Structural Dynamics
Prof. P. Banerji
Department of Civil Engineering
Indian Institute of Technology, Bombay

Lecture - 23
Multi Degree of Freedom Structure Equations of Motion

Hello there, we been discussing for the last couple of lectures on Multi Degree of Freedom System Equations of Motion. And we looked at essentially a rigid bar with 2 springs and we showed, how to develop the equations of motion for different kinds of situations. Today, what we going to be looking at is we are going to continually looking at multi degree of freedom system problems equations of motion, and what we are going to do then is, look at for flexible bodies.

I mean, how do you put together the equations of motion for a structure, which is made out of flexible parties. So, to start off with what I am going to do is, I am going to start off by talking about essentially, how to develop the, should I say the k matrix and the mass matrix for an element or a member in a typical frame. So, let us look at that and so let us look at a frame structure.

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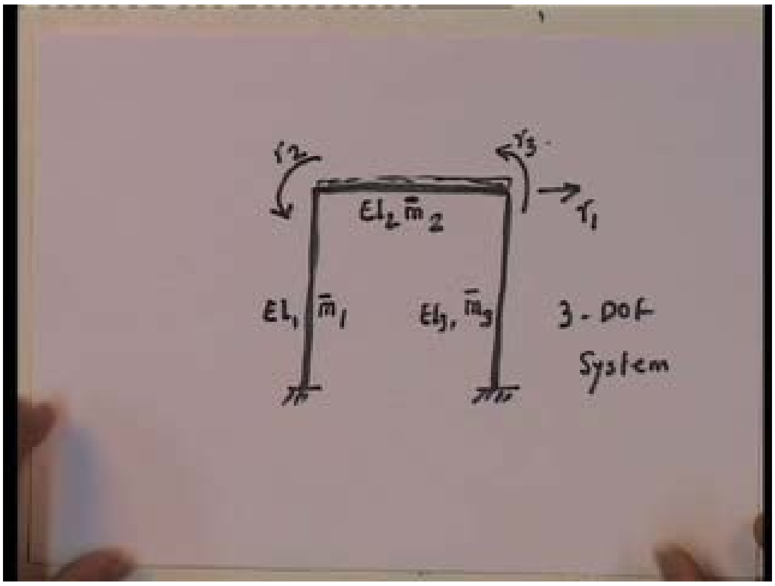

I am going to look at exactly the same structure that I started with, which was one bay single story single bay frame, when we were looking at the single degree of freedom what did I say, I said look the columns are massless, all the mass is at the floor level and this beam is a rigid beam and therefore, we got a single degree of freedom system. Now, what I am saying is, look this is m bar 1, this is m bar 2, this is m bar 3, there may be an applied mass kind of a thing over here, another UDL which is included in this m bar 2.

So, this is not just m bar 2 of the beam, it is also the mass that is coming from the slab, etcetera, so those are the things. Suppose, you have this kind of thing and now, these are all flexible bodies, so I will say that, all of them have E I 1, E I 2, E I 3. Now, how many degrees of freedom does this structure have, if I take each of these as members and I still continue to say that, all of them are what, all of them are actually inextensible. Then from structural analysis we know that, this is a three degree of freedom structure and the three degrees of freedom are this.

So, in other words, what we have over here is, I will call this r 1, r 2, r 3, so there this is a three degree of freedom system. Now, the question becomes that, just like we did in the classical mechanics structural analysis problem, what did we do, we found out the stiffness matrices of each member and then used these member stiffness matrices to find out the structure stiffness matrix. And what I am going to do here is exactly the same, I am going to first start of by looking at a member. And defining here, it is not just a stiffness matrix, you have to define the mass matrix also and we will see, how these are defined.

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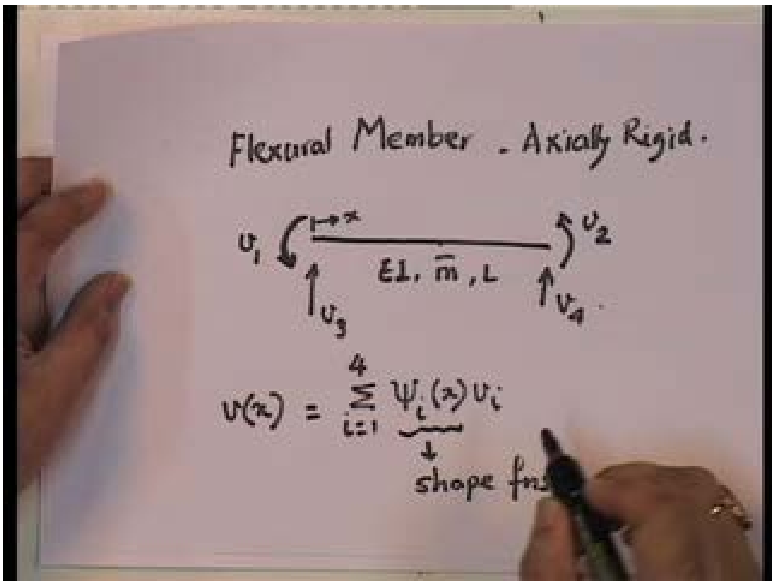

So, let us first start with the definition of the L t or a member, now this member it is actually inextensible. So, how many degrees of freedom does it have, so it has 4, so this is a member and it is a flexural member, which is actually rigid. And what I have is that, this member has E I and m bar as the flexural rigidity and the mass per unit and these are my degrees of freedom and I will call them in this way. I call them, does it is really matter v 1, v 2, v 3, v 4, so I will call these are the four degrees of freedom and so here what we are finding out is the following that, v at x, x starts from here and this is of length L.

So therefore, what we say is v at any point x, in a classical sense if you look at the stiffness approach what is v of x, it is given in terms of this and here it is equal to summation i going from 1 to 4, phi i x into v i, this is what we say is v x, where the psi i that I have here, psi i is nothing but the shape functions. So, each one has it is own shape function and again without having to develop this, because this is done in a structural analysis course. So, we are going to continue, we are not going to develop these i's over here, the structural analysis course are probably develop this shape functions. Here, I will use the standard shape functions that I used in, which are the cubic Hermitian polynomials and if we look at, what the cubic Hermitian polynomials are, they are the following.

