Structural Dynamics
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Lecture - 15
Response Analysis of Single Degree of Freedom Systems for Earthquake Loads

Good afternoon everybody, today we are going to be talking about totally different thing after now we have looked at a variety of loads without really looking at where there actually come from all though the beginning I did talk about and along the way I have talked about some practical situations, where specific loads r c. Today we are going to be looking at earth quake ground motions and the response analysis of single degree of freedom systems for earth quake loads.

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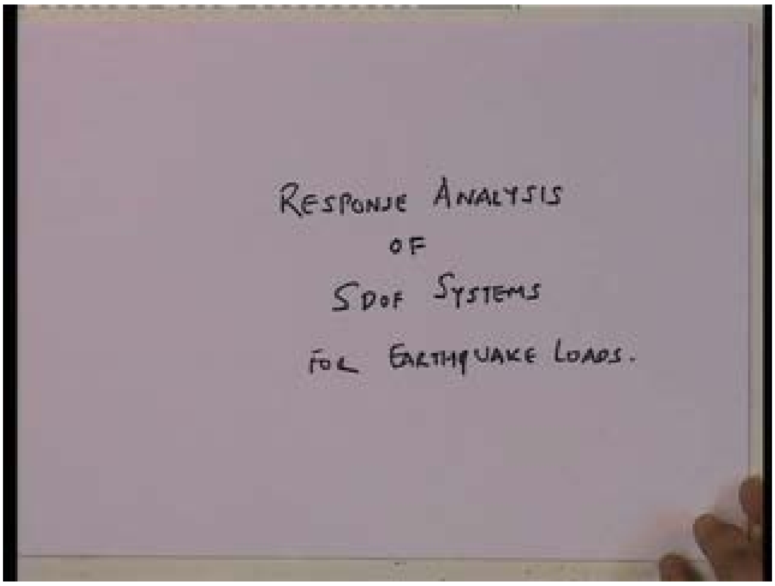

So, we are going to be looking at. So, this is what we are going to be talking about now before we start talking about the response analysis it is always a good idea to have some clue about the background of what exactly is in earth quake load. Typically what happens is you have all over the world because of specific fracture zones, which are known as falls always certain plateaus in the entire world is made up of plateaus different plateaus, and these plateaus are actually floating on mountain magma.

So, because of certain stresses etcetera for example, you to give an idea in the Indian context what is happening is why are the Himalaya's so high, they are so high basically, because you have the erosion plateau sitting here, and you have the Indian plateau which is going under the erosion plateau and pushing it up and that is how you get the Himalayas.

So, what we have in India it is called the main boundary fault the main boundary fault is something that runs west to east along the the terai region all the way from the western part as best as Himachal Pradesh, all the way through the top of Uttarakhand all throughout Uttar Pradesh through Bihar, and all the way through Assam and up in to Burma, and then that plateau again comes down through the paths of the bay of Bengal through the Andaman and Nicobar and indo Indonesia that is the zone of the fault line large zones.

So, now what happens is typically you have rocks and as the plateau moves for example, the Indian plate is moving under the erosion plateau at a rate of approximately about 5 in on an average about 5 centimetres a year. So, what is happening, due to this momentum the plateaus are moving, but on top of the plateaus are rocks. So, as the plateaus move the rocks are subjected to stress, and anything that it can take a particular amount this as it is moving this strain and due to that strain, this stress the stress build up and certainly you have a fracture when the fracture happens that is the earth quake.

So, essentially the fracture event is the earth quake, and due to these due to this sudden jerk and release of energy you have waves propagating through the geological rocks, and then on to the that the soil, and that is what we feel at the top. Now, in the beginning the various kinds of waves that are developed. For example, the fastest waves are known as the longitudinal waves, the p waves they are the fastest propagating waves and they pretty much or the first ones to hit at a particular site.

They are the first ones to hit although by enlarge by the time it comes to the surface, it is fairly complex thing, but, if let us assume that it is a homogenous body, then the fastest travelling wave is the p wave post. This p wave is what is known as shear waves shear waves p waves are what longitudinal waves are waves where the particle moves in the direction of the propagation. Shear waves are wave the wave is travelling in this direction, but the practical is moving in the transverse direction those are known as shear waves. Shear waves are the next fastest waves, and the next ones to arrive, and then we have the surface it self's starts on dilating. And you have rally waves and love waves and all those kinds of other kinds of surface wave that hit, and then the slowest propagating thing.

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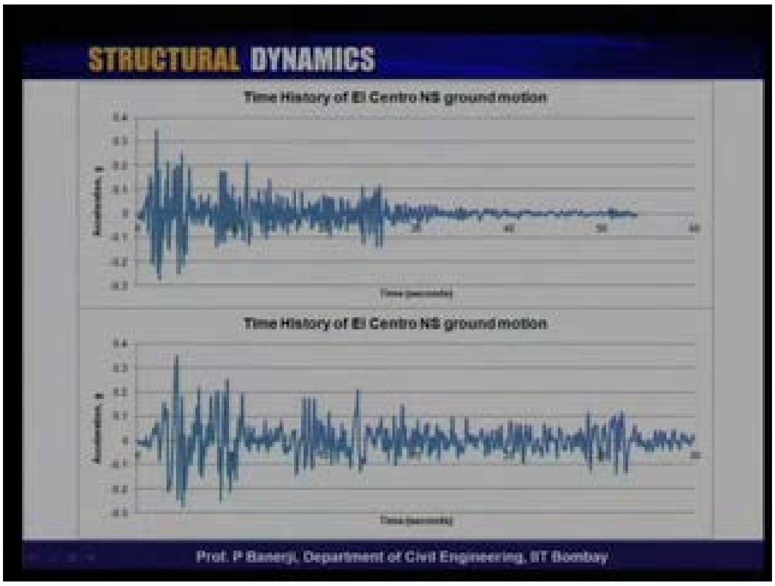

So, if you look at a particular ground motion, this is, I mean all though this is a time history at a station a called l central during the 19 40 imperial valley earth quake, this is an one central station. So, that is where this record was done. So, this is the path, when the p waves come. This is, when the major horizontal shaking this is the this is the horizontal ground motion north south ground motion is essentially the horizontal ground motion of, and this is how it looks.

