Soil Dynamics
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Module - 5
Machine Foundations
Lecture - 32
Use of EHS Theory for Analysis

Let us start our today's lecture on soil dynamics, we were continuing with our module 5 that is, machine foundations.

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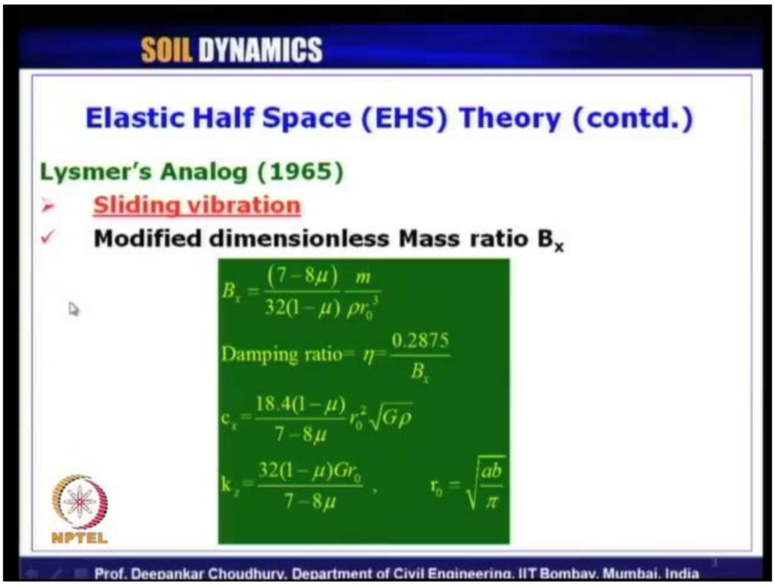

A quick recap of what we had studied in the previous lecture, we have seen the expressions for sliding mode of vibration using Lysmer's simplified analog in elastic half space theory.

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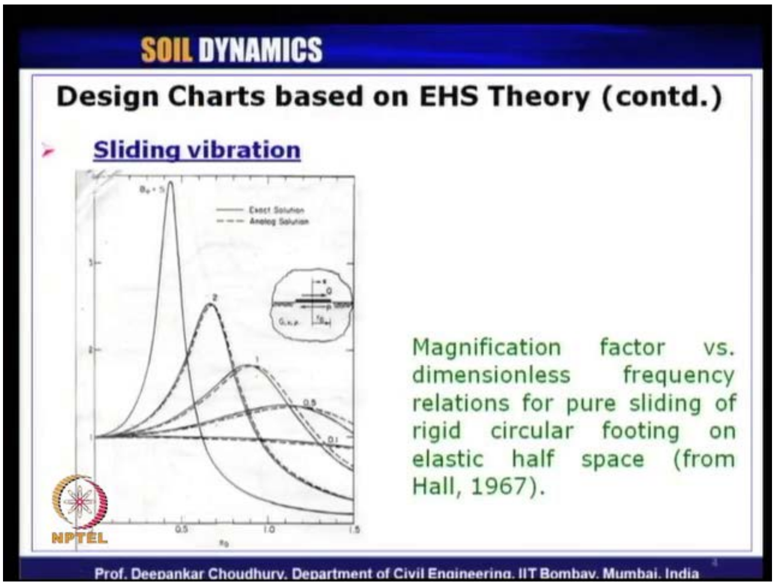

And the design charts corresponding to that.

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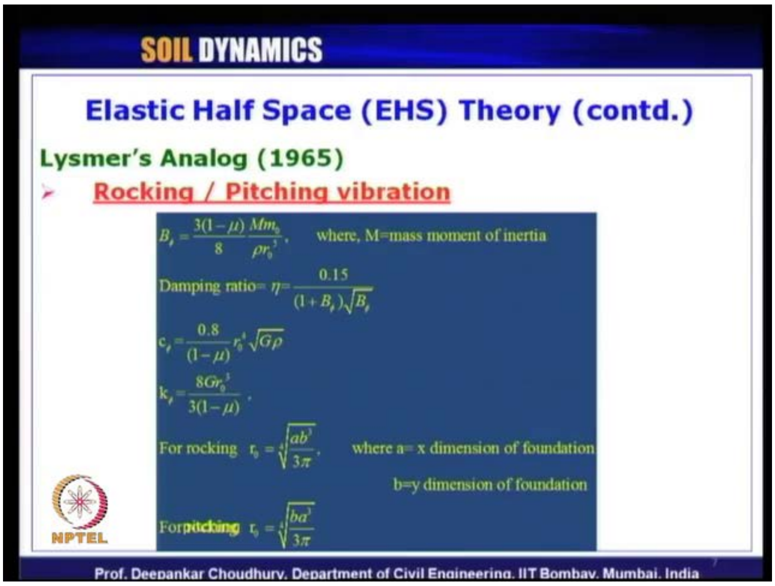

Also for rocking or pitching mode of vibration we have seen, what are the expressions for modified mass ratio damping ratio and so on.

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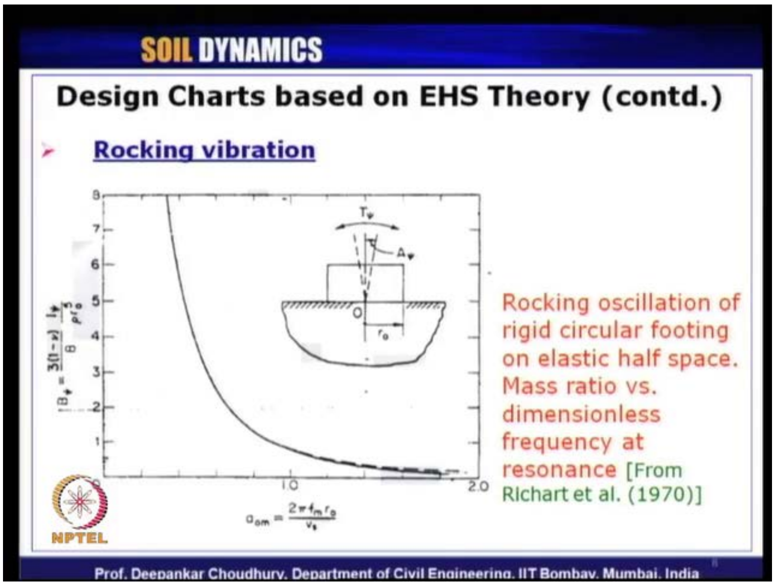

Then the design charts, to find out what is the frequency at resonance.

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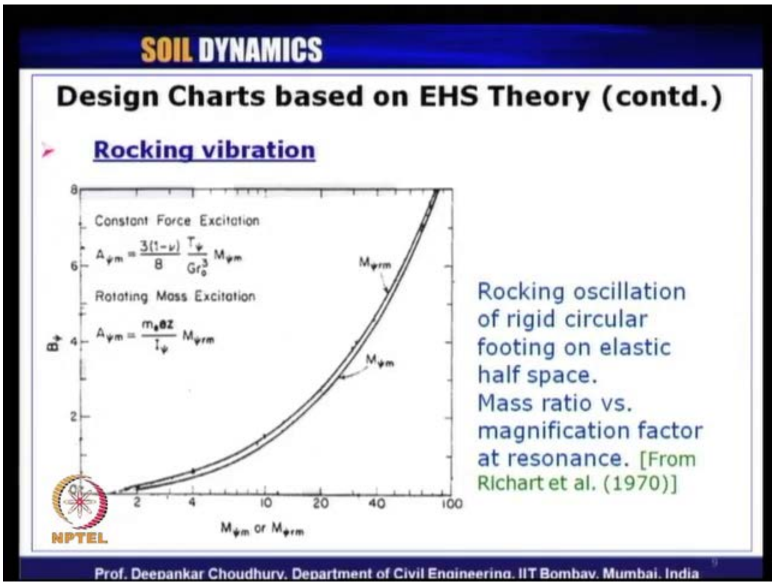

Also, to compute the displacement magnitude at resonance.