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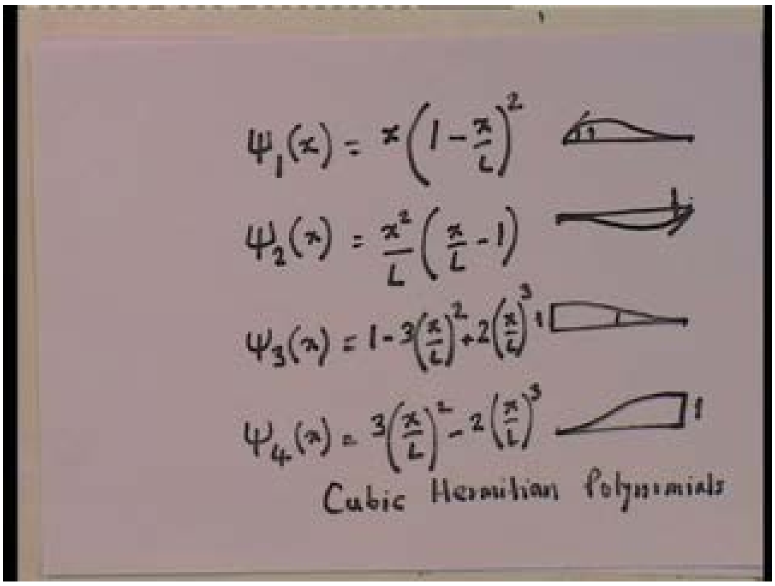

If you look at, if I am given my v 1 as this, this is my v 1 then my psi 1 x is equal to x into 1 minus x by L square. If you look at this, what is this suppose to be, if you look at psi 1 x, this is where, v 1 is equal to 1 and all others are equal to 0. So, this one has to satisfy the following and that is that, if I put at x equal to 0, what is this going to be, it is going to be 0. So, because that is v, v is 0 here and at this point if I put x equal to L, if I put x equal to L, this becomes 0, so this is 0, so it satisfies the boundary conditions.

And similarly, if you put psi 1 x is equal to psi 1 prime x that you differentiate this, you will find that, at x equal to 0, this is equal to 1 and at x equal to L, x is equal to 0. I am not going to go into those details, I am not going to find out again and since psi 2 is this one, so psi 2 is equal to x squared upon L into x by L minus 1. And this one, if you look at it, see this if you look at it, this one becomes like this, where this is 1. So now, here if you notice that, why is it x L minus 1, because if you see, if you put in any value of x which is less than L, it become negative.

You see that it is negative, here if you put x anything less than L, you will get positive, as you see this is positive. Then we have psi 3 x, which is 1 then all other displacement 0 then this one is equal to 1 minus 3 x upon L the whole squared plus 2 x upon L the whole cubed, this is my psi 3 x. And finally, my psi 4, if you look at it again, this is another one, in which if you put x equal to L, you will get 0. And if you put x by L any other value, you will get it as positive.

I mean for example, substitute L upon 2, this becomes half, half become one fourth, this is 3 by 4 and this one becomes half one eighth. So, this is 1 minus three fourth plus one eighth, so that is one fourth plus 1 eighth that is, three eighth, so the value over here is three eighth again. So, if you look at this particular one, this is in this fashion 1 and this is psi 4, psi 4 x is equal to 3 x upon L squared minus 2 x upon L cube. If this one you put x equal to L, you will see this is equal to 1, if you put x equal to 0, you will see that this is 0.

And then you differentiate them, you will see that, both of the slopes are 0 at this point, which is the boundary condition that is supposed to satisfy, so these are actually the classical cubic Hermitian polynomials. Now, with this, how do I find out my k i j, k i j is what, let us let us look at it now, let us start defining certain things.

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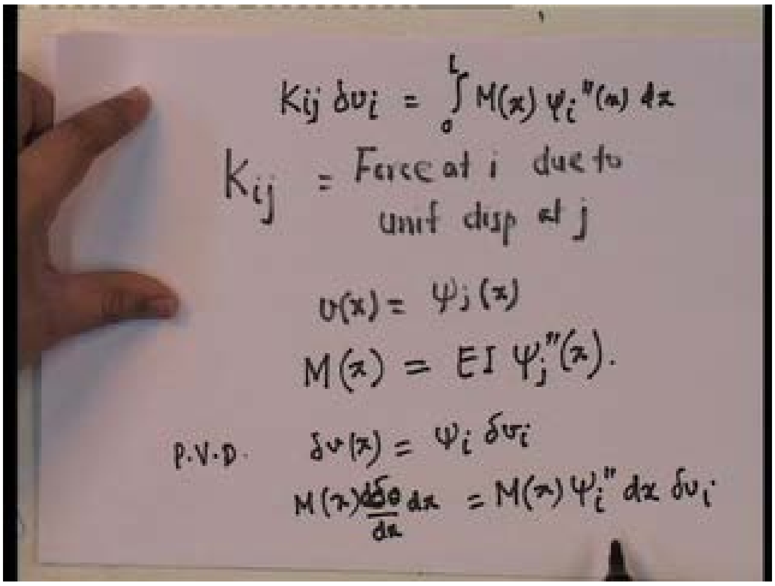

K i j is force at i due to unit displacement at j, so let us look at this, if I give a unit displacement at j then what is the corresponding v x, v x is equal to psi j. And if v x is equal to psi j then what do you have, you have the situation, this is psi j x. And what happens here is that, if v x equal to psi j x then you see what happens to M at x, M at x by definition is equal to E I psi j double prime into x. Now, how do I find out the force at i, I use the principal of virtual displacement so that means, I give a virtual displacement pattern, which has i.

So, what is del v upon del x, it is going to be equal to psi i into del v i, so if I have that then see M x has to be multiplied by what, M x has to be multiplied by the delta theta x by d delta by d x d theta into d x. So, the work done is this, because this is the, what is this one, this is the kind of the change, this one refers to the change of the virtual angle due to this virtual displacement. And we know that, that is equal to what, this is equal to M x into psi i double prime, because it is rate of change of theta is nothing but curvature into d x.

So therefore, if you look at this then by definition, K i j into delta v i is equal to integral from 0 to L M x into psi i double prime x d x. Because, that is at a particular infinitesimal length and then this is to be integrated over the whole length. So, if I look at that then this one will, there is a del v i, because this is a del v i here.