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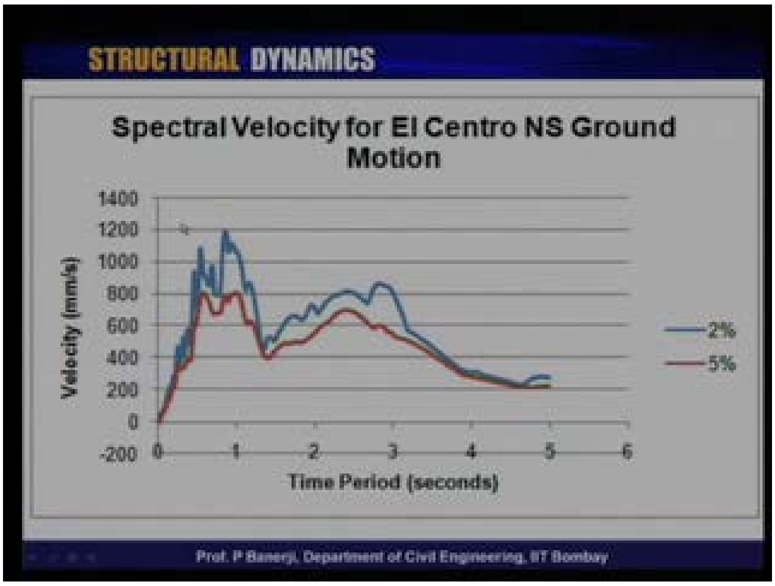

So, you basically have a very small part, then you have a slow build up then it kind of I am sorry, then it slowly builds up, it starts shaking and then you have a period of intense shaking, and then you know less shaking, and then slowly the shaking dies out. So, this is how a typical ground motion, and this is the acceleration of the ground this is how it is it is measured by an accelerometer and this is how it looks, and if you really look at it this is same one this is the full shaking that must seen at the station, and this is really you know these paths are something that the structural engineer is not really interested in we are really interested in the strong motion part and the strong motion part is approximately about thirty seconds.

So, this same thing, just this part have been zoomed up and gives you better idea of all the shaking occurs. So, if you look at it starts then it builds up and then you get the peak. The peak ground motion in this particular case was thirty four percent of g which basically means, three point four meters per second squared was the approximate grand motion that you had, and then you get waves motion and then so, the that the peak ground motion happened about two and half seconds in to the motion now that can vary, but typically, if you look at it the strong motion part stays about fifteen odd seconds, and then it starts going down slowly, and this is really the surface waves coming, and that is it then you start getting very low g values, and you do not really worry about.

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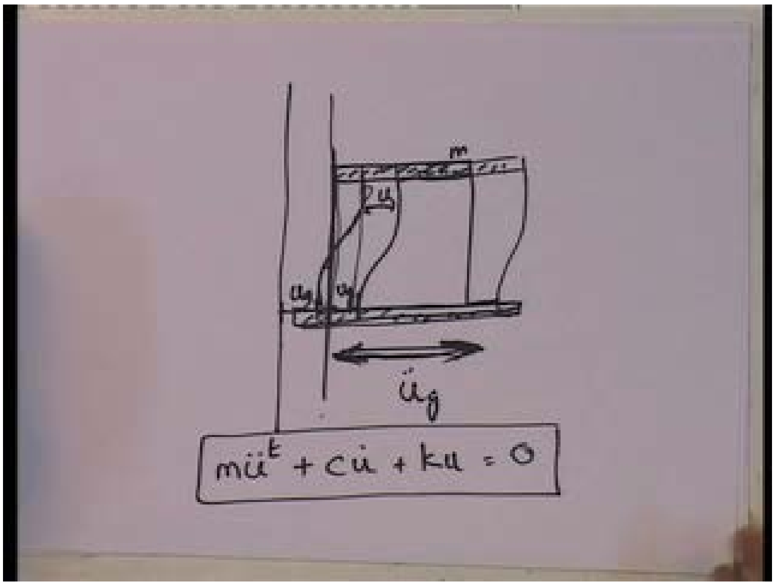

So, that is how a ground motion looks, and this is the ground motion that we are interested in. So, here I have my single degree of freedom with its foundation, and the foundation is this is how a typical single steel storey, and how what is the ground motion the ground motion is essentially this motion. So, in other words, the earth quake when it comes actually earth quake is fairly complex, because this is three d dimensional there will be two kinds there come it will shake in this direction.

It will shake in this direction, there also be vertical shaking, but you know let us just look at unidirectional shaking. For now, because you know this is the simplest kind of situation, and this is what is given as the acceleration time history, and this is the time history that we have over here that we are looking at this. So, this is u double dot g. So, if you have this right and you have. So, this equation looks like this then right, you see, let us look at inertial reference system. So, this is inertial reference system. Now, at a particular instant of time, the ground it is of the base that is the base itself will have moved by a particular amount u g.

Now, due to the ground motion this will sorry not this way this will have moved this much and this will go like this, will have moved this much, and this will go like this, and so essentially what you have is due to the ground motion you have one part, which is u g, which is the, which is the inertial reference system, which is the un-deformed reference system, you have u g and on top of that you have u, which is the deformation of the. So, if you really look at it the acceleration that this mass m sees is m u total plus c u dot plus k u is equal to 0. This becomes the equation of motion for this particular structure. And again this is something that we have already developed last time.

