Structural Dynamics
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Lecture -16
Earthquake Response Analysis for Single Degree of Freedom Structures

Hello, in the last lecture we talked about the earthquake loads and in this lecture two, we are going to be continuing discussion, just to kind of reiterate.

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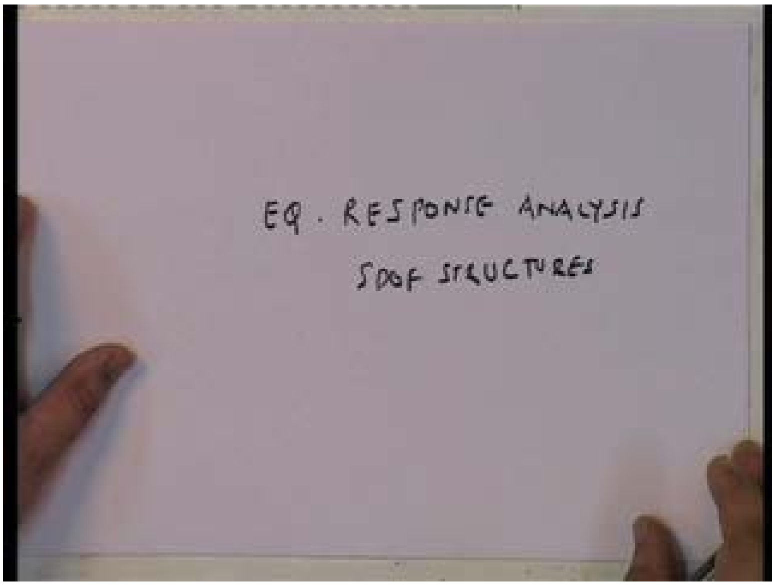

This is we are doing earthquake response analysis and this is again for single degree of freedom structures. So therefore, there is no question of that, we are still looking at single degree of freedom structures, excepting that we are looking at earthquake. And last time we saw that, in this particular case, there are certain aspects just to revise very quickly.

The acceleration time history is, what we typically define and this is again this is of specific, but note that, you know there are certain aspects to it and that is that, there is toing and froing of the ground motion. So, if you look at 0, it goes to plus, it goes to minus, it goes back, so one of the things about earthquake ground motion is that, the structure is subjected to many reversals of motion, many many reversals. Just look at the reversals that you have in this particular case now.

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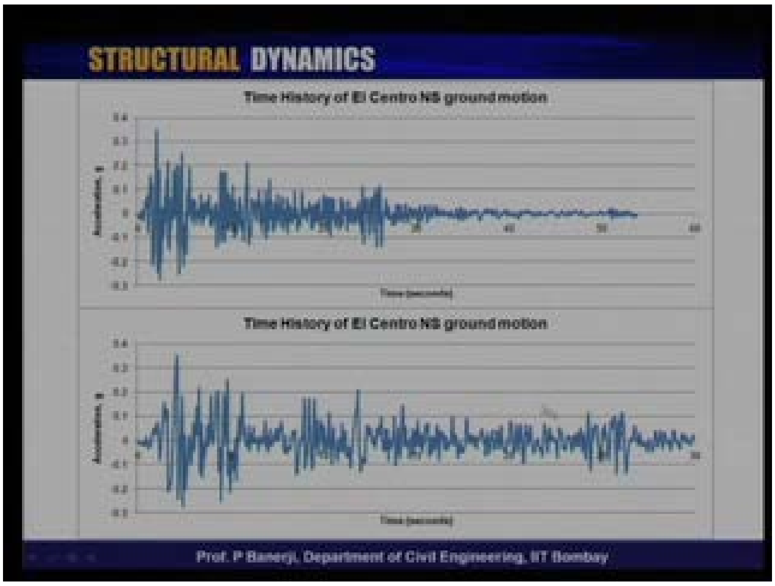

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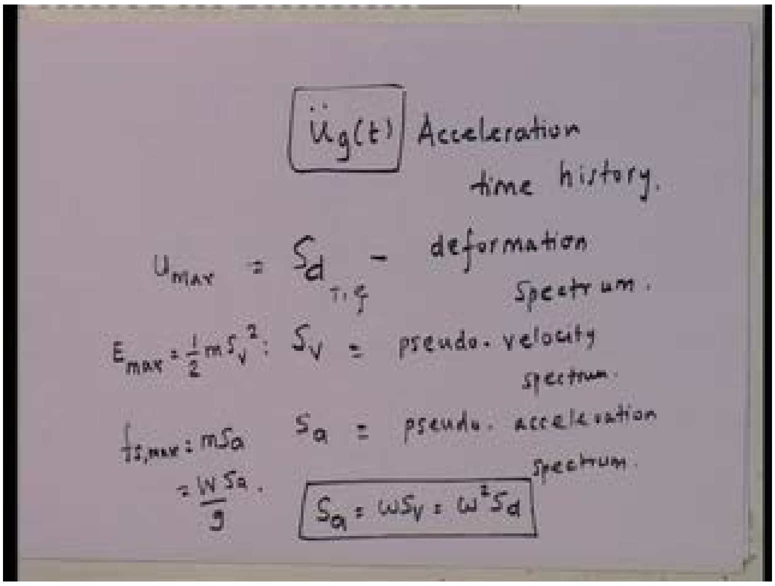

