Soil Dynamics
Prof. Deepankar Choudhury
Department of Civil Engineering
Indian Institute of Technology, Bombay

Module - 7
Dynamic Soil - Structure Interaction
Lecture - 37
Behaviour of Subgrade Soil below Railway Track

Let us continue our lecture on soil dynamics. We were discussing on module 7 that is, dynamic soil structure interaction.

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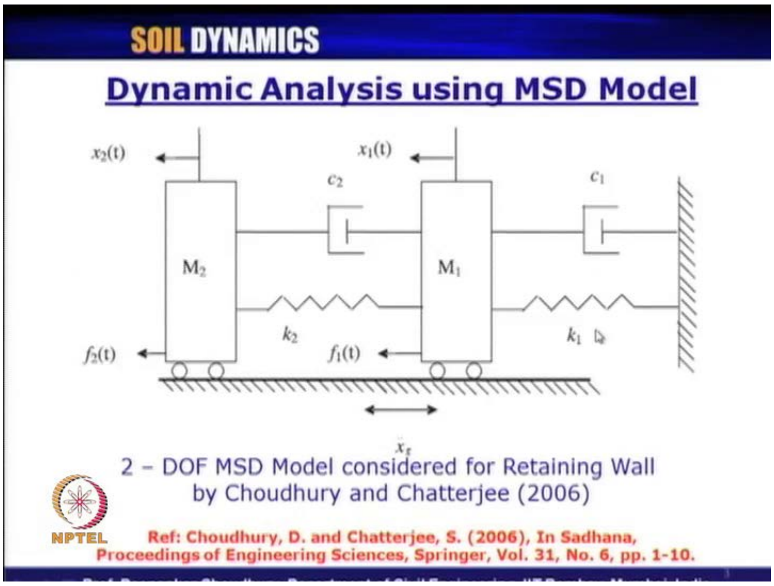

A quick recap of what we had studied in the previous lecture that for rigid retaining wall. How to model it using the mass spring dashpot model as given in this paper, this journal paper. So, retaining wall and soil has been modelled like this equivalent mass of rigid wall and soil with corresponding damper and stiffness for wall material and again damper and stiffness for soil material with the different applied dynamic loads.

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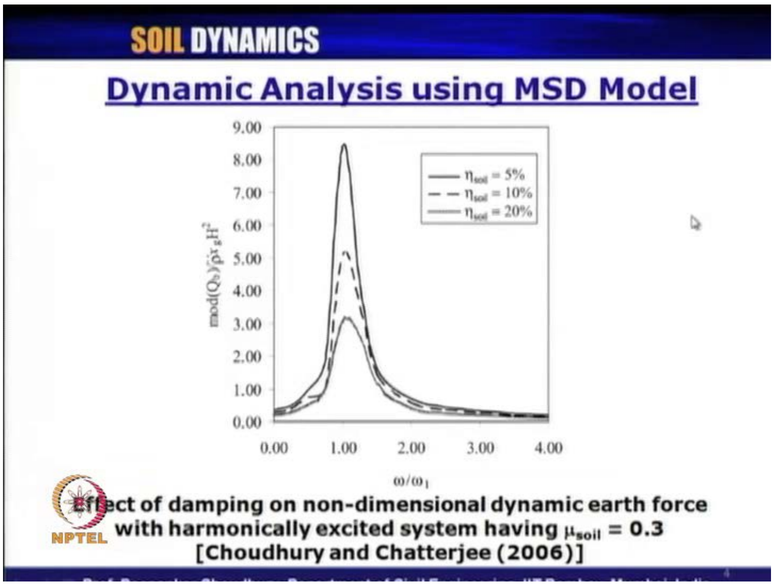

Finally, the results have been obtained in terms of non dimensional parameter that is, Q b which is nothing but the dynamic active earth pressure acting on the rigid wall mod of that, divided by density of the soil x double dot, g is the acceleration at ground surface due to the earth quake. And h square and this x axis we had considered as the frequency ratio that is applied dynamic load frequency and the natural frequency of the system for different damping ratio of the soil.

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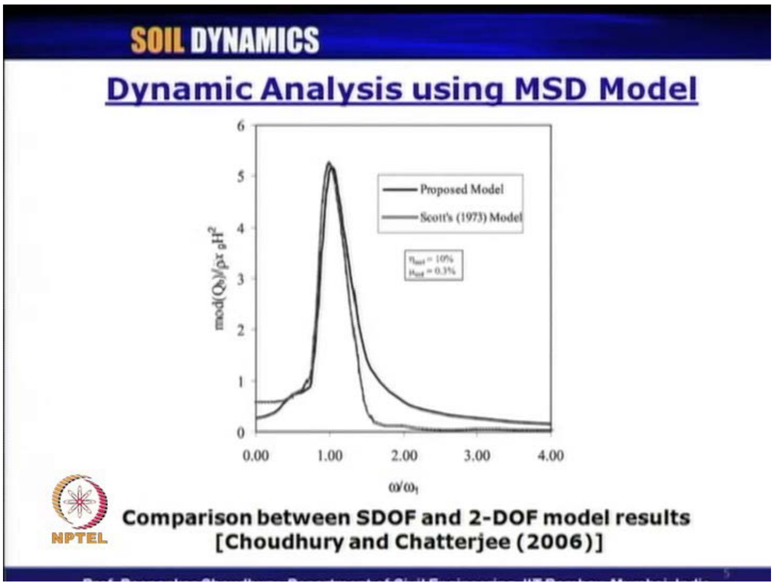

Then, that comparison with 2 degrees of freedom model and the previous single degree of freedom model has also been shown and what are the differences at high frequency ratio that also have been pointed out in this result.

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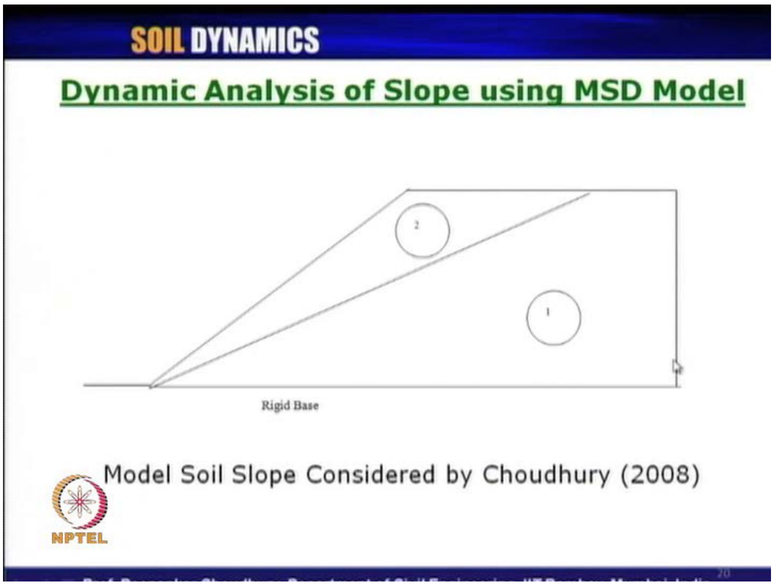

