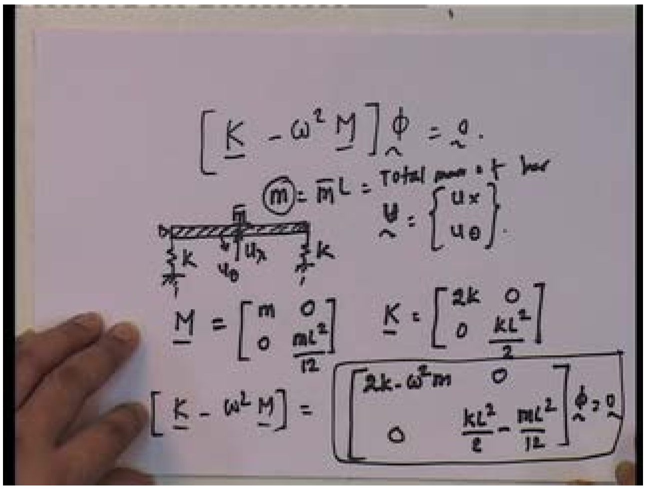
Structural Dynamics
Prof. P. Banerji
Department of Civil Engineering
Indian Institute of Technology, Bombay

Lecture - 26
Free Vibration for Multi Degree of Freedom Structures

(Refer Slide Time: 00:30)

Hello there. So, we have been talking about free vibration and again in this lecture, also I shall continue to talk about free vibration of multi degree of freedom structures or systems. So, let us look at other things. Let us start by looking at what we have already stated and that is essentially we need to solve, easy to solve this equation.

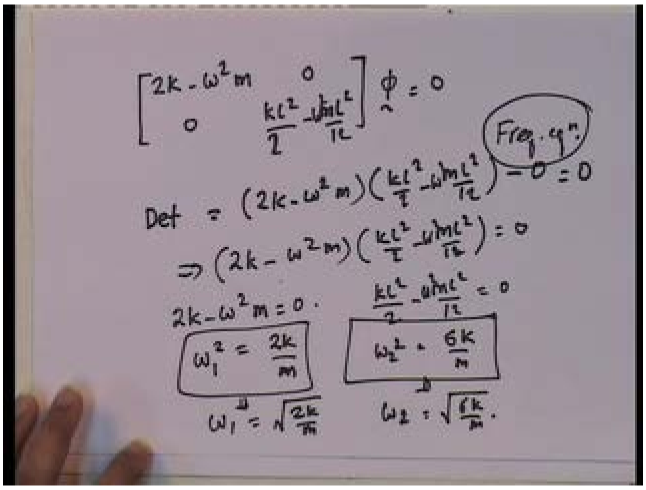
(Refer Slide Time: 01:03)



So, let us start by looking at a problem that we have in a way looked at earlier, and that is let me take the rigid. In this lecture, what I am going to be doing is you know up till now I have been talking in abstraction. I have been saying solve. This is how you get it. Now, I am not talked about how you solve it, and that is a very important part of the entire procedure. So, let us start by taking a simple problem m bar note. This is not allowed move in this direction, and I will call this k and right now, I will call this K. I will develop 2K. Later on let us start with k. We are done k and 2K over here, and we had defined the degrees of freedom one at you know this u x and u z.

So, let us see with v and as u x u theta and again, I have already derived the mass metrics. The mass metrics looks like this, and I am going to use a specific, I am going to say m is equal to m bar L, that is a total mass of bar. So, I am going to use this. So, if you look at it, m bar L k, so therefore, m bar L becomes m. This is 0 L cubed by 12. So, it becomes m L square by 12, and you can derive k the way you know we derived it in this particular case. It turns out to be 2K 0 0. You can derive K i j the way I have derived. I am not going into deriving this thing at every point. So, this is my k. So, now, I have got my m n k. So, what I need to do is write down this k minus omega square m. This is going to be equal to 2K minus omega squared m. Then 0 0 because these are both 0 and this is also 0. So, this is 0 and I have KL squared by 2 minus m L squared by 12 into. So, this is equal to this. So, this into phi is equal to 0 is the solution that I need to do. How do I do that? Well, remember that for us to do that.

(Refer Slide Time: 05:20)



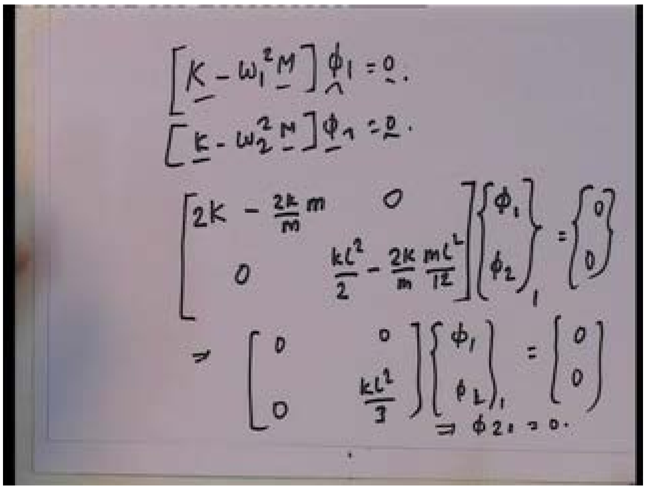
So, I am going to rewrite this here, 2K minus omega squared 0 0 KL squared by 2 minus m L squared by 12 phi is equal to 0. So, obviously determinant of this metrics is equal to 0. So, what is the determinant? Determinant is equal to 2K minus m omega square into KL square by 2 m L squared by 12 minus 0, and this is the determinant that is equal to 0. That is what we are given that for this to be valid determinant of this has to be 0 which basically implies that 2K minus omega square m into KL squared by 2 m L squared by 12 is equal to 0.

Now, note these are factors. So, what does that mean? It means that I have two values. So, let me just write it down first. This one gives me what 2K minus m is equal to 0, and this gives me KL squared by 2 and m L squared by 12 is equal. So, there are two possibilities. This could be 0 or this could be 0. So, if this is 0, then this is what happens. So, therefore, omega 1 squared is equal to 2K 1 m and over here, sorry this is oh my god omega square omega square omega square omega square. So, from this I get omega 2 square is equal to KL square upon 2 upon this. So, this becomes 6 k over m. So, this is equal to 0. Then you know for this to be 0, omega has to be equal to 6 k by m and for this to be 0, omega has to be equal to 2K by m.

So, what are my natural frequencies? My natural frequencies note that this natural is indeed the lowest frequency. So, this is equal to square root upon 2K by m and this one is indeed the higher frequency which is 6 k by m. Note that this problem has two frequencies. It is a two degree of freedom and it has two frequencies. Both are positive and both are distinct from each other. One is root k 2 root upon 2K upon m, and other one is root 6 k upon m.