So, that was essentially what we saw was that, the acceleration time history is one way of defining a ground motion. The other way S d, which is U max then, S v which is and these are functions of time and psi. S v is given such that, the maximum strain energy is given by half m S v square or you are given by the pseudo acceleration spectrum, which is S a, pseudo acceleration spectrum and this essentially gives us the peak force that the structure subjected to and that is equal to m s or W S a by g, where S a is given by this.

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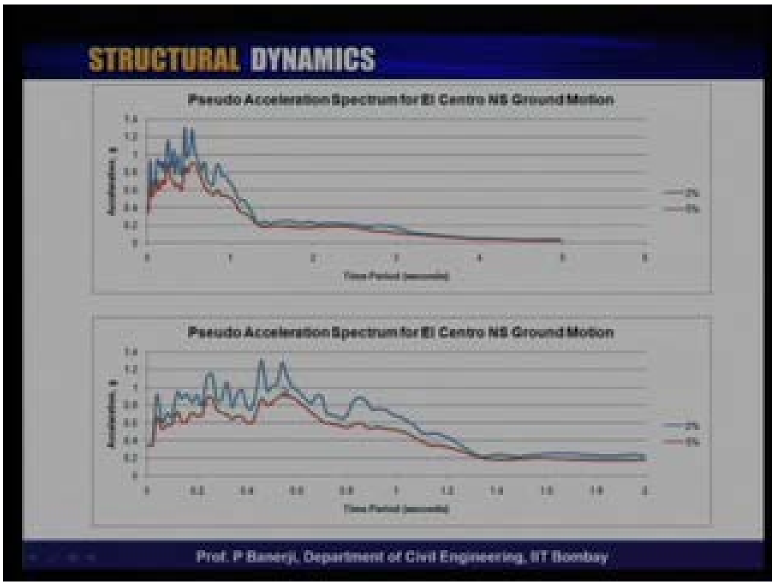

We also discussed the specific forms that, ((Refer Time: 04:41)) S d becomes 0 at T equal to 0 and S equal to u g max at T tending to infinity. S a is equal to u double dot the peak ground acceleration at time t equal to 0 and is equal to 0 at the t equal to infinity. So, we looked at them and S v, the way S a, S v are connected to each other is that, S a is equal to omega S v is equal to omega square S d or this is how, they related to each other. So, understand that, this is the two deformation spectrum, the other two are pseudo, but however they are important because each one represent this particular aspect of it, so much for this, now if I were to plot S a, S v and S d together.

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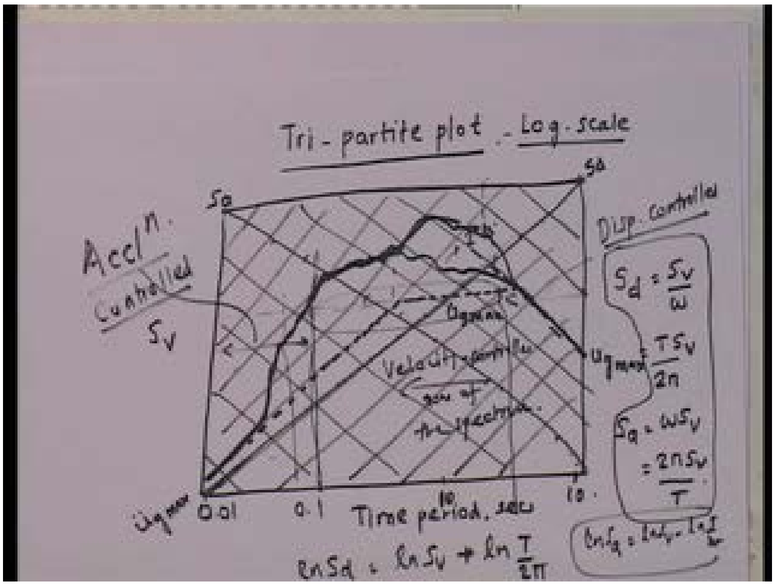

This is known as the tri partite plot, so if I were to plot it on a tri partite plot, how does I plot it, this is the way we typically plot it. S v is plotted here, time period in seconds is plotted here, along this axis is your S d. So, what we have is, so this is S d and along the 45 degree other axis is the S a, so this is what a tri partite graph looks like, now why I have able to draw this well, because of the fact that, if you were to look at it, S d is equal to S v upon omega.