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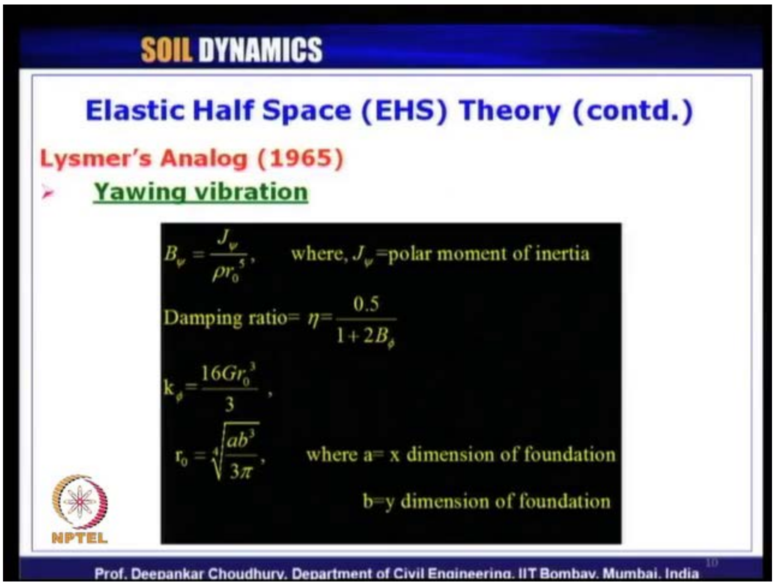

Then, for yawing or torsional mode of vibration using Lysmer's analog, the expression for modified mass ratio damping ratio and the stiffness was mentioned.

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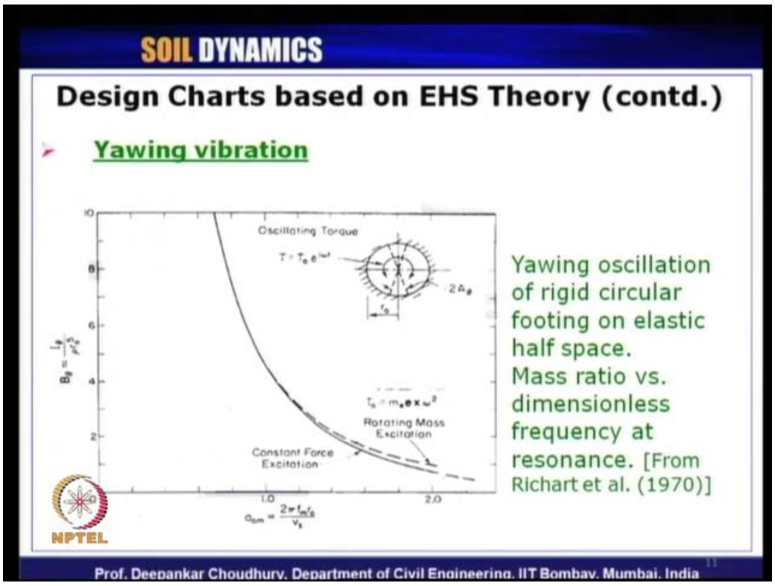

Then, the design charts we had discussed for different types of excitation, constant force type or rotating mass type, how to obtain the frequency at resonance and then the displacement amplitude at resonance.

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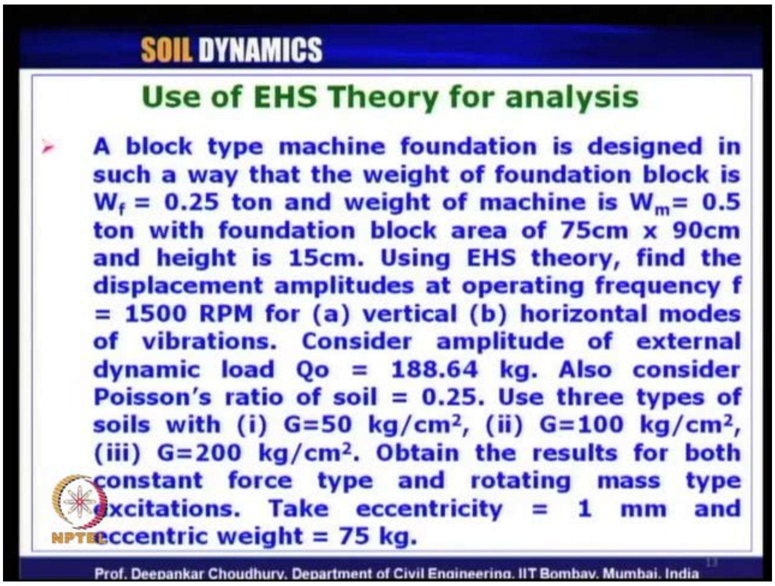

Then, we have solved one design problem, kind of a cross checking that is, one block type of machine foundation was designed with some dimension, known foundation weight and machine weight. Then using the elastic half space theory, it was asked to find out the displacement amplitudes at operating frequency for vertical and for horizontal modes of vibrations.

The dynamic load was given for three different types of soil with three different G values were given to us to compute the displacement, that operating frequency and also, for both constant force type and rotating mass type excitation.

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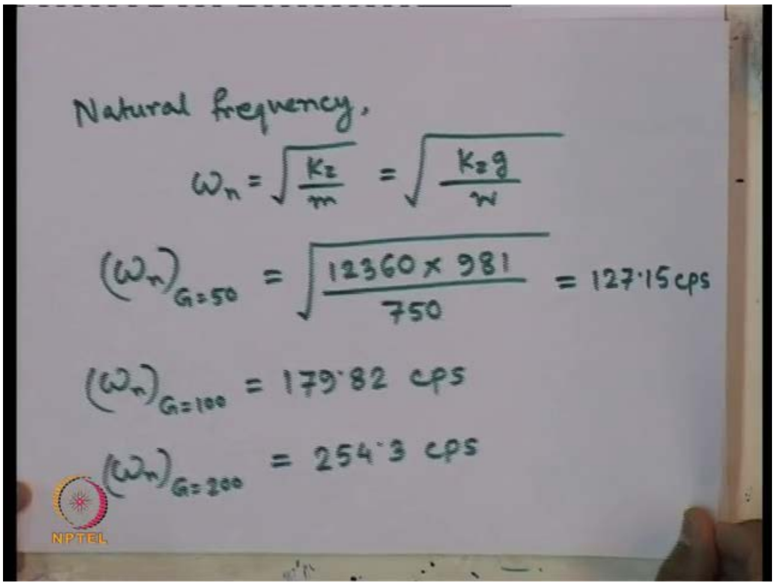

Now, we are ready to calculate the displacement amplitude for two different types of excitation so let us start with constant force type excitation.

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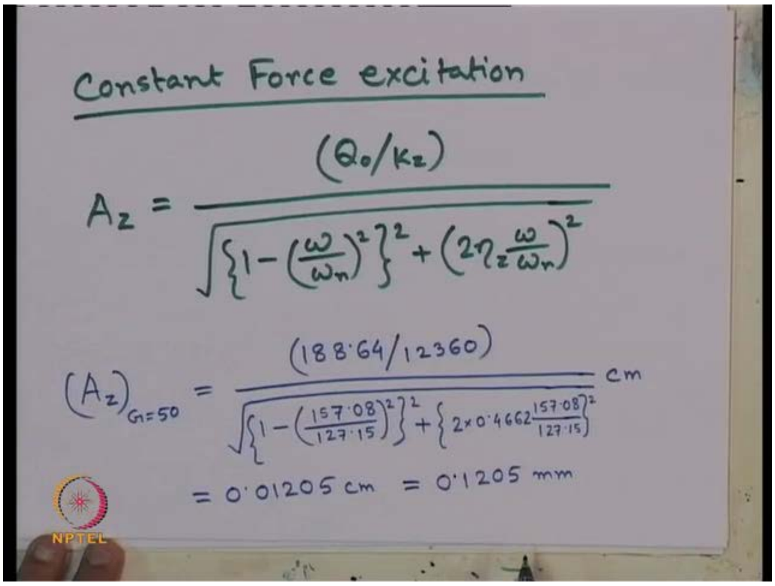

Constant force excitation with this, the expression as I said the basic equation of whether it is Reissner or Lysmer is nothing but similar to the equation of mass spring dashpot model. So, the expression for A z also will be obviously same because it is the solution coming from that equation. So, A z can be expressed like this, Q naught by K z by root over 1 minus omega by omega n whole square, that entire thing square plus 2 eta z omega by omega n whole square, that is the expression known solution to us. So here, we have calculated all these values based on the Lysmer's analog.

So, let us put those values here and get the value of A z for three different types of soils so for A z with G equals to 50, will be Q naught is given to us that is, amplitude of dynamic load 188.64 kg force, K z we have calculated for G equals to 50 as 12360. So, whatever unit we will get, we will be guided by this unit right, this is in kg per centimeter, this is kg.

So, we will get finally, this as centimeter because other things are just ratio root over 1 minus one, operating frequency is 157.08 and omega n is 127.15 for G equals to 50 plus 2 into how much is eta z we have calculated for G equals to 50,0.4662 into this one, 57.08 by 127.15 this square. Let us calculate this how much it is giving us, it is coming0.01205 so much of centimeter am I right, so in millimeter unit it is 0.1205 millimeter. Why I am converting it to millimeter because we know, our final check is it should be less than 0.2 millimeter. So, that is the check you can make here similarly, for other two types of soil also, we can calculate what is the value of A z.

