Soil Dynamics
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Module - 5
Machine Foundations
Lecture - 29
Problems on use of MSD Model for Analysis
Rocking Mode of Vibrations

Let us start our lecture on soil dynamics. We are continuing with our module 5 that is machine foundations. In the previous lecture, we had solved two problems; one for vertical mode of vibration and another for sliding or horizontal mode of vibration. How to design machine foundation rather than, I should say design it was basically a foundation size was given to us also the dynamic loads etcetera were proved to us.

It was a cross check type of problem or prove check type of problem, where the displacement amplitude has to be cross checked with respect to the permissible limits. As per the permissible displacement guideline, also as per the qualitative assessment that is as per Richart’s chart.

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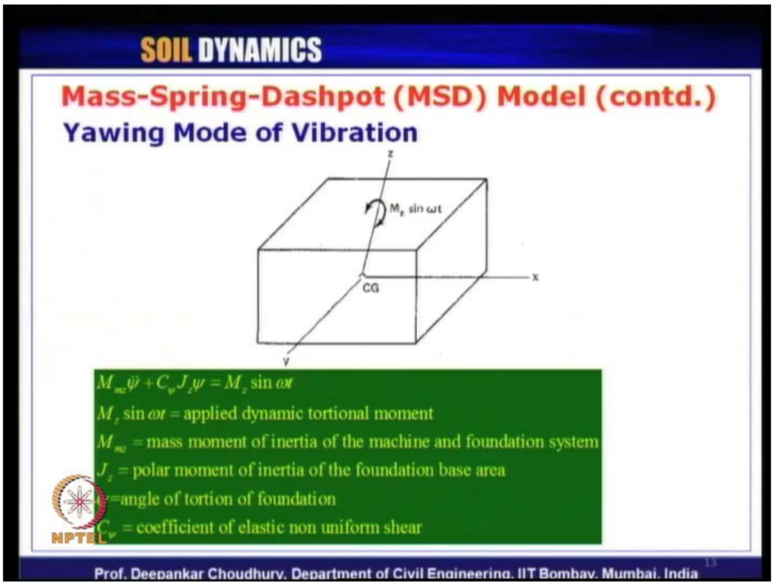

So, continue with similar type of problem using mass spring dashpot model, what we had discussed in the previous lecture a quick recap of that.

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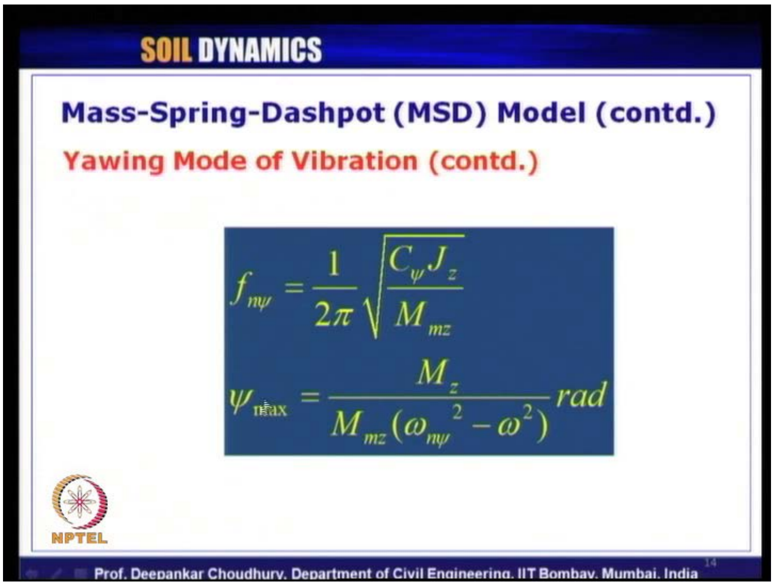

We had discussed about yawing mode of vibration using mass spring dashpot model or MSD model and the equation to compute the natural frequency of the system also the amplitude of maximum rotation in case of yawing.

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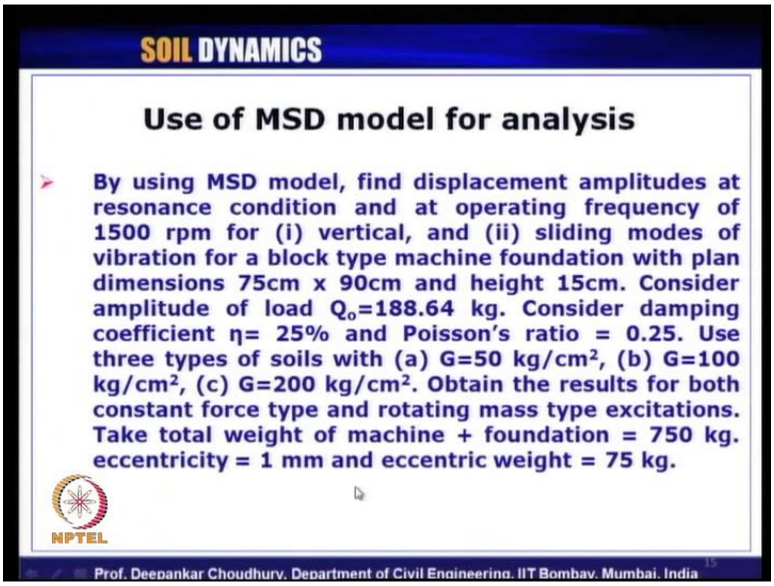

Then we had solved this problem for both vertical and sliding mode of vibration, we have computed displacement amplitude both at resonance condition as well as at operating frequency condition, for two different types of foundation that is constant force type and rotating mass type excitations with three different types of soil having G values like this.

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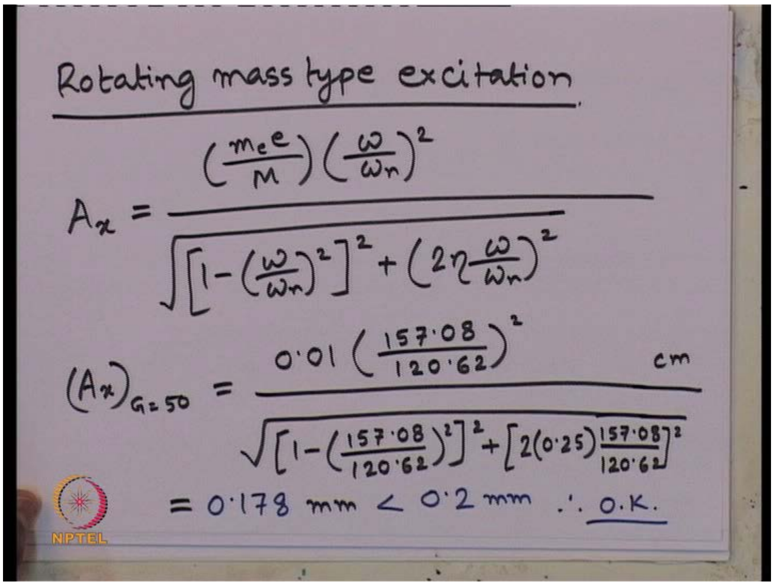

So, now let us go for rotating mass type excitation, this is for constant force type excitation we had completed. For rotating mass type excitation what is the expression to compute the value of A x? That is given by m e e by M, omega by omega n whole square by root over 1 minus omega by omega n whole square whole that square plus 2 eta omega by omega n whole square.

This is again from our module two vibration theory we get this expression, basically it remains same with the vertical mode of vibration also only changes are wherever the natural frequency etcetera has been changed. This mass disbalance is creating the horizontal type of vibration that is another important point to be noted, though we have kept in the problem the amplitude of load same. That means for vertical mode of vibration it is acting vertically, whereas for horizontal or sliding mode of vibration it is acting horizontally that is the only change.

So, for A x at g equals to 50 first type of soil, m e e by M we had considered already calculated as how much? 0.01 centimeter, right? Omega is 150 7.08 and omega n for g equals 20 we got 120.62 whole square, root over 1 minus 150.08 by 120.62 whole square that square plus 2 0.25 times 157.08 by 120 0.62 that whole square, so much of centimeter. If we calculate this in terms of millimeter, it comes out to be 0.178

millimeter which is less than our permissible limit of 0.2 millimeter, therefore the design is okay.