Now, coming to the next application of this mass spring dashpot model for the case of analysis of stability of slope that is, how to know how much displacement of the slope is going to occur when we are doing the stability analysis of any slope. Like in static case what we do we can estimate the factor of safety for the stability of a slope very easily and also if required we can find out the displacement when it goes beyond the factor of safety less than 1 then, how much the slope is going to move or slide. So, in this case this was the model considered in 2008, this is the slope, a zone has been again considered assuming this as rigid base. Only 1 type of failure we have considered that is, 2 failure and failure surface as you know it can be assumed as circular, it can be assumed as a log spiral, it can be assumed as any carved, it can be assumed as linear, it can be assumed as multi linear several different. So, for the simplicity at the beginning we started with the assumption of planar failure surface. So, this zone marked with number 2 is nothing but the failure zone of the soil that is, the slope which is failing at the condition of limit equilibrium we are applying and this zone 1 is the static zone which is not failing not moving. Now, truly speaking it is not moving not like that, it is not failing, I should say the word this is failing, this is not failing zone. However, at far distance there is no effect of this earth quake or dynamic load. So, this is again the zone of influence which we need to find out by method of trial and error.

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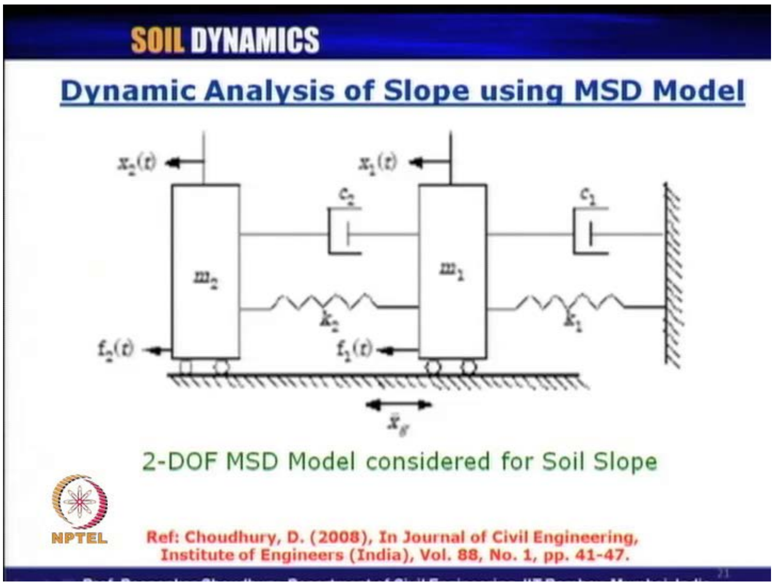

So, in this case the same way we had modelled it in 2 degrees of freedom model which is quite obvious, because 2 of them will behave completely differently though they are of same material. Mass of m 2 will be completely different zone which is participating in the sliding or failure of the soil and mass which is not failing m 1. And corresponding k 1 and c 1 can also be calculated, x 1 x 2 at the corresponding displacement of this different zone subjected to different dynamic loading of f 1 t and f 2 t. So, this is the reference of the paper which is given here in journal of civil engineering institute of engineers India this paper.

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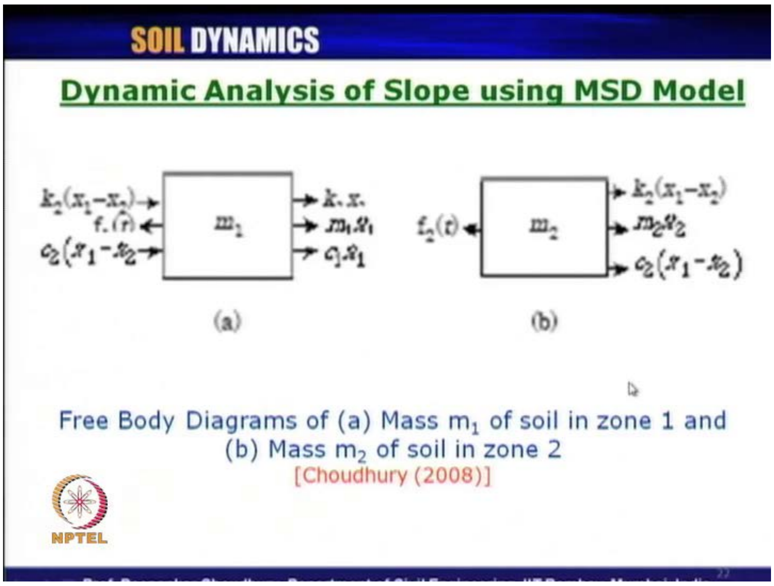

Again the forces acting on each of this element that is mass m 1 and m 2 are shown that is stiffness force, damper force and inertia force along with the externally applied dynamic load how they are acting this free body diagram can easily be drawn.

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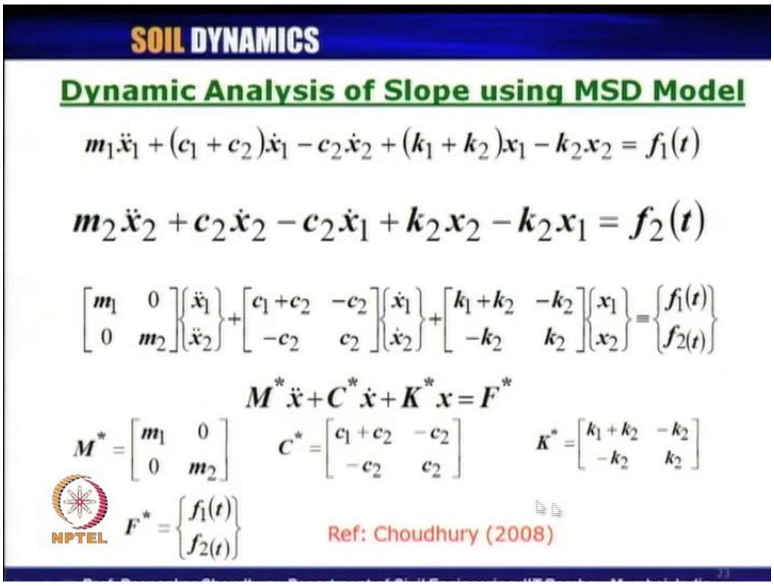

From this free body diagram we can use de Alember's principle and write down these equations, as we have already done earlier for our previous modules. And finally, express it in terms of this matrix form, the governing equation of motion which can be in simplified way can be mentioned in this way, say M star is nothing but mass matrix, C star is damping matrix and K star is stiffness matrix, F star is applied dynamic load matrix and x double dot is acceleration vector, x dot is velocity vector and x is displacement vector. So, these are different matrices which are expressed in this slide.

