Structural Dynamics
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Lecture - 11
Pulse Load Response Characteristics and Impulse Loading

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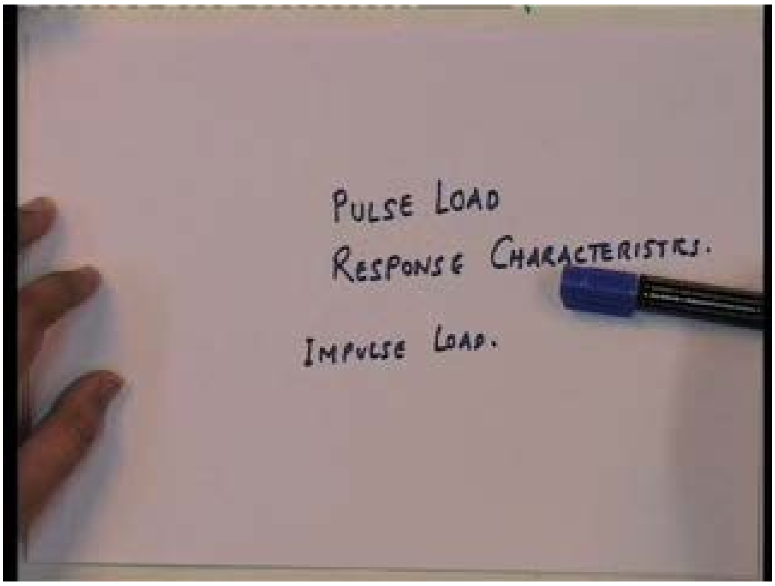

In the last lecture, we looked for the first time at short duration loads and I introduce the concept of the shock spectrum and also we solved some you know typical kind of solutions. In this lecture, we are going to be looking at the Pulse Load Response Characteristics, and also we will introduce the concept of Impulse Loading Now, pulse load response characteristics. Actually to illustrate this, I would like to take specific kind of loading and let us go through the process of having gone doing this.

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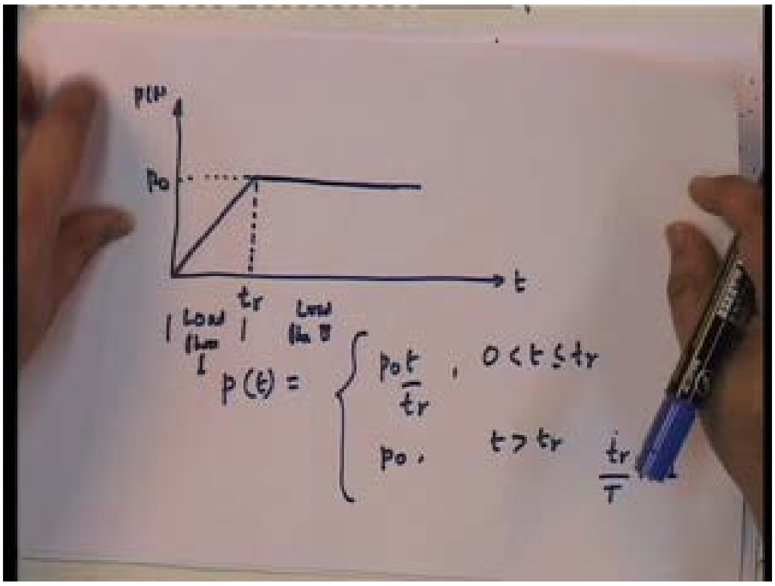

And for this, I will actually solve a problem which does not look like a pulse load, this is t, this is p of t, continues like this. Now, does it continue like this, actually does not, but it does not matter now, let us assume that, it continues and this I will call as t r. So, and this one will be p naught, and so therefore p of t can be written as t r, for 0, t, t r into and it is equal to p naught for t greater than t r. So, in other words, there are two solutions and what am I going to do is, I am going to now resume that, t r by t is let us say, less than 1.

Actually, substantially less than 1 and therefore, the duration of the load does not really matter because t r by t is substantially lower than 1. So, let us look at this solution now, there since the load is a two types actually, I have now here again this is different, this is not phase 1, phase 2 as we know it, this is just that the load phase 1 and load phase 2. And t r is the rise time of the load from 0 to maximum that is why defined as t r, whereas earlier add duration of load.

But, there is specific reason, why I am presenting this, because this will give me some idea about what I am talking about characteristics. So now, let us look at how to solve and now the thing is that, since it is very small, I am going to still continue to solve the undamp system. Because, understand that, since t r is much less than 1 and even in phase 2, the peak will occur somewhere in the beginning. So, for all practical purposes, damping does not come into the picture.