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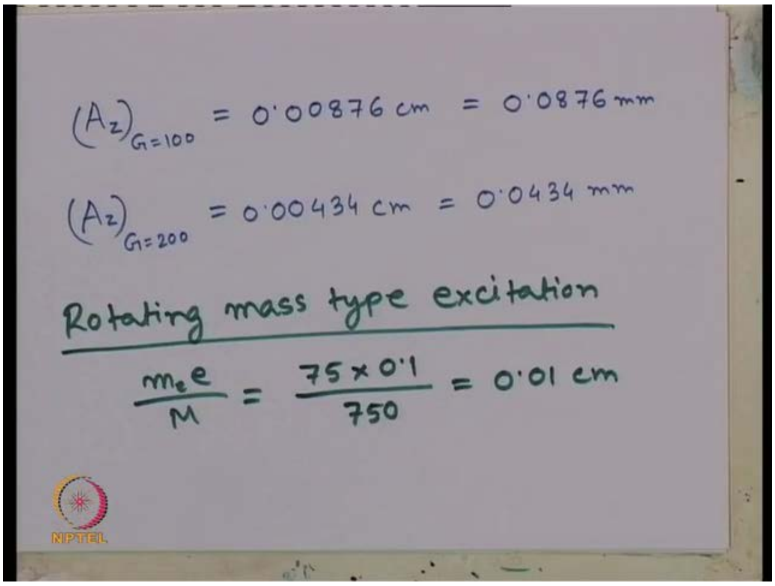

So, A z for G equals to 100, what are the changes will be there, Q naught remains same, K z change it will become double, omega remains same, omega n changes, eta z changes and then we will get the value of A z at G equals to 100. So, how much it is coming, 0.00876 centimeter which is nothing but 0.0876 millimeter, which is again much lower than the permissible limit. And A z with G equals to 200, 0.00434 centimeter so 0.0434 millimeter, all are within permissible limit. Now, let us do for rotating mass type excitation, rotating mass type excitation in that, we need to find out m e e by M, this ratio. What is eccentric mass 75 kg it is given, 0.1 centimeter is the eccentricity that is, 1 millimeter and 750 is the total mass so it is 0.01 centimeter.

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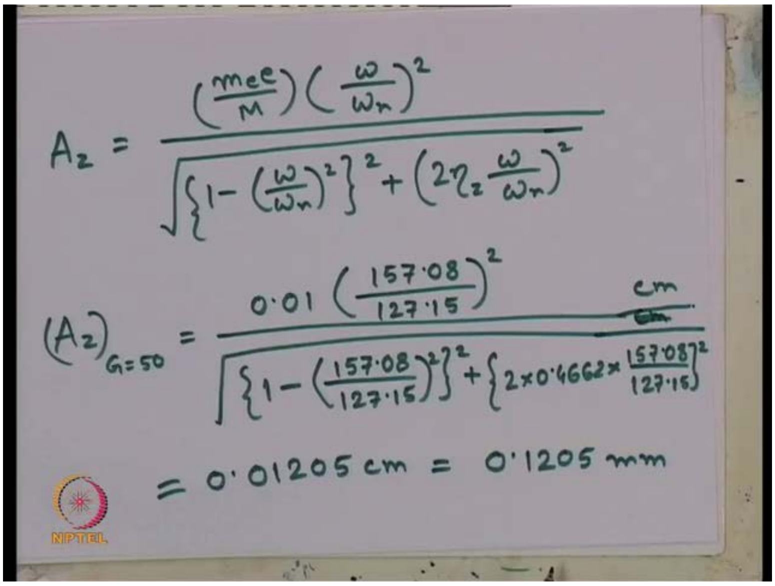

And expression for A z in this case was, m e e by M times omega by omega n whole square by root over 1 minus omega by omega n whole square, that square plus 2 eta z omega by omega n whole square. So, for first type of soil A z with G equals to 50 should be, this terms remain same 0.01 centimeter so it will be unit will be centimeter all others are ratios, this is 157.08 by 127.15 is for the first type of soil. The natural frequency we have calculated already and here, 1 minus 157.08 by 127.15 whole square plus 2 into 0.4662 into 157.08 by 127.15 whole square.

So, if we simplify this how much it is coming, 0.01205 centimeter which is nothing but 0.1205 millimeter, there is no need to calculate this one actually, why. Do you remember, earlier also I have said, if the load in constant force type and rotating mass type, they are same then at the same operating frequency, the displacement amplitude will also be same. You can do a cross check actually, you can take any problem, take the load same for both constant force type and rotating mass type. You will see that, at operating frequency, their displacement amplitude is exactly same.

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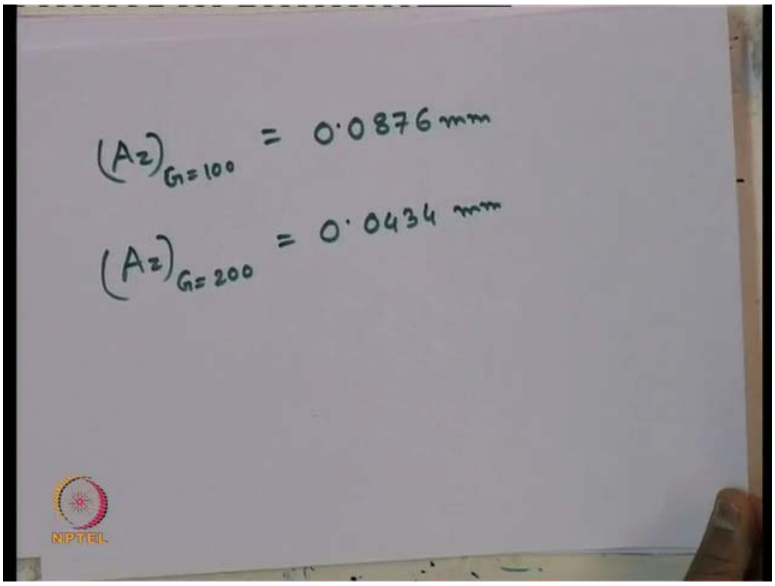

So, for other two cases also, we need not to calculate we can directly say, A z for G equals to 100 will be 0.0876 millimeter and A z for G equals to 200 will be 0.0434 millimeter. So, that completes our calculation of this displacement amplitude at operating frequency for vertical mode of vibration similarly, let us do this for next part that is, for horizontal mode of vibration.

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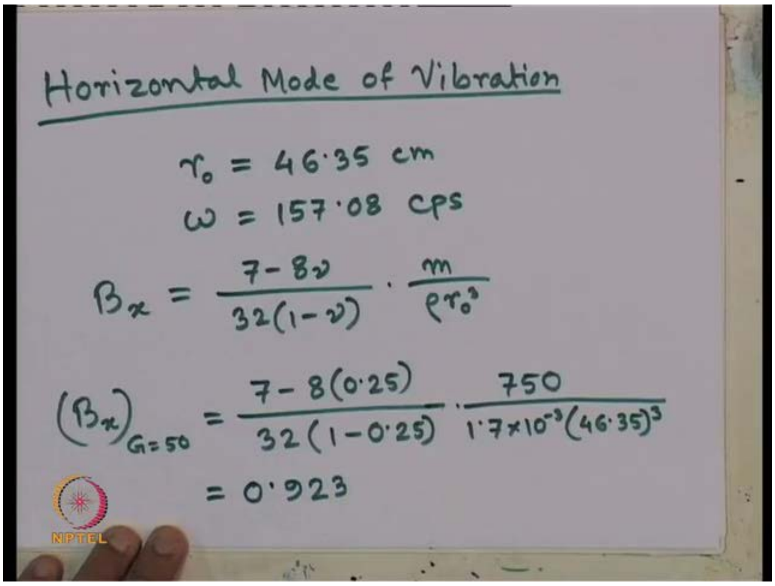

So, for horizontal mode of vibration, let us calculate all the required parameters using Lysmer's simplified analog. Equivalent circular radius r naught will be same like vertical one right, because the expression is same for translation of any cases 46.35 centimeter. Omega also we have already calculated 157.0 8 corresponding to that 1500 rpm, if you convert it to cps, it will give this. Now, the mass ratio modified mass ratio capital B x, what is the expression, 7 minus 8 mu by 32 times 1 minus mu into m by rho r naught cube.

So, for first type of soil B x with G equals to 50 kg per centimeter square, how much will be the value of B x, 7 minus 8 into mu is 0.25, Poission's ratio by 32 into 1 minus 0.25. And m by rho we can write as w by gamma no problem, w is total 750 kg force and gamma is 1.7 into 10 power minus 3 kg force centimeter cube and this is 46.35 centimeter cube. So, it will finally, give us the non dimensional mass ratio value, how much it is coming let us calculate, it is coming 0.923 am I right.