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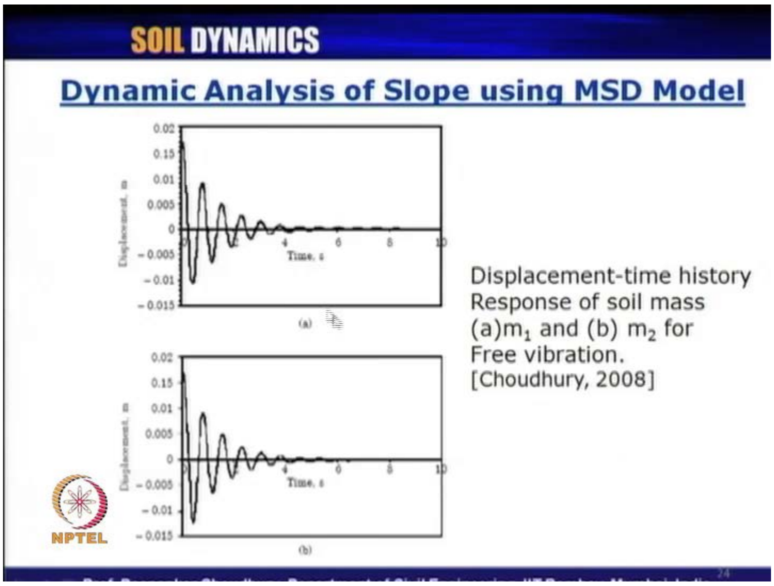

Now, we had carried out different with different modes of dynamic load. For the first case we have used free vibration how they behave as it is expected after giving an initial displacement. How much initial displacement we had given? By considering the static loading condition we can easily get the static displacement and that static displacement we had given as initial displacement and further we had calculated how much the displacement is going to follow for different input parameters.

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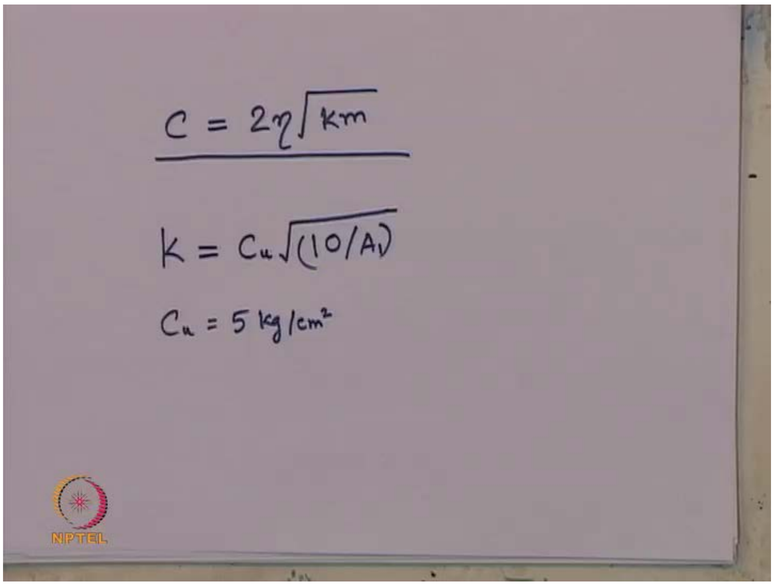

And for input parameters in this case, I would like to mention the stiffness was calculated using this expression which is also another standard way C u root over 10 by A 1 using this expression. I had already mentioned earlier in the design of machine foundation that k stiffness can be calculated using this C u. What is C u? Coefficient of elastic uniform compression or dynamic subgrade modulus. Do you remember? So, that time we have mentioned how to calculate this value in the field using cyclic plate load test. So, this equation was proposed by bar ken. So, bar ken's equation we had used where, A 1 is nothing but the area of the region which is under consideration, we are considering the projected area of the region and in this relation an equivalent area of 10 meter square was initially considered and a C u value we had considered for our analysis as 5 k g per centimetres square per meter length. So, per unit length in the other direction and C that is the damping constant was calculated using this expression for different damping ratio. The damping ratio like for the wall case also for slope we have considered 5 percent, 10 percent and 20 percent.

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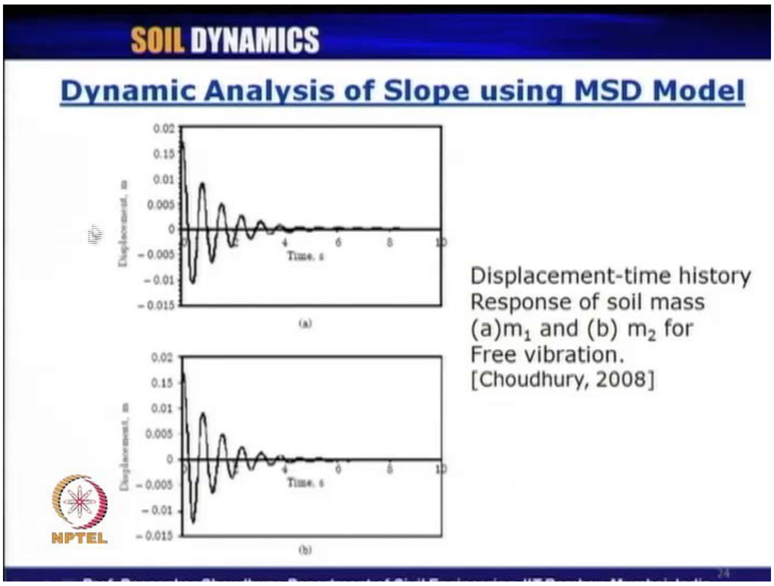

Again for coming to this slide, this was the response for mass m 1 and this was the response for mass m 2 that is both the failed zone and non failed zone we get the response.

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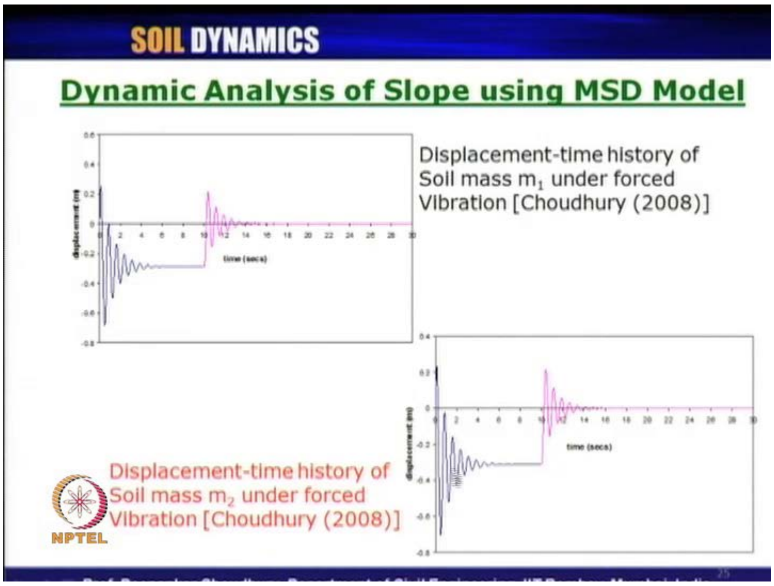

And the displacement time history for different masses under forced vibration. In forced vibration we assumed a particular sinusoidal load as if it is acting on the wall. In that case, about a 12 kilo newton amplitude of load was considered with a sinusoidal variation with the input parameter of lambda was used as 20 pi and it was considered to act for 10 seconds. So, till 10 seconds it was acting that is why the response you can see in this blue colour it is coming for the forced vibration up to 10 seconds and beyond 10 seconds the load was removed. So, after 10 second it is behaving as a free vibration. So, obviously it vibrates about this original rest line. Similarly, for the case of mass m 2 this is the vibration or response of displacement with respect to time we can obtain.