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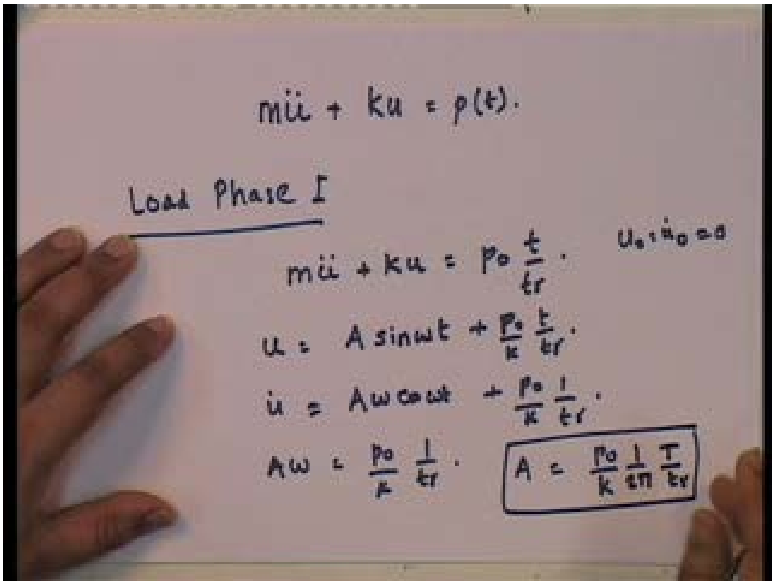

And so I wish I shall only solve, continue to solve this problem because I am looking at pulse load characteristic, response characteristics and that is why, to highlight that, I will continue with this same thing. So, load phase 1 what happens, let us see you are solving m u double dot plus k u is equal to p naught t up on t r. And so if you look at it, particular solution is really p naught up on k in to t in to t r and so without going into to any details, since this is a t r, what will get is, in a cosine term will the have to 0.

So, only the sin term will exists and so it becomes essentially u of t is equal to A sin omega t plus p naught understand that, u naught equal to u dot equal to 0 at rest initial conditions. So, this is the solution and this solution by plugging in u dot, which is equal to A omega cosine omega plus p naught k 1 upon t r. So, essentially what happens over here is that, A becomes equal to I mean, A omega is equal p naught upon k. So, A omega is equal to p naught upon k into 1 upon t r and so this becomes p naught upon omega k, omega k is 2 pi upon t. So, this basically A becomes p naught by k and what you have, you have 2 pi over here so it is 2 pi and on top I have t upon t r. So, this becomes my A, and so therefore the solution of the first load phase solution essentially becomes, what it becomes u of t.

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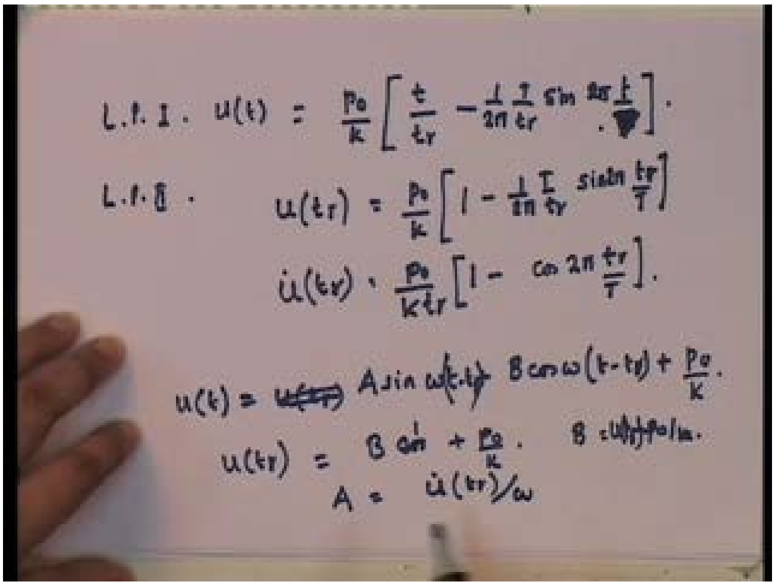

So, this is load phase 1 u of t is equal to and I can put p naught upon k outside and inside I have t, this becomes the following, I made a mistake this plus this is equal to 0 so this becomes minus. So, this becomes p naught k t upon t r minus 1 upon 2 pi T upon t r sin 2 pi t upon T, this is T. So, this becomes the solution of the first equation, load phase 1, so now load phase 2, how do load phase two. Load phase 2 essentially, I have to have u t r and u dot t r and then look at the rest of the thing.

So, what is u t r, u t r is equal to t upon t r. So, this becomes p naught upon k, t r upon t r is 1, minus 1 upon 2 pi T upon t r sin 2 pi t r upon T. So, this is what I get for my u t r and similarly u dot t r, if I double differentiate this, I get p naught k. And I am going to put t r outside so I get 1 minus, you see this one becomes cosine 2 pi upon T, so this goes. So, this 2 pi 2 pi cancels, t and t cancel, 1 upon t r goes outside, so this becomes 1 minus cosine of 2 pi t r upon T.

So, the point to write down is that, in load phase 2 I get u t r so these are my u t and u t r so my u t becomes A sin omega t. Again omega t is nothing but 2 pi t upon T so I am not writing it down cosine omega and this one, I am going to make it is t minus t r, B cosine t minus t r. Reason why, I am taking t minus t r is because then t r will become 0, that is all I am doing really. So, A cosine t r and then plus what is the thing, it will be p naught, so it will be p naught upon k u of t.