So, it is equal to T upon 2 pi into S v, so in other words what happens is, the S d is plotted along this line and if you look at S a, S a is equal to omega S v, which is equal to 2 pi S v upon T, so this is plotted along this line. So, essentially you know this is, in other words, these are lines of equal S a and these are lines of equal S and typically tri partite graph is typically plotted in log scale. So now, there are certain interesting things that happens that, if you were to look at this one, what will happen.

At this is time period t equal to 0, almost 0, because this is log scale, so it probably be 0.01 and you have 0.1, 1 and then, 10, so this is your time period. So, at this point, if I were to plot it, if I were to plot S a, how will it look, it will go something like this, where in this portion it remains very close to the acceleration and then, it starts climbing goes somewhere here and then, it goes in this fashion. And so if I were to plot the, let me call this as my u g max peak ground acceleration, along this is my plot of peak ground displacement and somewhere over here, I am going to plot the peak ground velocity.

So, along this is my plot of my peak ground acceleration, because that is along a constant S a, this is along the constant S d. So, this is a plot of the u g max the peak ground displacement and S v represent a velocity, a constant over this, actually represents the peak ground velocity. So, if I plot those and if I plot this curve at very large time periods what happens, at very large time periods, the deformation spectrum becomes the same as the ground spectrum.

At very low time periods, the spectrum essentially is u g max and in between, this plot that I have drawn over here is really the same as this particular plot, which I have showed you. All these three plots are actually plotted here, if you look at it from this perspective, this is what I have plotted, this is of course for a particular say, 2 percent. So, if you look at it from this perspective at S v, this is what I have plotted, if you look at it from here, what I have plotted. See it is going to 0, this is 0.01, at 0 it will go to 0 and it is going up, up, ((Refer Time: 13:37)) up, up, up and then, the maximum value is really your... So, here actually in this region, it probably goes somewhere across and then, comes down, that is how it is.

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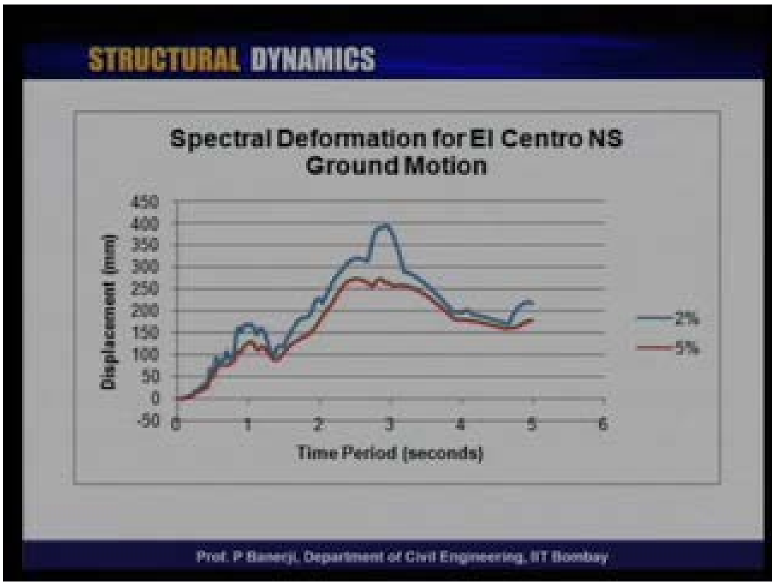

So, this will go if it is 2, it goes to 4 and comes down, so this is how in real fact it looks, if you look at spectral deformation, this is what you are looking, this is this plot. If you look at in this direction, this is what, this is the plot that you are getting.

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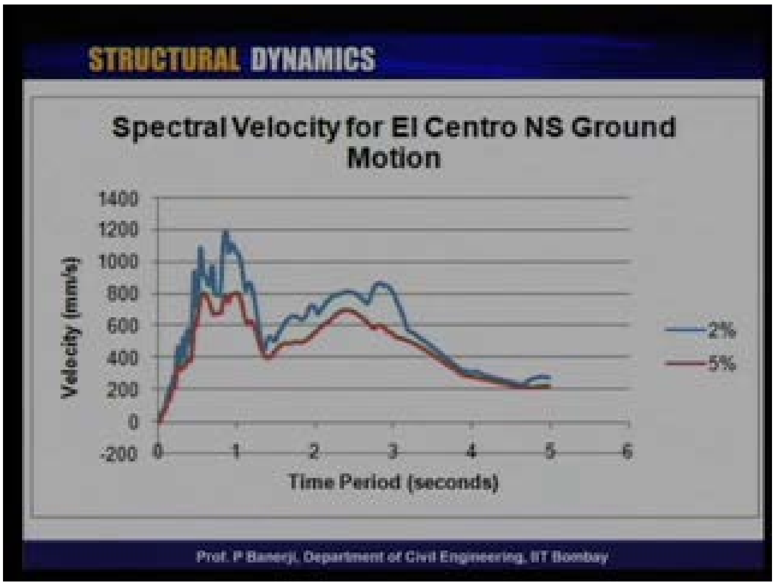