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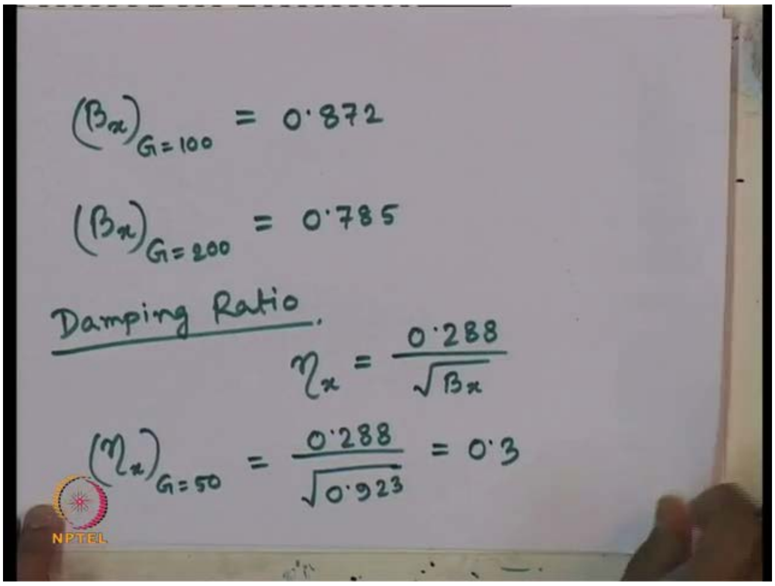

Next, for other two values of G that is, other two types of soil, B x will be, what are the changes let us see here in this expression, this Poisson's ratio we are considering same for all soils. So, this remains same, this remains same 750, only change will be here it will be 1.8 and this also remain same so only change is in the value of gamma. So, it is coming as 0.872 and B x for G equals to 200 how much, only gamma we will be changing 0.785.

Now damping ratio, next parameter we are going to calculate damping ratio, what is the expression, eta x for horizontal mode of vibration 0.288 by root over B x right. So, eta x for G equals to 50 first type of soil 0.288, what is the B x for first type of soil 0.923 right we have calculated just now, it is giving us how much is the value 0.3 fine. Eta x for G equals to 100 that is, second type of soil, how much it is coming 0.308.

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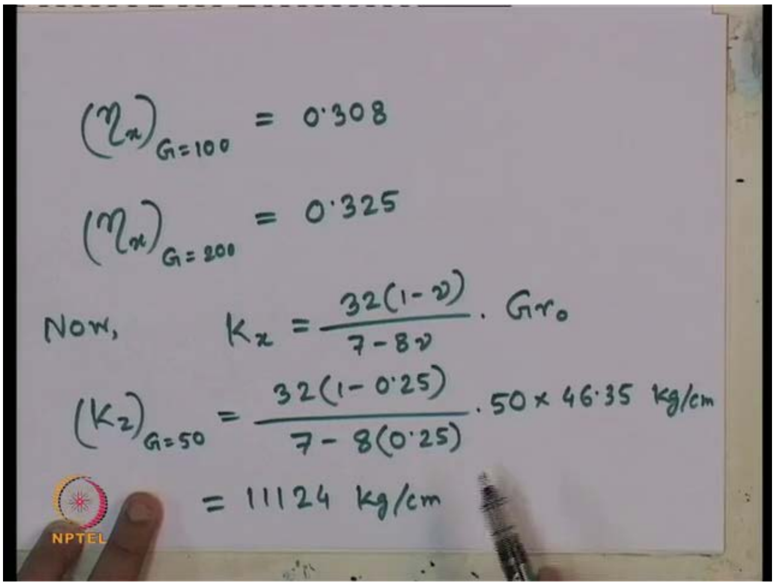

And similarly, eta x for G equals to 200 is coming as 0.325 and now, spring constant we need to calculate. Spring constant what is the expression, K x is 32, 1 minus mu by 7 minus 8 mu times G r naught. So, for first type of soil with k z for G equals to 50 kg per centimeter square will be 32, 1 minus 0.25 is Poisson's ratio, 7 minus 8 times 0.25 times G is 50, r naught is 46.35 so much of kg per centimeter, that is the unit. If we calculate it, how much we are getting, 11124 kg per centimeter.

So, similarly for other two soil G equals to 100 and G equals to 200, what are the changes, Poisson's ratio remains same. So, this remain same, this also remain same, only this changes and linearly related so for 100 it will be double, 200 will be double of that.

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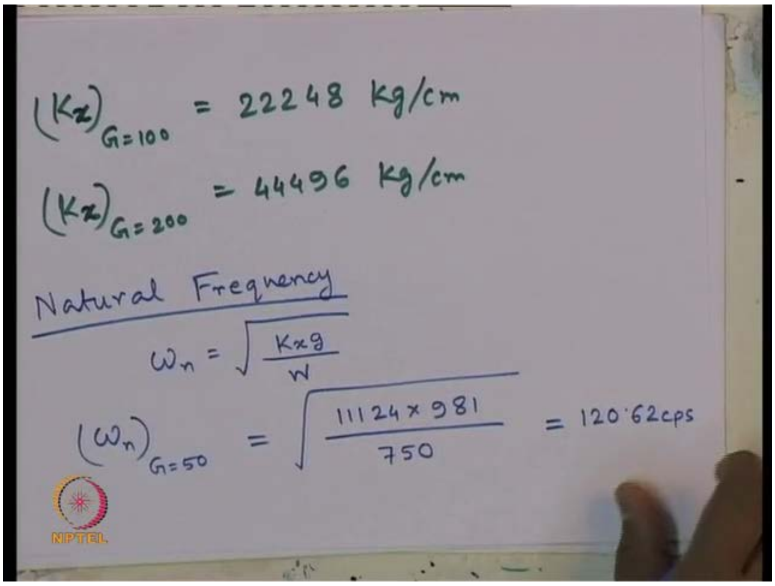

So, k z with G equals to 100 is 22248 kg per centimeter and k z with G equals to 200 will be double of this, so 44496 kg per centimeter is it. Now, what is the next we need to calculate, natural frequency of the system, once we get the k that is spring constant, we are now ready to calculate natural frequency. So, the natural frequency can be calculated omega n is nothing but root over k x I am sorry this will be k x, all will be k x because we are talking about horizontal vibration, this also k x not z, k x so k x g by W.

So, omega n for G equals to 50 will be how much, k x for the first type of soil is 11124, g is 981 centimeter per second square acceleration due to gravity and W is 750. How much it is giving, 120.62 cps similarly, for other two types of soil, only the k will change other two remains same.

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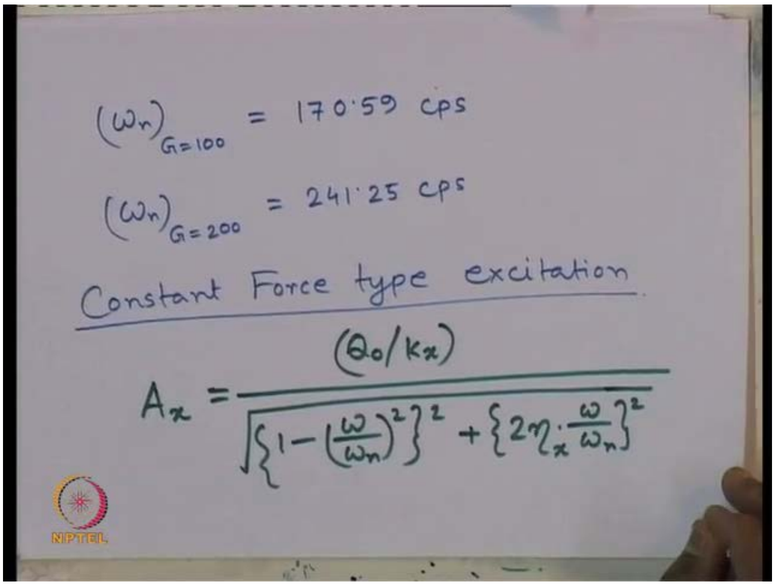

So, omega n for G equals to 100, we are getting 170.59 cps and omega n for G equals to 200 it is coming how much, 241.25 cps. Now, for constant force type excitation, what is the expression for A z when we calculate A x here, it will be expression for A x, will be Q naught by k x by root over 1 minus omega by omega n whole square, that square plus 2 eta x omega by omega n whole square fine. So, with this expression now, let us put the values and get for three types of soil, what are the displacement amplitude?

