Soil Dynamics
Prof Deepankar Choudhury
Department of Civil Engineering
Indian Institute of Technology, Bombay

Module - 5
Problem on Tschebotarioff's Method Continued
Lecture - 27
Machine foundations
Mass - Spring-Dashpot (MSD) Model

Let us continue our lecture of soil dynamics. In today's lecture, we are continuing with our module 5, that is machine foundations.

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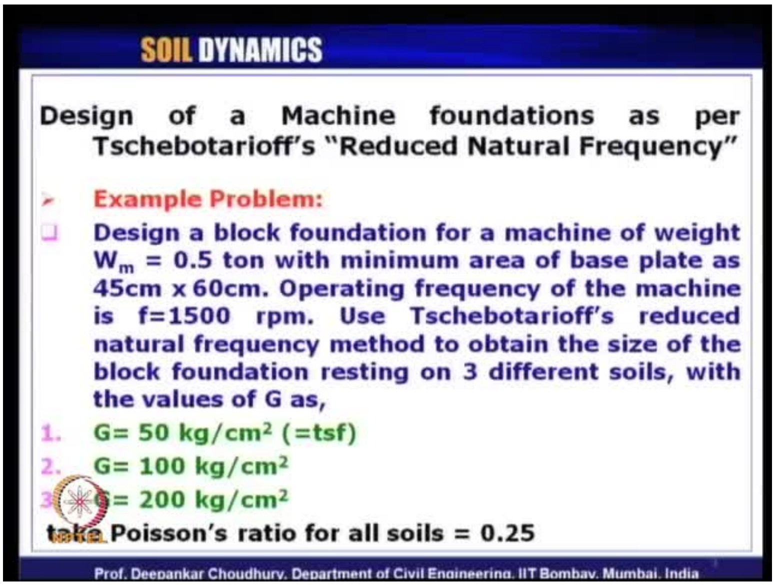

A quick recap of what we had studied in the previous lecture, we have solved a model example problem for design of block type machine foundation using Tschebotarioffs reduced natural frequency method. The problem statement was given to us with the size of the base plate, then weight of the machine, operating frequency of the machine and it was asked for three different types of soil. How we can design the machine foundation?

So, with this knowledge of K, which is required of course to calculate the natural frequency, because natural frequency we need root over K by m, m of course, we can calculate from the size provided, we know the m what is the mass of the footing, but K we need to compute, that is computed using this expression based on theory of elasticity.

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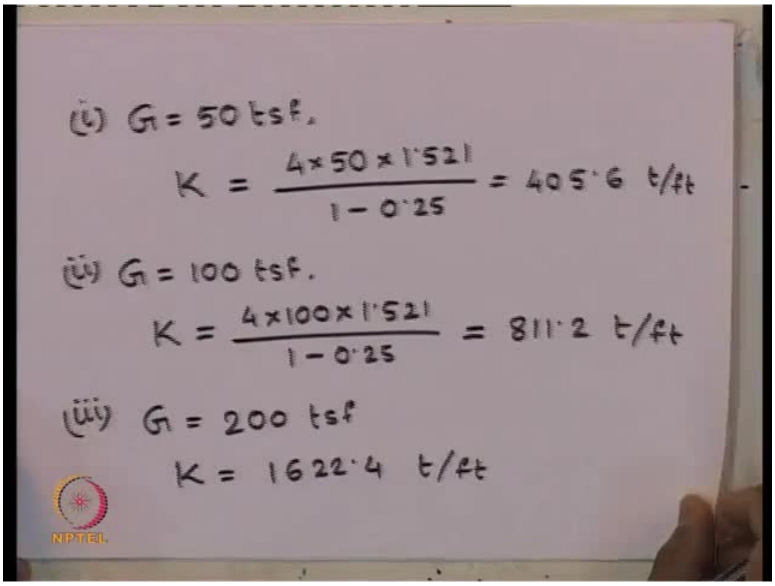

So, for first type of soil that is for G equals to 50 t s f. K is calculated as 4 into 50 into r naught is 1.521 by 1 minus mu is 1.25, which is coming as around 405.6 in the unit of ton per feet. For second type of soil that is with G as 100 t s f. K is computed as 4 into 100 into 1.21 by 1 minus 0.25. You can see these are linearly related 811.2 ton per feet. For the third type of soil with G as 200 t s f, K is coming out to be 1622.4. Now, in the next step, we have to calculate the natural frequency for both the cases.

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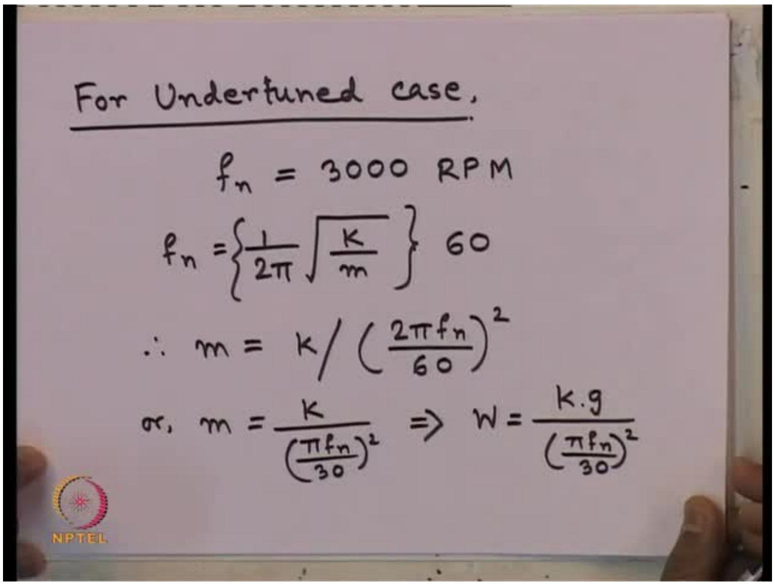

Let us start with under tuned case, for under tuned case f n is how much? 3000 RPM, f n we can write it like this K by m into 60, do you agree with me? Omega is root over K by m, omega 2 f is omega by 2 pi and revolutions per minute that to second, so that is why 60, clear? Therefore, m comes out to be K by 2 pi f n by 60 whole square or m equals to K by pi f n by 30 whole square, which will give us weight w is K times G pi f n by 30 whole square, with this equation. Now, K for each of the G value of each of the soil parameter, we know K value and f n value is known. So, how much weight is required we can calculate back. So, let us do that.

