Soil Dynamics
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Module - 7
Dynamic Soil - Structure Interaction
Lecture - 36
Force - Based Analysis, Dynamic Analysis using MSD Model

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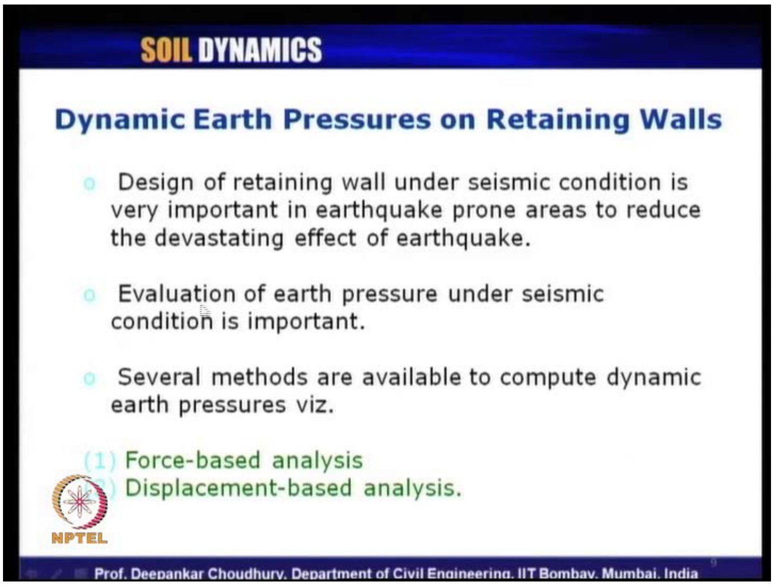

Now, let us start our next module that is, we are starting module 7 dynamic soil structure interaction within this module there are several sub topics has to be covered. So, let us start with the topic of dynamic earth pressures on retaining walls so, why that computation of dynamic earth pressure on retaining walls are necessary just few introductory remarks like design of retaining wall under seismic condition is very important in the earthquake prone areas to reduce the devastating effect of earthquake. So, evaluation of earth pressure under this seismic condition which is nothing but, giving a dynamic load is important for such cases and basically we have several methods available to compute these dynamic earth pressures methods means analysis wise. So, analytical methods can be classified in two broad categories what are those two broad categories, one is called force based analysis another is called displacement based analysis.

So, as the name suggest in the force based analysis we, are considering mostly the forces acting at equilibrium conditions limiting equilibrium conditions etcetera, on the failure zone on the wall etcetera to find out what is the earth pressure acting on the wall whether, it is passive earth pressure, whether it is active earth pressure or earth pressure at rest for different cases. We can get the through the force based analysis.

Let, us look at the slide so, in case of displacement based analysis, what is the difference based on the movements of the wall, how much earth pressures are going to generate because, we know the generation of earth pressures are functions of movements of the wall.

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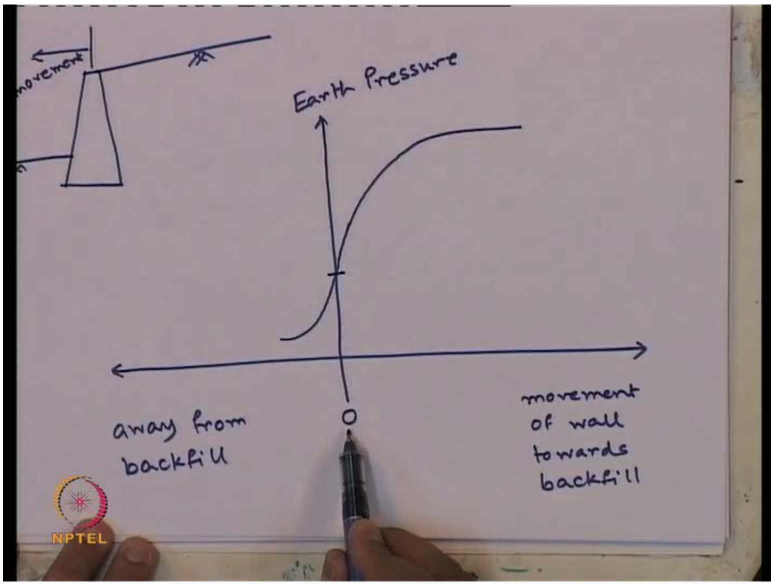

If you remember the basic diagram if I draw this for any rigid retaining wall suppose, we have back fill here and some back fill here this is dredge level this is the rigid retaining wall. So, for this rigid retaining wall which is retaining back fill soil at this height at 1 end other side at this end so, this is the dredge level we are calling.

So, depending on mode of movement of the wall what we see say suppose this is the main back fill if we say if the wall displace in this direction. So, movement direction if in this direction of the wall then we say on this side active earth pressure is going to generate on this side passive earth pressure is going to generate. So, basically this we called movement of wall this is towards back fill and this side we say away from back fill and this is earth pressure. So, typically we know the variation is something like this.

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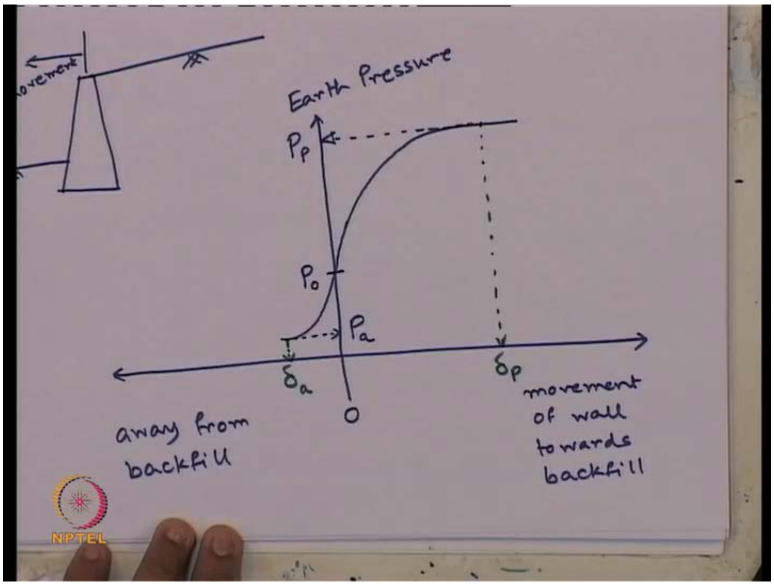

So, this magnitude of earth pressure where there is no movement of the wall that is this axis will be nothing but p not that is earth pressure at rest this magnitude of earth pressure after reaching the steady state that value of earth pressure, when the movement is towards back fill this is we call passive earth pressure and this magnitude of earth pressure when wall moves away from back fill is called active earth pressure.

So, these are the three states of earth pressure we can have for a rigid retaining wall for development of these magnitudes of earth pressure as you see, some amount of displacement is required here we need this much amount of displacement delta a to have this full value of active earth pressure for this also, we need certain amount of displacement delta p to have this full value of passive earth pressure.

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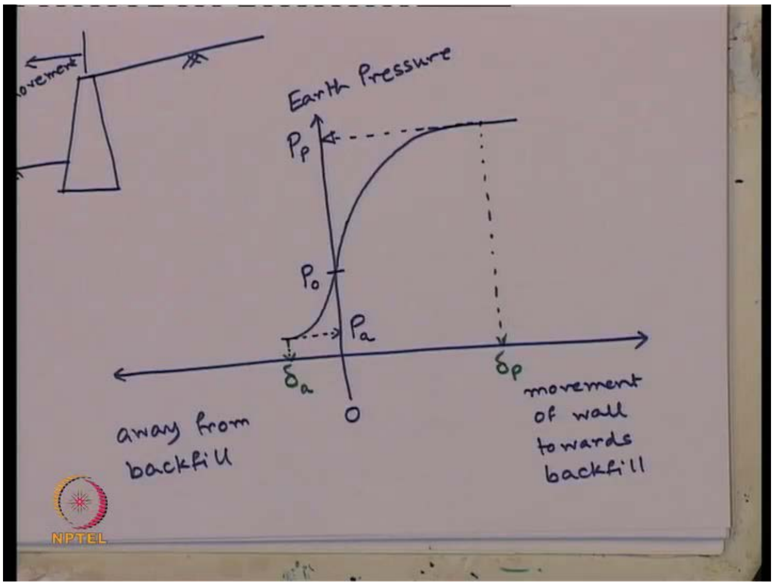

If the wall does not move minimum this amount of movement what will happen the earth pressure is going to generate as partial not full active or full passive suppose, it move up to this obviously we will get this much earth pressure, which is in between earth pressure at rest and earth pressure at active condition also, if the wall on this side moves up to this the developed pressure will be something in between at earth pressure at rest and full passive earth pressure.

So, these are called partial passive earth pressure zone and partial active earth pressure zone. So, that is why movement of wall is necessary and beyond these values of course, it will lead to a failure condition after a certain displacement.

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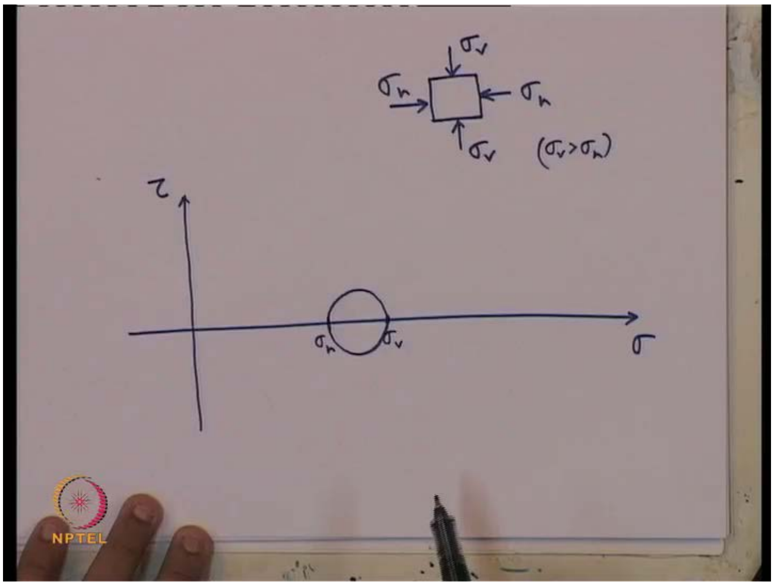

And how to explain this theory using a simple mode circle, we know that a state of stresses for a small element soil element considering the principle plain for all these cases, that is these are the principle stresses we are considering sigma v is vertical stress sigma h is horizontal stress. So, at any particular point we are plotting shear stress versus normal stress for any soil element like this so sigma v let us say sigma v is greater than sigma h at the starting condition. So, this is our sigma v and this is our sigma h so, the mode circle will be something like this is at any state of earth pressure sigma h is nothing but, the horizontal pressure which we can consider as earth pressure now how to get the active state or passive state.

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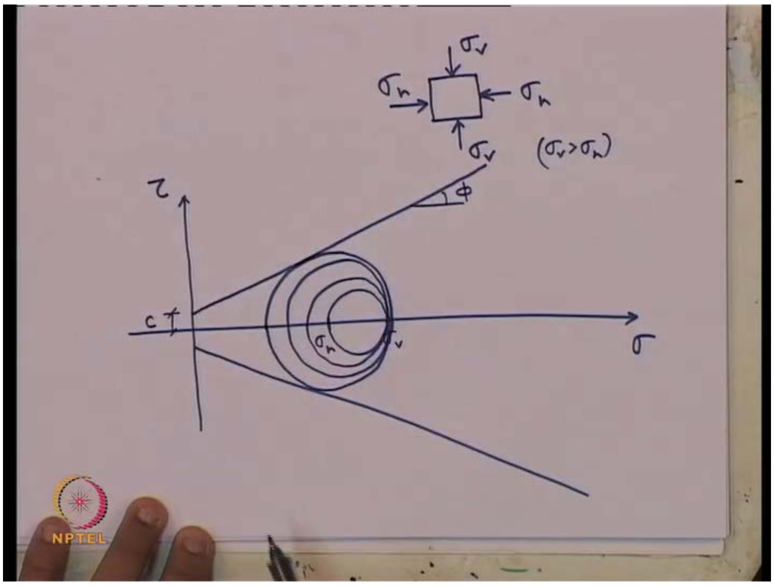

Suppose, if this is a c phi soil or phi soil depending on that we will have if it is the c phi soil the more (( )) failure envelop will go like this if we draw it on both the side this is our phi friction angle of the soil and, this is the intercept c coefficient of the soil. So, this is failure envelop this more (( )) failure envelop to generate the active state of earth pressure wall is moving away from back fill. When wall is moving away from back fill obviously, this sigma h is decreasing because wall is moving away from back fill so, sigma h is decreasing. So, what is happening sigma v remains constant because there is no change in sigma v what is happening sigma h is decreasing sigma h is decreasing as wall is moving sigma h is decreasing like that way it reached a particular state when this portion touches the failure envelop.