What is the next step? The next step is this. So, I have to find out these frequencies. So, the determinant that is equal to 0 is known as the frequency equation. So, we solve the frequency equation to get the frequencies. So, you have got the frequencies. Now, next what? Well, for each frequency I need to find out its mode shape. How do I do that? Well, the way I do it is the following. I have got this, right.

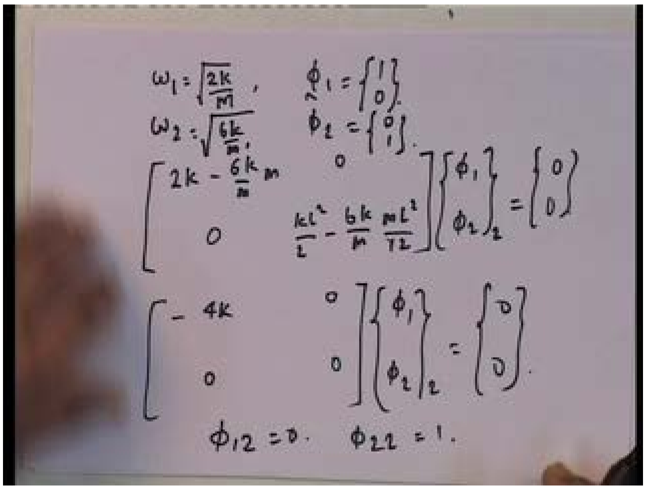
(Refer Slide Time: 09:10)



So, what I have is k minus omega square m into phi 1 is equal to 0, and I have k minus omega 2 squared m phi 2 is equal to 0. So, the first one I plug in omega 1 squared, I plug in omega 1 squared in here. So, if you plug in omega 1 squared in here, what do I get? Well, let us see. So, k, sorry 2K minus omega 1 is 2 K upon m into m 0 0, and this one is equal to KL squared upon 2 minus 2K by m into m L squared by 12, and here phi 1. So, this is phi 1, phi 2 and this is the first mode shape is equal to 0 0. Plug it in here. What you get? This becomes 0 0, and the bottom one become 0. What is this one? M m cancels and this becomes KL squared upon 6, and if you do KL squared upon 6, this becomes KL squared upon 3. KL squared upon 2 minus KL squared upon 6 is nothing but KL squared upon 3.

What is the first equation? It gives you 0 into phi 1 plus 0 into phi 2 is 0. Well, obvious it is a troublous solution. We get nothing from phi 1 and phi 2. What is the second one gives you? The second one gives you that KL squared 3 into phi 2 0 p 1 plus KL squared phi 2 of 1 is equal to 0. So, what is that mean? This implies that phi 2 of 1 is equal to 0. Well, we know that is equal to 0. So, what is phi 1 1? Well, you know everything is in terms of phi 1. Remember I said that you cannot find out any one of them. So, therefore, in this particular case for want of better thing, I take it equal to 0.

(Refer Slide Time: 12:15)

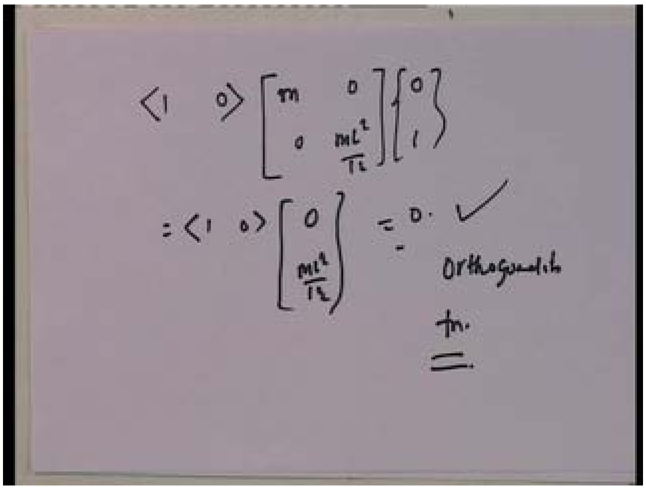


So, that means, I have found out omega 1. Omega 1 is square root upon 2K upon m and phi 1 is equal to 1 0. Now, this is not ortho-normal. It is just a mode shape. Let us look at plug in omega 2. Omega 2 is therefore I have this 2K minus 6 k upon m into m 0 0 KL squared upon 2 minus 6 k upon m into 2 m L squared by 12, and this is the second mode. So, this is the first and the second mode, and this is equal to 0 0. Look at the first equation. What is it says? This becomes equal to minus 4 k 0 0, and look at this 6 k. M m cancel, 6 upon 12 KL squared by 2 KL squared by 2 is 0.

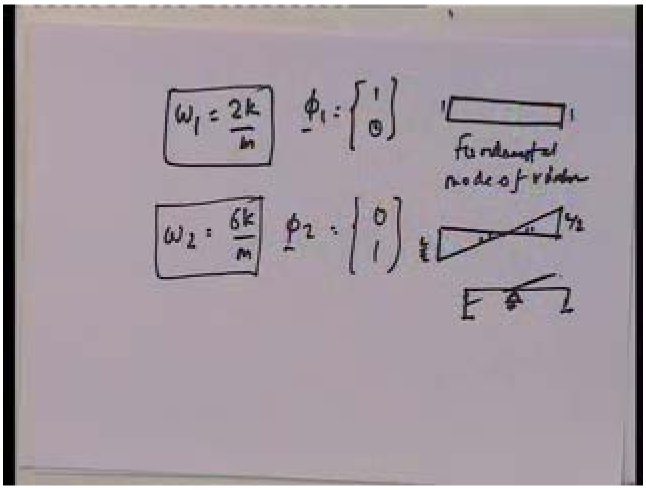
What is the first equation? Tell me that phi 12 is equal to 0, and what is phi 22. Well, the second equation does not give me phi 22 is equal to 1. So, that means, if you look at this omega 2 is equal to square root 6 upon m phi 2 is equal to 0 1. Now, let us look at what these mean. Now, first of all let me try to see if these are orthogonal with respect to each other. Well, let us see are they orthogonal.

Let us put it in 1 0 into the mass. Mass is what mass metrics is m 0 0 m L squared by 12 into 0 1, right. 1 0 is phi 1. So, it is phi 1 transpose m into phi 2. So, if you look at this, this one gives me what this is equal to 1 0 0 into 0. So, this is 0, this 0 into 0 into m L squared by 1 2. Now, do 1 into 0 into 0 is 0. Check orthogonality done. So, orthogonality function is done. Now, next what is it? Well, what is the next step? Well, let us draw what these look like. Let us draw this and if I draw them, let us see what it looks like omega 1 2K by m phi 1 is 1 0.