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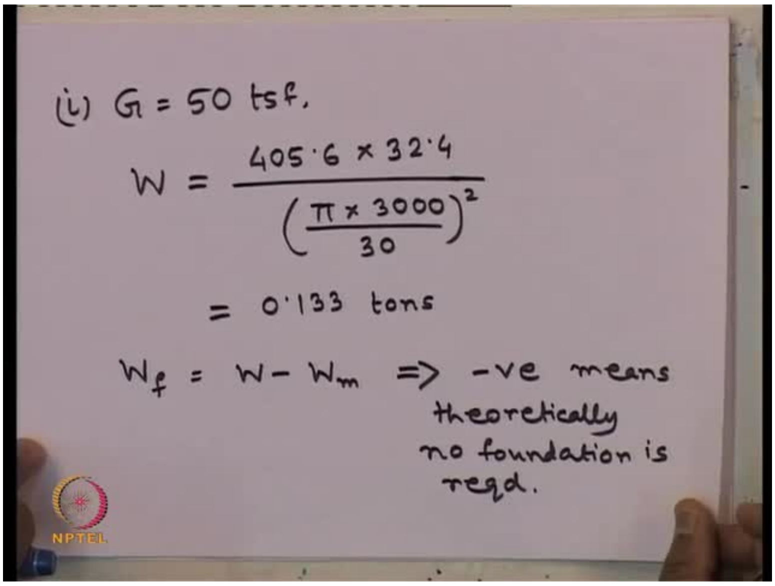

For first type of soil G equals to 50 t s f w is coming out K, how much we calculated? 405.6, G is 32.4 pi into f n is 3000 by 30 whole square. How much we are getting it is coming 0.133 tons. So, weight of the foundation required is w minus w m, which is giving us negative means theoretically. No, foundation is required so, but as I said we can provide minimum size. So, whatever minimum sizes is provided for this case of under tuned is good enough. Now, the same thing if we want to do for over tuned. Let us see, how it comes?

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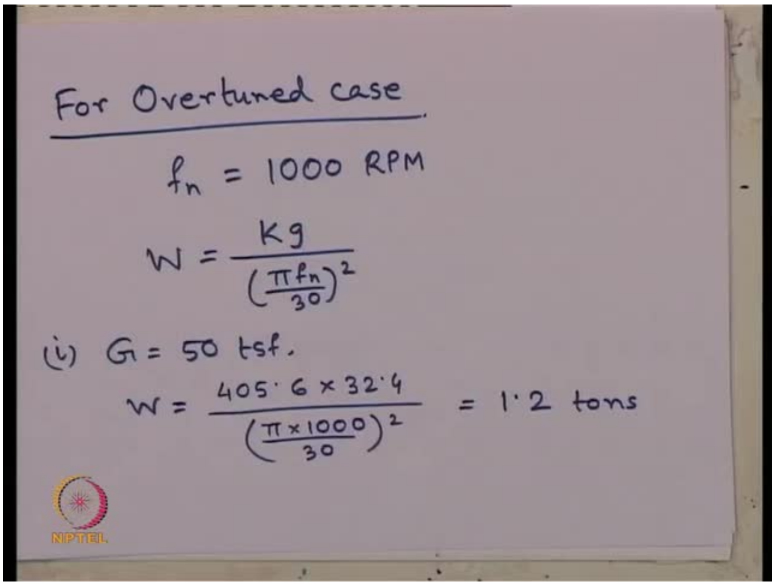

So, for over tuned case, we have f n as 1000 RPM, right? The expression remains same that W equals to K g by pi f n by 30 whole square. So, for the first case of soil, which G equals to 50 t s f w, we are calculating 405, 6 into 32.4 by pi into 1000 by 30 whole square. it is coming 1.2 tons. Let me clear this point, the G unit what I had told at the beginning; that is K g per centimetre square, what is that K g? K g f, k g force unit, then only it is equivalent to t s f in weight unit not in mass unit. Do not think that is a mass unit in the G.

Tschebotarioffs method is sufficient, not fully. It says after providing the foundation size, in this case it is far away from the whatever we are providing. So, it is sometimes requires to cross check weather really the no resonance criteria has been satisfied or not? So, hence to do that, we have to recalculate back. So, another procedure is from the provided section to calculate back your f n and check the ratio of a verses f n. If it is far away from 1 we are the safer side; that is the reason why we are doing this.

So, what we found for G equals to 50 t s f in both the cases of under tuned and over tuned, theoretically no foundation is required to be provided, but for the practical implications we provided the minimum size. That is why the minimum height of the or minimum thickness of the footing as 150 millimetre is provided. Coming to the third case, I am not doing the second case; that is repetitive. For the third case, so that I can show the difference at least. Let us see, what it comes?

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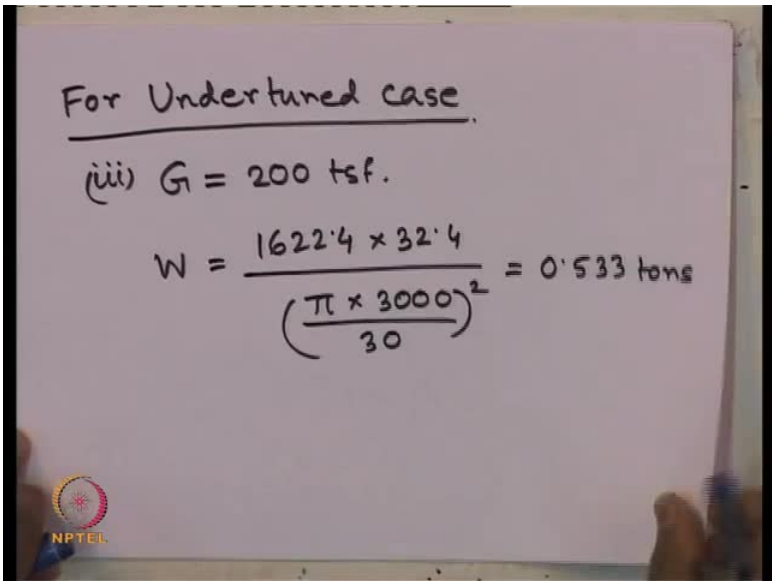

For under tuned case, with case three of the soil that is G equals to 200 t s f, we are getting W as, how much is K value for 200 t s f? 1622.4, right? 4 divided by pi into this by 30 whole square, how much it is giving us? 0.533 tons. Again theoretically, nothing is required. Let us check the over tuned case.

