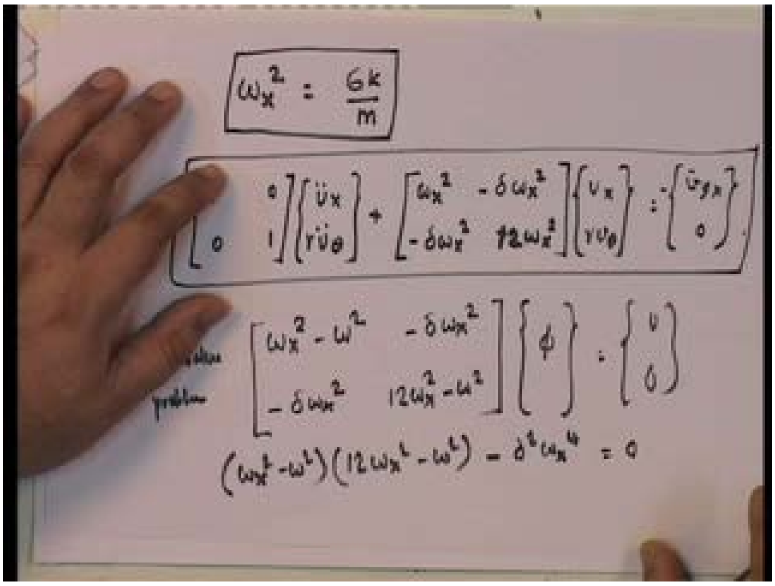

I will define omega x squared and I will define omega x squared by 6 k upon m, I will explain I mean later on we just substituting to make it simpler. So, if I take m out and I take you know m out, and I divide throughout by m my whole equation becomes the following 1 0 0 1 r V theta plus omega x squared minus delta omega x squared minus delta omega x squared and this become seventy 2k upon m. So, this becomes 72 omega x squared 72 by 6. So, this becomes 12 12x squared V x r V theta and is equal to.

Now, I am dividing throughout by m. So, this also disappears. So, this becomes this becomes the equation now this equation is now I am going to how I am going to solve this I am going use mode super position. So, for mode super position I have to first solve the Eigen value problem, let us look at what is the Eigen value problem is the Eigen value problem becomes omega x squared minus omega squared minus delta omega x squared minus delta omega x squared, and here I have 12 omega x squared minus omega squared into phi is equal to 0 0, this is the Eigen value free vibration problem. So, if I put this; that means, this the equation my equation becomes this omega x squared minus omega squared twelve omega x squared minus omega squared minus delta squared omega x fourth is equal to 0.

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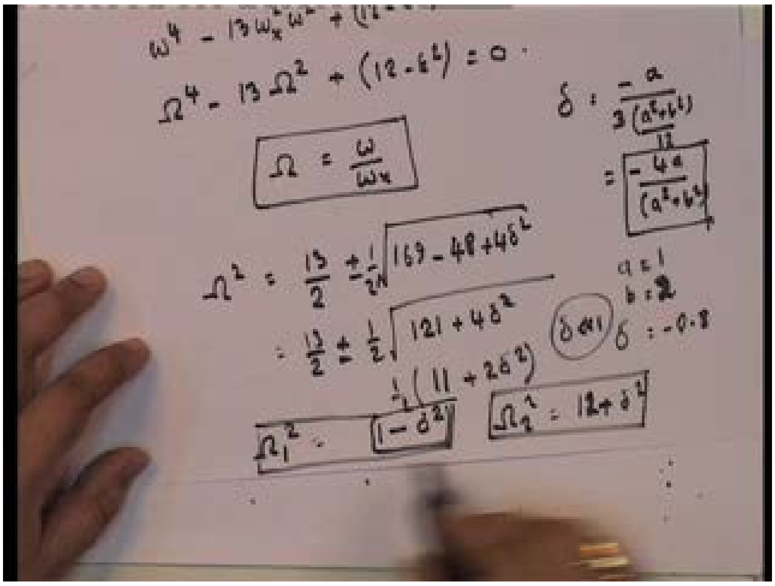

So, now I have the first one becomes. So, my equation my frequency equation becomes the following it becomes omega fourth and then I have minus12 minus. So, this becomes minus 13 omega squared omega squared and then I finally, have omega x fourth. So, that is going to be equal to plus 12 minus delta squared x 4th is equal to 0 now I can actually divide throughout by x fourth. So, what I will get I will divide throughout x fourth. So, I get my equation as in this form where omega is equal to omega upon omega x.