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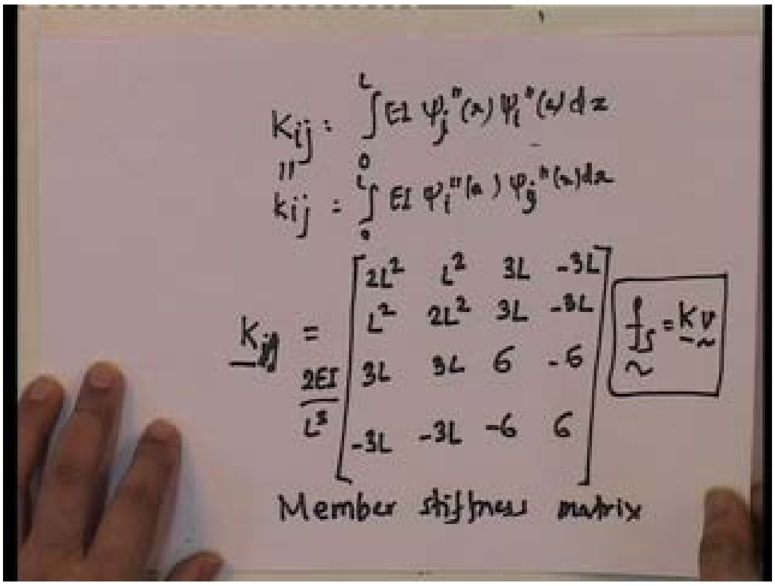

So, if you look at it, M x is equal to this and ultimately what is K i j equal to, ultimately K i j is equal to integration from 0 to L, E I psi j double prime psi i double prime d x, this is what we get. Now, note that, this one represents M x and this one represents the virtual kind of thing. So, K i j is actually this, if you look at this, K i j is equal to 0 to L, now here the real moment is given by this and this is the virtual displacement, we will see them actually the same thing.

So therefore, this is equal to this and remember that, we talked about this mass matrix means symmetric, even here the I mean, the stiffness matrix and mass matrix are symmetric, here also they are symmetric. So, if I have done that then ultimately what does my equation look like, my K i j, not K i j, K the stiffness matrix, the stiffness matrix looks like this, it is got... Now, if I am going to put it in this format then I am going to put 2 E I upon L cubed outside, I am going to put this outside.

So, if I put this outside then inside this becomes, actually this one is nothing but 4 E I by L, this one is 2 E I by L, this is 2 E I by L, so you will see that, this is what you get, this is 4 E I by L. We know this, I am just filling this up, so this is what it is and then this one if you look at the left hand, it is equal to, when you go this, this one becomes positive. So, this is equal to 6 E I by L squared, this is minus 6 E I by L squared, similarly this one becomes when you do this, again this one goes this way, so this is 6 E I by L squared and this is minus 6 E I by L squared.

And if you look at this one, this is 6 E I I mean, these have to be symmetric with respect to each other, so this is what you get and what do you have here, 12 v i by L cubed, so this is 12 v i by L cubed and this one is minus 12 v i by L cubed, minus 12 v i by L cubed, 12 v i by L cubed. So, this is what my K looks like and what I have ultimately is that, my f s, the forces are given by K into the v. So, this is my the stiffness matrix corresponding this, this is the member level stiffness matrix. And I have just gone through these steps, purely because this is something that we know from earlier, but it was a good idea to go through the process.

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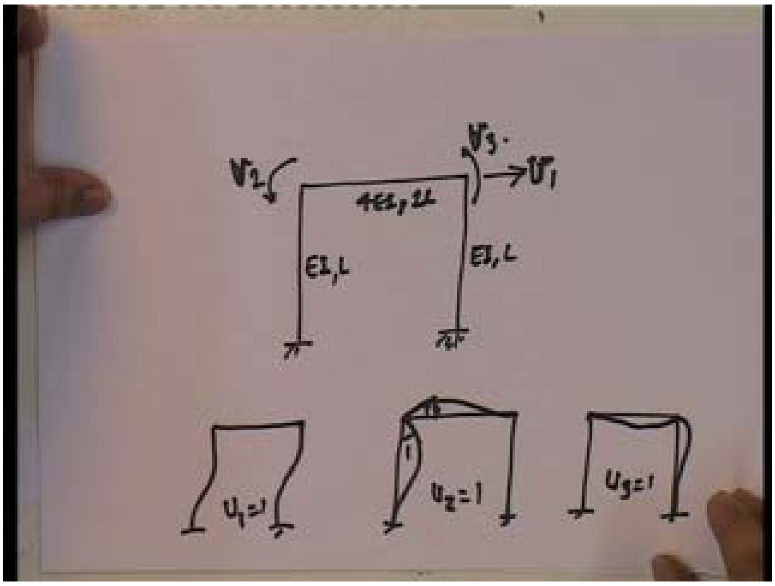

So now, let me look at the situation, let us look at then I define this as r 1, r 2 and r 3, I will call them now for purposes of this thing as I mean, because I am defining them as structure. So, I will make them v, so v 1, v 2, v 3, but these are structure level and the question then becomes that, how do I find out. Let us see what do I do, I just give each one, so this is v 1 equal to 1 then I have v 2 equal to 1 gives me what, that this one goes like this, this one rotates, this is v 2 equal to 1.

And finally, we have v 3 equal to 1 is this way, so this is v 3 equal to 1, where this angle is 1, this angle is 1. And so therefore, if I were to look at the structure stiffness matrix, I know each one, I know that, let us look at this that, this let me just define some numbers, let me say call this as E I L, this is also E I L and I will call this as 4 E I and 2 L. So that means, the length is 2 L and the flexural rigidity is 4 E I. So, if I do that then all I need to do is, I have got the stiffness matrix for each member, so all I need to do is, I need to put these together. And ultimately, if I put these together properly, if you look at this one what will I get, here these two move, because of that I have 12 E I by L cubed, coming in 12 EI by L cubed, 12 EI by L cubed and so 24 E I by L cubed.

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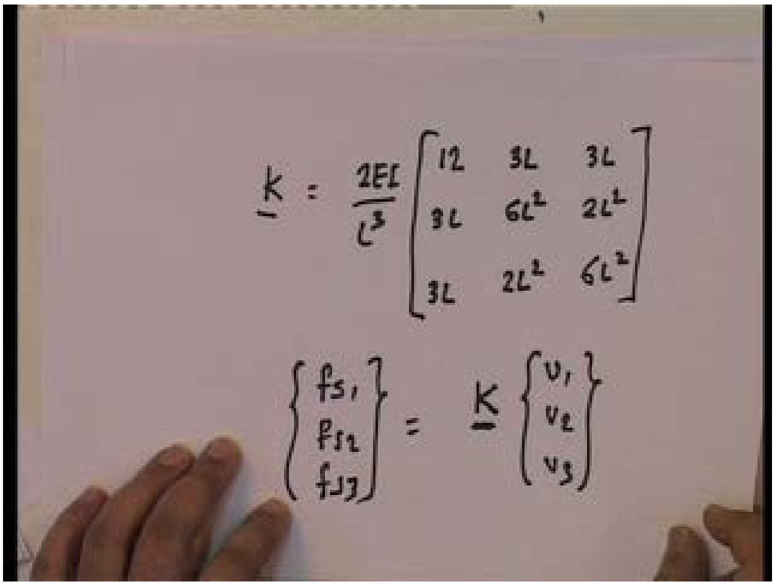

And this way, I will leave this to you as an exercise, but the K matrix turns out to be equal to again in terms of 2 E I by L cubed outside, this one basically becomes 12 3 L 3 L, here it is a symmetric, so these will be 3 L 3 L. It becomes 12 E I by L, because what you have is, I will just will give you one example of this one, this will come out as 4 E I. So, 4 into 4 that is, 16 E I upon 2 L, so that is become 80 EI by L and from this one, I get 4 EI by L, so the total this thing over here will be 12 E I by L.