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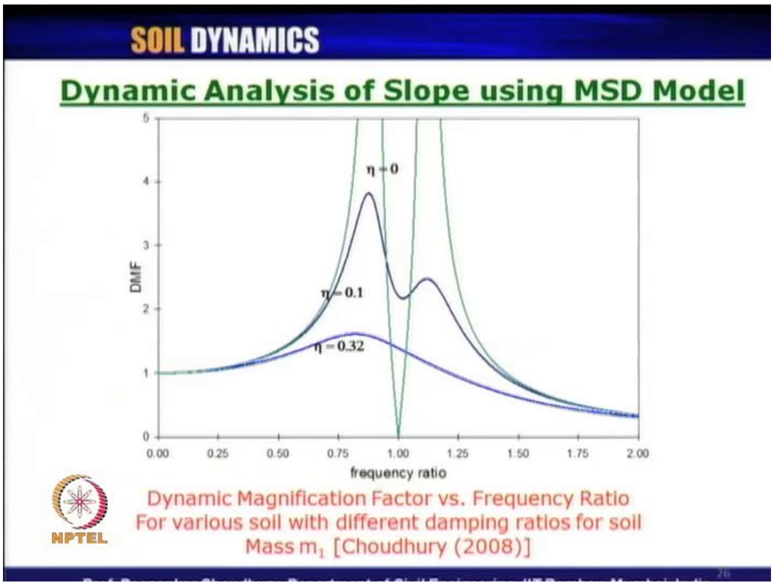

Finally, what we try to obtain the dynamic magnification factor, how it varies the dynamic displacement with respect to various values of frequency ratio for various soil with different damping ratio that is, when no damping is considered that is un-damped case and also for 10 percent and 32 percent these two are cases have been compared because in Timoshenko’s book also you can get a similar analysis but using different approaches so the for comparison purpose it has been used. And as you can see there is likely to be 2 mode shapes for 2 degrees of freedom model so, here we are able to capture beyond this 0.2 peaks were considering 2 modes. So, with this we have seen the application of mass spring dashpot model for dynamic analysis of soil slope also as well as retaining wall how we can do the calculations etcetera.

End of part A

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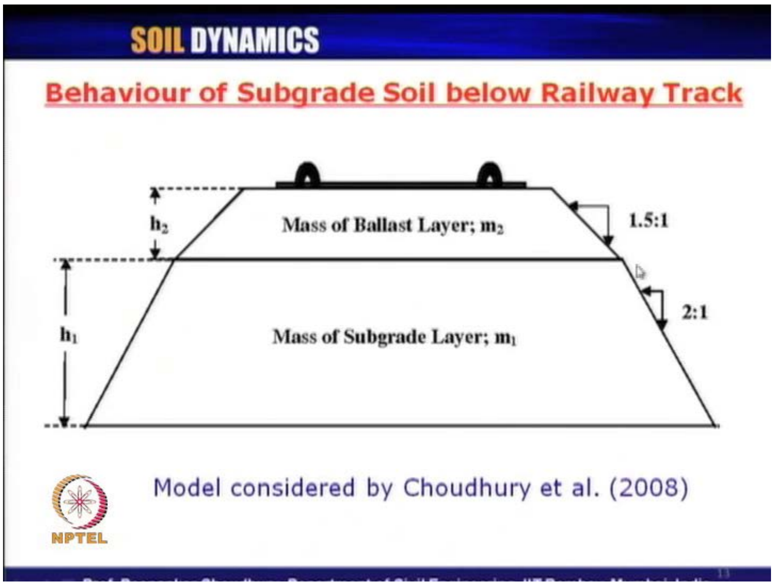

Now, coming to another application for today's lecture, what we are going to study behaviour of subgrade soil below the railway track. So, that we are going to model using our concept of this mass spring dashpot model because why we are going to apply that concept of dynamic analysis using mass spring dashpot model for a problem related to railway track. We all know railway track is going through the cyclic load due to the moving rail right. So, this is the typical cross sectional view of a railway track. So, these are the tracks, this is the sleeper, below this sleeper we have a zone with very big size of material which are known as ballast material. Right? So, this zone we are referring as ballast zone or ballast layer below that we have different types of soil. So, which altogether we can name them as we generally call them technically as 'subgrade layer'. And how to design this subgrade layer? You know using the similar values etcetera.

So, the design technique everything under the static condition is very well known. It is similar in the case of pavement also, in the case of runway also. The same model can be used for the analysis of pavement also for the analysis of runway because those are also subjected to moving load on the ground surface like this. So, we have shown in this case only the application related to railway track. So, we have the load acting on this track which is getting transferred through these sleepers to the ballast layer and then, from ballast layer it is finally going to the subgrade soil layer. And different slope of these layers are available as per the design standards of different countries. In India also we have several design standards given by RDSO. RDSO is the organisation which gives all the engineering guidelines, design guidelines for design of a railway track and the supporting system.

So, that is why from the RDSO specification we get a typical slope dimensions for ballast layer and the subgrade layer like this and different heights can also be considered. So, this result was carried out by this author choudhury et al 2008.

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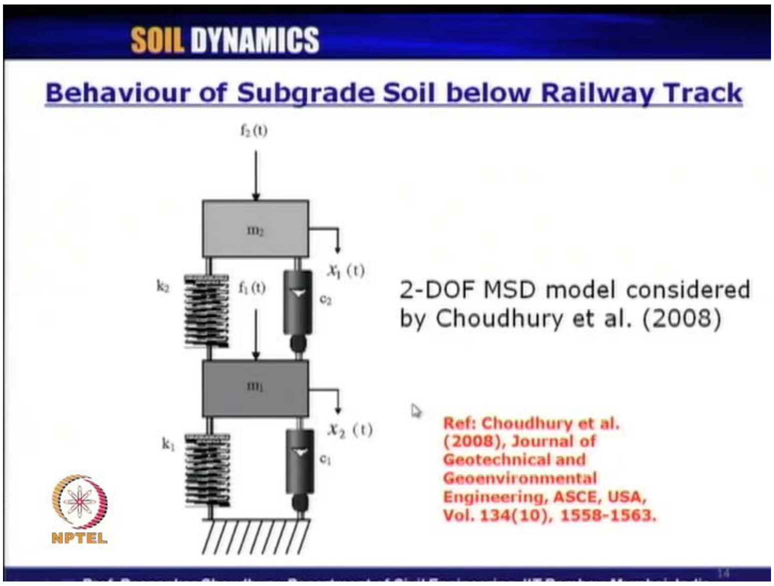

And it was available in this journal of geotechnical and geo environmental engineering of ASCE USA, 2008, 134 volume, issue number 10 from this page to this page. So, this is the ASCE journal paper. How the model has been simplified using a concept of mass spring dashpot model for this case?

