Soil Dynamics
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Module - 5
Machine Foundations
Lecture - 31
EHS Theory, Vibrational Control

Let us start our today's lecture on soil dynamics. We were continuing with our module 5 that is on machine foundations. A quick recap of what we have studied in the previous lecture.

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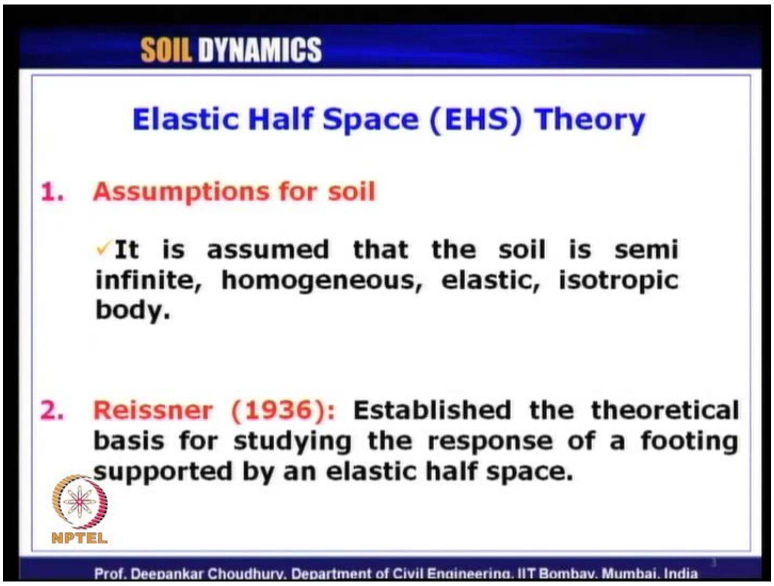

We have started the theory of elasticity that is elastic half space theory and the use of that, for design of machine foundation. Assumption for soil was, it is assumed that the soil is semi infinite, homogenous, elastic, isotropic body.

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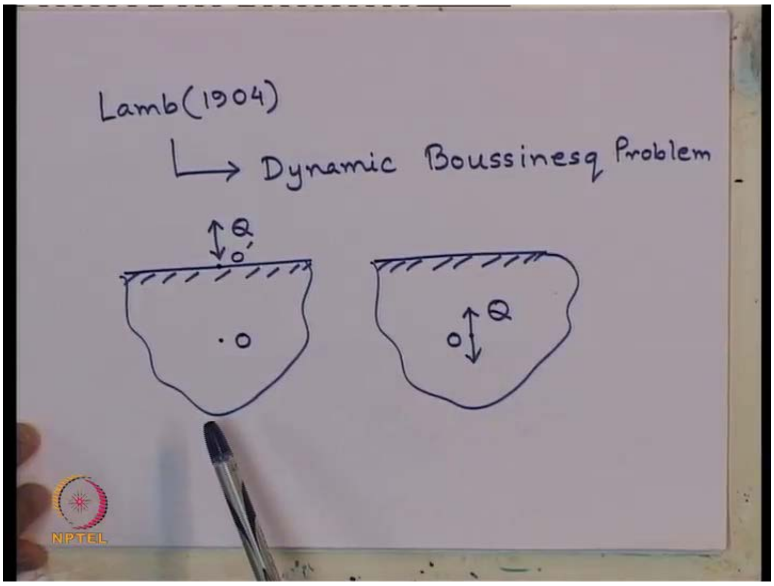

To start with this problem, we have seen that the elastic half space theory have proposed by Lamb in 1904 by considering it as Dynamic Boussinesq Problem, in the case of static, we know what is Static Boussinesq Problem to obtain the stress below any particular load at any depth and also at any radial distance. So for the dynamic load, it has changed to Dynamic Boussinesq Problem, like for vertical type of vibration, this is the module shown by Lamb. For horizontal loading this was the model.

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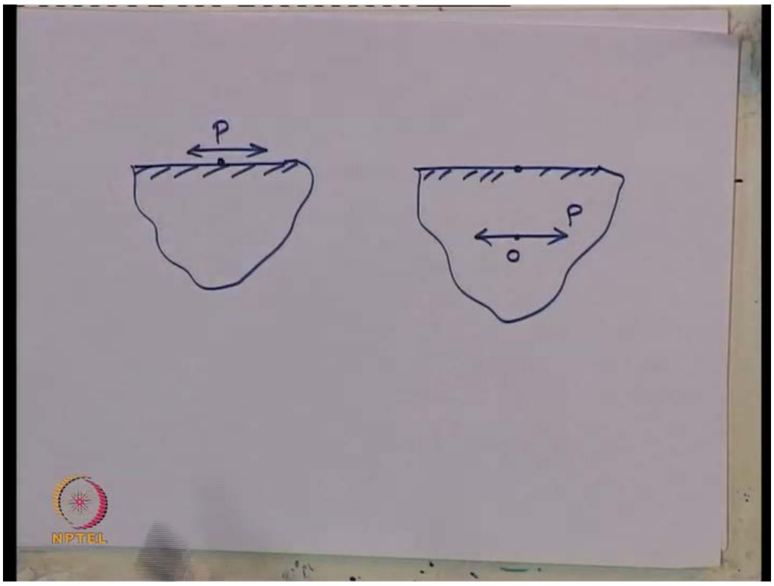

Then we have seen that Reissner in 1936, established the theoretical basis for studying the response of a footing supported by an elastic half space.

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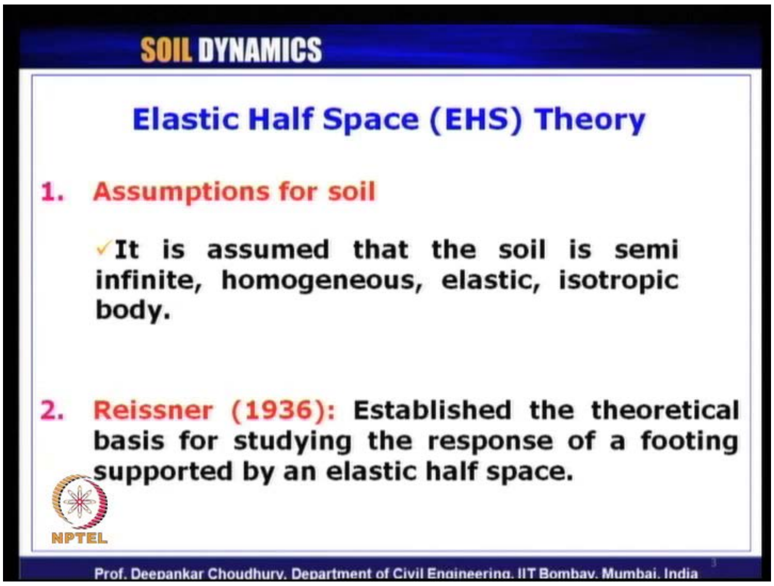

The basic model considered by Reissner was like this.

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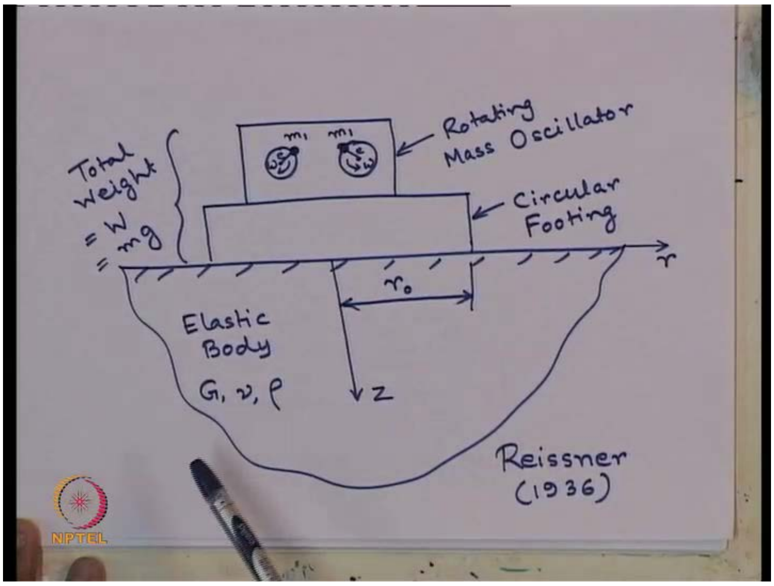

We have seen the picture of the model, that for any machine, suppose it is a rotating mass type oscillator having a circular footing with a radius of r naught at any depth z, we can find out its displacement conditions etcetera. The total, considered as elastic half space or elastic body with these are the properties shear modulus G, Poisson's ration nu, and rho as the density of the soil.

The theory of Quinlan and Sung both have been given individually in the same year 1953, how to apply Reissner's model of 1936 to compute the vertical displacement below the vertical mode of vibration for any machine foundation. So this is the expression which was given, z naught is nothing but the vertical displacement, P naught is amplitude of the dynamic load vertically applied, exponential i omega t where omega is external frequency or the exiting frequency, G is shear modules of the soil, r naught is equivalent radius of the circular footing, and these two terms f one plus i f 2, f 1 and f 2 are called as Reissner's Displacement Functions. These Reissner's Displacement Functions are nothing but, we have seen these are the functions of Poisson's ratio and another term a naught. How a naught was defined? We have seen that also for vertical vibration, the dimensionless frequency factor is called a naught, which is expressed like this, that is omega r naught by v s, where v s is the shear wave velocity in the soil media.

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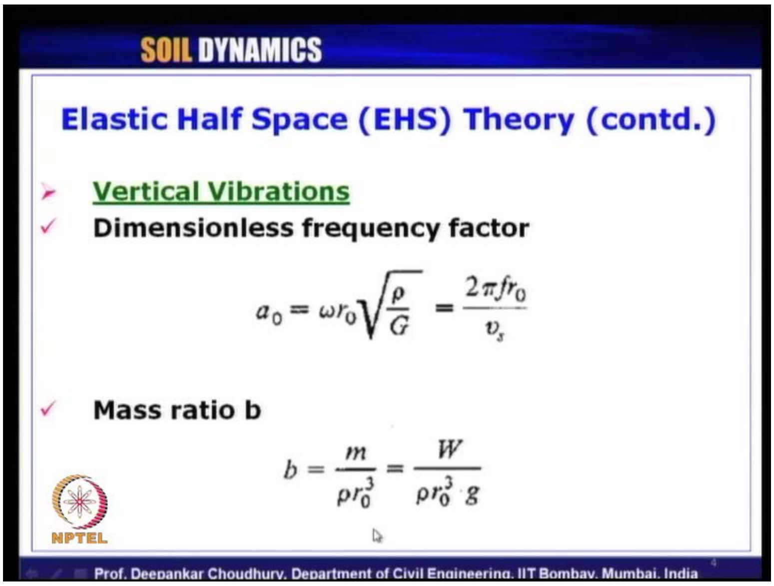

So in terms of f, it is 2 pi f r naught by v s. So this is non-dimensional frequency factor. Another non-dimensional parameter was defined by Reissner that is called mass ratio small b. Mass ratio is expressed like mass of the total machine plus foundation system divided by density of the soil and r naught cube, where r naught is the radius of that circular foundation. Then in relation to these we have seen what are the different expressions available by using theory of elasticity for different modes of vibration by considering rigid circular footing resting on a elastic half space.

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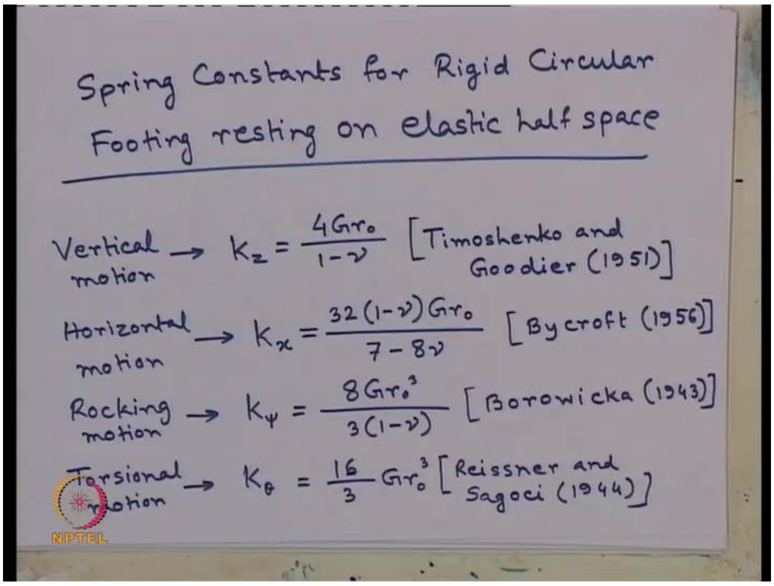

So for a vertical motion we have seen this is the expression for spring constant k z, that is 4 G r naught by1 minus mu, where mu is Poisson's ratio, G is shear modules and r naught is equivalent radius of circular footing. It is given by Timoshenko and Goodier in 1951. For horizontal vibration the expression for k x is this given by Bycroft in 1956. For rocking mode of vibration the expression for k psi is given by this. It is given originally by Borowicka in 1943. For torsional or yawing motion this is the expression for k theta, the spring constant.