So, if you look at this, this is nothing but 12 E I by L, similarly you can find out all the other ones, I just gave you some specific numbers, how to get them, you can always get them. As long as you remember that, K i j is equal to force at i due to unit displacement at j, so this is my K matrix and ultimately, the way this goes is that, my f s 1 f s 2 f s 3. For the structure, is equal to K into v 1 v 2 v 3 for the structure. So, I found out my structure K matrix, now the question becomes, what are the other things that we have to find out. We have to find out the mass matrix, remember that, and so therefore if you look at the mass matrix, what happens in this particular case.

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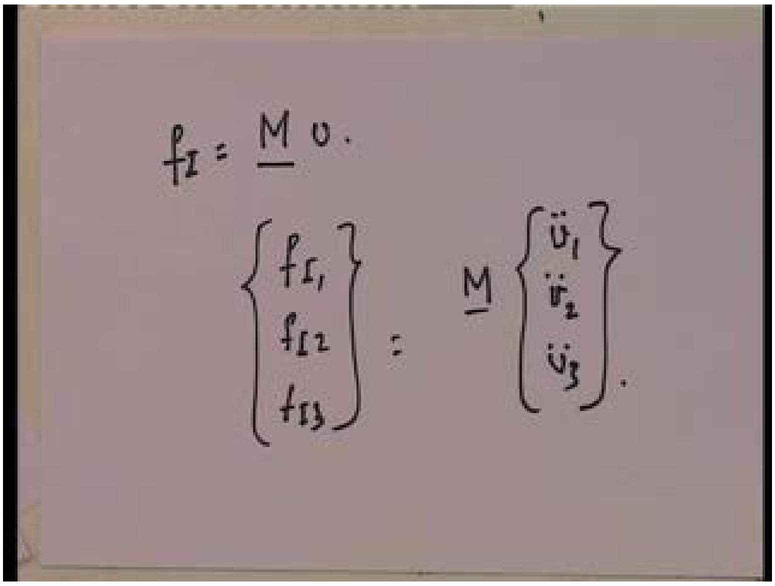

So, I have to find out the mass matrix, which is what, what is the mass matrix, this is f i is equal to into v. So, essentially what do I have, I have f i 1 f i 2 f i 3, for the structure is equal to mass matrix into v dot v 2 dot v 3 dot. Now, this is the mass matrix, now you see up till now I have only talked about, how to get the stiffness matrix. Now the question becomes, how do I get the mass matrix.

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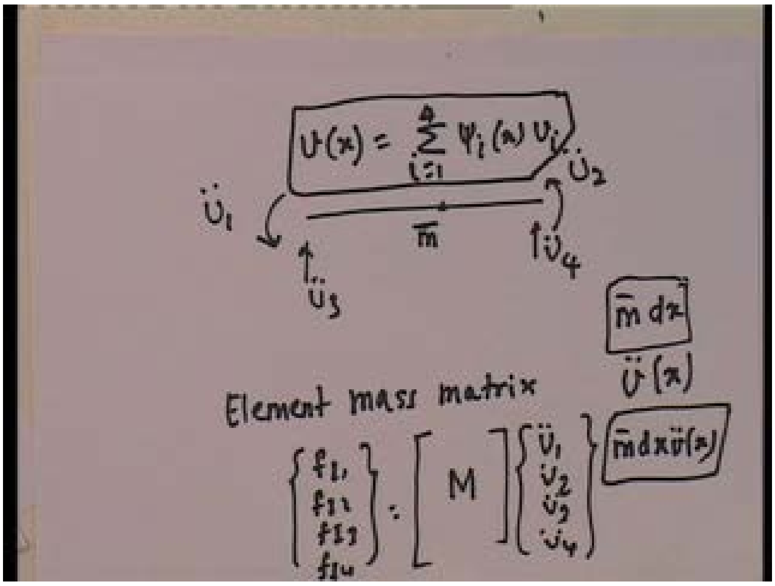

Now, if you look at the mass matrix, all we have to do now is that, if I go back to the elemental level, now I am computing mass matrix. So, this I have m bar, now note that, I have v 1 double dot, v 2 double dot, v 3 double dot, v 4 double dot. Now, here these are the I mean, if I want to find out for the element mass matrix, if you want to know the elemental mass matrix, what is the element level mass matrix here, the element level mass matrix becomes the following f i 1 f i 2 f i 3 f i 4 is equal to M into v 1 double dot v 2 double dot v 3 double dot and v 4 double dot.

So now, the question then becomes, to find out the element mass matrix, I have to look here, because since I have m bar, if I take elemental length at a distance x, which the mass is m bar d x, I need to know what is v double dot x. Because, ultimately the inertial force at that infinitesimal level is nothing but m bar d x into v double dot d x. Now, you see earlier, what had we said, we had said that, v x is equal to summation i going from 1 to 4, psi i x into v i, this was what we had used to develop the, what you may call it, the stiffness matrix. Now, this is absolutely correct, this is what you have to develop the stiffness matrix, but this is not one way. So therefore, let me put it this way, that one way of saying is, look obviously, if this is true then we can say that, look this is not a function of t.