Now, if you look at it straight, this is the plot that you get and around 1, it goes high and then, it comes down. And if you look at it from this ((Refer Time: 14:33)) perspective, again the difference between these that, this is the log scale plot and therefore, this is the linear scale plot. But, if you look at it, this is what happening, goes up, it peaks at about 0.3, 0.4 seconds and then, it comes down and it goes to 0 as you go higher and higher. So, you see all these three plots, I have plotted it in one particular plot here and it actually shows me something very very interesting.

And that is that, there is a zone, this zone is about till here, where it is acceleration, the response spectrum is acceleration controlled. There is a zone here, where it is displacement controlled and central region if you look at it, this is somehow in the zone where u g is max. So, this zone is the velocity controlled zone of the spectrum, so you see the tri partite plot is a very very powerful plot, it actually here what I have done is I have plotted the spectrum, all three spectrum.

You can see as I told you, look at it this way you see the acceleration spectrum, you look at it this way you see the deformation spectrum, you look at it this way you see the pseudo velocity spectrum, all three spectrums are on the same. Basically because they are all related to each other, this is how it is, so you know in this particular case, l n of S d is equal to l n of S v plus l n of T upon 2 pi. So, S d basically becomes in a way, you know why, this is the S d, it basically becomes m x.

If you look at this, this is nothing but m x by T plus y and so here, and S l n of S a is equal to l n of S v minus l n of T upon 2 pi, which is why it is along this direction. So, the tri partite plot is a log scale graph, this is very important to remember that, all the scales are in log log scale and this is in meters per second, this is in meters or milli meters and this is in meters per second square, these are the units. And if the way this graph is plotted, it gives you very much, where the acceleration controlled region is, where the displacement control region is and the central is where the velocity.

So, in other words, inertia controls, all structures in this area is controlled by inertia, so therefore, in this region what you have is that, m into S a is going to be determining, how the structure behaves. In this region, the displacement controlled region m into S a is going to be very small, it is not inertia, it is displacement controlled. In this zone, the peak displacement, ground displacement is what is going to determine the peak response of the structure and in this region, it is really energy controlled.

So therefore, short period structures are acceleration controlled, in other words they are force driven, inertial force driven when very long period structures, short period structures, inertia driven, acceleration determines the peak I mean, that is where the acceleration determines. So, S a determines, what you get then, you have the very long period structures, where S a does not give me anything, it is displacement controlled, the peak ground displacement gives me. And therefore, it is really the deformation that determines, how much the structure, what kind of forces of structure subjected to.

And in the middle, you have a situation, where it is really the velocity controlled and therefore, the energy is important as the energy observed by the system is important and need a... So, it is it is essentially driven by the absorbed energy from the earthquake that determines this region. So, you see and so therefore, between extremely short and extremely long is where, the energy determines, what the response of the structure is. So, this is very very interesting and once you have this, we can go directly on to something very, very important.

And that is, ultimately understand all that we talked about till now is the response spectrum for a particular ground motion and then, it become very important that, how do we define the design response spectrum. Because, after all you never designed a structure for a particular ground motion, because once a ground motion is occurred, you cannot design a structure for it. And it is very interesting that, even at l central, there was an earthquake in 1940 and that is the most famous one, which I have just shown you ((Refer Time: 22:12)) this is the ground motion that you get at the same place.

So, let us assume a building is built at this place, 1940 ground motion gave us this, the 1971 earthquake gave completely different ground motion, so it be completely different response spectrum. Then, in 1994, North Rich earthquake, you had a completely different ground motion again at the same site. So obviously, you cannot design, even if you have past earthquakes at a particular site and you want to design, see understand that all that we are doing in structural dynamics, let me review the point.

All that we are doing in structural dynamics is not do dynamics for dynamics sake, it is to determine the peak response of structures to given dynamic loads. So therefore, if you have an earthquake load, you want to be designing a structure for earthquake load and the problem that happens here is that, this is not a deterministic, it is not a load that I know. For example, I know my weight, so I can say that, look this is a known load, however an earthquake as I told you, the same station, their central station.

In 1940 saw a different earthquake motion, in 1971 saw an another completely different motion, in 1994 North Rich earthquake it is a completely different. So therefore, even for a particular site, it becomes very very problematic as to, how to design a structure for a given earthquake. And there are various other ways of doing it, but one of the first ways of doing this particular problem was to define, what is known as a design spectrum.

