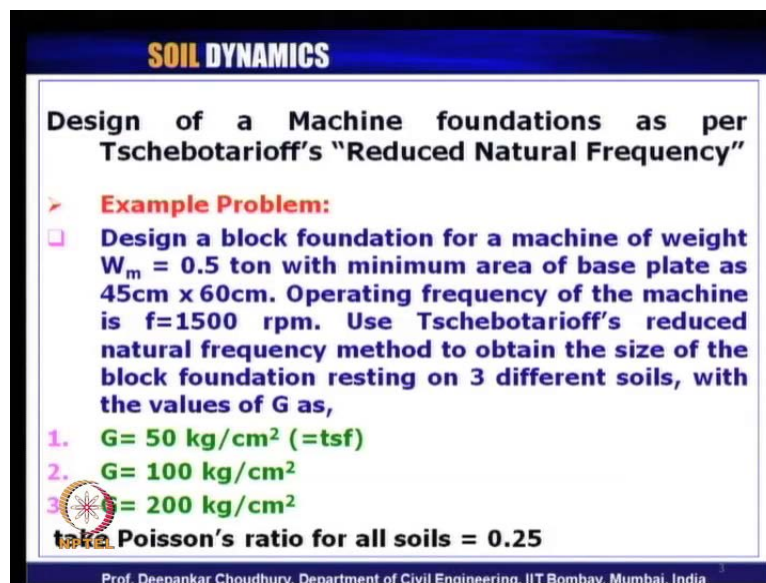


Soil Dynamics
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Department of Civil Engineering
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Module - 5
Machine Foundations
Lecture - 28
MSD Model - Yawing Mode of Vibration
Use of MSD Model for Analysis

(Refer Slide Time: 00:36)



SOIL DYNAMICS

Design of a Machine foundations as per Tschebotarioff's "Reduced Natural Frequency"

➤ **Example Problem:**

- ❑ Design a block foundation for a machine of weight $W_m = 0.5$ ton with minimum area of base plate as 45cm x 60cm. Operating frequency of the machine is $f=1500$ rpm. Use Tschebotarioff's reduced natural frequency method to obtain the size of the block foundation resting on 3 different soils, with the values of G as,
 1. $G = 50$ kg/cm² (=tsf)
 2. $G = 100$ kg/cm²
 3. $G = 200$ kg/cm²

take Poisson's ratio for all soils = 0.25

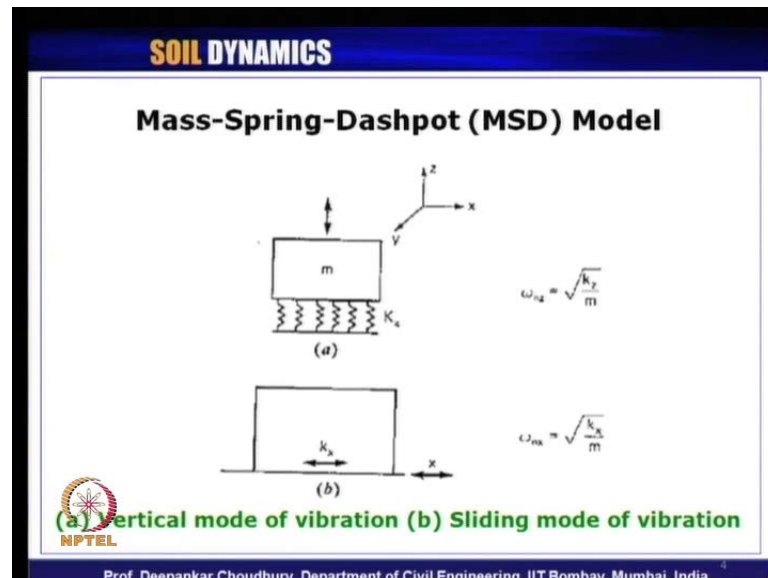
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Let us start out lecture on soil dynamics, we are continuing with our module 5 that is machine foundations. A quick recap of what we had studied in the previous lecture. We have solved a model example problem, for design of block type machine foundation using Tschebotarioff's reduced natural frequency method. The problem statement was given to us with the size of the base plate, then weight of the machine, operating frequency of the machine, and it was asked for three different types of soil, how we can design the machine foundation.

And after doing all the design analysis using Tschebotarioff's reduced natural frequency, we found that expect for one case in most of the other cases. The minimum required area based on the dimensional criteria, that is the minimum base plate size with 15 centimeter clearance all around. And minimum thickness of the footing, whatever is required that is 15 centimeter is sufficient enough to place this machine, on top of the block foundation.

And for the third type of soil that is with high value of G , there were found that we need to provide a larger size of the foundation.

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And then we had started with our second method of analysis using mass spring dashpot model or MSD model in that case we have discussed. What are the expression to compute the natural frequency of the machine plus foundation system, for each individual mode of vibration that is for vertical mode of vibration. The expression we got ω_{nz} equals to root over k_z by m , where k_z is the vertical spring constant and for sliding mode of vibration ω_{nx} is equals to root over k_x by m , where k_x is the horizontal spring constant.

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SOIL DYNAMICS

Mass-Spring-Dashpot (MSD) Model (contd.)

(c)

$$\omega_{n\phi} = \sqrt{\frac{k_{\phi}}{M_{m\phi}}}$$

(d)

$$\omega_{n\psi} = \sqrt{\frac{k_{\psi}}{M_{m\psi}}}$$

(c) Rocking mode of vibration (d) Yawing mode of vibration

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Then for rocking mode we have seen that $\omega_{n\phi}$ the natural frequency is calculated using $\sqrt{k_{\phi} / M_{m\phi}}$, where k_{ϕ} is the torsional spring constant. And $M_{m\phi}$ is nothing but mass moment of inertia about this point of rotation. So similarly, for pitching also we know the expression will be of similar kind only thing, the direction will be changed. For the case of yawing mode of vibration the natural frequency is computed using $\omega_{n\psi}$, $\omega_{n\psi}$ equals to $\sqrt{k_{\psi} / M_{m\psi}}$ where k_{ψ} is torsional spring constant about this z axis, and $M_{m\psi}$ is nothing but mass momentum of inertia about this z axis.

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SOIL DYNAMICS

Mass-Spring-Dashpot (MSD) Model (contd.)