So, this is the solution, I am writing it in this fashion purely to ensure that, you get your this thing. And so therefore, what do I get, if I substitute in here I get u t r is equal to B note that, sin t minus t r at t r is going to be equal to 0. So, this is going to disappear, this is going to be B cosine omega t upon t r that is 1, plus p naught by k. So, your B is equal to minus p naught by k and what you have is, if you look at differentiation this becomes sin, this disappears, so this one becomes what, A is equal to u dot.

So, B becomes u t r minus p naught upon k and this becomes u t r upon omega and so ultimately the phase 2 solution becomes the following. Please follow the mathematics because the mathematics is important to what I am discussing. But later on, after the mathematics is over, the physical part is what we are going to be concentrating on. So, the mathematics is important only in so far, as to get as to where we have this, so let us look at this.

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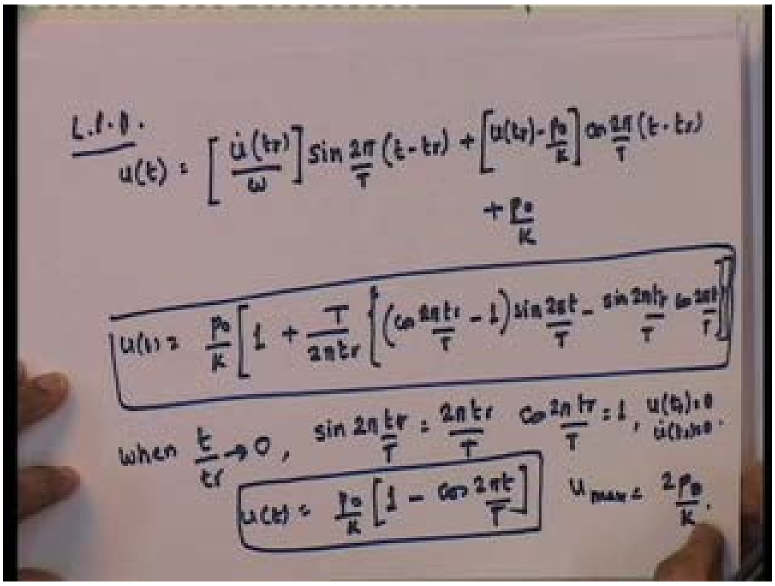

So, this becomes then ultimately, the u of t becomes equal to A, so this is going to be u dot t r upon omega into sin 2 pi by T into t minus t r plus u t r minus p naught upon k cosine of 2 pi t minus t r and plus p naught upon k so this is load phase 2 u of t. So now, the question becomes that, let us expand this and I am not going to subject u to the stress of, how to expanded. But if you write it down then I get the following, I can take p naught upon k outside and I have 1 plus T upon 2 pi t r cosine 2 pi t r by T minus 1, sin 2 pi t upon T minus sin 2 pi t r upon T into cosine of 2 pi t upon T and this then gets.

So, that is your u of t for load phase 2, so therefore for load phase 1, this is my u of t and for load phase 2, this is my u of t. So, what does that give me, let me try to look at it, when t by t r tends to 0, where will the peak occur note that, if t by t r tends to 0, load phase 1 practically disappears. So, it essentially is that, you are in load phase 2, so the peak will occur in load phase 2 and if you look at load phase 2, what do you get, you get sin 2 pi t r upon T is equal to 2 pi t r upon T. Then cosine of 2 pi t r upon T tends to what, is equal to 1 and essentially, u t r and u dot t r tend to 0.

Because, if you look at this, sin 2 pi upon t r this is become 2 pi upon t r 2 pi upon t r 1, 1 minus 1, 0 and this becomes 1 minus 1, 0. So, both of them are 0 and this is what happens, if you have that then these disappear, this become 1 minus 1 this disappears, this becomes 2 pi, so this can cancel with that and so what I have is 1 minus; So, my u of t becomes p naught upon k into 1 minus cosine 2 pi t by T and what is the maximum for this, 2 p k naught.

Very interesting and that is, that let us see what does that mean, if I look back at this all I am saying is that, this basically becomes like this and that as we know is like the triangular pulse where, the triangular pass lost long enough, u max is equal to 2. We know that long enough, this as long as it last more than t d by T, it becomes 2 pi we derive this through this concept.

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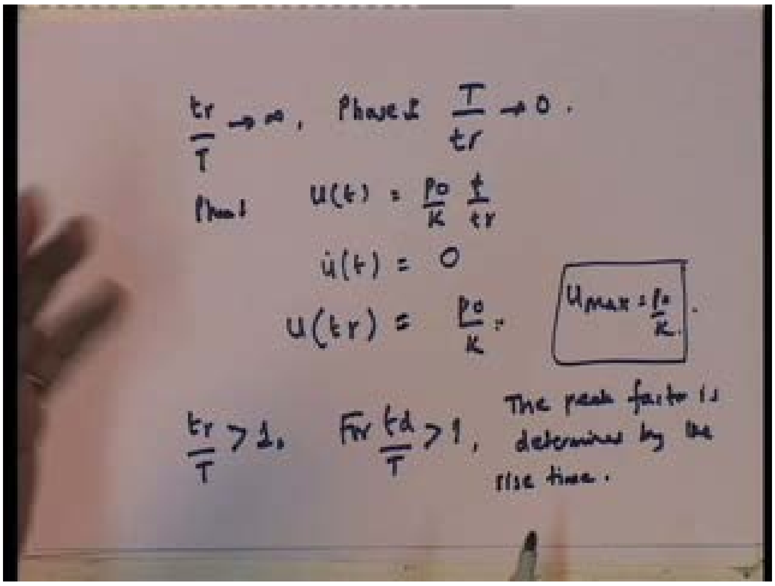