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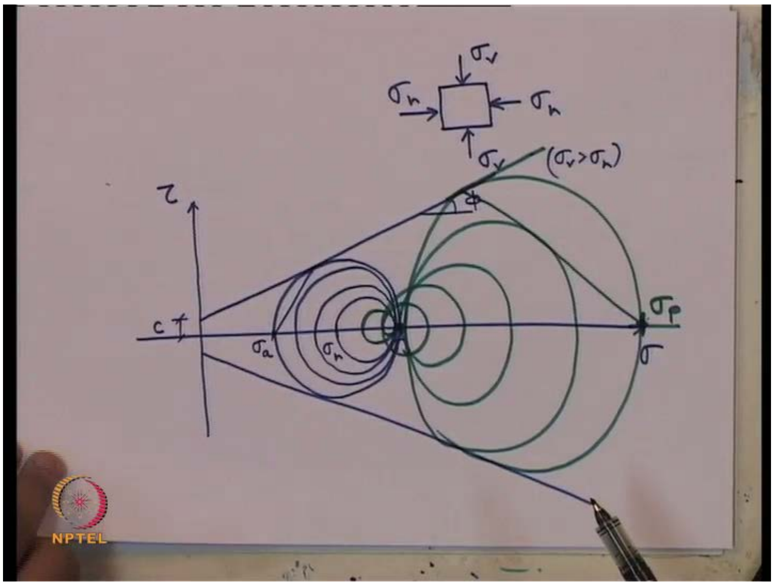

So, it cannot go beyond this failure envelop as we know so, obviously this is the minimum possible earth pressure which is active earth pressure. So, this value will give us sigma a, this is the way we calculate the value of sigma a and how to get the failure plane angles if you join this with respect to this tangential point, where the failure envelop goes through that inclination will give you the inclination of your failure plane. Now similarly, when the wall is moving towards the back fill what is happening sigma h is keep on increasing. So, sigma h is keep on increasing again remember sigma v remains same there is no change in sigma v so, what is happening sigma h is keep on increasing sigma h is increasing sigma v remains same here itself. So, when it touches this failure envelope once again this is the value of maximum earth pressure which is sigma p which is called passive earth pressure and that, point if you join with this failure envelope line this is the inclination of your failure plane.

These are the basics of finding out the earth pressure at active condition at passive condition and their corresponding failure plane inclinations. Now, these are by considering different amount of displacement which can be classified as displacement based analysis as i have mentioned here in this slide so, for dynamic case also we can do either force based analysis or displacement based analysis.

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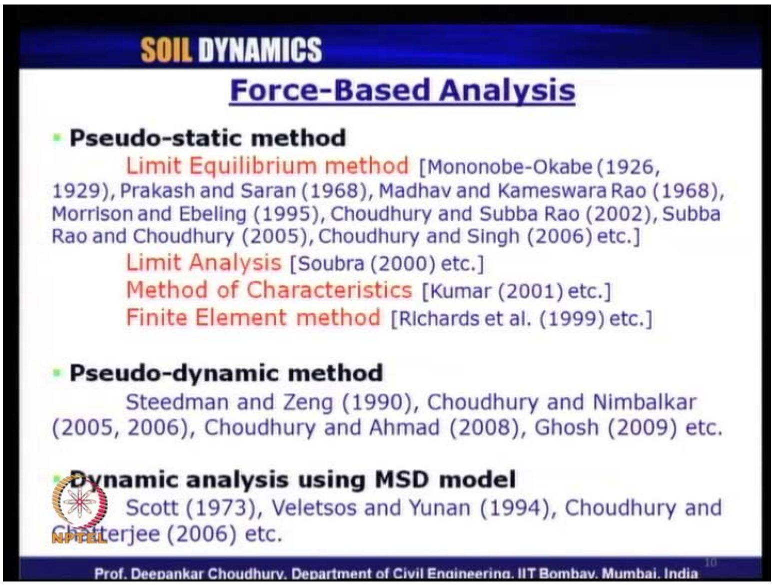

Let, us see what are the common methods available in force based analysis there are basically, if we can categorize them in three basic methods. There are several other methods but, these are the major 3 methods i can mention now, pseudo static method, pseudo dynamic method and dynamic analysis using mass spring dashpot model. So, for our course on this soil dynamics we will concentrate on this dynamic analysis using mass spring dashpot model, other two methods generally we cover in other course on geotechnical earth quake engineering.

Again the sub classification within pseudo static method, we can sub classify them as limit equilibrium method is 1 method limit analysis is 1 method. Method of characteristics is 1 method and finite element method is 1 method there are several researchers who have worked on this different methodologies to compute the seismic earth pressure using pseudo static method. Also there are several researchers who have worked on it and computed the seismic active and passive earth pressures on rigid retaining walls coming to dynamic analysis, using mass spring dashpot model what we have learnt in this course researchers like Scott in 1973 he had proposed as a pioneering research in this using this model of mass spring dashpot. How to compute the seismic earth pressure or dynamic earth pressure on a retaining wall in the active case further Veletsos and Younan they in 1994 computed then again active dynamic earth pressure on rigid retaining wall and Choudhury and Chatterjee in 2006 they computed using 2 degrees of freedom model.

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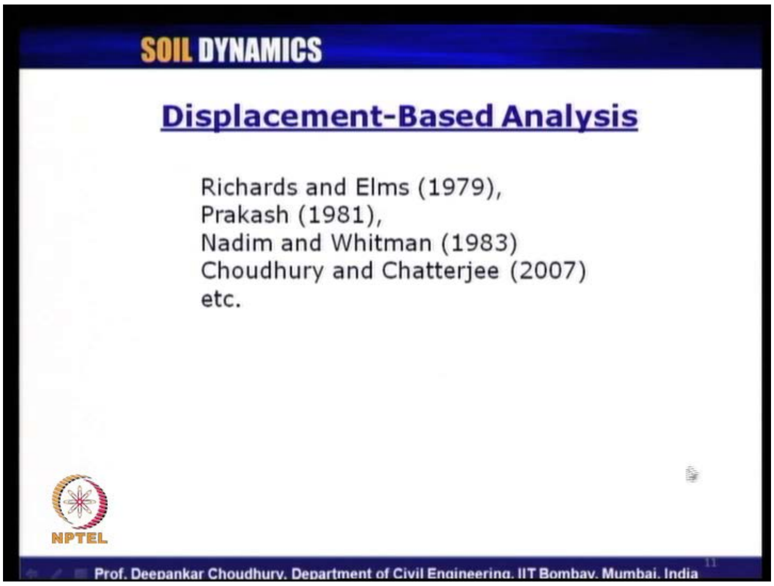

These two were the single degree of freedom model and this was using the 2 degrees of freedom model of mass spring dashpot model. How to come to the dynamic earth pressure on wall and in displacement based analysis? Also, several researchers have done the work i am not going into details of these things because they come under the scope of another course.

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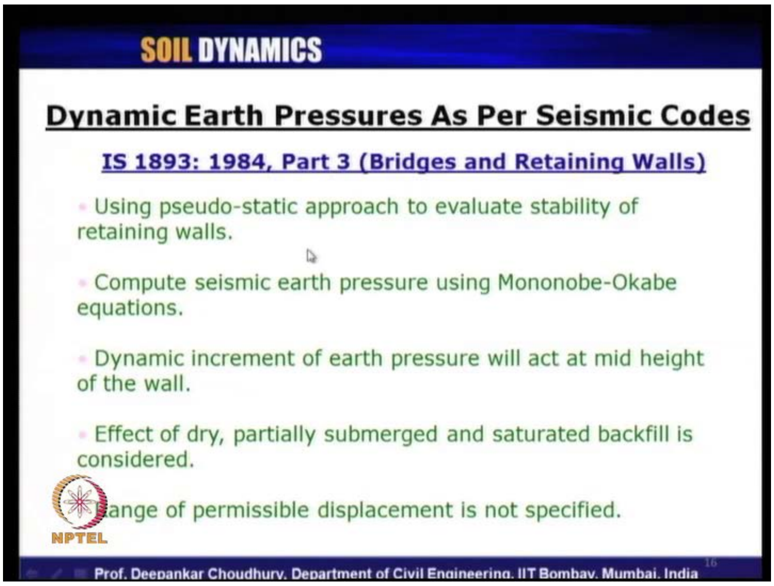

So, I will skip these methodologies also the design guidelines as per the seismic design code of i s 1883 1984 part 3 considers this earth pressure design. So, it is still 1984 other part 1 has been revised at 2002 others parts are still under revision not yet come. So, how to compute the earth pressure there are several guidelines like pseudo static approach has to be followed Mononobe-Okabe's equation has to be followed and so on.

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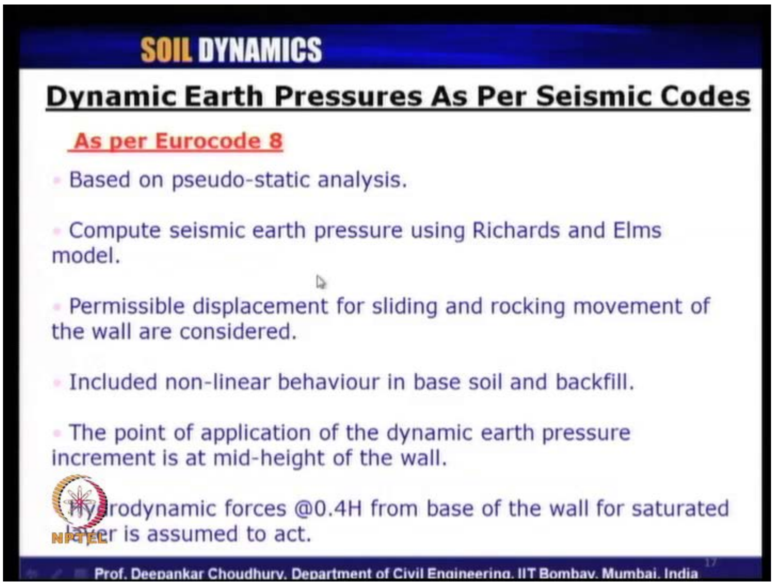

Similarly, other design codes of euro code the code followed in entire Europe euro code 8 consider, how to compute the dynamic earth pressure on rigid retaining wall.

