Structural Dynamics

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Lecture - 10
Pulse Loading

Hello again, so we have finished looking at in the last lecture on how to deal with periodic loading, so till now we have been looking at loads, which are essentially therefore very long duration. In other words we know we only consider a situation, where the loading is there. For example, harmonic loading only think that we says that harmonic load is that time period, but then the number of cycles we know goes on forever. Similar, if a periodic load also we say that we do not know when it started, we do not know when it ends, we just taking particular period during the time taking a window.

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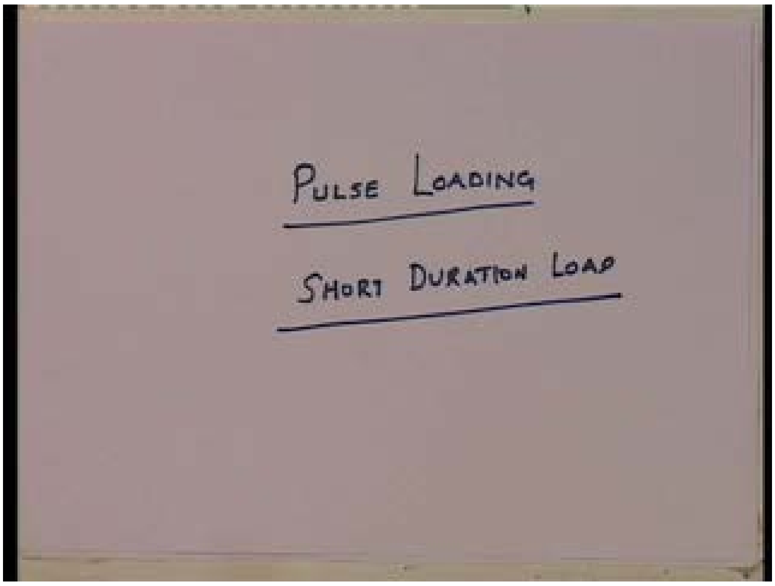

Today, we are going start looking at something that is known as pulse loading, pulse loading is essentially it is a short duration load. In other words the loading starts at a specific time and ends at the specific time, and what are pulse loads, these are typically what do we mean by short duration loads.

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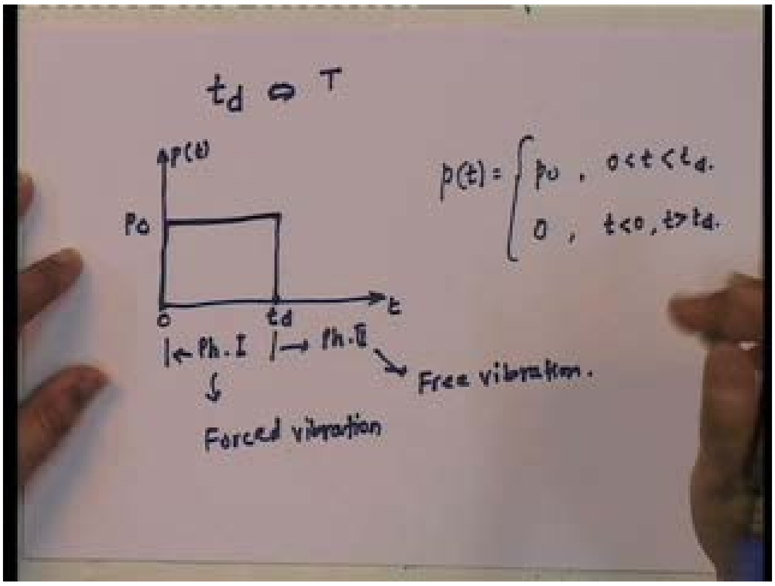

Short duration essentially implies that if I call t d, as the duration of the pulse, this is of the same order of magnitude as the time period of the structure, when I say same order of magnitude I essentially mean that they are similar to each other. It is not as if t d is equal to it just means that t d could be t by 2, could be t by 4, could be 2 t, but not more than 2 t for sure.

So, if you say that then what happens let us look at, so let me take and here what I am going to look at is we are going to look at mathematically defined loads. So, that we can get close form solutions and let us start off with the simplest kind of load, and that is a rectangular pulse. So, this is starts at 0, this end the t d the loading is p 0, so essentially it is starts at 0, and it is p naught till t d and then t d it goes to 0. Obviously, the rise and the fall or instantaneous, this is the mathematical abstraction reality all we say is that the risen fall are very short compared to the time duration.

So, in other words p of t is equal to p 0 between 0 and t d, and 0 for t less than 0 and t greater than t d, so now, the question becomes well what how do we solve this problem. Well, immediately there is no difference in the I mean you know we know, how to solve any you know partially differential equation. We know all of that, but there is there are 2 interesting aspects to this problem, one that I have to solve this is the 2 phases problems, phase 1 and phase 2.