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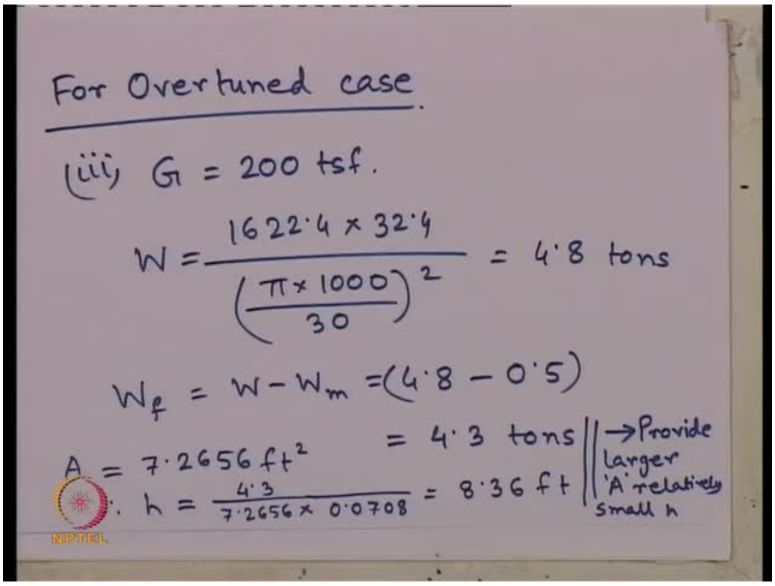

With G value of 200 t s f w is 1622.4 32.4 by pi into 1000 by 30 whole square 4.8 tons. So, in this case weight of foundation required is W minus W m 4.8 minus 0.5, 4.3 tons. This much weight required to maintain no resonance criteria, from which what we will get area. Provided a is how much? We had provided area 7.2656 square, right? So, height we had assumed 15 centimetre. Now, we have to determine height, so that this weight is provided. How much height is required then? 4.3 divided by 7.2656 into unit weight of concrete is 0.0708 so much of feet it is coming how much?

See 8.36 feet is the height, now your dimension, horizontal dimension in the two directions x and y and the height that is proportionate isn't it? So, as a good designer what we should do? Remember what we said earlier a is this a is minimum a to b provided, it is not necessary that we have to keep the clearance of 15 centimetre only. We can keep more clearance, so the base design will be from this, what it follows that provide larger a and relatively small h, that is the designed decision. So, that this weight of 4.3 tons.

Whatever is required is maintained and another practical implication, when we are providing the sizes of this foundations. We have to be careful that we generally provide in terms of workable dimensions, whatever workable dimensions in terms of generally in multiples of 25 millimetre, is it not? That is why 50 millimetre, 100 millimetre, 150 millimetre these are the standard sizes. If the thickness, if you provide you can now we have to do a little bit of iteration, which will give you a good design.

You can try with say 200 millimetre thickness of the footing, then how much area is required? Let us say you keep all around same clearance, so we will get the idea of area and your width breath ratio based on your size of the base plate is known, you will get the dimensions. Convert them to close possible multiples of that 25 millimetre. So these are the design guidelines you can note down, which I am not writing. So, as a practicing design engineer, you have to follow this aspect. After getting the weight get, how much is the height is required?

You have not provided this height, so you have to increase that height. Hence, it is required to be provided a bigger area and a relatively smaller height, instead of giving a very long height and small area. So, let us provide a bigger area with a reasonably smaller height, but not less than 150 millimetre always. That is the minimum thickness as required, so this is the way we do the design of machine foundation size using Tschebotarioff's chart. As we had already discussed earlier, this is the easiest method because just using Tschebotarioff’s chart you are getting all these values of area and cross checking it with no resonance criteria.

But drawback is, no where you can calculate the displacement. On the previous day, when I have shown the plot over tuned and under tuned, so which zone we know, so it is safer to go? Like, if we remain always on the left most part, very close to the 0 axis of the natural frequency axis, it will be safer in that side. If we go on very higher side of the mass, in that case when the operating speed or machine is getting started, there is a chance that it goes through some times the resonance criteria also.

Because here operating frequency it is not that it instantly picks up. It will start from 0 to that value. These are different conditions you have to check and which one will be the safer design guidelines as a engineer, you have to decide. Yes, I have given here for the sake of completeness both the method, but when you are using intuition for design you go for the safest mode.

Also as I said if it is mentioned reciprocating or rotating type of machine, it is well understood whether over tuned or under tuned, which one you have to take? Then there is no need to check for both the cases, only for intermediate operating frequencies only we do this kind of both calculations. So, as a designer we are always getting the choices or options for which mode to go on continue. So, let us move to next module.

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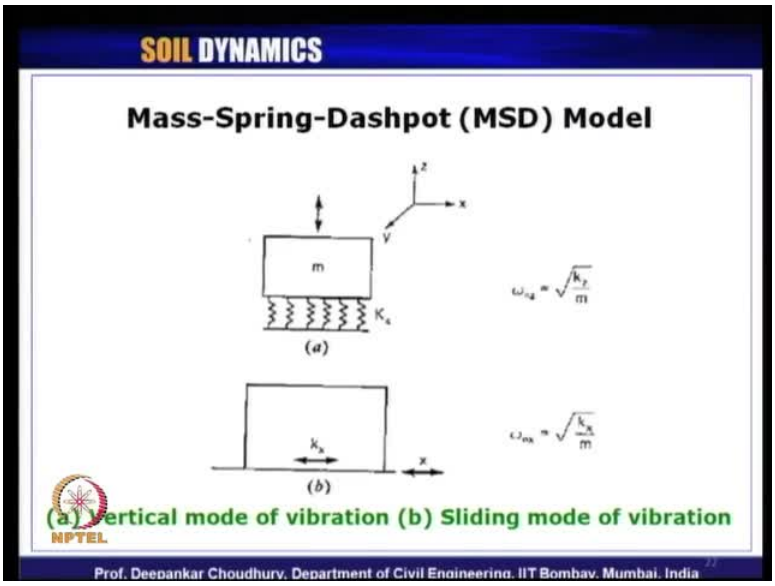

Our next design procedure is using MSD module, mass spring dashpot module. In mass spring dashpot module for different modes of vibration, the soils spring stiffness and the natural frequency how they are calculated, how they are modelled, it is shown in this slide. The first figure this figure a is showing for vertical mode of vibration, so this is the applied dynamic load. In the z direction it is vibrating, so the springs are considered having a spring stiffness of k z, that is in the vertical mode of vibration, vertical spring constant.

Natural frequency of this system can be computed as natural frequency for vertical mode of vibration, that is why omega n z, which is equals to root over this k z by mass of the machine plus foundation system. Whereas, the second figure it shows sliding mode of vibration that is it can be lateral in the direction of x or longitudinal in the direction of y with spring constant k x in this direction. Applied load is dynamic load in this direction, so the natural frequency of the system can be calculated omega n x as root over k x by m.