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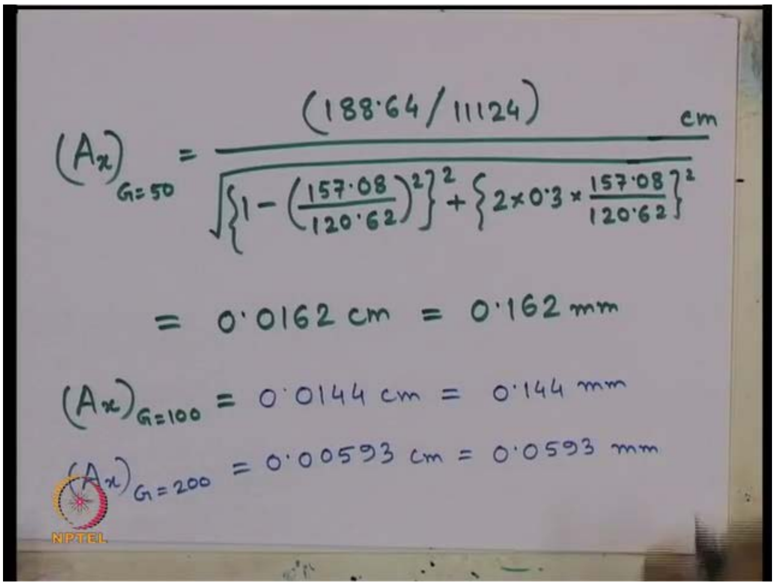

So, A x for G equals to 50, Q naught is 188.64 kg force, k x we have calculated as 11124 kg force per centimeter. So, it will give a centimeter unit, root of 1 minus 157.08 is the operating frequency and omega n for first type of soil we have calculated 120.62 plus 2 into eta x, we have calculated how much 0.3 for first type of soil then 157.08 by 120.62 so much of centimeter. Let us calculate this, it is coming about 0.0162 centimeter right, which is 0.162 millimeter, again you can cross check it with permissible limit of 0.2 millimeter.

Similar way, we can calculate A x for G equals to 100 in that case, changes are for this k x will change and this omega n will change and this eta x will change fine. So, how much it is coming, 0.0144 centimeter which is 0.144 millimeter. And A x for third type of soil G equals to 200, this k x will change, omega n will change, eta x will change, is coming 0.00593 centimeter so 0.0593 millimeter and this is for constant force type.

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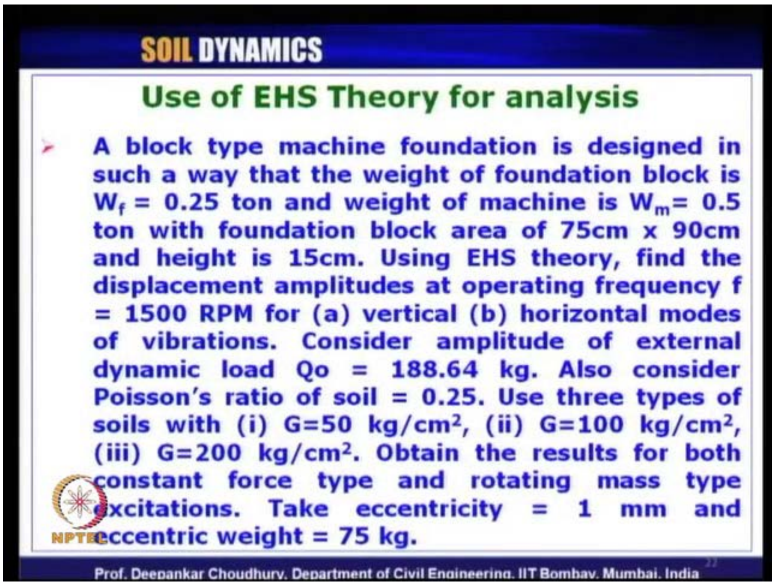

If we do this for rotating mass type excitation…

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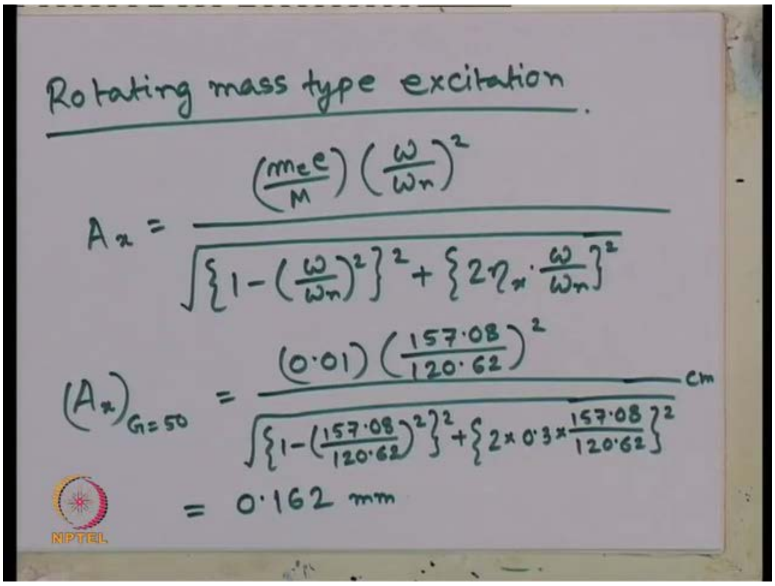

So, for rotating mass type excitation, we have this expression for A x as, m e e by M omega by omega n whole square root over 1 minus omega by omega n whole square, whole that square plus 2 eta x omega by omega n whole square. So, for A x with G equals to 50, it should be already we have calculated this value, 0.01 centimeter, this is 157.08 by 120.62 whole square root over 1 minus 157.08 by 120.62 whole square, that square plus 2 eta x is 0.3 and 157.08 by 120.62 that whole square so much of centimeter.

And as I have already mentioned, it should give the same value as the constant force type because loading is same and operating frequency also same. So, it will be 0.162 millimeter similarly, for the other two cases also, the values of A x will be like.

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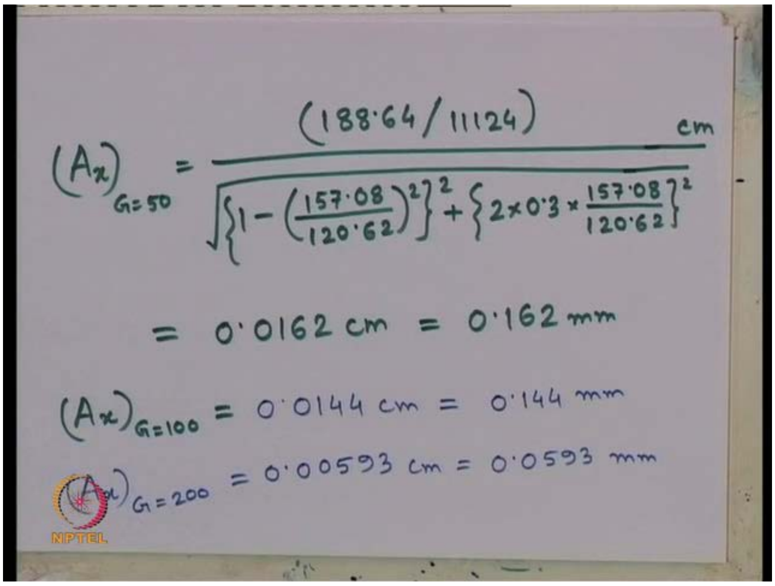

If we look back here, the value of A x for G equals to 100 will be 0.144 millimeter and A x for G equals to 200 will be 0.0593 millimeter so that completes our problem this one.

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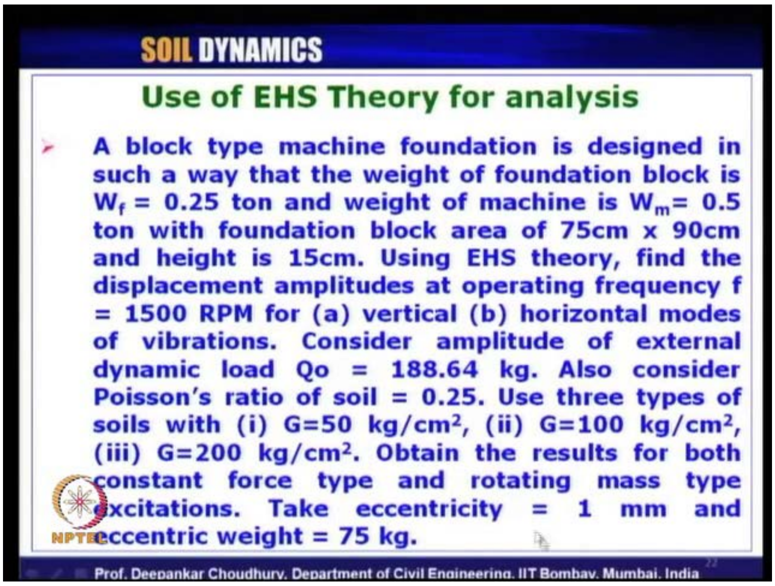

That we are able to calculate the displacement amplitude at operating frequency for both vertical and horizontal mode of vibration for different types of excitation, constant force type and rotating mass type, using elastic half space modeling.