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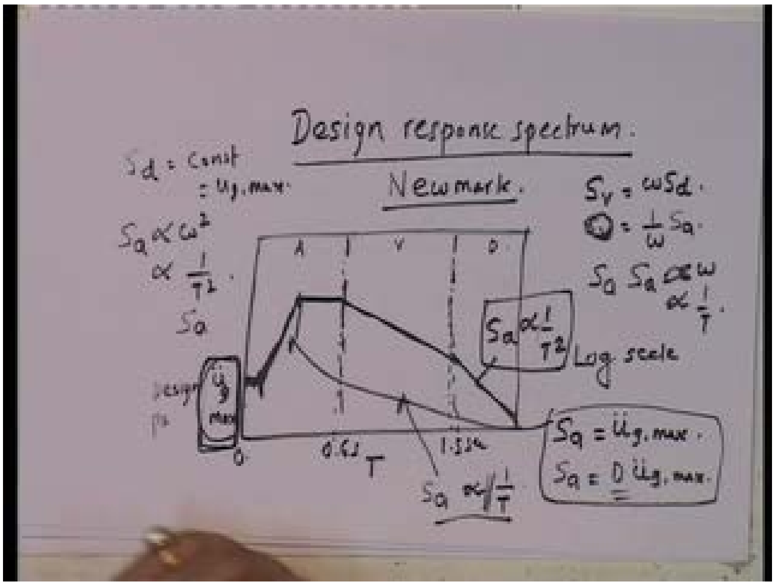

So, we define a design, now how did we design a response spectrum and here, I am looking at something that Newmark developed almost 50 years ago. So, I am going to look at something that Newmark developed and look at it from that perspective. And what did he say, he said the following that, look and now I am going to plot the acceleration spectrum, he talked about the acceleration spectrum. So, I am going to plot S a by T, he said that, look I am going to break up and this is based on, what we discussed over here ((Refer Time: 25:42)).

And from these tri partite plots for variety of ground motions, I am plotting them, they discovered that look, there is a situation where I have an acceleration control. So, if it is acceleration controlled, so acceleration controlled region, so this is acceleration, this is velocity controlled, this is displacement controlled. Again if we plotted this way, what he said was that, look in a portion of this, what is this, this is definitely the peak ground acceleration.

Now, there is a completely different aspect and this I am not going to go into and that comes from seismology, which determines design peak ground acceleration. So, I mean, this is something that we know, I am not go into that detail, this is structural dynamics course. In the earthquake engineering course, we could spend some time on discussing this, but here this is a known design parameter. So, Newmark said that, look this is what happens then, we have this kind of a situation and in the acceleration controlled region, part of it has this and then, part of it is this way.

Then, I come to the velocity controlled region and this is the log scale, in a velocity controlled region, it is going to be a constant velocity and in the displacement controlled region, it is going to be this way. Now, the question becomes in centring S a, S a in this zone is equal to u g max then, you have a transition zone and then, it is a constant S a, where S a is equal to some D times u g max. Then, we come to the zone where velocity controlled, in velocity controlled S a is proportional to 1 upon T.

Why 1 upon T, let us look at it, S v is equal to omega S d and S v is equal to 1 upon omega S a. So, in this particular portion, this is a constant, in the velocity controlled this is a constant, so what you have is S a is equal to constant into omega. So, that is, so that means, it is proportional to omega which means, it is proportional to 1 upon T. Then, in this, in the displacement controlled region, S d is a constant which is equal to u g max. If S d is a constant then, S a is proportional to omega square and so that is proportional to 1 upon T square.

So, in this zone, S a is proportional to 1 upon T, in this zone it is given in this fashion that, there is a transition zone, for one part it is a constant, it is u g and then, for another part, it is this way and then, here this goes down S a is proportional to 1 upon T square. And the proportionality constants of course, depend on this D, this proportionality constants, all of them actually depend on psi, they depend on psi and the transition points also depend on psi.

And Newmark actually developed a specific way of determining these from past earthquakes. So, but in a sense, the point still remains that, you have an acceleration controlled zone, a velocity controlled zone. Acceleration controlled zone S a is a constant for our practical purpose, excepting for transition zone, because we need to go from a peak ground acceleration to sometime some number of peak ground acceleration. So therefore, there is a transition zone, but in the velocity controlled zone, S v remains a constant and in the displacement controlled zone, S d remains a constant.

So, automatically S a is equal to some constant time 1 upon T and S a equal to some constant time 1 upon T square and these in a sense, is your designed a spectrum. And interestingly enough, if you really look at the pseudo acceleration response ((Refer Time: 32:16)) spectrum, now if you look at one of them, it becomes very very difficult. But, you know can actually see that, if you look at this, that is this zone, this is almost a constant.

Note that, this is linear and what I had plotted over here, was a kind of a log scale, this was a log scale kind of a thing, so try to see in a log scale, in log scale this part becomes very small. If you look at this, this is the part where the displacements starts controlling and you see over here, the way it is going, it is almost going down as 1 upon T in like a hyperbolic kind of a situation and beyond about 1.2 seconds you will see that, it starts going down at about 1 upon T square.