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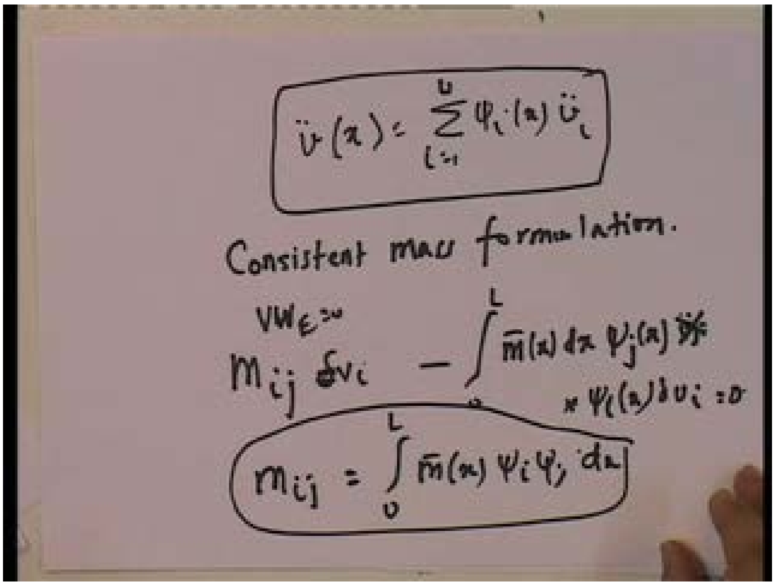

So therefore, we can go ahead and say that, look v double dot x then obviously, is equal to i equal to 1 to 4, psi i x into v i double prime, this seems obvious. If we start off with the displacement, now the question is, this approach is ok and this approach is known as the consistent mass formulation. So, here this becomes trivial, because once we have this then I can find out any m i j, what is m i j, this is the force at i, the inertial force at i due to unit acceleration at j.

So, if I look at unit acceleration at j, what do I get my this thing as, I get it as m bar x d x into, now v double prime basically becomes what, it becomes equal to psi j x into, which is equal to 1. So, this one disappears, so this is my inertial force at a level now using power, this is again using principle of virtual displacement, this force is multiplied by the displacement at that particular point due to the unit this thing. So, this will become psi i x del v i that is, m i j and this is equal to m i j into del v i, so if you look at virtual work equal to 0, this into del v i minus this, is equal to 0.

So, automatically from this and this integrated over the whole length, so automatically m i j becomes equal to 0 to L m bar x psi i psi j into d x. This becomes the consistent mass formulation and if I develop that this thing with uniform mass and using the same psi i that I have defined earlier, you go through the process. I am not going to go and detail out the process, but if you do this then what we get is the following.

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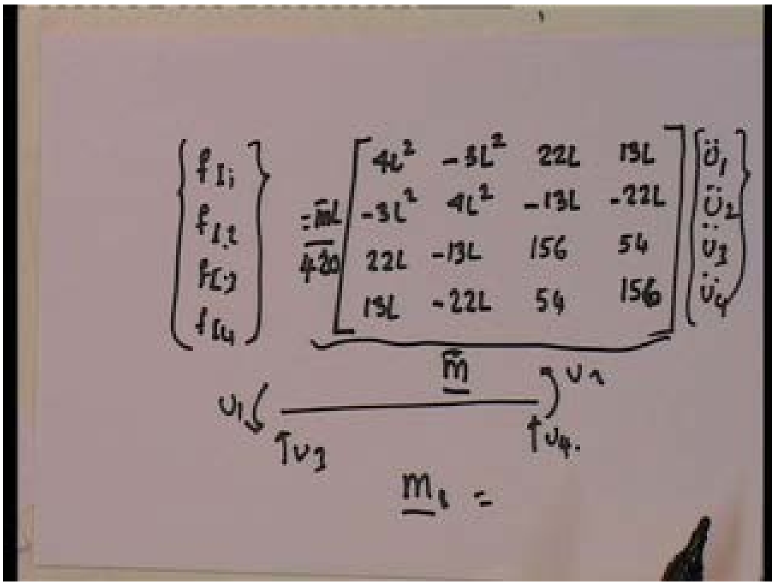

We get f i 1 f i 2 f i 3 f i 4, these are the rotational degrees of freedom and these are the displacement degrees of freedom. What you get is the following, m bar L is the total mass of the system, this becomes equal to 4 E I minus 3 L squared and then 22 L 13 L, this becomes obviously, minus 3 L squared 4 E I by L squared minus 13 L minus 22 L and then what we have here is, automatically I get 22 L 13 L minus 13 L minus 22 L and over here what we get is, 156 54 54 156.

Now, if you look at this, this 156 if you look at, we have done, we have got this number 156 upon 420, actually turns out to be 0.38 something. And this number we got in the last one earlier when we developed the equations of motion for a generalized single degree of freedom problem. So, this here note that, this into v 1 v 2 v 3 v 4, again let me draw, this is v 1, this is v 2, this is v 3, and this is v 4. So, this is my element level mass matrix, so this is one and if you look at these terms, this is called sponding moment due to unit rotation.

What is that, that is like a mass moment of inertia and you will see that, the units are mass moment of inertia. The direct linear displacement, these are direct mass, so this is a very consistent formulation and once you have this, look this is the inertial force, now all you have to do is, find out the end accelerations. So, you can do it exactly the way you get the structure stiffness matrix, you can get the mass stiffness matrix corresponding to this. And then you got your k and mass for a given this thing, so if we go back to the problem that we have looked at and use this consistent mass formulation that same problem, I want to go back to that.

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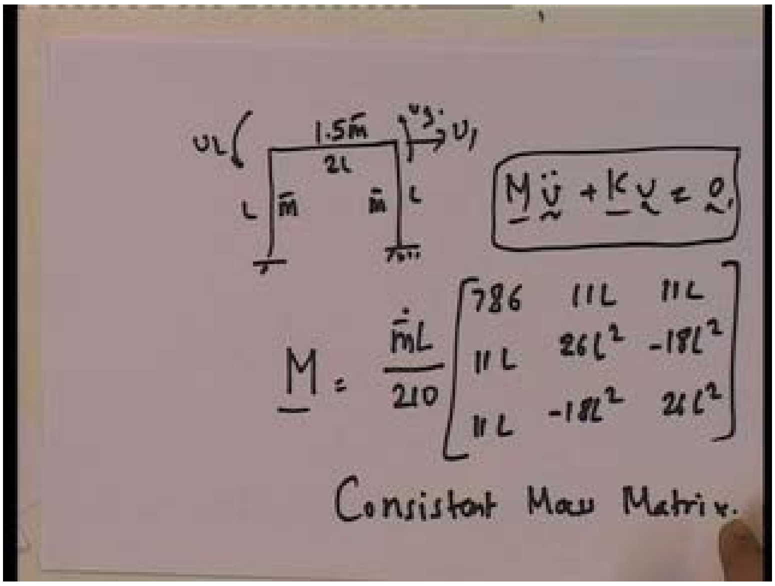

So, if I look at this problem and we say that, look this is m bar, this is 1.5 m bar, this is 2 L, this this is L. So, all I am saying is that, this one has 1.5 times the mass, that is all and we have this as my v 1, this as my v 2, this as my v 3 and going through the same procedure, the mass matrix comes out to be equal to m bar L by 210 into 786 11 L 11 L 11 L 11 L 26 L squared minus 18 L squared minus 18 L squared 26 L squared, at this mass matrix is known as the consistent mass matrix. And if you look at it, now we have done this, so this problem becomes, since there is no load I can actually say this, there is no load. This is the equation of motion, where M is given this way and K is given the other way. Now, there is an issue, the issue is that, for stiffness, I have to use the cubic Hermitian polynomials.