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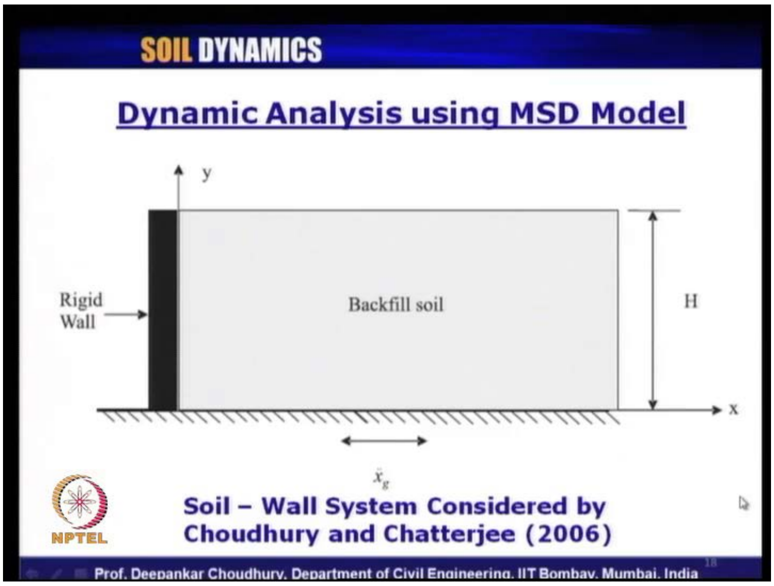

Now, coming to the model as we were discussing so, dynamic analysis of rigid retaining wall using mass spring dashpot model if we draw the typical picture this is the rigid retaining wall and this is the entire back fill. We are considering simple vertical phase retaining wall and a horizontal back fill and different excitations of dynamic load. We can apply here at the base of it so, this is the acceleration we are applying x g double dot this was considered by Choudhury and Chatterjee in 2006.

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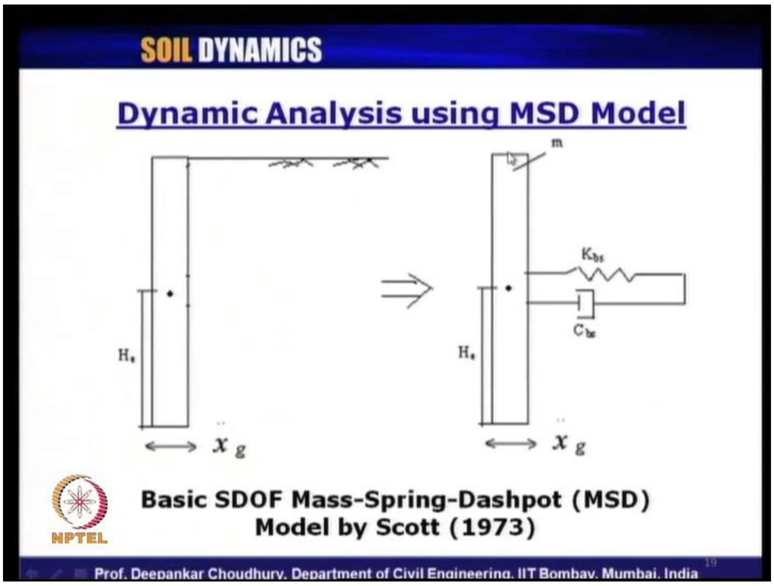

Similar, model was taken by the pioneering work as i have mentioned by Scott in 1973 that was basic single degree of freedom model. What Scott considered this is the rigid retaining wall the point of application H e at the height of H e from the base of the wall and wall is shaken with this amount of acceleration ground acceleration.

And the soil this back fill soil has been modeled as the stiffness and damper material and m is the mass of this wall fine. So, that was the model considered the single degree of freedom model mass spring and dashpot with 1 degree of freedom that is the movement of this 1 right movement of the wall how much it is moving.

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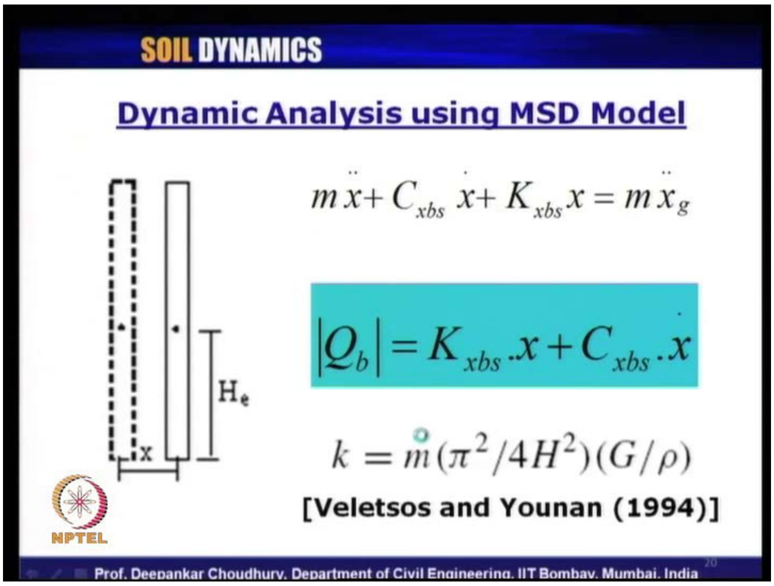

Veletsos and younan also had considered the similar way. So, it has been considered like this wall is moving by amount x so, basic governing equation of motion can be written like this m x double dot plus that, damper of the soil times velocity plus stiffness of the soil times displacement will give us mass times whatever is the base acceleration in this expression. Actually, we have seen for the case of earth quake analysis of frames also, in the very beginning how to model them and how to get the magnitude of the earth pressure the magnitude of earth pressure that is why this mod is used Q b is called which is nothing but…

If you look back the single degree of freedom model once again, how much pressure is acting on this wall or how much force earth pressure force is acting on this wall that is the force generated from this spring and force generated from this damper will be nothing but, the force acting on the wall.

So, that is why this stiffness times displacement plus the damper times velocity will give us the value of the how much dynamic earth pressure is acting on the wall. And how to compute this stiffness the expression to compute the stiffness is given here which is given in veletsos and younan and used by choudhury and chatterjee also, where from this expression has come.

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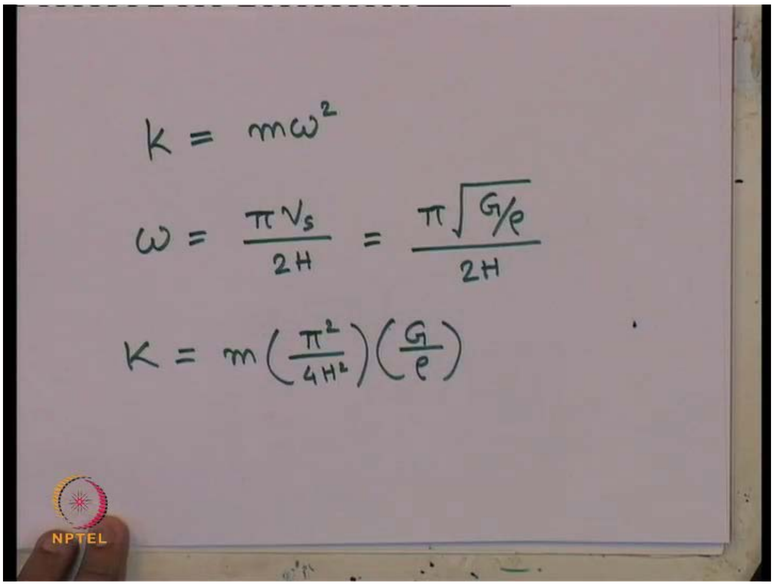

If we remember the value how we got this expression for k the k is nothing but, m omega square so, omega is given by this expression omega is given as pi v s by 2 h is the expression to compute the omega natural frequency and this v s is nothing but, root over g by rho 2 h hence, k can be written as m omega square. So, this becomes pi square by 4 h square times g by rho that is the expression to be used to calculate the value of k.

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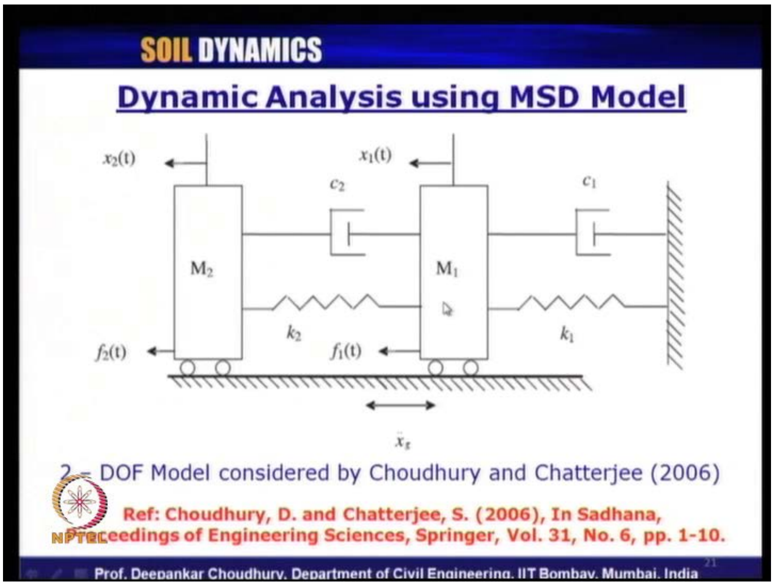

The same expression is given here also, if we look at the slide now modifying that single degree of freedom model choudhury and chatterjee this is the reference in the journal

Sadhana that, is proceedings of engineering sciences springer journal publication in 2006 volume 31 number 6 page 1 to 10. It proposed instead of considering a single degree of model it should be considered as 2 degree of freedom model 2 degree of freedom model. Why this is the wall rigid wall this is the mass of the soil, which is participating during the failure of the wall so, that is why the soil and wall they have different properties of course, so it should be considered as 2 degree of freedom model considering the mass of wall from wall properties to get the damping constant and stiffness for the wall.

So, c 2 and k 2 are correspondingly damping constant and spring constant of the wall m 1 is the mass of the soil which is participating and c 1 and k 1 are the soil damping and soil stiffness and this, f 1 t and f 2 t. What are these things these are applied dynamic load this can be computed from this x g double dot. So, looking at this which is quite logical because at the end we are considering it as rigid or fixed because at large extent in the horizontal direction at far field no effect will be there so, that is why we have to now, understand how to calculate the same 1 that is zone of influence has to be considered.

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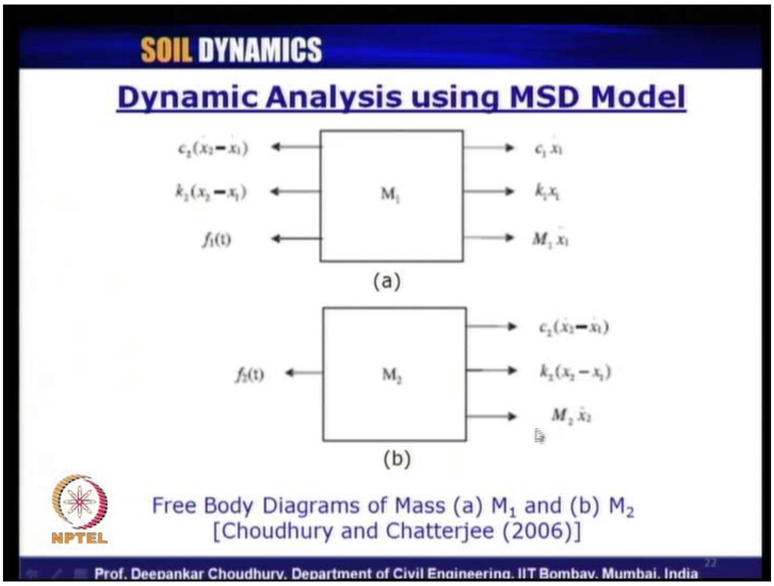

So, considering this basic 2 degrees of freedom model what was done the free body diagram of mass free body diagram of mass m 1 and free body diagram of mass m 2 m 2 is nothing but, wall so on wall we have externally applied dynamic load f 2 t the damper force will be c 2 times the relative velocity between the 2 that is x 2 dot minus x 1 dot.

Why it is 2 degrees of freedom once again let, me tell you these are 2 degrees of freedom. X 1 t is the movement of the soil x2 is the movement of the wall and stiffness force, will be the relative displacement between the 2 so, k 2 times x 2 minus x 1 and the inertia force will be m 2 times x 2 double dot these are will act in this direction.