Now if we do not have a circular footing which is quite common in practice, many a times we have either rectangular footing or the square footing. Then we can convert the dimensions of rectangular or square footing to equivalent circular radius.

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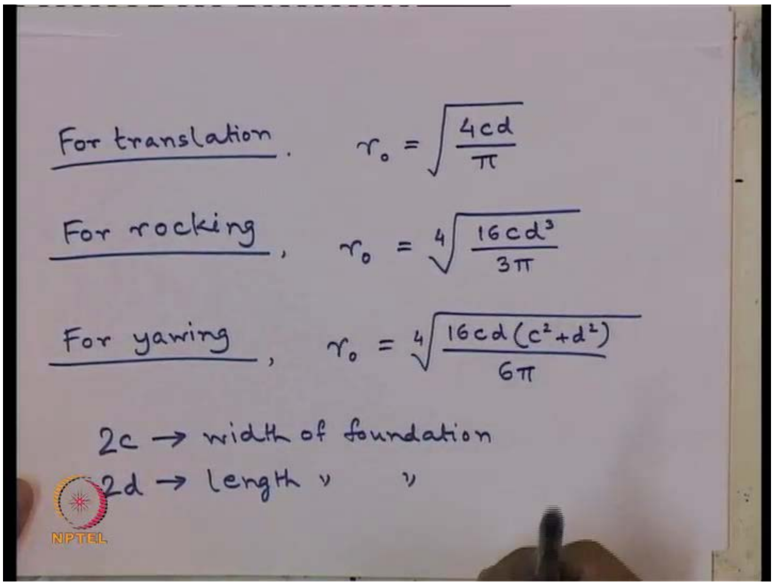

So how to convert that, like for translational vibration the expression is like this. For rocking mode of vibration r naught is computed using this expression. For yawing mode of vibration r naught is computed by using this expression where this c or d are defined like this, 2 c is nothing but total width of the foundation, so c is nothing but half of the width of the foundation, and 2 d is the total length of the foundation, so d is half of the length of the foundation. How these expressions were derived? Just for translation mode, area has been equated for a rectangular footing to a circular footing. Then for rocking mode and yawing mode inertia has been equated corresponding to its axis of rotation. Also we have seen that for rectangular footing the expression for spring constant for different modes of vibrations were given by different researchers.

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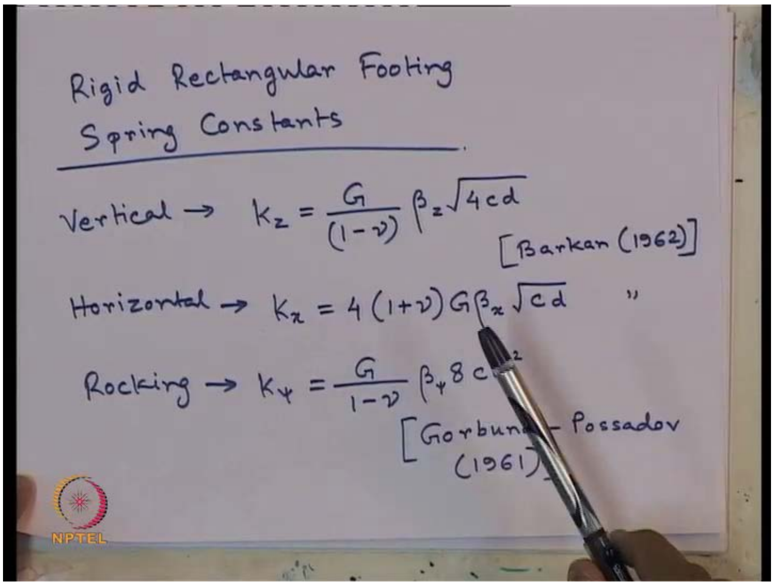

But, however it is always better to use the basic expression for circular footing by using the transformation of the dimensions from rectangular to the circular dimensions and this was the design chart as per the Reissner's model.

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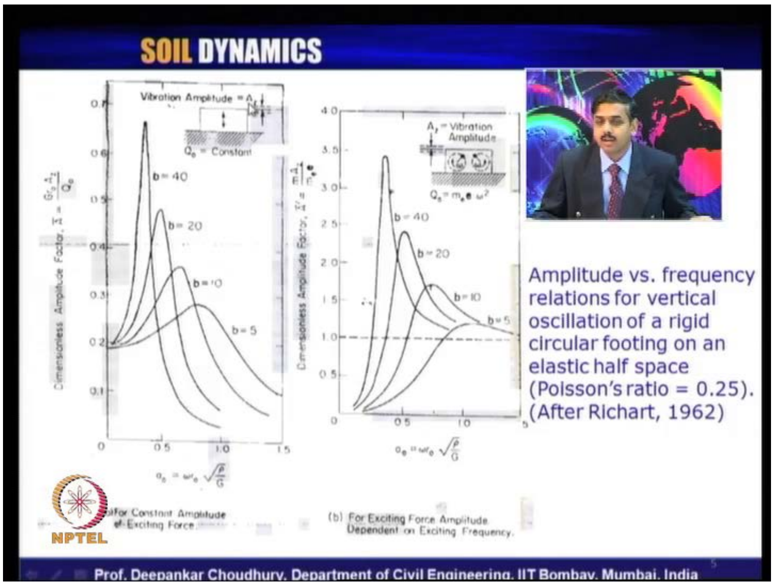

It is given in Richard 1962 for Poisson's ratio, value of 0.25, as we have seen it is a function of Poisson's Ratio. So, for a particular value of Poisson's ratio, how with respect to this non dimensional frequency factor a naught, the dimensionless amplitude factor where vibration amplitude A z can be computed using this design chart for various values of mass ratio b, and this is for constant force type vibration with q naught like this. And this design chart is for eccentric mass or rotating mass type of vibration with q naught is m e omega square with this 2 rotating mass and A z can easily be computed using this expression because in this expression m, m e e, all are known from design chart. We can compute e naught for calculated value of b. Using this chart we can read this y axis value and from which you can get the value of A z. So that is the use of Reissner's Model.

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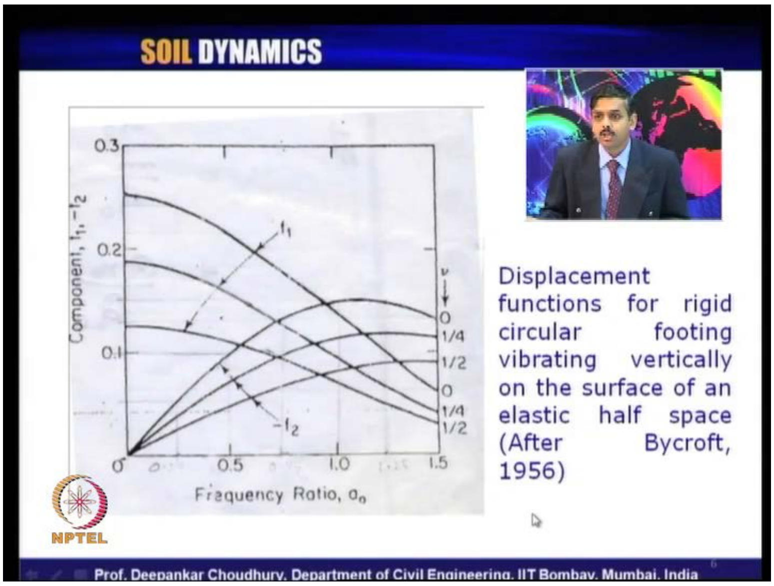

Also I have mentioned that either we can directly use the Reissner's model in the previous design chart as we have discussed just now or we can use this design chart given by Bycroft in 1956. It gives how f 1 and f 2, this factors that is Reissner's Displacement Functions, they vary with respect to this dimensionless frequency ratio a naught and different Poisson's ratio. So that also we can read from this graph and can use in the expression for computation of z naught as I have already mentioned. Then we have started discussing another simplified analog proposed by Lysmer's in 1965. Lysmer's Analog of 1965, what it does.

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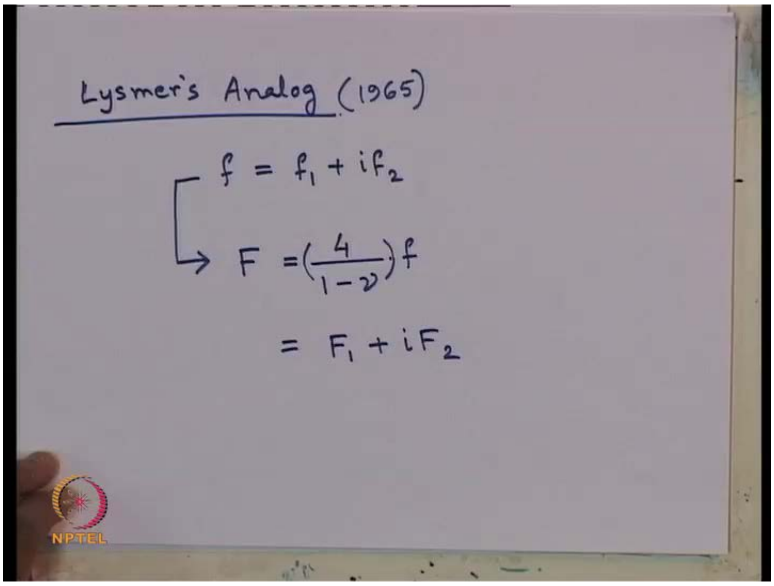

The Reissner's displacement functions was given by small f, expressed like f 1 and f 2. It was converted to capital F just by using this multiplication factor that is 4 by 1 minus mu, that is Poisson's ratio just to take care of the effect of Poisson's ratio, because earlier we had seen Reissner's displacement functions are function of both Poisson's ratio as well as dimensionless frequency factor.

So to get rid of the Poisson's ratio term, Lysmer has modified it to multiply with, with respect to this expression so that it is now independent of the Poisson's ratio. It is already taken care of within these factors capital f 1 and capital f 2. And for different modes of vibration we have started discussing from the Lysmer analog for vertical mode of vibration.

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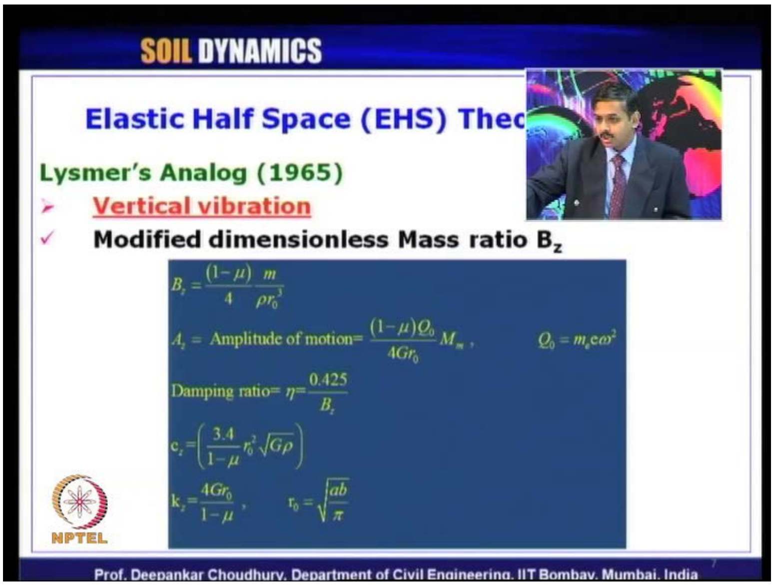

Again the mass ratio has been modified by Lysmer where ever Reissner's mass ratio was small b, but now here it is capital B for vertical mode. It is B z, which is expressed by this expression 1 minus mu by 4 into m by rho r naught cube and amplitude of motion can be calculated like this. Damping ratios c z k z all can be calculated using this expression.