And once again as I have mentioned for vertical case here also, A x value remains same in both constant force type and rotating mass type provided the exciting load remains same in both the cases, as it is the case here. Also we need not to calculate always like this we can just simply say that as the exciting load is same in both the cases the displacement amplitude at operating frequency will be same. Hence, for other two cases.

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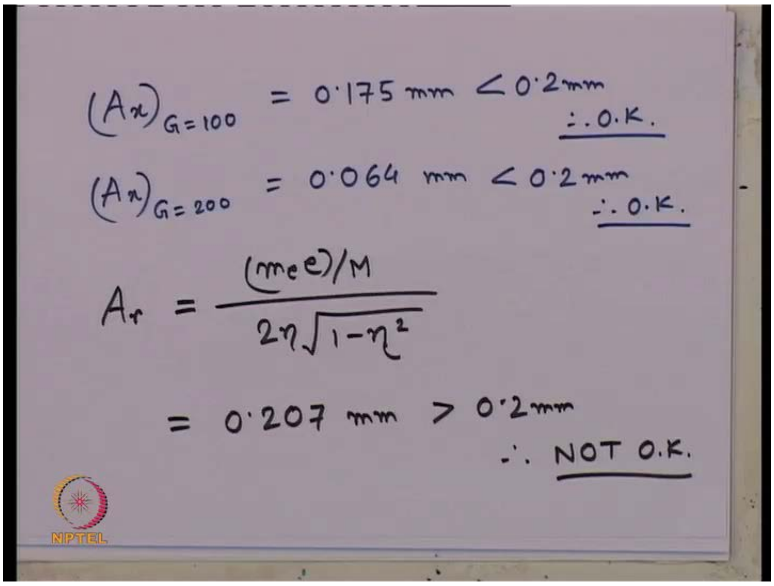

That is with A x at g equals to 100 t s f will be same the previous case 175 millimeter less than 0.2 millimeter. Therefore and A x for g equals to 200 t s f is 0.064 millimeter, which is less than 0.2 millimeter. Therefore the design is okay. This is at operating frequency, now the same exercise we can do for resonant frequency condition also. For that we have seen for rotating mass type the expression for displacement amplitude at resonance is given by m e e by M divided by 2 eta root over 1 minus eta square.

So, it is remains independent of soil type, so this will gives us a single value irrespective of three types of soil. So, if we put the values how much we are getting, so much of millimeter which is again greater than 0.2 millimeter. Therefore, not okay. The design is not okay at resonance condition. So, when you are checking any design as a consultant in the design office, as a design engineering you should check both the conditions. As I have mentioned both at operating frequency that mass to be always less than the permissible limit also at resonance frequency preferably that also should be within the permissible limit.

Then you are safe and ensure that your machine foundation is never going to exit the permissible displacement and it is safe throughout whether machine is picking up the speed or starting from 0. Then slowly building up the r p m or something like that, but still your machine foundation will remain safe. So, these are the checks needs to be done and that can be easily done by using the simple mass spring dashpot model, what we had learnt in our module 2 also in vibration theory.

So, both no resonance criteria as well as the displacement criteria and also remember all these displacement calculated you can cross check using your Richart chart for a qualitative assessment. So, you can check in which zone this points are lying? Suppose, if you refer back to that Richarts chart at this case our operating frequency is 1500 r p m. So, in x axis you go for 1500 r p m, then you have already computed. Suppose a z the vertical displacement in millimeter unit and whatever Richarts chart is given in the corresponding f p s unit you have to convert it.

Then in the y axis, you find out that value you can match. These two values and find out which point it is coming and that point is located in which zone? That is whether it is not noticeable to persons or very (( )) to persons in which zone it is lying, based on that you can again make another comment based on this qualitative assessment based on Richart chart. So, that also needs to be checked as well as commented in the design report. So, with this we will continue further with our different models in the next lecture.

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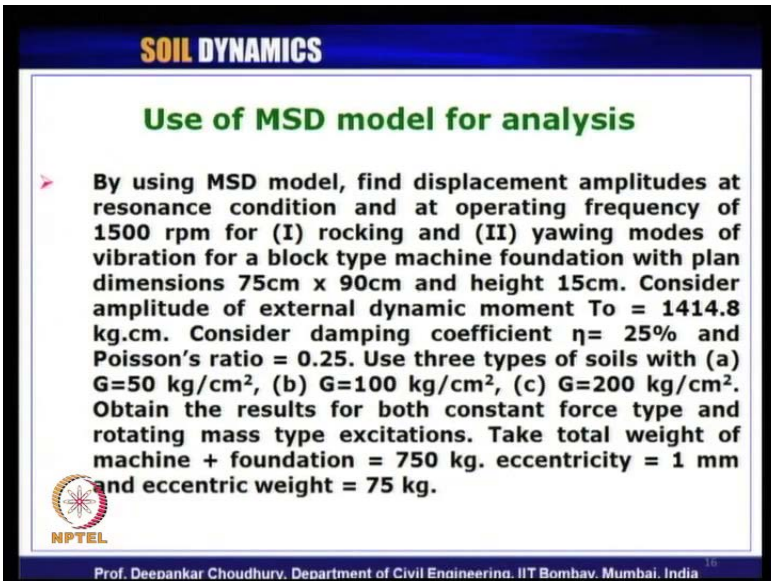

Now, today let us solve another similar problem that is by using mass spring dashpot model, find the displacement amplitudes at resonance condition also at operating frequency of 1500 r p m for rocking and then yawing modes of vibration for a block type machine foundation with plan dimension of 75 centimeter by 90 centimeter and height of 15 centimeter. It is given to us that the external dynamic load or dynamic moment amplitude is given as 1414.8 kg centimeter. Of course, this is kg 4 centimeter.

Consider damping ratio or damping coefficient is 25 percent, Poisson's ratio is 0.25 again for three types of soil with g value as 50 kg per centimeter square 100 kg per centimeter square and 200 kg per centimeter square. We have to obtain this displacement amplitude for both constant force type as well as rotating mass type excitations.

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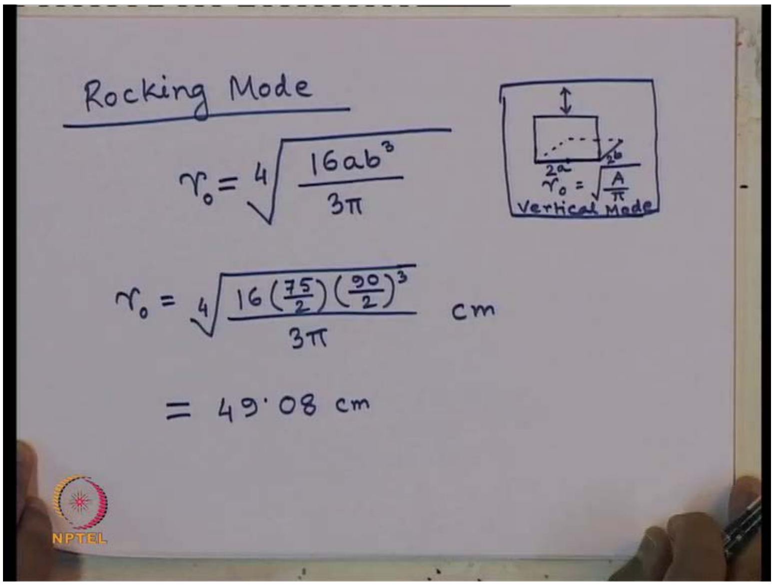

And for rotating mass type excitation it is given that eccentricity and eccentric weight are given to us, whereas the total weight of machine plus foundation system is 750 kg forces provided to us. So, let us start with rocking mode of vibration solving the problem, in rocking mode we have to find out the equivalent radius of a circular base. In this case the r knot is not like previous equation, that is root over a by pi not like that, why?