Phase 1 is known as during that period, when the duration when the loading is non 0 and phase 2 is after the duration of the load, we have what is known as a free vibration phase, so this is forced vibration, and this is free vibration. So, now, let us look at this particular problem here what we are going do is we are going to solve for this rectangular pulse, we are going solve both the phase 1 and phase 2 problems. And note that again our sole interest here is in getting the peak value of the response, that is all we are interested in because we are not interested in the time duration.

Now, interestingly enough one particular point that requires and this we say when we are look at damping, remember when I was looking at free vibration I say that damping takes some time to start off. And the viscous damping after 1 or 2 cycles of vibration, that is only the time when viscous damping takes of now since, t d is at the same order as t we are not going to get two cycles of vibration at all. So, for all practical purposes we can solve this problem as an un-damp problem both for the force vibration and free vibration, because again understand we are not interesting in the time history at all, we interested only in the peak value why because the peak value is what we need to design a structure for it.

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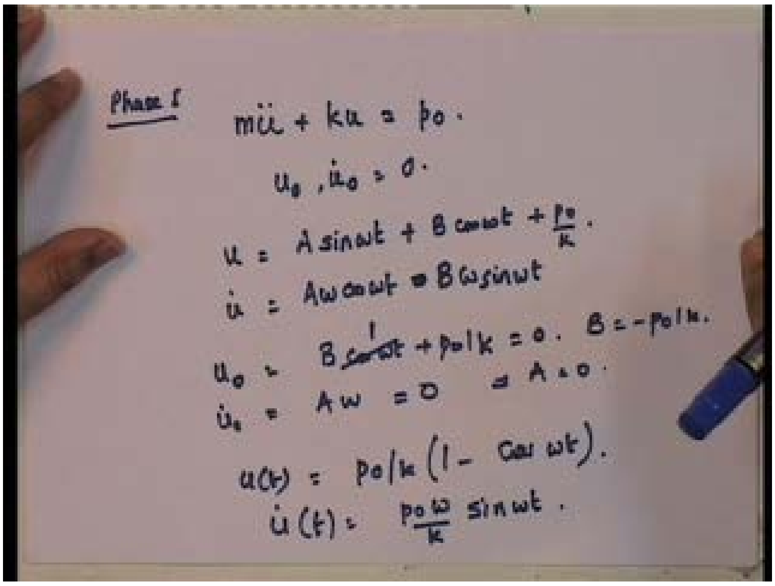

So, let us now look at it, so the problem is no longer the solution of m u double dot plus c u, but we can drop the c u term, why not because c u is 0, but because the viscous damping takes time to happen and t d is not is not sufficiently long enough for the loading to happened. So, this is the problem that we solve and the problem here is one of now, note that since it is a short duration load u 0, that is and u dot 0 which are what, initial displacement, initial velocity.

These are important and we will solve the problem as A, so I am looking at phase 1 solution right now, so how do I solve it particular solution, particular solution is p naught upon k and the homogeneous solution is well u is equal to a sin omega t plus b cosine omega t plus p naught upon k and A and B I get from here. So, if you look at it if I differentiate it u dot is equal to A omega cosine omega t sorry, minus B omega sin omega t in the plus note that these is a constant, so differentiation is 0. So, if we put u 0, since sin omega t, t equal to 0.

What we get is B cosine omega t plus p naught upon k is equal to 0? And this cosine omega t is 1, so you may get that b is equal to minus p naught upon k and again if we took u dot 0 is equal to A omega which equal to 0 this implies that a equal to 0. These implies that u of t is equal to p naught upon k into 1 minus cosine omega t, so this is the solution of the problem and again this is the reason I am writing now u dot of t and these is equal to p naught omega upon k into sin omega t, so this is the solution.

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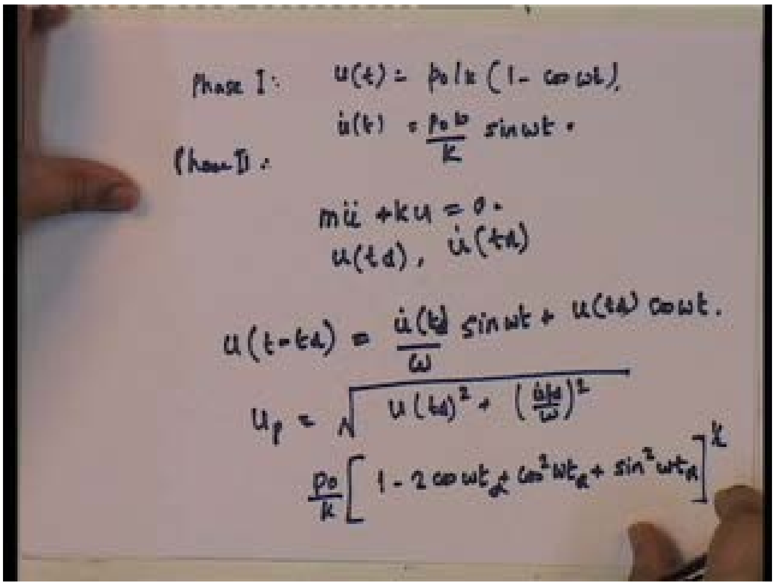