Let us look back here. So, we have a dynamic load or cyclical load acting on this layer coming to the ballast layer and then, subgrade layer. Now, considering the mass of this zone is negligible compare to the mass of this ballast and subgrade layer. We had tried to model this mass of ballast layer as mass m 2 and mass of subgrade layer as mass m 1 with an assumption of rigid fixed based at a large distance. So this h 1 h 2 can also be calculated accordingly.

So, this mass m 2 is nothing but as I expressed, equivalent mass of the entire ballast layer, m 1 is the equivalent mass of that subgrade soil layer. So, if you have different layers of soil we have to calculate m 1 accordingly, if you have homogeneous soil layer for the design then, it is perfectly alright we can easily calculate it. Then, we had considered the damper c 2 and the stiffness k 2 for the ballast material and then, c 1 and k 1 are the damper and the stiffness for the subgrade soil layer. So, 2 degrees of freedom are this one: x 1 and x 2 and these are the applied dynamic load acting on the railway track system which can also be calculated easily. Now, from this model of 2 degrees of freedom mass spring dashpot model for railway track. What we can do?

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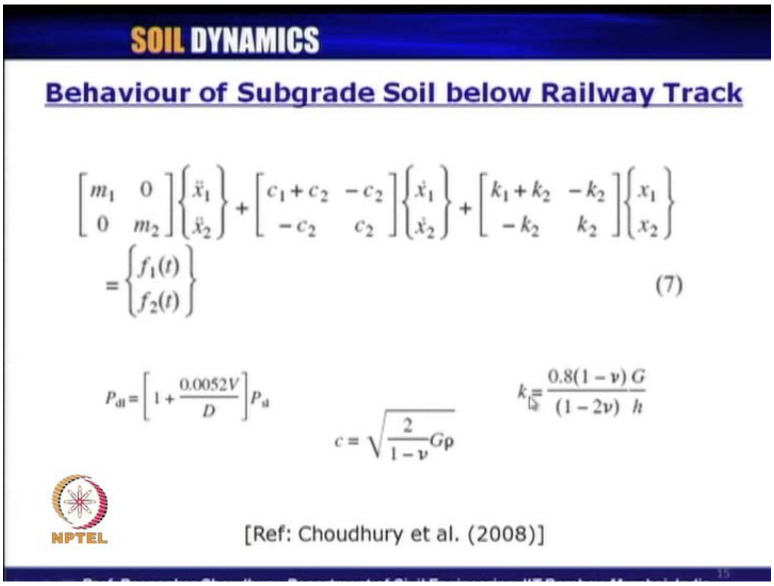

We can then draw the free body diagram of each of these mass. And then finally we can mention that, the equation of motion after considering the free body diagram of both the masses and combining the 2 equations the combined equation of motion will come in this format in the matrix form on simplified version it will come like this. So, this is the mass matrix with acceleration vector, this is the damper matrix with velocity vector, this is the stiffness matrix with displacement vector and this is the applied forced vector. Now, how to calculate this forces etcetera for this case? In this case, if you notice we are going to calculate the force f 2 t which is getting applied coming from the railway track. Right? So, basically f 1 t is will be either a function of f 2 t after the dispersion of the load or you can directly consider if you consider only direct applied load only the f 2 t component. Now, there are certain formulae available to consider where, from this concept of analysis has come.

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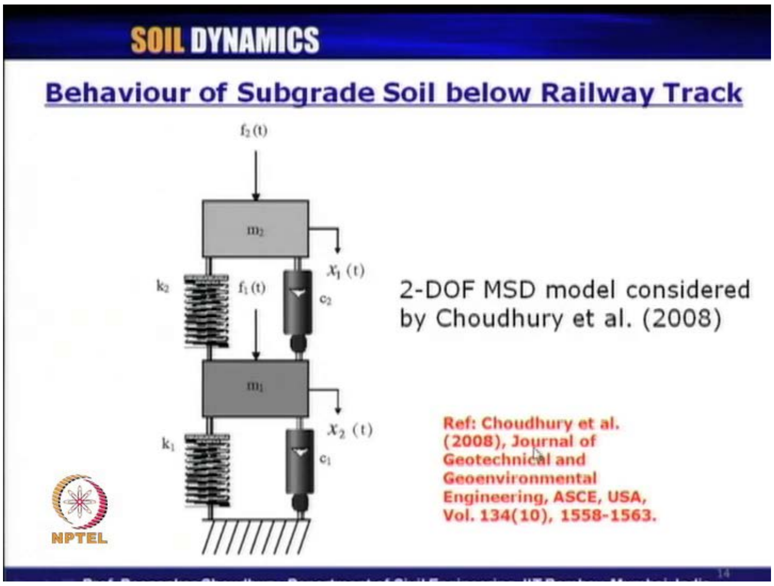

You can see the details in this paper for the derivations of all the equations. You can see this one.

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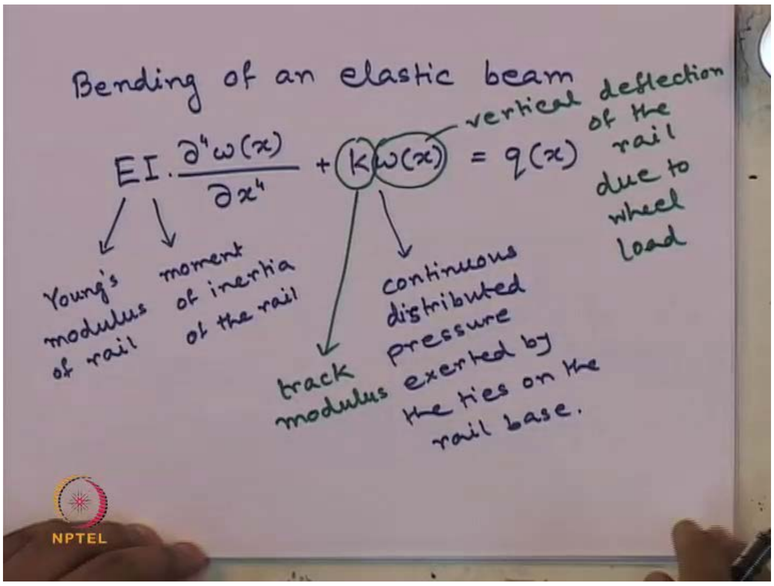

I am now writing here, what is the basic concept? The classical equation of bending of an elastic beam was considered here that is E I partial differential. So, that is the governing classical equation of bending for a elastic beam which you know. In this case what we have? E is the young's modulus of the railway track, I is the moment of inertia of the track, this K w x is nothing but the continuous distributed pressure exerted by the ties on the railways. So, this is young's modulus of rail means track material, this is moment of inertia of the rail, this one is continuous distributed pressure, distributed pressure exerted by the ties on the rail base and within this the part K is nothing but, this k is nothing but, track modulus. And this function that is omega x is nothing but vertical deflection of the rail due to wheel load. Okay.