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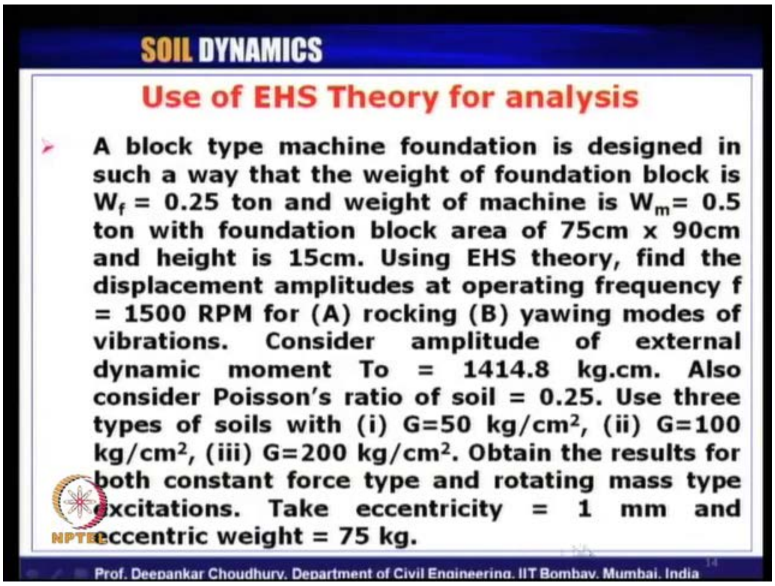

Now, let us continue further in today's lecture that is, block type machine foundation is designed in such a way that, the weight of foundation block is 0.25 ton and weight of machine is 0.5 ton with foundation block area of 75 centimeter by 90 centimeter with a height of 15 centimeter. The same problem actually we are considering, what is asked, using elastic half space theory, find the displacement amplitudes at operating frequency of 1500 rpm for so this is the only change for rocking mode and for torsional or yawing mode of vibrations.

And in this case, as you know the applied externally dynamic load should be either moment or torque. So, that amplitude of external dynamic moment is given to us as 1418.8 kg force centimeter. Remaining all other things like previous problem that is, Poisson's ratio is considered for all types of soil as 0.25 and the three types of soil, for which we need to compute this displacement amplitude or with G value. Shear modulus as 50 kg force per centimeter square, another is 100 kg per centimeter square, another is 200 kg per centimeter square.

And obtain the results for both constant force type as well as rotating mass type excitation and for rotating mass type excitation, the eccentricity is 1 millimeter and the eccentric weight is given as 75 kg. So, with this problem statement, let us see how to solve the problem for rocking and yawing modes of vibration using elastic half space theory at operating frequency.

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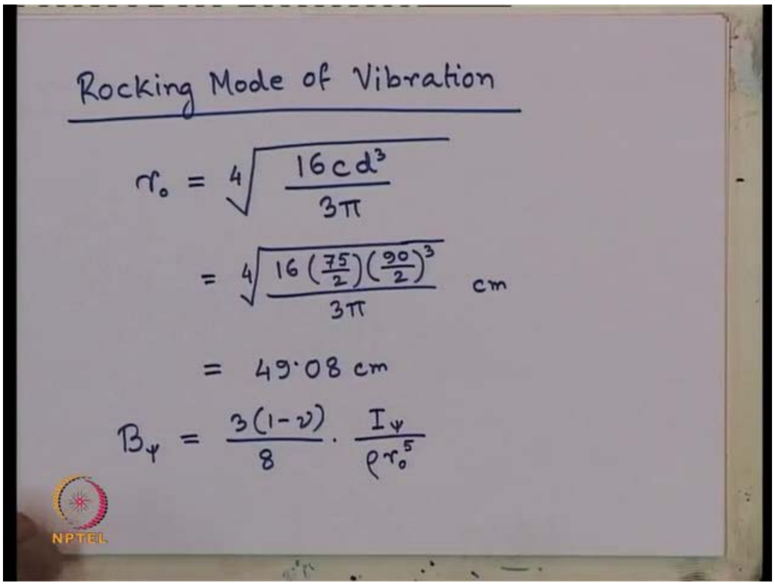

So, let us first start with rocking mode of vibration, now the first type for rocking mode of vibration is, to compute the radius of equivalent circular area and the expression for r naught. For rocking mode, we know that expression, we are already discussed in previous lecture, fourth root of 16 c d cube by 3 pi am I right where, c is half of the width and d is half of the length. So, fourth root of 16, how much is c, 75 by 2 am I right and d is 90 by 2 divided by 3 pi so much of centimeter.

If we calculate this, how much we are getting the equivalent radius for rocking mode, it is coming 49.08 centimeter. Now, next step is to obtain the dimensionless mass ratio that is, for rocking mode B psi, what is the expression for B psi, 3 into 1 minus mu by 8 I psi by rho r naught to the power 5 am I right, this is the expression for rocking mode of vibration now, what is the expression for I psi.

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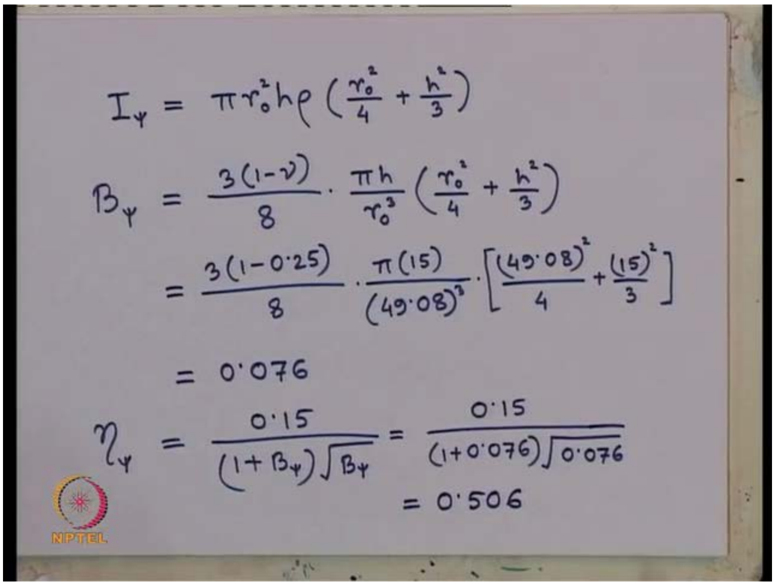

I psi is nothing but mass moment of inertia that is, pi r naught square h, r naught is equivalent radius of the circular formation, h is the thickness or depth of the foundation or footing, times rho times r naught square by 4 plus h square by 3. So, this is the expression to compute the mass moment of inertia about it is axis of rotation for rocking mode of vibration. So, the expression for B psi can be written like this, 3 into 1 minus mu by 8 into pi h by r naught cube, r naught square by 4 plus h square by 3.

Here, I have made 1 assumption, this rho will be rho for foundation and whereas, in the other case of B psi, the expression the rho is for rho of soil. So, I have considered both are almost of the same order and equal so I canceled from the expression to simplify the expression for B psi. So, remember this, otherwise also you can take the exact value, there is no harm, truly speaking.

Now, if you look at the expression for B psi carefully, it is a function of Poisson's ratio of the soil, which we are considering 0.25 for all the three types of soil, h is the depth of the foundation, which is constant given to us 15 centimeter, r naught already we have computed. So, it is not a function of different G values, so B psi unlike the case of vertical and horizontal mode of vibration, in case of rocking mode of vibration by simplifying in this fashion, we can get a constant B psi.

But, if you plan to use different rho value for corresponding to different g then you will get marginally different B psi value. So, 3 into 1 minus 0.25 by 8 pi into 15, this is 49.08

whole cube then this is 49.08 whole square by 4 plus 15 square by 3. How much is the value we are getting, 0.076 am I right, what is the unit, no unit, this is mass ratio so obviously, it will not have any unit. So, you can cross check actually centimeter, centimeter square, centimeter cube.