(Refer Slide Time: 14:25)



(Refer Slide Time: 15:42)



That means u x equal to 1, and u theta is equal to 0. What a kind of mode shape is that? That means the vibration is in this fashion that this is the starting. It goes here, comes down, goes here, come down. Exactly this is what it does. This is how the vibration of, free vibration of this is and omega 2 which is 6 k upon m phi 2 is equal to 0 1. Now, u x is equal to 0, u theta is equal to 1. What kind of a moment is that? So, this one is like 1 1 and this one is like L over 2 L over 2, but if you look at L over 2 L over 2, what does that mean. This we can also say that all that means is that if this unit is, these are L by 2, but that means what kind of a vibration we are looking at about the center, it going to do this only.

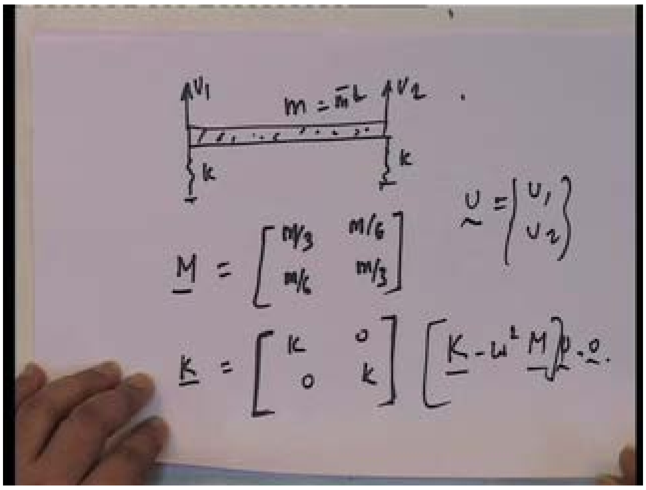
So, you see, remember I had solved the problem which was like this single degree of freedom with this at the center, and we got that this is the only degree of freedom to you.

See this looks like that degree of freedom does not, it is only rotating about the center. So, that is the second mode. This notes that lower frequency is a lower energy mode. This is the easiest thing for it to do, vibrate higher frequency, higher energy. You know the two springs are being rotated in opposite directions that so much higher frequency mode. So, it has a higher frequency.

So, that is the point the reason why we put the lowest. You know there is a reason why we order it whether lowest frequency is the first frequency, and as you go up, you get the increasing order. Reason behind it is that the fundamental, this is the fundamental mode of vibration. See all structures are as lazy as we are, and that is the reason why a structure if allowed to do to vibrate by itself, it will always vibrate in the fundamental mode because that is the lowest frequency mode. You have higher frequencies are associated with higher strain energies, higher energy mode. Difficult to do this. So, rather not do that.

So, that is the reason why the fundamental mode is called the fundamental mode. Why it is called fundamental? Fundamental because left to itself not being subjected to any kind of random motion, left to itself it will always and if you just hit it, ultimately it will just try to vibrate in this fashion. This particular structure we will try to vibrate only in this way.

(Refer Slide Time: 20:19)

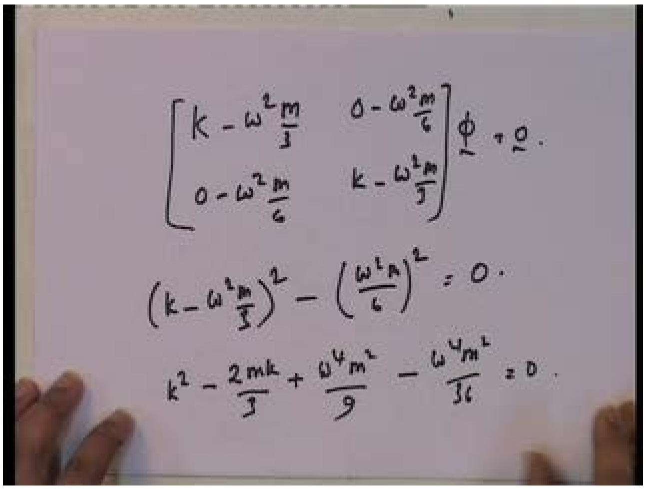


So, that is called the fundamental mode of vibration. A fundamental mode of vibration has a frequency associated with it, and a shape associated with it. So, you understand what the shape means. The shape is not some numbers. It actually means as how does structure vibrates. So, now, let us look at this particular same problem. Let us look at this problem, and let me solve this problem using another degree of freedom, using two other degrees of freedom. Remember we did that. These were defined at the geometric center.

Now, I am going to write this, the same problem, exactly the same problem with mass m. Now, I am saying the mass, uniformly distributed total mass is m which is equal to m bar L, this is k, and my degrees of freedom are defined as v 1 and v 2. Now, again I am not going to go into the details. We have already evaluated the mass metrics for v 1 and v 2, and the mass metrics looks like this m by 3 m bar. It is m bar L by 3 if you remember. So, that I have said is M equals to m by 6, m by 3. I am just using m as easier than m bar L that is all it is not as if there is a mass and that is why I am putting m. It is just that it is m bar l.

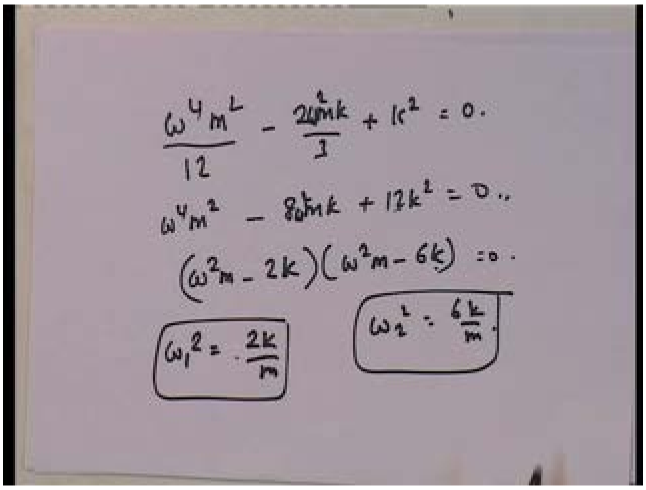
So, I am just replacing it just for writing. It is easier. The k vector you will see just like the way we evaluated is this. So, now corresponding to v, these are the masses differentiated. They look completely different from the mass and stiffness metrics corresponding to the degrees of freedom v x, and v theta defined at the geometric center, completely different. So, I have stated this over and over again that your mass and stiffness and load this, you know I am doing free vibration. There is no load. All of them depend on, what they depend on? They depend on the degree of freedom. The way you define the degree of freedom what you know. Now, let us look at the free vibration. So, here I am going to do k minus omega squared m into phi is equal to 0. I am going to do this. If I do this, what is it become? It becomes k minus omega squared m by 3.