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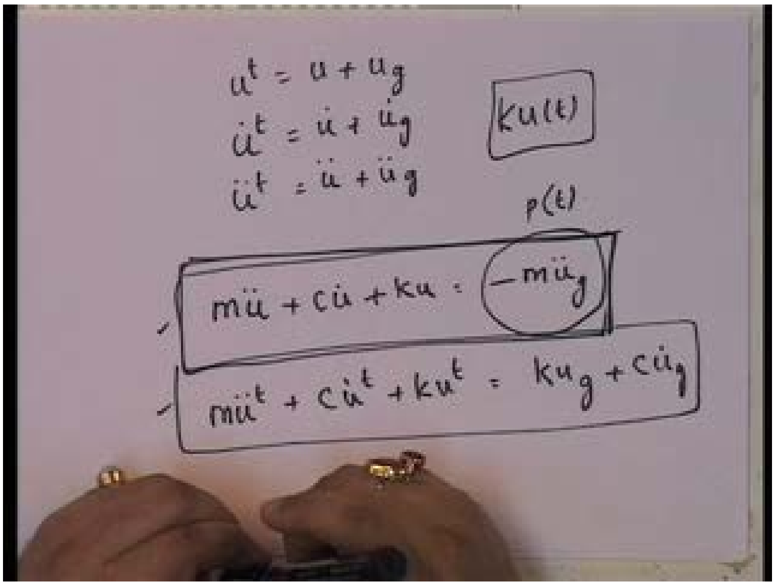

And so, here if you really look at it what we have is the fact that u total is equal to u dot plus u g u dot total is equal to u dot plus u dot g and u double dot total is equal to u double dot plus u g. So, if you substitute this and write this in terms of the deformation, then this is the equation that you get of course the other way of looking at it if you really look at this basically this part is your p of t.

The other way to write it again in terms of the total is, this is the other way to write the same ground motion. This is a lateral ground motion and so therefore, these are degrees of freedom or the lateral degrees of freedom. So, therefore, either one of these are valid, but this is the one that we typically, use why do use this one because remember, I had said that use this one, where we are really interested in k u, the force the equivalent static force that is occurring due to the earth quake. So, therefore, this is the equation that we normally look at.

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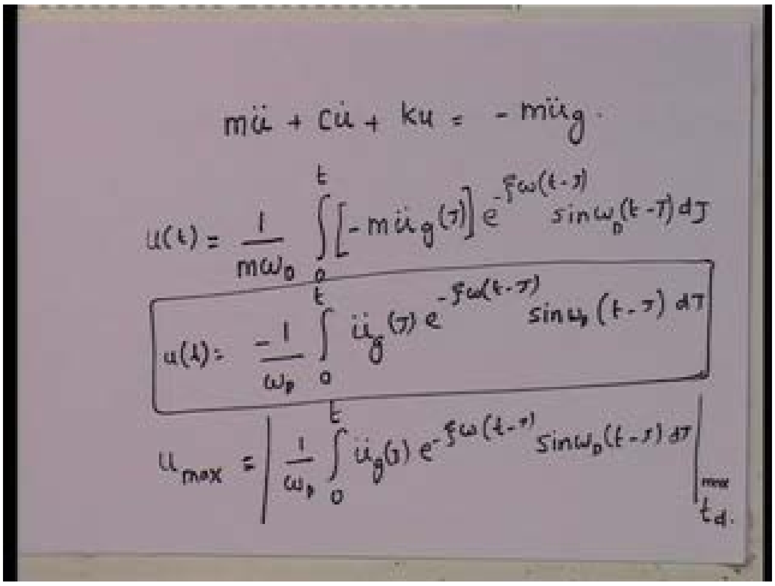

So, looking at that particular equation then what do we get we get that, m u dot plus c u dot plus k u is equal to minus m u g, and considering the duhamel integral, where this is p tau what does happens then if you look at it the solution u of t the duhamel integral approach is equal to 1 up on m omega d 0 to t and the p of t is minus m u g tau that is p of tau in to exponential minus psi omega t minus tau sine omega t minus tau in to d tau.

So, this is u of t and, if I re-write this, that m goes out minus goes out. This becomes the following u of t is equal to minus one up on omega d 0 to t. This is the response of the this thing and if I really look at it, if I look at u max the maximum value of the deformation. This is remember this we called this either the deformation or we call it as the motion of the mass relative to the ground. So, this is either deformation.

So, if you look at maximum deformation maximum deformation is nothing, but over the total duration of the earth quake. So, we find this out, and whatever is the maximum value over the duration of the earth quake is the maximum deformation. So, of course, do we evaluated this way well not quite. Now, let me do something, let me just go ahead and try to define do this for an un damped system. This is the reason why I am doing this for an un damped system just humour me.

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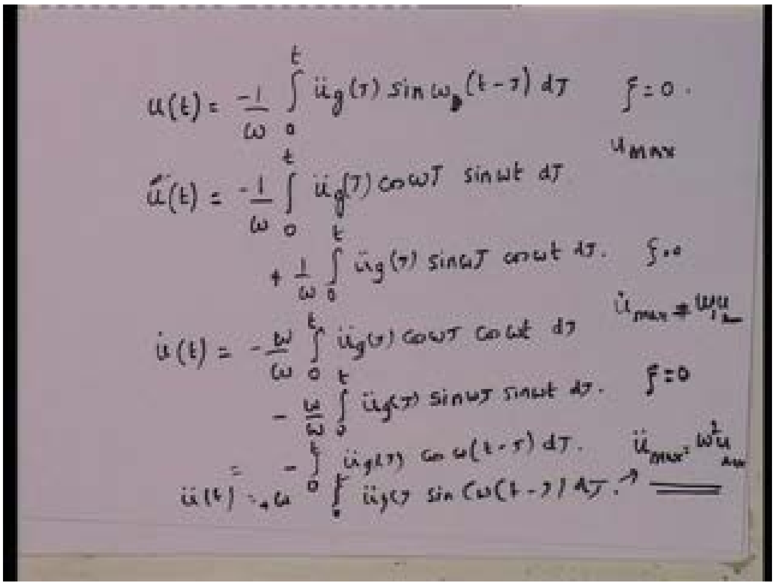

So, then your u max a u of t for an un damped system is going to be equal to 1 up on omega 0 to t u g tau sine omega t sorry, this is omega omega d becomes omega tau sine d. This is the un damped system response un damped system, where psi is equal to 0 hmm. So, now, let us try to differentiate this with respect to t what do we get, we get u dot is equal to 1 up on m. Now, let us look at this sine omega t minus tau if you really look at it, this is going to be become equal to 0 to t tau cosine omega tau sine omega t d tau and minus. By the way this is minus this is minus this becomes plus 1 up on 0 to t.