In the previous case for vertical or horizontal mode of vibration let me show you here, when a block type machine foundation is subjected to a vertical mode of vibration in that case what we did? This plan area we had considered and to find out equivalent radius r not equation, we had used root over A by pi, where A is area of this plan area. In the case of vertical vibration that is the projected area, whereas in case of rocking mode, this equation cannot be used. This was for vertical mode, in case of rocking mode it is going to rock about say a central line of this axis, so we have to find out equivalent radius by equating not the area, but area moment of inertia for a rectangular footing with a circular footing, is that clear?

So, in this case what we are finding out? The r knot should be by equating the area moment of inertia, so the formula gives us 16 a b cube by 3 pi where this dimension is 2 a. This dimension is 2 b, in that case this is the formula which we equate the moment of inertia for a circular footing with respect to a rectangular footing, so in this case r knot will be 4 root of 16 75 by 2 and 90 by 2 whole cube by 3 pi. So, much of centimeter, because this 75 centimeter and 90 centimeter are the total dimensions in x and y direction.

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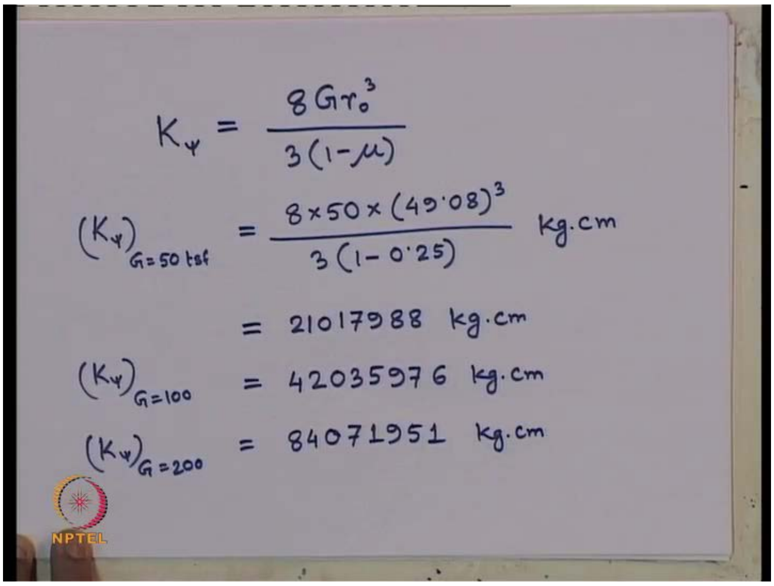

So, for rocking this will be the equation, whereas for pitching it will be a cube b. So, if you compute this, how much we are getting r knot is coming as 49.08 centimeter, so check with our yesterday or previous lectures calculation, what we got in our previous lecture r knot for vertical and horizontal mode of vibration it came about 46.35 centimeter was the equivalent radius of a circular footing for the case of vertical and horizontal mode of vibration, whereas for rocking mode it has changed to 49 centimeter, so we have to remember this expression and how we are obtaining that also I have explained just now.

And from theory of elasticity to obtain the expression for k let us denote it as k psi as for rocking, earlier we have mentioned k psi for yawing so now I am changing the symbol so it is just the change of symbol nothing else, but still we are discussing the rocking mode, because in Richarts book where we are finally checking the dimensions there k psi is used as a rocking symbol it does not matter it is just the matter of symbol.

So, for the case of rocking mode of vibration k psi can be computed using this expression 8 G r knot cube 3 times 1 minus mu, this is again obtained using theory of elasticity given in Timoshenko and Budierso, so in this case G is shear modulus of the soil, r knot is equivalent radius of circular footing, mu is Poisson's ratio, now for three types of soil let us find out k psi for this 50 t s f how much we are getting, 8 into 50 into r not just now we have computed 49.08 divided by 3 into 1 minus Poisson's ratio is given as 0.25.

So, much of what should be the unit? Unit is k g centimeter, Unlike earlier the unit of k was k g per centimeter, because that was linear spring, where are these are rotational spring ok, so note it that the unit has change now because these are rotational spring constant not the translational spring constant, so how much we are getting for this first type of soil 21017988 so much of k g centimeter right, for other two types of soil G equals to 100 it will be again linearly related with respect to G when r not and mu remains same so just double of this 42035976 kg centimeter and k psi for G equals to 200 will be double of this which is 84071951 kg centimeter ok.

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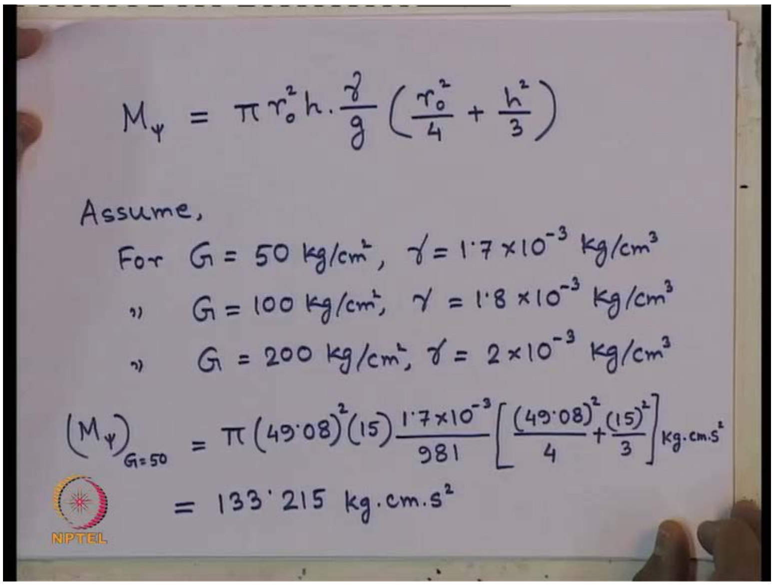

Now, what is the mass moment inertia that we need to find out m psi we need to find out to obtain the natural frequency of the system, now m psi can be calculated mass moment of inertia using this expression pi r knot square h times gamma by g times r knot square by 4 plus h square by 3, what are different components let me explain to you r knot already we have mentioned this is the equivalent radius of the circular footing, h is the height of the footing which is 15 centimeter given to us, gamma is unit weight of the soil, g is acceleration due to gravity, so this is basically will come the density and r knot is already defined, h is also already defined, in this case what we do not know gamma value we do not have with us right.

So, we have to assume some suitable value of gamma corresponding to different types of soil, so let us assume for G equals to 50 kg per centimeter square first type of soil, gamma is 1.7 into 10 to the power minus 3 kg per centimeter cube, remember this k g is k g force that is why it is unit weight we are considering, for second type of soil G equals to 100 kg per centimeter square let us assume gamma as 1.8 into 10 to the power minus 3 k g per centimeter cube and for g equals to 200 kg per centimeter square the gamma value is 2 into 10 to the power minus 3 k g per centimeter cube.

These are the based on how we have assumed this gamma value reasonable value we need to assume, as G increases we know it is the steeper soil so that is why we are using standard unit weight of soil in increasing order as the G increases, so with this now let us go back to this equation to compute the mass moment of inertia, so m psi for G equals to 50 should be how much let us put the value pi r we have calculated 49.08 whole square, then h is 15 centimeter giving to us, gamma is 1.7 into 10 to the power minus 3 in k g force power centimeter cube unit we have to divide it by g in centimeter per seconds square unit which is 981 times r knot 49.08 whole square by 4 plus h is 15 square by 3 and what is the unit if you put all the units you will get it will give us kg, centimeter, second square.