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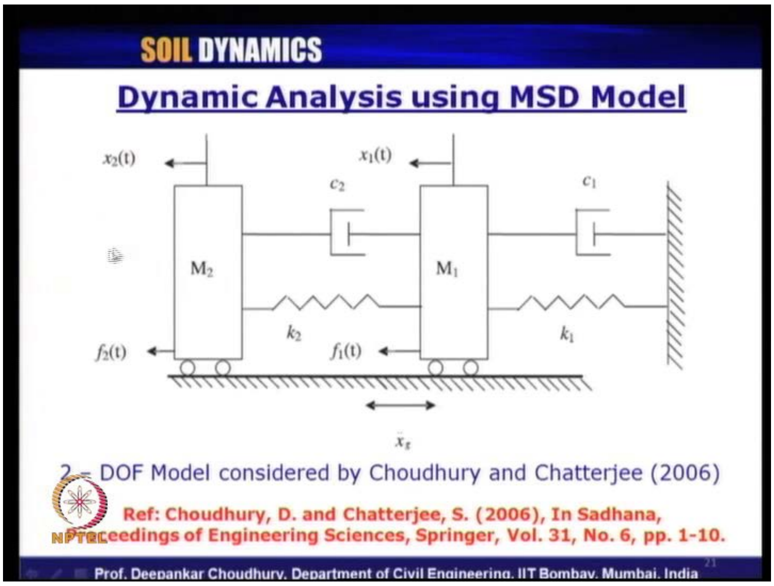

Because, we have considered out x 2 t in this direction so, obviously at an instant of time all these are resistive force acting in the other direction.

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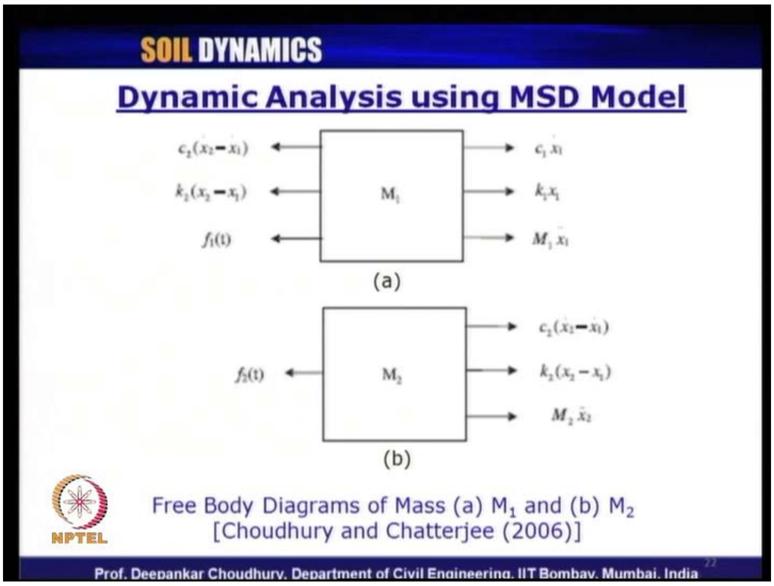

Now, for mass m 1 that is on soil what are the things acting on this side we have the damper force c 1 x 1 dot. We have spring force k 1 x 1 we have inertia force m 1 x 1 double dot and the balancing this damper and spring force. We have c 2 times x 2 dot minus x 1 dot in this direction because, here it is in this direction and this force will be k 2 times x 2 minus x 1 in this direction whereas, here it is in this direction.

So, this will be internal forces balancing each other and f 1 t the externally applied dynamic load on this component will be acting in this direction fine. So, with this basic model it can be further extended to compute the dynamic earth pressure acting on the wall.

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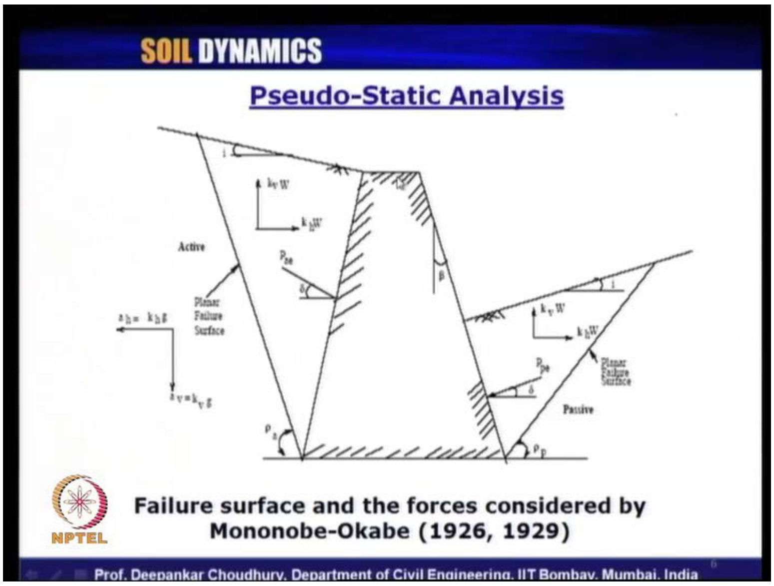

Now just a brief about the basic pseudo static method of analysis using limit equilibrium method, as was proposed by the first work done by Mononobe-Okabe as I said which is known as Mononobe-Okabe method of computation of seismic active. As well as passive earth pressure this is the line diagram of a rigid retaining wall, which is retaining a back fill soil up to this height at 1 end and on the other side the height of the soil is at this up to this level. So, considering the wall movement occurring from left to right in this direction, what we can say active state of earth pressure is going to get generated on this side whereas, on the other side where the wall is moving towards the back fill we can have the passive state of earth pressure getting generated.

So, these two generation of state of earth pressure. We had discussed already how they are going to occur this is because of changing of the value of the horizontal stress condition the vertical stress remains same and then, considering the Moore Coolum failure theory. When it reaches to the failure state then we consider full active earth pressure and full passive earth pressure has been developed. So, that is why in limit equilibrium method it is quite correct to assume some kind of failure surface because, we are handling the condition at the state of failure. So, that is why this different failure surface can be assumed by different researches Mononobe-Okabe had proposed to assume planar failure surface like this for both active as well as for passive case of earth pressure dynamic earth pressure and earthquake accelerations were mentioned like this a h as horizontal seismic acceleration with which can be represented by this k h g.

K h is nothing but, coefficient of horizontal seismic acceleration and g is obviously, acceleration due to gravity similarly, a v is considered as vertical seismic acceleration, which can be expressed as k v times g where k v is called the coefficient of vertical seismic acceleration. Now as the method suggest it is a pseudo static or quasi static method so, along with the static forces acting on this failure vague. We know how to analyze the failure vague to compute the earth pressure acting on a retaining wall from Coolum's theory. So, Mononobe-Okabe theory is basically the extension of Coolum's earth pressure theory in the case of static loading so, in Coolum's earth pressure theory whatever, forces were acting they considered cohesion less soil. So, obviously 3 forces were acting weight of this failure vague then, this active earth pressure which is getting generated and the soil reaction coming from this side.

So, these three forces has to be in equilibrium and they will meet at 1 point. So, the force triangle has to be satisfied at equilibrium condition now, in case of pseudo static analysis. What are the extra forces coming into picture these two extra forces that is the horizontal inertia force because, of this horizontal seismic acceleration. So, k h times the weight of this failure vague k h times w is nothing but, horizontal inertia force and the vertical inertia force is nothing but, k v times w which will act in the vertical direction remember, being these are earthquake accelerations coming out from the earthquake acceleration they can act in both the directions, that is from left to right as well as from right to left also. This k v w can act in both the direction towards upward and towards downward now, among these four combinations because if you take these two forces then obviously, considering two directions for each of them we can get four combinations. So, among those four combinations we are suppose, to find out which 1 is the critical combination which is giving us the design active earth pressure and what is design earth active earth pressure.

We know from the Coolum earth pressure theory. What we do to find out the active earth pressure we, consider different failure surfaces at different angle of inclination with respect to horizontal and then, we optimize or maximize the active earth pressure with respect to this angle that is the methodology we adopt in Coolum's earth pressure theory. So, in this case along with this optimization we have to do 1 more optimization additional, that is involved with this four combinations of the seismic forces. So, the combination which will give us maximum value of this p a e that is active earth pressure under earthquake condition will give us the design value of seismic active earth pressure. Similarly, if we talk about the passive earth pressure what we do in the case of passive analysis in Coolum's theory, when there is only static forces are involved three forces will be there for cohesion less soil weight of this vague soil reaction and this passive earth pressure getting generated.

So, we use different angles for this different planer failure surface and then we optimize means in this case of passive we minimize the value because this passive is the resistance. So, get a design value we have to minimize the value of passive earth pressure, which will be the design value and then in the case seismic forces what we are involved here additional two forces similar to this with four different combinations. So, these 2 seismic inertia forces will come into picture and among those again, we have to do 2 optimizations 1 with respect to this failure angle another with respect to the direction of the combinations of the seismic forces acting on this failure zone. That will give us the absolute minimum value of this p p e which will be the design seismic passive earth resistance. This is sometimes called passive earth resistance because it is truly speaking is not the pressure acting on the wall but, it is the resistance offered by the soil to the wall. So, this was the basic equation proposed by Mononobe-Okabe you can see this is nothing but, Coolum's equation with an addition term of additional term of this portion which considers the earth quake acceleration in vertical direction also. And the equation closed form solution for this coefficient of active and coefficient of passive seismic earth pressure is expressed by this equation, where different parameters I can mention here beta is nothing but, angle on inclination with respect to vertical the phase of the wall i is the angle.

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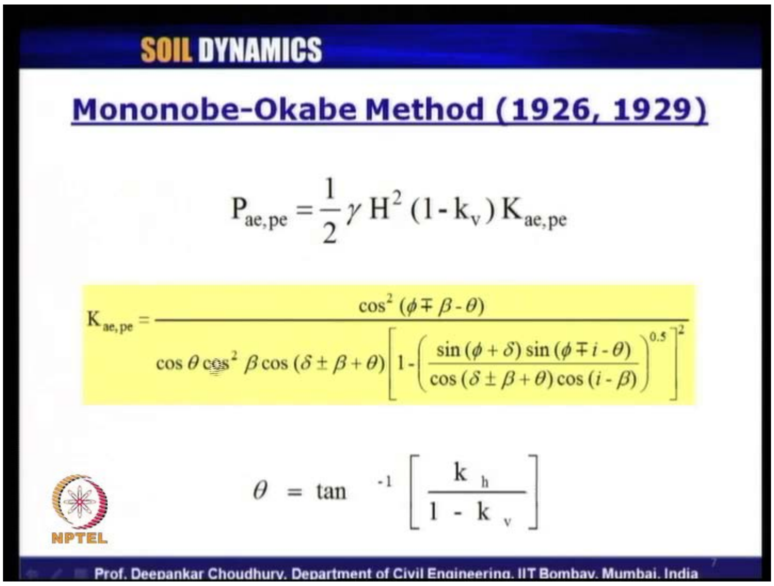

Here for the back fill soil and delta is the wall friction angle you can see here, this is delta and phi is friction angle soil. Friction angle and theta is another angle which can be estimated by using this relation of tan inverse k h by 1 minus k v it satisfies actually, the criteria of shear. If you remember we talked about during the chapter or during the module when, we discussed about liquefaction of soil to take care of the stability of the soil this is the criteria. We have to satisfy so, from that criteria this theta comes and for stability phi should be greater than this theta that is soil friction angle should be greater than this theta.