Now, that this solution means what, let us look at it and that is that we have a situation, where phase 1 is u of t is equal to p naught upon k into 1 minus cosine, let us look at phase 2. Phase 2 is what well little put down u dot t only here, is equal to p naught omega k sin omega t, so now, if we if we you put together what do we get we get it the following phase 2 basically becomes a problem on m u dot plus k u is equal to 0. Why?

Because, essentially you have situation where you know, again we looking at only peak value, so the peak value will happen in first one or 2 cycles, so damping does not come into picture, so this we know the solution to this solution is and where you have u of t d and u dot of t d as initial conditions. So, then I can write I know the solution to this, these if I just put it shift from t d and this solution basically becomes, u dot t d upon omega into sin omega t plus u t d cosine omega t, again the values itself is irrelevant, so what we want is we only want the peak value.

So, the u peak is equal to u t d squared plus omega the whole square, so that is the peak value, in phase to this the peak value, so I can plug in these are my response, I can plug those in and if take p naught upon k outside I get u dot t d is this plus whole squared. So, these basically becomes 1 minus 2 cosine omega t plus cosine squared omega t and u upon omega, so this is omega again p naught goes, so this becomes plus sin squared omega t d that is the peak value. So, if I look at this becomes 1, so it becomes 2 minus square root, so if I look at that particular solution this is 1, so it becomes 2 minus 2 cosine squared.

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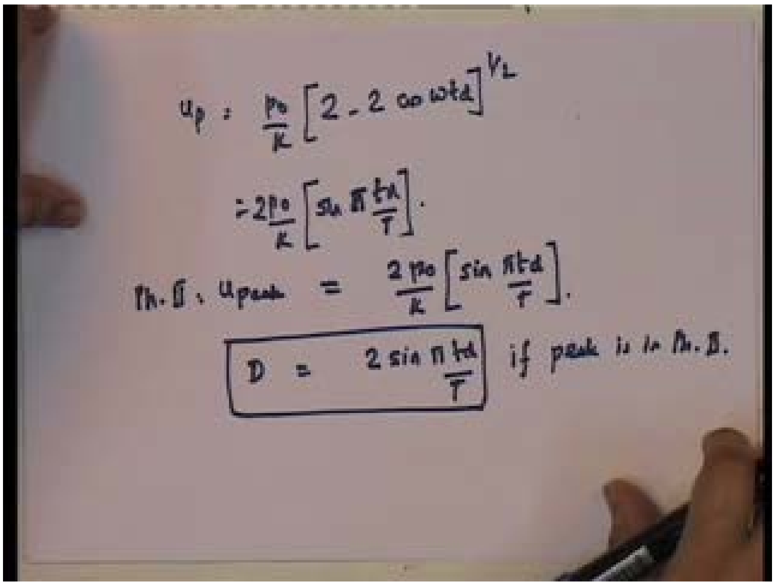

So, this becomes peak value becomes u p is equal to p naught upon k 2 minus 2 cosine omega d t squared, so this is equal to from trigonometric equality, this becomes 2 p naught upon k into sin. Now, it becomes omega upon 2 sin omega upon 2, so omega is 2 pi upon t, so this becomes sin pi t d upon t so; that means, if this is the peak value, let me just write this down. So, the peak value in phase 2 is equal to 2 p naught upon k into sin pi t d by t, and if I rewrite this as dynamic factor, then this is 2 sin pi t d upon t.

These basically comes from the fact that this into this essentially becomes 2 times 1 minus cosine is equal to sin squared pi you know omega t d upon 2, so that is way this becomes square root of sin squared becomes a you know the 2 two here. And so it becomes 2, so it becomes four square root of 4 is 2 and so this is what at comes out, so these is the dynamic amplification factor if peak is in phase 2.

Now, let us look at phase 1, suppose when will now let us look at the phase 1 displacement, understand that for the maximum to occur in phase 1 this has to equal to 0, do you understand that. So, I am going to write the following and that is that we have a situation where the peak value for the peak to line phase 1 p naught omega upon k into sin omega t is equal to 0, and omega is what omega is equal to, so now note for this to be equal to 0, since these cannot be 0 this has to be equal to 0.