Vertical Mode of Vibration

(a)

(b)

$$m\ddot{z} + k_z z = P_0 \sin \omega t$$

(c)

(d)

$$k_z = C_u A$$

Where, C_u = coefficient of elastic uniform compression

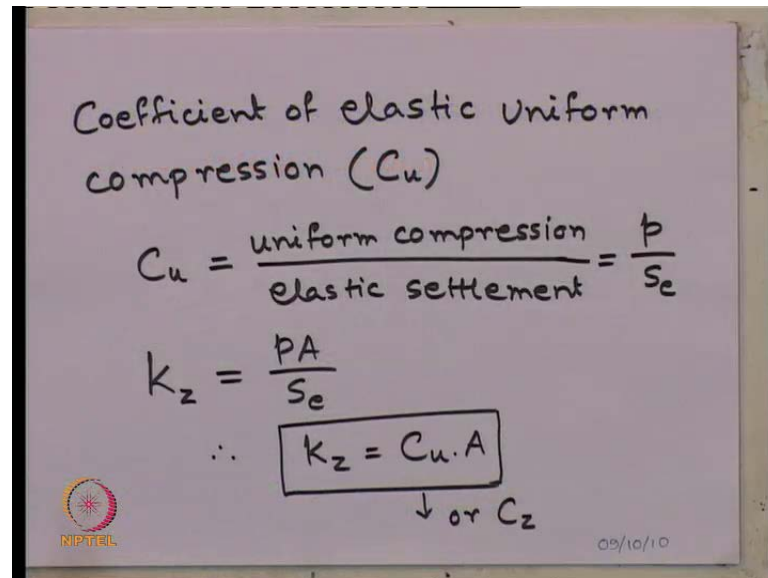
$$\omega_{nz} = \sqrt{\frac{C_u A}{m}}$$

Block foundation under vertical vibration. (a) Block resting at depth D_1 . (b) Block resting at surface of the ground. (c) Soil replaced by equivalent spring K_z . (d) Equivalent spring mass system for analysis.

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Then we had discussed with the simple equation of motion considering single degree of freedom model. For the mass spring dashpot model for vertical mode of vibration, when the externally applied dynamic load is some harmonic like this $p \sin \omega t$. Then our governing equation of motion is $m \ddot{z} + k_z z = p \sin \omega t$ ignoring the damper, and k_z is computed using this expression $C_u \times A$.

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Handwritten slide content:

Coefficient of elastic uniform compression (C_u)

$$C_u = \frac{\text{uniform compression}}{\text{elastic settlement}} = \frac{p}{s_e}$$
$$k_z = \frac{pA}{s_e}$$

\therefore $k_z = C_u \cdot A$

\downarrow or C_z

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So, in this discussion what we had defined some parameters we had defined which are known as coefficient of elastic uniform compression. That is denoted by this C_u , C_u is nothing but uniform compression by elastic settlement and k_z is computed by using this p by A times A . Hence, k_z can be expressed as C_u times A , this we had discussed earlier also in the cyclic plate load test analysis.

(Refer Slide Time: 05:00)

SOIL DYNAMICS

Mass-Spring-Dashpot (MSD) Model (contd.)

Vertical Mode of Vibration (contd.)

Amplitude of motion A_z is given by,

$$A_z = \frac{P_0 \sin \omega t}{C_u A - m\omega^2}$$
$$A_z = \frac{P_0 \sin \omega t}{m(\omega_{nz}^2 - \omega^2)}$$

Maximum amplitude of motion is given by,

$$A_z = \frac{P_0}{m(\omega_{nz}^2 - \omega^2)} < 0.2mm$$

(Check Richart chart)

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Then we had seen that what is the basic expression to compute the amplitude of motion in the vertical direction A_z is given by this expression, and from this we get the maximum amplitude that is this s in term should be considered 1. So, A_z becomes p naught by $m\omega_{nz}^2 - \omega^2$, it should be less than 0.2 mm as per the I S design code specification also. This A_z value has to be checked using the Richart's chart.

So, that the qualitative check about no disturbance for the people working in the surrounding vicinity, or not damaging to surrounding structures is ensured. Now, this is for the case of undamped system. Whereas, all practical systems as we know will be the damped system in that case the expression to compute is it, we know already in our module 2, we have derived that expression a need to be used that is p naught by k root over $1 - r^2$ whole square plus $2\eta r$ whole square that is the expression, and that value of a_z also needs to be checked with respect to these specifications of 0.2 mm and Richart's chart qualitative assessment.

(Refer Slide Time: 06:27)

SOIL DYNAMICS

Mass-Spring-Dashpot (MSD) Model (contd.)

Sliding Mode of Vibration

$P_x \sin \omega t$

$m\ddot{x} + k_x x = P_x \sin \omega t$

$\omega_{n,x} = \sqrt{\frac{k_x}{m}} = \sqrt{\frac{C_r A}{m}}$

Where, C_r = coefficient of elastic uniform shear

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Then we had seen the sliding mode of vibration. What is the equation of motion and how $\omega_m x$ is computed, using this expression $k_x x$ equals to $C \tau A$, where $C \tau$ is defined as coefficient of elastic uniform shear.

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SOIL DYNAMICS

Mass-Spring-Dashpot (MSD) Model (contd.)

Sliding Mode of Vibration (contd.)

Amplitude of motion A_x is given by,

$A_x = \frac{P_x \sin \omega t}{k_x - m\omega^2}$

$A_x = \frac{P_x \sin \omega t}{m(\omega_{n,x}^2 - \omega^2)}$

Maximum amplitude of motion is given by,

$A_x = \frac{P_x}{m(\omega_{n,x}^2 - \omega^2)} < 0.2mm$

(Check Richart chart)

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And the expression to compute maximum amplitude A_x is given by this, that again needs to be checked with respect to the specific guidelines.

(Refer Slide Time: 06:58)

SOIL DYNAMICS

Mass-Spring-Dashpot (MSD) Model (contd.)
Rocking Mode of Vibration

Moment due to soil reaction,

$$M_R = - \int_A dR dA l$$

$$= - \int_A (C_\phi \phi) dA l$$

$$= - C_\phi \phi \int_A dA l^2$$

$$= - C_\phi I \phi$$

and acts in an anticlock wise direction, where
 dR = soil reaction acting over small area dA
 l = distance of dA from center of rotation
 ϕ = angular displacement of block
 I = moment of inertia of contact area about an axis passing through centroid of base contact area (centroid of rotation in this case and perpendicular to plane of vibration)


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Next, we had discussed rocking mode of vibration for rocking mode of vibration, we had this externally applied rocking or moment on the system, which is considered as harmonic $m \sin \omega t$ is amplitude of the moment times sign ωt . And because of this at any instant of time the C G of the machine plus foundation system, will be displaced from here to here.

And suppose, if the C G is located at a distance l from the base of the footing in that case, what is the pressure distribution going to be developed below the footing is static uniform pressure. And then this variable dynamic pressure at the base of the footing because of that what are the moments are getting generated because of this dynamic pressure, unbalanced pressure some moment will be generated.

So, moment due to soil reaction is nothing but that component then from the displaced position of the C G, some moment will get generated and this is externally applied moment. So, some of these three moments will get generate the inertia component of the system, which is nothing but mass moment of inertia about this point of rotation times. If the angular displacement is ϕ then angular rotation is $\ddot{\phi}$.