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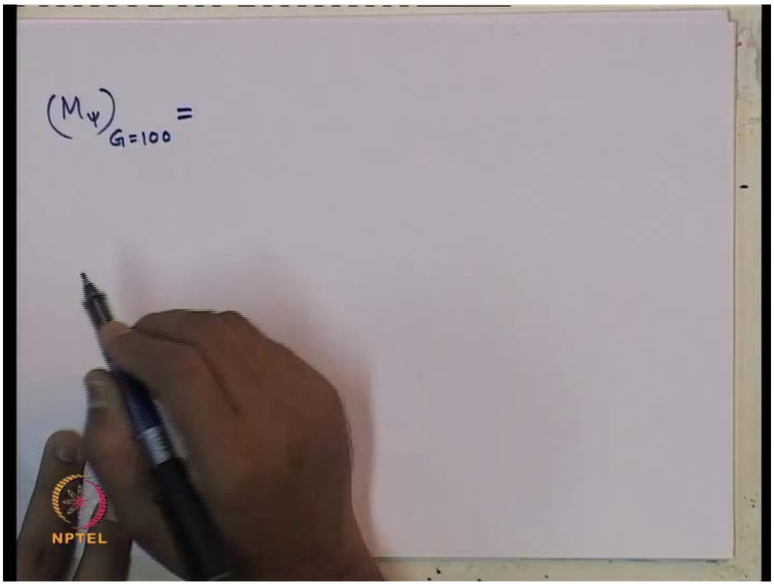

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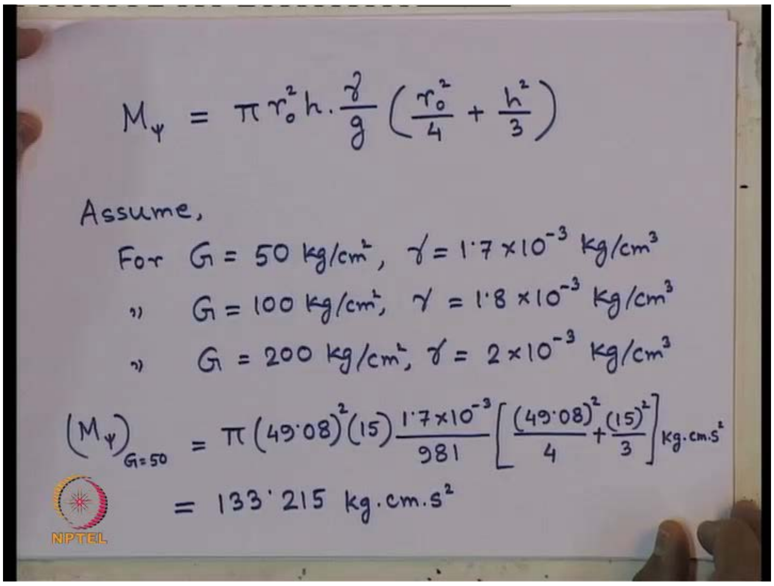

Now, let us calculate this expression and see how much we are getting 133.215 kg centimeter second square, so that is for first type of so, now similarly let us find out for other two types of soil the mass moment of inertia m psi for g equals to 100 will be in this expression what are the changes now we should make r remains same, h remains same, gamma changes as the g changes, so it will be now 1.8 rest of the thing all remains same,.

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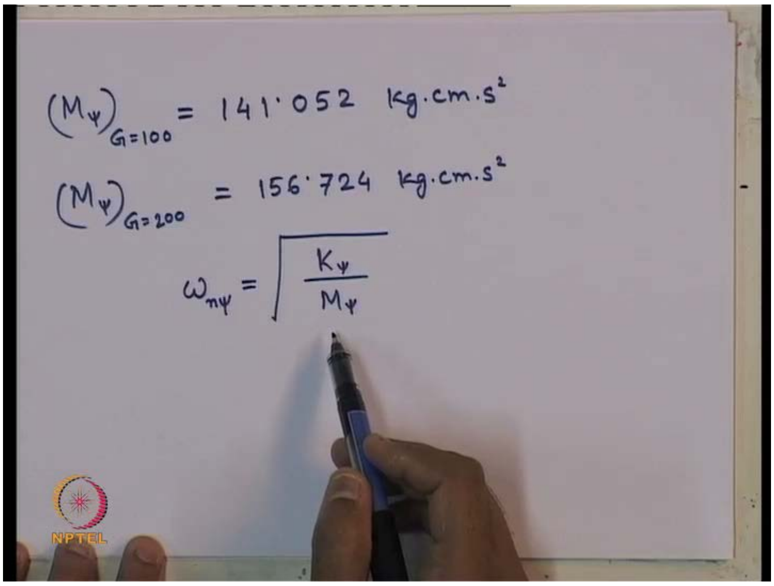

So, only change is in the value of gamma 1 41.052 and for third type of soil with G equals to 200 again the change will be only in gamma value 156.724 kg centimeter second square, why we have calculated these values because we know the natural frequency of the system omega n psi should be equals to root over k psi by m psi, am I right that rotational spring constant divided by mass moment of inertia.

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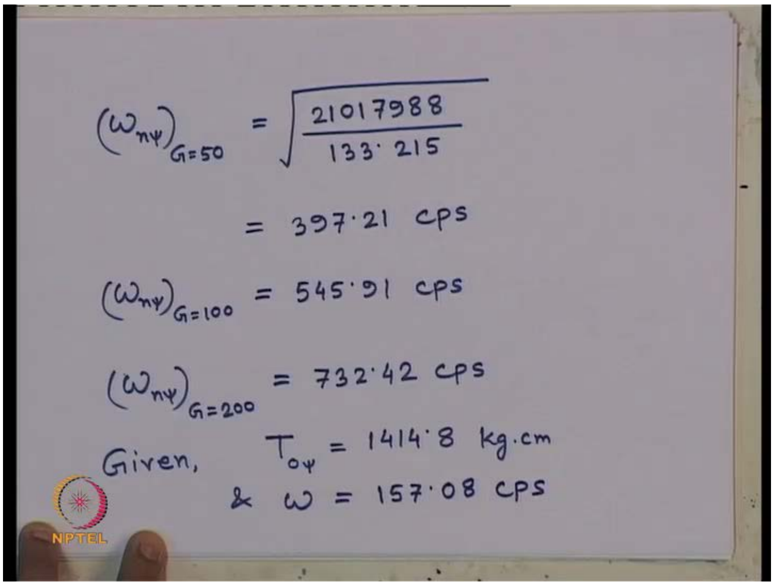

Therefore let us compute omega n psi for let us compute omega n psi for first type of soil that is G equals to 50 that will be k psi how much we had calculated 21017988 divided by m psi how much we got 133.215, so if we calculate this we are getting 397.21 cycles per second, then omega n psi for G equals to 100 the next type of soil, how much we are getting? This value will be double and this one will become now 141.052 by putting those values we are getting 541.91 c p s and omega n psi for third type of soil G equals to 200 this will become four times and this value will now be 156.724 as we have calculated.

So, how much it is coming 732.42 c p s so these are the natural frequencies of the system and what are the values given to us, given that t knot psi is 1414.8 k g force centimeter of course the amount of amplitude of moment externally applied and what is operating frequency omega we have calculated in the previous problem also it will remain same for fifteen hundred r p m it is 157.08 c p s right.

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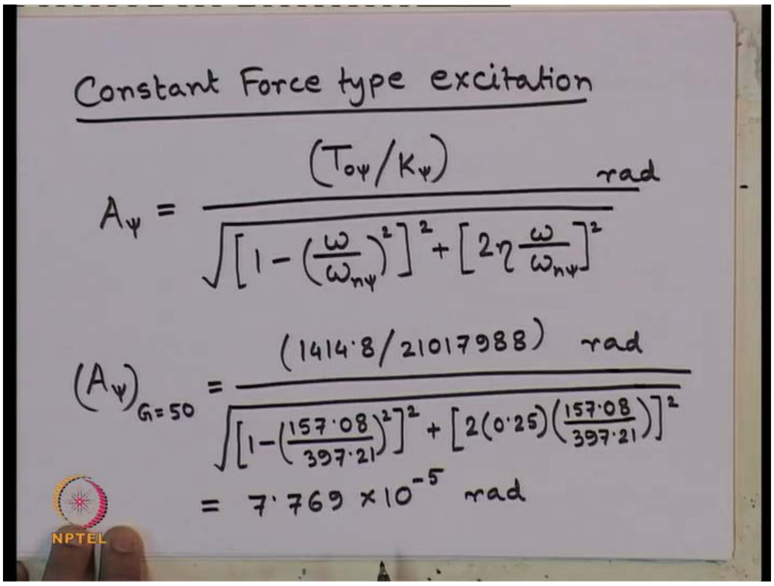

Now, let us do the calculation for both constant force type and rotating mass type, so for constant force type excitation what is the expression for A psi that is the amplitude of rotational displacement is equals to t knot psi by k psi divided by root over 1 minus omega by omega n psi whole square that square plus 2 eta omega by omega n psi whole square.