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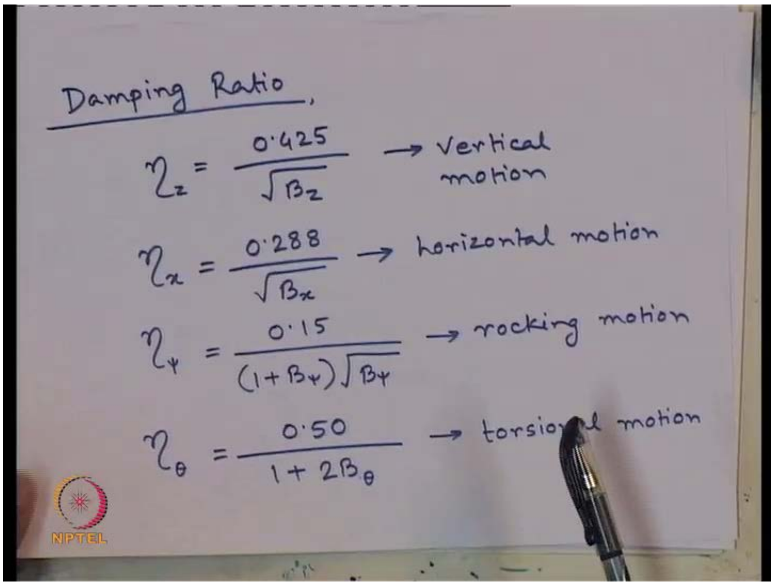

Once again, we have discussed the expressions for damping ratio for different modes of vibration as per Lysmer's simplified analog, that is eta z is the damping ratio for vertical mode of vibration, is given by this expression. B z is that modified mass ratio, eta x for horizontal motion where B x is the modified mass ratio for horizontal mode of vibration, eta psi is for rocking motion and eta theta is for torsional motion. So, these are expressions for damping ratio.

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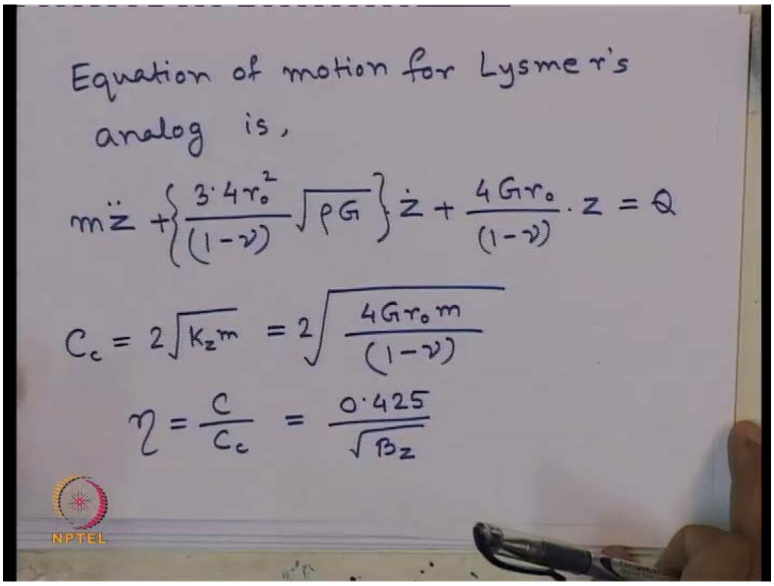

For vertical mode of vibration we have also seen what is the basic equation of motion for using Lysmer's analog. It is nothing but, similar to our mass spring dashpot model that is m z double dot plus this is the expression for c z that is the damping coefficient for vertical mode of vibration times z dot. This is our k z because we have seen the spring constant for vertical mode of vibration is nothing but, 4 G r naught by 1 minus mu times z equals to that externally applied load. How to calculate the damping ratio for which we have just now seen the expression? This is the way, C c the critical damping constant can be calculated as 2 root over k m, where k here is k z. So, we can use the expression for k z here and then if we apply this eta equals to nothing but C by C c, damping ratio and on simplification we can get this expression.

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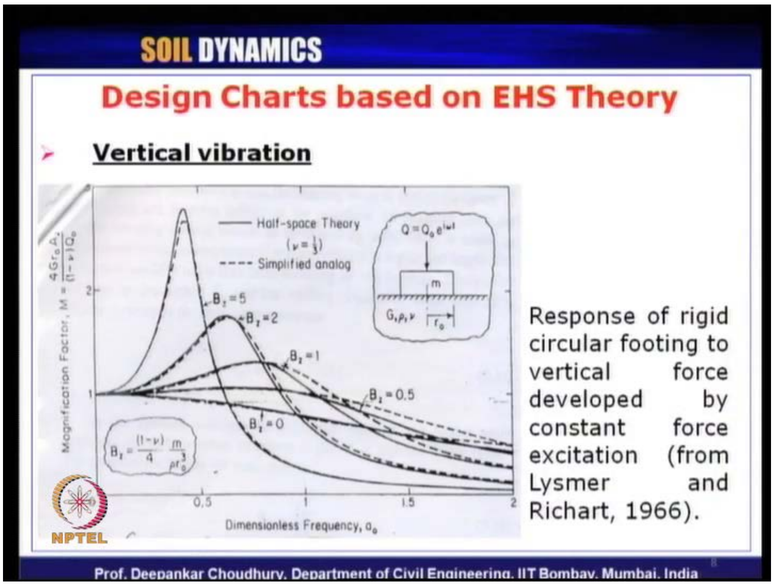

How to compute the different values of amplitude of oscillatory motion? These are the expressions which we will see soon in the design chart.

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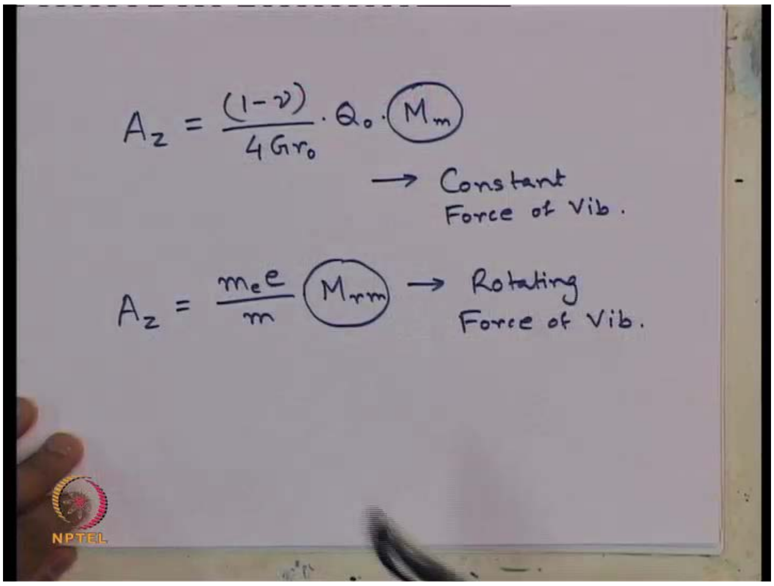

A z for constant mode of vibration and A z for rotating mass type vibration, these are the factors which the design charts are available.

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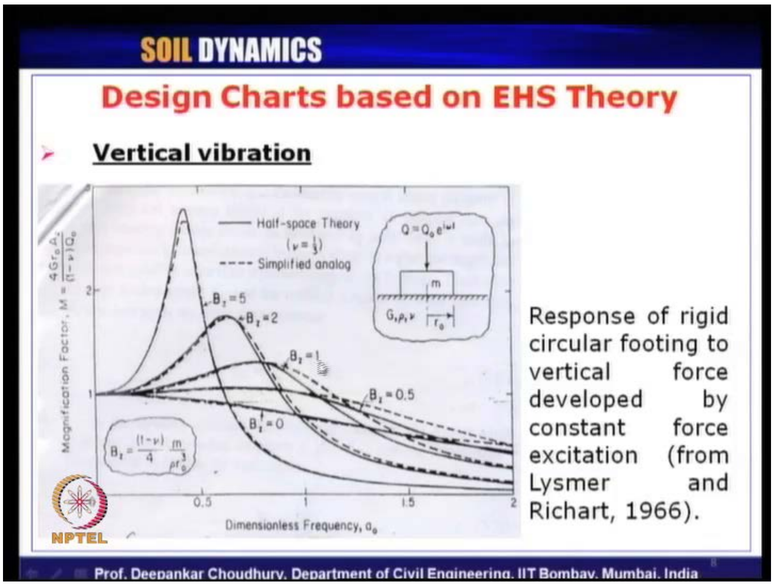

So looking back at the vertical vibration here, a competition of elastic half space theory that is basically a Reisnner's Model and simplified analog given by Lysmer has been compared for different values of dimensionless frequency factor with magnification factor. It is nothing but, this is our k. so k A z by q naught, for a constant force type vibration.

For different values of B z, it has been found that they are essentially giving almost a good comparison with the exact half space theory. So that is why instead of using complicated half space model we can also use Lysmer's simplified analog for using elastic half space theory for the design purpose of machine foundations.

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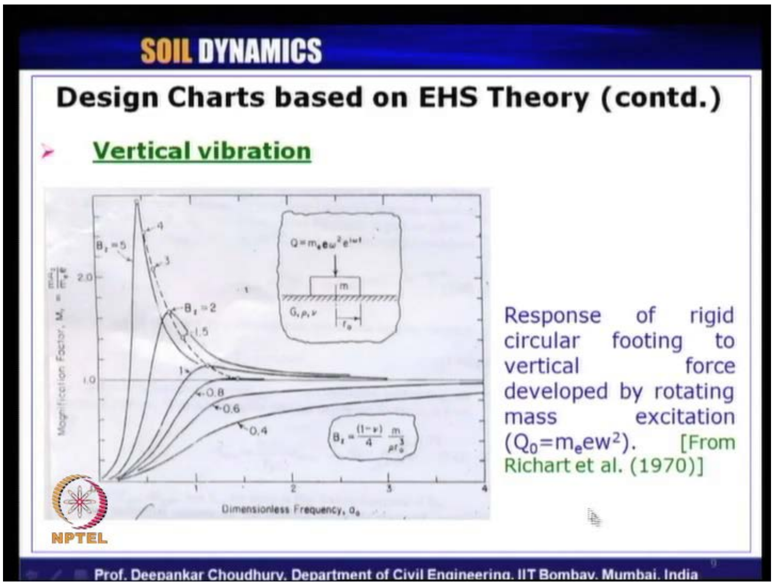

This is again for rotating mass type excitation, how the variations of magnification factor with respect to different values of frequency dimensionless frequency factor and modified mass ratio B z that has been expressed.

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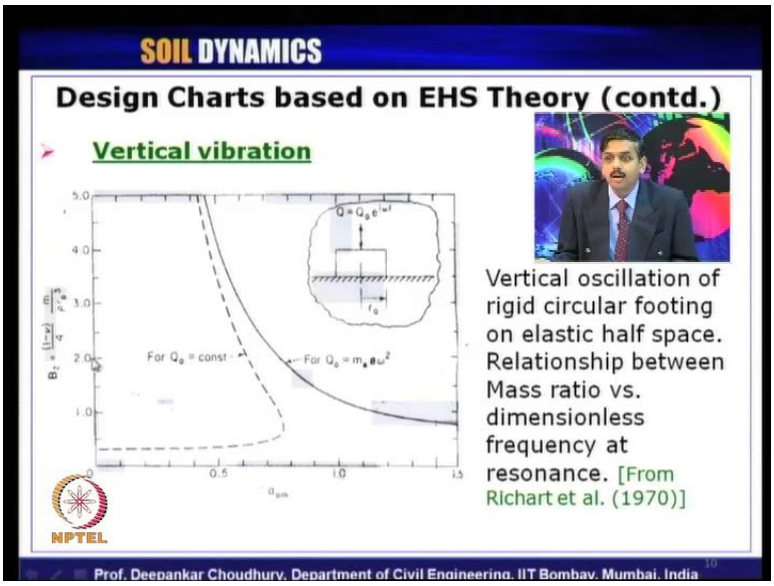

This was the design chart I had mentioned. This is for what? This is for resonance condition, that is how to obtain the resonance frequency and the displacement at resonant condition. The previous 2 cases where for operating frequency condition.