So, this is not u dot this is u u g t, then you have sine omega tau cosine omega t d tau again, the sine omega t and cosine omega t comes out side and so, when we do this is not u dot of t, this is just u of t u dot of t basically becomes the following minus 1 up on omega and note that this omega becomes outside. So, this becomes omega 0 to t, and this becomes cosine omega tau, and this becomes plus the cosine omega t, this becomes minus omega up on omega 0 to t.

This will become sine omega tau sine omega d tau and so, if you look at this is nothing, but equal to 0 to tau and u double dot, if you notice is going to be equal to omega 0 to tau, again minus, this will become plus omega u double dot tau sine omega t minus tau d tau. Now, the question that becomes that, if you look at it, if you look at peak, if you look at the peak, if this gives you u max look at this one u double dot max is nothing, but equal to omega squared u max now if we look at so, therefore, if psi is equal to 0.

But note that, u dot max is not equal to omega in to u max, why because, if you look at this has cosine. So, cosine cannot be. So, even for psi equal to 0 u dot max is not equal to omega, But u double dot is equal to omega of course, when e to the power of minus I comes in it becomes a little bit more complicated, and we cannot we know I mean I do not want to get in to those details right now because those are not of relevance.

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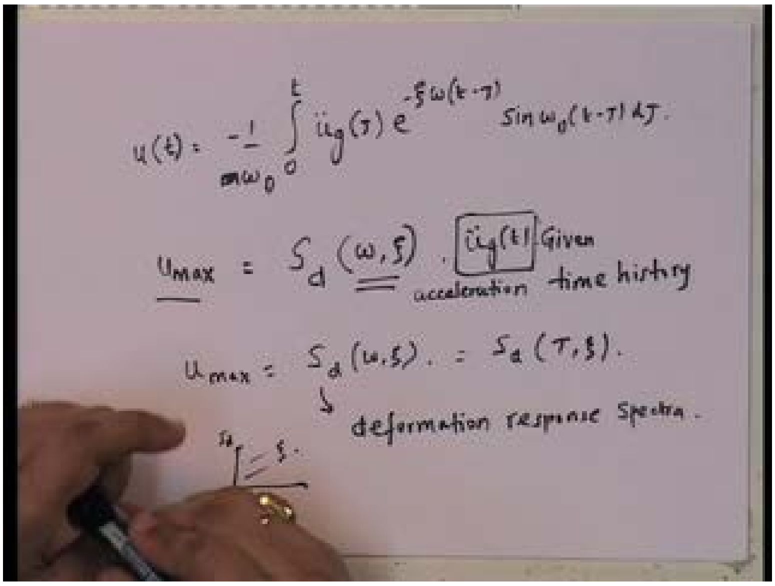

Now, the question, then comes back, if I go back to the original equation in the original duhamel integral. It is the m goes away its 1 minus omega d 0 to t u tau e to the power of minus psi omega t minus tau sine omega d t minus tau d tau see. If you look at this, what does the u max depend on of course, it is its u max I will call define this, as what is this depend on, it depends on omega, psi, and of course, it is a function of this. But if given this, if you are given this, this is a function of omega and psi. This is just a function of omega and psi, and nothing else, nothing else, no other, if we look at omega d omega d is also a function of omega and psi. So, omega psi omega d psi and the only other term that you have is this.

So, given u max is a function of this and. So, if I plotted this as a functional omega n s this is known as deformation response spectra, why spectra well the way this deformation spectrum is plotted is that is a function of omega for different values of psi that is how the deformation response spectrum is plotted and, if we look at this, then for the l central ground motion, which I had shown here, if we, if we solved this and got the peak values. So, therefore, I need a if you look at this, it is a function of omega in other words it is also a function of time period and omega right omega and time period.

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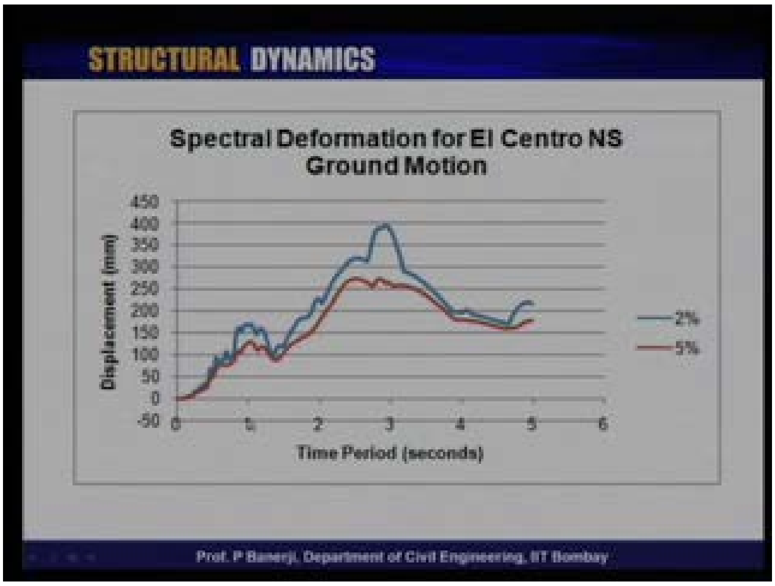

And typically, one way of plotting, it easily is with time period, where this time period actually refers to the fundamental, the time period of natural vibration of a structure. So, this is the time period of the structure vibration, and if you look at this is a displacement and so over across the time period for different values of psi is the spectral deformation or the deformation response spectrum for the one central north south ground motion. So, this is for a particular ground motion, this is how it looks something very interesting comes out of it, and that is that what we see is the following that, if you look at s d omega and psi.