So, this is the expression as we had obtained earlier also in our module 2 that is vibration theory, the expression for displacement amplitude remains similar only the type of forces etcetera changes and in this case this is moment by the rotational spring constant, so whatever value we will get that will be guided by this expression here it will come in which unit radian so much of radian, because it is a rotational displacement amplitude.

So, for first type of soil that is a psi with G equals to 50 should be how much this value is 14114.8 by k psi is 21017988 divided by root over so much of radian 1 minus omega is how much 157.08 by omega n psi for G equals to 50 we obtained as 397.21 whole square that square plus 2 into damping ratio is 0.25 times 157.08 by 397.21 that whole square.

All the values where from we got this and let us calculate this how much we are getting, so it is coming about 7.769 into 10 to the power minus 5 radian and when we need to check this value as we have mentioned how this rocking mode of vibration the displacement is going to take place.

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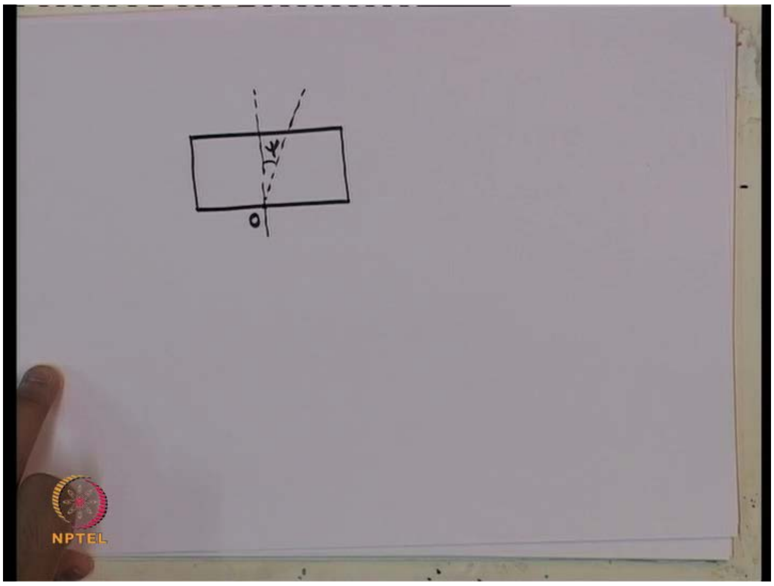

If we try to look at the picture the block type of machine foundation is something like this about this point it is going to be rotated by this angle if we say psi is the rocking angle about this point o it was rocking right that is what we had seen earlier also in the picture.

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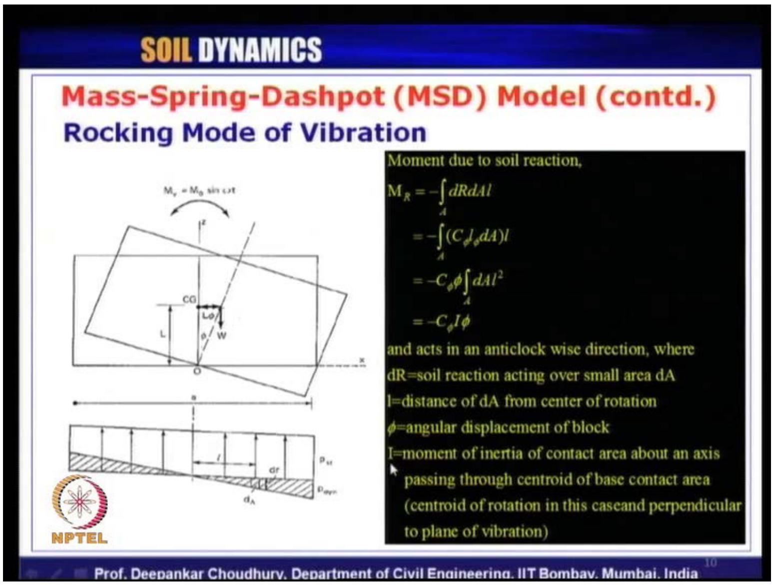

Let us look at the slide here so that it will be clear to us once again look at here, so for rocking mode of vibration it is rocking about its base so this much is the maximum amplitude, so how is the linear displacement is going to occur you can multiply it with respect to the height of the foundation, that will give you how much this displacement is going to occur here in this direction, that needs to be checked with respect to 0.2 millimeter and with respect to the Richarts chart for qualitative assessment so remember that.

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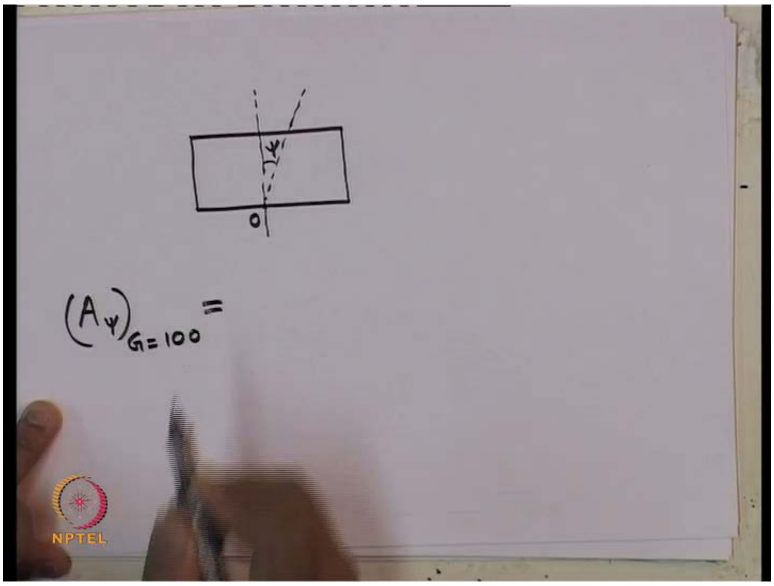

So, let us find out the other two cases for how much is A psi we are getting, similarly in the previous expression what are the changes now will occur let us see.

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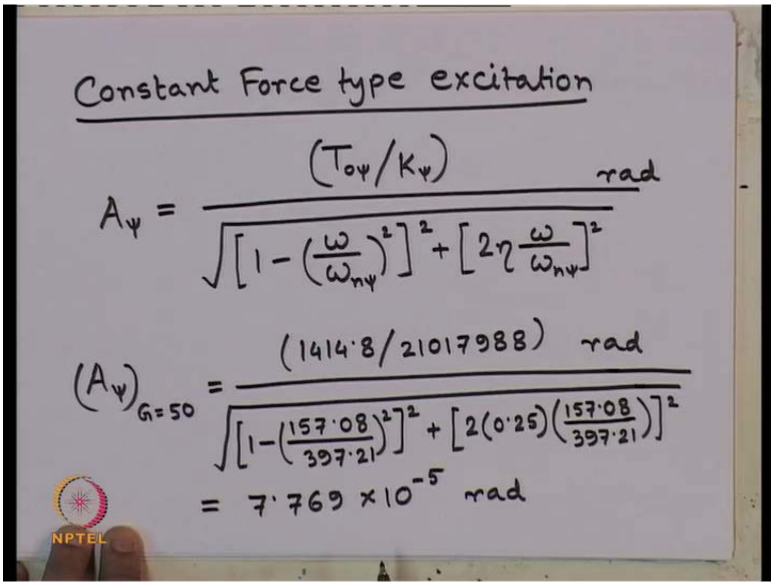

This t knot psi remains same, k psi changes as the soil changes to 100 this will became double, omega remains same, whereas omega n psi change to 545.91 here also eta remains same ,omega remains same ,omega n psi changes to 545.91.