Now for resonant condition, how to use this design chart of Lysmer for vertical mode of vibration B z? We can easily calculate because all these parameters are given to us. The known total mass of the machine plus foundation is m, rho of the soil is known density, r naught is known, mu of the soil is also known, so B z is known to us.

So, y axis value, suppose if it is 2, if we calculate, then depending on what type of dynamic loading we are considering, whether it is a constant force type or a rotating mass type, we can go to that particular graph and drop the line to calculate a naught m. What is a naught m? It is nothing but, dimensionless frequency ratio and a naught m is the dimensionless frequency ratio at resonance. So, this value of a naught m will give us the value of resonant frequency. So, if you use the expression of a naught in the a naught m, whatever f you will get finally, that will give you nothing but, the resonant frequency for that type of design foundation. Now, how to use this value again for calculation of displacement amplitude at resonant frequency?

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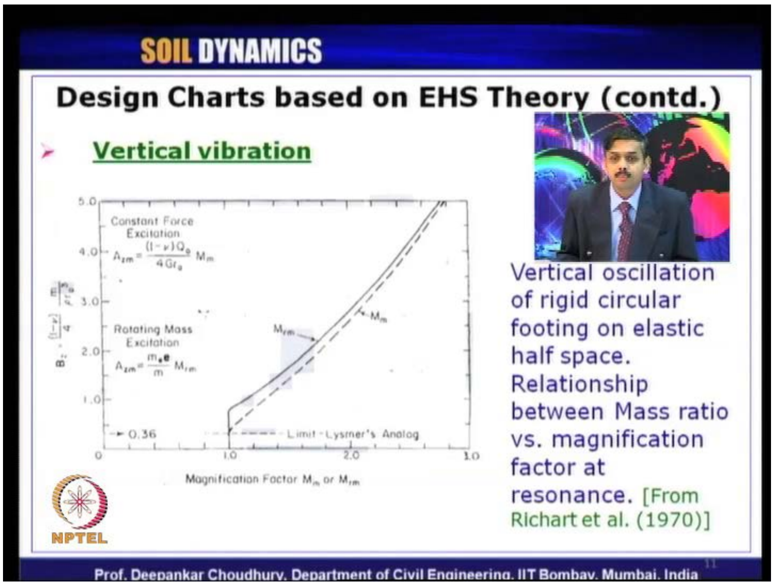

This is another chart. B z again we know depending on whether it is a constant force type or arotating mass type. These are the different lines. Constant force type is with m, m is the magnification factor that is, this dotted line. For rotating mass type excitation it is m r m that is this solid line. So, knowing the value of B z go to that corresponding curve depending on your type of loading, drop it here. You can read the value of M m or M r m depending on type of loading. Put that in this expression, you will get the displacement amplitude. Remember this displacement amplitudes are nothing but, displacement amplitude at resonant conditions, like in Mass Spring Dashpot model also we have computed displacement amplitude at 2 conditions, 1 at operating frequency and another 1 at resonant condition. So, here also using the first 2 graphs we can find out the displacement amplitude in case of operating frequency. In this case using the last 2 graphs we can find out the displacement amplitude at resonant frequency condition.

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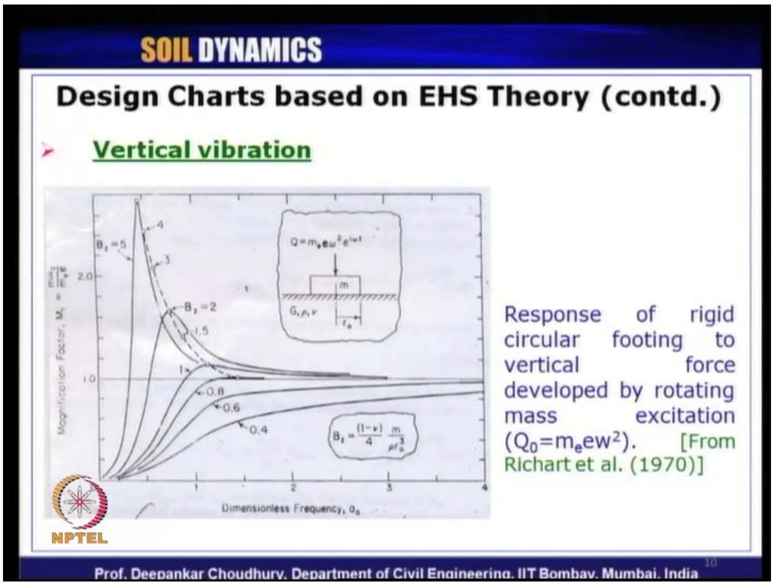

Now let me give you the expression for damping ratio. As for the vertical mode of vibration, already I have given the expression of damping ratio and how it is obtained I have shown. From the equation of motion you know what is your C z. Then you divide it by C c. C c you can compute from your k z and then ratio of C z by C c, you will get the damping ratio.

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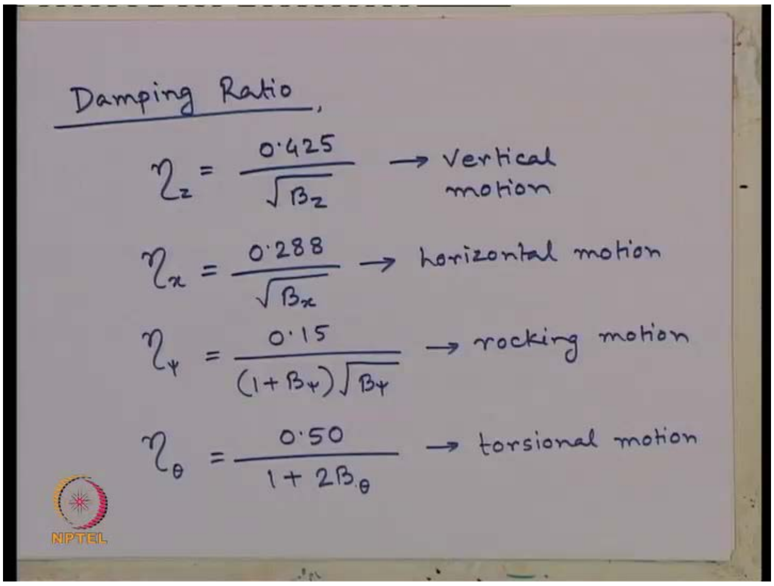

So, in this case for vertical mode eta z we have already seen 0.425 by root over B z is the expression for vertical motion. Then eta x is 0.28 by root over B x. This is for horizontal vibration. For rocking, eta psi is given by 0.50 by one plus B psi bracket into root over B psi. Then eta theta is expressed as 0.5 by one plus 2 B theta for torsional motion. So, these are the expressions of damping ratio that has to be used for corresponding vibration. Whatever we are going to analyze, how we can analyze this thing?

There are two ways again, either we can use direct chart of Lysmer to get the magnification factors for particular value of a naught and then do the calculation or we can use the expression for amplitude of vibration similar to mass spring dashpot model because equation of motion, the shape of equation of motion is same. So, the solution for the equation also will be same. Is that clear? So, either analytically you can compute or from reading the graph you can get the results. So, let me tell you how to read the graph from the design charts.

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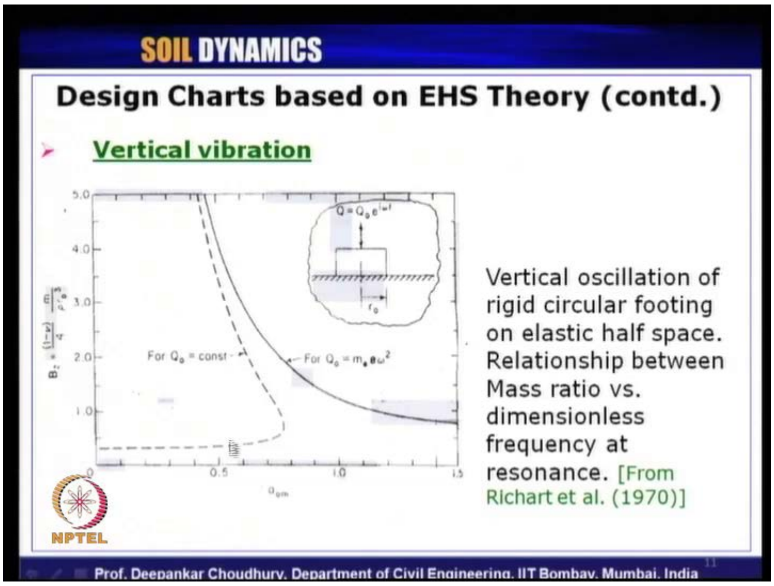

So this is 1 of the design chart. First design chart I should say, for vertical mode of vibration with rigid circular footing on elastic half space. It gives the relationship between mass ratio verses dimensionless frequency at resonance. So, this is at resonance condition. Remember that it is not at operating frequency.

There are different curves, 1 for at resonance condition another will be for, at operating frequency conditions. First you calculate B z knowing the mass of your machine plus foundation. You are going to design the section in the trial which you have taken. So, knowing the mass rho of the soil, you know r naught. You have chosen some dimension for the footing. So, this is known to you, mu of soil, Poisson's ratio is known. So, B z is known to you. How you can use this curve? B z is known.

This dotted line shows the design curve for constant force of vibration and this solid line shows the design curve for rotating mass type vibration. So, first you calculate this B z for vertical mode of vibration. Suppose it is coming to for, if it is a constant load type you are addressing, go to this curve and drop it here. Calculate the value of or read the value of a naught m from this chart. Remember this a naught m is not the same as a naught. Why? The a naught you have calculated, the dimensionless frequency factor, is using your exciting frequency or operating frequency, whereas a naught m is at resonance condition. So, in this way, what you are getting from this design chart? You are getting the resonance frequency for the machine plus foundation system. Then, what you are doing? Go to the next chart.

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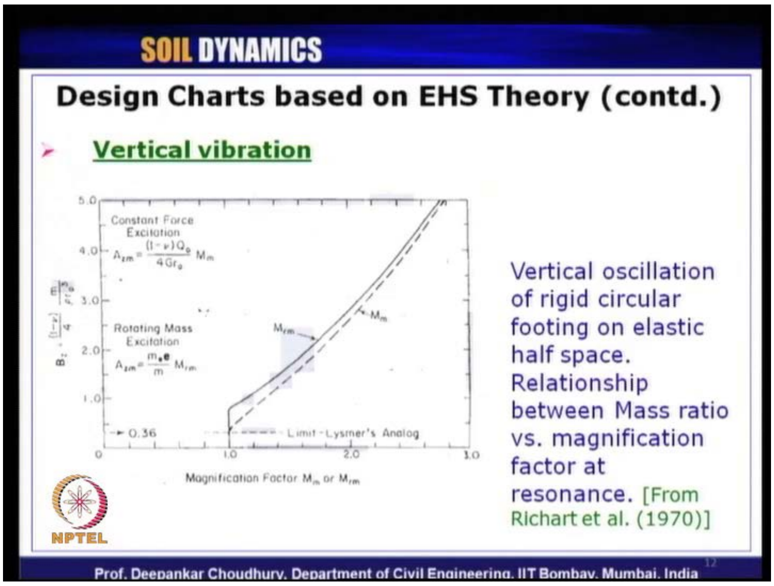

This is showing the relationship of mass ratio verses magnification factor at resonance. So, again there are 2 lines. 1 solid line, that is for rotational rotating mass type excitation and the dotted line for constant force type excitation. So, for constant force type excitation we need to use this expression to get the amplitude. For rotating mass type excitation we need to use this expression to get the amplitude. B z is known to us, take suppose it is 2, take it here, if it is rotating mass type. Come to this line and drop it here. You will get the value for rotating mass type. We will get M r m. That value of M r m, you put here. m e e and m are known to you. So you can easily get what is the amplitude of motion. Is this clear, how we are using this Lysmer's design chart for our design purpose?

This way, whatever displacement amplitude you are getting, what is that amplitude is not at operating frequency, it is at resonance condition, clear? So, for operating frequency, you can use the expression as we have used for Mass Spring Dashpot model. Only thing in that case, the damping ratio value you have to calculate using Lysmer's analysis and other expressions for k z or B z, everything will be as per Lysmer's analog.