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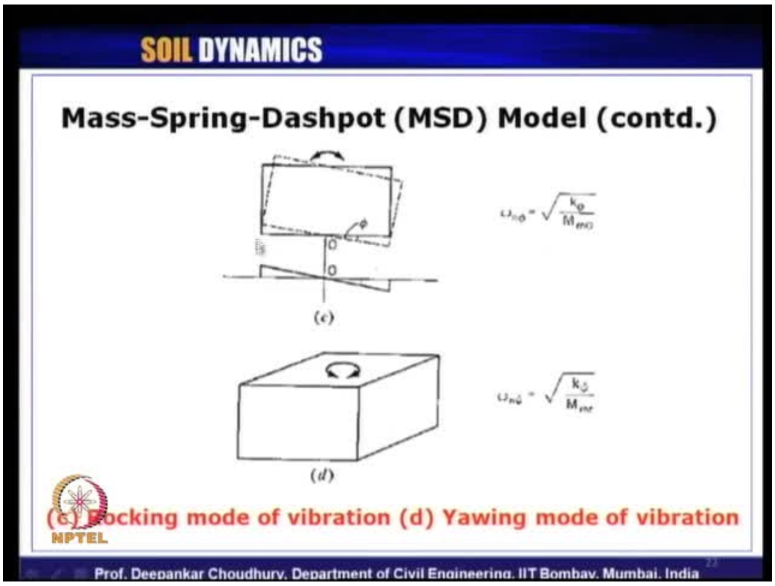

Coming to other two modes rocking mode of vibration, this picture shows rocking mode of vibration, applied dynamic load is acting like this. It is rotating about this point o and angular or rotational displacement is 5. So, in this case the natural frequency of the system is computed omega and 5 as root over k 5. k 5 is nothing but rotational spring constant in this rocking mode divided by m naught; m naught is nothing but mass moment of inertia about this point o, above this rotational point. So, similarly, for pitching also the similar expression will hold good only, it is about y axis, the other one will be about x axis.

But basically the expressions and the nature of equations everything will remains the same. Whereas, for yawing mode of vibration, yawing means when it is rotating about z axis, applied load is in this direction, the natural frequency of the system is calculated like this. Omega n phi sorry, omega n psi, this was phi. Omega n psi is equals to root over k psi by m m z, m m z is nothing but mass moment of inertia of the system calculated about this z axis. So, this is mass moment of inertia calculated about the z axis k psi is torsional stiffness of the system. Now, let us define some terminologies.

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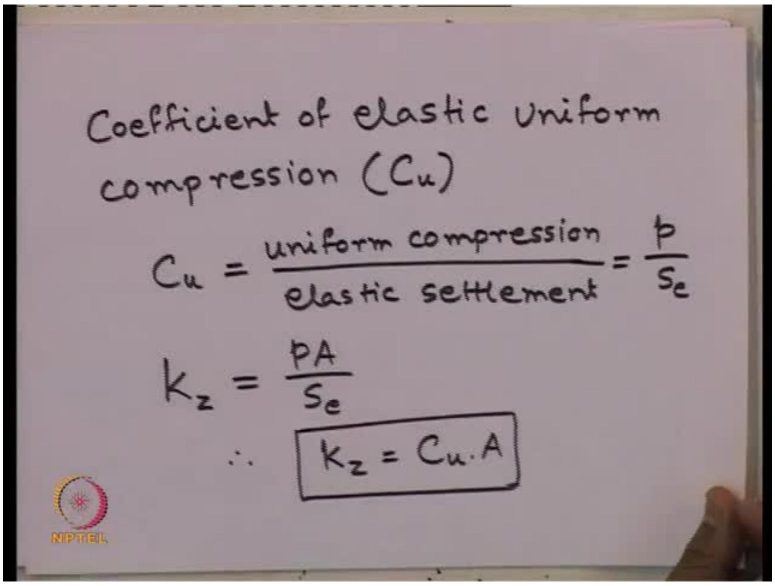

Coefficient of elastic uniform compression this is generally denoted by C u. So, what is C u mathematically. If we express it is nothing but uniform compression by elastic settlement. So, say small p by S e, which actually we had seen for cyclic plate load test. Can you recap this is uniform compression is nothing but load per unit area by elastic settlement. So, using this what we can compute k z that is spring constant for vertical mode of vibration that is given by p A times A e therefore, k z is C u times A. That is the use of this coefficient of elastic uniform compression from which you can compute vertical spring constant for vertical mode of vibration. A is the area of the base plat another terminology.

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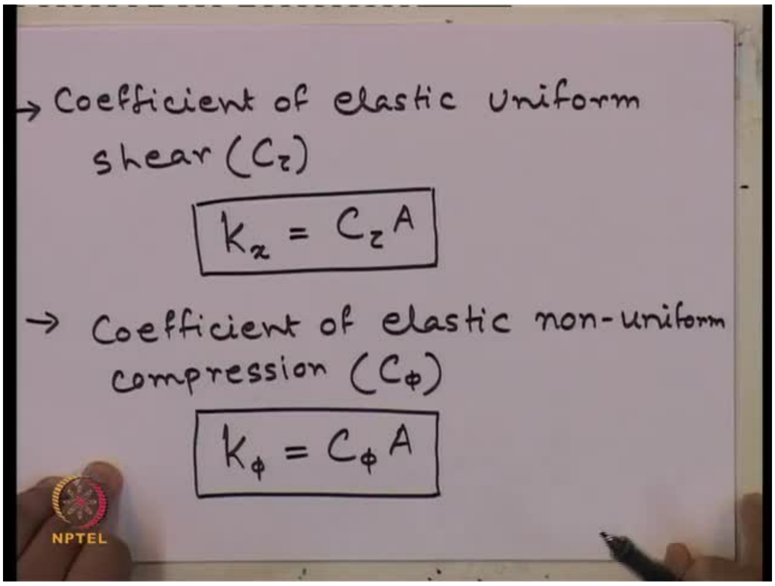

Coefficient of elastic uniform shear C tau here the relationship is k x is equals to C tau times A. So, using coefficient of elastic uniform shear C tau we can get the horizontal spring constant k x or k y in whatever direction. We are considering the shear multiplying with the area a next another terminology coefficient of elastic non uniform compression that is C phi denoted generally by C phi k of phi is related to this as, so k of phi calculated using coefficient of elastic non uniform compression times the area projected.