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SOIL DYNAMICS

Mass-Spring-Dashpot (MSD) Model (contd.)
Rocking Mode of Vibration (contd.)

$$\omega_{n\phi} = \sqrt{\frac{C_\phi I - W L}{M_{n0}}}$$

In practice, $C_\phi \gg \gg W L$

$$\text{so, } \omega_{n\phi} = \sqrt{\frac{C_\phi I}{M_{n0}}}$$
$$k_\phi = C_\phi I$$


Max. displacement A_ϕ

$$A_\phi = \frac{M_0}{M_{n0} (\omega_{n\phi}^2 - \omega^2)} \text{ rad}$$
$$I = \frac{b a^3}{12}$$
$$f_{n\phi} = \frac{1}{2\pi} \sqrt{\frac{C_\phi b a^3}{M_{n0} 12}}$$

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So, that way we had equated the expressions and finally, we got the expressions how to compute the omega n phi using this expression. The maximum displacement amplitude A_ϕ is calculated using this expression it will come in radian, and we know this radian can again be converted in linear displacement by multiplying the dimension. And this I which is nothing but area moment of inertia is calculated $b a^3$ by 12, where a is the dimension in the x direction, when we are talking about rocking. In case of pitching it will just get interchange in that case, it will be $a b^3$ by 12, where b is the dimension of the y axis.

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→ Coefficient of elastic non-uniform shear (C_ψ)

$$K_\psi = C_\psi A$$

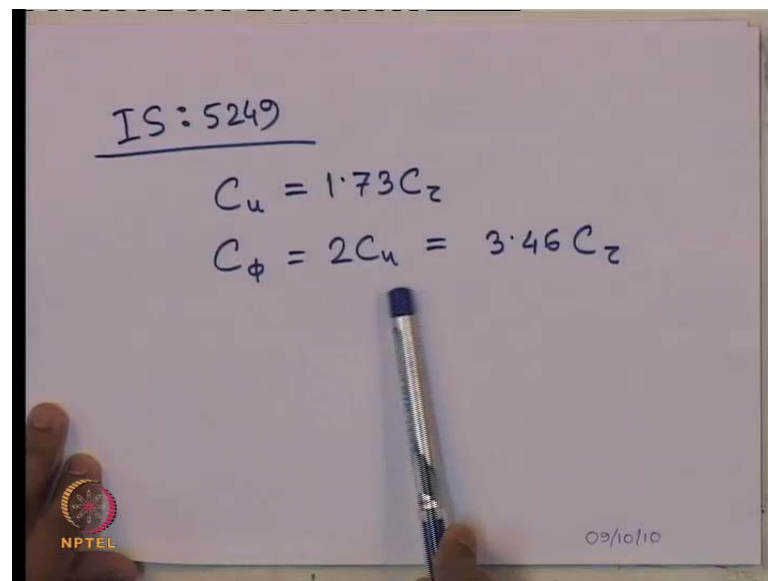
Barkan (1962)

$$C_u = 2 C_z$$
$$C_\phi = 2 C_u$$
$$C_z = 1.5 C_\psi$$

09/10/10

Then we had also seen in the previous lecture, what are the guidelines to compute these values of C_u . C_u can be computed experimentally from cyclic plate load test, we had seen earlier and from the relationship between C_u with other coefficients, like with C_τ it is related like this C_u equals to two times C_τ , C_ϕ is equals to two times C_u and C_τ equals to 1.5 times C_ψ . This is given by Barkan where as our Indian standard code gives.

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$$C_u = 1.73 C_\tau$$
$$C_\phi = 2 C_u = 3.46 C_\tau$$

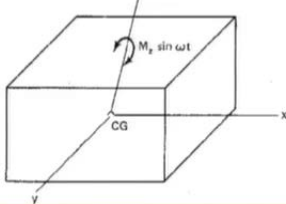
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These expressions that C_u will can be considered as 1.73 times C_τ and C_ϕ is equals to two times C_u equals to 3.46 times C_τ . So, using these relationships knowing C_u from the field test, we can get each of these co-efficient. And why it is required? Once we can compute this values of coefficient, we can get the stiffness corresponding stiffness whether k_z , k_x , k_ϕ or k_ψ that can be used in our expression to compute the natural frequency and so on the displacement extra.

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SOIL DYNAMICS

Mass-Spring-Dashpot (MSD) Model (contd.)
Yawing Mode of Vibration



$M_{sc} \ddot{\psi} + C_v J \dot{\psi} = M_z \sin \omega t$

$M_z \sin \omega t$ = applied dynamic torsional moment
 M_{sc} = mass moment of inertia of the machine and foundation system
 J = polar moment of inertia of the foundation base area
 ψ = angle of torsion of foundation
 C_v = coefficient of elastic non uniform shear

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Now, coming to today's lecture yawing mode of vibration is another mode of vibration in which case, it is the block type of foundation is rotating about this z axis and about z axis, the externally applied moment or torque is given which is $M_z \sin \omega t$. So, in this case the equation of motion governing equation of motion by considering a mass single degree of freedom, mass spring dash pot model can be written like this. If suppose, ψ is the rotational displacement about the z axis because of this applied dynamic load in that case, we can say $\ddot{\psi}$ is nothing but the rotational acceleration. So, $M_{sc} \ddot{\psi}$ is nothing but inertia force plus $C_v \dot{\psi}$ is nothing but it is giving us the spring force, which will be equals to $M_z \sin \omega t$ that is externally applied dynamic load.

So, in this case it is given here $M_z \sin \omega t$ is nothing but applied dynamic torsional moment M_{sc} is mass moment of inertia of the machine. And foundation system about this z axis J is polar moment of inertia of the foundation base area, about z axis and remember this is the area moment of inertia. Whereas, M_{sc} is mass moment of inertia. So, these two things we have to be careful where, which component needs to be used mass moment of inertia will be associated with inertia component.

Whereas, for calculating the stiffness force, we need to consider the area moment of inertia and ψ is angle of torsion of the foundation that is about z axis. How much angular twist is going to occur for the foundation, and $C_v \dot{\psi}$ is nothing but already we

had defined this in the previous lecture, it is called coefficient of elastic non uniform shear, which can be obtained from the relationship between C_u and C_ψ and C_τ .

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The slide contains the following text and formulas:

SOIL DYNAMICS

Mass-Spring-Dashpot (MSD) Model (contd.)