Because, if you look at ((Refer Time: 33:14)) it, the spectral velocity, this is the zone in which the spectral velocity is controls, so that is between about 0.6 seconds till about 1.4 seconds, this is where the velocity controls ((Refer Time: 33:32)). And if you look at this, this is where the displacement starts, so about 1.6 seconds, this is where displacements starts controlling. And so therefore, if you look at it, in this particular case, we can almost say this goes from 0 to about 0.6 seconds, this goes to about 1.5 seconds.

So, approximately this is what we see, spectral displacement from about 1.5, 1.6 seconds is where displacement starts controlling. If you look at this, this is where acceleration controls from about 0.6 till about 1.5 and ((Refer Time: 34:23)) acceleration between about 0 to 0.6. You see, how, what we are looking at, really comes out even in one single this thing and although the design response spectrum is not based on one ground motion, it is on based on many many ground motions.

And so if when you do these kinds of peaks and values that you see in any one particular ground motion, actually gets kind of even out and you will see that, you will get somewhere over here. This is 0.3, for 5 g it is about 0.8, so it is about 2.67, if you go to 2 percent, this is about 1, so that is about 3.2. So, you see, you start getting these numbers 3.2, 2.6 for 5 percent, it is about 2.67 for 2 percent, it is about 3.5, 3.6. And you start getting numbers that, the design response spectrum that Newmark talked about, you start seeing them in this particular case.

So, that is the interesting part of, how the design response spectrum is developed and all of this became probable, because it was put in the tri partite log log. And it became very obvious that, the tri partite log scale gave us the acceleration controlled region, the velocity controlled region and the displacement controlled region. So, you see how step by step and by using the characteristics of the response spectrum, you can go step by step from a time.

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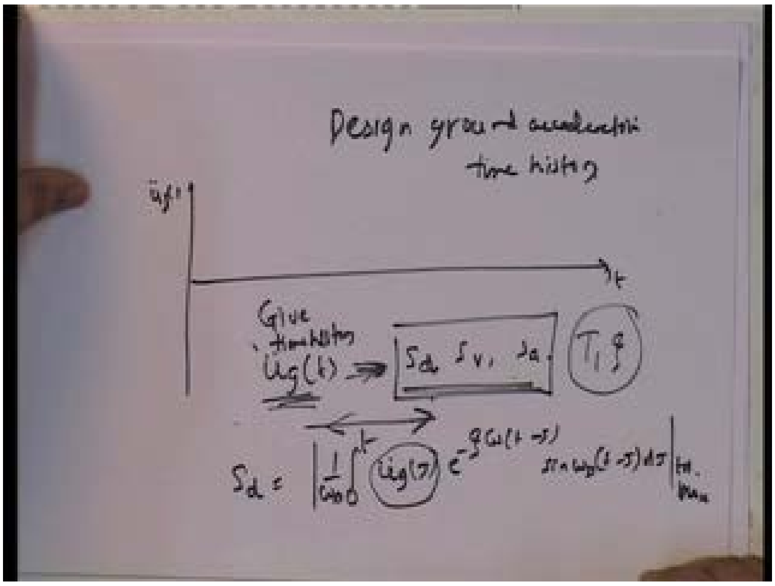

Now, you know it is very difficult, you see it is much easier to define a design response spectrum than to define a design ground acceleration time history. How and earth am I going to be able to get any defining, unless I define, see understand that, if you are looking at design. When you looking at design, you need to look at certain characteristics in, whatever you are looking at, to be able to design to define a design load. So, you always, whenever even when you are looking at loads, any loads for example, live load, you look at certain characteristics of live load and based on that, that is what you get, how you develop your designed load.

So, we saw how we went step by step through the entire process by looking at specific aspects from specific behaviour of structures. We went to see, how each response spectrum had specific characteristics then, we put it together in a tri partite log plot. And we saw that, in certain zones, certain paths of the ground motion are controlled and from that, we went one step forward to getting the design response spectrum. So, there unless something has a characteristic built in, you cannot get any designed load for that.

Now, let us look at the ground motion, look at this ground motion, does it look like anything, can you define anything. For example, the only defining characteristic is the peak and that also is something that we know, that we can define, but can we define the time history of this. So therefore, your time history is something that, it is impossible to define a design ground motion history. What is done, I will explain that, you must have heard somewhere, especially if you have been working in a particular organization that does earthquake response analysis.

And you would heard of, there are design non motions, these are called spectrum compatible ground motions. And here the point becomes that, ground motions do not have characteristics that we can pick up, what we try to do is, look at the spectrum where we know that, we can get a design spectrum. And you see, there is a given at time, if you are given this going from this to S d, S v, S a is unique, why because all you have to do is that, S d is equal to...