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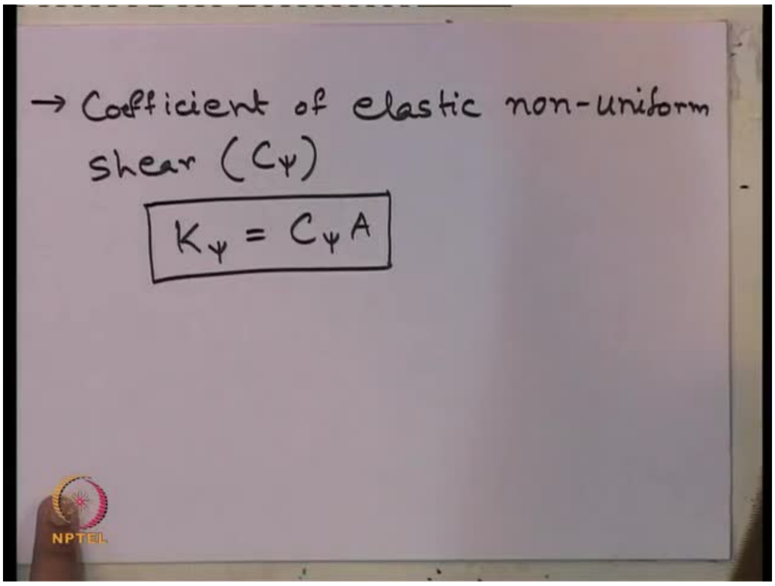

Whereas coefficient of elastic non uniform shear is denoted by C psi and k psi is calculated as C psi times A. So, these are the four terminologies or four parameters, which can be obtained from test. Then they can be used to compute your spring constant for respective mode of vibrations, whether vertical whether horizontal whether rocking or pitching or whether yawing. How it is done? Generally, what is the standard procedure for from the test field? Test we compute, the first one that is C u or sometimes it is called C z also.

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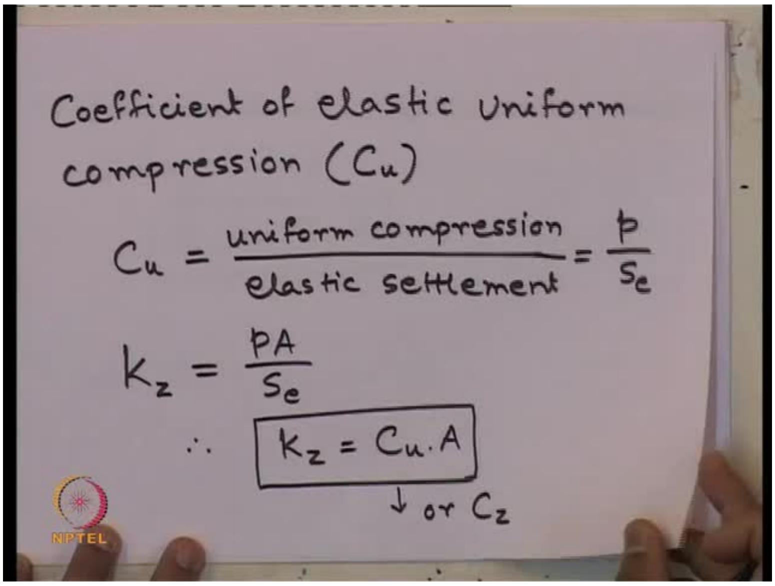

Let me put it back here either C u or C z earlier we had used C z u. Remember for your lecture on cyclic plate load test, we have used C z also that is also used. So, we compute C z and other remaining parameters we generally compute from the relationship between this individual coefficients. It is easy to determine the relationship between coefficients of elastic uniform compression and coefficient of elastic uniform shear and other coefficients. So, you can note down these coefficients are correlated.

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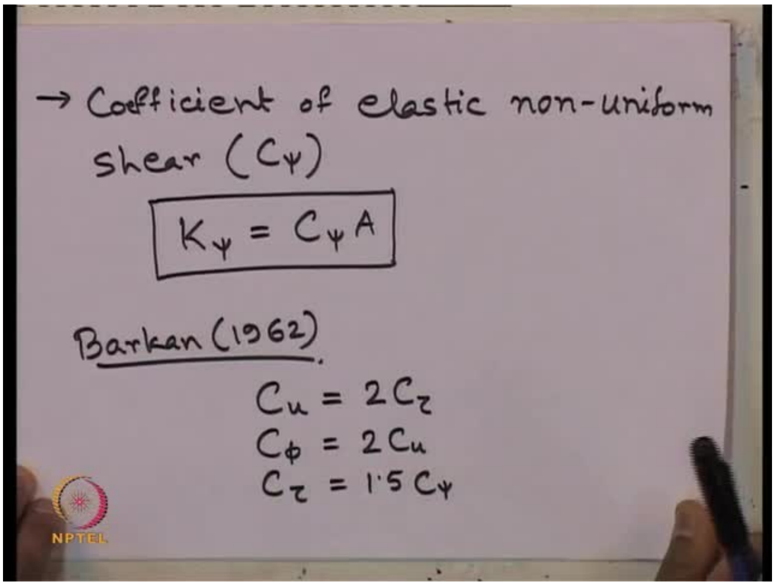

Now Barkan in 1962, it proposes some correlations of C u with other parameters C u is about 2 times of C tau C phi is about 2 times of C u and C tau is about 1.5 times of C psi. So, these are the expressions, which can be easily derived just by using the compatibility of different modes equating the spring constant and their relationship for different modes of vibrations from which we will get in the sight. As I have mentioned, we just determine C u.

Then using this expressions we calculate C tau by using this expression, once the C tau is known C psi also can be obtained and from C u. We can get C phi also, this is the way we calculate generally all these coefficients. Then from this coefficients we will get the spring constant in our analysis using mass spring dashpot module because in mass spring dashpot module. We must know the spring constant, then only we can proceed further.