(Refer Slide Time: 22:41)



Now, 0 into omega squared m by 6, then 0 minus omega squared m by 6 and finally, k into omega squared m by 3 this into phi is equal to 0. So, determinant is nothing but k minus omega squared m by 3 squared minus. This is the determinant. The determinant is this into this minus this into this, right. I mean I am not going to find that out. So, now, let us see what this comes out to be equal to? This turns out to be equal to k squared minus 2 m k by 3 plus omega fourth m squared by 9 minus omega fourth m squared by 36 equal to 0. So, therefore, if this is there, see one this is 1 upon 9, 1 upon 36. So, if you take 36 below 4 and minus 1, so that is 3 upon 36. That is twelfth.

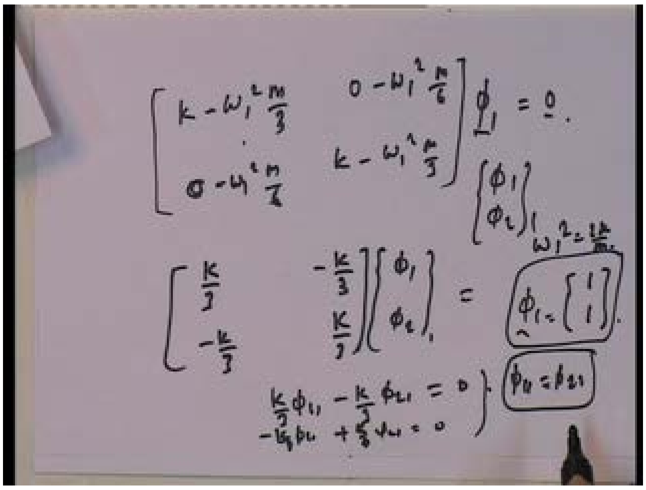
(Refer Slide Time: 24:29)



So, basically this equation then becomes nothing but the following omega fourth m squared upon 12 minus 2 m k by 3 plus k squared is equal to 0. Now, you know I am going to multiply both sides by 12. So, then this becomes w L fourth m squared and then 12 comes here that is 4. So, that becomes 8 m k plus 12 k squared is equal to 0 becomes 12 into 0, still 0. So, now, if you look at this, you know you can factorize it 6 into 26. So, this becomes factorized as omega squared m minus 2K, and omega squared m into 6 k. See that this becomes 12 k and this becomes 2K. Sorry, this omega is a, omega squared here this is omega squared here, omega squared 2 m k upon 3. I always forget that.

So, this is omega squared. If we do that, then this becomes 2 m omega squared m k 6 k 3. You get this is equal to 0. This means that either this or this. So, if this is equal to 0 omega 1 squared is equal to 2K by m omega 2 squared is equal to 6 k by m. Note that both the frequencies are identical, whether we used u v x v theta or we used v 1 and v 2, we got exactly the same expression and that is very interesting. So, therefore, remember I said that the frequencies are invariant. They do not depend on the way we define a degree of freedom, and this has been conclusively proved. We use two 2 m's, 2 k's, completely different. When you find out the Eigen value, you get omega 1 squared is equal to 2K by m omega 2 squared is equal to 6 k by m irrespective of any of the methods that you chosen to do. Now, let us look further. Let us try to find out the frequencies.

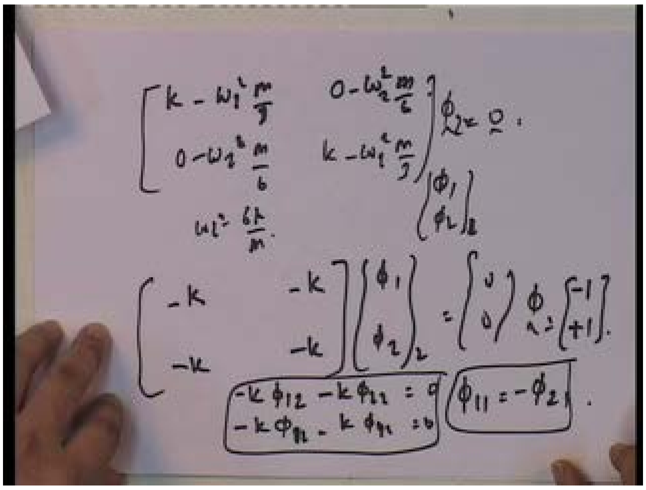
(Refer Slide Time: 27:21)



The mode shapes. How do I find out the mode shape? Well, I plug in k minus omega 1 squared m by 3 0 omega 1 squared m by 6 0 minus omega m by 6 k minus omega 1 squared m by 3 into phi 1 is equal to 0, and a phi 1 is given by phi 1 phi 2 1. So, therefore if you look at this, what you get? Well, let us see put in omega 1 square omega 1 squared is 2K by m 2K by m. M m cancels 2 k. So, this becomes k by 3 omega 1 squared 2K by m. So, this becomes minus k by 3.

Similarly, this becomes minus k by 3 and this becomes k by 3 phi 1 phi 2 1. So, the first equation says that k 3 phi 1 1 minus k by 3 phi 2 1 is equal to 0, and the lower equation gives minus k by 3 phi 1 1 plus k by 3 phi 2 1 is equal to 0. Both of them give that phi 1 1 is equal to phi 2 1. That is all they give. So, what does that mean? My phi 1 is equal to if I take phi 1 1 as 1, my phi 2 1 is also 1. That is all. That means, that is the mode shape corresponding to 2K by m, and the second one we will see what this looks like. You know the mode shape looks very different. The other one you got 1 0. Why? Well, we will see this is the mode shape as it is given. It looks is dependent on the degrees of freedom because here it is v 1 and v 2.