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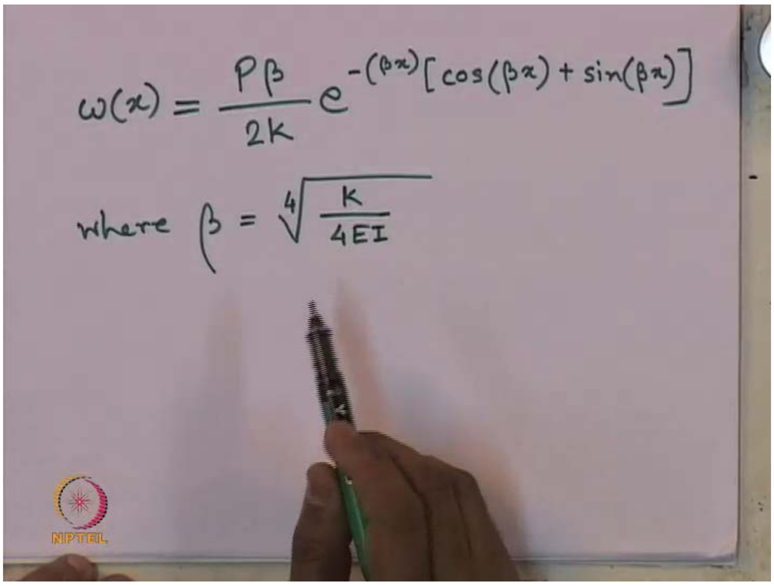

Now, this is how to calculate this omega x? Omega x is calculated, the solution p beta by 2 K e to the power minus beta x cosine of beta x plus sin of beta x. In this case where, this beta is nothing but a parameter which is expressed like this 4th root of K by 4 E I. Can you relate this parameter? For the foundation settlement analysis we had used this parameter also. If you refer to the Indian standard code for foundation settlement analysis, you will find this is a parameter expressed as modulus by this is subgrade modulus equivalent to subgrade modulus. Say, in this case we have this is as track modulus and E I of the rail. So, equivalent to rail this beta parameter can be calculated.

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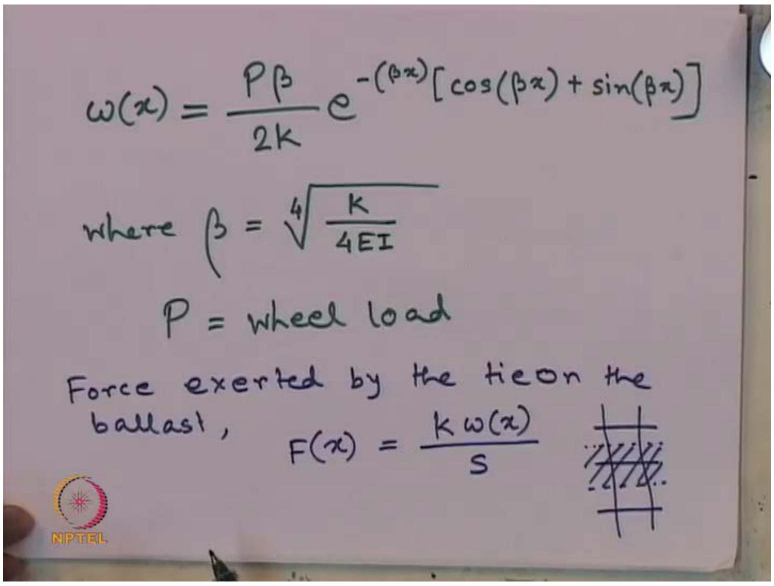

And P is nothing but wheel load. And in this case the force exerted by the tie on the ballast can be calculated like this. Let me tell you, force exerted by the tie on the ballast that is, F of x can be written as K omega w x by S. What is s? In this equation S is the new parameter we are introducing. S is nothing but spacing between 2 adjacent sleepers so this is on the plan if you see. So, this is the track we have sleeper so actually basically it is taken as influenced area from a sleeper. So, S by 2 S by 2. So, basically the spacing between the sleepers will give the total force exerted by the tie on the ballast from the track modulus and the displacement of the track considering it as a elastic beam.

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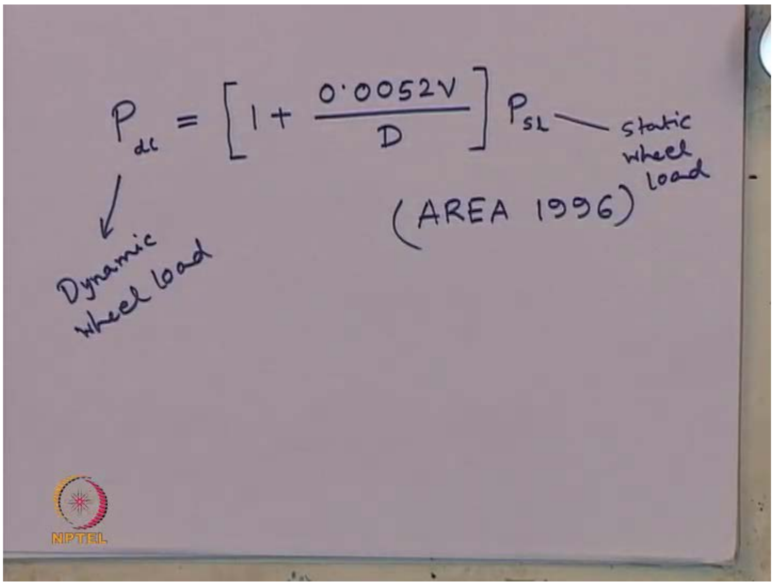

Now how to get the wheel load because that is required to calculate the displacement. So, to have the value of the wheel load the expression is P d l is nothing but 1 plus 0.0052 V by D times P s l. This formulae is given by AREA 1996. What is AREA? It is a abbreviation form it is American railway engineering association.

So, their guideline gives this formulae to calculate what is the dynamic wheel load. P d l is nothing but this is dynamic wheel load and P s L is static wheel load. The static wheel load of course, we know for any countries railway system the static wheel load specification will be known to us. This is the formula to be used to calculate the dynamic wheel load. And do you remember what our i r c c says, that we can consider this as a factor we can multiply with a factor of sometimes `1.2 or something like that. Right? So, the at least one way of getting the dynamic load. Otherwise, you to get an exact value this, the latest methodology.