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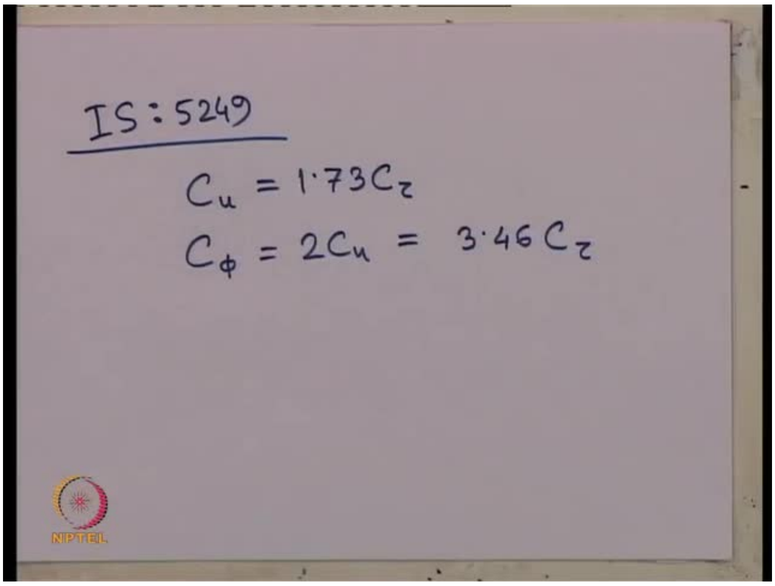

Whatever Indian standard code says, Indian standard 5249, it recommends the value or relationship of C u as 1.73 times C tau and C phi as 2 times as C u equals to 3.46 times C tau. So, these are the relationships given by Indian standard, these are nothing but factor relations, which can be used for the design purpose. Now, using the mass spring dashpot model for each of the modes of the vibrations. Let us see, how the design can be done, let us look at here first.

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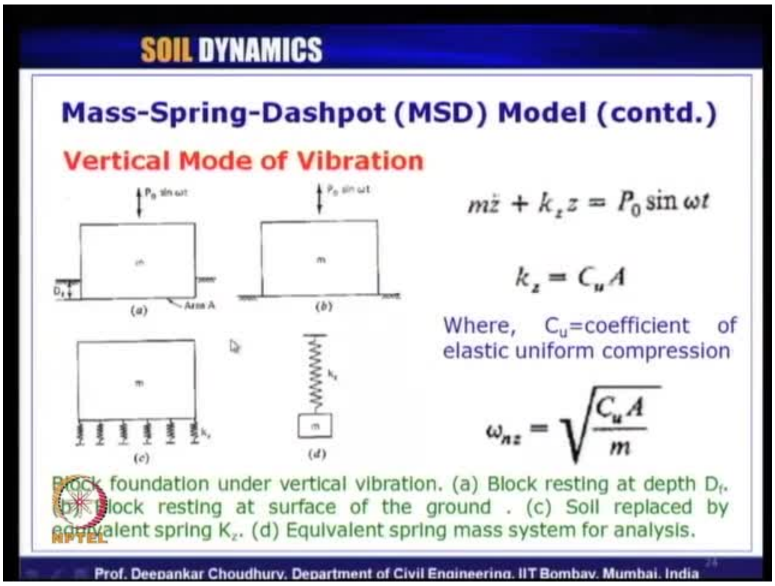

We have to do the analysis in vertical mode of vibration, this is the block type of foundation block resting at the death of d f below the ground. This is the embedment depth base area is A. This the diameter of p naught sine omega t. In the second picture it is showing block resting on the surface on the ground, which is always obviously not the case we have to provide a minimum embedment depth always of 150 millimetre soil. If it is replaced the soil is replaced by the equivalent spring, then k z at the spring stiffness in the vertical mode of vibration.

If we put it in our conventional single degree of freedom mass spring system, it will look like this. Obviously if we add the dashpot, dashpot also will come into picture. So, what is the equation of the motion for single degree of freedom, ignoring dashpot it will be m z double dot. That is z double dot is nothing but acceleration in z direction and k z time z is p naught sine omega t and k z. We have seen equals to C u times a is C u is nothing but coefficient of elastic uniform compression.

We have already mentioned, so omega n z that is natural frequencies calculated root over k z by m, which is nothing but k u a by m. Now, C u is already obtained from the field test as I have mentioned. So, you can get your k z from which you will get your omega m z.

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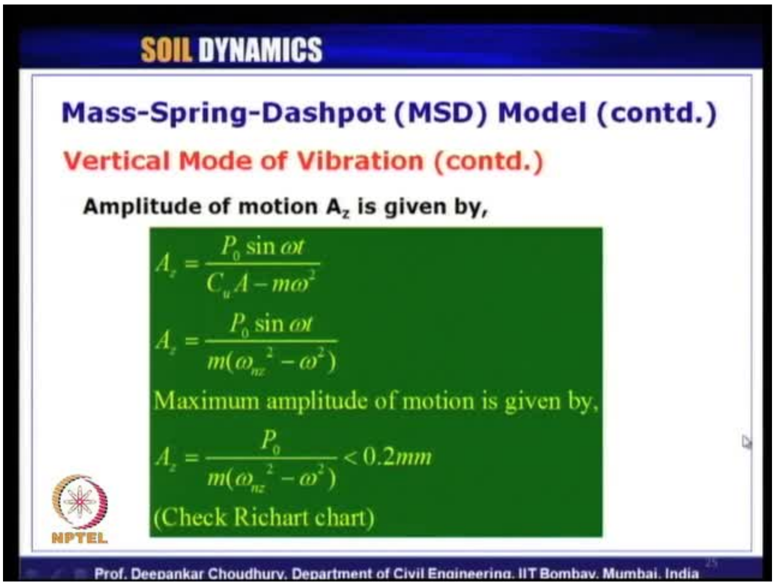

How the amplitude of the motion is calculated? Amplitude of motion A z, why A z? Because in the vertical direction, this amplitude will come, right? So, this will be p naught sine omega t by k minus m omega square. This we had already derived in our mass spring dashpot model analysis. If you put the expressions here, it will be m times omega m z square minus omega square, where this omega is exiting frequency.

Omega m z is natural frequency of the system in z direction, this maximum amplitude will be obviously naught the sine of omega t will be involved. So, p naught by m times omega m z square minus omega square. This value should be less than 0.2 millimetre as per our Indian standard design code guideline. Also this A z value has to be checked with Richarts chart for the third criteria.

That is it is not annoying to the people working in surrounding vicinity or to the adjacent structure. So, these two has to be checked. Now, if the damper is involved, which is the practical case obviously this equation will involve the damping ratio also. You now that equation right the expression will be A z that p naught by k root over omega 1 minus r square whole square plus 2 whole square. So, that is the expression. So, remember it is nothing but the same whatever we have studied in our module 2, that is to compute the amplitude in the vertical direction. That is nothing new as such only the design procedure as to be followed and where from you will get that k value? These are the guidelines coming to the next mode of vibration.