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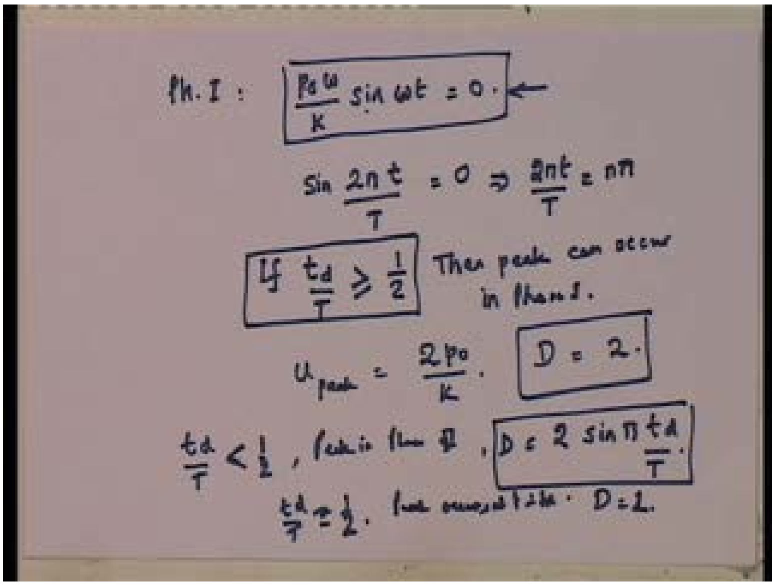

So, in other words let me rewrite this sin 2 pi t upon t is equal to 0, so what does that mean this implies that your 2 pi t upon t is equal to let us look at it what does 0 imply, 0 can be well it cannot be 0 because t equal to 0 is 0 is a trivial solution. So, it has to be what is the next one, it is n pi now; obviously, now the duration is small, so which is the first one, so in other words if t d by t is greater than look n pi the first one is pi upon 2 is half.

So, if t d by t is greater than half then peak can occur in phase 1, and what is that peak then well 2 pi upon t, so omega t is equal to pi. So, if I take omega t is equal to pi look at this, this is cosine omega t, so cosine of pi what is cosine of pi minus 1, so this is becomes 2. So, you are u peak is equal to 2 p naught upon k, so this implies that if t is greater than half, then peak can occur and this is the peak; however, if t d by t is less than half peak in phase 2 and what is that peak value the peak value is equal to.

So, in other words D is equal to 2 and peak is in phase 2 D is equal to 2 sin pi t d upon t, note very interestingly that if t d by t equal to half. Then where does the peak occur if t d by t equal to half then the peak occurs at t equal to t d and what is the value put t d equal to t here what you get sin pi, sin pi is what sorry, made a mistake here. Here, is if t d by t is equal to half, this is sin pi by 2 sin pi by 2 is 1, so d is equal to 2, so it exactly is D equal to 2. So; that means, what we have is if t d by t is greater than or equal to half D is equal 2 and if t d is less than half then D is equal to, so we have solved the problem of the peak.

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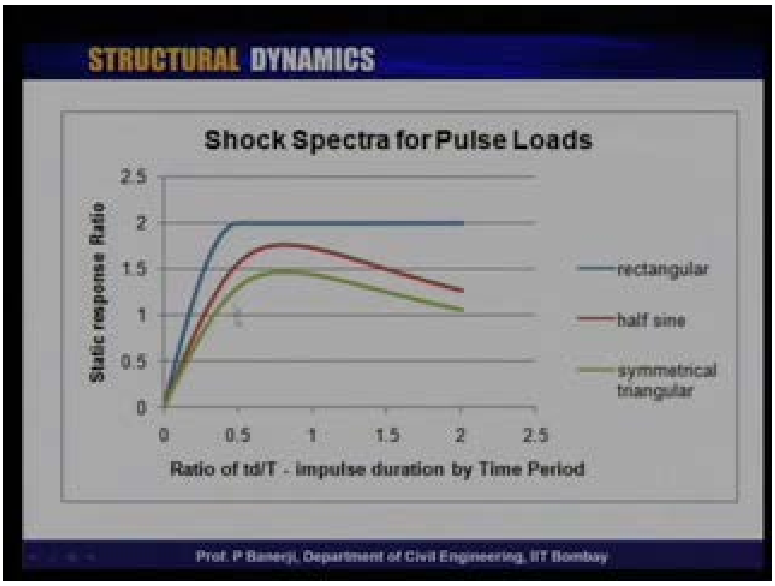

And if you look at this, here you look at this one, this one is a sinusoidal function which is this sinusoidal function, goes here at t d upon t equal to half it is 2 which is what we get here. And after that if it is longer duration, then t d is equal to 2 which is what we get here, so this one you look at it, this is what is known as the shock spectrum for a rectangular load.