Yawing Mode of Vibration (contd.)

$$f_{m\psi} = \frac{1}{2\pi} \sqrt{\frac{C_\psi J_z}{M_{mz}}}$$

$$\psi_{\max} = \frac{M_z}{M_{mz} (\omega_{m\psi}^2 - \omega^2)} \text{ rad}$$

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Now, for the yawing mode of vibration, the natural frequency $f_{n\psi}$ can be calculated like this $\frac{1}{2\pi} \sqrt{\frac{C_\psi J_z}{M_{mz}}}$. If we express in terms of circular natural frequency, then $\omega_{n\psi}$ will be $\sqrt{\frac{C_\psi J_z}{M_{mz}}}$ because $C_\psi J_z$ will give us the torsional stiffness. That is k_ψ and M_{mz} is nothing but mass moment of inertia and to compute the maximum torsional displacement that is ψ_{\max} , can be calculated like this the amplitude of externally applied torsion.

That is M_z divided by mass moment of inertia M_{mz} times, this natural circular frequency $\omega_{m\psi}^2 - \omega^2$ exciting frequency. That is nothing but it will come in radian because this is the angular displacement we are computing. Again this angular displacement, we can project it in terms of linear displacement by multiplying with the linear dimension of the foundation, and that has to be checked again with respect to that 0.2 mm, the I S code guideline and also the Richart's chart for qualitative assessment. So, with this background. Now, let us start solving another model design problem for all these modes of vibration using this mass spring dashpot model, or MSD model.

(Refer Slide Time: 15:13)

SOIL DYNAMICS

Use of MSD model for analysis

➤ By using MSD model, find displacement amplitudes at resonance condition and at operating frequency of 1500 rpm for (i) vertical, and (ii) sliding modes of vibration for a block type machine foundation with plan dimensions 75cm x 90cm and height 15cm. Consider amplitude of load $Q_0=188.64$ kg. Consider damping coefficient $\eta=25\%$ and Poisson's ratio = 0.25. Use three types of soils with (a) $G=50$ kg/cm², (b) $G=100$ kg/cm², (c) $G=200$ kg/cm². Obtain the results for both constant force type and rotating mass type excitations. Take total weight of machine + foundation = 750 kg. eccentricity = 1 mm and eccentric weight = 75 kg.

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So, use of M S D model for the analysis let us see the problem statement, what it says by using M S D model find displacement amplitudes at resonance condition, and at operating frequency. So, displacement amplitude need to be obtained at two conditions, one at resonance condition another at operating frequency of 1500 rpm. In this case it is a kind of design check problem not a full design problem because in this case, the dimension of the foundation block type machine foundation is given to us. What it says that for vertical and sliding modes of vibration for a block type machine foundation with the plan dimension of 75 centimeter by 90 centimeter and height of 15 centimeter.

That is whatever we had designed by using Tschebotarioff's reduced natural frequency method. Now, we are checking for that foundation as we know using Tschebotarioff's reduce natural frequency, we cannot check for displacement. So, that is why using M S D model the same foundation now, we are checking for displacement whether it is satisfying the permissible limit or not or the Richart's chart condition extra.

And it is given to us that amplitude of the load, the dynamic load which is applied externally on the foundation is given like 188.64 k g of course, this is k g force remember, this is k g force I am talking about. Consider the damping coefficient eta as 25 percent and Poisson's ratio of the soil as 0.25, again let us use three types of soil. First type which is G shear modulus as 50 k g per centimeter square equals to 50 T S F, the

second type of soil with G equals to 100 k g per centimeter square, and the third type of soil with G equals to 200 k g per centimeter square.

Obtain the results for both constant force type and rotating mass type excitations, in our module 2 that is vibration theory. We have seen for mass spring dashpot model for any machine foundation, we can have two cases, one is when the machine is subjected to a constant harmonic load. So, that is called constant force type machine foundation and another is when, there is some imbalance present in the system.

So, because of some rotating component we can get also the dynamic load on the system. So, for rotating mass type excitation also we need to find out the displacement. So, here we will use the expressions whatever we had derived earlier for our module 2 in vibration theory. And it is also given take the total weight of machine plus foundation system as 750 k g, weight means this will be k g force.

And eccentricity is 1 millimeter and eccentric weight is 75 k g these two are of course, required only for this rotating mass type excitation case for constant force type, it is not required as we already know. So, these problem once again as I had mentioned just now it is a cross check whether, the design foundation is good enough in terms of the displacement criteria or not, that is what we are going to check.

So, in this problem as you must have notice there are several combinations are have been asked, within vertical mode of vibration. We have again two cases with constant force type and rotating mass type and within that also we have two cases each at resonance condition amplitude and at operating frequency amplitude. Similarly, for the sliding mode of vibration. So, let us start solving this.

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The image shows a whiteboard with handwritten mathematical derivations for vertical mode vibration. The text is as follows:

Vertical mode

$$Q_0 = 188.64 \text{ kg}$$
$$f = 1500 \text{ RPM}$$
$$\omega = \frac{1500}{60} \times 2\pi = 157.08 \text{ CPS}$$
$$K_z = \frac{4Gr_0}{1-\mu}$$
$$A = 75 \text{ cm} \times 90 \text{ cm}$$
$$\therefore r_0 = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{75 \times 90}{\pi}} = 46.35 \text{ cm}$$

In the bottom left corner of the whiteboard, there is a small circular logo with a starburst pattern and the text "NIPTEEL" below it.

For vertical mode let us do it first, for vertical mode given Q naught is 188.64 k g, k g force of course, we are talking about the operating frequency is 1500 r p m. Hence, if we find out in terms of circular frequency ω will come about 1500 by 60 into 2 pi. So, 157.08 cycles per second. Now, in case of vertical mode of vibration, what is the expression to compute stiffness K_z , that either can be obtained from cyclic plate load test or using the expression for K_z as given by Timoshenko and Budier. This one K_z equals to 4 G r naught by 1 minus μ and in this case, we have area base area of the foundation given to us 75 centimeter by 90 centimeter. Therefore, r naught that is radius of the equivalent circular base will be root over a by pi 75 times 90 by pi. So, it gives us how much it is coming about 46.35 centimeter.

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Handwritten calculations on a whiteboard:

$$(K_z)_{G=50} = \frac{4 \times 50 \times 46.35}{1 - 0.25}$$
$$= 12360 \text{ kg/cm}$$
$$(K_z)_{G=100} = 24720 \text{ kg/cm}$$
$$(K_z)_{G=200} = 49440 \text{ kg/cm}$$
$$W = 750 \text{ kg}$$

The whiteboard also features the NPTEL logo in the bottom left corner.

Hence K_z we will get for three different types of soil with G equals to 50, we will get 4 into 50 into 46.35 divided by 1 minus μ is Poisson's ratio given as 0.25. So, it is coming about 12360 in the unit of kg per centimeter, other k also let us compute here itself for other two types of soil with G equals to 100 T S F, it will be just double of this 24720 and K_z for G equals to 200 will be double of this once again, 49440 kg per centimeter. Now, how much is w total weight of machine plus foundation system is 750 kg, 750 kg force, it is given to us so.