Secondly, interestingly enough, if we try to get, if we take t r upon T to infinity, in other words what it does mean, that the life time is exceedingly slow. So therefore, note something that, just by setting up the load time, just setting up the rise time, so I am saying that, one rise time is really, really fast and the other rise time is very very slow, t r by t tends to infinity. Actually, it does not tend to infinity, it just long enough, if it is long enough then what happens, now let us see again.

We know that, this will be in phase 1, if this tends to 0 this implies that, T by t r tends to 0 so if T by t r tends to 0 then u of t in phase that I mean, it will be in phase 1 obviously and phase 1 u of t is equal to p naught upon k t by t r. Let us solve it, let us look at it mean, go through it, let us this look at load phase 1, load phase 1 is this and when T by t r tends to 0, this term drops out and what you have is this term. So, it is p naught T upon t r and u dot t now, let us see what is u dot t, 1 minus cosine.

So, this t r upon T, when I put t r in here and I put T here, I get that this becomes essentially equal to 1 minus 1 because this term is goes to 0. So, essentially u t goes to 0 so what is the value at the end of the phase 1, p naught upon k. So, if I look at maximum, maximum will be at t equal to t r because if I differentiate this, I get that this is equal to 0 so that is going to be so this is the maximum. So, in other words, u max is equal to p naught upon k what does that means, if the rise time is slow for all practical purposes, u max is the static response.

So, we have this situation that, you have that if t r by T is greater than 1 then max will be in phase 2. So now, the question then become is this that, for t d by t larger than 1 because you see I showed this one that, this one goes on this particular value, it goes on for long time. I mean, it is t d by T greater than 1, what we get is the peak factor, is determined by the rise time. So, if t r by T is very fast in other words, the rise time is very fast means, t r by T is tends to 0, you get 2 p naught upon k.

If rise time is very slow, t r by T becomes very large then d is equal to 1 so in other words, what are the things that comes out of this characteristics for pulse load is that, if t r by T is much large; I mean, if the duration of the load is such that, the peak during the first phase, which is the loading phase then dynamic amplification factor peak value is actually dependent on, how fast it rises. It can vary between 1 to 2, depending on how fast is rises and how slow it rises, see if you look at this, this is this will give you an idea.

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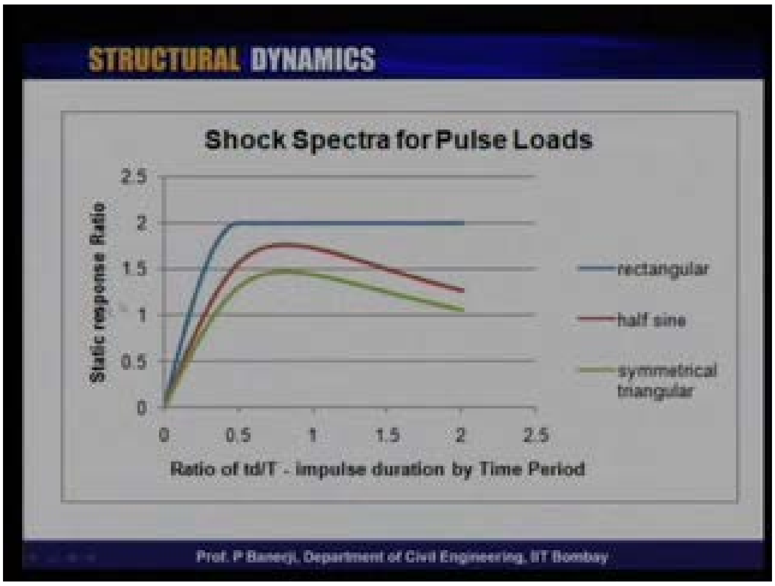

If you look at this particular shock spectrum that I showed you, if you look at the shock spectrum, rectangular really goes up fast. So, that is why, in this particular, if you have very large duration I mean, t d by T greater than 1, look at this, this has the highest value. Half sin goes up, you see the sin goes up faster than triangle, so that is why this is larger, triangle goes up the slowest, so it is a lowest. If I get the triangle to go really, really slowly, see it goes to 1.

So, in other words, this triangle if t d by T is large, the triangle really goes up slowly, it tends to 1. So, all of these points that I have made, come out directly from this shock spectrum and that is the reason why, I showed this specific problem that I looked at, the rise time, so that is a very interesting point of the pulse characteristics.