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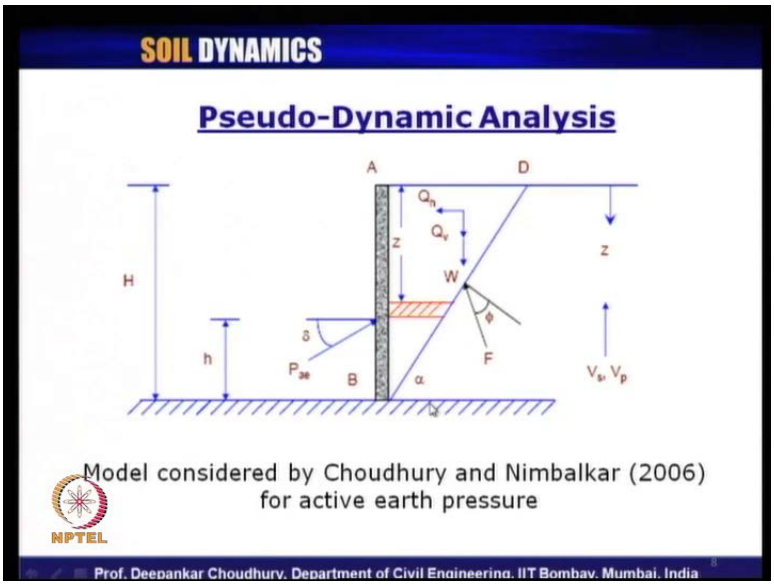

Then how the pseudo dynamic method was proposed means the changes from the pseudo static method are like this here the shear wave and primary wave, which are travelling during the process of an any earthquake are also, considered effect of them and this q h q v are the horizontal and vertical inertia forces, which can be calculated based on the variation of the earthquake acceleration with respect to death also, with respect to time and by considering the frequency content of the considered earthquake. So, that is why as i said all the dynamic nature of earthquake forces can be considered in this pseudo dynamic method. Hence instead of considering a single coefficient or single value attached to the weight of this failure zone it is considered as infinitesimal small element or specimen at any depth z with a height or thickness of d z and that on that element, we have considered how the acceleration in both vertical and horizontal direction they are creating the inertia forces and those inertia forces are further integrated over the inter height of the wall.

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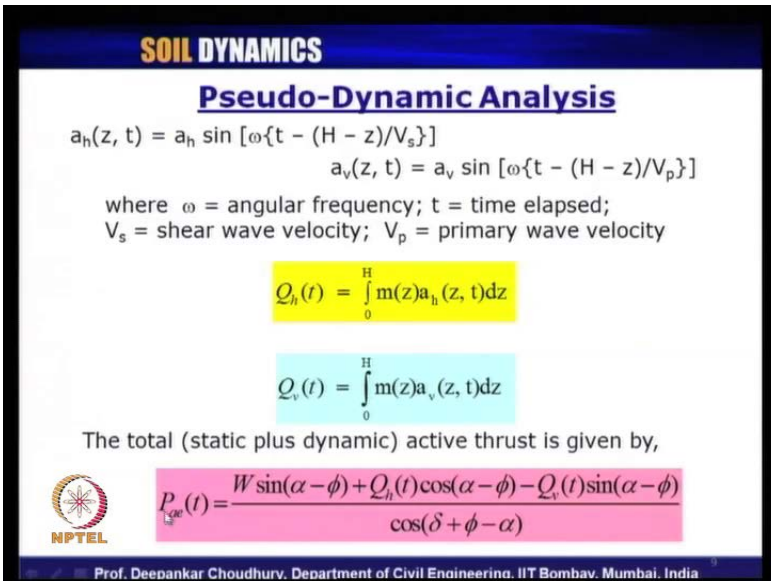

So, these are some of the expressions, how the horizontal seismic acceleration is approximated it will be obviously, random in nature but, for obtaining a closed form solution. We need to consider some equivalent form of the equation so, that it leads to a generalized solution, which can give us a design chart so, that is why any random motion, we can approximate by considering the area under the curve and different methodologies that equivalent sinusoidal wave was considered in or was assumed a h is the amplitude of that motion omega is the frequency of the earth quake h is total height which we are considering for the height of the wall z at a particular height, where we are considering for that infinitesimal element and t is the time for which the earthquake is occurring. The duration of earthquake and v s is shear wave velocity which is creating. We have assumed that v s is creating the horizontal acceleration and v p that is primary wave is creating the vertical acceleration this also, I have discussed, what type of direction of excitation of to the particles these different waves generate in the module of wave propagation.

So, similarly, the vertical acceleration is expressed in this way and the inertia force in horizontal and vertical directions are nothing but, mass of that slice times accelerations. Mass times acceleration will give us the force so, that is inertia force we have to integrate it over the height of the wall and then we will get different expressions and solution. Finally, we used limit equilibrium method to get the expression for p a e and remember this needs to be optimized with respect to the different failure plane and different combinations of this directions of q h q v.

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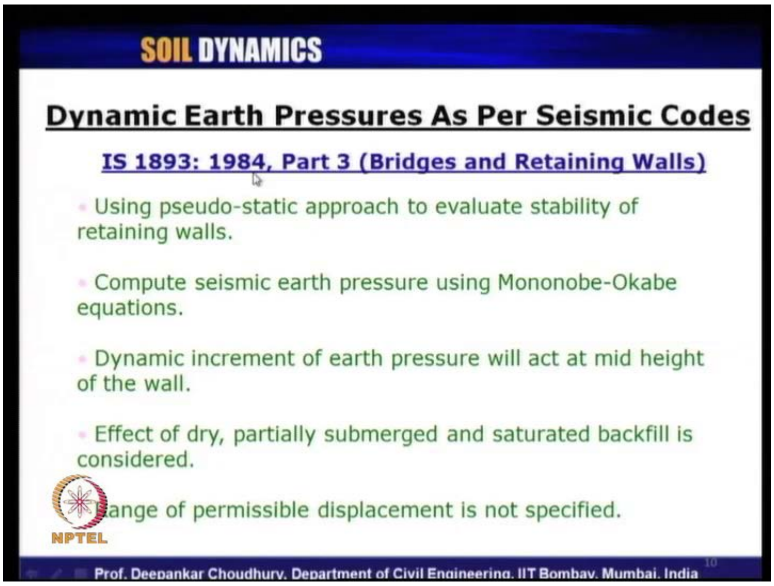

And what the design code mentions the i s code Indian design standard i s 1893 seismic design code the part 3 handles the portion about this bridge and retaining wall under seismic condition. It is 1984 version as I mentioned that i s 1893 part 1 has been revised in 2002 but, other parts are still under revision.

It has not yet come out so, what it says that using pseudo static approach we can evaluate the stability of retaining wall by considering the seismic earth pressure. So, it proposed to use the Mononobe-Okabe equations dynamic increment of earth pressure will act at mid height of the wall and effect of dry partially submerged and saturated back fill is considered. And range of permissible displacement is not specified in the code there are several drawbacks as you can see and approximations as in this code because, it was framed and came out long back. So, after that several research output and advancement has taken care of and that is why different country seismic codes are have been revised but, Indian code are still under revision so, hopefully it will also come out with some good guidelines.

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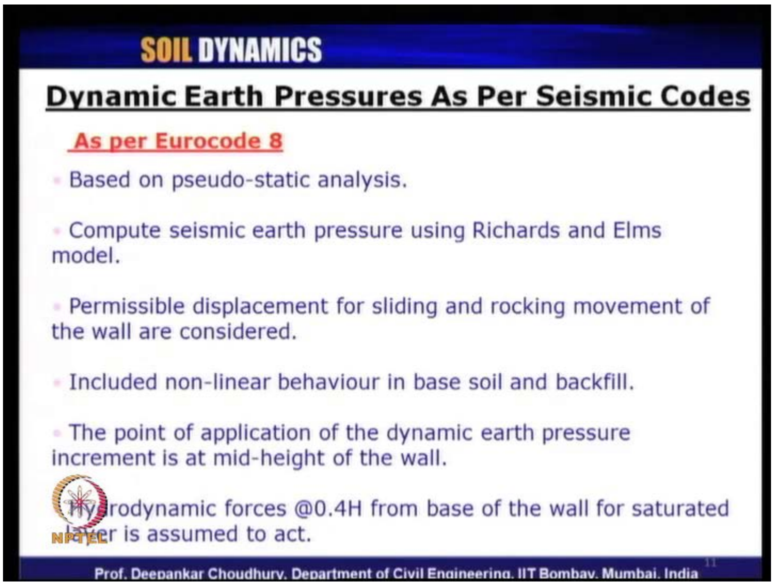

And as per Europe code euro code 8 that is also based on pseudo static analysis but, the beauty of this euro code they considered the Richard elms model to take care of the displacement also, which is very important.

We should know how much displacement we should allow. So, this is based on the displacement base criteria so, permissible displacement for sliding as well as rocking movement of the wall have been considered in this and it includes the non-linear behavior of the soil and back fill also, which is another advancement and the point of application of dynamic earth pressure it still considered at mid height and different hydro dynamic force. If it is a saturated layer is assumed is mentioned in this code euro code 8 also they had 1998 version later on they revised in 2003. So, like that frequently they revise their code as per the research developments keep on going across the world.

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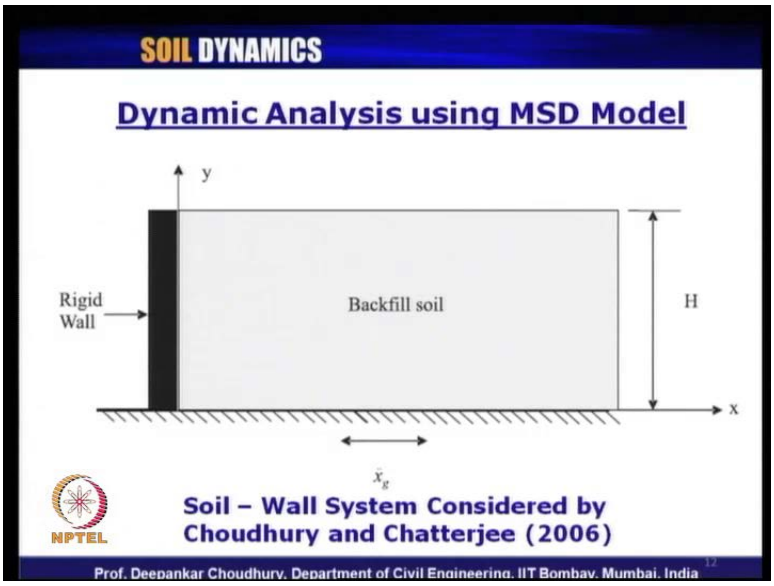

Similarly, the Nihar code is available of u s and u v c is also, available for us those are also considering mostly Nihar is considering most geological guidelines and Nihar code 2003 is probably the latest 1.

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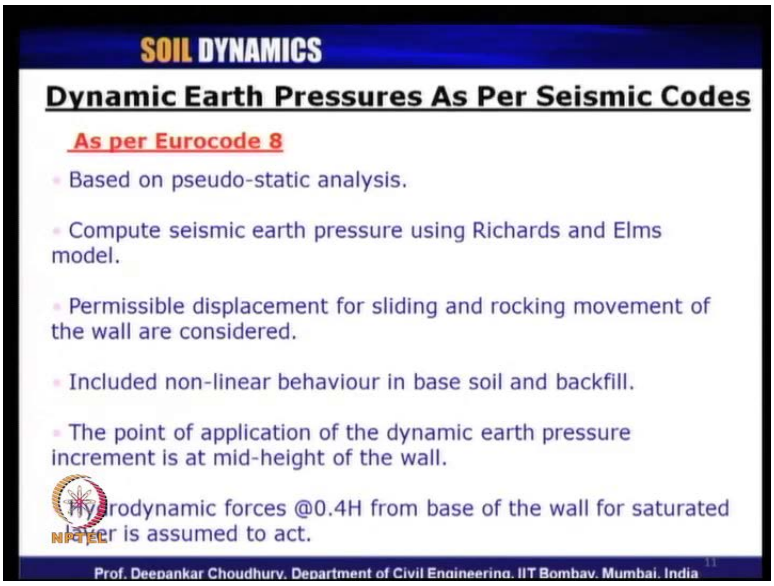

So, that considers all the effects of permissible displacement of the wall under seismic conditions and different calculation methodologies for earth pressure and 1 extra important thing i forget to mention in euro code 8 they mention to calculate the soil amplification also, which is absent in the Indian standard code soil amplification during the earthquake process the amplification factor. It can be considered in our pseudo dynamic analysis, where as in pseudo static analysis there is no scope for considering the amplification as such.