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Handwritten calculations on a whiteboard:

$$\omega_n = \sqrt{\frac{K_z \cdot g}{W}}$$
$$(\omega_n)_{G=50} = \sqrt{\frac{12360 \times 981}{750}} \text{ CPS}$$
$$= 127.15 \text{ CPS}$$
$$(\omega_n)_{G=100} = 179.82 \text{ CPS}$$
$$(\omega_n)_{G=200} = 254.3 \text{ CPS}$$

The whiteboard also features the NPTEL logo in the bottom left corner.

What is natural frequency ω_n that is nothing but K_z times G by W , K_z by m . So, natural frequency ω_n for three different types of soil will be depending on three different values of K_z for three different types of soil. So, 12360 into 981 by 750 so much of cycles per second. If we calculate this it is coming 127.15 c p s. Similarly, ω_n for other two types of soil, if we put k_z for g equals to 100 T S F we will get the value and ω_n with third type of soil, it is coming about we will put K_z corresponding to G equals to 200 T S F, 254.3 c p s. So, we have computed the natural frequency also. Now, we have to solve this problem for two different cases, one is constant force type excitation and another one is rotating mass type excitation.

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Constant Force

$$A_z = \frac{(Q_0/k_z)}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\eta \frac{\omega}{\omega_n}\right)^2}}$$

$$\therefore (A_z)_{G=50} = \frac{(188.64/12360) \text{ cm}}{\sqrt{\left[1 - \left(\frac{157.08}{127.15}\right)^2\right]^2 + \left(2 \times 0.25 \times \frac{157.08}{127.15}\right)^2}}$$

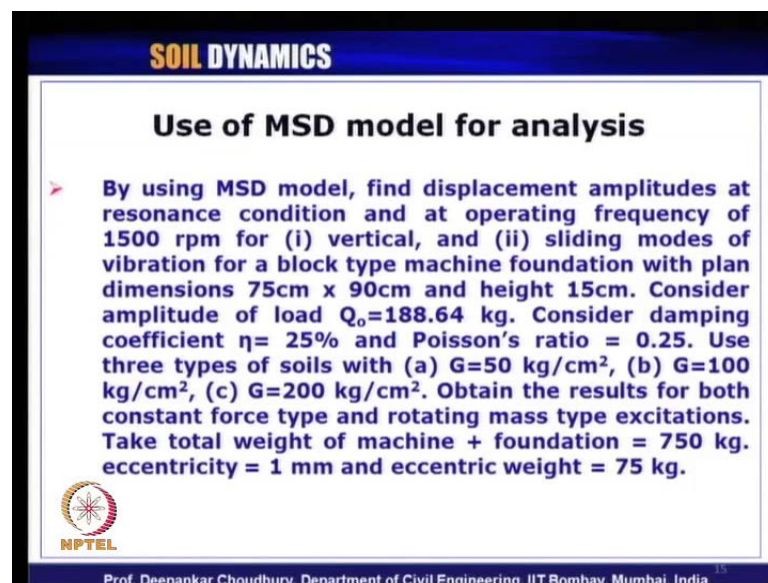
= mm

So, let us start doing for constant force type first, in constant force type excitation what is the expression for A_z . The amplitude of or maximum dynamic displacement in z direction that is given as Q naught by K_z root over 1 minus ω by ω_n whole square, whole square plus 2 eta ω by ω_n whole square right, this we had defined earlier as r frequency ratio. So, 1 minus r square whole square plus 2 eta r whole square, in our module 2 vibration theory we had derived this expression it is the maximum amplitude otherwise, there will be some harmonic function how it varies.

So, only we are concerned about the maximum value therefore, A_z for G equals to 50 should be Q is how much 188.64 k_z is how much, 12360 and these values ω is 157.08. That is operating frequency in c p s, and ω_n for this soil condition of G

equals to 50 T S F is 127.15 plus 2 eta is 25 percent. So, 2.25, 157.08 by 127.15 in which unit we should get it, in centimeter because our K z is in centimeter unit k g per centimeter unit. So, much of centimeter. So, let us calculate this and find it out in millimeter because finally, we have to check against millimeter and remember this displacement, what we are obtaining is that operating frequency right two cases we need to check.

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SOIL DYNAMICS

Use of MSD model for analysis

➤ By using MSD model, find displacement amplitudes at resonance condition and at operating frequency of 1500 rpm for (i) vertical, and (ii) sliding modes of vibration for a block type machine foundation with plan dimensions 75cm x 90cm and height 15cm. Consider amplitude of load $Q_0=188.64$ kg. Consider damping coefficient $\eta= 25\%$ and Poisson's ratio = 0.25. Use three types of soils with (a) $G=50$ kg/cm², (b) $G=100$ kg/cm², (c) $G=200$ kg/cm². Obtain the results for both constant force type and rotating mass type excitations. Take total weight of machine + foundation = 750 kg. eccentricity = 1 mm and eccentric weight = 75 kg.

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One at operating frequency another at resonance frequency. So, in this case we are calculating it at operating frequency.

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Constant Force


$$(Q_0/k_z)$$

$$A_z = \frac{(Q_0/k_z)}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\eta \frac{\omega}{\omega_n}\right)^2}}$$

$$\therefore (A_z)_{G=50} = \frac{(188.64/12360) \text{ cm}}{\sqrt{\left[1 - \left(\frac{157.08}{127.15}\right)^2\right]^2 + \left(2 \times 0.25 \times \frac{157.08}{127.15}\right)^2}}$$

$$= 0.188 \text{ mm} < 0.2 \text{ mm}$$

\therefore O.K.



0.188 millimeter right and we need to check this with respect to 0.2 mm. Therefore, comment is the design is fine. So, this is the design check problem as I said so, it is found that design is alright. This is for first type of soil now for other two types of soil.

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$$(A_z)_{G=100} = 0.1536 \text{ mm} < 0.2 \text{ mm}$$

O.K.


$$(A_z)_{G=200} = 0.0552 \text{ mm} < 0.2 \text{ mm}$$

O.K.

Displacement amplitude @ resonance

$$A_r = (Q_0/k_z) \cdot \frac{1}{2\eta \sqrt{1-\eta^2}}$$

$$(A_r)_{G=50} = (188.64/12360) \cdot \frac{1}{2(0.25)\sqrt{1-(0.25)^2}} \text{ cm}$$

$$= 0.315 \text{ mm} > 0.2 \text{ mm} \text{ NOT O.K.}$$


Let us compute A z again. So, A z for G equals to 100 T S F is coming how much, in that case we have to put Q naught will remain same, K z will change K z now as per G goes to 100 T S F. We have to use omega will remain same operating frequency. Whereas, omega n will change omega n now, for the G equals to 100 T S F we have to use so here

also η will remain same ω will remain same, but this ω_n will change. So, what were the values K_z for G equals to 100 T S F is 24720, here we need to put 24720.