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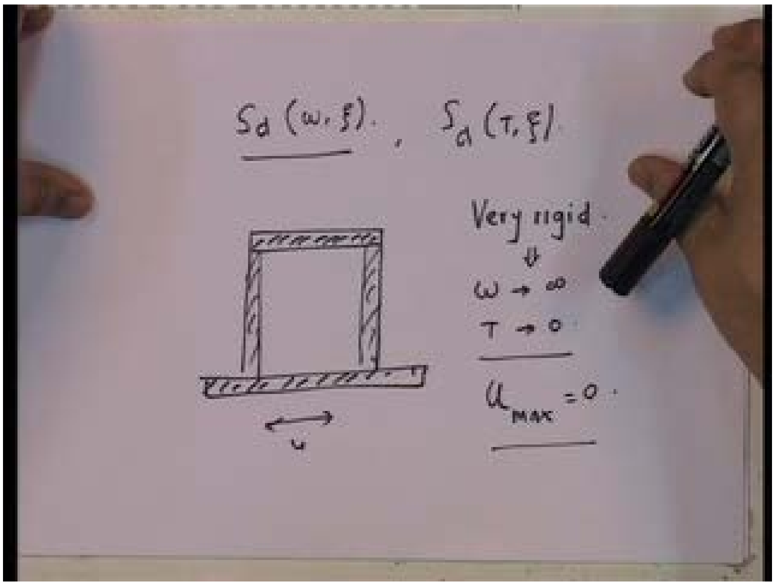

Let us look at a structure and it is, and where the ground motion happens here. This is the structure. So, I mean s d r s d t and psi. So, let us assume, let us look at a very rigid structure very rigid, if we have very rigid structure what is that mean, omega tends to infinity t tends to 0. Now, think of it is a very rigid structure.

It is not being allowed to move at all, what would the deformation be, let us look at let us look at, it this is vibrating, but this is a very rigid structure. So, what is gonna happen to this, what is the deformation, deformation occurs in this. Now, if it is very rigid what is happen happening this is rigid right its rigid this cannot deform. So, it cannot deform. So, what is my deformation the peak deformation equal to, its equal to 0. Look at this, as time tends to 0 at time t equal to 0, the displacement is 0. This is irrespective of what your psi value is you notice.

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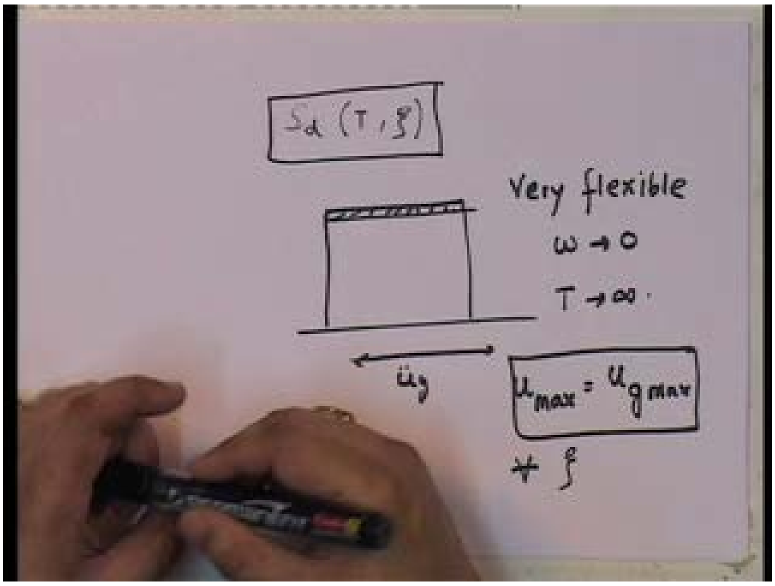

Let, us look at another part and that is let, us look at a very flexible structure and let, us look at S d as a function of t and psi. So, if you look at this and I have a very flexible structure, a very flexible, very flexible what is that mean, very flexible implies that omega tends to 0 and t tends to infinity. What happens let, us look at this is extremely flexible as this moves what is going to happen, the inertia of this is going to keep this is here and this is what is going to happen. So, what is the peak deformation, the maximum deformation, if we look at the maximum deformation, this does not move why because this is infinitely flexible right infinitely flexible.

So, as this moves, the mass has inertia right and the ground motion is actually not being transmitted, because these are. So, flexible they cannot transfer any force. So, this what is going to happen as for as the mass is concern, it does not see. So, what is the peak motion, the peak displacement peak deformation is going to be what, this is here right. So, peak deformation peak deformation is going to be equal to the peak ground displacement, again this is for all psi.

This is for all psi, and if we look at this here, this is of course only 5 seconds, if you keep going you will see that both of these are converging to a certain value, and that value is of the order of about 200 millimetres, because that was the peak ground displacement 2 metres was the peak ground displacement in the l Central north south ground motion of 19 42 19 14 imperial value earth quake.

So, you see the deformation spectrum has a specific form characteristic that it has, which is very very interesting, and that is that it does not, it has specific characteristics for example, at time t equal to 0, S d will be 0. So, if s d is not 0 there is something wrong because by definition if you have a very rigid structure that structure cannot deform. So, if you cannot deform u max, which is a deformation peak deformation is obviously 0, and if, it is extremely flexible; that means t tends to infinity.

Again, if extremely flexible, the ground can vibrate, the structure does not even know that the ground is vibrating, because the columns are so flexible. They do not transfer any load. So, what is the peak value of the deformation well, whatever the ground that maximum ground deformation is going to be the maximum deformation of the column, because the base is moving, and this is the the mass stays where it is; obviously, the peak deformation is going to be the peak ground displacement.