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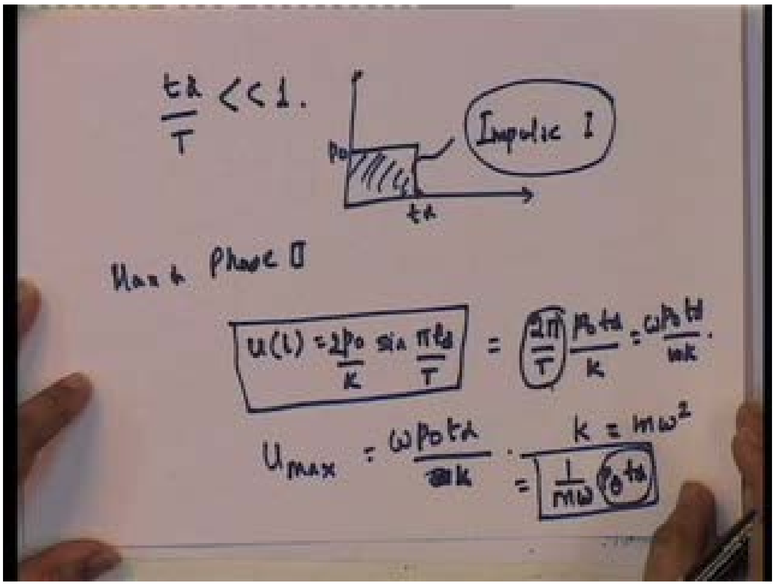

Another characteristic to note is that, if you look at when t d by T in a pulse load is very much less than 1. In other words let us look at the rectangular, this t d for all practical purposes, it cannot even see yet. So then let look back at our response of the rectangular thing and let us just look back at the rectangular response, what did we get. Note that, if t d by T is very much less than 1 where, the max will always be in phase two, because it can never get a chance to rise. And so therefore, what we have is that, u max is equal to 2 p naught k sin pi t 1 T where, this is t d.

Now, when t d by T is very, very small, what does this become, this becomes 2 pi p naught t d upon T upon k, this is nothing but omega. So, I will call that as p naught t d upon omega k so u max is equal to p naught, omega is one term, omega t d upon k. Now, you know k is equal to m omega square, I am going to substitute that in here and so this is going to be equal to m omega square comes here, so this becomes 1 upon m omega p naught t d.

So, u max is really note that, m and omega are characteristics of the structure so what is loading, if you look at it, it is p naught t d. What is p naught t d, p naught t d if you look at it, is really, really under the curve, which is known as impulse I. So, u max is equal to p naught t d upon m omega so now, let us look at this and that is, that let us see if we can get that equation from somewhere else.

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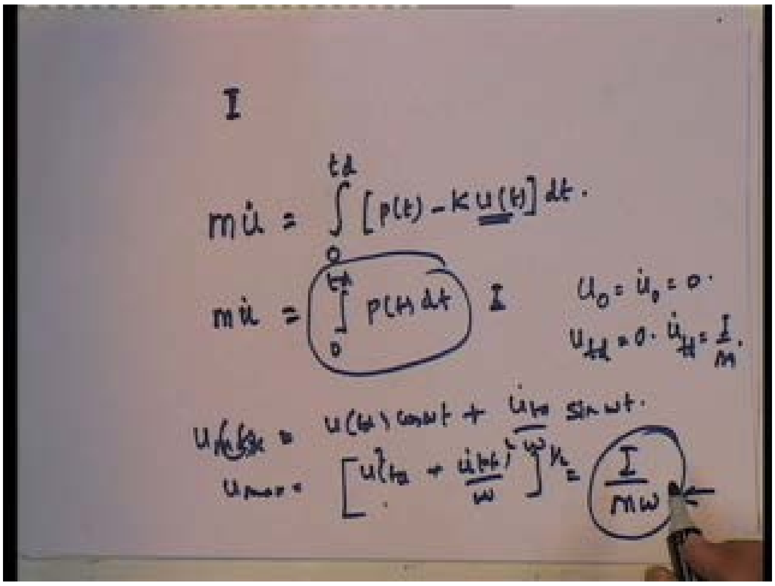

Now, let us consider that the structure is subjected to an impulse I then if you really look at it, this if you look at the impulse moment relationship, what does it say, it says m into the velocity. So, what is this, this is the m into u dot is the momentum, the momentum is equal to, 0 to t d the net force. So, the net force is p t minus k u t d t so now, if you really look at it, if you have a very short duration load, u t practically 0. So, this basically becomes m u dot is equal to, 0 to t d p t d t and what is this, this is nothing but the impulse.

So, essentially what we are saying is, change of momentum is equal to impulse so if you take before, before this impulse was there, it was addressed so addressed meaning, u 0 is equal to u dot 0. After the impulse is happen, which is over at duration t d, what is it, u of t d still 0 because it is too short at duration for velocity to come in to picture. However, u dot t d is equal to I upon N so now, since u t d is practically, for all practical purposes almost not there I mean, it is almost 0.