So this is the equation I have done all of this to kind of simplify the equation that I get and this is just a quadratic in omega squared. So, omega squared turns out to be equal to minus b upon 2. So, that becomes 13 upon 2 plus or minus b squared 169 minus 4 minus this minus 4 what is four 48 1 upon 2 here minus 4 that is 48 a is 1. So, this become minus 48 plus four delta squared. So, this basically becomes thirteen by 2 plus or minus half this becomes equal to 121 right plus 4 delta squared and let us look at delta is what delta is equal to minus let us look back at what we have defined it as that was equal to minus a upon 3 r what was r r was a squared plus b squared upon 12.

So, this becomes equal to minus 4 a upon a squared plus b squared this is what my delta is. So, now,; obviously, this 1 I cannot do anything about depending on a and a b squared we can always find out what this is, but essentially if you look at it you will see 2 parameters suppose I put let me put a equal to 1 and b equal to 1 what do I get a equal to 1 and b equal to 2 squared. So, you get here you get 1 over here and this becomes 5. So, this becomes minus 0.8 and. So, therefore, if you plug this in delta becomes 0.8 if you plug this in delta squared this term is actually quite small term.

So if you look at this basically I can neglect this you know and put this as equal to 11 plus 2 delta squared. So, if you look at this in essence half over here it becomes if you look at it just becomes nothing, but 1 plus delta squared. So, if you look at it this one becomes minus if I take the minus 1 this becomes minus. So, it becomes 1 minus delta squared. So, omega 1 squared becomes 1 minus delta squared and if you look at omega 2 squared which is a addition 1 you get 22 you get 11 11 plus 11 plus 224. So, you get 12 12 plus delta squared. So, these if because delta small delta squared becomes small compared to this. So, you can say delta small.

Ok. So, therefore, you get this kind of a term very interesting 1 minus delta squared plus 1 minus delta squared and you can just go ahead and do it and you will get it. So, essentially what we get is the following and that is that you do not. So, this become omega 1 squared and lets plug in these values to get their phi's. So, let us let us plug in this value. So, we will we will see what we get.

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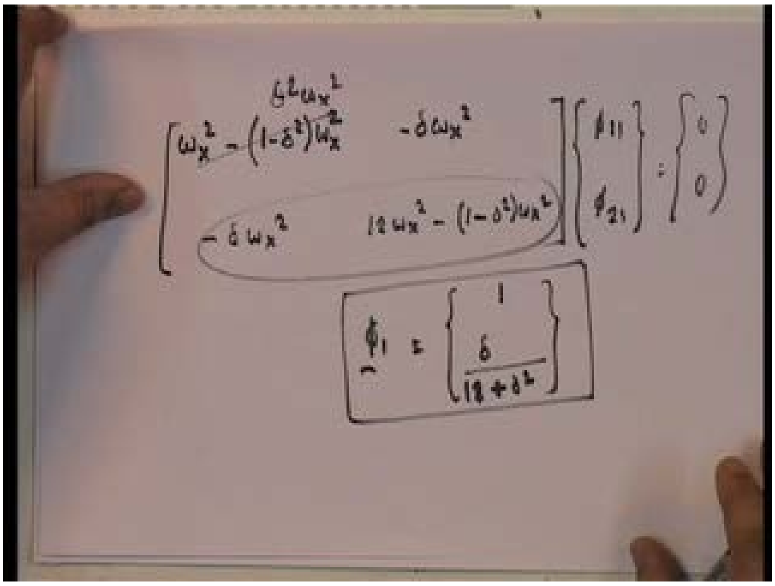

So, if I plug in to this what was the original equation that we have. So, I have omega x squared right. So, this is going to be equal to minus omega squared. So, this is omega 1

So, this becomes 1 minus. So, this becomes 1 minus delta squared omega x squared the other one turns out to be delta omega x squared this becomes minus delta omega x squared and this one if you look at it becomes 12 omega x squared minus 11 minus delta squared into omega x squared and this one if you look at it this is phi 11 phi 2 1 equal to 0 zero now look at this becomes 0 and this is very small. So, this actually goes to 0 or I can actually say well it does not go to 0 it goes to delta squared omega x squared, but you see if delta is small this term actually disappears if this equation that gives you the value. So, what you get is the following you get the following that this minus this upon this gives you if. So, you put a phi 11 what is phi 11 phi 11 is v x. So, you want to put that equal to 1. So, if you put that equal to 1 then the other term basically becomes this plus this.