So, these are aspects that are very important. Now what about in the middle well, it is not it is not very easy to say, what is happening in the middle; obviously, you can see that in the middle there is some kind of resonance, because if you look at it the ground, the ground displacement about 200, the peak displacement that you have is about 400. So, there is a some dynamic behaviour; obviously, this is dynamic behaviour, this is about 3 seconds so, that is I mean there is nothing more we can say about the deformation response spectrum.

So, deformation response spectrum is a very very important, but now, let us look at certain other points which are important from the perspective, what are the other things that one aspect I am interested is u max, but u max for sake does not interest me what interests me is the following. Let us look at this.

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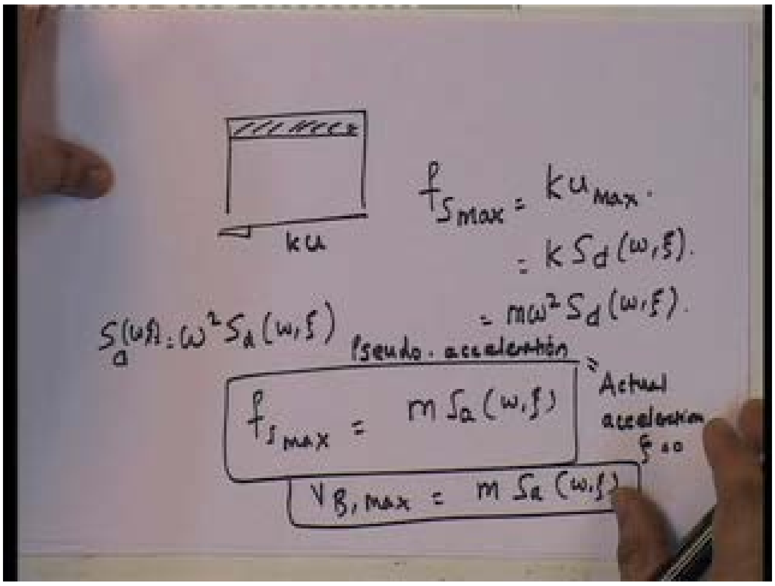

So, this is the mass, I am looking at edge deformation and so, its deformation what you have here is K u and that is the shear that is being transmitted to the base from the structure due to the ground motion, and that is the that is a very important parameter and. So, this parameter, which is the elastic force, so I will call it as the elastic force peak is; obviously, equal to k in to u max there is nothing there is nothing wrong with that right now the question then becomes is that. So, this is equal to k times s d which is a function of omega and psi now can I put k I know is equal to m omega squared.

So, if I take m omega squared s d what is the term omega squared s d equal to what are the units of this millimetre and omega squared is radians per second squared. So, this is become millimetre per second squared its units of acceleration and this is called as s a which is also a function of omega and psi and. So, if we call this then f s the max is equal to m in to s a and this is also equal to the peak b shear which is equal to m in to s a. so what is this s a the s a is related to the deformation spectrum and in a way force is equal to mass in to acceleration.

So, it is like a pseudo acceleration because it is does not represent the true acceleration the only case where it represents the two at acceleration is if you remember we looked back at that particular function did not we looked back and we saw that when you have 0 psi equal to 0 the peak u is equal to omega square in to d. So, in a way the pseudo acceleration the pseudo acceleration is equal to the actual acceleration total acceleration if psi equal to 0 only for psi equal to 0 is a pseudo acceleration equal to actual otherwise it is not; however, it is that pseudo acceleration spectrum is actually more important to us because what it does do it actually represents. If you look at this gives me the equivalent force that is applied to the structure that is applied to the structure due to the ground motion.

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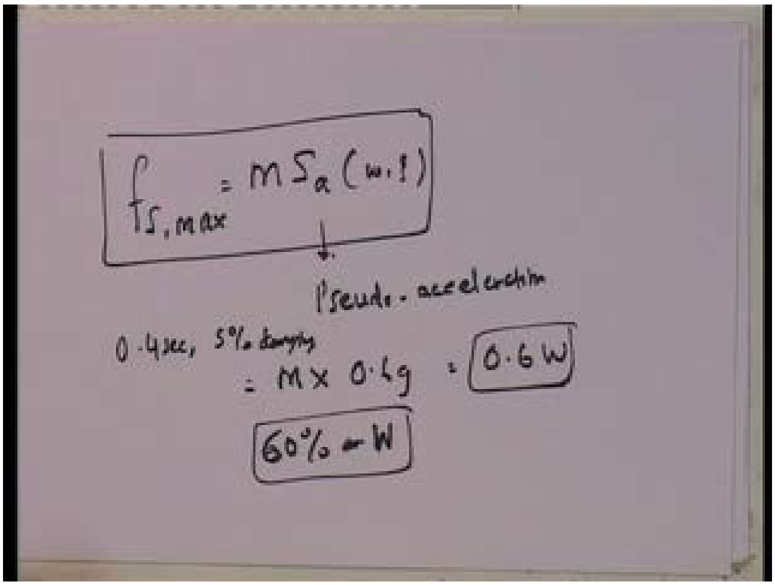

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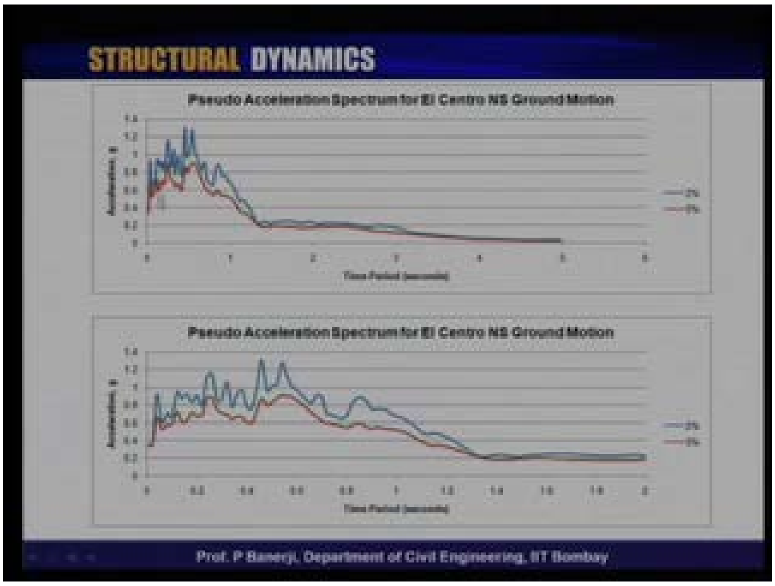

So, in other words I have made the ground motion you see that the earth quake ground motion has been transformed through a specific form of the response spectrum in to a mass in to acceleration kind of. So, the load is due to mass in to acceleration and this of course, acceleration is a pseudo acceleration and if you draw it with respect to omega or t.