End of part-a

Now, let us star today with sliding mode of vibration for Lysmer's analog. So, in case of Lysmer's analog for sliding vibration ,the modified dimensionless mass ratio expression will be B x, where B x is given by this expression.

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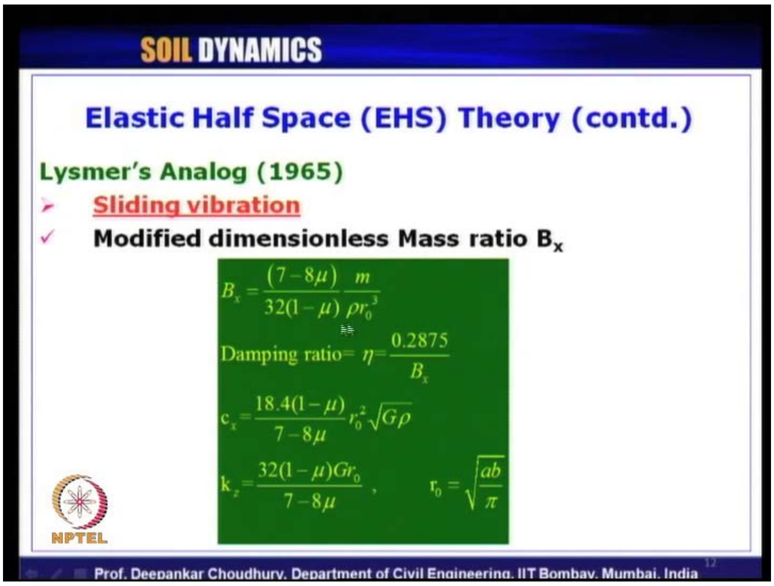

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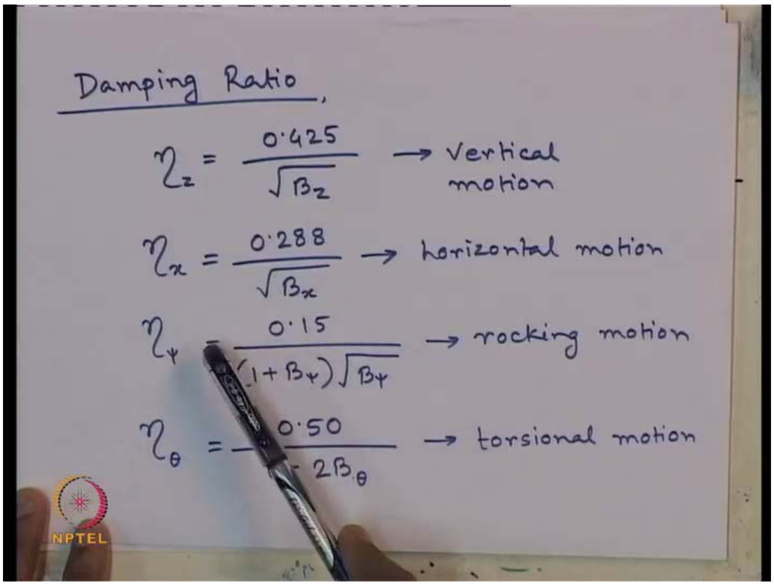

The damping ratio already we have discussed. Let us look back here, eta x is 0.288. Truly speaking it is 0.2875 but, you can approximate it as 0.288 by root over that B x.

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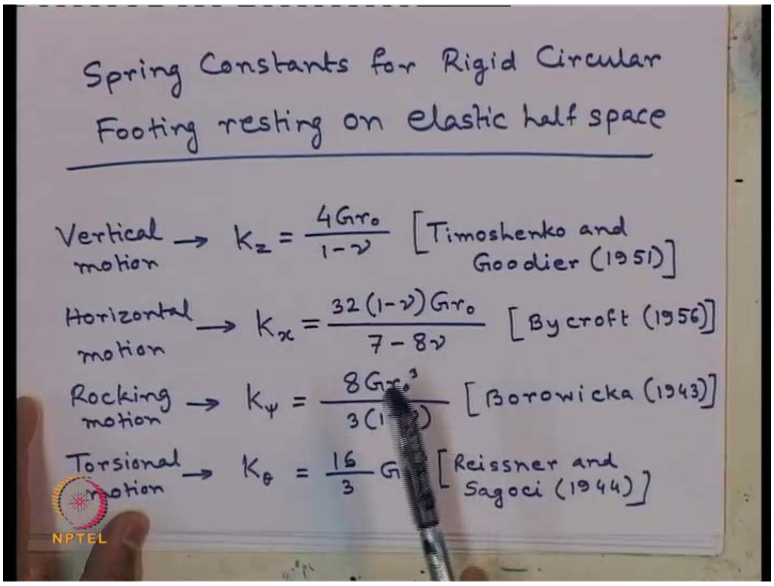

The expression for k x. Also we have seen for horizontal mode of vibration, this is the expression for k x.

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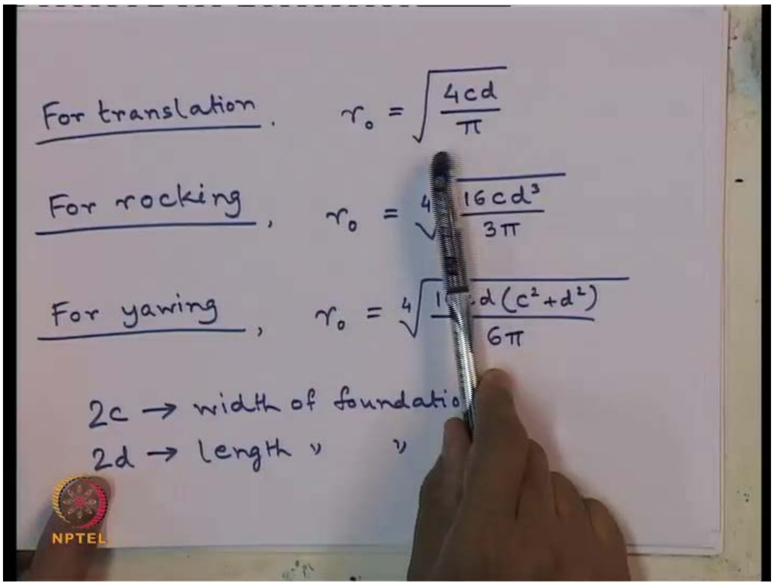

How to compute the r naught? That is, for translational mode, that also we had discussed earlier. So, this is the expression to compute r naught if our original foundation is in rectangular shape.

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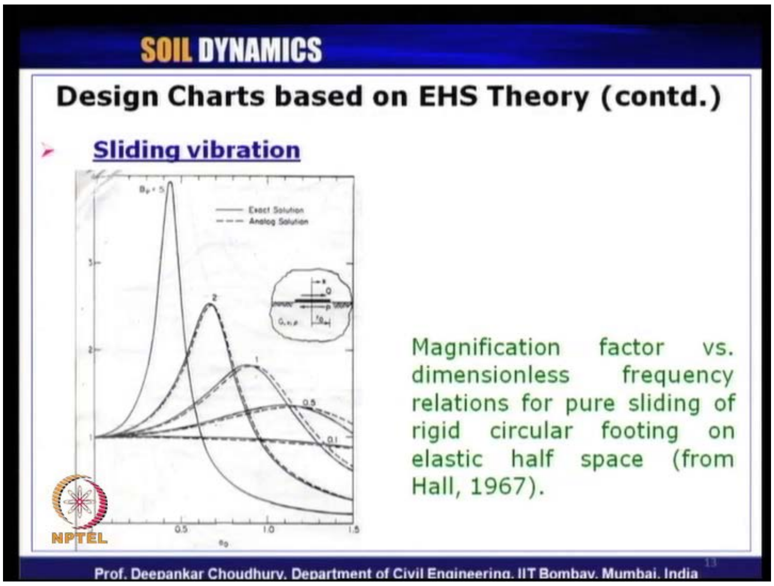

Now, how to use the design chart? This is again, you can see for different value of B 5 with respect to different a naught, the amplitude can be computed. Here the exact solution, that is using elastic half space theory and the dotted line shows the Lysmer's analog . They compare very well. So, it shows that Lysmer's analog can be used for the design purpose.

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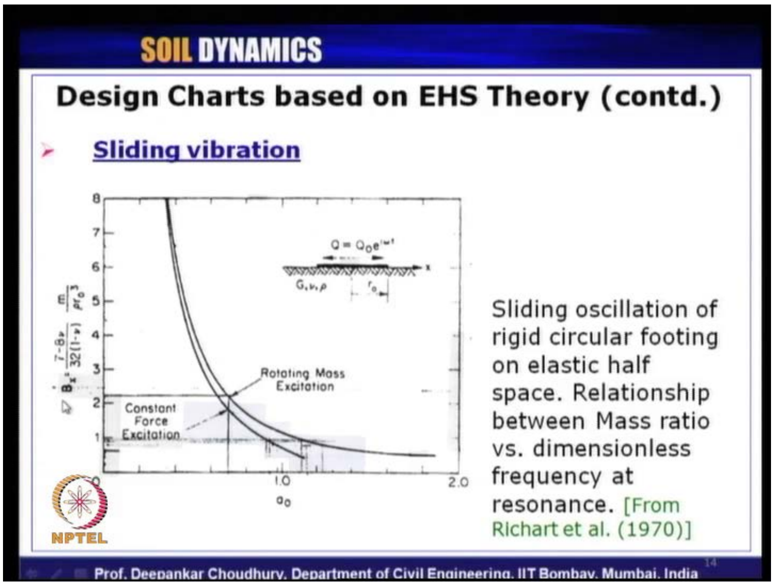

This is the way at resonant condition, 1 can find out what are the resonant frequency for a designed foundation section and what are the displacement at resonant condition. This is the expression for B x which we can easily compute for the given data. Depending on constant force type or rotating mass type vibration, we can drop it here. We can calculate a naught m. This is your a naught m that is, frequency at resonant condition.

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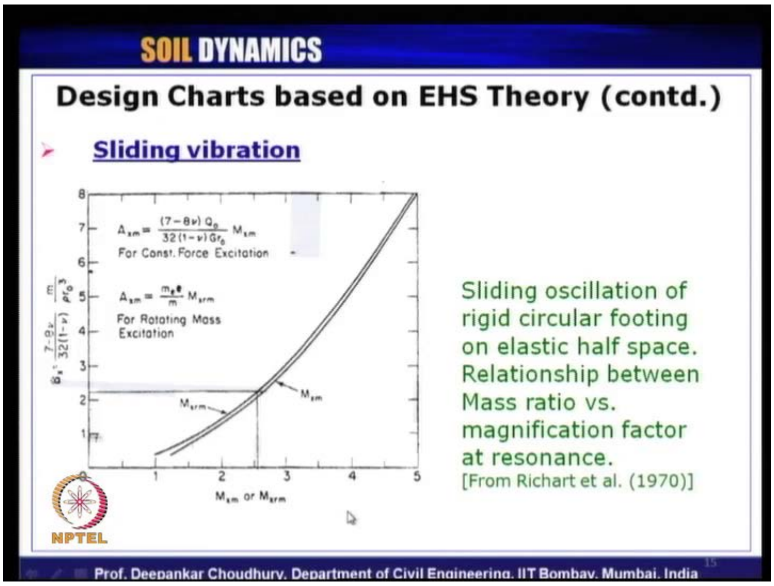

Then we will use this design chart knowing the value of B x. Depending on case of whether constant force type or rotating mass type, we will get the magnification factor. Using that in this expression, either of these 2, we can get the displacement amplitude at resonance.