Now, next step is to obtain the damping ratio, what is the expression for damping ratio, for this rocking mode of vibration, it is given by 0.15 by 1 plus B psi times root over B psi, that is the expression. So, now, let us put the value of B psi, which we have computed just now 1 plus 0.076 times root over 0.076. How much it is coming, if we calculate it 0.506 right, that is the damping ratio.

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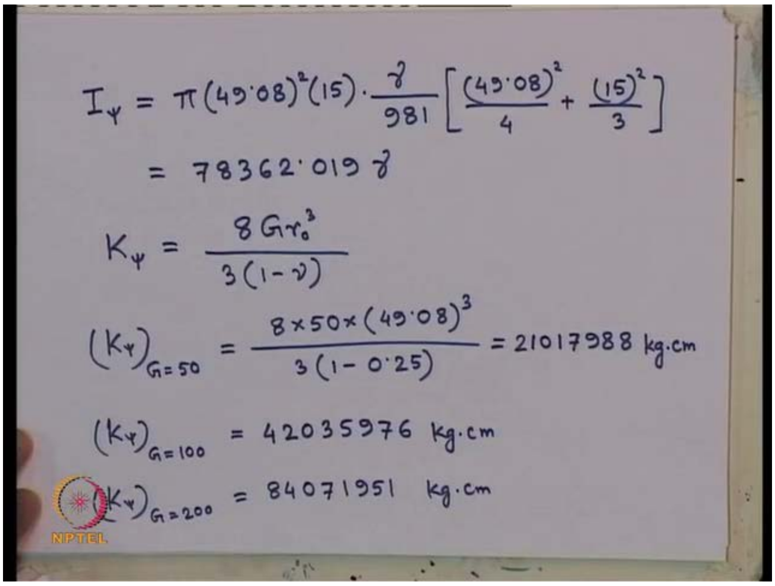

Now, next let us calculate I psi also because that will be required later on that is, mass moment of inertia. I have already given the expression for I psi, which is pi 49.08 whole square times 15 times gamma by 981 that is, rho 49.08 whole square by 4 plus 15 whole square by 3, it is coming as 78362.019 of gamma. I am keeping it in terms of gamma so that, for different values of gamma, I can get this I psi and what is the stiffness k psi, the expression for stiffness that is, 8 G r naught cube by 3 into 1 minus mu, this is the expression for k psi.

So, for first type of soil with G equals to 50 kg per centimeter square, this will be 8 into 50 into r naught is 49.08 whole cube 3 into 1 minus 0.25 so how much we are getting, 21017988, what is the unit, for this unit for this will be kg force centimeter, why. This is a rocking stiffness equivalent to the torsional stiffness, this is the not the linear spring rotational spring so do not forget the units, these are important also. You can cross check also, this G is in kg per centimeter square here, you have centimeter cube so you will get kg centimeter.

And similar way, we can get k psi for G equals to 100, will be just double of this because they are linearly related with G. Other values remain same, it will be 42035976 so much of kg force centimeter am I right and k psi for G equals to 200, that will be again double of this so 84071951 so much kg force centimeter is that. Now, next step is to calculate natural frequency of the system, that is what we did for the previous problem also right. So, let us take the expression for natural frequency for this rocking mode of vibration.

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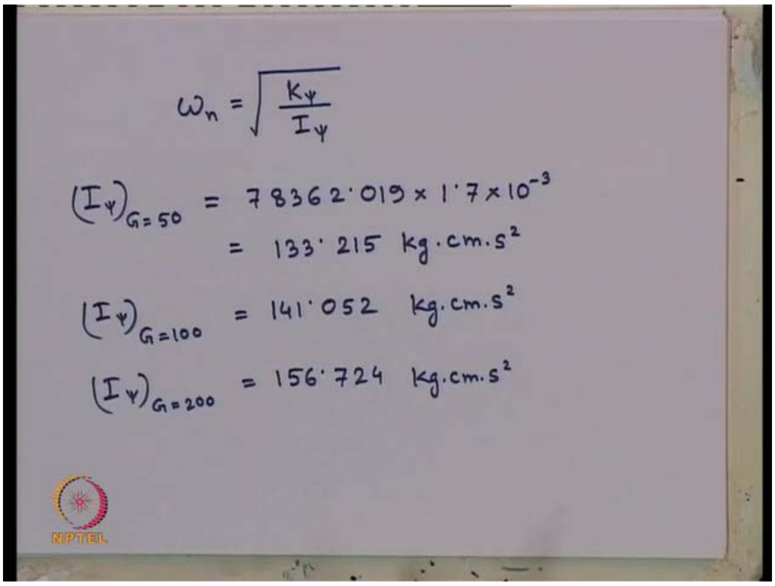

The expression is omega n is root over k psi by I psi where, I is mass moment of inertia so in translational case, it is root over k by m, whereas here it is torsional spring or rotational spring by that mass moment of inertia. So now, we can write down I psi for G equals to 50, the first case using the assumed gamma value, so 78362.019 into how much we considered, 1.7 into 10 power minus 3. So, it will give us 133.215 what should be the unit of I psi, kg centimeter seconds square, where from this seconds square appeared, in the unit of rho we have make it gamma by G, from G it has come.

I psi G equals to 100 will give us, this into 1.8 into 10 power minus 3 so 141.052, these are k g force actually kg centimeter second square and I psi for G equals to 200 will be, this into 2 into 10 to the power minus 3 so we will get 156.724 kg centimeter second square. Now, if you want to cross check the unit of omega n, omega n should be cycles per second or radian per second, that should be the unit.

Now, you put here, kg centimeter is the unit of k psi and I psi, you are getting unit of kg centimeter seconds square so kg centimeter gets cancelled, 1 by second square root of that 1 by second which is nothing but you are getting actually radian per second or cycles per second. So, it is tallying so nowhere we have make mistake in the dimensionless analysis dimensional analysis clear. So, this way it is always better, at least for this torsional and rocking mode of vibration, you cross check with the units also because there are possibilities of getting mixed with the units.

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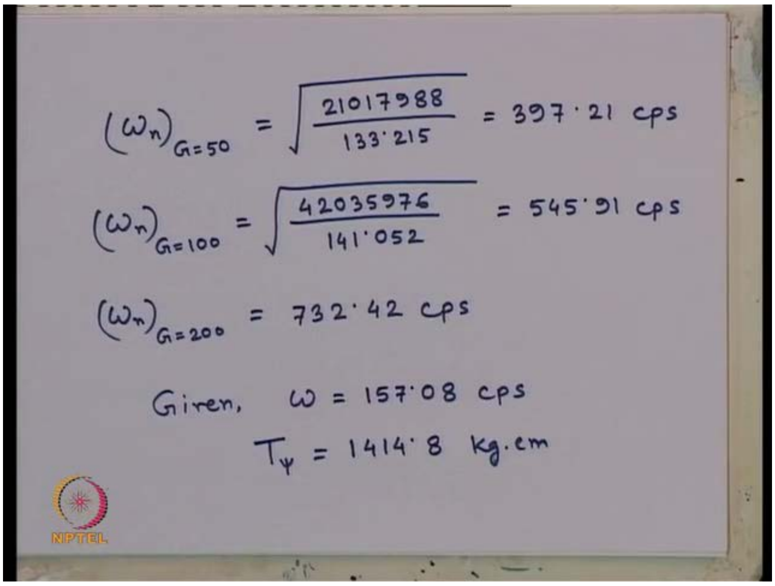

So now, we can get omega n for three different types of soil, for the first type of soil with G equals to 50, it will be k value is 21017988 and I psi we have calculated 133.215. It is coming how much if we calculate, 397.21 so much of cycles per second, omega n for G equals to 100, the k value is double of this so we have calculated it as 42035976. And what is I psi, I psi we have calculated 141.052 so this will give us 545.91 cps and omega n for third type of soil with G equals to 200, you put corresponding values we are getting the value of omega n as 732.42 cps.

And given to us, given omega is how much, exciting frequency is given to us 1500 rpm, after converting that, we already got it in the previous lecture 157.08 cps right, that is the operating frequency. And another value is given T psi that is, amplitude of applied moment, which is 1414.8 kg force centimeter, so using these values now, we can calculate the displacement amplitude at operating frequency.