Then this is what it looks like if you look at the pseudo acceleration spectrum this is what the entire thing looks like the pseudo acceleration spectrum there are certain interesting things about this is of course, again the same thing this is across all time periods and this is where you have taken the peak one and shown it blown it up and shown it here this is this is the same thing expecting that I am really taking this part and showing it little bit larger and this is something very interesting and that is that the acceleration has a specific role to play note that this is the pseudo acceleration spectrum pseudo acceleration spectrum across time period for different values of psi.

So, now the point becomes that you see the earth quake ground motion the mathematics has been left behind and what we have seen is that it essentially becomes a problem of where well if you are looking at it purely from a peak perspective the peak force in the columns and of course, on the base is equal to mass in to a pseudo accelerations value which given a structure if we look at this. So, therefore, this becomes the following that suppose at a one second structure and it was 5 percent damping so. So, now, let us look at it at here 0.4 second time period 0.4 second time period is to 0.5 hertz.

So if we look at this 0.4 second and I was psi of 5 percent I come over here look at this value look at this value this is 0.6 g. So, 0.6 g. So, therefore, the peak ground acceleration becomes m in to 0.6 g sets point six times the weight of the structure so; that means, I know that the force due to the l Centro ground motion is equal to the lateral force is equal to 60 percent of the weight of the structure I did not have to do I immediately know this I did not have to do any computation I did not have to do any structural dynamics I did not have to do anything. So, in other words if we have for a particular ground motion if we have the pseudo acceleration response spectrum we can directly get the forces that the structure is subjected to directly you know I have just done it you know for a particular.

So, this was this was for a 0.4 second structure with five percent damping and the force is 60 percent the weight of the structure very interesting the response spectrum and its use gets highlighted through this procedure. Now again just like the way I tried to get your for the displacement let me look at a situation where I want to look at the what happens to the pseudo acceleration response spectrum in the limits.

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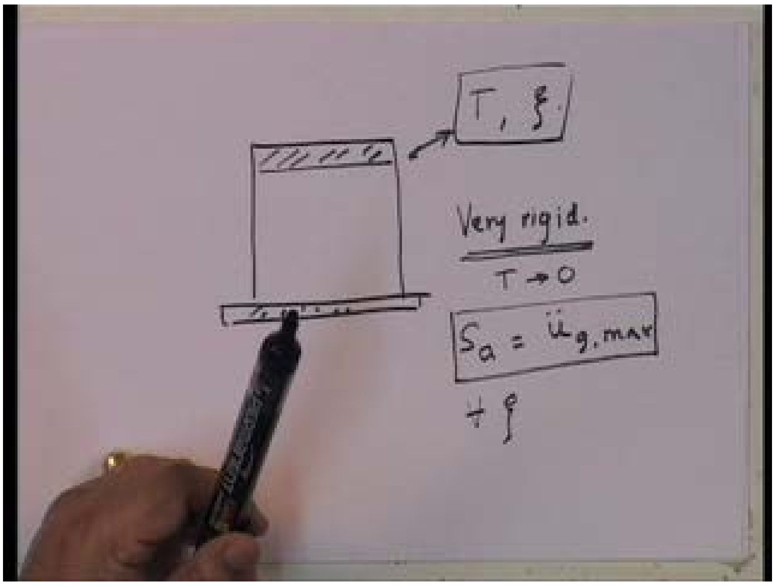

So, in other words I have. So, it is given by a certain t and psi. So, this structure is defined through some t and psi. So, let us assume that its very rigid let us look at this what happens when it is very rigid the base is vibrating this is vibrating with it why well because it is very rigid there is no deformation. So, therefore, if you look at it what is happening the top is moving exactly as the base is. So, in other words what does s a become for very rigid as t tends to 0 s a becomes the same as. This is when it is very rigid s a becomes equal to u g max let us look at this lets come back here at t equal to 0 what is the value if you if you read it off here you will see it is equal to 0.33 g lets go back you see this it is 0.33 g that is the value at this 0.33 g.

So, in other words we have satisfied ourselves that you have very infinitely rigid structure s a is equal to and this is not a function of this is for all psi and this you note you will see here is that irrespective of two percent or 5 percent in this zone it goes to point three four g. So, that is something that is very interesting; that means, the 0 time period 0 time period pseudo acceleration spectrum is equal to the ground acceleration.

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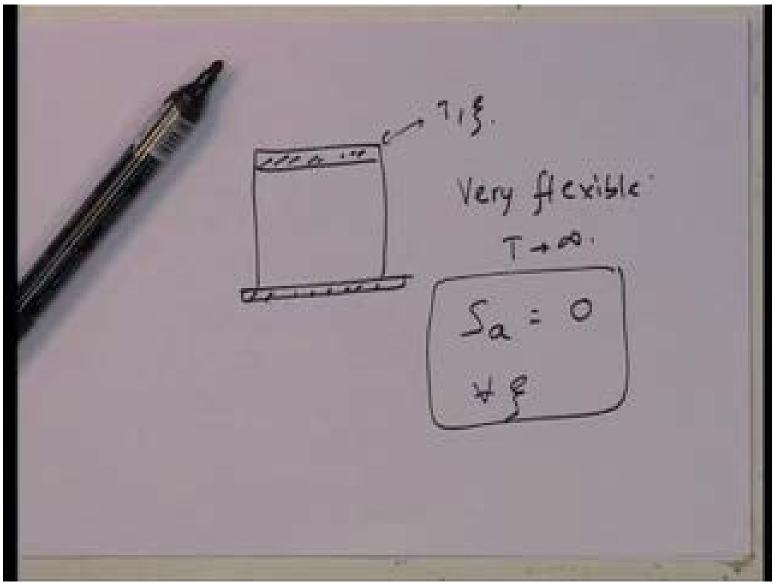