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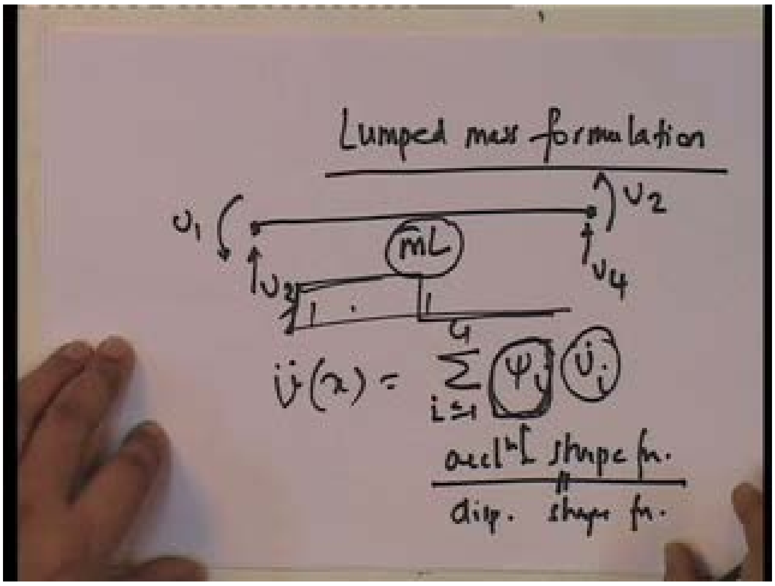

But, for mass, look I can put it this way, this is mass, I can say that, look the total mass of the structure is this way. And I will say that, look I am going to say that, the contributory mass over here corresponding to this degree of freedom. So, I will say that, look all I am going to say is that, this mass in other words, I do not, see no where that, v double dot x is a function of this, is no where considered. So, the point then I am trying to make is, as far as mass is concerned, I can say that, look this total mass is equal to m bar L.

And what I am going to do is, I am going to put half the mass here and half the mass here as point masses. If I put them and this is known as lumped mass formulation, see understand that, even for the acceleration what we say is, consistent mass formulation. People may say, that is absolutely the correct way of doing it, but you see, who says that this is valid, who says that the acceleration follows the same shape function. So, in other words, what we are saying is that, the acceleration shape function is the same as the displacement shape function.

Note, the displacement, the cubic Hermitian polynomials are exact, where you have a static situation. So, we are saying that, look we know that the cubic Hermitian polynomials are exact for a uniform beam for a static situation, no question that we have already developed in a structural analysis course earlier and you can look through it. There is no issues associated with that, it is the true kind of the shape functions or the cubic Hermitian polynomials.

But, understand that this is not a static problem, even we are looking at it, we are looking at it as a dynamic problem. And all that we are saying is that, since we do not know what else to do with stiffness, we will use the cubic Hermitian polynomials to develop the stiffness, nothing wrong with that. Although again, the K matrix that we get for solving a dynamic problem, is approximate even for a uniform cross section, because we are saying that, look under the dynamic issue, the psi remains the same as the original.

Now, for displacement shape function, there is no other thing that we can do about, but for the acceleration shape function, it is not obvious that we have to follow the same procedure. So, the consistent mass formulation follows this procedure no question, the consistent mass formulation says that, these are equal to each other. What is the lumped mass formulation say, lumped mass formulation says that, look I am saying that, here the formulation is the following way I mean, it looks like this. Under v 1 this is, now you may say, how does these accelerate by 1 in this, it is as good as anything else. So, the lumped mass formulation says that, look this is it and so if you look at it, the lumped mass formulation is a relatively easier formulation. Because, it has point masses here, point masses do not have any rotatory inertia.

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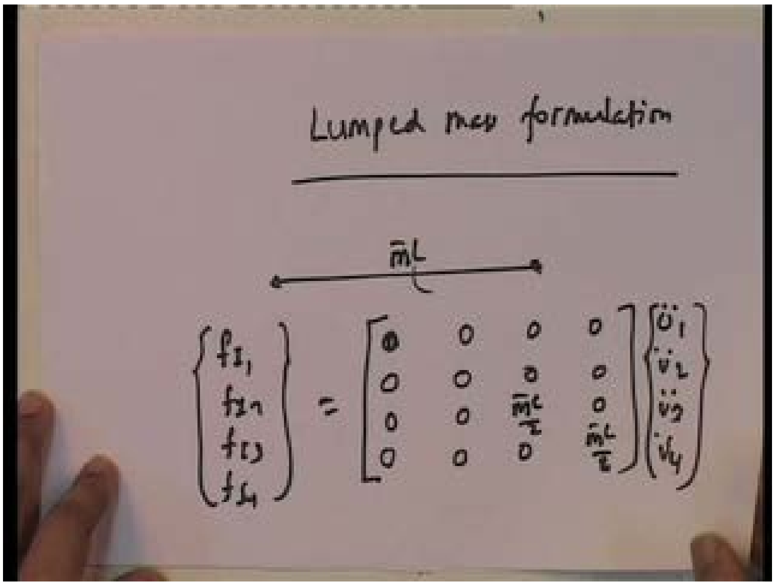

Since they do not have any rotatory inertia, what is the mass matrix look like, in a lumped mass formulation what we have is that, m bar L is a total and so my inertia f i 1 f i 2 at the member level, f i 3 i 4 is equal to 0 0 0 0 0 0 0 0 0 0 0 0 m bar L by 2 0 0 m bar L by 2. Note that, in the lumped mass formulation, since I put point masses here and I have taken the kind of displacement for v 3 as this and for v 4 as this, I get m bar L and they do not contribute any force over here.

So therefore, the lumped mass formulation is two point masses as a point, when this point mass moves up, this point mass goes no where, so this become m bar L upon 2 m bar L upon 2, total mass is m bar L. And since point mass should not have any rotatory inertia, this is a rotatory inertia terms, a very simple formulation. Now, my question is, now you may well ask that, how good is this formulation and will show you later that, this is a reasonably good for most situations.

Why because most situations especially for frame buildings, in most situations the mass that comes from each member is much smaller than the floor mass. So, how we approximate the member mass is not very relevant, that is the key point to note. There is the lumped mass formulation was developed for building structures only, where the floor mass essentially completely over whelms the entire thing.