(Refer Slide Time: 29:53)

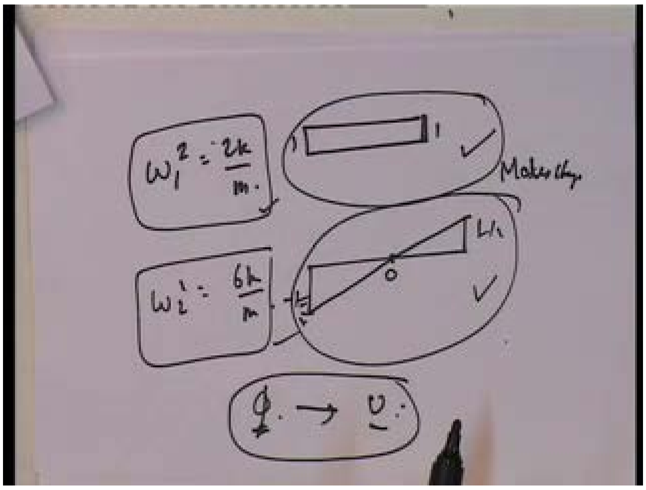


So, therefore, let us go back and try to solve the second one. The second one will give what k minus omega 2 squared m by 3 0 minus omega 2 squared m by 6 0 minus omega 2 squared m upon 6 k minus omega 2 squared m by 3 onto phi 2 is equal to 0. Now, phi 2 is phi 1 phi 2 of second mode. This is corresponding to v 1, this corresponding to v 2 and so therefore, if we put in omega 2 squared is equal to 6 k by m, we plug it in. Let us see what we get? You plug in 6 k by m, so 6 k here, m m cancel 6 k and this is 6 2. So, this becomes minus k here 6 k upon m by m into 6 is minus k.

Similarly, this is minus k, this is minus k phi 1 phi 2 of 2 is equal to 0 0. The first equation gives what minus k phi 1 2 minus k phi 2 2 is equal to 0, the second equation gives phi 1 2 minus k phi 2 2 is equal to 0. Both are identical. You see this is a rank two metrics. You know m and k, but because the determinant what is put equal to 0, you see what is the rank become and minus 1.

So, it becomes rank one metrics. What does that mean is it has only one independent equation. Both equations say the same thing. So, if I put it in, what does it say? It says that phi 1 1 is equal to minus phi 2 1. So, what is phi 2? Well, if I put this as 1, this I get as minus 1 or if I put this as minus 1, I get this as plus 1. It does not matter. You know you can choose any one of them to be one and choose other one to be other, whatever the relationship gives you.

(Refer Slide Time: 32:31)



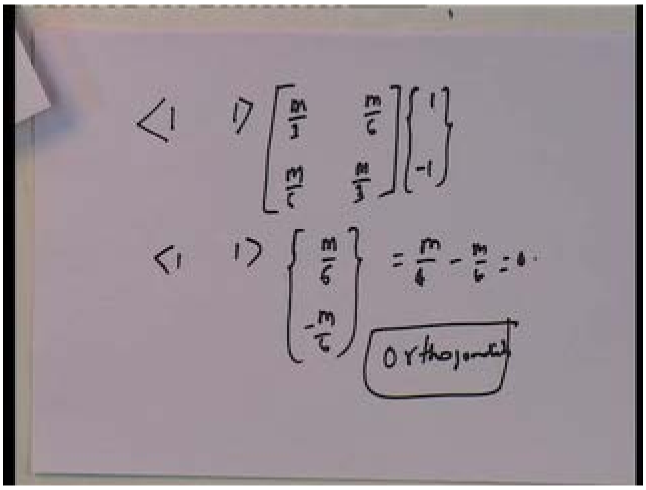
So, let us now look at this mode shapes. It is going to be very interesting. Omega 1 squared 2K by m, this is exactly the same and let us look at what is the mode. The first mode says 1 1, right. What does that mean is 1 1. What is the mode shape? See this and that is the beauty. You know even though the phi's do not look the same, they actually are the same. It is going like this.

What is omega 2 squared say? Omega 2 squared say 6 k upon m. If I do it plus 1 minus 1 plus 1 minus 1, draw it, what you get is 0 at the center. What kind of this thing is? It is exactly the same as what we got earlier and that reference is what it is this property metrics. The m and k, the mass metrics, stiffness matrices and the load, they are corresponding to a specific factor v. So, if you define your phi differently, see attempted on the same problem. I have taken it is a two degree of freedom. So, I can choose any two independent degrees of freedom as long as I do that. It is fine.

I choose first the one as v x and v theta as the two degrees of freedom. Mass metrics was completely different from when I chose v 1 and v 2. Even the k metrics was completely different, but then when I had solve the Eigen value problem and found out the frequencies and mode shapes both of them gave the same omega 1, same omega 2 and the same shapes, and this is what I mean by shape mode. Shape is not the specific. Phi that you get, phi corresponds to a v. The values of phi correspond to a v. Now, you may say well this one is not the same, the other one was L by 2 and minus L by 2. Again, it is a factor.

So, you know this is what I said at the beginning. I could have chosen this as L upon 2, and this would become minus L upon 2 and I get exactly the same thing. It is a shape; the values themselves do not matter. It is only when you get the orthonormal mode, it does matter. It will give the same result. So, that is essentially what you get out of this so. So, therefore, the mode shape and that thing are properties of the structure. Given a structure, the frequencies, the natural frequencies and mode shapes of vibrations are independent of how you define your degrees of freedom. Here, let me show something else. I have got 1 1 and 1 minus 1, and let us see if it is mass orthogonal and stiffness orthogonal. Well, they would be right because they are the same shape, but let us just go through the processes.

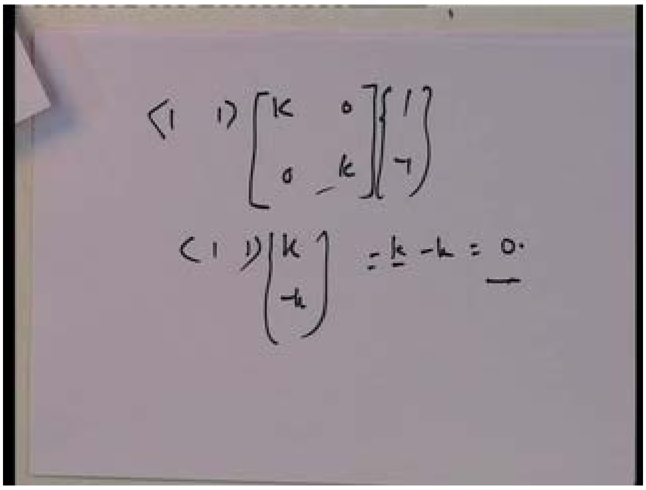
(Refer Slide Time: 36:08)



The processes is phi 1 is equal to 1 1. What is the mass metrics? Mass metrics is m by 3, m by 6, m by 6, m by 3 and phi 2. So, it is phi 1 transpose m phi 2. Phi 2 is 1, minus 1. So, let us see what happens. Let me do this thing first. It is m by 3 minus m by 6. So, this becomes 1 1 and this becomes m by 3. Minus m by 6 is what m by 6. So, this becomes m by 6. Similarly, this is m by 6 minus m by 3. This becomes minus m by 6. If you do this becomes m by 6 minus m by 6 0. So, orthogonality is obtained.