So then what does it become, my u max becomes equal to u t d cosine omega t plus u dot t d upon omega sin omega t. So, that is u t so u max is going to be equal to u t d squared plus u dot upon omega square. So here, u t d is 0, this is 1 upon so half square root so what do I get, I upon m omega. U max, u max is I upon m omega, let us look at this what do I get, I upon m omega. So therefore, the question then becomes the following and that is, that if your response…

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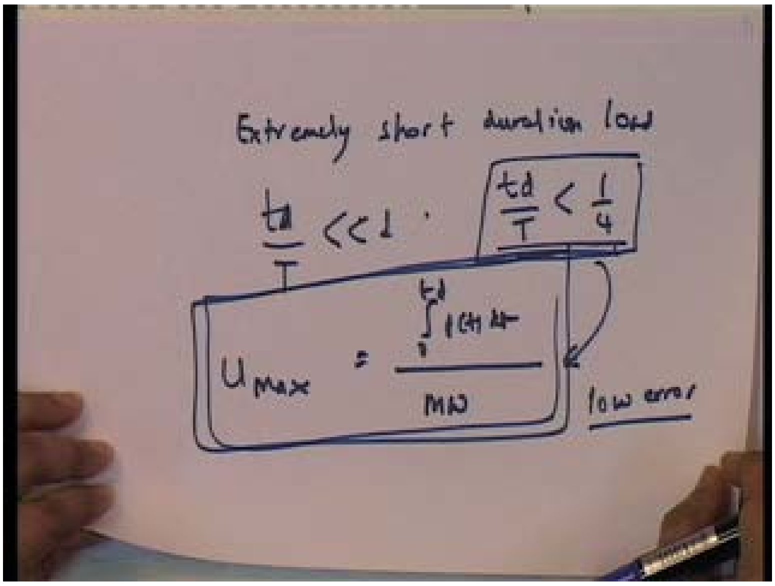

If your load is extremely short duration, extremely short duration load where, t d by T is very much less than 1 then u max is equal to integral 0 to t d, p t the area under the curve upon m omega, that is what u max depends on, that is what u max is. So therefore, note that, it becomes very, very simple, find out the area under the curve, divide by m omega and that is your maximum value of this thing. And it is seen that, in reality, as long as for the pulse load t d upon t is less than 1 by 4, this gives very low errors.

So, this approximation is valid as long as t d upon T is less than 1 upon 4 so this becomes a very, very interesting part of the entire characteristics. So, the pulse load characteristics give us two parts, what is that. If t d by T is much greater than 1 or at least greater than 1 then the dynamic amplification factor is essentially a function of the rise time for example, the rise time for rectangular pulse is very large, so it is true. For half sin pulse wave, it is relatively less than the rectangular but more than the triangular.

And so therefore, you we saw that, as your rise time goes down, your dynamic amplification factor keeps going down. And further more that, if the rise time relative to the time duration is very large then for all practical purposes, the dynamic amplification factor is 1that mean, the responses static. So therefore, dynamic amplification factor varies between 1 and 2, as long as the duration of the load relative to time period, t d by T is greater than 1, this is valid.

If t d by T is much less than one forth, then the maximum is given actually by the impulse divided by m omega. So, this is something that, we get from the pulse load characteristics and now, we also looked at how to characterize impulse loads directly from that. Now, what I am going to do for the small time that I have is, introduce you to the basic concept for generalize loading.

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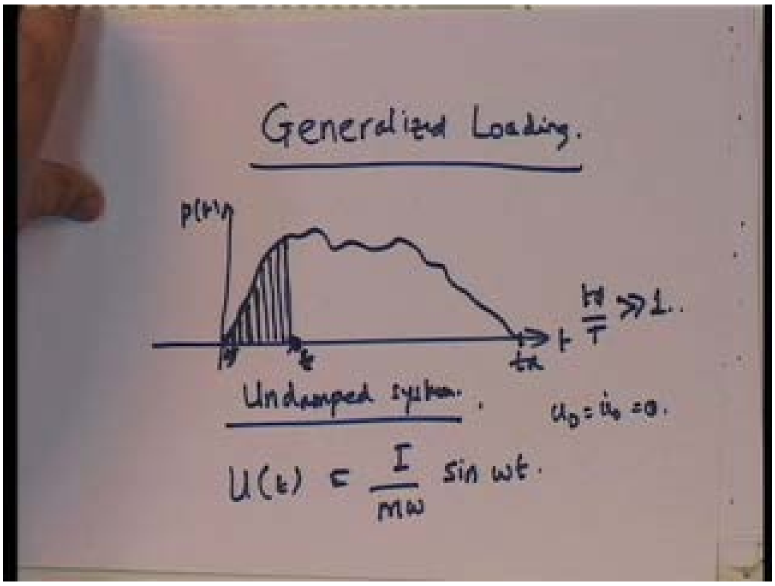

I shall spent lot more time on this in the next couple of lectures talking about it but since I have introduce the concept of impulse loading, I will just give an idea of how this procedure is created. Generalize load is one which and let us say, it stops, it has a duration but t d by T is sufficiently larger than 1. So then what happens, how do I get the response so this is my p of t. So, generalize loading essentially means that, I have a loading that last for a particular duration, but the duration is sufficiently large.

So that, why am I doing saying that, is just that I am saying that, look I do not have to look in phase 2 because my peak will occur in phase 1 itself and let me worry about the phase 1 itself because that will give me the response completely. So, how do I solve this problem, the way I solve this problem is the following and I am going to first solve, write down this solution. For an undamped system, what was the impulse load if you remember, u of t is equal to I upon m omega into sin omega t, that is the response.