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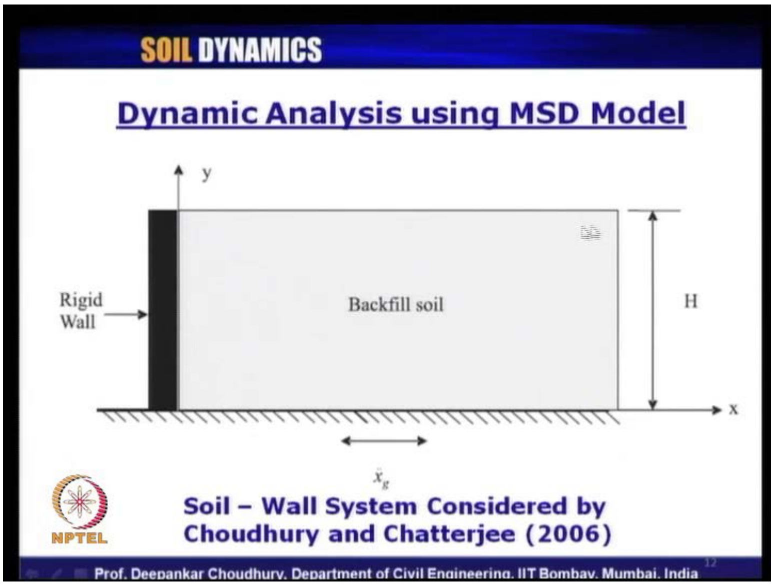

Now, coming to the methodology which is involved for our course that is dynamic analysis using mass spring dashpot model this is the basic model. We had considered by Choudhury and Chatterjee in 2006 the rigid retaining wall schematic diagram and this is the back fill soil zone, which is getting excited under some dynamic loading.

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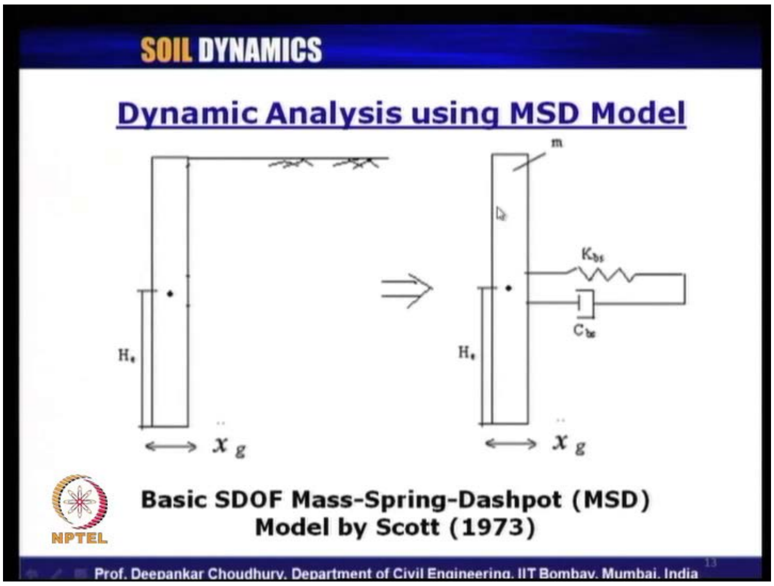

And what was the basic model proposed by Scott in 1973 he considered single degree of freedom mass spring dashpot model. Instead of 2 degrees of freedom he considered this is the rigid wall and this is the excitation it was equivalent model with by considering. This effect of the soil was taken by using a spring and dashpot that is soil was replaced fully by a spring and a dashpot and that was considered to be attached with the wall and wall mass was considered in that single degree of freedom model at this retaining wall.

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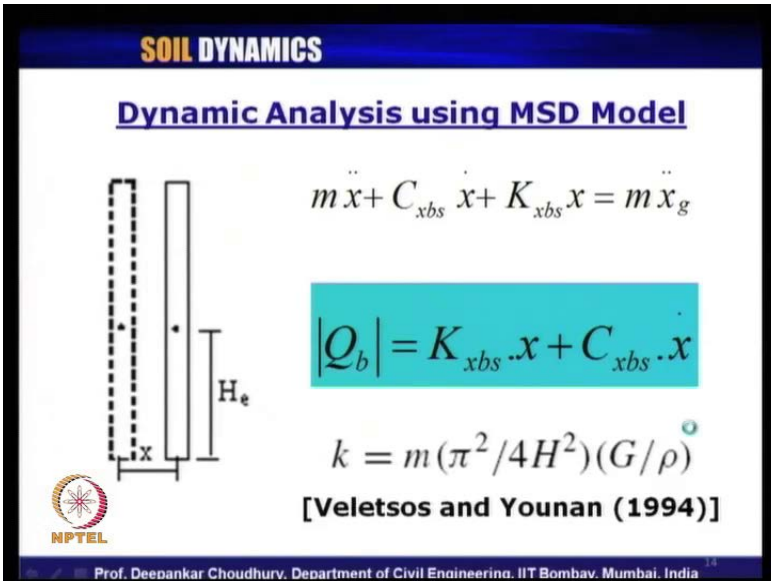

Also, this model was continued and followed by veletsos and younan in 1994 and they used better expressions to compute the stiffness values for soil involved in this basic governing equation for single degree of freedom. And how to compute the magnitude of earth pressure they also, came out with that magnitude of earth pressure acting on the wall is nothing but, coming from this spring force and the damper force.

But, always it can be questioned whether the mass of the soil is only involved in this process of dynamic analysis or mass of the soil also should be involved because in our other analysis, when we do any limit equilibrium analysis or limit analysis as such for earth pressure calculation. We take the zone of failure within the soil mass and both of them are experiencing through the earthquake forces and that has to be considered its effect also has to be considered which is directly coming on the wall.

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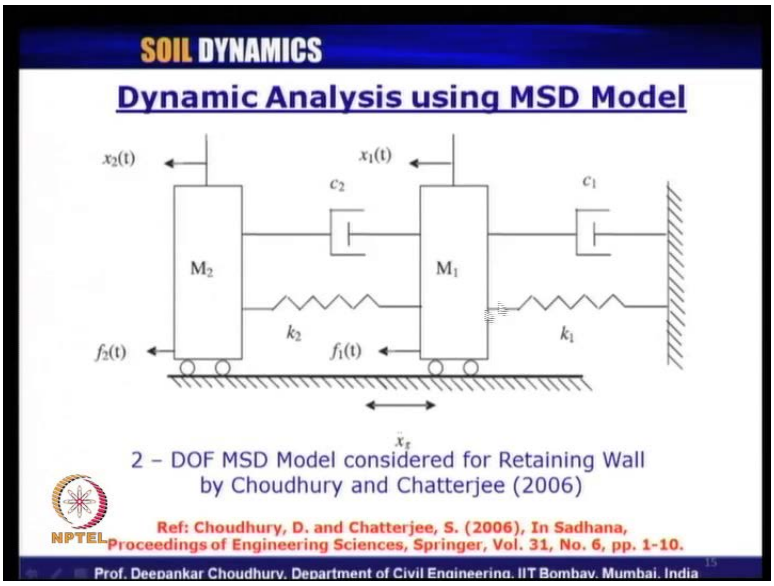

So, that is why further modification was done by this work of Choudhury and Chatterjee this wall has been modeled as m 2 mass of the wall and wall properties mostly for concrete or r c c wall or whatever different types of wall material in use depending on that damping constant and the spring constant can be evaluated and m 1 is the mass of the soil involved taking part into the analysis or which is taking dynamically vibrated along with the wall and the soil stiffness and soil damping has been also considered.

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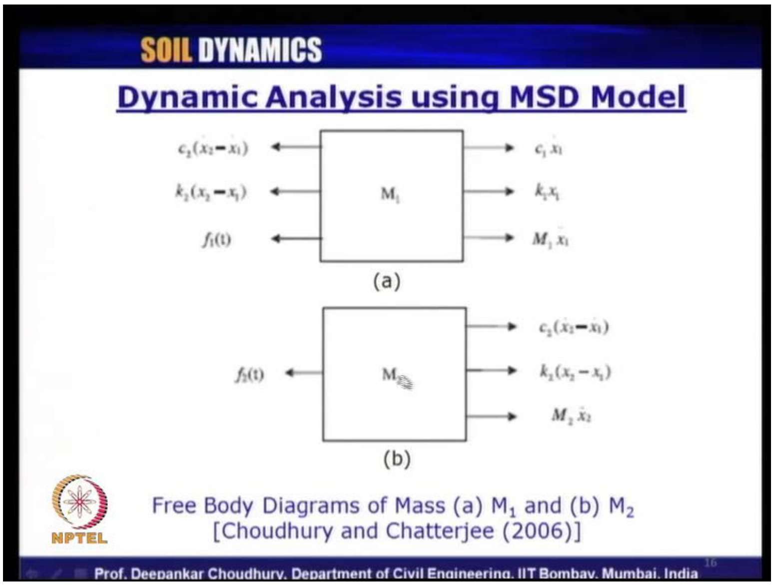

Now, if we draw the free body diagram of each of this mass that is the m 1 is for wall m 2 is for soil. We can easily see that this is the boundary has been considered as rigid because at far field conditions we can always consider it as a rigid boundary. These are the forces externally applied dynamic load inertia force these are the damper force and spring force these 2 are the balancing forces between the 2 material and this m 2 x 2 is nothing but, the wall inertia force. So m 2 is for wall and m 1 is for soil so, this is for soil and this is for wall and f 2 t.

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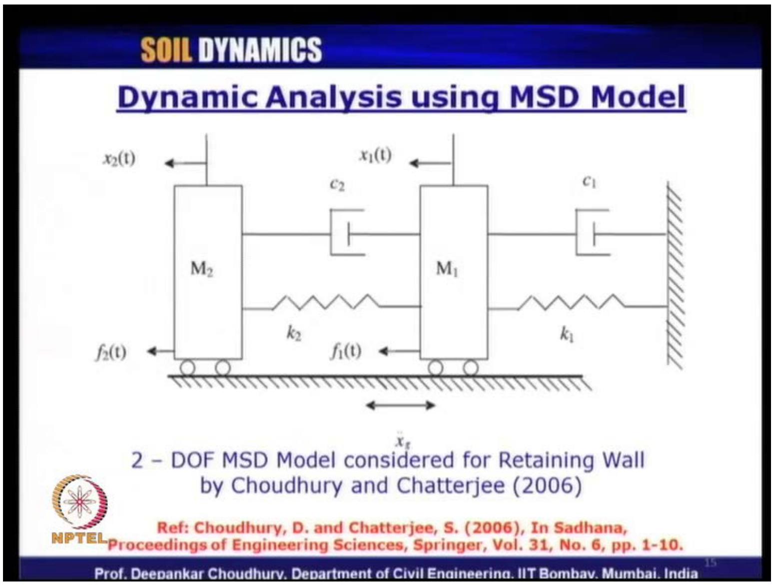

Now, in this case the question automatically arises. We can easily compute m 2 because for a chosen wall section, which we are going to design or suppose, we have designed considering the static loading for all stability. We know the section of the wall so, m 2 can be calculated and that we can do the dynamic analysis and see how it performs in terms of its displacements etcetera, whether it is within permissible limit that can be checked. So, we do not have any doubt about m 2 what about the other parameters.