ω_n at G equals to 100 is 179.8 to c p s we need to put it here, here also 179.82 c p s. So, with this we get A_z value as 0.1536 millimeter, which is again less than 0.2 mm. So, the provided size of the foundation is similarly, A_z at G equals to 200. Similarly, we can change the value of K_z and ω_n only for this third type of soil, and we will get the value is coming 0552 mm, which again less than 0.2 mm therefore.

Now, these are the displacement amplitudes at operating frequency, what about the other condition what is asked that displacement amplitude at resonance condition. So, what are the expressions for displacement at resonance condition, again let us refer to our module 2, vibration theory displacement amplitudes at resonance. That is $A_{\text{resonance}}$ that is displacement amplitude at resonance is given by $\frac{Q_{\text{naught}}}{K_z} \times \frac{1}{2} \frac{\eta}{\sqrt{1 - \eta^2}}$. Do you agree with me, this we get from the maximum displacement at D M F that is dynamic magnification factor.

Dynamic magnification factor is nothing but the dynamic displacement by the static displacement. How much is the static displacement? This $\frac{Q_{\text{naught}}}{K_z}$ right. So, we have to multiply this with respect to $\frac{Q_{\text{naught}}}{K_z}$ to obtain the component of dynamic displacement. Otherwise this divided by this is nothing but the maximum D M F, $\frac{1}{2} \frac{\eta}{\sqrt{1 - \eta^2}}$. So, that way we will get what is the maximum dynamic displacement at resonance.

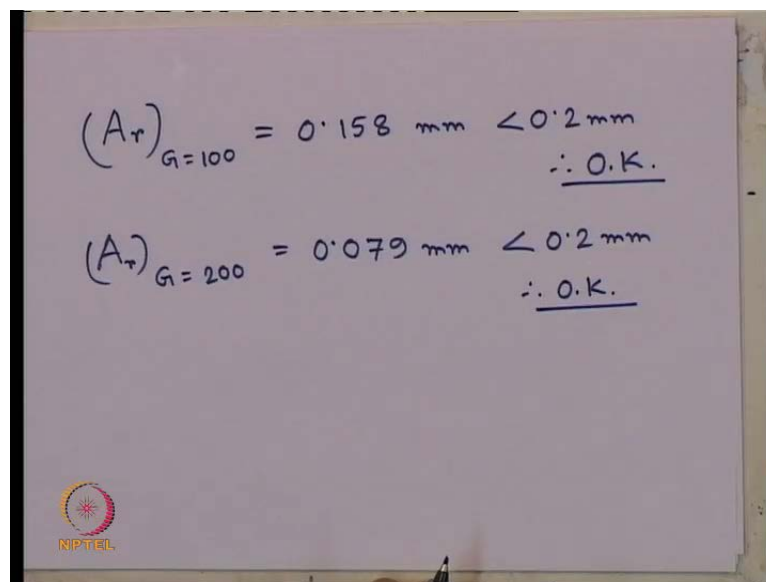
Now, let us calculate this for three difference soil conditions A_r for G equals to 50 will be Q_{naught} is 188.64 by 12360 times $\frac{1}{2} \frac{\eta}{\sqrt{1 - \eta^2}}$ is 0.25 root over 1 minus 0.25 square and how much we are getting from this 0.315 millimeter, this will give us in centimeter we have to convert it to millimeter. So, what is the check, we can do here this is greater than 0.2 mm, which is not okay.

Now, here I want to mention you one thing, this displacement is actually not going to occur, which displacement is going to occur displacement at operating frequency because the machine will always run at its operating frequency. So, whatever is the displacement for the machine plus foundation system is going to occur that will be because of the operating frequency, but still why we need to check displacement amplitude at resonance because by chance. As I said in previous lecture also for machine

foundations we cannot take any risk, by chance if any condition similar to resonance arises in that case, what is our maximum displacement amplitude that also it is better if we cross check. So, in this case we found it will not be within our permissible limit.

Suppose, while starting the machine itself if we go through a frequency same to as our natural frequency of the system. It will be almost close to resonance condition that time this displacement is going to occur, which is beyond the permissible limit. So, we have to be very careful about this aspect as well, though it is not that common because machine will operate obviously at its operating frequency hardly, it will go to the resonance condition. We do not want to keep it at its resonance condition.

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The image shows a whiteboard with handwritten calculations. The first calculation is $(A_r)_{G=100} = 0.158 \text{ mm} < 0.2 \text{ mm}$ followed by $\therefore \text{O.K.}$ with a horizontal line underneath. The second calculation is $(A_r)_{G=200} = 0.079 \text{ mm} < 0.2 \text{ mm}$ followed by $\therefore \text{O.K.}$ with a horizontal line underneath. In the bottom left corner of the whiteboard, there is a logo for NIPTEEL, which consists of a circular emblem with a sun-like pattern and the text 'NIPTEEL' below it.

Let us check the A_r values that is displacement amplitude at resonance for other two values of G . So, we need put in this expression now Q naught will remain same, K_z will change this also remains same. So, only change is in K_z value and K_z value is linearly increasing as G is increasing for 100, it will be double. So, the magnitude will be half right.

So, it is pretty simple to calculate these things directly, here it is less than 0.2 mm. Therefore, for second type of soil it is still okay, if it goes to the close to resonance condition also still the displacement amplitude at resonance also below our permissible limit. And A_r for G equals to 200 that will be again half of this 0.079 millimeter, which is less than 0.2 mm. Therefore, here also design is so for second and third type of soil,


what we can say the design is perfectly alright. Whereas, for first type of soil caution will be it should never be close to its resonance condition. So, these are the discussions or comments, we need to do after doing this design check. So, this is for constant force type excitation.

(Refer Slide Time: 38:34)

Rotating Mass type excitation

$$A_z = \frac{\left(\frac{m_e e}{M}\right) \left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\eta \frac{\omega}{\omega_n}\right]^2}}$$

Now, $\frac{m_e e}{M} = \frac{(75/g) \times 0.1}{(750/g)} = 0.01 \text{ cm}$



Now, if we consider the other case that is rotating mass type excitation, what are the values of this displacement amplitude let us see. So, rotating mass type excitation, in this case the expression for A_z is given by $\frac{m_e e}{M} \frac{\omega^2}{\omega_n^2}$ by root over $1 - \left(\frac{\omega}{\omega_n}\right)^2$ squared plus $2\eta \frac{\omega}{\omega_n}$ squared that is the expression. If we recap our module 2 vibration theory, there we had derived that expression for displacement amplitude at operating frequency, this is at operating frequency is given by this expression. Where this m_e is nothing but eccentric mass, e is amount of eccentricity M is total mass of the system, ω , ω_n are exciting frequency, and natural frequency.