So, S d is, if you have this time history, this is a known time history, by solving this now, whether you solve this or whether we have already talked about various ways of solving a problem, the Duhamel integral can be solved numerically or you can do time margin schemes to get S, whatever. Once you get your deformation time history, from the deformation time history, you take the peak value and that is, S d. Once you know S d, you know S v, S a, so at a particular for a given structure with a time and this you going from this to this is unique.

In other words, given this you can find these out, that is not an issue, the issue becomes that, typically what is our design. Our design, for a given time history, acceleration time history, you can find out the response spectrum. We have already done that in the last lecture and we shown it in these kind of things at given this, given this time history of l central ground motion, you can get the spectral deformation, get the spectral velocity, get the pseudo acceleration spectrum, you can get these, these are unique, you can get them.

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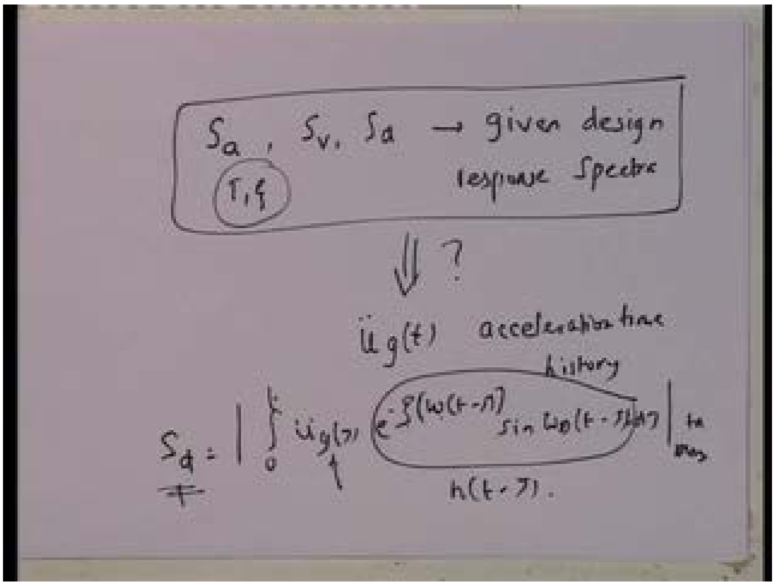

The problem that happens is, now as I said, a design spectrum gives you what, it gives you S a and from S a, S v and S d, these are given design response spectrum. I just talked about it, so this is given, now given this, can you find out u g of t, which is the acceleration, can you find it out, problem, there is no equation like the one that I had given, which was this is equal to... So, the problem becomes, if I know this and this is as a function of T and psi, so S d as the function of psi and omega, this is known.

So, this is like a problem, when I know this and can I find this, impossible. If there was something that said that, look u g double prime t is equal to S d into something, something, something. If that was there then, I would be able to find out u g of t, but there is not a single equation that gives u g in terms of S d or S v or S a. So, you have a problem, in which this is known as, what is known as a classical inverse problem. If you know one thing, finding out the other thing is really, this is input, the input is the time history, you have a kind of a box, box that represents the structure. And what is that, that is given by h of t, which is the impulse response function, unit impulse response function h of t. H of t is nothing but this, this is h of t minus tau remember.

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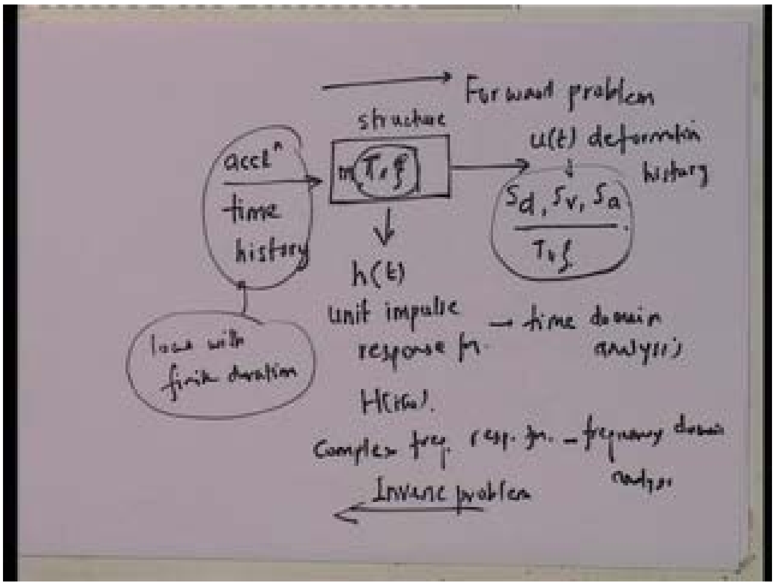

So, what happens is, input is let me just draw this in a, it will be interesting for you to look at this that, what we have essentially is something like this. Mathematically, input is acceleration time history and output is S d, S v and S a. Actually the original output is u of t, this deformation history and from that, we derive these things, these are of course, for various. So, in other words, how to get S d, S v, S a is, this gives us only one value of S d, S v, S a for a given T and psi, so we have to keep doing it.