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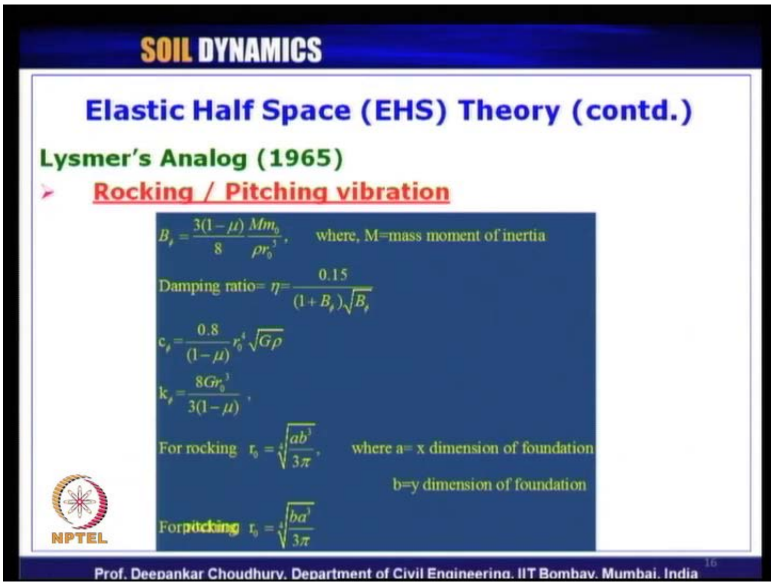

Now coming to the Rocking or Pitching mode of vibration using Lysmer's analog, the expression for dimensionless mass ratio for rocking mode of vibration is given like this. B phi is given by this expression, where M m naught is nothing but mass moment of inertia in this case, and remember here it is rho r naught to the power 5 not 3. This is because, this mass moment of inertia, whatever unit it will have, to make it non dimensional, we need to multiply it with respect to r naught to the power 5 so that it will become kg centimeter square and here also it will be kg centimeter square.

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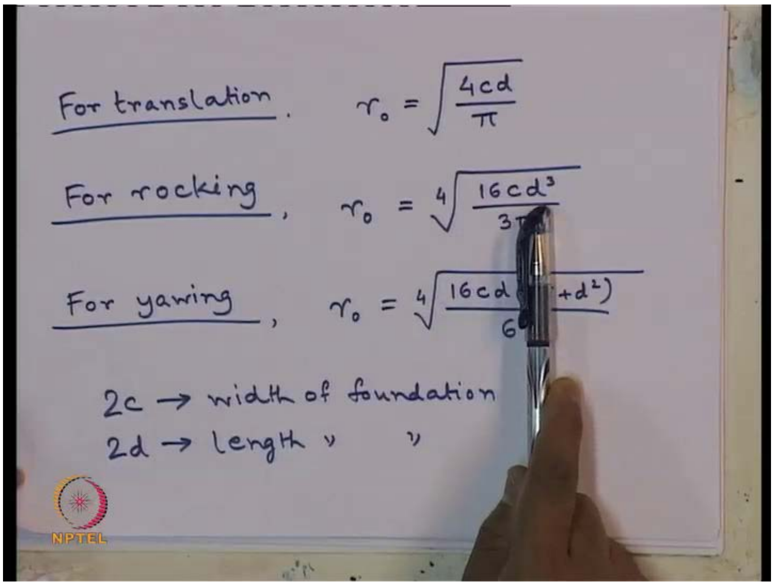

How to compute that r naught for rocking mode of vibration? This is the expression to calculate r naught for rocking mode of vibration. For pitching, what will occur ? Here c and d will interchange. The dimension in the x axis is c when we are considering rocking and d is the dimension in the deduction of y axis. So, for pitching it will be 16 d cube c by 3 pi.

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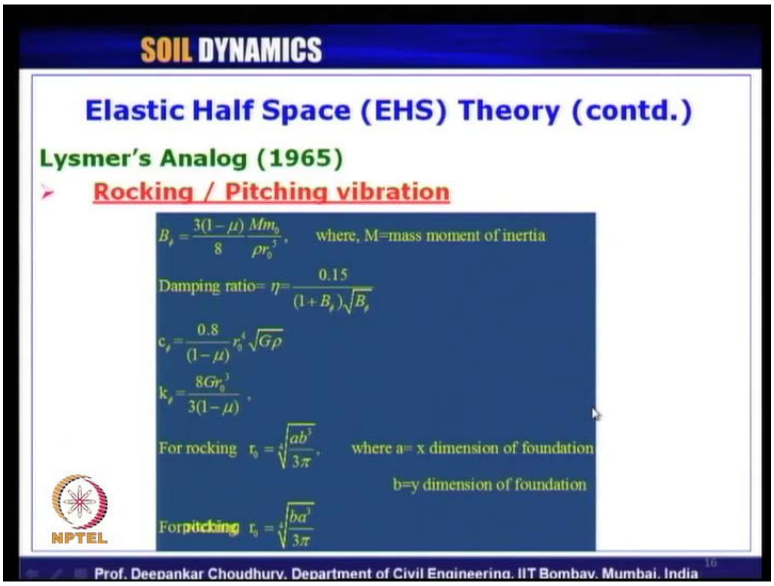

Now let us look back here. For calculating k phi this is the expression. Also c phi can be calculated like this.

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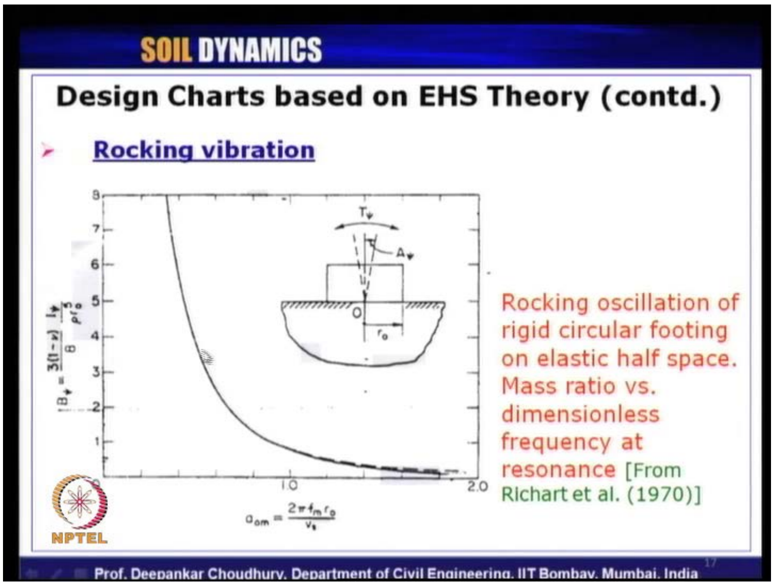

How to use the design chart again in the similar way for other two cases? For resonant condition B psi, we can easily calculate for rocking mode of vibration using this expression .Then depending on whether constant force type or rotating mass type, look here in this case, both the cases are giving exactly the same line only except at this very high value of a naught m. Otherwise there is essentially a single line. You can get the value of resonant frequency.

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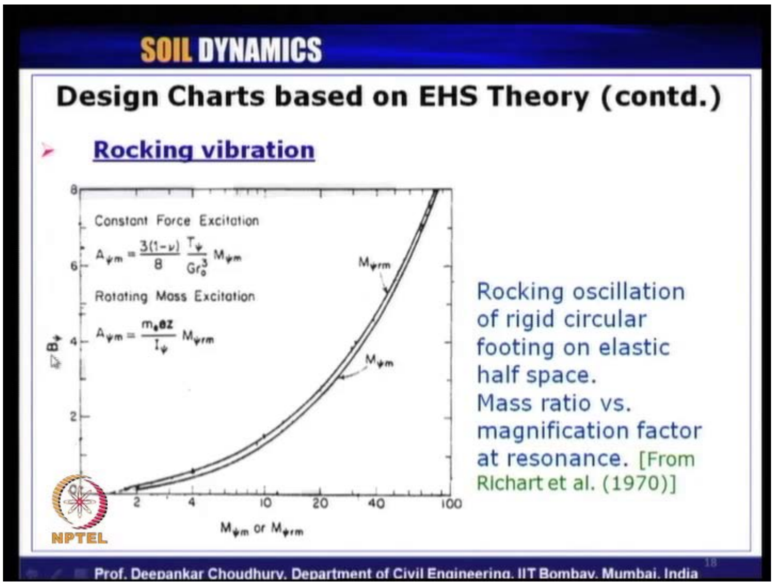

Then, using this chart we can get the value of magnification factor, this M psi m or M psi r m. Depending on type of excitation from the known value of B psi, we can calculate this A psi m for the different exciting loads of dynamic load.

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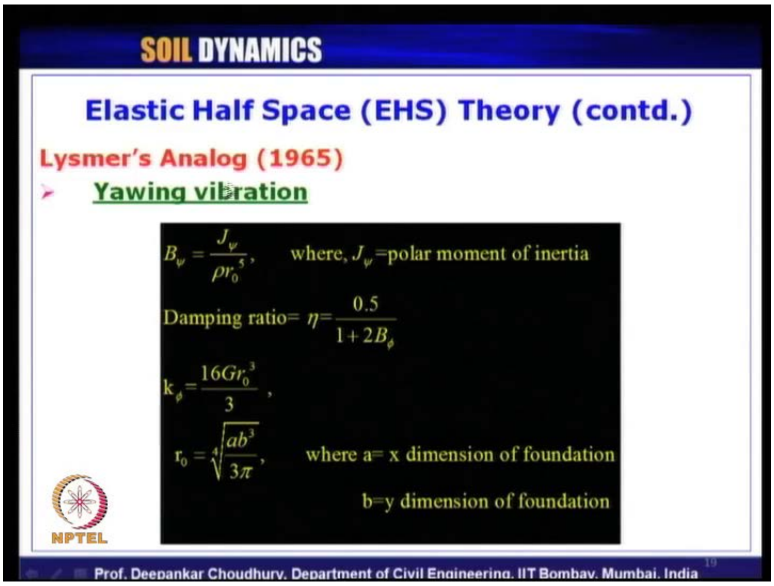

Coming to yawing mode of vibration using Lysmer's analog, this is the expression to calculate the non dimensional modified mass ratio of Lysmer where J psi is nothing but, polar moment of inertia divided by rho r naught cube. The expression for damping ratio is given by this expression. For spring constant it is given by this. For r naught, it has to be calculated for yawing mode of vibration, this is the correct expression, look at here.

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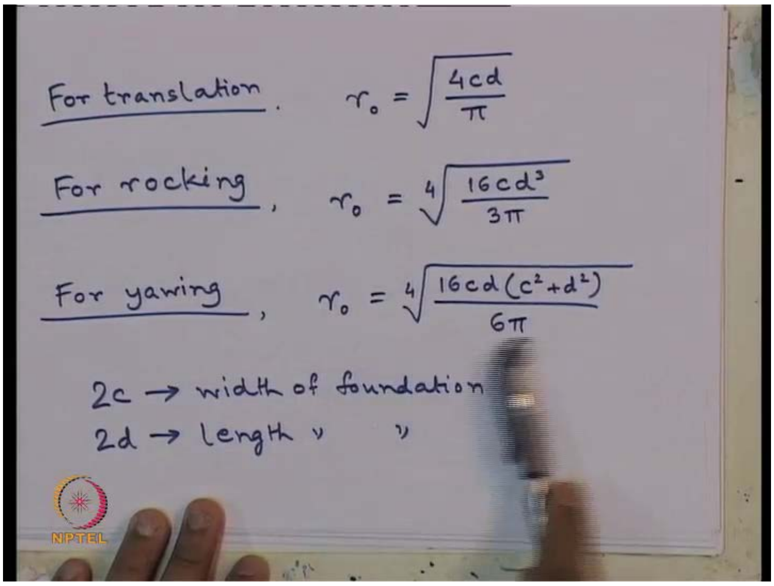

This is the expression to calculate the r naught for yawing mode of vibration.

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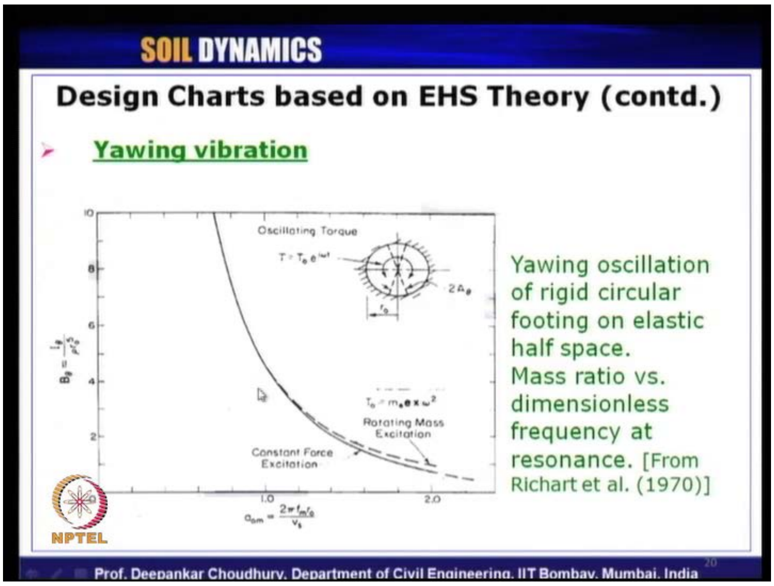

So, similar design charts are also available for yawing mode of vibration. So B theta can be calculated here also for constant force type. Rotating mass type curves are essentially same till this value of a naught m. Then they slightly vary. You can get the value of a naught m, that is, at resonant frequency you can calculate it.