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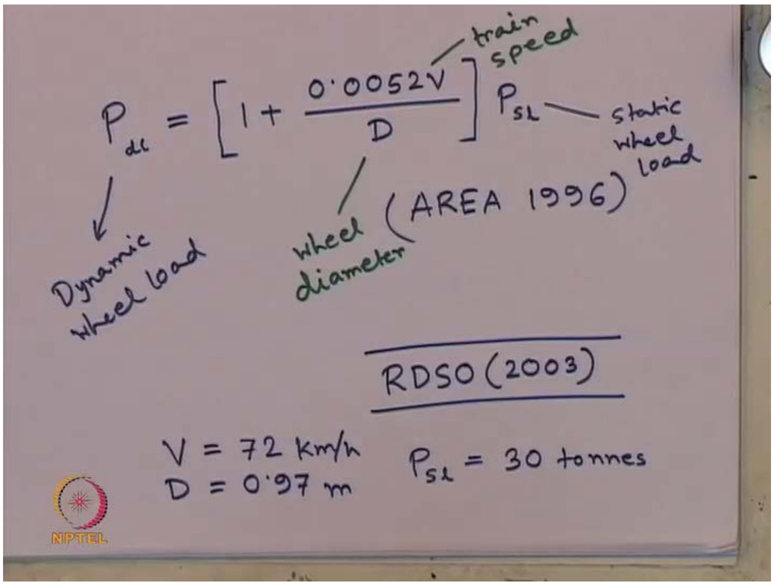

So, this is also used worldwide it can be used here. And in this equation what are these terms V and D? V is nothing but train speed and D is wheel diameter. So, now RDSO suggest that is Indian design specification for railway system. RDSO 2003 gives us the dimensions to be considered for V and D. It is mentioned that consider the design speed of railway is about 72 kilometre per hour and D is 0.97 metre and static wheel load P s l for Indian railway system can be considered as 30 tonnes. So, these are the specified values in RDSO 2003 for Indian railway system. So, finally we have calculated of course, in terms of our Indian railway system values in this paper. So, knowing these parameters we can easily calculate the dynamic wheel load and knowing the dynamic wheel load now we can put it back in the expression for the calculation of displacement coming from the railway track to ballast that is, w x and from that w x we can use it on the expression of force exerted by the tie on the ballast and that force will be nothing but the load which is acting on the ballast layer.

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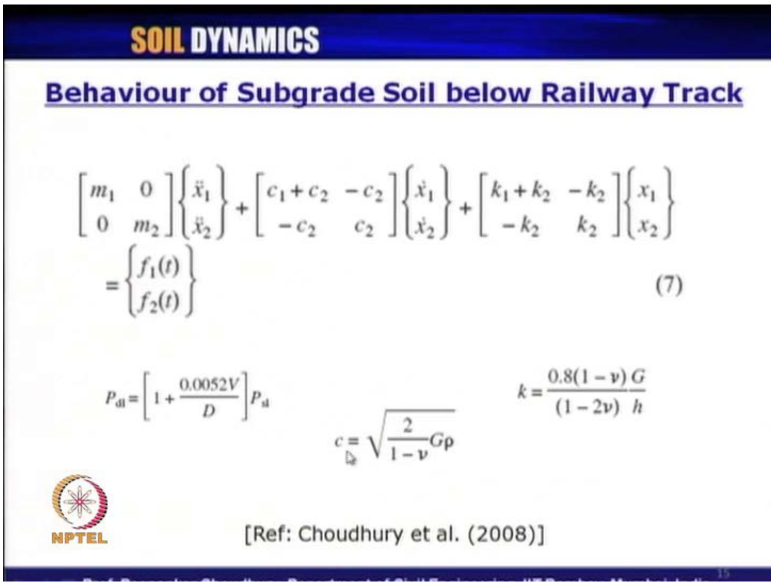

So, this is the way we can calculate and the typical formulae can be used to calculate the damping and the stiffness are given by this. It is based on the shear modulus of that particular material whether it is ballast or soil subgrade layer, h is the thickness of that particular layer and here and nu is the poisson’s ratio of that particular layer. So, for ballast layer and for the soil layer what are the different properties we had considered? I will mention you. So, this equation was given by basically Veletsus and Yunun in 1994. As I had mentioned earlier also, for the case of rigid retaining wall analysis these equations for computation of k and computation of c.

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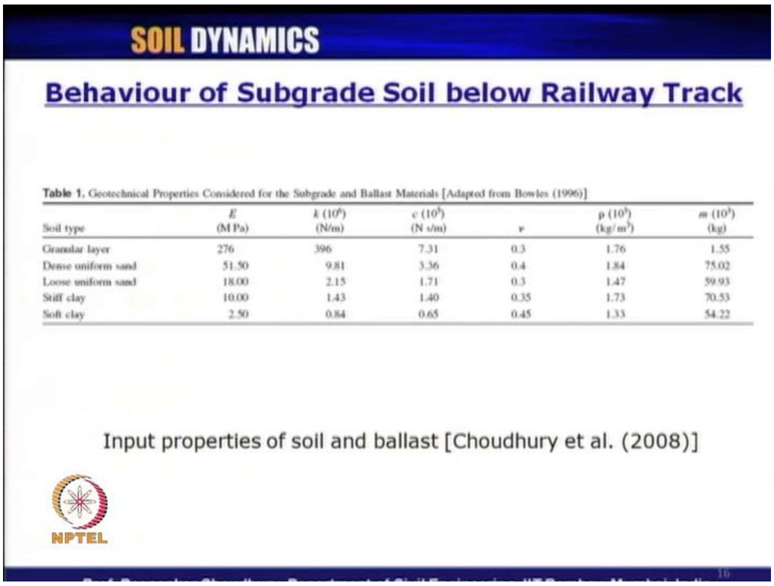

So, now let us look at what the different geotechnical properties considered for the upgrade material and ballast material in this paper by us. The material properties have been taken from the reference of bowl’s book like different types of soil you can see here. The granular layer is nothing but we have considered it as ballast layer the first one is the ballast layer and then, 4 different types of subgrade material we had considered: dense uniform sand as a subgrade layer, loose uniform sand as a subgrade layer, stiff clay as a subgrade layer and soft clay as a subgrade layer. So, we wanted to see how the displacement profile changes if we have different types of subgrade soil. So, that was our basic idea. So, what will be the effect of this dynamic load on the railway track because for granular layer ballast material throughout India is exactly has to follow the similar pattern or similar properties however the changes occur only in the subgrade soil. So, that is why different types of subgrade soil we have studied of course, there is a scope to consider several different aspects of the subgrade soil but these are the important properties we had considered.

E young’s modulus in M Pa, k that is stiffness we had calculated. Once we know e what we have done poisson’s ratio is also known to us these values standard values we had taken. You can see for ballast layer it is 0.3, for dense uniform sand 0.4, loose uniform sand 0.3, stiff clay 0.35 and soft clay close to 0.5. So, 0.45 layer considered knowing this poisson’s ratio values and the young’s modulus values of this material we can easily calculate the shear modulus. We know the expression between the shear modulus and the young's modulus. Once we know that G value, what we can do? We can calculate the stiffness and the damping constant for these particular materials using the formula proposed by Veletsus and Yunun and the density of this material are also given. Mass can be calculated by considering the design height of different layers as per the design specifications of RDSO and that can be estimated easily from the cross sectional view in terms of sleeper spacing we can easily calculate the mass in k g. Am I right?