Now, this structure is defined by m, T and psi, and this is the structure, which is represented either by h of t, the unit impulse response function or h i omega bar, which is the complex frequency response function depending on, whether you do time domain analysis or frequency domain analysis. Note that, acceleration time history actually is like a load with finite duration, so all the things that we talked about two lectures ago, all of them become valid and given this, you can find this and then, all of these.

Now, the reverse problem, where you know these and you want to find out this, is actually this, this is a forward problem and this is known as an inverse problem. So, given S d, S v, S a, finding out acceleration time history is actually a classical inverse problem and what is even worse is that, S d, S v even if I had the deformation time history, there would be some ways that, would be actually a classical inverse problem. Given the deformation history, finding out the acceleration time history is a inverse problem.

What makes it even doubly difficult is that, look we do not even have the deformation time history, because all we are picking up from the deformation time history is the peak value and that is my one S d value. And so when I give the spectrum, all I have it is with time period, it is all for different structures and for different damping, that is my response spectrum. How on earth, it does not even represent that, that spectrum does not even represent a particular state, it is actually many many states that we are representing on one graph.

So, the inverse problem is actually practically impossible, however people have tried to solve this problem. And therefore, you must have heard, I do not want to go into the details of, how they have solved the problem. You need to go in to a deep mathematics to be able to solve it and what you get then is, what are known as spectrum compatible acceleration time history.

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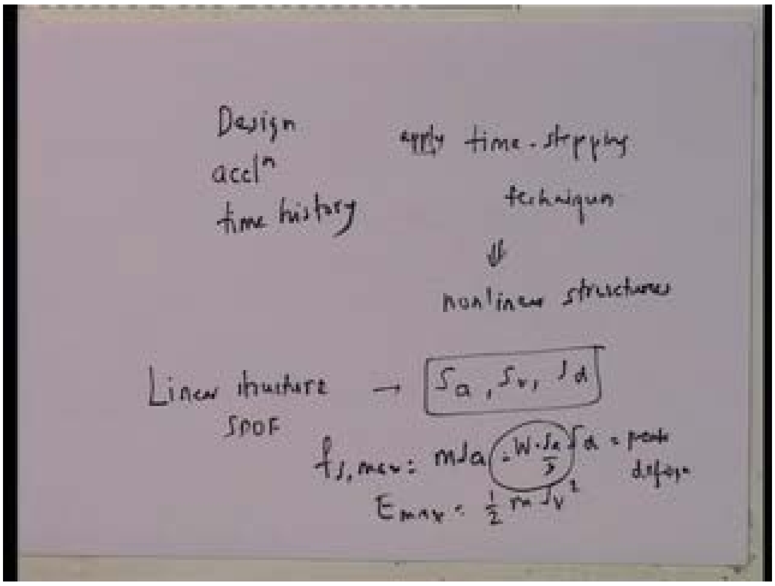

Now, my final question to all of you is, does it really matter, where do you require acceleration time history. The only place where you require design acceleration time history is, where you need to do time stepping need to be apply, need to apply time stepping techniques. And I did talk about, where time stepping techniques are required, they are required for non linear structures. If you have a linear structure, this is of course we are looking at single degree of freedom system.

If you have a linear structure, S a, S v, S d is all that you require, because why, f s max is equal to m S a, S d is my peak deformation and peak strain energy is equal to half m S v square. You see if you have a single degree of freedom linear structure, the design spectra themselves give me all the responses that I am interested in. The design response is that I am interesting, the peak value of the force, what do I design the supporting structure for, for m into S a.

What is the peak deformation that I am likely to see so that, I can then design certain systems to take the deformation, it is S d. What is the energy that is absorbed by the columns so that, there are certain things, in which if they are exceeded that fails that is, half m S v square. S a, S v, S d gives me all the information that I have required and these I have already looked at design spectrum. And therefore, if you are done with response analysis for earthquake, if you are given design response spectra, you do not need anything else.

You do not to be need to know, where those response spectra have come from, the design response spectra, use them and that is what is used if you look at the Indian code IS 1893. IS 1893 is the earthquake code, if you look at it, 1893 2002 part 1 if you look at it, you will see design response spectrum. And you will see that, you design a structure for certain percentage of the weight and that is nothing but weight into S a by g. Thank you very much, that brings me to the end of all the kinds of loads that I am going to look at for a single degree of freedom system. In the next lecture, I shall start of with looking at, structures that are not obviously single degree of freedom systems, but we can treat them as single degree of freedom systems.

Thank you very much, bye bye.