So, shock spectrum essentially if I give you this, if you know t d the duration of the load of a rectangular load and you know t you just go here read it of and that gives you the dynamic amplification factor and multiply that by p naught upon k and you have your response. So, in other words the shock spectrum you know replaces I have done the mathematics here, over here I have done the mathematics, but if I had this I did not need the mathematics the shocks spectrum if someone give me the shock spectrum I could not then look at it and say this is my duration look at it read it that is the duration. For shock spectrum actually gives you us full idea about how to go about it, so now, I will just do another specific duration short duration load, before I move on to some other kind of loading interesting loading that you can do and that is let us look at the shock duration.

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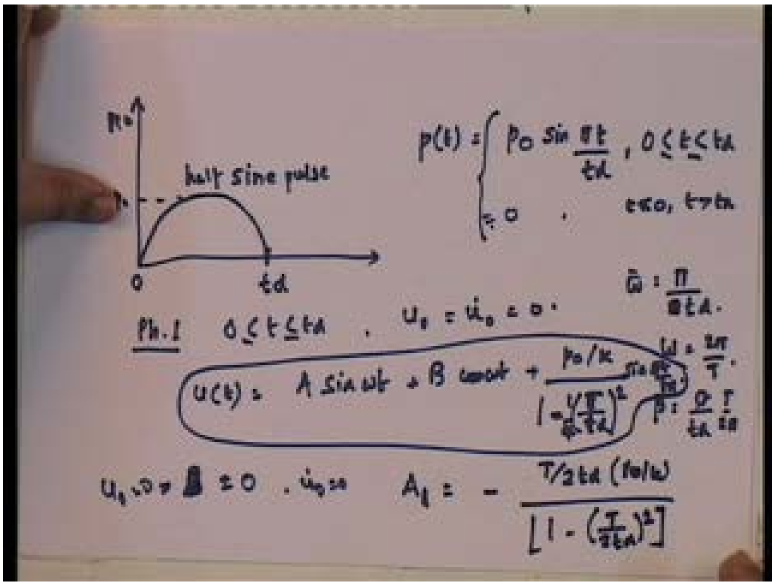

Another kind of shock duration load is the one that I have draw here which known as the half sin curve, so let us try to get these shock spectrum from all mathematics, what is the half sin, half sin is essentially this. Where this is duration this and this is given as this is p naught, so p t is equal to this is p of t p of t is equal to p naught sin pi t upon t d for 0 less than and is equal to 0 for t less than 0 and t greater than t d, this is my half sin pulse.

So, now, how do I solve for this well again, I go through the same step solve the phase 1 solution and phase 1 solution for phase 1 is basically 0 t d this. Actually, this particular case we can do this and the solution and of course, you know we have the fact that u 0 is equal to u dot 0 is equal to 0 at rest initial conditions. And therefore, u of t is equal to and again I am solving what the un-damp problem, so un-damp problem becomes A sin omega t plus B cosine omega t plus the solution of this, the solution of this is nothing but if I put this in and solve by this thing.

I already know what this is, this is equal to plus p naught upon k into 1 minus beta squared, beta squared is omega. Now, the omega is what pi upon t d is omega upon omega, which is 2 pi, so this becomes what it becomes t by t d squared, so I am going to put this as 1 over 4, because I have remember this pi upon 2 and the other one is 2 pi.

So, omega bar is pi upon 2 t d and omega is equal to 2 pi sorry, pi upon t d omega bar is pi upon t d and omega bar is 2 pi upon t, so beta is equal to pi upon t d upon omega bar upon omega, so this becomes then t upon 2 pi. Pi, pi cancels, it becomes 2, so this is what it have has and then into sin pi by t t d, this is my solution of the first equation and I can solve for A 1 and A 2. If I solve for A 1 and A 2, I get that B sorry, B is equal to 0 and because note that if I put t equal to 0, this one becomes b this goes to 0 this goes to 0, so therefore, B is equal to 0, when, u 0 equal to 0 implies that B is equal to 0 and from u dot equal to 0, I get that A 1 is equal to minus T by 2 d into p naught by k upon 1 minus I am going take this inside t upon 2 d the whole square, so this is A 1. And so therefore, if I look at the total solution this becomes the following, I am just now going to substitute A 1 here and take out the p naught upon k outside.

So, this is going to be equal to u of t is equal to p naught upon k and then note that I have 1 upon 1 minus t upon 2 d this is 1 minus beta square, you already solve this I am not going to solve it again, so that one also comes out. So, inside I have what I have sin pi t upon t d minus T upon 2 d into sin 2 pi by t, and again this 2 pi by t can be actually written as T d upon t sin pi the radial matter, let not a confuse the issue here, so this is the solution in phase 1, for a half sign pulse.