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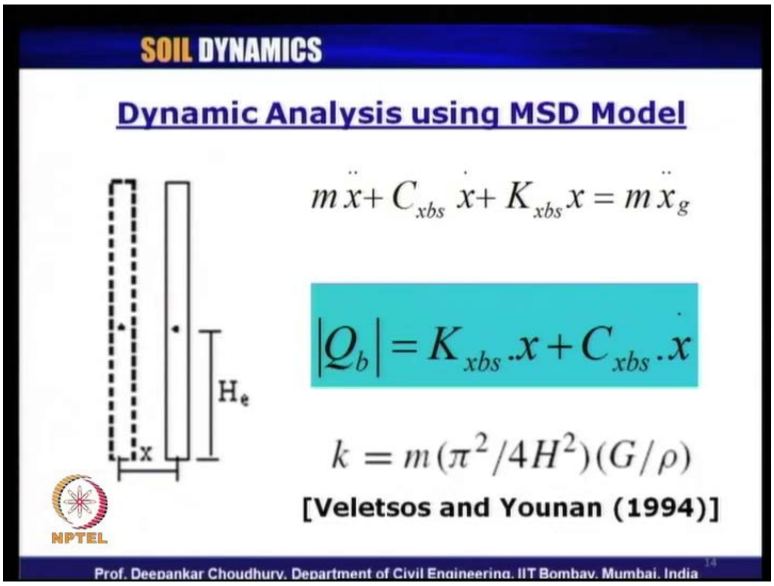

How to compute k 2 c 2 these parameters let, us go back to the expression of this k is given by this m times pi square by 4 h square times g by rho m of the corresponding material for which we are calculating suppose, k of wall we want to compute. So, here m of the wall has to be considered h is the height of the wall g is the shear modulus of the wall material. So, for concrete if you are using the concrete wall then concrete shear modulus has to be used and rho is the density of the wall material so, rho of wall material has to be used in the expression what about c.

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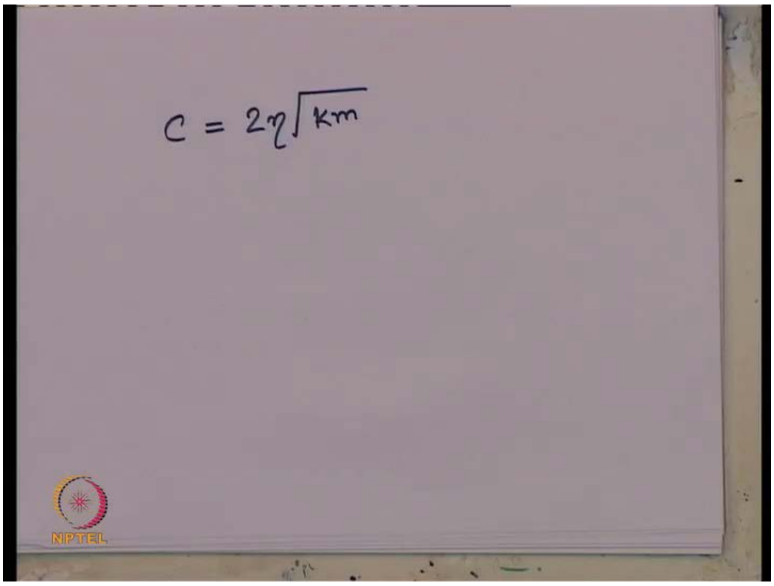

How to calculate the c damping this is 2 eta root over k m. Do you agree with me that this is the expression? we know the standard expression the damping constant can be estimated using damping ratio stiffness and mass. So, for the estimation of damping constant of concrete or wall material we should know the damping ratio of the concrete. So, damping ratio of the concrete is more or less known mostly it is about 5 percent. So, 5 percent if you consider the damping ratio k just now we have estimated m of the wall also, we can calculate. So, c of the wall will be known to us.

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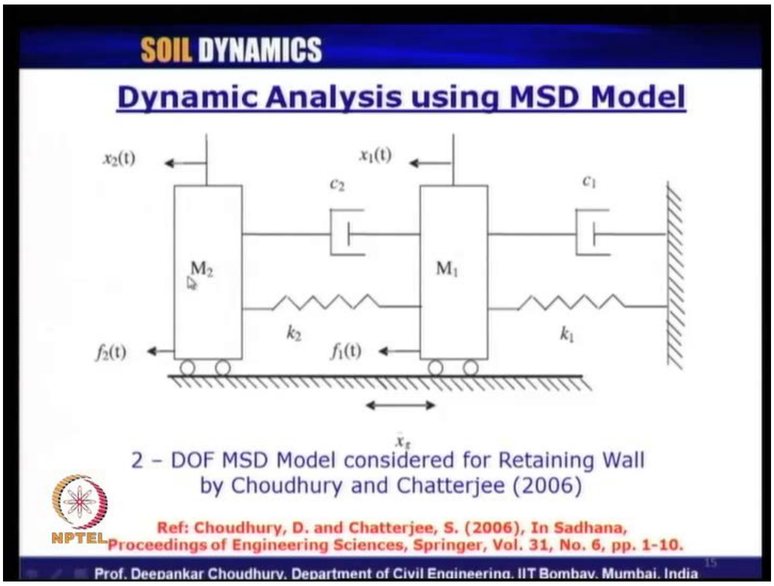

So, this way we can estimate m 2 c 2 k 2, which is fine now, the question comes what way we can calculate m 1 c 1 and k 1 expressions are available but, the first thing we should calculate is m 1.

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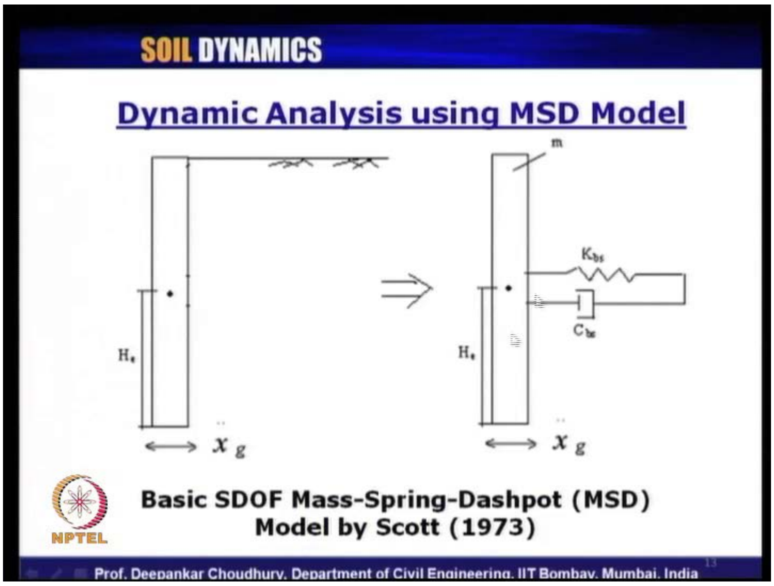

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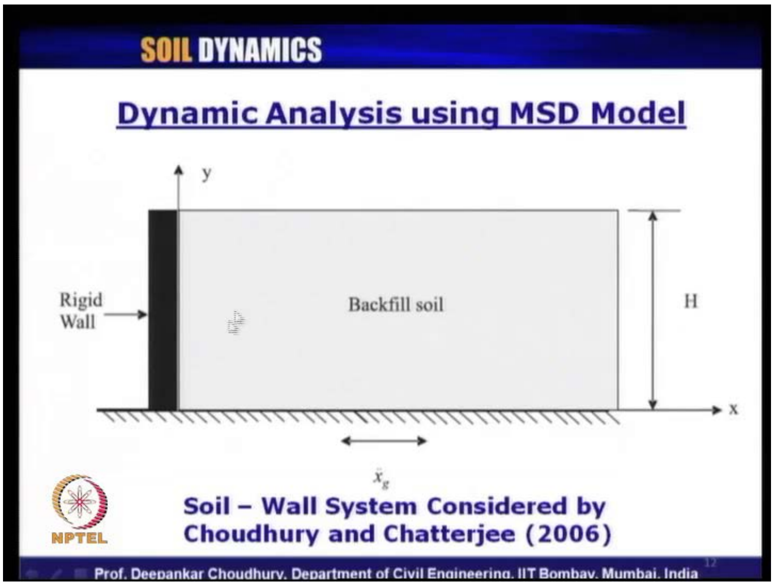

That is why Scott avoided probably the calculation of m 1 because this is not so clear. So, how to get the value of m 1 for that what is required we should know what is the influence zone. How we can do in this paper what we have done the analysis? We have considered different distances this horizontal distance. We have considered in terms of some factors of this height h suppose, 2 h 3 h 5 h up to 10 h and we have done a trial and error process of analysis by using different zone of extension and tried to obtain the final output of the displacement function. And when after a particular distance horizontal distance, we found that there is hardly any change significant change in the value of the displacement function. What we can say that beyond that it is hardly having any effect on the wall so, that is nothing but, the way to compute the zone of influence for the estimation of this soil mass clear so, if you go through this paper which I have already given the reference this is the paper you will get all the details and that way finally, we have found out for different conditions of course, the applied forces dynamic forces.

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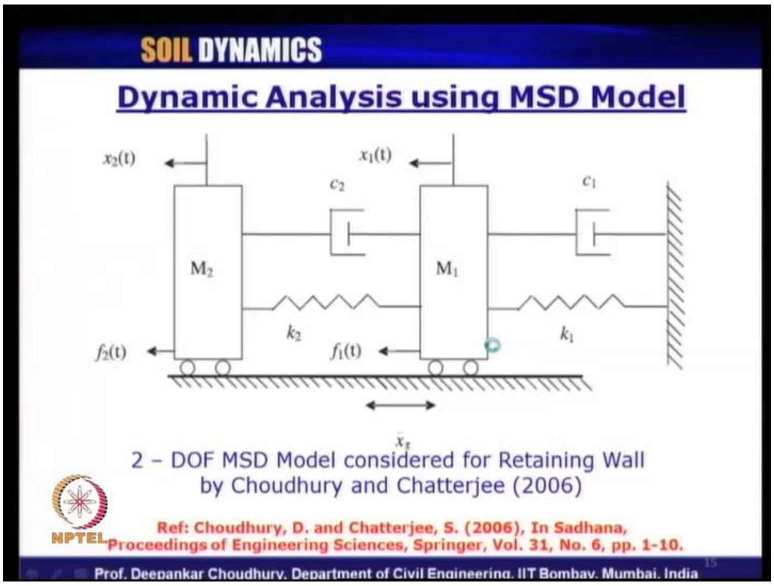

What is the different zone of influence and from that now, we are ready to calculate m 1 very easily.

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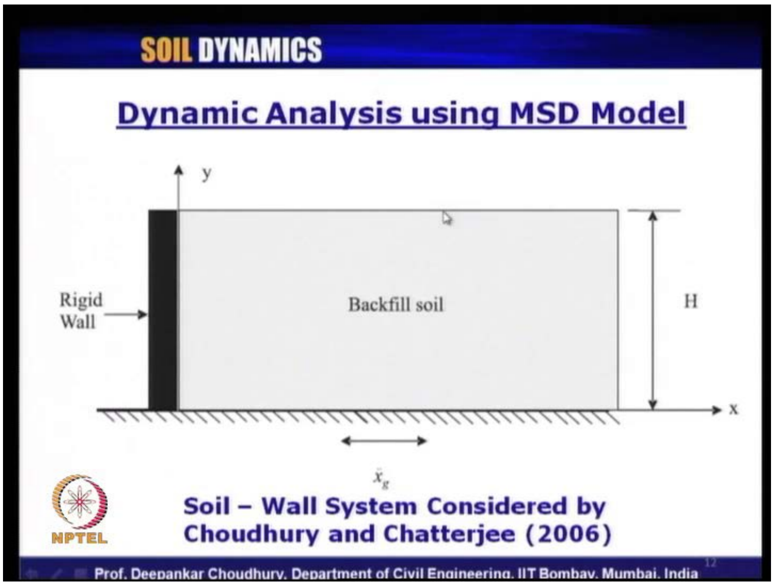

Because, it is a simple rectangular area, we have considered in this figure in our basic model height is known this influence zone is now known and other dimension is per meter always. We consider for retaining wall plane strain problem.