So, phi 2 1 becomes this upon this. So, essentially what you have is phi 1 becomes equal to 1 and the other 1 becomes delta upon 12 omega x squared and omega x squared cancel out 12 minus 1. So, that becomes 11 plus delta squared that becomes your first term phi 11 and similarly we can get phi 12 the question then becomes is this that lets see let us assume what will happen is if you take this as alpha.

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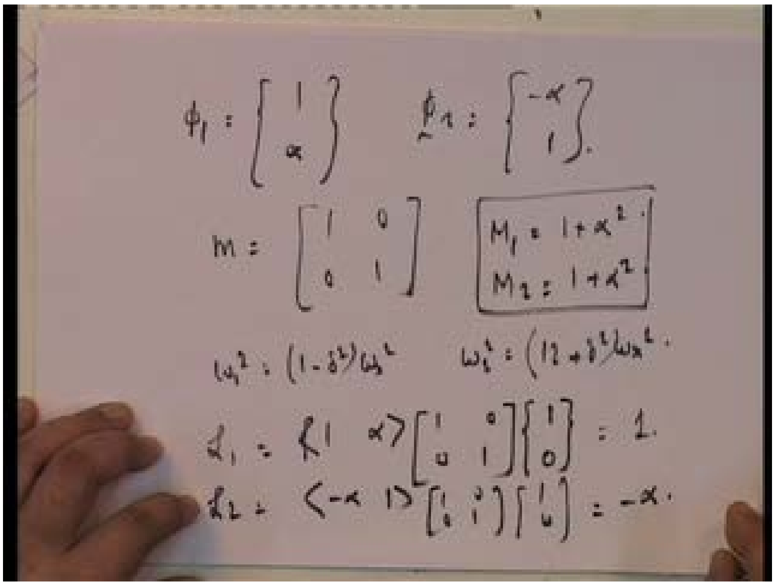

You will see that phi 1 is equal to 1 alpha and phi 2 will be minus alpha 1 and since m is equal to 1 0 zero 1 you will see that they satisfy the conditions and what you get as m 1 is equal to 1plus alpha squared and m 2 is also equal to 1 plus alpha squared where alpha is equal to delta upon 11 plus delta. So, this is what you get as your expression. Now the question then becomes the following and that is that. So, we have got m 1 m 2 we've got phi 1 and phi 2 we have got omega 1 and omega 1 is 1 minus delta squared omega x squared omega 2 is equal to you know 12 plus 2 delta squared omega x squared. So, these are the 2 frequencies these are the m's and. So, what you get is all you need to do is find out l 1 which is what which is equal to phi 1 which is 1 alpha into 1 mass into the r which is equal to 1zero right because r theta is 0 and r the x is 0. So, this becomes then what it becomes 1 this is the unit. So, this becomes 1 alpha. So, l1 becomes equal to 1 and l 2 which is equal to minus alpha alpha 1 0 0 1 1 1 0 becomes equal to minus alpha. So, these are l one and l 2 and so therefore l 1 upon m 1.

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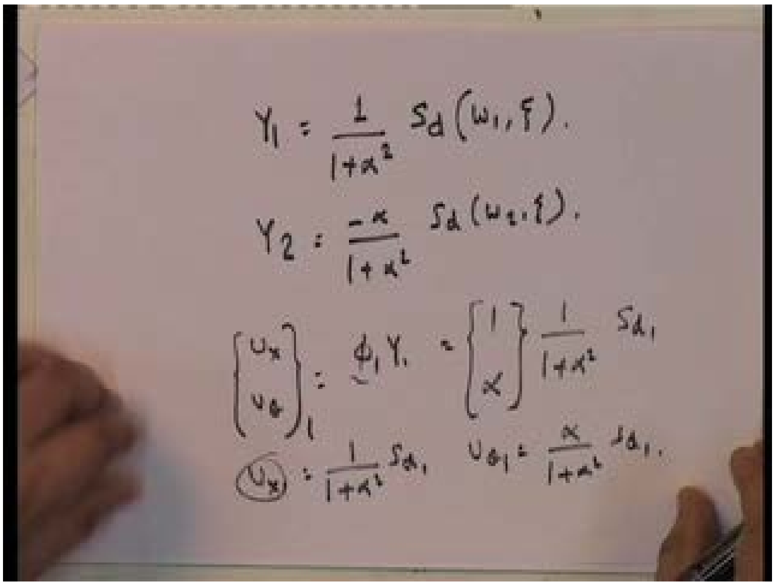