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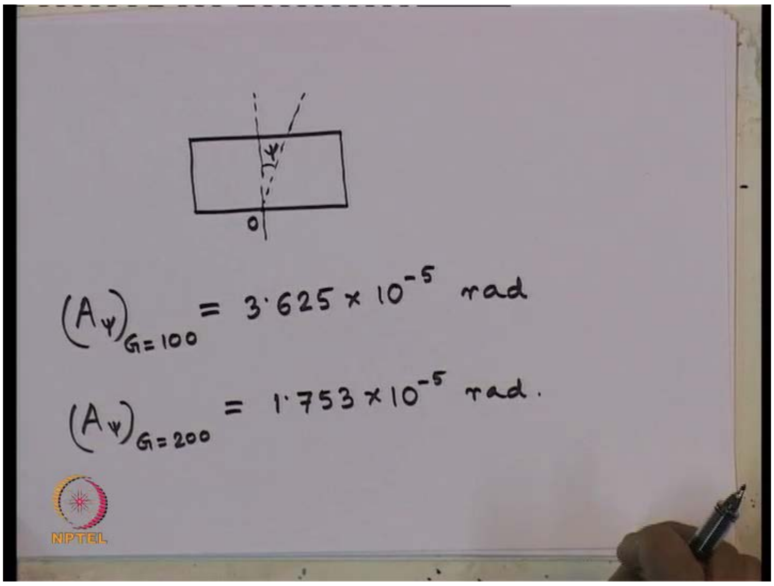

So, by putting those values how much we are getting for A psi 3.625 into 10 to the power minus 5 so much of radian, also for the third type of soil with G equals to 200 if we put the corresponding value of k psi and omega n psi for third type of soil G equals to 200 kg per centimeter square we will get 1.753 into 10 to the power minus 5 so much of radian, this displacement what we have computed those are at operating frequency.

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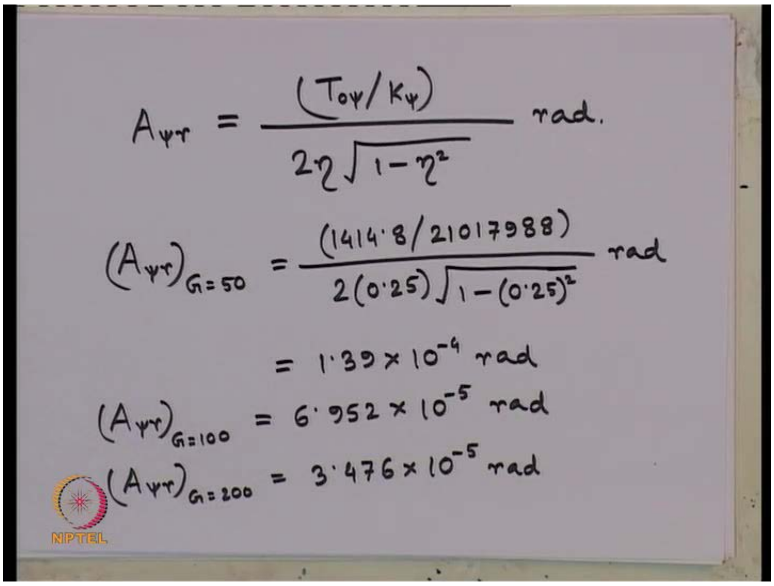

Now, we need to find out the displacement amplitude at resonance frequency also, at resonance frequency based on the expression of d m f we can find out how much is A psi r means at resonance, the expression for that will be t knot psi by k psi by 2 eta root over 1 minus eta square, that is the expression from d m f we got.

Again it will be guided by this unit which will give us radian, so let us compute it A psi r for three different types of soil for G equals to 50 kg per centimeter square it will be 1414.8 by 21017988 divided by 2 into eta is 0.25 root over 1 minus 0.25 whole square so much of radian, if we calculate this how much we are getting 1.39 into 10 to the power minus 4 radian.

And for other two cases at G equals to 100 the changes will be this remain same k psi will become double, this remains same, so it is basically linearly will decrease by half from this value, so it will be 6.952 into 10 to the power minus 5 radian and a psi at resonance for G equals to 200 kg per centimeter square will be half of this again 3.476 into 10 to the power minus 5 radian.

And one observation which you must have noticed the values of A psi r will always be greater than the values of A psi, which is quite obvious that is displacement amplitude at operating frequency will be less than the displacement amplitude at resonance, so this is a kind of cross check you can do you can check whether the obtained or calculated values are really for resonance condition are bigger than the operating frequency or not if it is not then probably some calculation mistake somewhere we have done, so for a design or a design check this way we have to carry out the calculations so this completes the rocking mode of vibration case.

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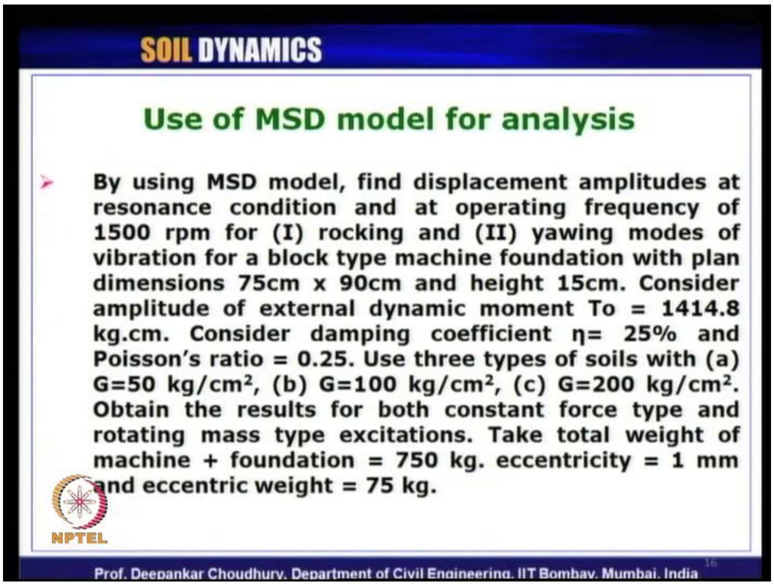

Now, let us move to next case that is this yawing mode of vibration let us come to now the rotating mass type excitation, we have calculated only for the constant force type excitation, so for rocking mode we have to now do the calculation for rotating mass type excitation.

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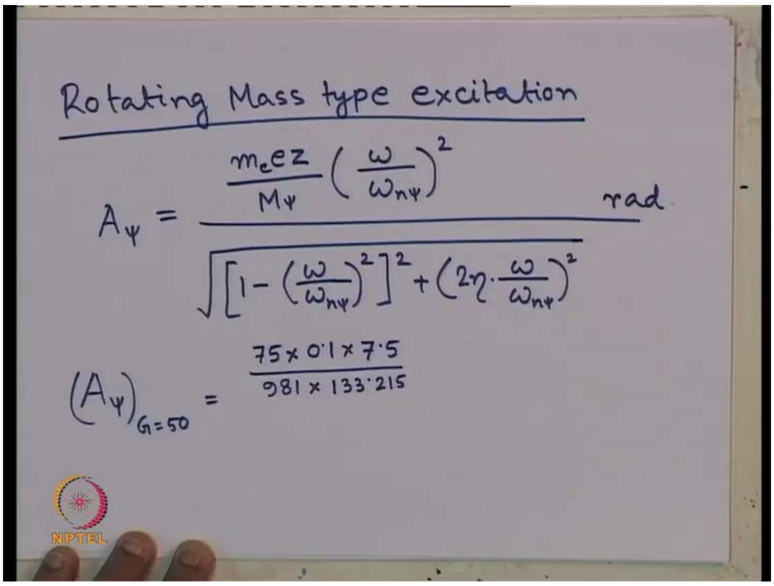

For rotating mass type excitation what is the expression for A psi that is m e e z by M psi times omega by omega n psi whole square by root over 1 minus omega by omega n psi whole square that square plus 2 eta omega by omega n psi whole square, this is the expression to get the displacement amplitude rotational displacement amplitude for rocking mode in case of rotating mass type excitation, these are well understood where from this has come this expression you can equate it with respect to in case of vertical and sliding mode it was m e e by m that is eccentric mass eccentricity divided by mass of the total system.

In this case it will change to equivalent moment because now it is the rocking displacement or rocking mode of vibration, so the external load which is applied on the system has to be a movement so how much is the moment applied because of the eccentricity that is m e e and z how much is z, z we can consider with respect to c g how much it has displaced so in that case z will be half of the height of the foundation.