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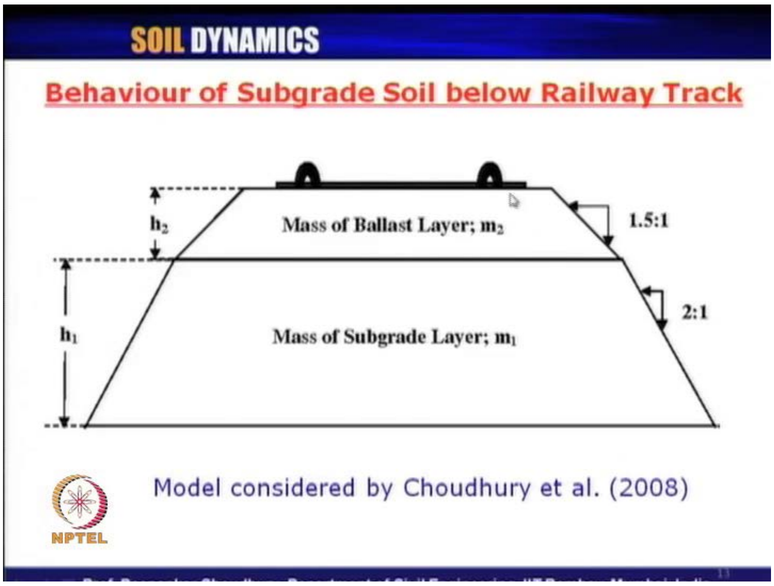

Because the sectional dimension is known to us, this dimension is known to us. The other dimension we are taking as the spacing. So, that is way it can be easily considered.

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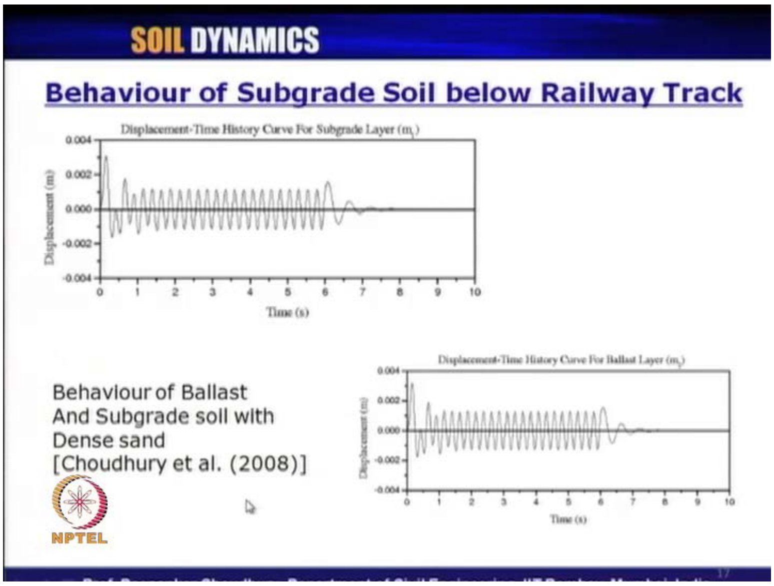

Finally, the results have been obtained you can see the profile of displacement for the subgrade layer with respect to time and this is for the ballast layer. So, from this you can get how much maximum displacement is going to occur we are concerned about this maximum value of the displacement. So, this is with the combination of ballast with the subgrade soil as dense sand. So, this is for dense sand and ballast combination. In that combination these are the results which are coming up.

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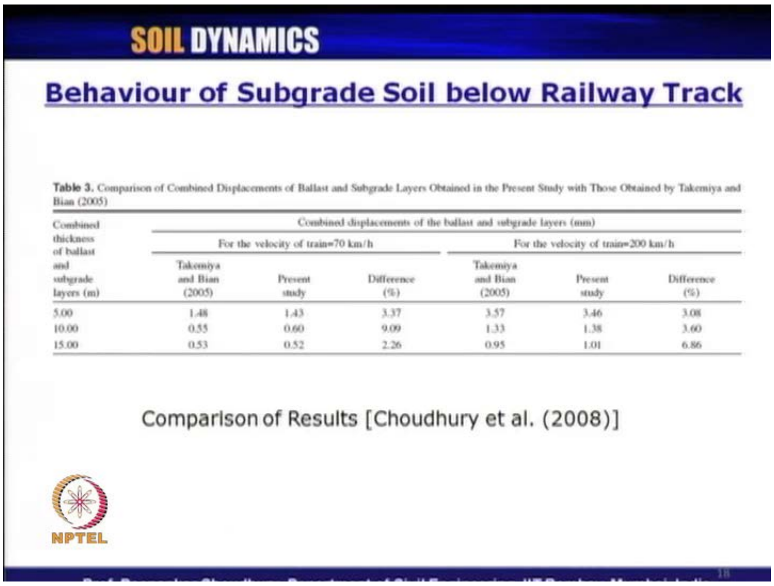

Table 3. Comparison of Combined Displacements of Ballast and Subgrade Layers Obtained in the Present Study with Those Obtained by Takemiya and Bian (2005)

Combined thickness of ballast and subgrade layers (m)	Combined displacements of the ballast and subgrade layers (mm)					
	For the velocity of train=70 km/h			For the velocity of train=200 km/h		
	Takemiya and Bian (2005)	Present study	Difference (%)	Takemiya and Bian (2005)	Present study	Difference (%)
5.00	1.48	1.43	3.37	3.57	3.46	3.08
10.00	0.55	0.60	9.09	1.33	1.38	3.60
15.00	0.53	0.52	2.26	0.95	1.01	6.86

Similarly, for all other types of soil also combinations of loose sand and ballast, stiff clay and ballast, soft clay and ballast the analysis have been done you will get the details in that paper. And finally, what we need to do? The validation of this model because these 2 degrees of freedom model has been first time applied here for the analysis of the railway track earlier in this way the analysis was not done.

So, we have to validate this model with available other analytical methods or experimental methods. So, we could get some experimental studied results as mentioned by Takemiya and Bian 2005 again in ASCE paper. So, for different train speed we had calculated one as per our RDSO specification at 70 kilometres per hour design speed and another case in this paper they have considered the speed of 200 kilometres per hour. So, in that formula of given by area we have used this velocity also suppose if Indian railway system is going to run the train in this speed then, what are the displacement is going to occur. So, the combined thickness of ballast and subgrade layer for different thickness of the layer the amount of combined displacement coming from ballast and subgrade layer. Because remember the displacement of individual layer x 1 and x 2 that is ballast and subgrade layer will be finally, added up to give the total displacement of the entire system because in this case finally, the how much settlement of the entire railway track is going to occur because of this operation of the rail or movement of the rail through the track has to be estimated.

So, these are the analytical value what we had estimated from our proposed results and these are experimental values as reported by these researchers for both the train speed you can find out that results are agreeing pretty well within 10 percent there and here for high speed it is given much lesser. So, it can be concluded that it is validated and well accepted method of analysis for a railway track to apply the mass spring dashpot model for this kind of cyclic load behaviour on a railway track. So, with this we will stop our lecture today here and we will continue further with some more discussions in the next lecture.