Let us look at a situation where you have the same structure now it is given by certain t and psi and I say it is very flexible very flexible means t tends to infinity very flexible t tends to infinity what happens here when it very flexible this is moving this is not moving right because it is very flexible none of this motion is getting transferred to the mass and. So, therefore, what is s a equal to s a equal to 0 for all psi well let us look at this you see this it tends to 0 and irrespective of what you know percent of psi that you have so in other words.

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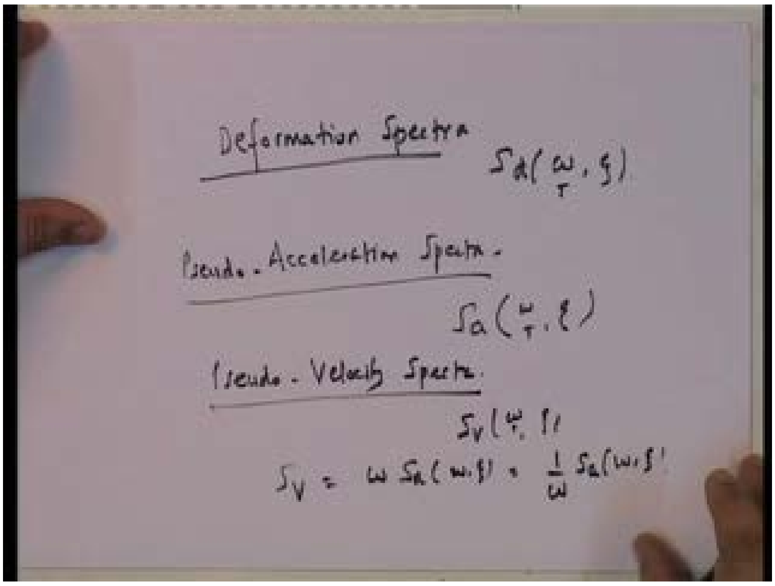

So, therefore, this form is such that this is going to be u g the peak ground acceleration and this is going to be 0 in between well we will see how the thing happens there are certain characteristics that we will a look at a little bit later now.

So, I have talked about you know I have given you a reason for the deformation spectra deformation spectra is s d omega r t of psi and I have looked at the pseudo acceleration spectra which is s a either omega or t function omega t and psi; these two are things that we have looked at and finally, there is the pseudo velocity spectrum and where does that come from that is called s v omega t, and s v is equal to omega s d which is equal to one up on omega s a because; obviously, this s v what does s v have a reference to s v if you really look at it.

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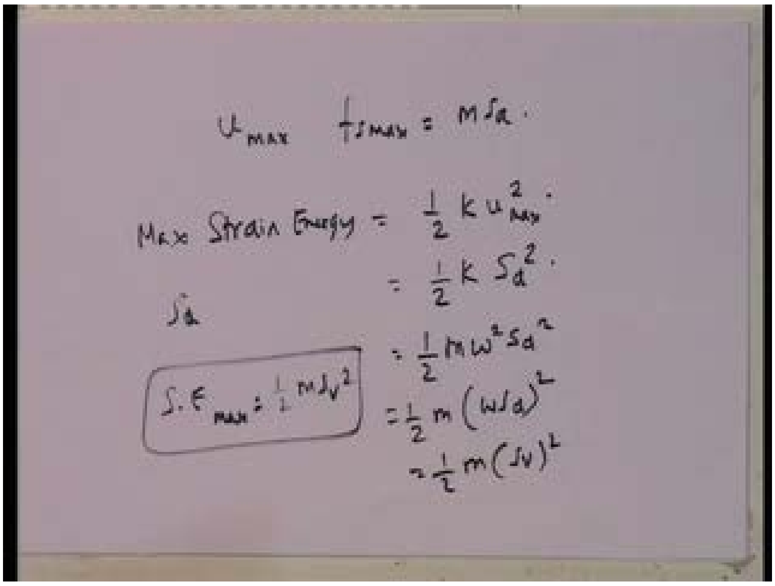

Let us look at this and that is that if you have u max displacement I want to find out the strain energy the maximum strain energy stored what is that equal to half k in to u max squared do you agree to this. That is the maximum strain energy stored and. So, this is equal to half k s d squared here right and what is l k equal to m omega squared s d squared. So, I will call this as half m omega s d squared and this omega s d i will call as s v squared and so, now, what are the units of s v; obviously, velocity what is the maximum strain energy its equal to half m in to the pseudo velocity spectrum. So, think about this looks like maximum strain energy is equal to maximum kinetic energy it looks like a kinetic energy term does not it excepting that it is not kinetic energy because the pseudo velocity spectrum.

So, in other words the you know what is s a required for because f s max is equal to m in to s a what is s v s d of course, is directly I know what s d is maximum deformation and the maximum strain energy is equal to half m s v squared. So, we have now define that in terms of for a earth quake we can define the earth quake either by its time history or by its deformation spectrum or its velocity spectrum or its acceleration spectrum note that all these three are related to each other. So, any one of them is good enough for me to find out the other typically what we give is this is what we give because here you know there are some specific things which I will talk about in the next lecture which give us certain aspects of, so therefore either the time.

So, the definition of ground motion either the time history of acceleration time history of ground motion or the response spectra of the ground motion. So, that is what we have talked about and once we have the spectra the reason why the response spectra are better is because we directly can find out response of a single degree of freedom system to earth quake loads directly from the response spectrum.

Thank you very much.