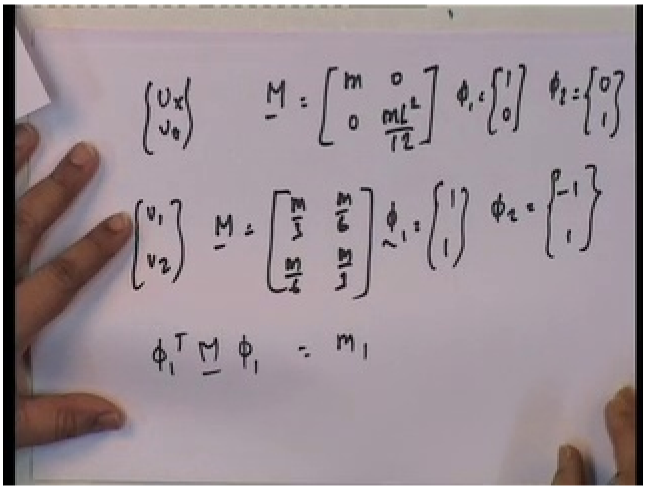
Now, I want to start off and remember I told you that if I use orthonormal mode shapes, you will get and similarly, by the way I did not do the stiffness orthogonality. Let me do this stiffness orthogonality. What is the stiffness metrics? In this particular case, it is equal to, so this is 1 1 and stiffness metrics is k 0 0 k 1 minus 1.

(Refer Slide Time: 37:37)



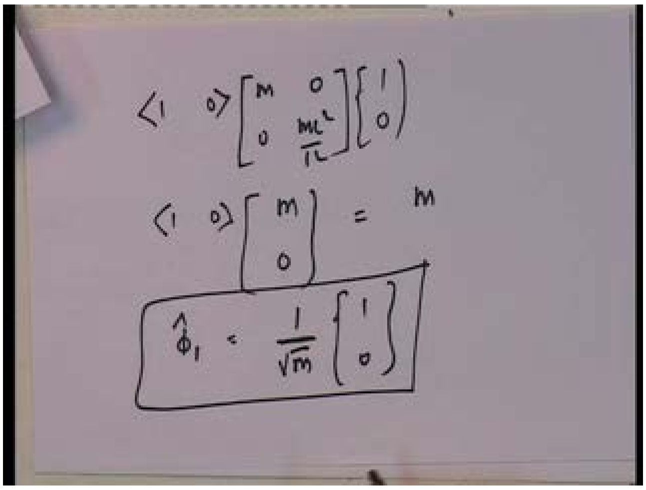
So, this becomes k, this one becomes minus k 1 1 is equal to k minus k is equal to 0. So, all orthogonality is maintained. Now, I want to go back and I want to look at the fact that, remember?

(Refer Slide Time: 38:11)



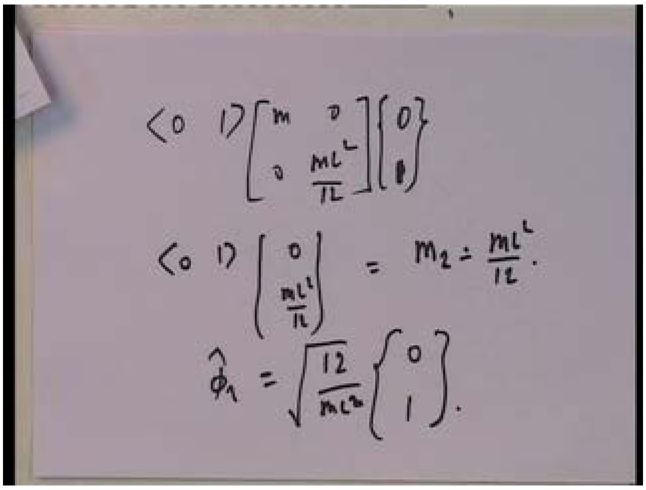
Let us define this where I define v x and v theta my mass metrics was m 0 0 m L squared by 1 2 and my phi 1 that I got was 1 0, and the phi 2 that I got was 0 1. When I chose v 1 and v 2, the two extreme displacements, my m metrics was m by 3, m by 6, m by 6, m by 3 and my phi 1 was 1 1 and phi 2 was minus 1 1. So, let us see whether I can generate orthonormal mode shapes. So, what do I do? Well, I actually do phi 1 transpose m phi 1 and find out m 1.

(Refer Slide Time: 39:36)



Let me do it for the first one. If I do it for the first one, what do I get? I get the following. I get 1 0 into m 0 0 m L squared by 1 2 and 1 0 phi 1 transpose m phi 1. So, let me see what this gives me. 1 0 and from the top, I get m. So, I take m, the bottom 1 0 into 0 0. So, what do I get? I get m. So, my m 1 is m. So, what do I need to do? Well, my orthonormal mode shape would be 1 upon square root of m 1 0. That is my orthonormal mode shape and similarly, my orthonormal second mode shape would be what?

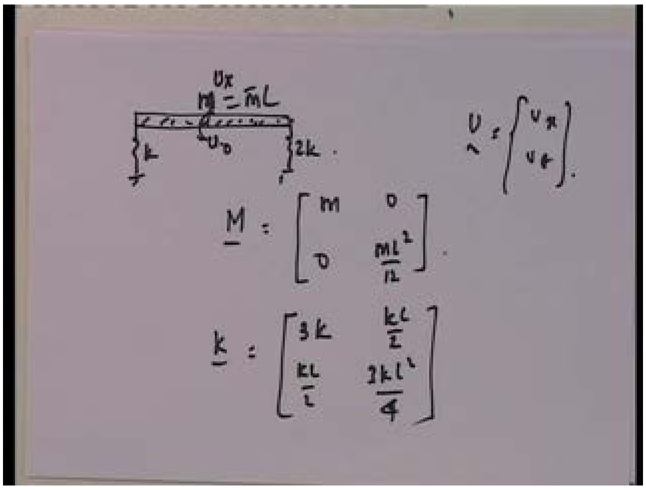
(Refer Slide Time: 40:40)



Let us see it is 0 1, that is phi 2 transpose into m. So, that is m 0 0 m L squared by 12 1 0, sorry 0 1 that is phi 2 transpose m phi 2. So, if I look at this, it become 0 1. Here this becomes 0 into 0. So, top is 0 0 into 1's has become m L squared by 1 2. So, if I do this, my m 2 is equal to m L squared by 1 2. So, what do I need to do? Square root. So, my phi 2 transpose turns out to be equal to 1 2 upon m L square root 0 1. That is my orthonormal mode shape. I am not going to you know. So, this I have just proved it to you and shown it, and I am not going to do any of this. Other one you can do yourself and find out.