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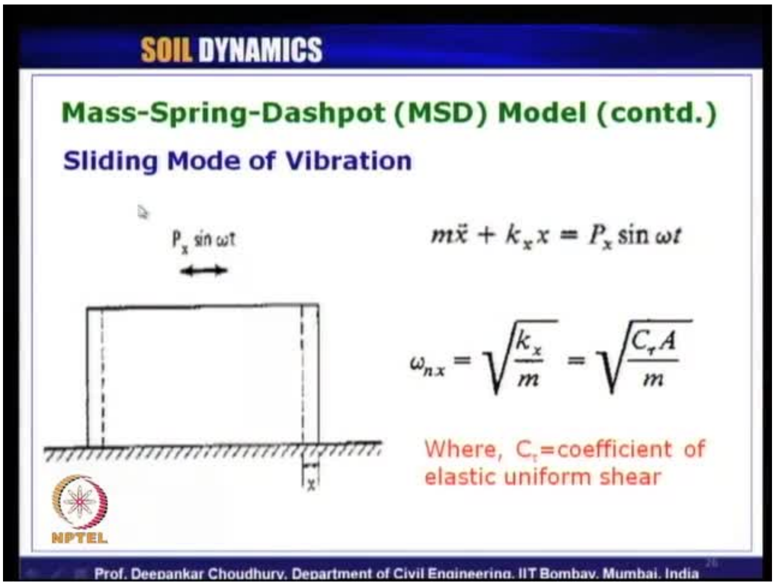

That is sliding mode of vibration, now in the case of sliding mode of vibration this is applied dynamic load and sliding displacement is x. So, our equation of our motion is for a single degree of freedom system m x double dot plus k x x equals to p x sine omega. t k x is that horizontal stiffness, which can be calculated as C tau a. We have seeing just know C tau is nothing but coefficient of elastic uniform shear. So, natural frequency of this system is omega n x is nothing root over k x by m equals to root over is C tau a by m.

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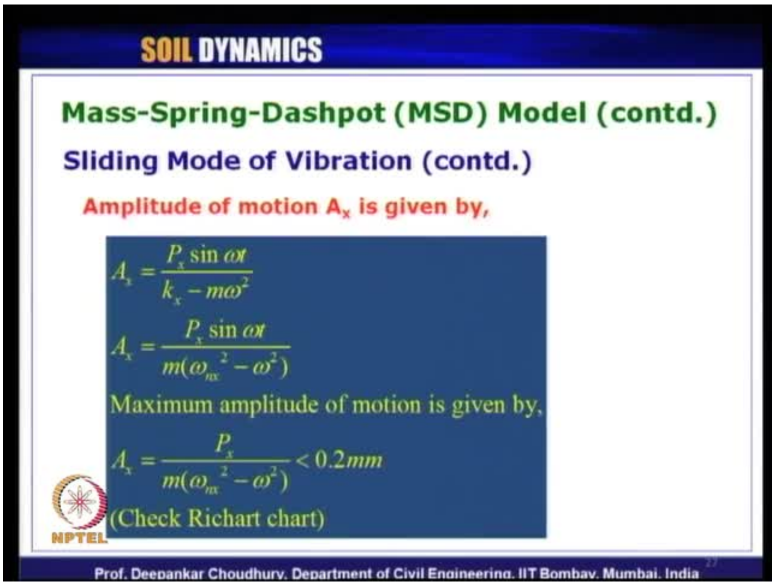

Here, how we compute the maximum amplitude of motion a x amplitude of motion equation will be exactly the same for vertical mode. Also, only difference will be the suffix will be in terms of the sliding direction, that is in x direction. If we are considering x k x and p x and maximum value will a x is p x by m omega n x square minus omega square, that also needs to be less than 0.2 millimetre. It again needs to be checked with Richart's chart. Original it is Lex chart, given in Richart's book, right?

So, for horizontal mode of vibration, that needs to be checked the same expression. See if damper is involved it will be p x by k x route over 1 minus r square whole square plus 2 whole square. There is no difference only instead of vertical here it is horizontal.

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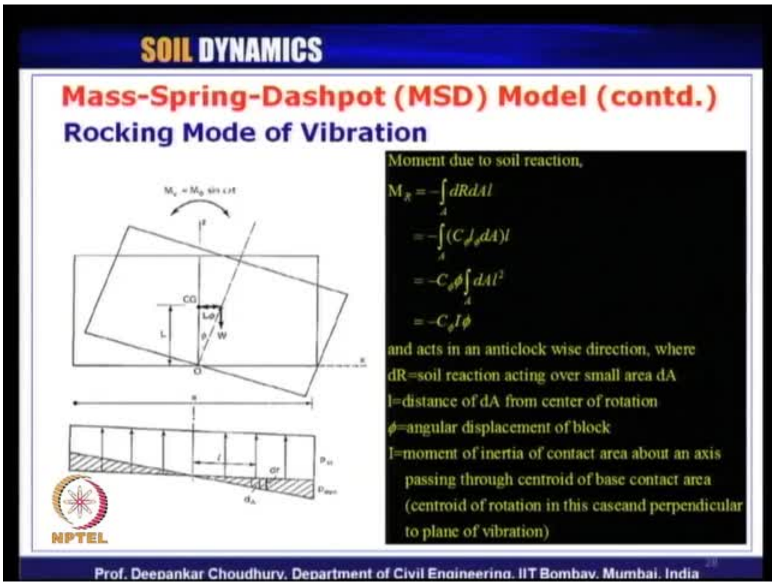

Now, let us come to rocking mode of vibration, rocking mode about this point o it is rocking. This is our x axis, so the axis perpendicular to this plane, that is coming out from this. This is nothing but over y axis. So, m y applied dynamic load m y is nothing but let us say it is harmonics m naught sine omega t m naught is the amplitude. So, that is why it is creating this rocking mode of vibration for the foundation and when it is subjected to this kind of rocking mode of vibration due to the load m y. The weight gets shifted from this C g to this C g.

Let us say l is the height at which the C g of machine plus foundations system was existing, l is the height at which from the base of your footing the C g entire. This foundation plus machines system is located that will be shifted here, let us say for small angular rotation phi is our angular rotation this distance will be l phi. How the pressure will get generated at the base of the footing because of this, type of rotation pressure will be pressured distribution will be this. Rectangular uniform pressure is nothing but static pressure right p s t and the dynamic pressure will be here compressional here tension.

So, that is why the hatched position is nothing but our dynamic pressure getting developed at the base of the footing. Now, let us consider that a is the dimension of the d footing in x direction l is this length at which we are considering an small infinite decimal strips, where the d r is nothing but the soil resistance or soil pressure. d d a is the area on which it is acting, so moment due to the soil reaction. Due to this soil reaction at this infinite decimal place it will be d r time d a times this l moment about this point o about this axis, isn’t that has to be integrated over the entire area?