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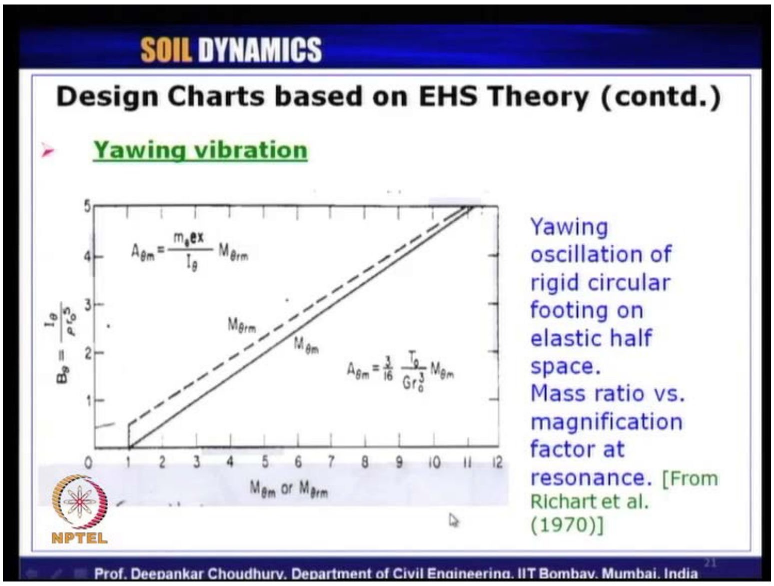

Then using this chart you can get M theta m or M theta r m, from which you will get the value of displacement amplitude at resonance condition depending on what type of excitation we are using.

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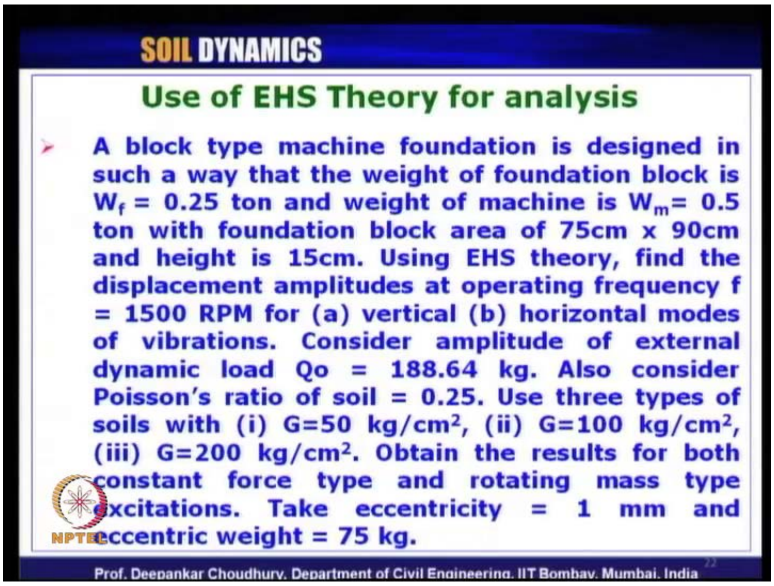

Now let us come to the application part that has to do with the design checks for block type machine foundation using the concept of this elastic half space theory. So, this is the problem statement. It says a block type machine foundation is designed in such a way that weight of foundation block is W f is 0.25 ton and weight of machine is 0.5 ton with foundation block area of 75 cm by 90 cm and the height of the foundation is 15 cm. So, it has already been designed. What we have to do using elastic half space theory? We have to find out the displacement amplitudes at operating frequency.

So, the first we are trying to find out the displacement amplitude at operating frequency only. What is the operating frequency? It is given as 1500 RPM, for these 2 modes of vibration, first 1 is vertical mode of vibration and the second 1 is for horizontal mode of vibrations. What are the other data given to us for the design ? Consider the amplitude of external dynamic load for both the cases, whether it is vertical mode or horizontal mode, it is 188.64 kilogram force and consider the Poisson's ratio of the soil as 0.25. Use 3 different types of soil, that is with G shear modules as 50 kg per centimeter square, then another type of soil with shear modules G equals to 100 kg per centimeter square, and another type of soil with G equals to 200 kg per centimeter square.

What has been asked? Obtain the results for both the constants, that is force type excitation and rotating mass type excitation. For the case of rotating mass type exciting we need some more input parameter. Those are also given, that is, take the eccentricity as 1 millimeter and the eccentric weight is 75 kg force that is, 10 percent of total of this machine plus foundation system which is 750 kg force.

Now comparing the previous problem statement, what we have used in case of mass spring dashpot model, are you able to find out? What is the basic difference from the given input data from MSD model and two EHS theory for the same problem? Because, the same design problem we have checked using mass spring dashpot model, now we are going to apply elastic half space theory. What is the basic difference, can you identify? Which input data is not given here but, it was given or it was required for mass spring dashpot model? Can you identify?

If you compare with the previous problem statement what we have used for mass spring dashpot model, if you look carefully there is one input parameter which is not given in this EHS theory or which is not required to be given as an input parameter for EHS theory, whereas that parameter is essential for using mass spring dashpot model. Yes, the parameter is damping ratio. Can you identify? In the case of mass spring dashpot model, we have solved the same problem, exactly same problem. We have checked using mass spring dashpot model that the value of damping ratio should have been given, otherwise we cannot proceed with our design check calculation. However, in case of elastic half space theory the damping ratio value need not to be given, why? This is because, the expression for damping ratio using elastic half space theory are already available, which needs to be used to compute the damping ratio, if we are using the elastic half space model. So, this is one important note.

So let us start now solving this problem in this case again. We need to assume the unit weight of the soil or the density of the soil. We will assume the same as we had considered for the Mass Spring Dashpot model. Also, that is, for these three different types of soil for G equals to 50 kg per centimeter square, we will assume gamma, that is unit weight as 1.7 into 10 to the power minus 3 kg force per centimeter cube. I will write it here.

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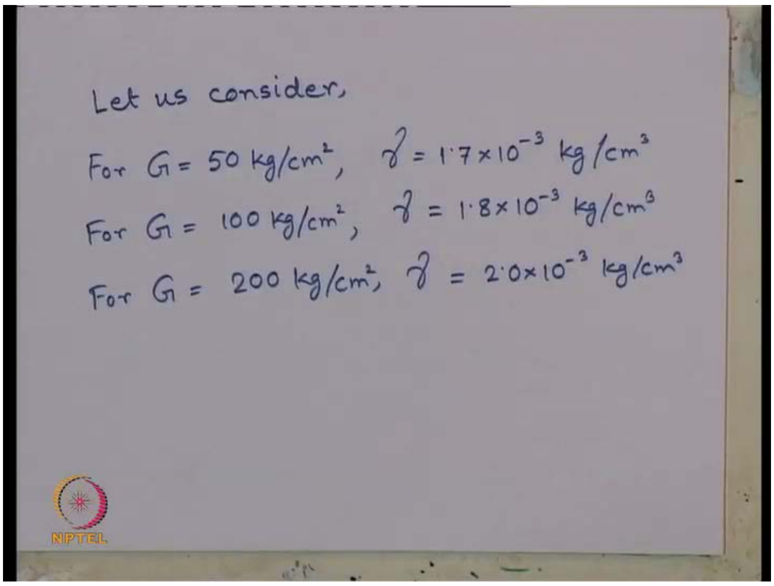

So, for the soil with G equals to 50 kilogram per centimeter square, let us consider gamma, that is unit weight 1.7 into 10 to the power minus 3 kg per centimeter cube. For G equals to 100 kg per centimeter square let us assume gamma is 1.8 into 10 to the power minus 3 kg per centimeter cube and for G equals to 200 kg per centimeter square let us assume gamma as 2 into 10 to the power minus 3 kg per centimeter cube. These are equivalent to, if you talk about SI unit, this is 17 kilonewton per meter cube, this is

18 kilonewton per meter cube and this is 20 kilonewton per meter cube, which are the standard values of unit weight for different stiffness of the soil which we can consider by looking at the G values. As it is becoming stiffer the unit weight we have considered or assumed as higher which is quite justified. So, now let us start the solution for the vertical mode of vibration first.

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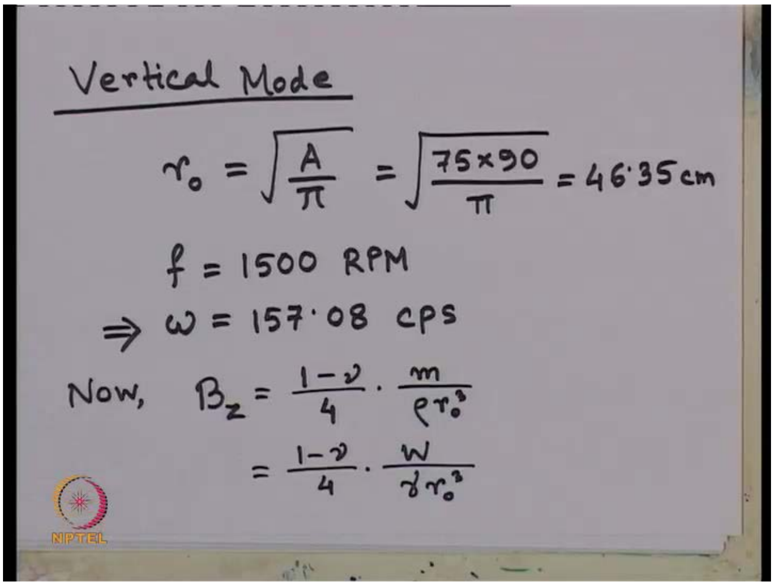

So for vertical mode of vibration the size of the footing is given. So, equivalent radius r naught we can easily calculate which is nothing but, square root of a by pi. So 75 by 90 centimeter is the area. So r, already we had calculated it earlier ,it is 46.35 centimeter. The operating frequency at which we need to find out the displacement amplitude is 1500 RPM which we had converted to omega, it gives you omega as 157.08 cycles per second.

That also we have seen earlier. Now we need to calculate using Lysmer's simplified analog B z ,vertical mode of vibration. So, B z we need to calculate. How much is B z and what is the expression? It is 1 minus mu by 4, times m by rho r naught cube which can be written as 1 minus mu by 4 into w by gamma r naught cube. I multiplied this by g, this also by g. So for 3 different types of soil we can calculate it like this.

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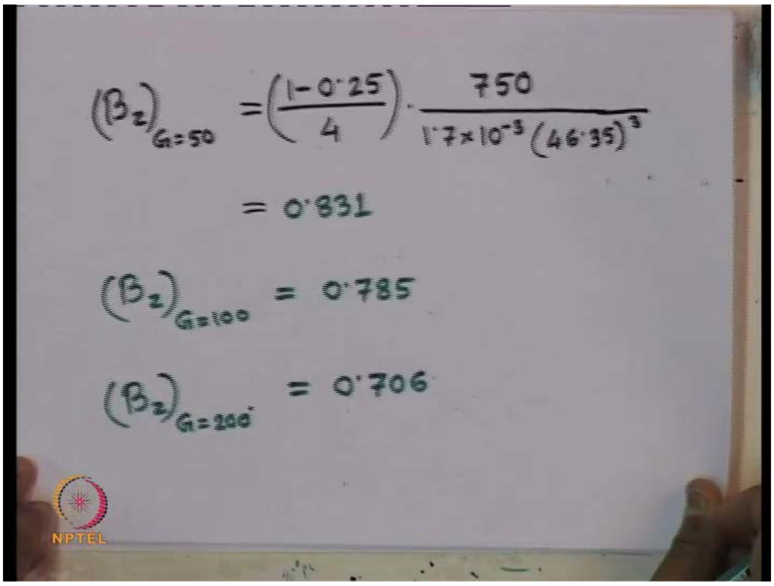

B z for G equals to 50 kg per centimeter square, will be 1 minus mu, Poisson's ratio is 0.25, by 4. This remains same for all soil. So you can keep this value constant. What is the total weight of the machine plus foundation system? It is 0.75 ton. So, see it is 0.25 ton is foundation weight and machine weight is 0.5 ton. So total is 0.75 ton. So it can be converted to 750 kg force. So 1.75 into 10 to the power minus 3 is the value of gamma, we have considered. What is r? That is 46.35. So, this is kg force, this is kg force per centimeter cube, this is in centimeter. So, centimeter cube we are getting. That is why finally, we are getting a dimensionless ratio as it is expected.