I am going to, now look at you see I have solved a problem in which the frequency equation could be factorized. So, I actually got you know omega omega squared minus 2K upon m into omega squared minus 6 k upon m is equal to 0. So, the factors themselves if I can factorize, it I have got my omega. Well, the reason I could factorize it was because that when I developed in terms of as a string, this is actually what is known as symmetric. You know system, it is a symmetric system, so that the lateral displacement and the rotational displacement are on cupped from each other, and therefore, I could factorize it. Let me take another problem in which I cannot do that.

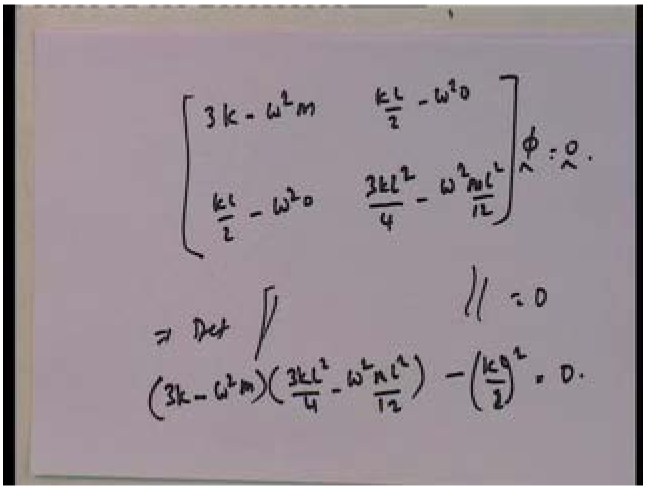
(Refer Slide Time: 42:55)



So, therefore, I go back to that original problem that we had solved in starting, and that was that m m bar. Of course, this was k and this was 2K. Remember that problem? That was the problem that we solved a few lectures ago, and the m metrics came out to be equal to m m bar L is m. Remember that I keep repeating that 0 0 m bar L cubed. So, that becomes m L squared k and k. Look back at your k metrics. You say it is equal to 3 K KL by 2 KL by 2 3 KL squared by 4.

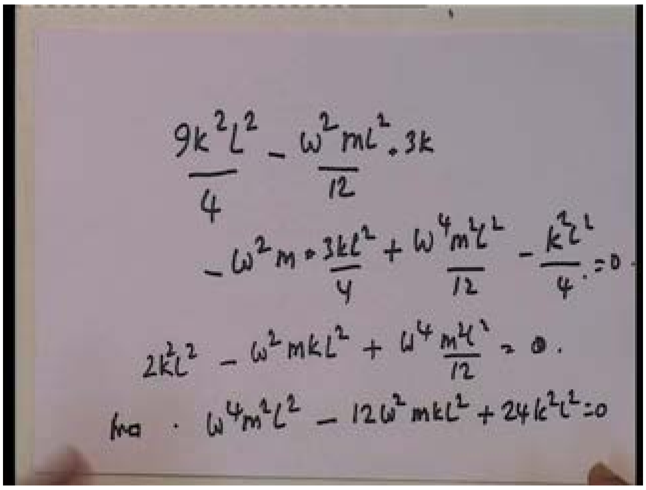
So, these were the property matrices that we got corresponding to v x and v theta. So, v was equal to v x, and v theta defined at the centroid. So, that was what we got. So, now, let us solve this problem. Let us solve the Eigen value problem. So, if we solve the Eigen value problem, this is what you get, k minus omega squared m.

(Refer Slide Time: 44:30)



So, this becomes k k is 3 k minus omega squared m. So, here this is KL by 2 minus omega squared 0, this is KL by 2 minus omega squared 0 and this is 3 KL squared upon 4 minus omega squared m L squared by 1 2. This into phi is equal to 0. This implies the determinant of this is equal to 0, and the determinant is equal to 3 k minus omega squared m into 3 KL k squared upon 4 minus omega squared and m L squared upon 1 2 minus this is 0 0. This becomes minus KL squared equal to 0.

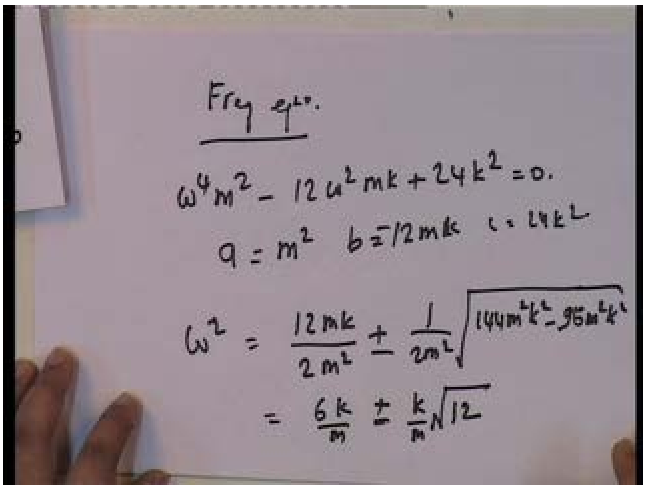
(Refer Slide Time: 45:50)



So, now, let me expand this. If I expand this, what do I get? I get 9 k squared L squared upon 4 minus omega squared m L squared upon 12 into 3 k. Then I have minus omega squared m into 3 KL squared by 4 and then finally, plus omega fourth m squared L squared upon 12 minus k squared L squared upon 4 is equal to 0. So, here if you look at this 9 upon 4 minus 1 upon 4 becomes 2 KL squared 2 K squared L squared, the omega square terms. You will see 3 upon is m L squared upon 4. This is minus 3 by 4 k m KL squared. So, minus 1 by 4, 1 by 4 minus 3 by 4 become minus 1. So, this becomes omega m squared m KL squared. These two added and then plus omega fourth m squared L squared upon 12 equal to 0.

Now, what I am going to do is, I am going to take the entire, I am going to pre-multiply both sides by 12. So, if I do that, this becomes finally the equation frequency equation is omega fourth m squared L squared minus 12 omega squared m KL squared plus 24 k squared. L squared is equal to 0. Now, you see L squared is there in every one of them. So, I can actually put L squared outside and taken the other side, so L squared disappears.