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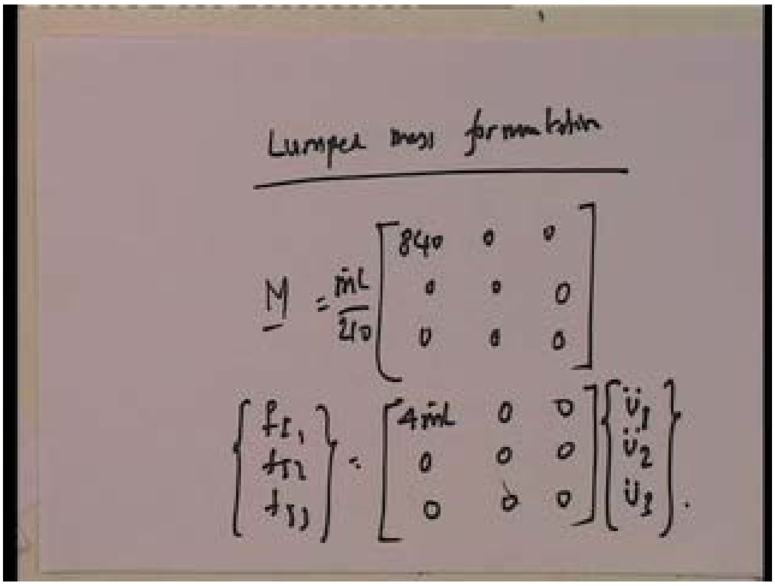

I will show you an example to this later on, so therefore, in this particular problem, that specific problem that we solved, this one. The consistent mass matrix was this and the lumped mass matrix, lumped mass formulation will give a mass matrix, which is of the same of this format. And I am going to put the same as I had put earlier 840 0 0 0 0 0 0 0 0, this is my mass matrix. In other words, what we are saying is f i 1 f i 2 f i 3 is equal to, I will put it as 4 M bar L 0 0 0 0 0 0 into, a very interesting thing happens.

You see even in the structure mass matrix, the rotational degrees of freedom in the lumped mass formulation have no terms associated with them. Now, this is how good is this, let us look at this that, they do not look at all close to each other, this is what if you look at this. I will just put this a little bit further up and you can see here, I will put like this, I have not look at it, here this is 840, this is 786.

And in a way, you see these terms are so much lesser than this term that, all we are saying is that, look these two when we put them together, you will see that, there is no too much of a difference in whatever we try to get, if we get response whatever, so there going be too much of difference. Now, note that, I have put both m bar L upon 1 210 outside in both cases, so this is the only thing that, all these terms are actually lumped into this over here.

Now, why do we do this, that is the question, why do are we so much interested in the lumped mass matrix. The reason why we are interested in the lumped mass matrix is, it gives as the following.

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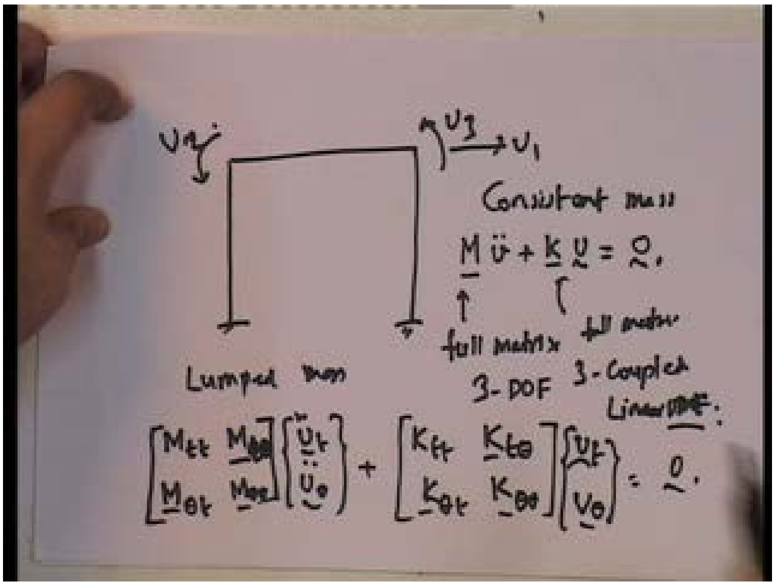

If you look at the structure level, let us look at this structure v 1 and v 2, v 3, so this is the structure that we have been looking at. Under the consistent mass formulation, what do we have, we have m bar v double dot plus k v and I am going to put 0 now, because I am not looking at loading at all at this particular situation. So, this and here, full matrix, this is the consistent mass formulation, this is the full matrix and so we have to solve a three degree of freedom.

So, three coupled linear differential equations, ordinary differential equations, this is what we have to solve, three coupled linear ODE's. If you use the lumped mass, what happens and here, I am going to put it in this way, I am going to put them in this format, the mass matrix. I am going to say that, look this is the m translation M t t, these are the t theta's, this is theta t, this is the matrix in this particular case, because this corresponds to and this is m theta theta, where v t is the translational degree of freedom and theta's.

So, I am kind of sub structuring the matrix and I am sub structuring the matrix to take the translation degree of freedom and theta degrees of freedom plus K t t into K t theta K theta t. These are matrices, these are actually vectors and this one is a matrix into v t ((Refer Time: 51:12)) this is acceleration, so these are this v theta, is equal to now I will put it as 0. Now, here if I use, so this is what I have and in the consistent mass, all of these are full, so you cannot do anything.

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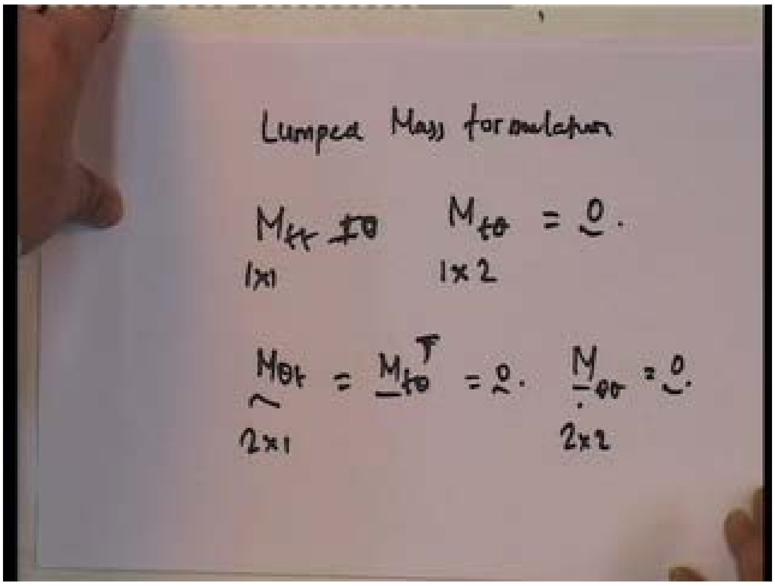