So, if we express it in terms the soil properties the reaction it will give us C phi l phi d a times l C phi phi over the entire area d a l square. It will finally, give the expression C phi times I times phi. It acts in the anti clock wise direction, where this d r is soil reaction acting over the small area d a l is the distance of d a from the centre of rotation phi is angular displacement of block.

I is moment of inertia of the contact area about an axis passing through the centroid of the base contact area. So, centroid of rotation in this case perpendicular to the plane of the vibration. So, remember in this case I is area moment of inertia, not mass moment of inertia. So, C phi I times phi this will give us the moment due to the soil reaction.

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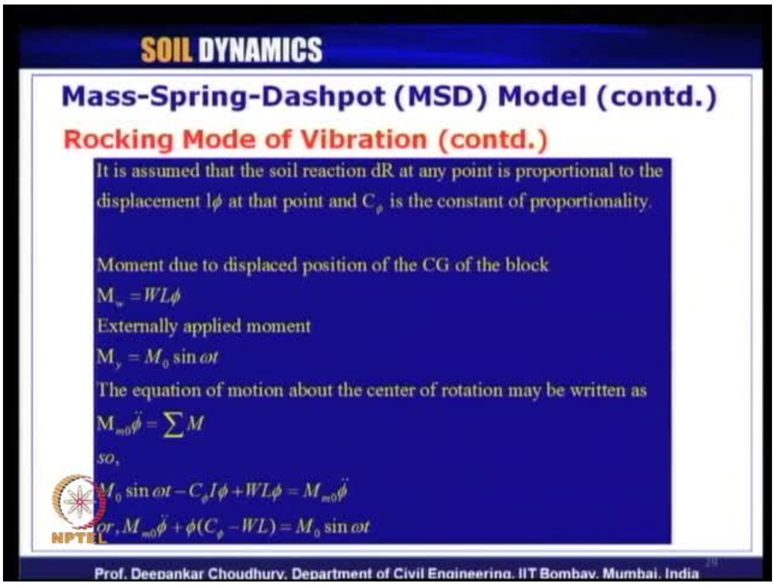

Now, in this case it is assumed that the soil reaction d r at any point is proportional to the displacement of l phi. At that point C phi is the constant of proportionality that constant of proportionality is nothing but what we have define as coefficient of non elastic non uniform compression, right? Moment due to displaced position of the C g because of this rocking mode of vibration the C g has shifted from here to here. So, because of that the moment generated is nothing but w into l phi for small rotation.

We assuming this we are neglecting this component, we are considering l phi, right? So, W times l phi is giving the moment due to displaced position of the C g. What is external? We applied moment that is dynamic load, nothing but m y is m naught sine omega t. Now, to maintain the equilibrium or in other words if we say if we apply that Deyalumbus principle it should give us that, whatever inertia force is coming for the foundations system, which is nothing but the rotational acceleration is phi double dot.

Mass moment of inertia m m naught, so m m naught you can note down it is mass moment of inertia about this axis about this point o. Whereas I is the area moment of inertia. So, do not get confused, this is area moment of inertia where m m naught is mass moment of inertia because here the inertia force is involved. This mass moment of inertia times acceleration will give us the inertia force, which should be equated with some of all other moments, which are getting generated externally externally, because of the reason of soil pressure displace location of C g.

The applied moment clear, so what is coming m naught sine omega t applied moment C phi I phi, we got it from soil reaction W l phi. We got to from displaced C g location some of them should equate with the inertia force getting generated in the system, which you simplify. You will get in the our conventional or canonical form of equation of motion, that m m naught phi double dot plus phi times C phi minus W l. So, it will be nothing but our resultant k phi am I right? This is nothing but resultant k phi equals to m naught sine omega t, the equation for a single degree of freedom system.

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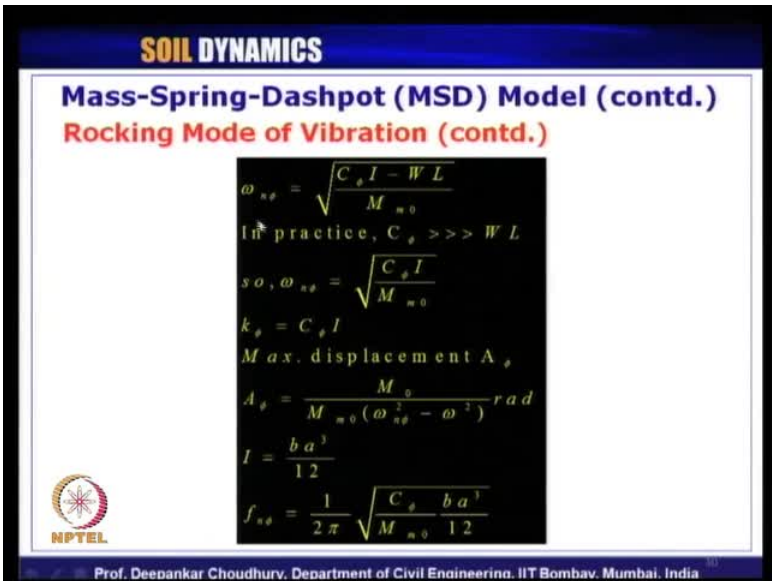

Now, this natural frequency omega m phi that is calculated as root over k phi by m m naught. Now, k phi is nothing but this 1. So, in practice, this C phi is much more than W l. So, we can consider this equals to root over C phi i by m m naught and maximum displacement a phi will be this m naught. That is externally applied moment amplitude divided by m m naught, that is mass moment of inertia of the system about axis o times omega n phi square minus omega square.

So, much of radian in this case, I will be what I was telling you. The area moment of inertia, that is nothing but for a rectangular foundation b a cube by 12, where A is the dimension in the direction of about which it is rotating, right? And B is the another horizontal direction in the perpendicular direction of the foundation, clear? So, A is the dimension in x direction and B is the dimension in y direction and f n phi obviously can be represented by this expression.

Now, if I ask you how we will check this maximum displacement A phi in terms of the guideline or stimulated displacement or the reference displacement? Can we do that? Yes, this is angular displacement, if you multiply this with respect to your height, you will get how much is the projected linear displacement, right? That will have to check against that 0.2 m m. Is this clear? See, if we multiply this with the height you will get the linear dimension of the maximum possible displacement; that needs to be checked with 0.2 m m, which is specified value. So, with this we will stop our lecture today here. We will continue further in the next class.