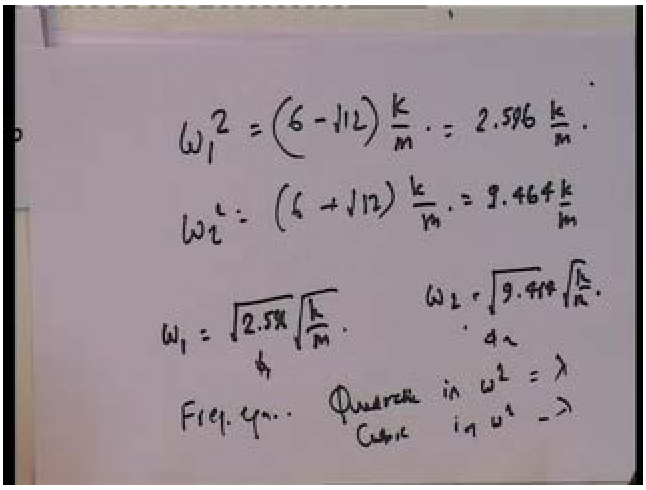
(Refer Slide Time: 48:36)



So, the equation then essentially becomes the following. The equation, the frequency equation essentially becomes the following. It becomes minus 12 omega squared m k plus 24 k squared is equal to 0. Now, this cannot be factor. I mean factorization is standard thing, but we can solve for omega squared, right. This is like a squared omega to the power; you know a omega to the power of fourth minus b omega squared plus c is equal to 0.

So, my a is m squared, my b is minus 12 m k, my c is 24 k squared. So, omega squared which is the basic quantity because omega fourth is omega squared square, this is the quadratic in omega squared. So, omega squared is equal to 12 m k upon 2 a. So, that is 2 m squared plus or minus 1 upon 2 m squared square root b squared. So, that becomes 144 m squared k squared minus 4 a c. 4 a c is 96 m squared k squared. So, this one if you look at, it becomes essentially 48 and 48 is 2 square root this thing. So, this becomes by rewriting, this will become 6 k upon m plus minus m squared k squared also comes out as m k. So, what we are left with is plus minus k upon m into square root of 12. So, these are my omega squared that I get.

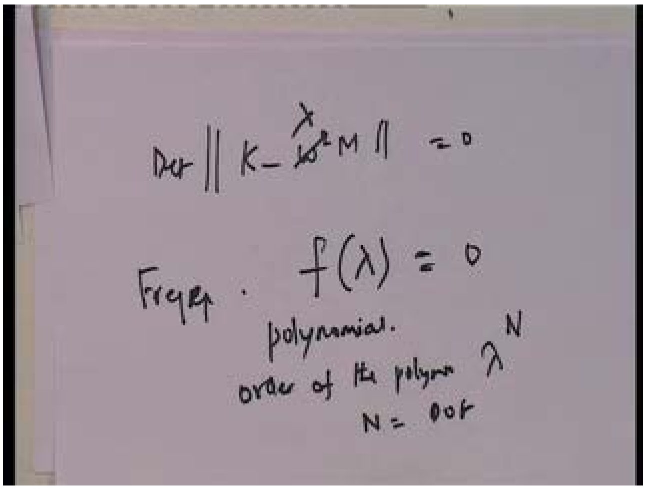
(Refer Slide Time: 50:59)



So, if I solve for this, what do I get? I get omega 1 squared is equal to 6 minus root 12 into k by m omega 2 squared is equal to 6 plus root 12. Obviously, right and if you do this becomes equal 2.596 k upon m and this one becomes 9.464 k upon m. So, omega 1 is equal to square root of 2.598 root k upon m, and omega 2 is equal to square root of 9.464 square root of root upon k. Obviously this is the lower one, this is the higher one. So, what I have shown is that you know every time you see this is the two degree of freedom.

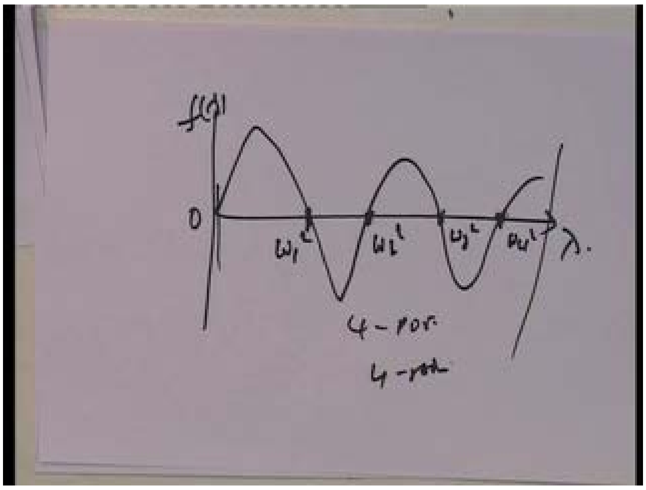
So, what you do is two degree of freedom, the frequency equation will be a quadratic in omega squared. If I have three degrees of freedom, the frequency equation will be cubic in omega squared. So, if I take omega squared as lambda, what I have is, I have ultimately now by the way I am not going to go into details of finding out phi 1 and phi 2. I have just shown you that I have just tried to evaluate omega 1 and omega 2 in a situation, where you do not have a factorization. So, you have to solve the quadratic and quadratic solutions are close form solutions.

(Refer Slide Time: 53:11)



So, I have got those. The question then becomes is that you see when I take the determinant of k minus omega squared m is equal to 0, what you generate is the frequency equation which is functional. I will call this as lambda. So, it is a function of lambda is equal to 0, and this is a polynomial and the order of the polynomial is lambda to the power of n where n is the degrees of freedom. So, you are actually solving a polynomial. You are finding out this basically becomes nothing but what it becomes finding out of the roots of a polynomial. In other words, what we are actually doing is if you look at it pictorially, what we are doing is a polynomial.

(Refer Slide Time: 54:17)



A polynomial function in the 0 in the lambda space. What happens is it is something like this. At 0, this is always 0. I mean you do this and what you are doing is, you are finding out the roots. You are finding out, this is f lambda. You are finding out the roots where f lambda is equal to 0 and what we are saying is that this is odd polynomial of order n. So, therefore, there could be n roots. So, if it is 4 degrees of freedom, there will be 4 roots and these 4 roots, this one is omega 1 squared, this one is omega 2 squared, this one is omega 3 squared, this one is omega 4 squared and this is the way.

So, in essence you know from the determinant, you get a polynomial and it is you know getting the frequency is just nothing but a root finding technique. So, numerically it is nothing but a root finding technique and there are many root finding, numerical root finding techniques. So, if I have n degrees of freedom, I can always find out the n roots and I know that since k and m are positive definite and symmetric that all the roots will lie in the positive lambda plain. That I know and they will be all distinct. So, this in essence is free vibration in it over view. Next time I will introduce you to a couple of basic types of problems and then we will look at how to do response analysis of multi-degree freedom problems.

Thank you. Bye.