So now, let me look at a p of t as a series of pulse strings, let me look at them as series of pulse strings and let me assume that, u dot and u dot 0 are 0. So then what happens then let me look at this, let me look at the response at time t. So, if I look at response time t then what happens, I can say that look, this what happens here, the response over here is starting at 0 due to all these loads upto this point load and that is the response. So, I can look at this load as a series of impulse strains where, each impulse is over something like d tau, I am using a dummy variable because t now specifies a specific values.

So, specific value, it is no longer a variable, t is a specific value so I am using tau as a variable from 0 to t. So, if I take it as d tau then I can look at it this way so if I take each one, what is the response due to each one individually. In other words, note that, at this point if I am looking at this, I am only looking at as if this existed then all these dint exist then if you look at it, all I have is at instant. In other words if I look at this particular thing then note that then system is linear I can assume that, these strings over here are, each of them has a separate load. And I could find out the response to each load and then I could superpose the loading.

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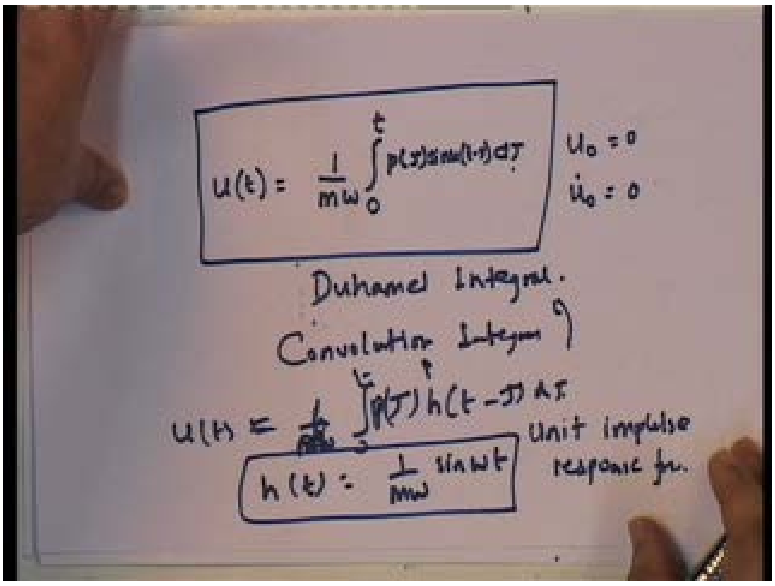

So, if I did that, let us look at each individual load, what would be u of t equal to, u of t would be equal to 1 upon m omega into p of tau because I am finding at a particular time tau, d tau this is the impulse, p tau del tau is the response at that particular instant or time. And from there, if I start my thing, what do am I doing, I am doing sin omega t minus tau because I am starting at time tau, I am looking only at this response, forgetting everything else.

So, what would it be, if I look at the response due to just this one, it would be if I look at this, if I look at time tau and I am looking at due to this alone. So, this one would be p tau because it is tau and then I have tau plus d tau so this one for all practical purposes I could take the central value and call it as p tau and call it as even. So, if it is even, it becomes what is the impulse, the impulse becomes p tau d tau and the response is going to be sin omega t minus tau because the impulse will only given at time tau.

So therefore, since time tau was when it was given, so therefore this is the shifted time, t minus tau is a shifted time. So, this is the response and therefore, if I consider all the trends, all I need to do is, integrate from 0 to t. So, in other words, see note that, for a general loading, my response is equal to this of course, this is assuming u 0 is equal to 0 and u dot 0 is equal to 0. I mean, this is valid only if you have addressed conditions. So, this is actually, if you look at it this is nothing but this is called the Duhamel integral.

And this is nothing but a larger convolution integral where, u of t is equal to 1 upon m omega, no not 1 upon m omega, the convolution integral is p tau h t minus tau d tau where, h t is equal to 1 upon omega into sin omega t. This h of t is what, let look at this, remember I upon m omega into sin omega t, so this is known as the unit impulse response function. And look at this, this is purely a function of m omega, the m and the dynamic characteristic, it is just a function.

So, it is basically a function which depends only on the structure dynamic characterstics because m omega, and since this is undamped, zi 0 anyway. So, the characteristics are only m and omega, this is the unit impulse response function and the huge of t is given as p t p tau where, p t is the load, h t minus tau d tau, 0 to t and this is the classical convolution integral, it is also call the Duhamel integral, and if we look at this, if I now extend this to damp system.