So, if we look at this, what does y1 turn out to be y1 turns out to be equal to let us see y1 is going to be equal to l1. So, that is 1 upon m1 l1 upon l1 into s d omega 1 into psi psi2 is equal to minus alpha 1 plus alpha s d omega 2 into psi and. So, if we want to find out if we want to find out our term and that term is I want to find out what v x and v theta are. So, this 1 into phi 1 right. So, that 1. So, in 1 is going to be equal to phi1 y1. So, that is going to be equal to if you look at it phi 1 is 1alpha into y1. So, that is 1 plus alpha squared s d1. So, if you look at it phi x1 is equal to 1 upon 1 plus alpha squared s d1 and v theta is equal to alpha upon 1 plus alpha. So, this is these are the peak values these are the peak values.

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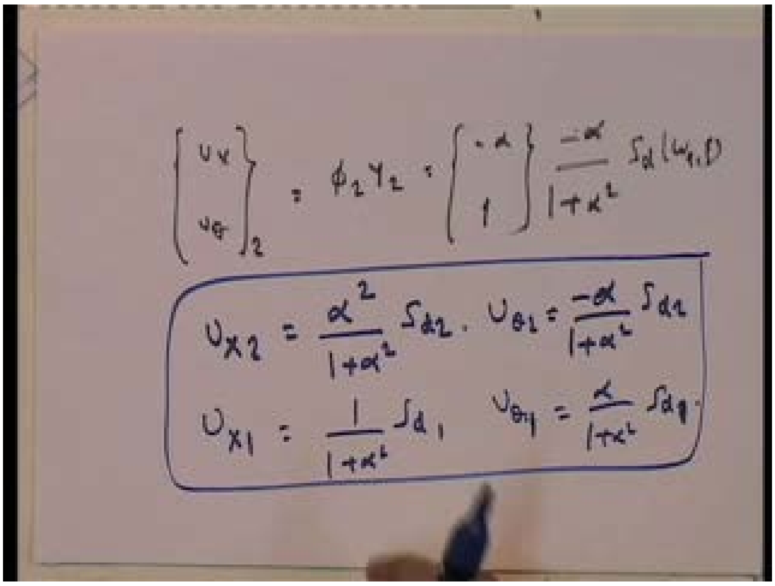

Similarly, if we find out if we find out the v x and v theta and 2 that is equal to phi 2 y2 that is going to be equal to minus alpha 1 into minus alpha into 1 upon alpha squared s d omega 2 psi. So, this becomes equal to v x v x 2 is equal to alpha squared upon 1 plus alpha squared into s d2 where is v theta 2 is equal to minus alpha into 1 plus alpha squared s d 1. So, now, alpha values are I have already plugged put in because you know them. So, you can plug them in and that in essence becomes your fundamental concept over here that v x1 is equal to 1 upon 1 plus alpha s d 1 v theta 1 is equal to alpha into 1 plus alpha s d 2 and essentially you read of your this is s d1. So, v theta 1. So, therefore, this is there.

So, once you read these of you can find out the displacement quantities given s d 1 and s d 2. So, and you know you can read out s d because you know omega 1 and omega 2 and you can find out all of these. so this in essence is what in a multi stored building you have you see you have actually the rotations despite there being only a translational rotation because there is lateral torsional coupling because that coupling you have these kind of terms and I have actually given a example problem to show you that how to go about it. So, now, in essence what you have is that if you have a symmetry in other words the centre of mass and the centre of stiffness do not coincide you have to do a three dimensional analysis and I just kind of went through the procedure of how a three dimensional analysis for a one storey a symmetric plan building is to be done.

So, I went through the entire steps in this also in a way words kind of dealing with response spectrum analysis for a multi storey for a multi degree of freedom problem I hope I have been able to give you an over view of a multi degree of freedom first you need to get the equations of motion I have done that in this particular case next you need to do the free vibration analysis I have kind of done that third you need to do the response analysis there are.

Two ways to do response analysis if you are given the time history of the load you can do response time history or else if you are given some form by which you can get the peak modal amplitude you can use mode you can use modal combination rule to get that of curse I have not applied the modal combination rules here this in over view is dynamic response of multi degree of freedom system problems thank you very much from next lecture I am going to start off on what is known as distributed parameter systems, bye.