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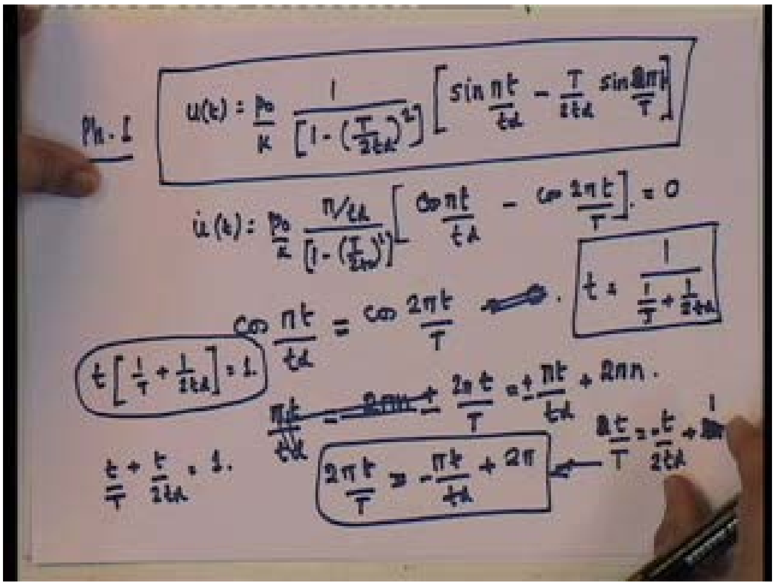

And once we have that I can actually get u dot also u dot turns out to be equal to it is easy to do p naught upon k and I will get pi upon t d and so that is 1, so that is cosine pi t upon t d minus cosine of 2 pi t upon T, so this is what I get as my solution for the equation. So, now, let me first start of by, so we why do I do the u dot t in the second phase, well that is because I mean sorry, in the first phase, because if the if maximum is in phase 1. Then this has to be equal to 0 and this is equal to 0, I am sorry this is this 1 minus t upon 2 d, this is still there and so over here what you have is.

So, I get cosine pi t upon t d minus cosine 2 pi t upon t has to be equal to 0, that is the; obviously, thing that you will land up getting and in other words, so note that for this to happen, this has to be equal to this, because this minus this as to be equal to this. So, if this has to be this all; that means, is that pi t upon t d is equal to 2 pi n plus or minus sorry, this is the one, so just this wrong.

This is not correct what you get is this one comes on this side, so it is basically what your getting is t 2 pi t is equal to pi t upon t d plus or minus plus 2 pi n, and note that this one will this is valid for the first n, so it is 2 pi. So, if I do that I get 2 pi t upon t is equal to pi t upon t d plus 2 pi, so then I get this side to be coming in and I can solve for t, over here. So, any way I mean this is either here nor there this one, I need to solve this for time t depending on my t d and t, so this is the time at which I have the peak and then that one I plug back into here and I will be able to get peak value of the displacement.

So, I am not going to substitute all of those, but so this becomes little bit more complex if you look at this, this one does not follow as clean a line as we got for the other one, it is a much more complex a solution. Now, the question then becomes that can I substitute in here, so I can just put over here and I get I can just take out the pi and I get t upon t is equal to sorry, 2 t I will just make it t upon t is equal to t by 2 t d plus sorry, is 2 pi is gone because I have, so plus 1 minus.

So, if I take this in I get t upon t plus t upon 2 d t is equal to 1 and therefore, I can just say that t into 1 plus T plus 1 by 2 t is equal to 1, so that is I am sorry that is what we get here. And I can solve for t by saying that look t is equal to 1 upon 1 plus t upon 1 plus 2 d, so now, depending on the omega and time duration and the time period we can compute the time at which this happens and then you plug this in here and you will get it. So, that is that is when it happens in phase 1 now the question comes that what happens in phase 2, well let see what happens in phase 2.

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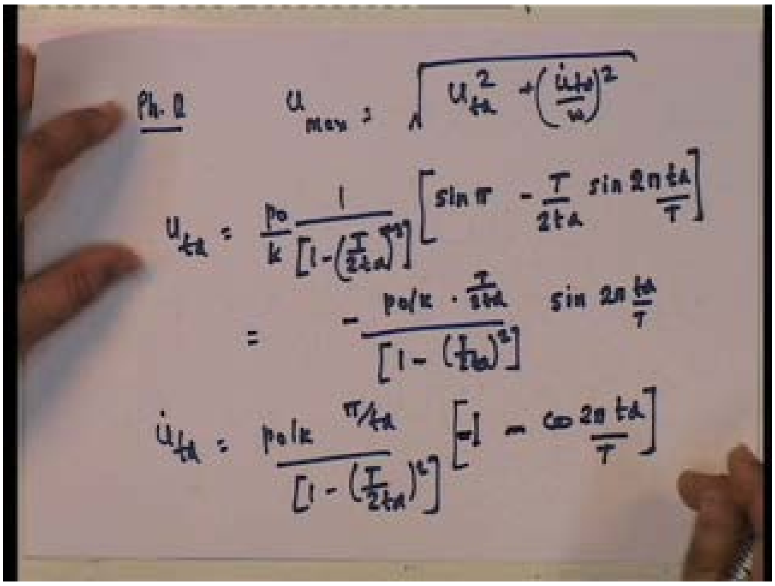