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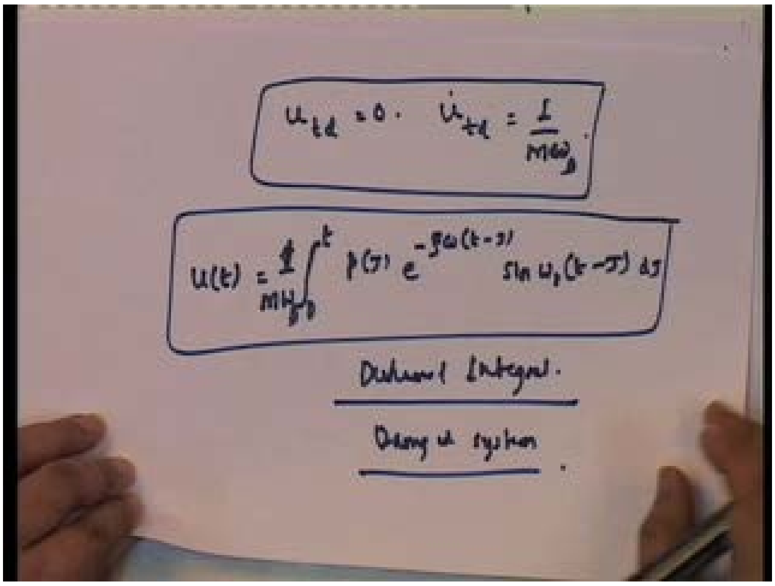

Let us look at this I mean, we did not see that, you had a situation where, u of t d is equal to 0, that impulse you know that, t d is at the end of the impulse and u dot t d is equal to I upon m omega, now this is the simple harmonic motion. So, if you have a damp system, for a damp system what does it land up becoming, for damped system it lands up becoming simply how do I get this. I get the response to be equal to the following, this is m omega d because that omega d is what you get.

So, I get u of t is equal to 1 upon m omega d, 0 to tau, p tau e to the power of minus zi omega t minus tau sin omega d t minus tau into d tau this is nothing but the simple harmonic motion. And so this is p tau d tau and so that, that is what you have, 1 upon this, that is the Duhamel integral for a damped system. So, that in a sense is brief over view of, how from the impulse response you can get to this thing. Now, just for completeness sake, I will continue with this but for completeness sake, I won not to actually solves a problem.

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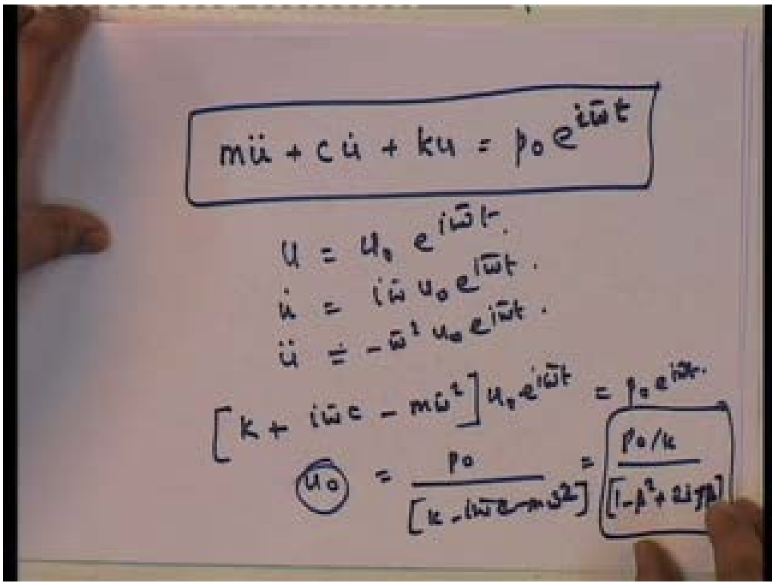

I want to go back to the harmonic problem and I want to solve a problem, which is known as, the reason why I am introducing this over here is the following. This is also harmonic, this is a simple harmonic motion, only thing is that, it is a complex, so how do I solve this problem, when I am looking only at steady state. Look at steady state, the steady state will also have the form u 0 into e to the power of I omega bar t, the steady state response.

So, when I substitute this u dot is equal to I omega u e and u double dot is equal to minus omega bar square u into... So, if I look at it, what do I get, I get the following, k minus I omega bar c minus m omega bar squared into u 0 e to the power of omega I t is equal to p 0 e to the power of I omega bar t. So, what do I get, I get equal to u 0 is equal to p 0 upon k minus I omega bar c minus m omega bar square. And if you look at this, this is equal to p naught upon k into 1 minus beta squared ((Refer Time: 51:23)) plus 2 I zi beta, that is my amplitude.

Very interesting and I should continue with this concept a little bit further because just like I showed, that the generalize loading can be a solution, we can use the solution as the Duhamel integral. I would now want to bring in another different concept, which is known as the fast Fourier transform. So, this is just introduction to that and therefore, I shall continue with this in the next lecture. So now, I just wanted to review, what have we done till now, we have looked at the free vibration response of a single degree of freedom system.

First we of course, develop the equations of motions, we have done the free vibration equations, free vibration response of a single degree of freedom of system. We have done harmonic response of a single degree of freedom system, we done response to periodic load, we done response to pulse loads, impulse loads and general loads. At least one approach, the Duhamel integral approach I looked at in brief, in other words we have pretty much looked across the board at all kinds of dynamic loads, that a single degree of freedom system can possibly phase.

In a more mathematical manner but at least we have looked at the variety of things, when we come to multi degree of freedom system of course, I still have a lot to look at. For my single degree of freedom point of view, when we come to multi degree of freedom problem, we will not solve any equation, no mathematics. We will only see, how we can use the characteristics, that we have studied for each response for a single degree of freedom system and look at it.

Thank you very much, bye.