So, that is the standard assumption we can take z as the h by 2 that is with respect to center of gravity of the system if we consider the eccentric mass is creating this much of momentum with respect to f c g and M psi already we have calculated that is mass moment of inertia, so instead of mass now mass moment of inertia has to be considered here again the unit of this will be guided by this unit which will come as radian, so for the first type of soil A psi for G equals to 50 kg per centimeter square how the calculation will be done 75 then 0.1 then 7.5 is h by 2 divided by 981 into M psi how much we got for G equals to 50 k g per centimeter square 133.215 where from this 981 has come.

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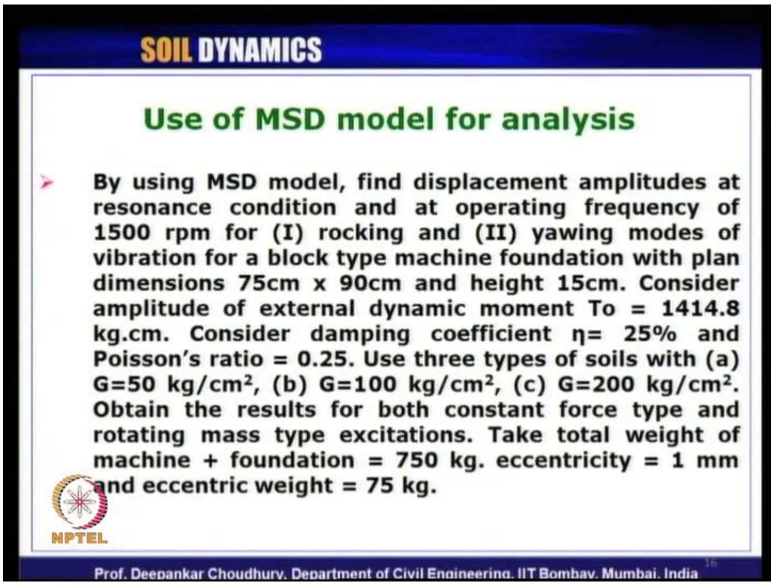

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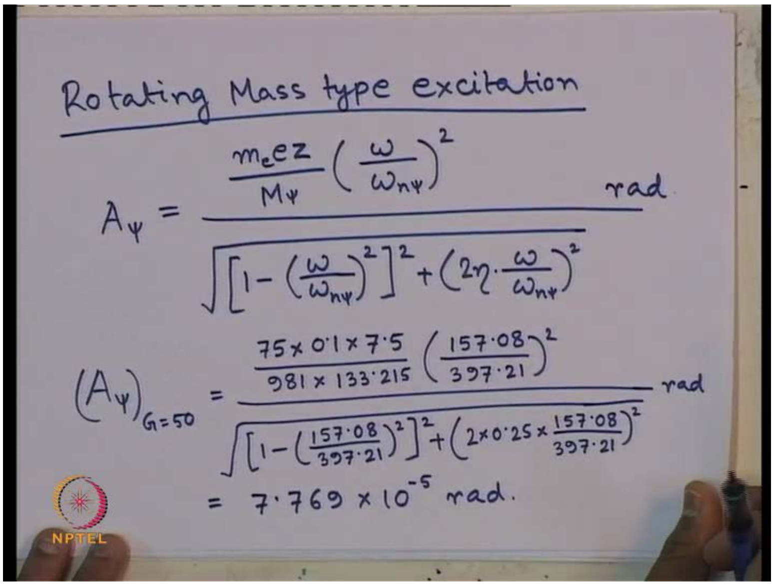

This was eccentric weight is given to us if you go back and look at the problem statement look at the slide, your eccentric weight is 75 kg as I have said weight means k g force, so to find out eccentric mass we need to divide it by G so that is why we have divided it by 981 here, omega is how much 157.08 operating frequency and omega n psi for g equals to 50 we had already calculated 397.21 that square now divided by root over 1 minus 157.08 by 397.21 whole square whole that square times 2 into 0.25 into omega is 157.08 omega m psi 397021 whole square so much of radian.

Now, let us calculate this how much it is coming, it is coming 7.769 into 10 to the power minus 5 radian again the observation is that in rotating mass type case also we got at operating frequency, the rotational displacement is same as the constant force type provided the externally applied moment is same in both the cases, so if you calculate this how much external force is getting generated how you will calculate q knot was m e e omega square that has to be multiplied with respect to the (( )) you will get the moment is that fine,

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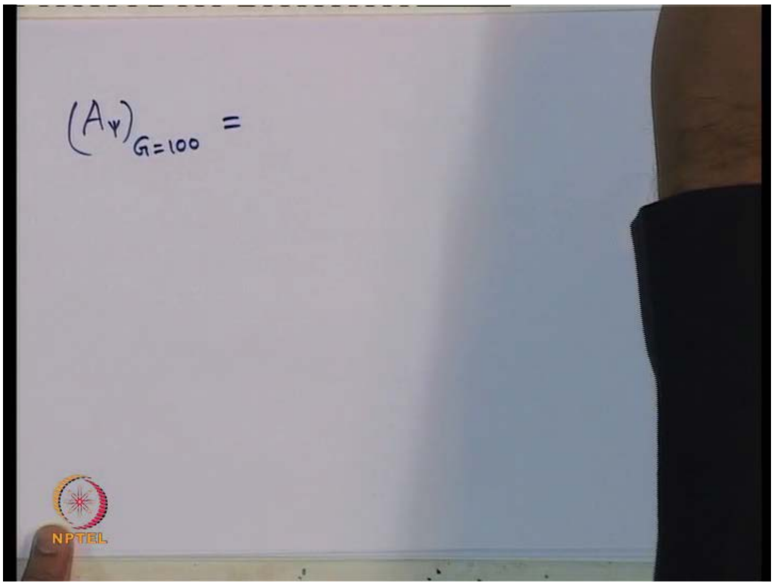

Now, let us calculate for other two cases of soil A psi for G equals to 100 what are the changes let us look here, this values will remain same only M psi will change as the soil type changes, also this omega remains same, but omega M psi will change as the soil changes, so by putting the corresponding value we should get the same displacement as we have obtained for the constant force type case.

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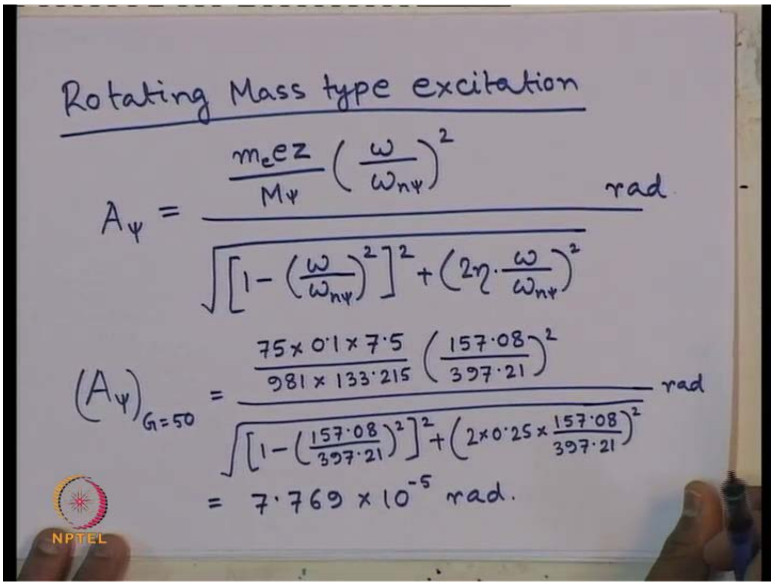