In phase 2, what are we know that it is going to be u max is equal to u t d squared plus u dot t d upon omega the whole square, so we have to do this. So, we have to find out what u of t d is, so u of t d substituting into that becomes p naught upon k 1 minus t upon 2 t d the whole squared into now sin of pi because t is equal to t d. So, t upon t d is 1, sin of pi minus t upon 2 t d into sin, now you see this is equal to 2 pi t d upon t, so sin of pi is what is sin of pi 0. So, what we get is u of t d is equal to minus p naught by k into t upon

2 t d all over 1 minus y of 2 t d squared into sin 2 pi t d by t this is u of t d and similarly substituting in here.

What we get it as is if I put t d t is equal to t d because cos pi and this becomes cosine t d upon t, so this essentially a substitute that u dot of t d is equal to p naught upon k pi by t d upon 1 minus t upon 2 t d the whole squared into cosine of pi what is cosine of pi minus 1. And so therefore, cosine of pi is minus 1 sorry, the cosine of 0 is 0 cosine of pi is 1 sorry, it is 1, so therefore, what we get is cosine of pi is 1 minus cosine of 2 pi t d upon t. So, that is what we get sorry making a mistake here, cosine of pi is minus 1, I am very sorry extremely sorry cosine of pi is minus 1, so it is minus 1 here and minus here.

So, u d and u t d is these, so now, if I write this is getting fairly complex, but just stay with me what you get is if I do u dot squared, and u dot is divided by omega. So, let me see what I get as u t d by omega implies, actually u dot t d into t upon 2 pi, so essentially if I put this in your upon omega this becomes essentially t upon 2 pi, so this becomes again t upon 2 t d.

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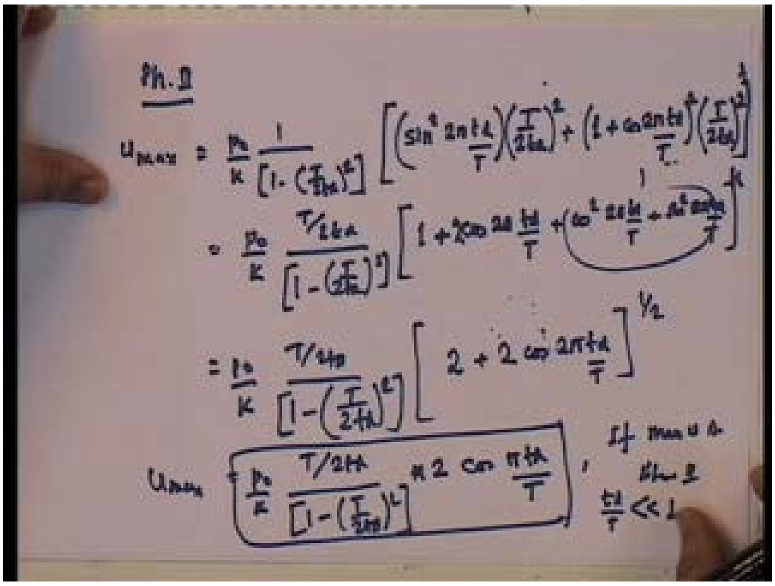

So, therefore, essentially what you get is u max in phase 2 is equal to you got I am going to take the entire p naught upon k into 1 upon 1 minus t upon 2 d the whole squared, I am going to keep this entirely outside and inside what I have is the following. I have sin squared 2 pi t d upon t and over here multiplying this into t upon 2 d the whole and on this side I have minus minus I get plus 1 plus cosine 2 pi t d upon t the whole squared.

And inside I have I have t upon 2 t d the whole square, again this is there in both of them second take that out, so this becomes again p naught upon k I will put t upon 2 t d outside inside 1 minus t minus 2 t d squared and inside I have 1 plus cosine sorry, 2 cosine 2 t d by t. And then plus cosine squared 2 pi t d by t plus sin squared 2 pi t d by t these 2 become 1, so this becomes and then of course, square root I forget this square root here is equal to p naught upon k into t upon 2 t 1 minus t upon 2 t d squared then into 2 plus 2 cosine 2 pi x t d square root.

This become if you look at this becomes 2 cosine of it becomes 2 into 2 cosine squared pi t d upon t x and so if I do this square root of that this becomes equal to p naught upon k into t upon 2 t 1 minus and the 2 comes out. So, I have essentially I will just re-write it t upon 2 t d whole squared multiplied by 2 the 2 into 2 4 comes out square root is 2, and the square root of cosine squared is cosine of pi d upon t, this is your u max, if max is in phase 2.