Now, for this $\frac{m_e e}{M}$ this value is given to us, what is given 75 kg force that can be divided by g into eccentricity is 1 millimeter. So, let us first use in centimeter everything because our previous values of K , Q everything we have used in centimeter 75 by g . so, it will be 0.01 centimeter, this term this will remain constant for a particular rotating mass type foundation. So, what will be the A_z value for three different types of soil let us see.

(Refer Slide Time: 41:06)

$$(A_z)_{G=50} = \frac{0.01 \times \left(\frac{157.08}{127.15}\right)^2 \text{ cm}}{\sqrt{\left[1 - \left(\frac{157.08}{127.15}\right)^2\right]^2 + \left[2 \times 0.25 \times \frac{157.08}{127.15}\right]^2}}$$
$$= 0.188 \text{ mm} < 0.2 \text{ mm} \therefore \text{O.K.}$$
$$(A_z)_{G=100} = 0.1536 \text{ mm} < 0.2 \text{ mm} \therefore \text{O.K.}$$
$$(A_z)_{G=200} = 0.0552 \text{ mm} < 0.2 \text{ mm} \therefore \text{O.K.}$$

Now, A_z for G equals to 50 should be 0.01 times ω_n by ω_n , what is ω_n exciting frequency that remains constant. And natural frequency that depends on which type of soil we have. So, for first case G equals to 50 T S F, that was 127.15 ω_n by ω_n whole square, that square plus 2 $\eta \omega_n$ by ω_n . So, how much it is coming 0.188, this is giving us in centimeter because this is in centimeter millimeter am I right.

And A_z for other two types of soil G equals to 100 is 0.1536 millimeter and A_z at G equals to 200 is coming at 0.0552 millimeter and these has to be checked again with respect to the permissible value of 0.2 mm. Therefore, the design is okay, this also less than 0.2 mm therefore, the design is okay, this also less than 0.2 mm therefore, design is okay. One small thing have you notice these values of A_z they are same as constant force type, how it become same?


(Refer Slide Time: 43:36)

Rotating Mass type excitation

$$A_z = \frac{\left(\frac{m_e e}{M}\right) \left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\eta \frac{\omega}{\omega_n}\right]^2}}$$


Now, $\frac{m_e e}{M} = \frac{(75/g) \times 0.1}{(750/g)} = 0.01 \text{ cm}$

$F_0 = m_e e \omega^2 = 188.64 \text{ kg}$



If you calculate using the given value of $m_e e$ and ω . What is Q naught? Q naught is nothing but $m_e e \omega^2$. So, if you operate that that is Q naught in this case is $m_e e \omega^2$, it will come as 188.64 kg force only by given the provided dimension. What does it mean? If the exciting load amplitude remains same for whether it is constant force type, or rotating mass type the displacement amplitude at operating frequency remains same. So, displacement amplitude at operating frequency for two different types of excitation constant force type, or rotating mass type remains same provided the amount of amplitude of external load remains same in both the cases.

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$$A_r = \frac{\left(\frac{m_e e}{M}\right)}{2\eta \sqrt{1-\eta^2}}$$
$$(A_r)_{G=50} = \frac{0.01}{2(0.25) \sqrt{1-(0.25)^2}} \text{ cm}$$
$$= 0.2066 \text{ mm} > 0.2 \text{ mm} \therefore \text{NOT O.K.}$$
$$(A_r)_{G=100} = 0.2066 \text{ mm} > \eta \quad \text{"}$$
$$(A_r)_{G=200} \quad \text{"}$$


Next, we have to calculate the displacement amplitude at resonance. So, for what is the expression for rotating mass type system that is $\frac{m e e}{M} \cdot \frac{2 \eta}{\sqrt{1 - \eta^2}}$ that was the expression through the dynamic magnification factor, which we had discussed in our module 2, vibration theory that maximum D M F. In this case also expression remains same $\frac{1}{2 \eta \sqrt{1 - \eta^2}}$. Whereas, in this case the D M F is defined as dynamic displacement divided by $\frac{m e e}{M}$, do you remember. So, for G equals to 50 how much this value is 0.01, $\frac{2}{0.25 \sqrt{1 - 0.25^2}}$.

So much of centimeter it is coming 0.2066 millimeter, which is greater than 0.2 millimeter therefore, not okay. And for other two values of G that is G equals to 100 or G equals to 200 they remain same. Why? If you look this expression carefully, they are independent of G value. So, in that case also the same comment holds good. So, for rotating mass type, the same block type of foundation in all the three cases of soil at operating frequency, they are perfectly fine because the displacement amplitudes are within permissible range. However, at resonance frequency it is not okay, for all the three different types of soil. So, with this we have completed only the analysis for vertical mode of vibration.

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Sliding Mode

$$K_x = \frac{32(1-\mu)}{7-8\mu} \cdot G r_0$$

$$(K_x)_{G=50} = \frac{32(1-0.25)}{7-8(0.25)} \times 50 \times 46.35 \text{ kg/cm}$$

$$= 11124 \text{ kg/cm}$$

$$(K_x)_{G=100} = 22248 \text{ kg/cm}$$

$$(K_x)_{G=200} = 44496 \text{ kg/cm}$$

Now, let us go for next mode of vibration which is sliding mode of vibration for sliding mode in this case. Let me give you first the expression for K_x , how K_x can be

computed. Once again I had, I had already mentioned it can be computed using K_x equals to $C \tau$ times A , $C \tau$ can be computed from C_u or C_z using cyclic plate load test data, or using analytical expressions based on theory of elasticity. The expression K_x can be used for this purpose in that case the expression is given like this. This is the expression given again in Timoshenkobudier that horizontal stiffness of the spring can be computed like this G shear modulus of the soil, r naught is equivalent radius of the base of the foundation, μ is Poisson's ratio of the soil.

So, by knowing this the K_x value for our three different types of soil can be computed easily for G equals to 50 T S F. The value will be Poisson's ratio is 0.25, r naught already we have computed 46.35 k g per centimeter unit. See r naught remains same that is equivalent circular radius remains same in both vertical vibration, and sliding mode of vibration or horizontal mode of vibration. So, lets us compute this how much it is coming right. So, for other two types of soil K_x per g equals to 100 it will be again just linearly increasing because G is linearly increasing, it is double of the previous one. And k_x for G equals to 200 T S F will be k g per centimeter. So, once K is obtained our next step is to calculate the natural frequency of the system.

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The image shows a whiteboard with handwritten mathematical equations. At the top, the formula for natural frequency is given as $\omega_{nx} = \sqrt{\frac{K_x g}{W}}$. Below this, three specific calculations are shown for different values of G :

- $(\omega_n)_{G=50} = \sqrt{\frac{11124 \times 981}{750}} = 120.62 \text{ cps}$
- $(\omega_n)_{G=100} = 170.59 \text{ cps}$
- $(\omega_n)_{G=200} = 241.25 \text{ cps}$

In the bottom left corner of the whiteboard, there is a logo for NIPTEL.