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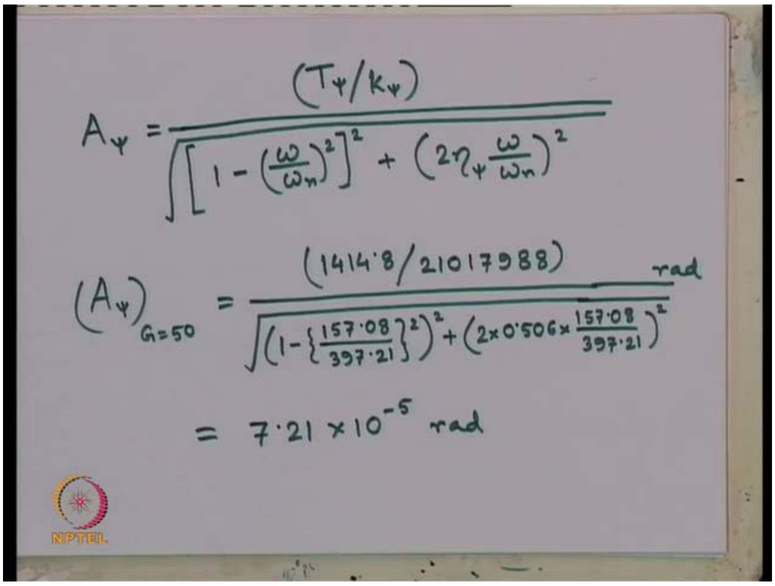

And what is the expression for that, A psi should be T psi by k psi, 1 minus omega by omega n whole square that square, this entire thing will be under root plus 2 eta psi omega by omega n whole square right, that is the expression. What should be the unit of a psi radian, it is the rotational displacement and look, if you have any doubt, the unit of T psi and k psi. T psi is in kg force centimeter, k psi is also in kg force centimeter so obviously, it will give the value only which is in radian.

So, let us calculate the A psi for first type of soil with G equals to 50 kg force per centimeter square 1414.8 by 21017988. And this value is 1 minus, omega is 157.08, omega n for first type of soil how much we have calculated, 397.21 whole square that square plus 2 into what is eta psi we calculated, 0.506 times 157.08 by 397.21 whole square, this much, radian is the unit. Let us calculate this and see, how much we are getting, it is coming 7.21 into 10 to the power minus 5 radian am I right.

Now, how to check this value with our permissible limit, this is the angular rotation going to occur when the rocking is going to takes place, we have seen the picture. So, we have to multiply this with respect to the length dimension, if you multiply with respect to it is height, you will get how much is the linear dimension is coming. So, that is the way, you will get the equivalent millimeter, how much is the displacement and that value should be below 0.2 millimeter clear.

So, that is the check, we need to carry out after obtaining this estimated value of displacement amplitude. Similarly, let us compute A psi for other two types of soil, value of T psi remains same, k psi changes, omega also remains same, omega n changes and eta psi also remains same in this case.

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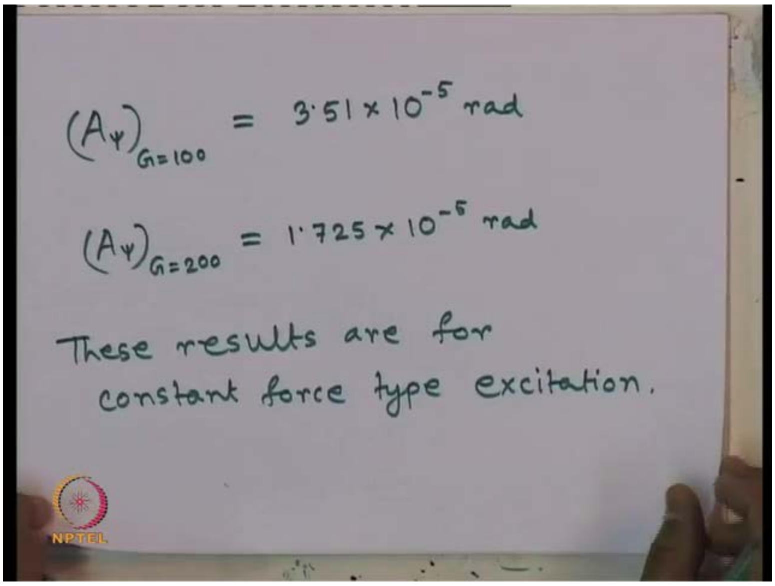

So, let us calculate the other two cases also, A psi for G equals to 100 kg per centimeter square, how much value we are getting, it is coming 3.51 into 10 to the power minus 5 radian. And for third type of soil, A psi with G equals to 200 kg per centimeter square, it is coming 1.725 into 10 power minus 5 radian so that is for constant force type excitation, these results are for constant force type excitation. Now, we need to check for rotating mass type excitation, let us see what is the expression for rotating mass type excitation.

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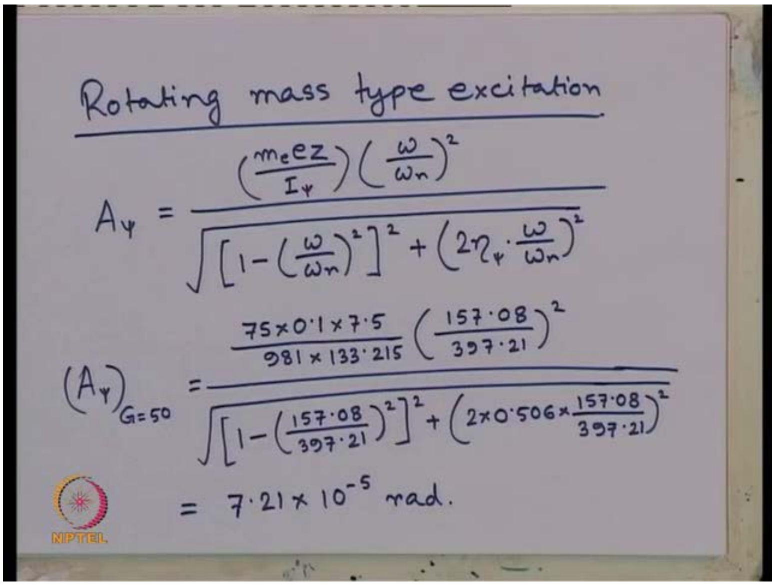

For rotating mass type excitation, the expression for A psi is nothing but m e e z by I psi, omega by omega n whole square root over 1 minus omega by omega n whole square, that square plus 2 eta psi omega by omega n whole square, that is the expression right. And for eccentric mass, how much is this z, we are considering from the cg so h by 2 right, this z will be 7.5 centimeter because total height of the footing is 15 centimeter we have, it is given in the problem.

So, if we put the values of the given and calculated terms, A psi for G equals to 50 first type of soil it will be, 75 into 0.1 centimeter is the eccentricity 7.5 centimeter is the (()) and 981 into 133.215 is I psi. Why 981 because m e is mass and we have been given with 75 k g weight so weight we are dividing by this so that is actually kg force, when we are talking about weight it is kg force. So, divided by 981 that is the reason, 157.08 by 397.21 whole square root over 1 minus 157.08 by 397.21 whole square, that square plus 2 into 0.506 into 157.08 by 397.21 that square fine.

So, let us calculate this so the value of A psi for G equals to 50 kg force per centimeter square after calculating it, it is coming 7.21 into 10 to the power minus 5 radian, once again it will be same because the amount of load we get same, for both constant force type and rotating mass type. That is why, at operating frequency, the displacement amplitudes are same, as we had discussed earlier also. So, no need to calculate for the other two cases because they also will be of same values.

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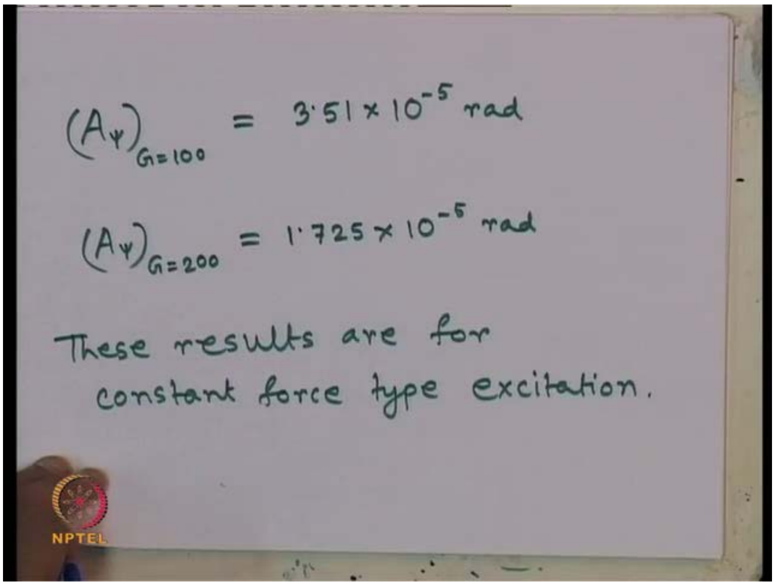

That means, they are A psi as 3.51 into 10 to the power minus 5 and A psi for G equals to 200 will be 1.725 into 10 to the power minus 5 radian, so that we will continue in our next lecture, following lecture.