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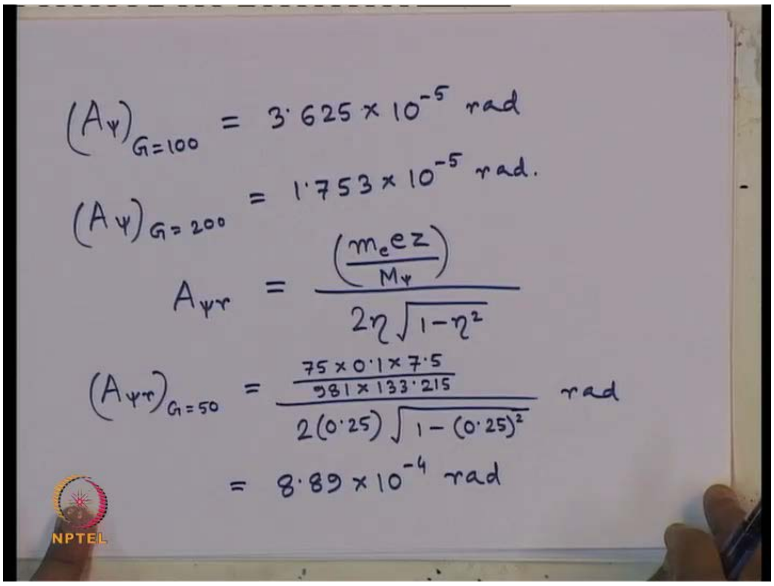

And A psi for G equals 200 is coming 1.753 into 10 to the power minus 5 radian, now let us find out how much is A psi r that is at resonance condition how much is the rotational displacement amplitude that is computed by using this expression m e e z by M psi divided by 2 eta root over 1 minus eta square, this is the expression once again we are getting from the expression of d m f, so A psi r for G equals to 50 will be 75 into 0.1 into 7.5 by 981 into 133.215 divided by 2 0.25 root over 1 minus 0.25 square so much of radian, if you calculate this we are getting 8.89 into 10 to the power minus 4 radian, which is obviously larger than at operating frequency how much we got A psi which is 7.769 into 10 power minus 5 radian.

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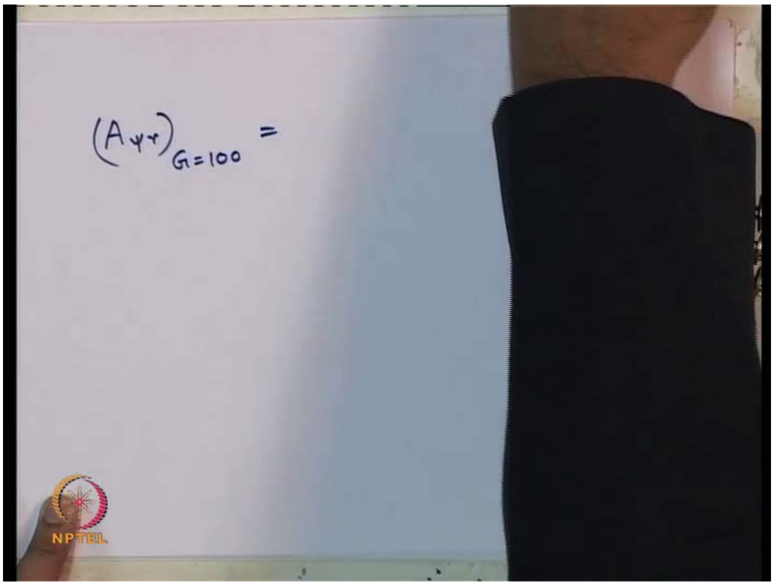

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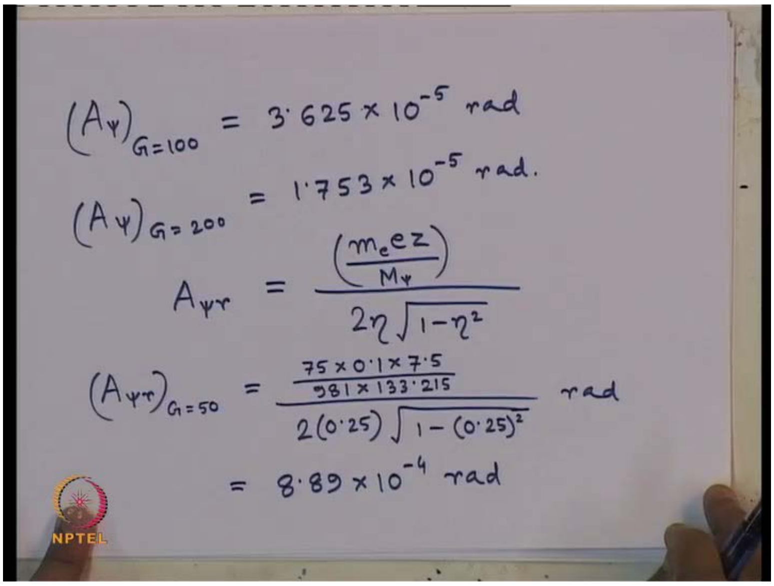

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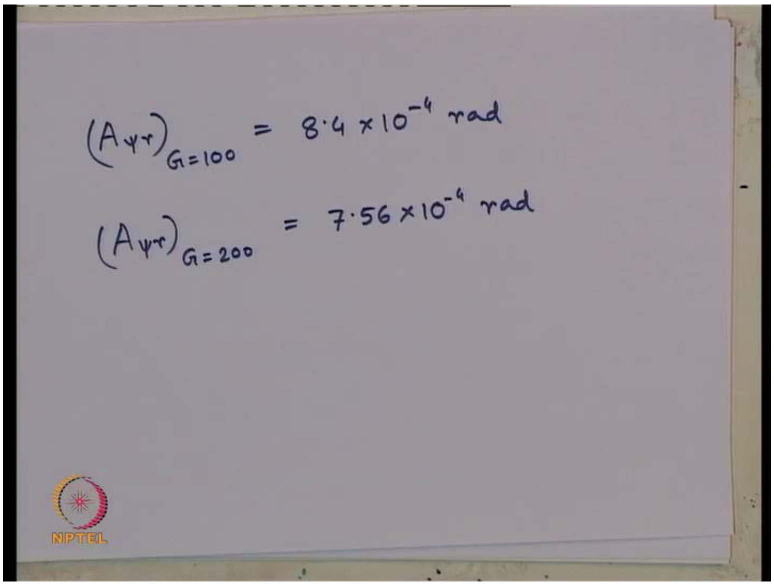

Now, let us compute for other two cases of soil that is G100 the changes will be only at this value of M psi will change all other things remains same, so by calculating we are getting 8.4 into 10 to the power minus 4 radian and A psi r for G equals to 200 we are getting 7.56 into 10 to the power minus 4 radian, so with this we are now actually completing this rocking mode of vibration results.

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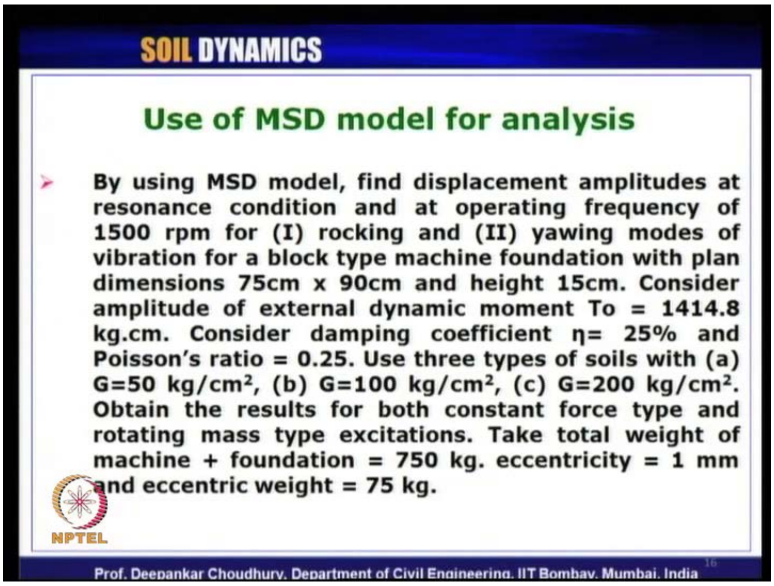

For rocking mode of vibration what are the displacement amplitude both at resonance condition and at operating frequency for constant force type and rotating mass type excitation with three different types of soil, so with this we will stop our lecture today here we will continue further in our next lecture.