But, in the lumped mass formulation what happens, in the lumped mass formulation if you looked at it, M t t is not equal to 0. But, if you look at in this particular case, what is the size of M t theta, see this is a 1 by 1 and it is non zero, we know in this particular case, we have seen it is 4 m bar L. Now, m t theta, what is the size of m t theta, it is a 1 by 2 vector and this is a zero vector. Similarly, m theta t which is a 2 by 1, which is actually m t theta transpose, it is nothing but I mean the transpose, because it is a symmetric matrix, is also a null matrix. And similarly m theta theta, which is a 2 by 2 in this particular case, is also a null matrix, so in other words, in the lumped mass formulation, the only thing that exists is M t t.

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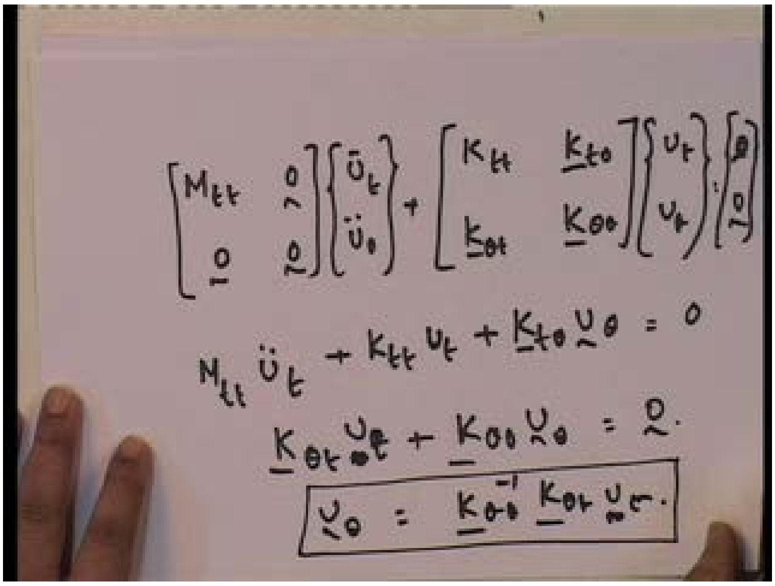

So, if M t t is the only thing that exists, what do we have, so then this equation becomes like this, M t t 0 0 0 into v t v theta plus, now K t t and K t theta, these are not non zero, because these can only be obtained through the consistent formulation. So, they are non zero and this is equal to I mean, 0 and 0, so if I do this, let us now write down the equations. The top equations comes this way, M t t into v t 0 into this, plus K t t into v t plus K t theta into v theta.

Note that, this is a 1 by 2 into a 2 by 1, so this is a 1 by 1, so this is actually a scalar vector and this is equal to 0 and the other one is equal to, note that 0 into v t plus 0, so this one does not show up, so this comes up as K theta t into v t. Now, note, what is this, this is v t, this is a 2 by 1 into a 1 by 1 that is, 2 by 1 plus K theta theta into v theta theta, this is a 2 by 2 into 2 by 1, so we get a 2 by 1 and this is, so these are actually two matrices, which we put together. Now, if you see, this is equal to 0, this is a static equation, so if we look at this, what we get is, if I take it this way, I get v theta is equal to K theta theta inverse K theta t into v t.

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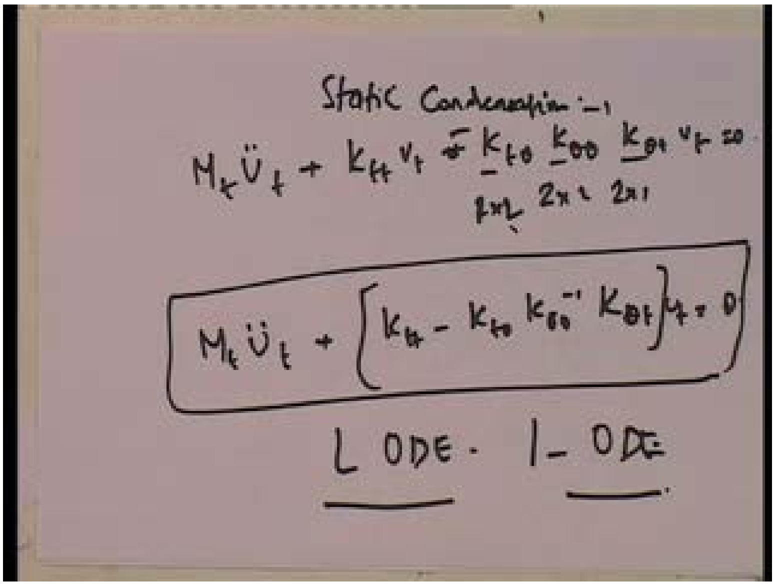

So, I have got a static relationship between v theta and v t, if I substitute that into the top equation what do I get, I get the following. I get M t into v t plus K t t into v t plus K t theta into K theta theta inverse k theta t into v t is equal to 0. Note, that this is a 2 by 2, this is a 1 by 2 and this is a 2 by 1, a 2 by 2 into 2 by 1 is a 2 by 1, 1 by 2 into in a 2 by 1 is a 1 by 1, so this is actually a scalar. So, in other words, this equation basically becomes K t t, there is a minus here.

So, this is minus, minus K t theta K theta theta inverse into K theta t into v t is equal to 0, this becomes the only equation, which is a linear differential equation. So, essentially the dynamic problem is solved with only one ODE, so for the purposes of the dynamic, you are solving a single degree of freedom become much easier and then you see v theta is not 0. But, once you find out v t, you can find out v theta, because v theta is given in terms of v t through this equation, so this procedure is known as static condensation.

And if you use the lumped mass matrix, you can use the static condensation procedure to simplify your ODE. See, now instead of solving, consider the mass formulation, you have to solve three coupled linear differential equations. If I use lumped mass formulation, it become one ODE and the other two I get through a static relationship, that is the advantage of the lumped mass formulation for framed structures. Please understand, that it was originally developed for framed structures, today I see people using lumped mass for all kinds of problems and going ahead with it. But, please understand, that the lumped mass formulation is reasonably accurate for framed structures, purely because members. The member contribution is negligible compared to the floor mass and the floor mass is a point mass, which only translates to the degree of freedom.

Thank you very much, I will stop here for today, bye.