Now, you know in the other case we got a beautiful situation, where we could find out a value of t upon t d or t d upon t remember at in rectangular, if t d upon t is less than half. It will in phase 2 and we know the value and if t d upon t is greater than half, then d is equal to 2 here if you note that you cannot really do that purely because this process. This is the t at which u t is maximum getting, this is the fairly complex a process, so what we do is how do we solve this problem, how do how do I plot this red curve which is for the half sin wave.

Well, the easiest thing is I know that at any time t d upon t I solve this, this I can solve given t d upon t I can solve for this and get of course, u max upon p naught upon k that gives me the dynamic amplification. This is completely dependent on t d by t for a particular value I know what this value is, so this is the maximum if it occurs in phase 2 then for same t d by t I can solve for this one. So, if solve for this one this one essentially, becomes something like I could rewrite this in this form that t is equal to 1 upon t into 1 plus.

So, note that this time duration does not only depend upon t d by t, it also depends on t itself, so therefore, this will be this maximal occur at different times for different t d’s t d by t. So, therefore, all you need to is find out this for given t d by t find out this for a given t d by t and of course, time you know you know that you get this t and for that t substitute into this and get the dynamic amplification which is u t upon p naught u max by substituting that, t in here.

And get that value and compare this value with that value, whichever is the what will it be whichever is the larger value is the one, that is operational. Typically, when you are very, very low very, very small this operates because when t d by t is very much less than 1, then you know it does not get a chance to do this. And therefore, since it does not get a chance to do this one it will always be this, so if you look at this particular part for a small part it is this, but once t d by t becomes slightly larger than what happens? When you have to compare this and this because of the solution that this solution does not only depends t d by t it also depends on t itself. So, that is the region why you find out both whichever is maximum is the value that you get, so much for this and similarly what we can do is if we come here.

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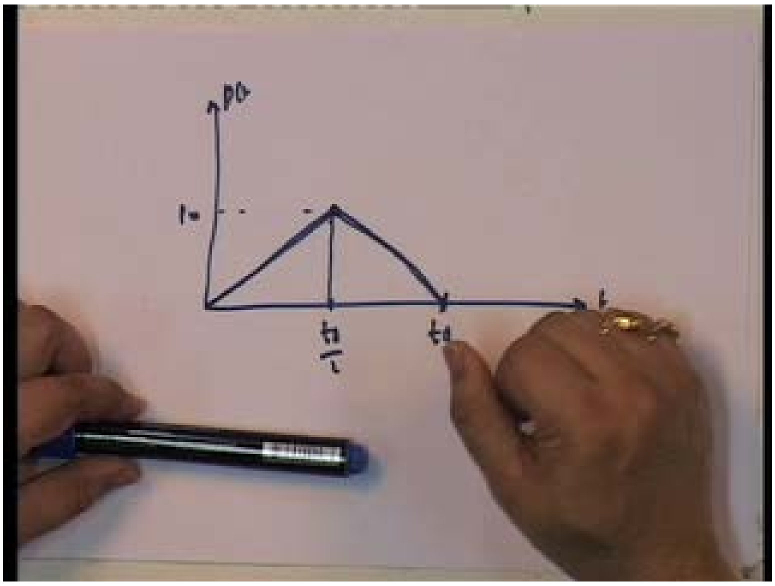

We draw another curve for a symmetrical triangular is typically of this form, I will just show it to you a symmetrical triangular is I am not solving this, I leave it up to you to solve with if you wish. To, p of t, t symmetrical triangular is t d t d by 2, p naught why do not you people you take this is an exercise, take this is an exercise and find out this show that this is the shock spectrum for a symmetrical triangular. Of course, again for this also you know the t d by t a value for which it is in phase 1 and phase 2 is not obvious of course, in the smaller part it will be in phase 1 that know for sure, because when t d by t is very small the duration of the load is not large, enough for it to be occurring in phase 1, so this area is always in phase 2. However, just like in this case we had half in this we did know when it was coming in this case also it is going to be slightly problematic, but you can plot it you can put in values of t d by t and t and you can get it.

So, that is what is known as pulse load, I have shown you that in pulse load short duration loads you did not only have one solution like we use to have for harmonic and periodic you have phase 1 solution phase 2 solution and how to compute the peak I also showed you. For the rectangular we knew exactly when it will occur in phase 1 minus occur phase 2 for the half sign I showed you well, for phase 2 you can get a close form solution for phase you should not a close form solution for the peak value, and so you do not know where it is.

However, the point that I am trying to make is that you do not have to solve anything, any mathematical solution, if your given the shock spectrum and you have given the particular kind of load. If you know what kind of load you have you can always read of here p d upon of 1.75 read 1.5 it is symmetrical triangular. And this is the value it is mean that and multiplied by p naught upon k and you have your may peak responses that is all you need, so shock spectrum actually is very, very useful for pulse loads were you do have solve any problem.

Thank you very much, see you next time, bye.