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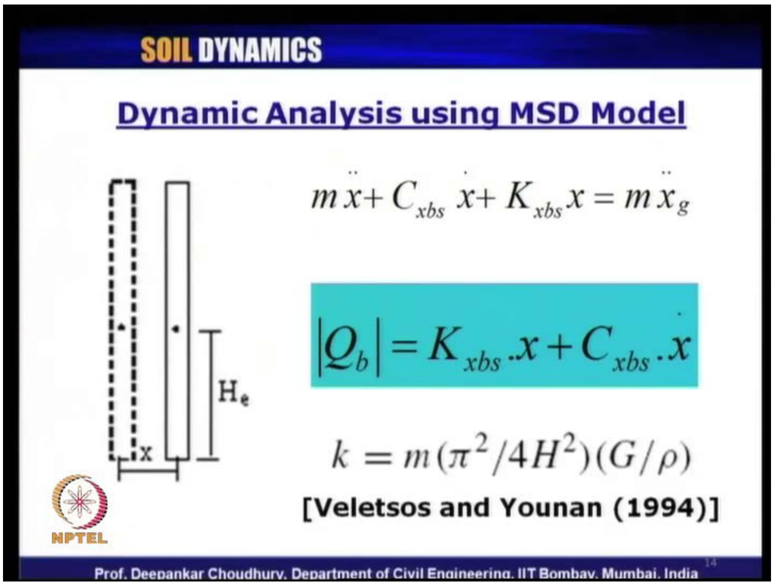

So, the total volume is known and soil density you know so, unit weight you know. So, you can find out easily the mass of the soil so, m 1 is known once m 1 is known now, we are ready to calculate k 1 also, that is spring constant for the soil using this expression so, here m 1 is now, we have calculated h is known g of soil has to be used here and rho of soil has to be used in this expression.

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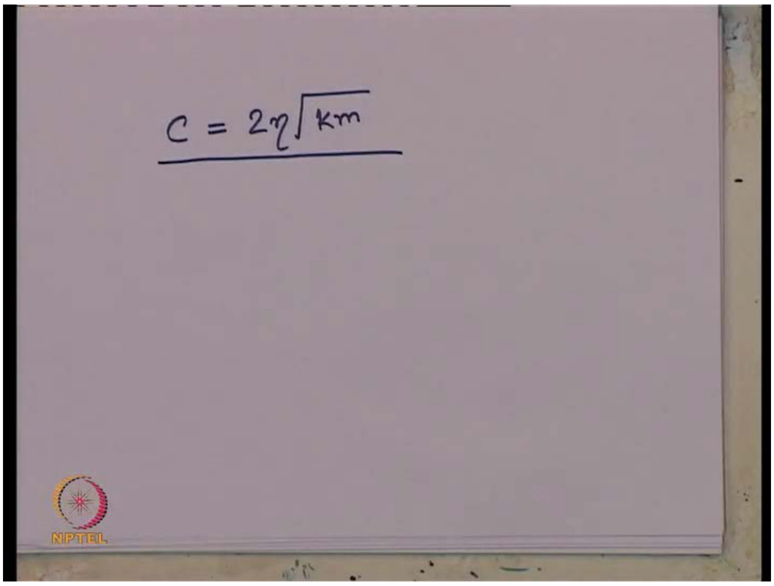

And then coming to the calculation of c 1 that is for soil damping constant again, we refer to this equation here eta damping ratio of the soil should be known to us that in our analysis. We have varied for different damping ratio you can easily get a design chart for different damping ratio or different types of soil. We have considered 5 percent 10 percent and 20 percent damping ratio, which are the practical ranges of different damping ratio for the soil material as we had discussed earlier also.

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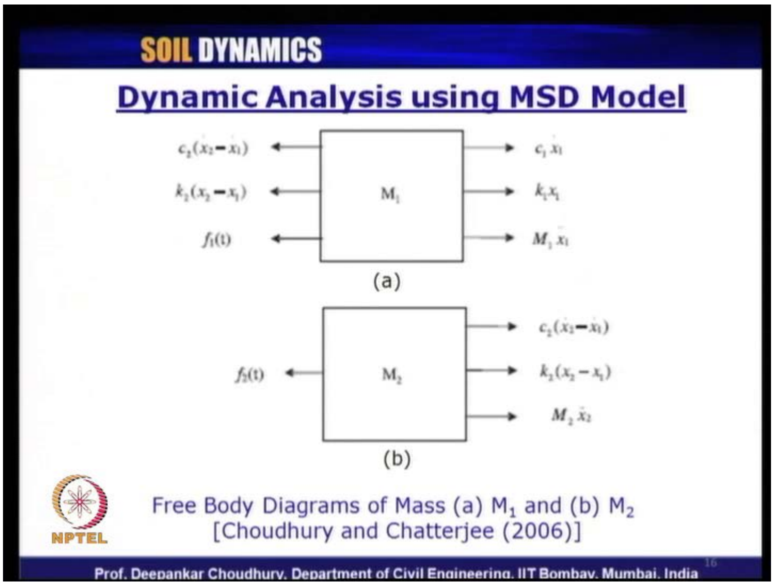

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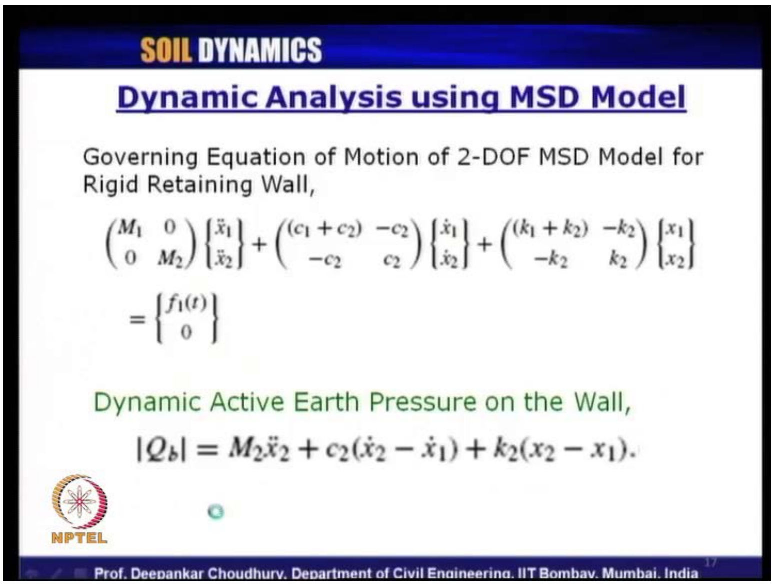

K already, we have computed k 1 and this will be m 1 that also, we have calculated. So, we can get c 1 very easily fine so, with this knowledge now next step after considering this free body diagram we can easily write down the by applying de alember's principle. What are the equations and those governing equation of motion for 2 degree of freedom model in matrix form. We can express in this format because it is a 2 degrees of freedom system.

So, in matrix form x 1 and x 2 we need to determine and that way we have and dynamic or active earth pressure on the wall has to be calculated based on q b will be remember q b is nothing but, on this whatever forces are coming these forces that is inertia force stiffness force and the damper force. Some of them will be nothing but, the force acting on the wall which is in horizontal direction nothing but, the dynamic earth pressure.

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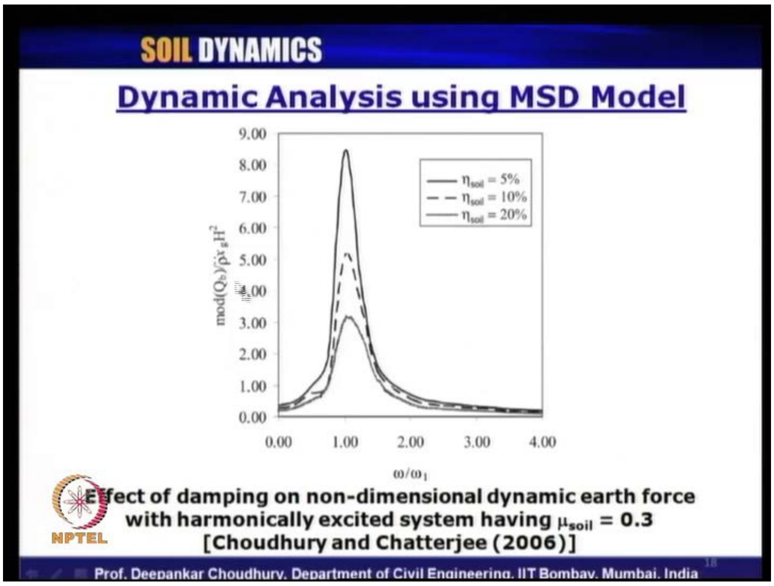

So, that way q b we had estimated and then the non dimensional dynamic earth pressure is plotted in this result non dimensional active earth pressure dynamic. Active earth pressure with respect to different frequency ratio different frequency ratio in this case which frequency ratio we have considered this omega is obviously, the exciting frequency or applied frequency different excitation you can consider always and omega 1 is the natural frequency of the soil wall system considering how many natural frequency we will get here. We have considered 2 degrees of freedom system so, obviously we will get 2 natural frequencies now, among them which natural frequency. We should use the fundamental natural frequency, what is fundamental natural frequency among the 2 natural frequencies? Whichever is the lowest 1 that is the fundamental natural frequency? So, by considering that fundamental natural frequency why it is required to consider.

It will give us the first mode shape. So, in that way the frequency ratio we had calculated here and how the earth pressure force dynamic earth pressure force is non dimensionalized. It is the value of q b mod of that divided by rho x double dot g x double dot g is the applied ground acceleration that is for a particular dynamic force or a earth quake force. Whatever acceleration we have considered and h square so, obviously this unit and this unit will be same. So, we can get a non dimensional value this is also non dimensional for different damping ratio of soil of 5 percent 10 percent 20 percent you can see the variation is something like this so, it is kind of a design chart because, once you know for your soil wall system what is the fundamental natural frequency and also the zone for which you are going to design or the dynamic load for which you are going to design your wall the exciting frequency will be known. So, this value will be known to you can go this chart knowing this damping ratio. You can get a value of this in this rho x g double dot and h square also will be known so, you can get the estimation of this q b.

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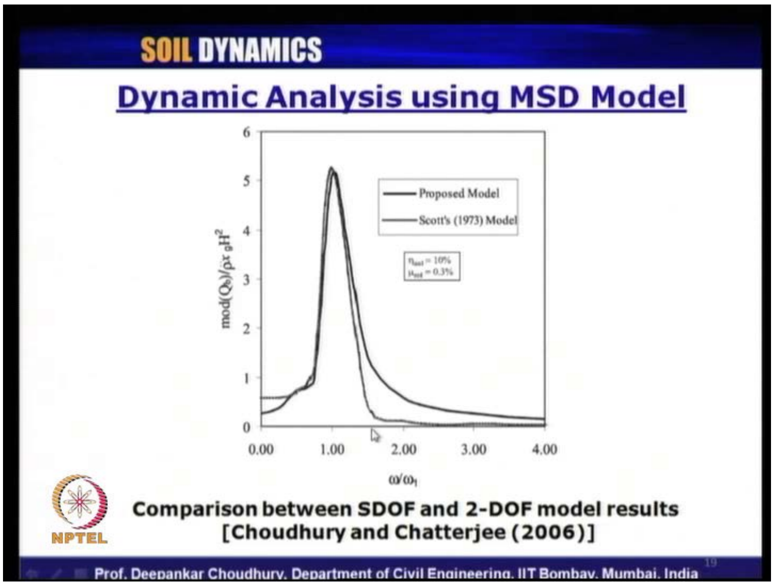

And also how to validate the model we have validated the our model with respect to basic single degree of freedom model or Scott's model you can see this is our proposed model considering 2 degrees of freedom model and this 1 is Scott's model by considering single degree of freedom model. You can see at higher frequency ratios Scott model or single degree of freedom model is giving a drastic change in the result. Whereas 2 degree of freedom model can replicate a smooth curve or smooth results which is expected actually the variation of the results of this dynamic earth pressure with respect to different frequency ratio otherwise, at the close to the resonance condition they are matching pretty well. However at higher frequency ratio this 2 degree of freedom model is capturing the better behavior or better result or better response of the wall than a single degree of freedom model. We will stop our lecture here today we will continue further in the next lecture.