So, let us calculate the natural frequency ω_n natural frequency of the system is calculated as $K_x g$ by w that we have seen for horizontal vibration, or sliding vibration this is the expression $\omega_n x$. Actually, earlier it was $\omega_n z$. So, ω_n for G

equals to 50 should be 11124 into 981 divided by ω is 750 it is coming about 120.62 cycles per second am I right, ω_n for other two types of soil 170.59 cycles per second is that and ω_n for third type of soil. So, after obtaining the natural frequency now, we are ready to compute the displacement amplitude for two types of cases constant force type and rotating mass type. Now, let us start with constant force type excitation.

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Constant Force type excitation

$$A_x = \frac{(Q_0/k_x)}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\eta \frac{\omega}{\omega_n}\right]^2}}$$

$$(A_x)_{G=50} = \frac{(188.64/11124) \text{ cm}}{\sqrt{\left[1 - \left(\frac{157.08}{120.62}\right)^2\right]^2 + \left[2(0.25) \frac{157.08}{120.62}\right]^2}}$$

$$= 0.178 \text{ mm} < 0.2 \text{ mm} \therefore \text{O.K.}$$

So, for constant force type excitation A_x in this case it is horizontal. So, A_x can be computed by this expression, from our knowledge of fundamentals of vibration theory. In the second module, we can write this expression Q_{naught} is given to us amplitude of external force remember in this case it is obviously horizontal, it has to be horizontal right then only the horizontal vibration will get generated. So, remember that so, our A_x for first type of soil that is G equals to 50 T S F will be 188.64 divided by K_x is how much for the first case, we got 11124 root over 1 minus.

What is ω ? ω is operating frequency that is 1500 r p m we have converted to cycles per second is 157.08 right. 120.62 whole square 2.25, 157.08 by 120.62 whole square, this will come in centimeter unit that we can convert it to millimeter unit because that is required for us to check mm right this is less than 0.2 mm, which is for us. Similarly, let us calculate what is the horizontal maximum amplitude of displacement for

other two types of soil, G equals to 100 T S F and 200 T S F at operating frequency these all are at operating frequency that is why, this ω by ω_n is coming into picture.

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Handwritten calculations on a whiteboard:

$$(A_x)_{G=100} = 0.175 \text{ mm} < 0.2 \text{ mm} \quad \therefore \text{O.K.}$$

$$(A_x)_{G=200} = 0.064 \text{ mm} < 0.2 \text{ mm} \quad \text{O.K.}$$

$$A_r = \frac{(Q_0/K_x)}{2\eta \sqrt{1-\eta^2}}$$

$$(A_r)_{G=50} = \frac{(188.64/11124)}{2(0.25)\sqrt{1-(0.25)^2}} \text{ cm}$$

$$= 0.35 \text{ mm} > 0.2 \text{ mm} \therefore \text{NOT O.K.}$$

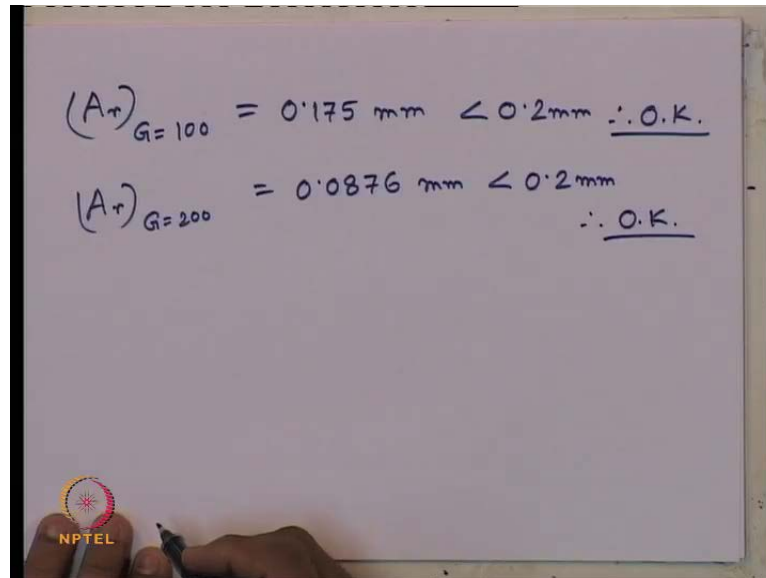
So, A_x for G equals to 100 is coming about the changes will be in this expression Q naught remains same. K_x will change, this will become double 1 minus ω remains same ω_n changes for the G equals to 100, here also ω remains same ω_n changes. So, that way if we calculate we will get A_x for the G so much of millimeter fine, which is again less than 0.2 millimeter therefore, the design is okay.

For the third type of soil with G equals to 200 T S F, if we put the values once again we will get this much horizontal amplitude of displacement, which is less than the permissible one hence. So, for third type of soil with G equals to 200 T S F, A_x that is displacement amplitude at operating frequency we are getting as 0.064 mm, which is less than given permissible limit of 0.2 mm hence, the design is okay.

Now, next step to compute the displacement amplitude at resonance condition. So, displacement amplitude A_r at resonance condition, what is the expression we know this Q naught by K_x by 2 eta root over 1 minus eta square therefore, A_r for G equals to 50 T S F will be Q naught is how much 188.64 K_x is 11124, 2 eta is 0.25, 1 minus eta square 0.25 square so much of centimeter. So, if we calculate this in millimeter how much it is coming 0.35 millimeter, which is greater than 0.2 mm. Hence, not okay at resonance

condition of course, as I have mentioned earlier also. We are checking for resonance condition as well fine.

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The image shows a whiteboard with handwritten calculations. The first line is $(A_r)_{G=100} = 0.175 \text{ mm} < 0.2 \text{ mm} \therefore \text{O.K.}$. The second line is $(A_r)_{G=200} = 0.0876 \text{ mm} < 0.2 \text{ mm} \therefore \text{O.K.}$. In the bottom left corner, there is a small circular logo with a starburst pattern and the text 'NPTEL' below it.

Let us find out this displacement at amplitude at resonance condition for other two types of soil that is with G equals to 100 T S F it is coming. What are the changes will be Q naught remains same, this $k \times$ only change, this denominator also remains same. So, only thing it will become half of this because $K \times$ getting doubled. So, 0.175 millimeter, which is less than 0.2 mm therefore, in this case in this type of soil it is okay, and A_r at G equals 200 T S F is half of this about so much of millimeter, which is less than 0.2 mm. Therefore, this case also it is fine. So, with this, we will stop our lecture today, we will continue further with our different models in the next lecture.