So, let us calculate this and see how much it is coming? It is 0.81. Similarly, the other 2 values of B z, also we can calculate B z for G equals to 100. The change will be only here. It will become 1.8, as we have considered all other things remains constant. So, how much we are getting? 0.785, right? And B z for g equals to 200, that will be 2.0. It is the only change. So it is coming 0.706.

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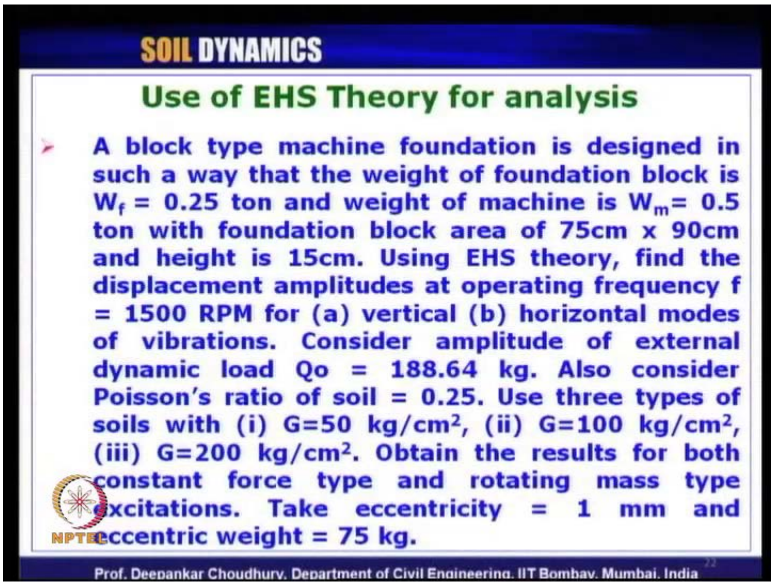

Now, next step is to calculate dimensionless frequency ratio because now we are handling with the operating frequencies. So, we need to calculate a naught. What is the expression for a naught?

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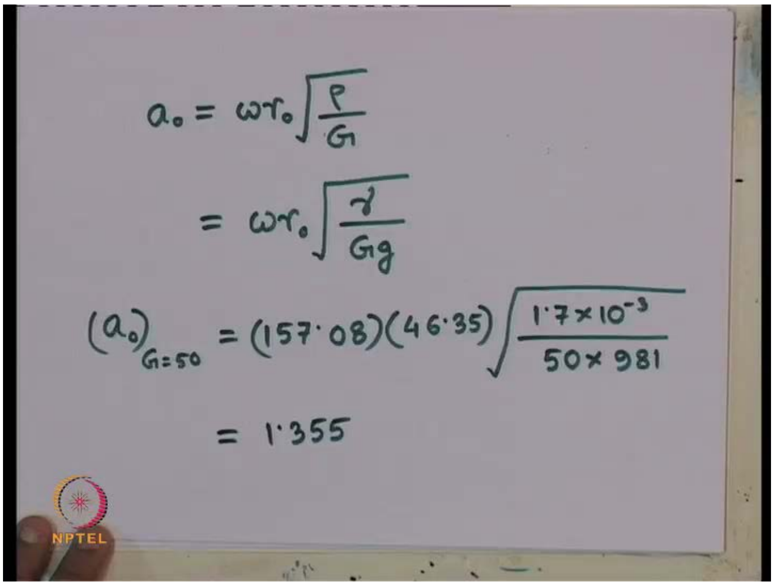

It is, a naught equals to omega r naught root over rho by G, Isn't it? So, omega r naught root over gamma by capital G small g. Small g is acceleration due to gravity. Therefore, for three different types of soil we will get 3 values of a naught that is, a naught for G equals to 50, the first type of soil. How much we will get? We have calculated omega as

157.8, r naught is 47.35 centimeter, gamma for this type of soil is 1.7 into 10 to the power minus 3, G is 50 and small g is 981 centimeter per second square. See if we calculate this, how much we are getting? We are getting 1.355. For other two types of soil, similarly, we can calculate a naught for G equals to 100. How much is it coming? In this expression, what are the changes that will occur? This remains same, this remains same, this will become 1.8, this will become 100 and this remains same. So how much a naught we are getting?

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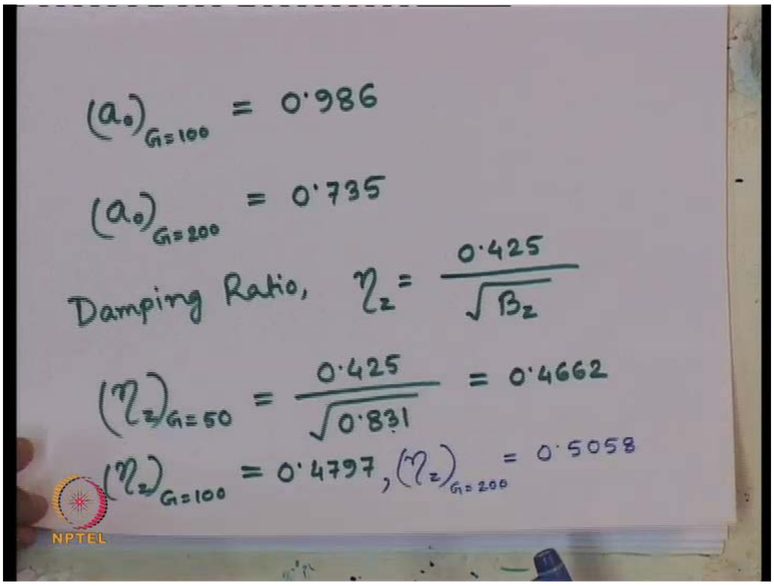

By calculating I am getting 0.986. Is it okay? And for the third type of soil a naught with G equals to 200, what is the value? Again the changes will be here, it will become 2, it will become 200.735. What is the next parameter we can calculate using this Lysmer's simplified analog? It is damping ratio. So, damping ratio for vertical mode of vibration, what is the expression? Eta z is equal to 0.425 by root over B z. B z already we have calculated for three different types of soil. So the eta z for three types of soil, we can easily calculate. For G equals to 50 it will be 0.425 by root over Bz. How much was the B z at G equals to 50? It was 0.831, right?

So, how much is damping ratio? We are getting 0.4662. Similarly, eta z for G equals to 100 is how much? B z for 100 was 0.785. So put here 0.785. Let us see. It is 0.4797, right? And the eta z for third type of soil G equals to 200, is how much? B z for G equals to 200 was 0.706. So put here 0.706, we will get the value 0.5058.

Now you can note one thing here. The same problem when we have solved using Mass Spring Dashpot model, what was our damping ratio we assumed? We assumed as 25 percent, 0.25, whereas, using Elastic Half Space model using this Lysmer's analog, the damping ratios are very high, about close to 50 percent, is just about the double, why? This is because in Mass Spring Dashpot model the elastic theory of both soil and machine plus foundation together was not considered effectively.

That is why the effect of damping ratio, which was considered was only for soil and that was assumed also. Whereas, if we use the Elastic Half Space model of Lysmer, in that case it will consider not only the damping ratio of the soil, but the entire system which is vibrating, which I had shown in the dynamic Boussinesq Model. However, after shifting a load it can be considered as the total system and the damping ratio of the entire system is considered in this model. So, this is another value of damping ratio which needs to be considered for the design using Lysmer's model. Next is, let us calculate the spring constant. Spring constant for vertical mode of vibration which is k z .What is the expression?

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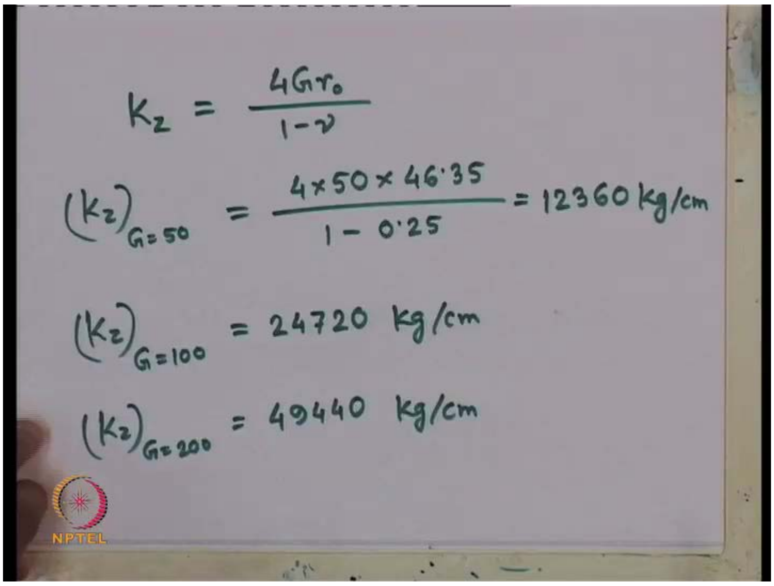

It is 4 G r naught by 1 minus mu. So k z for first type of soil with G equals to 50 is how much? It is 4 into 50 into r naught, where r naught is 46.35, divided by 1 minus Poisson's ratio, which is 0.25. It is coming 12360 kg per centimeter and value of k z for other 2 types of soil, that is G equals to 100, only thing this is linearly proportional. So, when it will become 100, it will be double of this, so 24720 kg per centimeter and k z with G equals to 200 will be again double of this so, 49440 kg per centimeter. Now operating frequencies are given to us. What about the natural frequency of the system, that we need to calculate, right?

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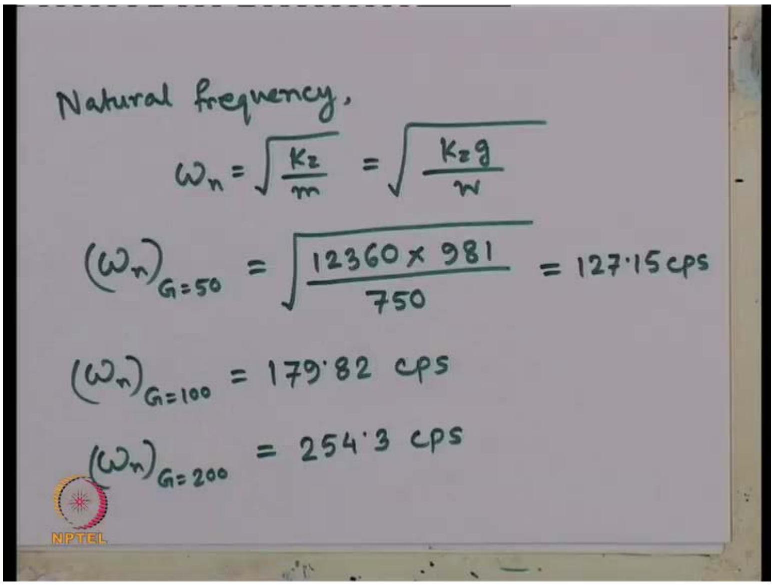

To calculate the displacement amplitude, we should know the frequency ratio. Operating frequencies are given. So, let us now calculate the natural frequency. So, natural frequency of the system omega n is nothing but that k z by m, so k z g by w. Therefore, for three different types of soil omega n with G equals to 50, that will give us, how much is k z? We have calculated just now 12360. G is this unit centimeter per second square is 981 and w is 750 kg force.

How much it is coming? It is 127.15 cycles per second. Also, omega n for other 2 types of soil with G equals to 100, we have to put k z for G equals to 100, other 2 things remains same. So, it is coming 179.82 cps. Am I right? Also, omega n for G equals to 200 will be, k z will change for G equals to 200, other 2 remain same, it is coming 254.3 cps. So, with this, we will stop our lecture today here. We will continue further in the next class.